Explaining Bias in the Foreign Exchange Market: The Case of Traded Volatility and Fractional Cointegration*

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Abstract

The persistence of the forward premium has been cited both as evidence of the failure of the unbiasedness hypothesis and as rationale for the forward premium anomaly. Exploring the nature of this may provide useful insights into issues of market efficiency. This paper examines the proposition that the forward premium and the conditional variance of the spot rate are fractionally cointegrated using traded volatility as a measure of the latter. A corollary of the results is that the risk premium is non-stationary. Although non-standard, it is not inconsistent with risk premia derived from sticky-price general equilibrium models.

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1. Introduction

The issue of whether the foreign exchange market is efficient has produced a voluminous literature. Market efficiency is based on the principle that asset prices reflect all publicly available information (Fama, 1970). Under the joint assumptions of risk neutrality and rational expectations, the expected returns to speculative activity in an efficient market should be zero. Therefore, in a forward or futures market the current price of an asset for delivery at a specified date should be an unbiased predictor of the future spot rate. Interestingly, recent literature has found evidence of long memory (see Baillie and Bollerslev, 1994) or unit root (see Kellard *et al.*, 2001) behaviour in the forward premium, suggesting a rejection of this unbiasedness hypothesis. In a similar vein, Maynard and Phillips (2001) propose that the literature should subsequently explore why the forward premium might display such time series characteristics.

Explanations of the determinants of the time series behaviour of the forward premium include persistent inflation differentials (Roll and Yan, 2000) and peso problems (Evans and Lewis, 1995). It can also be shown, under certain assumptions, that international CAPM implies that the conditional variance of the spot rate, possibly in combination with a risk premium, may provide an alternative rationale. Indeed, Baillie and Bollerslev (2000) exploit this possibility by simulating a model of the foreign exchange market where the assumed long memory behaviour of the conditional variance is inherited by the forward premium. Interestingly, the simulated results are broadly consistent with the empirical features of the forward premium puzzle.

The motivation for this paper is to examine empirically, adopting a fractional cointegration methodology, whether the time series properties of the forward premium *are* inherited from the conditional variance of the spot rate. To our knowledge this has not been hitherto attempted in the literature. Employing daily data from five major currencies, the possibility of fractional cointegration between the forward premium and the conditional variance of the spot rate is examined using the recently developed semi-parametric technique of Hassler *et al.* (2002). Under certain assumptions they prove that a residual-based log periodogram estimator, where the first few harmonic frequencies have been trimmed, gives rise to limiting normality. Thus, this methodology provides a novel asymptotically reliable testing procedure for fractional cointegration. In a further extension, we employ data for implied volatility that is traded on the market (and hence is directly observable) as a measure for the conditional variance of the spot rate[1]. Complementing our analysis, novel Monte-Carlo simulations quantify the effect of the magnitude of component variables on the reliability of standard testing procedures for the integration order of a composite variable. These results are shown to support the primacy of employing forward premia to examine the extent and causes of bias in the foreign exchange market.

For all five currencies, and in contrast to the supposition of Baillie and Bollerslev (2000), it is shown that the forward premium and the conditional variance of the spot rate are not fractionally cointegrated. As a corollary, international CAPM posits the existence of a non-stationary and possibly fractionally integrated risk premium. This is a interesting proposition given the general assumption of a stationary risk premium in the literature. Furthermore, it is shown that a possible explanation for persistence in risk premia can be found in sticky-price general equilibrium models outlined by Engel

(1999). Specifically, derived risk premia are generated solely by the variance of money growth and previous empirical work, including Thornton (1995), cannot reject the possibility that the volatility of money growth is persistent.

The paper is divided into six sections. Section 2 describes the theoretical foundations and the empirical approach to be adopted. Section 3 describes the data, including the novel use of implied volatility to measure expectations about the future variance of the spot exchange rate. Section 4 examines the market efficiency issues within a conventional cointegration testing framework and then moves to a fractionally integrated framework. Section 5 explores the implications of the results for market efficiency and in particular, the properties of foreign exchange risk premia. Finally, section 6 concludes.

2. Theoretical foundations and empirical approach

Following Zivot (2000), the unbiasedness hypothesis under rational expectations and risk neutrality is given by

$$f_{t-1} = E_{t-1}(s_t) \qquad (1)$$

where s_t and f_t are the natural logarithms of the spot and forward rates at time t and $E_t(.)$ is the expectations operator conditional on information available at time t. Moreover, equation (1) is commonly expressed as the levels relationship

$$s_t = f_{t-1} + \varepsilon_t \qquad (2)$$

where ε_t is a random, zero-mean variable. From (2) and considering that spot and forward prices are generally found to be non-stationary (see Meese and Singleton, 1982), a necessary condition for market efficiency is the existence of cointegration between spot and lagged forward rates. The cointegrating regression can be specified as

$$s_t = \beta_0 + \beta_1 f_{t-1} + u_t \tag{3}$$

Clearly, the unbiasedness hypothesis requires $\beta_0 = 0$, $\beta_1 = 1$ and that u_t is not serially correlated. Tests for cointegration generally confirm that spot and lagged forward rates are cointegrated (see Kellard *et al.*, 2001). However, contradictory evidence has emerged on whether the restrictions of the unbiasedness hypothesis are appropriate (see Engel, 1996).

An equivalent approach for assessing the unbiasedness hypothesis comes from noting that the residual term in (2) can be expressed as

$$\varepsilon_t = s_t - f_{t-1} = (s_t - s_{t-1}) - (f_{t-1} - s_{t-1}) \tag{4}$$

Given the stationary behaviour[2] of the spot return $(s_t - s_{t-1})$, the order of integration of the forecast error $(s_t - f_{t-1})$ is determined solely by the lagged forward premium $(f_{t-1} - s_{t-1})$. Thus Maynard and Phillips (2001) note that inclusion of the spot return introduces unnecessary noise that may cause finite sample bias. Additionally they suggest that this finite sample bias is of particular significance because, as reported by

Newbold *et al.* (1998), the forward premium is so small in magnitude that the time series properties of the forecast error can be easily dominated by those of the much larger spot return. However, as with evidence from the forecast error, cointegration and unit root tests on the forward premium have provided conflicting results. Hai *et al*. (1997), Horvath and Watson (1995) and Barnhart and Szakmary (1991) reject the presence of a unit root in forward premia series. However, Crowder (1994, 1995), Kuersteiner (1996) and Kellard *et al.* (2001) provide evidence to the contrary.

Inconsistent evidence has led some to suggest that neither short memory nor unit root models are entirely appropriate to model the data. Specifically, Maynard and Phillips (2001) and Baillie and Bollerslev (1994) find that a fractionally integrated model fits the forward premium adequately and reason that this provides an explanation for the dichotomy in the literature. Of course, long memory *or* unit root behaviour in the forward premium imply persistence in the forecast error, allowing it to be predictable from past values. This provides a rejection of the unbiasedness hypothesis. Thus, Maynard and Phillips (2001) propose that the literature should subsequently explore why the forward premium might display such time series characteristics.

A generalisation of (1) can be achieved by noticing that under the assumptions of constant relative risk aversion (CRRA) and log normality, the international CAPM simplifies to

$$E_{t-1}(s_t) = f_{t-1} - \frac{1}{2}\mathrm{var}_{t-1}(s_t) + \mathrm{cov}_{t-1}(p_t, s_t) + \rho\,\mathrm{cov}_{t-1}(s_t, c_t) \qquad (5)$$

where c_t, p_t and ρ denote the logarithm of consumption, price level and degree of relative risk aversion (see Obstfeld and Rogoff, 1996). Under rational expectations, equation (5) can be expressed as the levels relationship

$$s_t = f_{t-1} - \frac{1}{2}\text{var}_{t-1}(s_t) + \text{cov}_{t-1}(p_t, s_t) + rp_{t-1} + \varepsilon_t \tag{6}$$

where $rp_{t-1} = \rho\,\text{cov}_{t-1}(s_t, c_t)$, is a time dependent risk premium. Subtracting s_{t-1} from both sides of (6), ignoring the covariance term due to very small size (see Engel, 1996) and rearranging leads to

$$(s_t - s_{t-1}) - (f_{t-1} - s_{t-1}) = -\frac{1}{2}\text{var}_{t-1}(s_t) + rp_{t-1} + \varepsilon_t \tag{7}$$

Given the short memory process of the spot return process and assuming, as the literature often does, a similar process for the risk premium, it is therefore possible that the time series properties of the forward premium are inherited from the conditional variance of the spot rate. Exploiting this possibility, Baillie and Bollerslev (2000) simulate (6), adopting a fractionally integrated (d = 0.75) GARCH model of $\text{var}_{t-1}(s_t)$. As noted earlier, the simulated results were broadly consistent with the empirical features of the forward premium puzzle.

The motivation for this paper is to examine empirically whether the time series properties of the forward premium *are* inherited from the conditional variance of the spot rate. From the above discussion, it becomes clear that this could be attempted in two sequential steps. Firstly, establish whether the salient features of the literature can be replicated using the chosen data. Specifically, test whether spot and lagged forward rates are cointegrated with the vector $(1,-1)$ by employing conventional cointegration and unit root methodologies. Secondly, as some in the recent literature have suggested that the forward premium and the conditional variance of the spot rate are fractionally integrated series, examine the possibility that they are fractionally cointegrated. This is particularly important as Gonzalo and Lee (2000) have demonstrated that if unit root tests incorrectly indicate that fractionally integrated variables are *I(1)*, then conventional cointegration tests are likely to find spurious cointegration too frequently. Of course, if the forward premium and the conditional variance are not fractionally cointegrated, (7) implies the possibility that the time series properties of the forward premium are analogous to the risk premium.

3. Data

Daily and monthly[3] time series of spot, one-month forward rates, interest rate differentials and conditional spot rate variances were constructed from daily data for the period January 1991 to March 2001[4]. The data for spot exchange rates, one-month forward rates and eurocurrency[5] rates were obtained from Datastream and calculated as the closing (London time) average of bid and ask quotes for five currencies: US dollar/Sterling, Yen/US dollar, Deutschmark/US dollar, Deutschmark/Sterling and Deutschmark/Yen. Finally, in a novel procedure, the conditional variance of the spot rate is proxied by the square of 'traded' implied volatilities which measure the market's expectations about the future volatility of the spot exchange rate[6]. The more common approach (see, *inter alia*, Baillie and Bollerslev, 2000) has been to generate the conditional spot return variances using a GARCH-type process. However, as currency volatility has now become a traded quantity in financial markets, it is therefore directly observable on the marketplace. The data used are at-the-money, one-month forward, market quoted volatilities at close of business in London, obtained from brokers by Reuters. The databank is maintained by CIBEF at Liverpool Business School. Since these data are directly quoted from brokers, they avoid the potential biases associated with the backing out of implied volatilities from a specific option-pricing model.

4. Testing

4.1 Cointegration of spot and forward exchange rates

To establish whether the salient features of the market efficiency literature are present in our data, the monthly data set is used, replicating the common frequency adopted (see, *inter alia*, Kellard *et al*., 2001). Whether a time series contains a unit root is frequently assessed using an Augmented Dickey-Fuller (ADF) test (see Dickey-Fuller, 1981), choosing the number of lags through general-to-specific testing at the 10% level, as recommended by Ng and Perron (1997). Following this approach Table 1 shows the results of ADF tests for the logarithm of spot and forward series. The Dickey-Fuller tests fail to find any evidence of stationarity for any of the series, in line with the results of Meese and Singleton (1982).

Testing for cointegration between s_t and f_{t-1} was carried out using the Johansen (1995) method of reduced rank regression. This technique specifies the vector error-correction model (VECM) of the *m*-variable VAR for a time series vector X_t as

$$\Delta X_t = c_0 + c_1 \Delta X_{t-1} + ... + c_{k-1} \Delta X_{t-k+1} + \Pi X_{t-k} + v_t \qquad (8)$$

where k is sufficiently large that v_t is vector white noise.

The technique then tests for the rank of Π, the m x m parameter matrix attached to the vector of the (lagged) levels of the variables. The lag length k was chosen by sequential reduction using the Schwarz Information Criterion (SIC). Assuming that X_t is a vector of *I(1)* variables, then ΠX_{t-k} has to be stationary for v_t to be stationary. The absence of cointegration implies that there are no linear combinations of the X_t that are $I(0)$ and the rank (r) of Π, is zero.

This case specifies $X_t = (s_t, f_{t-1})$ so that Π has a maximum rank of 2. Using the Johansen λ–max (maximal-eigenvalue) and trace statistics, the technique sequentially tests for $r = 0$ and $r \leq 1$. Table 2 presents the cointegration results from the application of the Johansen method of reduced rank regression to the cycling datasets. For all exchange rates, the null of no cointegration ($r = 0$) is rejected at the 1% significance level. Furthermore, in four of the five currencies, the null of reduced rank ($r \leq 1$) cannot be rejected and cointegration is unambiguously accepted. However, for the Dollar/Sterling the null of reduced rank is rejected. It is well to remember that although SIC is a consistent estimator if the correct model is under consideration, it is not efficient. When a lag length of 2, identified by AIC, is used in the unrestricted VAR, cointegration is accepted for the Dollar/Sterling. Table 3 displays the standardised cointegrating vectors for each currency and suggests, for only the Mark/Dollar, that matched spot and forwards have a cointegrating vector of $(1,-1)$. Like similar work in the area, this provides limited evidence in support of the unbiasedness hypothesis[7].

A theoretically equivalent test of unbiasedness, given the stationary behaviour of $(s_t - s_{t-1})$, is to examine the order of integration of the forward premium $(f_{t-1} - s_{t-1})$. Under covered interest parity (CIP) the forward premium is equal to the interest rate differential. Maynard and Phillips (2001) demonstrate that the differential is a much cleaner series than the forward premium and thus is preferred for use in empirical analysis. Hence, the ADF results are displayed in Table 4 are those for the interest rate differential, π_t, of each of the currency series. The null hypothesis of a unit root can only be rejected for the Mark/Yen at the 5 per cent level of significance and thus again provides clear evidence against the unbiasedness hypothesis.

Interestingly, the conclusions of the testing procedure on the forward premium and the forecast error do not concur for the Mark/Dollar. As observed earlier, Newbold *et al.* (1998) and Maynard and Phillips (2001) provide an empirical explanation for such a phenomenon by suggesting that the forward premium is so small in magnitude that the time series properties of the forecast error are dominated by those of the much larger spot return (see Figures 1, 2 and Table 5). It follows that an examination of the time series properties of the forecast error may tell us very little about the time series behaviour of the forward premium.

The effect of the magnitude of component variables on the efficacy of testing procedures for the integration order of composite variables can be evidenced by Monte Carlo methods[8]. For example, the following series can easily be generated

$$x_t = x_{t-1} + \varepsilon_{1t} \; ; \qquad \varepsilon_{1t} \sim N(0,1) \tag{9a}$$

$$y_t = \eta\varepsilon_{2t}; \qquad \varepsilon_{2t} \sim N(0,1),\ y_t \sim N(0,\eta^2) \tag{9b}$$

$$z_t = x_t + y_t \tag{9c}$$

By definition x_t is $I(1)$, y_t is $I(0)$, the composite variable z_t is $I(1)$ and η is a scaling parameter that controls the magnitude of y_t. Table 6 shows Dickey-Fuller critical values for z_t, at the 5 and 10 per cent levels of significance, employing different values of η. Initially allowing $\eta = 0$ in the experiment, the standard critical values are replicated. However, as the scaling parameter η increases, the critical values for both levels of significance decrease dramatically. Even at quite small levels of η, using standard critical values will result in many more rejections of the null hypothesis of non-stationarity for z_t than is warranted. This result substantiates the conjecture of Newbold *et al*. (1998) and Maynard and Phillips (2001) and additionally demonstrates the sensitivity of standard tests for the properties of time series to the magnitude of any component variables.

4.2 Fractional integration of the forward premium

The previous section demonstrated our monthly data display some of the salient features of the market efficiency literature. In particular, conventional tests could not typically reject the presence of a unit root in the forward premium. The implications of this result can be clarified by considering a simple autoregression

$$y_t = \beta_o + \rho y_{t-1} + \varepsilon_t \tag{10}$$

Conventional unit root tests examine whether $\rho = 1$. This imposes the discrete choice that the time series is *I(d)* with $d = 0$, corresponding to the case where $\rho < 1$, or $d = 1$, the case of $\rho = 1$ (a unit root). These can be considered as two extreme hypotheses because with $d = 1$, shocks are permanent, whereas if $d = 0$, shocks disappear geometrically. Clearly, results thus far suggest innovations to the forward premium will be permanent.

However, some in the literature have suggested that the forward premium is neither an *I(1)* nor an *I(0)* process, but a fractionally integrated or *I(d)* process. The introduction of the autoregressive fractionally integrated moving average (ARFIMA) model by Granger and Joyeux (1980) and Hosking (1981) allows the modelling of persistence or long memory where $0 < d < 1$. A time series y_t follows an ARFIMA (*p*, *d*, *q*) process if

$$\Phi(L)(1-L)^d y_t = \mu + \Theta(L)\varepsilon_t, \qquad \varepsilon_t \sim iid(0,\sigma^2) \tag{11}$$

where $\Phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$ and $\Theta(L) = 1 - \theta_1 L - \ldots - \theta_q L^q$. Such models may be better able to describe the long-run behaviour of certain variables. For example, when $0 < d < 1/2$, y_t is stationary but contains long memory, possessing shocks that disappear hyperbolically not geometrically. Contrastingly, for $1/2 < d < 1$, the relevant series is non-stationary, the unconditional variance growing at a more gradual rate than when $d = 1$, but mean reverting.

The memory parameter d can be estimated by a number of different techniques. The most popular, due to its semi-parametric nature, is the log-periodogram estimator (Geweke and Porter-Hudak, 1983; Robinson, 1995a) henceforth known as the GPH statistic. This involves the least squares regression

$$\log I(\lambda_j) = \beta_0 - d \log\{4 \sin^2(\lambda_j / 2)\} + u_j, \quad j = l+1, l+2, ..., m \qquad (12)$$

where $I(\lambda_j)$ is the sample spectral density of y_t evaluated at the frequencies $\lambda_j = 2\pi j/T$, T is the number of observations and m is small compared to T. *Inter alia*, Pynnönen and Knif (1998) and Hassler *et al*. (2002), note that the least-squares estimate of d can be used in conjunction with standard t-statistics. For the stationary range, $-1/2 < d < 1/2$, Robinson (1995a, 1995b) demonstrated that the GPH estimate is consistent and asymptotically normally distributed. Additionally, Velasco (1999a, 1999b) shows that when the data are differenced, the estimator is consistent for $1/2 < d < 2$ and asymptotically distributed for $1/2 < d < 7/4$.

Following Maynard and Phillips (2001), the d parameter of the forward premium will be estimated for daily series. For the Mark/Dollar, Mark/Yen and Mark/Sterling this runs from 2nd January 1991 to 31st December 1998 and totals 2023 observations. For the two other series the data run from 2nd January 1991 to 16th March 2001 and total 2594 observations. GPH statistics were estimated using differenced data[9] and Ox version 3.3 (see Doornik, 1999) and are shown in Table 7[10]. Agiakloglou, Newbold and Wohar (1992) note that GPH estimation may suffer from finite sample bias in the presence of strongly autoregressive short memory. Thus, for comparative purposes, also shown in Table 7 are ARFIMA (*p, d, q*) models computed by exact maximum likelihood (ML)[11]. These estimates are less robust in a large sample but explicitly modelling the autoregressive and moving average terms in (11), less susceptible to finite sample bias.

Table 7 contains some striking results. Firstly, the GPH point estimate of fractional differencing in the forward premia are spread over the range 0.84 to 0.98. Reassuringly, similar conclusions can be drawn from the estimated ARFIMA models. These point estimates are similar to Maynard and Phillips (2001) but much higher than those of Baillie and Bollerslev (1994), whose values range from 0.45 to 0.77. Despite the closeness of their point estimates to unity, Maynard and Phillips (2001) concur with Baillie and Bollerslev that the forward premium is a fractionally integrated series. Table 7 indicates that when the standard errors of the GPH point estimates are considered this conclusion can be considered doubtful. Specifically, three of the five series cannot reject the null of a unit root[12].

4.3 Fractional Cointegration

A non-stationary, possibly fractionally integrated forward premium indicates a failure of the unbiasedness hypothesis. Maynard and Phillips (2001) note that an explanation of this failure may stem from a fractionally cointegrated relationship between the forward premium, conditional variance of the spot rate and perhaps, a risk premium. As the risk premium is unobservable, the following analysis explores the empirical relationship between the forward premium and the conditional variance of the spot rate.

Fractional cointegration can be defined by supposing y_t and x_t are both *I(d)*, where *d* is not necessarily an integer, and the residuals, $u_t = y_t - \beta x_t$, are $I(\delta = d - b)$. When $b < d$, where *b* is also not necessarily an integer, series are fractionally cointegrated. Testing for fractional cointegration can be accomplished using a multi-step methodology (see Hassler *et al.*, 2002) where (1) the order of integration of the constituent series are estimated and tested for equality and (2) the long-run equilibrium relationship[13] is estimated and the residuals examined for long-memory. Alternative methodologies include the joint estimation of memory parameters of the constituent series, the cointegrating residuals and the equilibrium relationship (see Velasco, 2003) or the use of bootstrap methods (see Davidson, 2003).

A frequently used approach is to adopt a multi-step methodology where the concluding step estimates the GPH statistic, δ, for the least squares residual of the equilibrium relationship (see Dittman, 2001). *Inter alios***,** Tse, Anh and Tieng (1999) experimentally noted that t-statistics associated with $\hat{\delta}$ might not be normally distributed. Hassler *et al*. (2002) demonstrate that $\hat{\delta}$ has a limiting normal distribution provided the very first harmonic frequencies are trimmed. Specifically, this entails setting $l > 0$ in (12). Of course given asymptotically normal estimators, standard inference procedures can be legitimately applied. Therefore to ensure robustness of our results, this important finding of Hassler *et al.* (2002) will be applied within a multi-step methodology to explore the possibility fractional cointegration between the forward premium and the conditional variance of the spot rate.

First, we tested for fractional integration in the conditional variance of the spot rate, measured by traded volatility. Table 8 shows the GPH point estimate for the conditional variance of the spot rate range from 0.63 to 0.88. Generally speaking, these values are not untypical when compared with those previously noted (see Baillie *et al.*, 1996). Tests for $d = 1$ show that, in particular, the traded volatility series are fractionally integrated with $0.5 < d < 1$. As for the forward premium, similar conclusions can be drawn from the estimated ARFIMA models.

It is interesting to note that the GPH point estimates are closer to unity for the forward premium than for the conditional variance of the spot rate for all currencies. To examine this in more detail we test that the fractional orders of the constituent variables are equal by applying the homogenous restriction

$$H_0 : PD = 0 \tag{13}$$

where $D = \begin{bmatrix} d_y \\ d_x \end{bmatrix}$ and $P = \begin{bmatrix} 1 & -1 \end{bmatrix}$. Robinson (1995a) noted the relevant Wald test statistic could be expressed as

$$\hat{D}'P'\left[(0,P)\left\{(Z'Z)^{-1} \otimes \Omega\right\}(0,P)'\right]^{-1} P\hat{D} \tag{14}$$

where Ω is residual variance-covariance matrix from (12) , $Z = \begin{bmatrix} Z_{l+1} & \ldots & Z_m \end{bmatrix}'$ and $Z_j = \left[1, -\log\{4\sin^2(\lambda_j/2)\}\right]'$. Table 9 contains the Wald test results. For the Mark/Dollar, Yen/Dollar, Dollar/Sterling and Mark/Yen the test indicates different fractional orders for the two variables of interest. Clearly, for currencies with unequal orders of integration, bi-variate fractional cointegration cannot hold. Only for the Mark/Sterling does the Wald test assert equivalence and it is thus necessary to examine the fractional differencing parameter of the possible cointegrating relationship.

Table 10 shows the estimation of the fractional parameter for the differences of least squares residuals[14] for the Mark/Sterling using (i) the GPH methodology with $l = 1$ in (12) and (ii) the conventional ARFIMA methodology discussed previously[15]. Interestingly, the point estimate of δ is almost exactly the same as the fractional parameter d of constituent series. Again, this clearly implies the non-existence of bi-variate fractional cointegration between the forward premium and the conditional variance of the spot rate.

5. Market efficiency and risk premia

As shown in (7) international CAPM suggests that the time series properties of the forward premium are related to those of the conditional variance of the spot rate and a risk premium. The results in the previous section suggest the forward premium and the conditional variance of the spot rate are not fractionally cointegrated as a bi-variate pair. Although contradicting the supposition of Baillie and Bollerslev (2000), these results do not necessarily imply market inefficiency. Assuming rational expectations, our findings suggest that we require a non-stationary, possibly fractionally integrated risk premium to balance the system in equation (7). This is a interesting corollary given the common assumption of a stationary risk premium in the literature. It is important to note that the uncertainty over the long memory properties of the risk premium come directly from the empirically observed uncertainty demonstrated in the forward premium.

The results presented here provide another piece of empirical evidence to assist those who would construct a rational expectations model of the foreign exchange market. Any theoretical model must be capable of delivering a non-stationary risk premium. One possible candidate is a time-varying version of the constant risk premium model derived by Engel (1999). To see this consider the salient features of a sticky-price general equilibrium model where there is pricing to market[16]. In the two country case, it can be shown that optimal risk sharing in equilibrium implies

$$\frac{S_t P_t^*}{P_t} = \left(\frac{C_t}{C_t^*}\right)^{\rho} \qquad (15)$$

where C_t and C_t^* are the consumption indices in the many goods case for domestic and foreign countries respectively, P_t and P_t^* are the price indices for domestic and foreign countries respectively and S_t is the domestic price of foreign currency. Clearly consumption levels in the two countries diverge to the degree that there are movements in the real exchange rate. Expressing (15) in logarithmic form gives

$$s_t = p_t - p_t^* + \rho c_t - \rho c_t^* \qquad (16)$$

Assuming a cash-in-advance constraint where home (foreign) residents must purchase all goods with home (foreign) currency then

$$m_t - p_t = c_t \qquad (17)$$

$$m_t^* - p_t^* = c_t \qquad (18)$$

where m_t and m_t^* are the logarithms of money supply for domestic and foreign countries respectively. Noting that goods prices are predetermined, (5), (16), (17) and (18) lead to an expression for the risk premium

$$\begin{aligned} rp_{t-1} &= \rho \operatorname{cov}_{t-1}(s_t, c_t) \\ &= \rho \operatorname{cov}_{t-1}(\rho(c_t - c_t^*) + p_t - p_t^*, c_t) \\ &= \rho^2 \operatorname{var}_{t-1}(c_t) = \rho^2 \sigma_m^2 \end{aligned} \qquad (19)$$

where σ_m^2 is the variance of the money growth, assumed constant. Equation (19) implies that it is monetary variability that causes the correlation between consumption and exchange rates. Engel (1999) suggests this is a particularly promising model, generating a risk premium of a magnitude that is reconcilable with the empirical evidence, an identified problem with much of the previous literature. However, constant money growth variance implies a constant risk premium. To generate a time-varying risk premium, assume a time-varying money growth variance $\sigma_{m,t}^2$

$$rp_{t-1} = \rho^2 \sigma_{m,t-1}^2 \qquad (20)$$

Moreover, (20) implies the time-varying variance of money growth will need to be non-stationary in order to generate a non-stationary risk premium. Certainly a prima facie case for this can be provided by previous empirical work examining money growth volatility. For example, Mehra (1989) noted that the assumption that money growth volatility is stationary is not supported by any evidence. Thornton (1995) applies conventional ADF tests to data from nine countries and results suggest some uncertainty as to whether the null hypothesis of a unit root can be rejected. This perhaps mimics the uncertainty in the forward premium stressed earlier. More recently, Radchenko (2003) uses a Bayesian framework to circumvent acknowledged uncertainty over the order of integration of money growth volatility.

Substituting (20) into (7) gives a new expression for an empirical version of the international CAPM

$$(s_t - s_{t-1}) - (f_{t-1} - s_{t-1}) = -\frac{1}{2}\mathrm{var}_{t-1}(s_t) + \rho^2 \sigma^2_{m,t-1} + \varepsilon_t \qquad (21)$$

Thus (20) and (21) suggest further empirical work regarding the time series properties of the variance of money growth and the possibility of fractional cointegration between the forward premium and the variance of money growth.

6. Conclusions

Recent literature has found evidence of long memory (see Bollerslev and Baillie, 1994) or unit root (see Kellard *et al.*, 2001) behaviour in the forward premium suggesting a rejection of the unbiasedness hypothesis in the foreign exchange market. However, such time series behaviour does not necessarily imply market inefficiency. Indeed, international CAPM implies that several variables, for example the conditional variance of the spot rate or a risk premium, may transfer their time series behaviour to the forward premium. Thus, Maynard and Phillips (2001) propose that the literature should explore *why* the forward premium might display particular time series characteristics.

This paper investigates foreign exchange market efficiency by examining the previously untested possibility that the time series properties of the forward premium *are inherited from* the conditional variance of the spot rate. Monthly and daily time series of spot, one-month forward rates, interest rate differentials and conditional spot rate variances are constructed from daily data for the period January 1991 to March 2001. The currencies examined are the US dollar/Sterling, Yen/US dollar, Deutschmark/US dollar, Deutschmark/Sterling and Deutschmark/Yen. In a novel procedure, the conditional variance of the spot rate is proxied by actual traded volatility which captures the market's expectations about the future volatility of the spot rate and, hence, is more reliable than the estimated GARCH volatility typically used in the literature.

Firstly, it is shown that the salient features of the literature are replicated in the data set using order of integration tests on both the forecast error and the forward premium. Specifically, results suggest a rejection of the unbiasedness hypothesis. However, it is noted that time series properties of the forecast error can be prejudiced by a large component variable. Novel Monte-Carlo simulations are used to quantify the effect of the size of component variables on the reliability of testing procedures for the order of integration of a composite variable. The simulations substantiate the conjecture (see Newbold *et al*., 1998, and Maynard and Phillips, 2001) that the forward premium is the preferred variable to be employed in examining the extent and causes of bias.

Secondly, it is demonstrated that the conditional variance of the spot rate is non-stationary and fractionally integrated. Similar evidence is found for the forward premium, though the fractional parameter is closer to unity than that for the conditional variance. Consequently the possibility of fractional cointegration is examined using the recently developed semi-parametric methodology of Hassler *et al.* (2002). Strikingly, the equivalence of the fractional orders is accepted for the Mark/Sterling only. Moreover, the fractional parameter of the cointegrating residuals is shown to be no lower than that of the constituent series. Although these results imply the non-existence of bi-variate fractional cointegration between the forward premium and the conditional variance of the spot rate they do not necessarily imply market inefficiency. Assuming the validity of the international CAPM, the possibility of a non-stationary and fractionally integrated risk premium to balance the system can be posited. This is an interesting finding given the general assumption of a stationary risk premium in the literature.

Finally, it is shown that a possible explanation for the implied non-stationary behaviour of the risk premium lies in an examination of the underlying sources of risk. Specifically, Engel (1999) uses a sticky-price general equilibrium model to derive risk premia generated solely by money growth variability. Interestingly, the observed time series behaviour of money growth volatility is not incompatible with the implied risk premium. Further empirical work is encouraged along these lines.

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Endnotes

[1] This contrasts with the typical approach in the literature where expectations of the future variance of the spot rate are generated by estimating a GARCH model. This approach has a number of weaknesses: first, it is purely a statistical method and does not provide any information about the underlying economic determinants of volatility. Second, the use of the same set of data to estimate both the volatility model and the exchange rate model raises problems of identification and degrees of freedom. Third, recent literature demonstrates that implied volatility outperforms GARCH models in forecasting future currency volatility (see, for example, Jorion, 1995; Dunis *et al*., 2000; Dunis and Huang, 2002).

[2] Engel (1996) notes that numerous studies have established that the spot return is *I(0)*.

[3] The monthly data sets were constructed to minimise biases in sampling at this frequency due to institutional considerations; specifically, following the description of Breuer and Wohar (1996), purchase and settlement dates for forward contracts were matched correctly and allowed to cycle throughout the data period. These authors show that this procedure minimises both the end-of-period overlapping problem and the end-of-period clumping problem. For consistency, other series were matched in the same way.

[4] In fact, the dataset for the Deutschmark/US dollar is curtailed at February 1999 due to the introduction of the Euro. The choice of start date was governed by the availability of implied volatility data.

[5] Eurocurrency rates are annualised rates so that a quoted interest rate of 5 per cent typically translates to a thirty-day rate of 0.05(30/360). The calculations assume that annualised rates for the dollar, Mark and Yen refer to a 360-day year, whereas annualised rates for Sterling refer to a 365-day year. Note also that the number of days used to calculate each monthly interest rate is correlated exactly with the length of the appropriate forward contract.

[6] Implied volatilities are also annualised rates so that a quoted volatility of 5 per cent typically translates to a monthly rate of $0.05(21/252)^{0.5}$. The calculations assume that annualised rates refer to a 252 trading day year. Note also that the number of trading days used to calculate each monthly volatility rate is correlated exactly with the length of the appropriate forward contract.

[7] We also tested for cointegration using both the tri-variate system (s_t, f_{t-1} and $var_{t-1}(s_t)$), and the bi-variate system (π_{t-1}, $var_{t-1}(s_t)$). In both cases we found evidence of cointegration, which appears to support the supposition that the forward premium and the conditional variance of the spot rate have

similar time series properties. However, given our results on the fractional integration of the conditional variance and the forward premium, reported in the subsequent sections, these conventional cointegration results are not valid.

[8] Maynard and Phillips (2001) suggest that the effect of the relative magnitude of component variables on testing procedures might be usefully analysed by the use of either simulation or small scale sigma asymptotics.

[9] The resulting estimate of d was then increased by 1. Also note that in (12) l is set equal to zero, indicating no trimming of the harmonic frequencies.

[10] Note that the GPH statistic was estimated at $m = T^{0.75}$ following Maynard and Phillips (2001). The estimated standard error of d is that derived by Geweke and Porter-Hudak (1983) and shown in equation (4) of Hassler et al. (2002), who show it to be more appropriate than the conventional and Robinson (1995a, 1995b) alternatives.

[11] Applied to the first differenced series to satisfy the stationarity/invertibility condition $-0.5 < d < 0.5$ and again the resulting estimate of d was then increased by 1. Following Davidson (2003), the model order was chosen by minimising the SIC.

[12] For completeness, we also tested for fractional integration in the spot exchange rates. However, the GPH point estimates were not statistically different from one, so we cannot reject the unit root null hypothesis.

[13] The long-run equilibrium relationship itself could be approximated by OLS, a fractional version of the Fully Modified method suggested by Kim and Phillips (2001), Gaussian semi-parametric estimation developed by Velasco (2003) or narrow band spectral estimates (see Robinson and Marinucci, 1998).

[14] The cointegrating vector is estimated by OLS. Ng and Perron (1997) examine the normalisation issue in two-variable models. They demonstrate that the least squares estimator may possess poor finite sample properties when normalised in one direction but can be well behaved when normalised in the other. As a practical suggestion, they advise using as regressand, the variable that is less integrated. Thus, we use conditional variance of the spot rate as the dependent variable.

[15] Hassler *et al*. (2002) demonstrate by application of Monte Carlo experiments that trimming only one frequency, $l = 1$, provides a satisfactory normal approximation for the distribution of GPH statistic.

[16] Producers set the price in the currency of the consumers. Therefore, the prices consumers face are invariant to the exchange rate.

Table 1

Augmented Dickey-Fuller Tests: $k_{\max} = 6$

	Series	t_{a_1}	k
DM/US$	s_t	-2.173	0
	f_{t-1}	-1.775	2
US$/UK£	s_t	-1.643	6
	f_{t-1}	-1.805	6
Yen/US$	s_t	-1.875	0
	f_{t-1}	-1.803	6
DM/Yen	s_t	-2.334	0
	f_{t-1}	-2.261	0
DM/UK£	s_t	-1.640	3
	f_{t-1}	-1.790	3

Note: t_{a_1} represents the Dickey-Fuller test statistic, and *k*, the number of lags chosen. The 5% critical value is –2.89.

Table 2

Tests of Cointegration Rank: $X_t = (s_t, f_{t-1})$

Currency	H_0:*r*	*l*-max†	Trace†	Lag Length	Comment
DM/US$	= 0 ≤ 1	237.57** 5.82	243.40** 5.82	1	Rank = 1 Reject non-cointegration
US$/UK£	= 0 ≤ 1	332.50** 12.67*	345.17** 12.67*	1	Rank = 2 Spot and forward I(0)
US$/UK£	= 0 ≤ 1	76.32** 8.06	84.39** 8.06	2	Rank = 1 Reject non-cointegration
Yen/US$	= 0 ≤ 1	389.57** 3.88	393.45** 3.88	1	Rank = 1 Reject non-cointegration
DM/Yen	= 0 ≤ 1	299.98** 6.60	306.58** 6.60	1	Rank = 1 Reject non-cointegration
DM/UK£	= 0 ≤ 1	218.70** 2.00	220.70** 2.00	1	Rank = 1 Reject non-cointegration
Critical Values (0.05)	= 0 ≤ 1	15.87 9.16	20.18 9.16		

† Using small sample degrees of freedom correction (Reimers, 1992)
** Reject null at 1% level
* Reject null at 5% level

Source: Osterwald-Lunem (1992)

Table 3

Standardised Cointegrating Vectors for $X_t = (s_t, f_{t-1})$

	s_t	f_{t-1}	intercept	$\chi^2(2)$	$\chi^2(1)$
DM/US$	1.00	-0.995	0.0012	2.062 [0.36]	0.172 [0.68]
US$/UK£	1.00	-1.028	0.0110	14.174 [0.00]	5.868 [0.02]
Yen/US$	1.00	-0.985	0.0716	18.417 [0.00]	6.555 [0.01]
DM/Yen	1.00	-1.002	0.0064	18.945 [0.00]	0.097 [0.76]
DM/UK£	1.00	-1.001	-0.0005	5.033 [0.08]	0.028 [0.87]

Note: The fifth column gives the test of the restrictions $\beta_0 = 0$ and $\beta_1 = 1$ in (3). The sixth column gives the test of the restriction $\beta_1 = 1$ only. The figures in square brackets are p-values.

Table 4

Augmented Dickey-Fuller Tests: $k_{\max} = 6$

	Series	t_{a_1}	k
DM/US$	$s_t - s_{t-1}$	-7.972	1
	π_t	-1.161	4
US$/UK$	$s_t - s_{t-1}$	-5.752	5
	π_t	-1.753	3
Yen/US$	$s_t - s_{t-1}$	-5.538	5
	π_t	-2.351	3
DM/Yen	$s_t - s_{t-1}$	-8.682	0
	π_t	-3.251	4
DM/UK£	$s_t - s_{t-1}$	-4.240	2
	π_t	-1.141	3

Note: t_{a_1} represents the Dickey-Fuller test statistic, and *k*, the number of lags chosen. The 5% critical value is –2.89.

Table 5

Sample Variances

	$\mathrm{var}(s_t - s_{t-1})$	$\mathrm{var}(s_t - f_{t-1})$	$\dfrac{\mathrm{var}(s_t - s_{t-1})}{\mathrm{var}(s_t - f_{t-1})}$	$\mathrm{var}(\Delta\pi_t)$
DM/US\$	9.31×10^{-4}	9.38×10^{-4}	0.992	1.55×10^{-7}
US\$/UK£	7.36×10^{-4}	7.28×10^{-4}	1.011	1.79×10^{-7}
Yen/US\$	1.40×10^{-3}	1.43×10^{-3}	0.980	1.66×10^{-7}
DM/Yen	1.12×10^{-3}	1.11×10^{-3}	1.010	1.60×10^{-7}
DM/UK£	4.85×10^{-4}	4.86×10^{-4}	0.999	1.67×10^{-7}

Table 6

Dickey-Fuller Critical Values for (9c)

η	5%	10%
0	-2.892	-2.579
0.1	-2.913	-2.583
0.5	-3.356	-2.941
1	-4.389	-3.911
2	-6.486	-5.976
5	-9.430	-8.971
10	-10.83	-10.32

Note: The critical values are calculated by drawing random samples of 100 observations for z_t in (9c). For each sample the Dickey-Fuller statistic for z_t, allowing a constant but no trend, is calculated. The process is repeated for 3000 replications.

Table 7

GPH/ARFIMA Tests for the Forward Premium

	d_{GPH}	$\tau_d = 1$	d_{ML}	*(p,q)*
DM/US$	0.9704045 (0.0386769)	-0.765	0.981906 (0.02052)	(1,0)
US$/UK£	0.980247 (0.0350188)	-0.564	0.9338433 (0.02)	(1,0)
Yen/US$	0.842711 (0.0350188)	-4.491	1.0076744 (0.05319)	(1,1)
DM/Yen	0.962448 (0.0386769)	-0.971	0.9929663 (0.03102)	(3,0)
DM/UK£	0.916755 (0.0386769)	-2.152	0.9338913 (0.01992)	(1,0)

Note: numbers in parentheses below the estimates for d_{GPH} and d_{ML} are standard errors (σ_d). Numbers in the third column represent the test statistic $(d_{GPH} - 1)/\sigma_d$.

Table 8

GPH/ARFIMA Tests for the Conditional Variance of the Spot Rate

	d_{GPH}	$\tau_{d=1}$	d_{ML}	*(p,q)*
DM/US$	0.83152 (0.0386769)	-4.356	0.789221 (0.03408)	(1,0)
US$/UK£	0.778716 (0.0350188)	-6.319	0.767546 (0.03077)	(1,0)
Yen/US$	0.631889 (0.0350188)	-10.51	0.741474 (0.03466)	(3,0)
DM/Yen	0.749957 (0.0386769)	-6.465	0.685983 (0.02729)	(0,2)
DM/UK£	0.881339 (0.0386769)	-3.068	0.883102 (0.01853)	(0,0)

Note: numbers in parentheses below the estimates for d_{GPH} and d_{ML} are standard errors (σ_d). Numbers in the third column represent the test statistic $(d_{GPH} - 1)/\sigma_d$.

Table 9

Wald Tests for the Equality of the GPH Estimates for the Forward Premium and the Conditional Variance Rate

DM/US$	US$/UK£	Yen/US$	DM/Yen	DM/UK£
6.6893 [0.0097]	17.522 [0.0000]	18.032 [0.0000]	14.560 [0.0000]	0.40197 [0.50806]

Note: the Wald statistic has a $\chi^2(1)$ distribution. The figures in square brackets are p-values.

Table 10

GPH/ARFIMA Tests for the Cointegrating Residual (DM/UK£)

δ_{GPH}	$\tau_{\delta=0.5}$	δ_{ML}	(p,q)
0.872423 (0.0403426)	9.232	0.883146 (0.01853)	(0,0)

Figure 1

Spot Return and Interest Rate Differential (DM/US$)

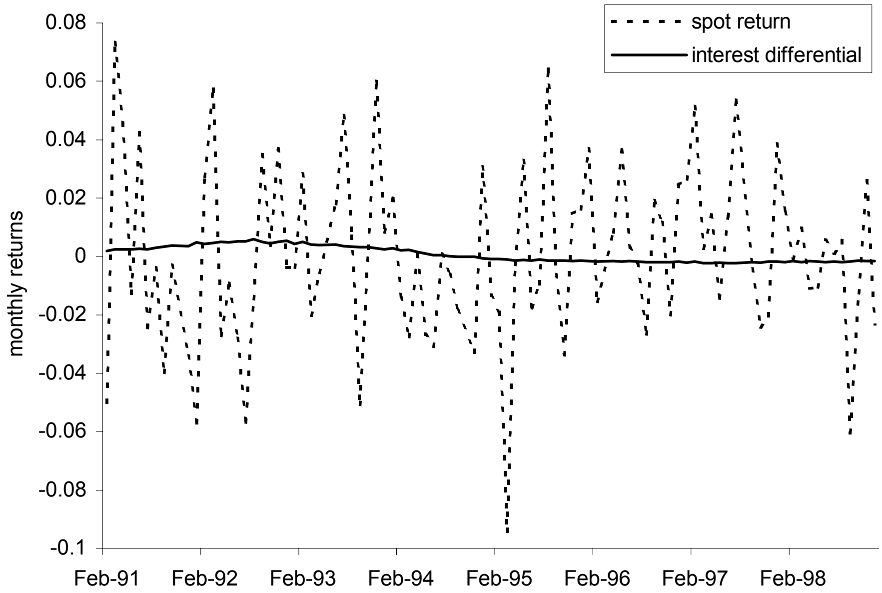

Figure 2

Spot Return and Forecast Error (DM/US$)