The behavior of the nominal exchange rate at the beginning of disinflations *

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Abstract

A standard rational expectations model would give strong predictions about the behavior of the nominal exchange rate at the beginning of a disinflation (a rise in interest rates): a substantial initial appreciation, followed by a steady depreciation. It largely conflicts actual observations, like the recent experience of Poland, Hungary, and Chile, where an initial appreciation was not followed by any systematic depreciation. The paper tries to explore whether rational expectations can be rescued by introducing noise and parameter learning. An optimistic learning case (worse than expected inflation data every period), or the combination of a pessimistic learning case (better than expected data every period) and a declining proportional risk content of the interest rate offers a potential explanation.

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1 Introduction

This paper addresses the inability of a frictionless rational expectations model (one that builds on interest parity) to match the observed behavior of the nominal exchange rate at the beginning of "interest-rate-based" disinflations (a surprise interest rate hike).[1] Interest parity would imply

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[1] The terminology "interest-rate-based disinflation" means that monetary contraction takes the form of an initial interest rate increase, and then a gradual return to normal levels. The monetary authority sets the nominal

a large initial appreciation (following the rise in interest rates) and then a gradual depreciation (reflecting the equalization of expected returns on domestic and foreign bonds, including the capital loss of holding domestic currency). Actual observations often show the initial strengthening, but then the exchange rate follows no clear reversal. Let me illustrate the phenomenon with three actual episodes: Hungary in 2001-2002, Poland in 1999-2001, and Chile in 1992-1994. Later on, the framing of the model itself will mostly reflect the Hungarian example.

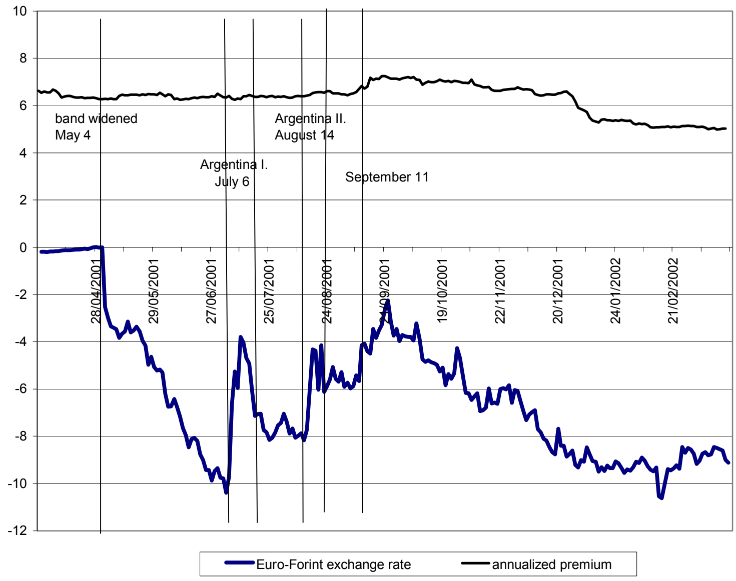

Figure 1: The Forint-Euro exchange rate and the interest differential

Figure 1 depicts the evolution of the Forint-Euro exchange rate and the difference of 3-month Forint and Euro benchmark yields, in 2001-2002. As shown on the picture, the current phase of disinflation started on May 4, 2001: the Forint, which used to have a $\pm 2.25\%$ band, was allowed to move freely within a $\pm 15\%$ band, and its full convertibility was introduced. The immediate response was a heavy appreciation (though part of it might have reflected an initial undervaluation), in line with interest parity (reflecting the attractive bond yield, which could not have appreciated the Forint further in the previous regime). Later on, however, actual and predicted behavior diverged: apart from three large depreciation episodes (two turmoils related to Argentina, and the consequences of September 11), the exchange rate showed a general

interest rate, and allows the exchange rate and money supply to be determined by markets. A purely "money-supply based disinflation" (when money supply is set, while the interest rate and the exchange rate is endogenous) might lead to a different interest rate behavior, thus a different exchange rate prediction as well. The exchange rate implications of an exchange rate based disinflation (the nominal exchange rate is set, and the other two quantities adjust) are obvious.

appreciating tendency (though after the first two episodes, there seems to have been a correction, but it clearly vanished after the third episode). On the other hand, the excess bond yield was stable and substantial. Though not shown on the figure, this episode indeed corresponds to a change in the disinflation process: after a gradual decrease since the mid-nineties, inflation became flat in 1999-2000, and then returned to a decline. This decline, however, was not homogeneous, it involved a partial halt in the beginning of 2002.

Figure 2 conveys the similar experience of Poland. The beginning of the sample is the first interest rate hike within the inflation targeting regime – the first indication of a true shift in monetary policy –, which was followed by three further contractionary steps. This period observed a gradual strengthening of the Zloty, up to June 2000 (a quarter before the last rate increase), which was followed by large swings, but without any tendency of reversal. Interest rate differentials, however, remained large, regardless of Zloty interest rate changes. Inflation, on the other hand, remained stubbornly high, initially it has even increased, which was finally followed by an abrupt drop in 2000 and 2001.

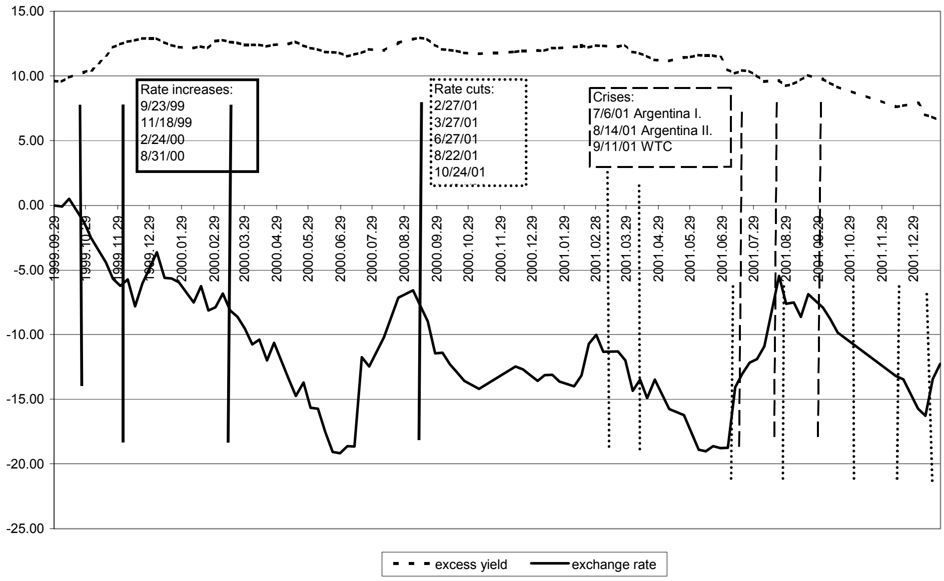

Figure 2: The Zloty-Euro exchange rate and the interest differential

At first glance, the experience of the Czech Republic around 2000-2001 shows an opposite behavior: low, sometimes nearly negative excess yields and a dominantly appreciating nominal exchange rate. If we are ready to assume a strong entry parity of the Czech currency in the EMU, this behavior is consistent with interest parity: low (negative) yields and a strong expected future exchange rate imply a gradual strengthening. Two qualifications, however, are necessary. The first is that the Czech National Bank, unlike its Hungarian and Polish counterparts, was actively intervening in the foreign exchange market. The second is that this period is already after the main disinflation episode of the Czech Republic. Looking back to 1998-1999, when inflation came down from 10% to 3-4%, we have in fact a similar picture to Hungary and Poland: impressive and stable excess yields during 1998, and a gradually appreciating currency. By the end of 1998, inflation was beaten, unlike in the other two countries.

Figure 3: Chile: the Peso-Dollar exchange rate and the interest differential (smoothed and adjusted for the crawling peg)

Chile had a similar experience in the early nineties: the exchange rate band was widened in January 1992. As Figure 3 shows, this was followed by a heavy appreciation of the currency, and apart from two depreciation episodes, the exchange rate was fairly stable afterwards, while the interest premium remained large. The first episode corresponds to mid-92, when significant capital controls were introduced. The other episode does not match any particular event, nevertheless, it coincides with a period when the crawl of the central parity was unusually low, and inflation became flat. One needs to be a bit careful in interpreting the actual numbers: unlike in Poland and Hungary, Chile operated under a nonzero crawl of the central parity (continuous devaluation). The exchange rate number corresponds to log deviation from this central parity. The same adjustment is applied to interest rates: the pre-announced crawl is subtracted. Since interest rates were indexed to realized inflation, Figure 3 displays smoothed series (applying a

Hodrick-Prescott filter). Just like in the other two countries' case, this period was part of a broad disinflation process. In particular, the band was widened and realigned after a reversal in the gradual decline of inflation. As a consequence, inflation returned to its downward path, although with some plateaus.

A similar puzzle is the delayed overshooting finding of Eichenbaum and Evans (1995). They document that after a monetary contraction, nominal exchange rates show a gradual appreciation, followed by a gradual reversal. The first finding is in line with the Hungarian, Polish and Chilean experience as well, but the similarity of the gradual reversal is not.[2]

As put by Obstfeld and Rogoff (1996), page 622: "While conventional wisdom holds the Mundell-Fleming-Dornbusch model to be useful in predicting the effects of major shifts in policy, its ability to predict systematically interest-rate and exchange rate movements is more debatable." The underlying force in that model is again the uncovered interest parity condition, which predicts the same strong but empirically questionable behavior of the exchange rate.

The puzzlingly poor track record of uncovered interest parity is well known: a classical documentation and interpretation is offered in Fama (1984), and surveyed in Froot and Thaler (1990) and Isard (1995), among others. This paper does not aim at any general evaluation or rescue of the UIP hypothesis: the narrow objective is to focus on marked disinflation episodes.

To frame the discussion and main points, I will adopt a forward-looking "small macromodel" of an open economy (along the lines of Svensson (2000)). The motivation for this choice is at least twofold: it allows for an explicit treatment of an interest-rate-led disinflation process, highlighting the relation between inflation, interest rates and exchange rate behavior; besides, this is also in line with the current major monetary framework for modeling inflation. Benczúr (2002b) offers a general but simple description of inflationary dynamics in small macromodels. In this paper, I will just summarize the necessary results, and using a substantially simplified version, I address the behavior of the nominal exchange rate, under Bayesian learning.

In particular, I want to investigate whether the following consideration can be a qualitatively and quantitatively important factor in determining the nominal exchange rate. At the beginning of a disinflation, it should be quite clear for investors that the currency will offer a medium-term (a couple of years) excess yield, thus leading to massive capital inflows, and a large initial appreciation (reflecting not just the immediate excess yield, but also its "persistence"). Apart from this "obvious" step, a disinflation then continues with many uncertainties: about the

[2] To give a full historical account, one should note that the 2003 evolution of the Forint is in fact similar to a delayed overshooting. After a speculative attack on the strong side of the exchange rate band in January, it has later depreciated to near-2001 levels.

determination of the central bank, the effectiveness of monetary policy tools (like the exchange rate pass-through, the effect of real interest rates on the output gap, and the disinflationary effect of the output gap, etc.), the persistence of inflationary expectations, just to name a few. This means that every major data announcement represents an additional surprise, which can counteract the trend depreciation of the currency. In a modeling language, this means that I maintain rational expectations, but introduce noisy signals: I relax the assumption of perfect foresight, model-consistent expectations.

A traditional channel for such uncertainties is the behavior of the central bank itself: how much costs it is ready to tolerate, consequently, how aggressively it would react to changes in inflation. In an inflation targeting framework, this asymmetry should be less serious, since the regime operates under a high degree of transparency. This does not mean that investors fully trust the announced disinflation path of the central bank: they might fear, for example, that there is a possibility of a regime realignment.

Even under the highest level of credibility, there is still room for uncertainties, which are in fact shared by the central bank and market participants (*symmetric, public information*): after a regime shift (moving into a disinflation, or changing the monetary policy framework), many of the monetary mechanisms might have changed, so their size or strength is not known precisely. The behavior and effect of fiscal policy, or the value of the equilibrium real exchange rate can be a similar source of uncertainty. As shown in Benczúr (2002b), the dynamics of inflation, hence the behavior of interest rates and exchange rates can be quite sensitive to the precise size of such effects. Among many others, the effect of the output gap on inflation, or the strength of the exchange rate pass-through can be an abstract source of such aggregate uncertainty.

An akin idea is followed by Lewis (1989): there is uncertainty about the change in the money demand process, and markets learn the new situation only gradually. That paper succeeds in quantifying the bias this learning caused (ex post), using actual data. Though my approach is similar, the focus is shifted in many ways: first, I want to concentrate on the specifics of disinflations, which is a clear restrictive shock, and it is only its effect but not the change itself that is uncertain. Second, by considering the links between inflation, interest rates and exchange rates, the source of uncertainty will be more structural. For this reason, I need to model their determinants and interdependence, in particular, to use an interest rate rule for the central bank. This also enables me to track the performance and the components of the uncovered interest parity condition itself (changes in the long-run nominal exchange rate, the cumulative excess yield, and its risk content). Third, in my model, there is a feedback from imperfect expectations to the inflation process, thus back to the signal extraction problem as well. Finally, current monetary regimes use the nominal interest rate as their policy tool, so the endogeneity of interest rates needs to be incorporated into the analysis.[3] Unfortunately, the short time period of potential observations, and the complications implied by my version of parameter learning does not allow a precise quantification of this argument.[4]

Since my objective is to explain a one-two year-long episode, the speed of learning is a key concern. One may accept that it took financial markets some weeks to digest changes, but is it reasonable to have learning even after a year? In my view, it is so: to learn about structural parameters of new mechanisms, one needs new observations. For aggregate links like between inflation and output, or real exchange rates, the relevant frequency is monthly, or rather, quarterly. Moreover, there is enormous noise in the observations. So even after an entire year, one still has only twelve (or four) noisy observations, which will not yield precise estimates. In Lewis (1989), where learning was about the money demand process, the data suggested a 1-3 years span of learning. For the determinants of inflation, I would expect the speed of learning to be even slower.

Can the analysis of such a particular episode add to the general uncovered interest parity debate? My story tells us that in these episodes, where a parameter changes (due to a regime switch), a previously inactive channel becomes operational, there is mistrust in the central bank's behavior, or market participants need to tell persistent supply and short-lived demand fluctuations apart, there is a substantial ex post bias in interest parity. If such episodes are relatively long – measured in years –, then we may run into sample size problems even with ten years of monthly or weekly data: the long-run average of the ex post bias is zero, but we will have only 5-6 such episodes in our data, insufficient for cancelling the bias. A further indication is the finding of Lewis (1989), that learning can explain half of the observed bias of interest parity in a nearly 3-year long period.

Another relevant question is whether such a learning story can be applied in policy simulation, or forecasting models. If the source of the uncertainty is some structural parameter, applicability is doubtful: it would require simulating the economy using the true parameter, but both market participants and the central bank would be restricted to learn this parameter only

[3] It is not straightforward to determine which interest rate should be used in the interest parity condition, or how such an interest rate is influenced by the rate decisions of the central bank. To avoid these complications, I assume that the central bank sets the relevant interest rate directly, and any additional interest rate can be obtained by using the expected future rates.

[4] The main complication is caused by the nonlinearity of the structural model: all variables are linear in inflation, but highly nonlinear in the underlying structural parameters. This makes the explicit calculation of conditional expectations practically impossible.

gradually. If, however, uncertainty is about the central bank behavior (less than full credibility of the interest rate rule, for example), then it can be inserted into a simulation: the central bank always behaves, but market participants do not take it for granted, and get to trust the bank only gradually.

The paper is organized as follows. The next section examines the perfect foresight behavior of the nominal exchange rate. Section 3 describes the small macromodel framework for endogenizing inflation, and then introduces parameter learning into this framework. The behavior of the realized exchange rate is analyzed in Sections 4 and 5, first under the assumption of no risk premium (a fixed proportion), and then allowing for a systematically changing risk premium content. Finally, Section 6 offers some concluding remarks, and the Appendix contains a formal but not fully rigorous treatment of the explicit learning process.

2 The rational expectations behavior of the nominal exchange rate

In this section I explore the behavior of the nominal exchange rate at the beginning of an *interest-rate-based* disinflation, under the assumption of model-consistent expectations (perfect foresight if there is no uncertainty, rational expectations if there is any noise). These results are completely independent from whether we believe that the economy is described accurately by a small macromodel or not: I will use only rational interest rate and inflation forecasts, and the uncovered interest parity condition (thus assuming risk neutrality and no market frictions), and some form of purchasing power parity.[5]

2.1 Perfect foresight (no uncertainty)

Start from the interest parity condition for the nominal exchange rate (in logarithmic form):

$$s_t = s_{t+1|t} - i_t + \phi_t,$$

[5] My assumption about the rationality of inflation expectations also involves a long-term stability assumption: inflation must converge to its equilibrium value sufficiently fast, thus the sum $\sum \pi_t$ – which defines the long-run price level – exists. With noise in the model, it applies to expected values. For a linear model, it is a relatively weak requirement: for the solvability and stability of such a model in general, inflation must disappear asymptotically, which happens with an exponential speed, so it is summable. The role of this assumption is to ensure that the nominal exchange rate has a long run value: with the real exchange rate reaching an equilibrium value, and the price level converging to some constant, the nominal exchange rate is also constant.

where s_t denotes the current value of the nominal exchange rate, i_t is the current nominal interest rate (in excess of world interest rates) ϕ_t is a risk premium term,[6] and $s_{t+1|t}$ is the expected exchange rate one period ahead. All time t expectations are taken at the *beginning* of period t.

At the beginning of disinflation, there is a surprise regime change: in the case of Hungary, it was to let the currency out of a narrow band and eliminate capital controls.[7] Using rational expectations again, market participants now have the ability to predict the central bank's behavior under the new regime, and all of its consequences (assuming that the wide band does not become binding). This means that they form an infinite sequence of expected interest rates, inflation, output gap etc. Iterate interest parity along this expected path:

$$s_t = s_{\infty|t} - \left((i_t - \phi_t) + \left(i_{t+1|t} - \phi_{t+1|t} \right) + ... \right). \tag{1}$$

The current exchange rate is the difference of the long-run expected exchange rate (we will see its existence later on) and the cumulative (risk-free) excess interest rate. Interest rates need to be adjusted for the *current ("expected") levels* of current and future risk premia. Since it is possible that new information emerges during the process of disinflation, for example there is learning about the strength of certain monetary effects, or how much costs the central bank is ready to accept, $\phi_{t+j|t}$ and $\phi_{t+j|t'}$ may be in general different from each other.

The long-run value of the nominal exchange rate is determined by the equilibrium level of the real exchange rate, implied by some form of purchasing power parity. Normalize this level to zero. Since the real exchange rate satisfies

$$q_t = s_t + p_t^* - p_t, \tag{2}$$

we must have

$$0 = q_{\infty|t} = s_{\infty|t} + p^*_{\infty|t} - p_{\infty|t} = s_{\infty|t} + p^*_{t-1} - p_{t-1} - \left(\pi_t + \pi_{t+1|t} + ... \right). \tag{3}$$

[6] One can include a nonzero risk premium even under "perfect foresight": markets may price in the probability of an event that changes the model itself. This event never happens along the equilibrium path, but its risk is incorporated into the exchange rate every period. Section 5 offers a more detailed interpretation and discussion of the risk premium term.

[7] Under the narrow currency band and less than full convertibility, the currency had no room for further appreciation, or a substantial reversal later on. This situation was altered by the increased bandwidth. Besides, there were signs of an undervaluation: see, for example, Halpern and Wyplosz (1997) on the real undervaluation in transition economies in general. Kovács (2001) estimates the initial undervaluation around 5%, which is smaller than the initial appreciation of 10%.

This implies

$$s_t = p_{t-1} - p^*_{t-1} + \left(\pi_t + \pi_{t+1|t} + ...\right) - \Big((i_t - \phi_t) + \Big(i_{t+1|t} - \phi_{t+1|t}\Big) + ...\Big). \tag{4}$$

The current level of the nominal exchange rate is thus determined by the current price level differential (it is only a matter of normalization), the cumulative excess expected inflation and expected interest rates. Therefore, the long run value of the nominal exchange is well-defined if there is a limit of the price level at infinity, i.e., the series of excess inflation is summable (converges to zero fast enough). In contrast to the real exchange rate and inflation, the long run value of the nominal exchange rate cannot be considered as an equilibrium variable: for example, it depends on initial conditions.

How does the exchange rate evolve through time? Assume that initially $p_{t-1} = p^*_{t-1}$, $q_{t-1} = 0$ (the process starts from the current equilibrium value of the real exchange rate[8]), then $s_{t-1} = 0$. Markets learn the regime change at the "middle" of period t, and then s_t is set by (4). It seems safe to assume that interest rates are overall restrictive, meaning that the riskless real interest rate is positive, then (4) implies an initial appreciation (nominal and real as well).

After this first surprise, without further news or noise – thus all expectations being equal to model-consistent realizations –, interest parity implies a steady depreciation:

$$s_{t+1} = s_{t+1|t} = s_t + i_t - \phi_t,$$

so if $i_t - \phi_t$ is positive, there is a depreciation. In the long run, the interest differential converges to zero, and the exchange rate becomes constant.

This constant level (s_∞) is necessarily weaker than the initial value $s_{t-1} = 0$: for the real exchange rate to return to its original level, any cumulative excess inflation must be exactly offset by the long-run nominal depreciation. We have positive excess inflation at the beginning, so unless it becomes heavily negative for quite some time, the increase in the domestic price level will be larger than that of foreign, so we must have a long-run nominal depreciation. Note that having a negative cumulative inflation is not necessarily unreasonable, since this would refer to a negative inflation on top of "structural" inflation, as implied by foreign inflation and the

[8] In case of an equilibrium real appreciation (or depreciation), let q_t denote the deviation from the equilibrium level. This appreciation then must be matched in the equilibrium path of inflation, thus π_t is also the deviation from the sum of foreign inflation and the structural excess inflation, caused by the real appreciation; and p^*_t is the hypothetical equilibrium price path, starting from the initial price level. Then (2) remains valid, and the long run behavior of inflation and the real exchange rate is still consistent with a fixed nominal exchange rate. This procedure essentially means that we think of inflation and the real exchange rate only of domestically produced and consumed goods.

Balassa-Samuelson effect (or any other factor that causes the equilibrium real exchange rate to appreciate). Therefore, it need not mean a true deflation, but only a smaller than "equilibrium" level of inflation.[9]

For a general value of q_{t-1}, the initial real exchange rate, and $s_{t-1} = 0$, (3) implies that

$$s_{\infty} = -q_{t-1} + \left(\pi_t + \pi_{t+1|t} + ...\right).$$

This means that if the initial real exchange rate was undervalued by, say, 5% initially ($q_{t-1} = 5$), then there can be an at most 5% long-run appreciation.

2.2 Introducing noise and learning

These considerations remain mostly unaltered if there is noise in the economy, but without any informational content. If both market participants and the central bank know the true parameters of the system from the very beginning, then we can write identical equations for the expected values of the same variables, and we get only mean zero deviations. The nominal exchange rate might show fluctuations around the depreciating trend, but its trend should be a gradual weakening.

To get a rational deviation from this strong prediction, one needs to introduce informative surprises into the economy (rational learning). A straightforward first interpretation is that there are many parameters uncertain in the economy, and each new observation leads to new estimates. If there is really new information in new data points, then this is a better estimate, but if the new information comes in the form of a noisy signal, then the new estimate is still not perfect.

Given the new estimate, market participants would rationally update their interest rate and inflation forecasts, and the new level of the exchange rate would reflect this information, thus being different from the level of the exchange rate predicted in the previous period.

The interest parity condition would still hold between the current and the expected next period exchange rate, but not between realized exchange rates.[10] Moreover, the forecast based on the old estimates will look biased *ex post*: knowing the direction of the update, one would have predicted the prevailing appreciation or depreciation correctly. Now suppose that information

[9] The equilibrium level of inflation is in general the foreign inflation level. Under an equilibrium real appreciation (due to excess productivity growth, for example), the equilibrium level of inflation becomes higher as well. See also footnote 8 on page 10.

[10] Looking at Reuters polls about market expectations of the HUF/EUR exchange rate, it shows no reversal either, thus casting doubts even on this form of interest parity.

is always about a faster than expected disinflation, matched by a more than proportional reduction in anticipated future interest rates (assuming a higher than one inflation coefficient in the reaction function of the central bank). Then every period comes with a decrease in cumulative real interest rates, thus an overall monetary easing, leading to a depreciation bias of the actual exchange rate.

Looking ex post at many periods of the exchange rate (current and predicted), one would find a significant bias in the predictions, but that statement involves an ex post conditioning on the fact that new information was always about an even faster disinflation. At any moment, the market formed its best forecast based on current observations, and this forecast was updated systematically in one direction. Ex post we do know that some uncertain parameter was higher than initially expected, but due to noisy signals, the best feasible estimate was only converging to this true value.

Gourinchas and Tornell (2001) explores a very similar, though more general idea behind the forward premium puzzle: in their scenario, market participants underestimate *ex ante* the persistence of interest rate changes, which leads to the failure of interest parity. In my model, people can be subject to a similar but *ex post* underestimation: based on their too optimistic expected disinflation path, they also expect interest rates to return to normal faster than the realization. The first key difference is that they can also overestimate interest rates ex post, if they were too pessimistic in their inflation forecast, and the second is that there are no ex ante misperceptions. One can also view my model as a structure behind interest rate expectations, and reinterpret misperceptions about the persistence of interest rate changes as misperceptions about the persistence of inflation shocks.

To give this rough idea a formal framework, I will have to be explicit about the determinants of inflation and interest rates. For this reason, I adopt a small macromodel description of the economy, and introduce gradual learning.

3 Parameter learning in the Phillips curve

3.1 The specification of the small macromodel

As explained in many studies (like Svensson (2000), or Gali and Monacelli (2002)), the Calvo sticky price model can be reduced to a convenient linear dynamic system. The key ingredient of these models is the "new keynesian Phillips curve"

$$\pi_t = \beta y_t + \lambda E\pi_{t+1|t}, \tag{5}$$

where y_t is the output gap, and λ is approximately one. This equation implies the persistence of the price level, but not inflation itself. For this reason, practitioners usually replace λ by one and $E\pi_{t+1|t}$ by $\alpha\pi_{t-1} + (1-\alpha)\, E\pi_{t+1|t}$. This form introduces the desired persistence of inflation, though it no longer has so solid microfoundations as the original expression (5).

For a closed economy, the model consists of an aggregate supply equation (Phillips curve), an aggregate demand relation, and a reaction function. The open economy extension includes an uncovered interest parity equation, and a role for the real exchange rate, either through the output gap, or inflation directly:

$$\begin{aligned}
\pi_t &= \alpha\pi_{t-1} + (1-\alpha)\pi_{t+1|t} + \beta y_t + \kappa q_t + \delta\left(q_{t+1} - q_t\right) \\
y_t &= \gamma y_{t-1} - \eta\left(i_t - \pi_{t+1|t}\right) + \phi q_t \\
i_t &= \tau\pi_{t+1|t} + \psi y_t \\
q_t &= q_{t+1} - \left(i_t - \pi_{t+1|t}\right).
\end{aligned} \tag{6}$$

All variables refer to deviations from steady state (i.e., the natural rate of output, foreign interest rates, etc.).

Inflation (π) is given by a Phillips curve relation: there is a pure persistence term $\alpha\pi_{t-1}$, and the remainder is determined by expected (future) inflation. Price rigidities, however, lead to a positive effect of output gap (y) on inflation. Inflation is potentially reduced by a strong real exchange rate (like in Leitemo (2000), Svensson (2000)), a real appreciation (like in Buiter and Clemens (2001)), and a strong real exchange rate also depresses output (through the worsening of the trade balance, for example).

Output gap is set by an autoregressive term y_{t-1}, the real interest rate and the real exchange rate. The real exchange rate is determined by interest parity, while the nominal interest rate follows a Taylor rule. Its essence is that for a system of two state variables, such a form contains all linear interest rate rules. As such, it also covers all optimal reaction functions corresponding to a fixed coefficient, quadratic objective function of the central bank.

For simplicity, I neglect the autoregressive term ($\gamma = 0$). As explained in Benczúr (2002b), the dynamic properties of the autoregressive system would remain identical. With $\gamma = 0$, the Taylor rule reduces to $i_t = \tau\pi_{t+1|t}$, and one can merge ϕ into κ, and δ into η. Plug this into interest parity and iterate to infinity:

$$q_t = q_\infty - (\tau - 1) \sum_{s=t+1}^{\infty} \pi_{s|t}.$$

Assume that the long run value of the real exchange rate is determined by purchasing power parity (an exogenous assumption), thus $q_\infty = 0$. Then the final form of the inflation equation is:

$$\pi_t = \alpha \pi_{t-1} + (1 - \alpha - \beta\eta (\tau - 1) - \kappa (\tau - 1)) \pi_{t+1|t} - \kappa (\tau - 1) \sum_{s=t+2}^{\infty} \pi_{s|t}. \tag{7}$$

Many important effects are neglected in the model, like fiscal policy, or the behavior of wages. One would definitely get richer but also more complicated dynamics with these factors being present, but the story of the nominal exchange rate would remain the same.

Benczúr (2002b) offers a detailed analysis of the convergence and stability properties of such models. Here I only need the following two results. The first is that the conventional choice of $\alpha > 0.5$, $\tau > 1$ leads to stable inflation dynamics. Moreover, the speed of disinflation (its half-life) is very sensitive to and "monotonic" in parameters (like β, κ). This offers the coexistence of large inflation surprises and slow learning.

The second is the recursive method of solving (7). We look for exponential solutions of the form $\pi_t = \pi_0 \lambda^t$. This yields the characteristic equation of the dynamic system. The infinite sum is summable if $|\lambda| < 1$, its value is then $\frac{1}{1-\lambda}$. Multiplying the equation with $(1 - \lambda)$ gives a cubic equation, which should have a single convergent root λ_1. Inflation is therefore $\pi_t = \pi_0 \lambda_1^t$, and all other contemporaneous and future variables can be written in a closed form as well.

3.2 The learning process

For simplicity, assume that the only influence of the central bank over inflation is through the composite real exchange rate channel (the coefficient κ from the reduced form equation). Now suppose that there is a "true" parameter κ in the Phillips curve $\pi_t = \alpha \pi_{t-1} (1 + \varepsilon_t) + (1 - \alpha) \pi_{t+1|t} + \kappa q_t$. Its precise value is not known to the market or the central bank, but they have a common prior distribution. Every period constitutes a new observation for estimating (learning) κ, leading to a common Bayesian update of the distribution of κ. As time goes on, this distribution should converge to the truth.

One can explicitly model this learning process: start from some prior distribution about κ, and a true value. We also need a source of noise, with its distribution. Then it is possible to derive the updating rules. This unfortunately gives only a random variable, a function of the per period realization of the noise. It should in general move towards the true value of κ, unless we have some extreme realizations of the noise. Simulating many potential time profiles, the sample average then describes the average evolution of inflation and the exchange rate (alternatively, one might be able to calculate this expected value explicitly). Note that this average is an expectation conditional on the true κ, so it is not equal to the inflation path expected by agents at any point in time.

A more convenient approach is to work with the certainty equivalent of this learning process. The Appendix shows a formal but incomplete method for defining a single parameter κ_t that matches the true average value of realized inflation, real and nominal exchange rates, and the nominal interest rate: calculating the true expected future real interest rate path and inserting it into real interest parity, the implied average q_t and π_t are equal to the values calculated using κ_t as a single model parameter. For any other variables in the more distant future (q_{t+1}, for example), one should use a different point estimate κ_t'. For such variables, using κ_t gives us a biased, but still reasonable forecast. Forecasted real exchange rates and real interest rates still satisfy the real uncovered interest parity condition.

This means that the market forms a point estimate κ_t of κ, and uses that parameter to obtain its forecast. That does not lead to fully correct expected value calculations in general: in period $t+2$, the expected value of inflation already depends on higher moments of the κ distribution. Using the point estimate implicitly assumes that these higher order terms are relatively small.

Asymptotic learning then requires that $\kappa_t \to \kappa$.[11] We can now insert this parameter sequence into the deterministic model: whenever there is a time t expectation term, expectations are obtained as forecasts, by replacing κ with κ_t.[12] There are two main cases for learning: pessimistic – when the starting value of κ_t is smaller than the truth, and learning means a gradual revision upwards ($\kappa_t \nearrow \kappa$), and optimistic – $\kappa_t \searrow \kappa$. Again, the Appendix contains a formal but not fully complete argument for the certainty equivalent κ_t converging monotonically to the true κ value.

As explained in Section 3.1, if $\alpha > 0.5$ (the backward looking term dominates in the Phillips

[11] Intuitively, it is clear that we have complete asymptotic learning: rewrite the Phillips curve as $\pi_t/\pi_{t-1} - \alpha - (1-\alpha)\,\pi_{t+1|t}/\pi_{t-1} = \kappa q_t/\pi_{t-1} + \varepsilon_t'$. All variables are observed every period, the parameter α is known, and ε_t' is an orthogonal mean zero error term, with a known distribution. As $t \to \infty$, even the OLS estimate of κ from this equation is consistent.

[12] One might want to call them predicted values, instead of expected values. Since κ influences most variables in a nonlinear way, no single, common point estimate can yield unbiased predictions for all variables. I will define κ_t in a way that the prediction of the infinite sum of real interest rates is unbiased, thus the resulting value of q_t (and hence π_t) is consistent with rational expectations.

curve), then the speed of disinflation is increasing in κ. So one would expect that the optimistic case would imply too low expected inflation (too fast expected disinflation), and each observation would push κ down, decreasing the expected speed of further disinflation. Optimistic agents are continuously subject to negative surprises, they keep updating their inflationary and interest rate expectations upwards; and exactly the opposite for the pessimistic case. The next subsection establishes this link between parameter-pessimism and inflation-pessimism.

3.3 Optimism, pessimism and inflation dynamics

What is the effect of parameter uncertainty and learning on the speed of disinflation? One can interpret the question at least two ways. The first concerns the effect of parameter uncertainty relative to full information. The answer is ambiguous: in the pessimistic case, expected inflation is large, but that implies a large real interest rate and also a stronger real exchange rate. The first effect indeed increases inflation (relative to the perfect foresight $\kappa = \kappa_t$ case), but the latter works against it. Everything is flipped in the optimistic case, but we get the same ambiguity.

The following example shows that the real exchange rate effect might dominate in the pessimistic case, implying that too much pessimism initially achieves faster disinflation than perfect foresight. If $\kappa_t \approx 0$, then $\pi_{t+1|t} \approx \pi_{t-1}$, so $\pi_t \approx \pi_{t-1} + \kappa q_t$. On the other hand, $i_{t+j|t} - \phi_{t+j|t} - \pi_{t+j|t} \approx (\tau - 1 - \phi)\,\pi_{t+j|t}$, and inflation converges to zero at a speed of $(1 - o(\kappa_t))^j$, so the sum of the geometric inflation series can be arbitrarily large as $\kappa_t \to 0$. This means that as $\kappa_t \to 0$, $q_t \to \infty$. The inflationary effect of pessimism relates to the coefficient $\lambda = 1$ of π_{t-1} (instead of $\lambda < 1$), and it is finite, while the anti-inflationary effect (the nominal and also the real exchange rate) can be arbitrarily large.

From our purposes, however, the relevant question is whether a smaller than true point estimate κ_t implies a lower than expected inflation. *For a given real exchange rate*, the pessimistic case indeed means that inflation is smaller than expected (due to $\kappa > \kappa_t$). There is, however, a link between inflation and the real exchange rate: using (4), we get

$$\begin{aligned} q_t &= s_t + p^*_{t-1} - p_{t-1} \\ &= p_{t-1} - p^*_{t-1} + \pi_t + \underbrace{(1 + \phi - \tau)\left(\pi_{t+1|t} + \pi_{t+2|t} + \dots\right)}_{I_{t|t}} + p^*_{t-1} - p_{t-1} =: \pi_t + I_{t|t}. \end{aligned}$$

Notice that under reasonable assumptions (cumulative inflation is positive, and $\tau > 1 + \phi$),

$I_{t|t} < 0$. Writing it back to the inflation equation, we get

$$\pi_t = \underbrace{\alpha\pi_{t-1} + (1-\alpha)\,\pi_{t+1|t}}_{J_{t|t}} + \kappa\pi_t + \kappa I_{t|t}$$
$$\pi_t = \frac{J_{t|t} + \kappa I_{t|t}}{1-\kappa}.$$

Similarly

$$q_{t|t} = \pi_{t|t} + I_{t|t}$$
$$\pi_{t|t} = \alpha\pi_{t-1} + (1-\alpha)\,\pi_{t+1|t} + \kappa_t\pi_{t|t} + \kappa_t I_{t|t}$$
$$\pi_{t|t} = \frac{J_{t|t} + \kappa_t I_{t|t}}{1-\kappa_t}.$$

Comparing realized and expected inflation, their relation is still not clear: if $\kappa > \kappa_t$ (pessimism), then π_t has both a smaller numerator and a smaller denominator (and the opposite for $\kappa < \kappa_t$). The relation $\pi_t > \pi_{t|t}$ boils down to

$$\kappa_t\left(\alpha\pi_{t-1} + (1-\alpha)\,\pi_{t+1|t} + I_{t|t}\right) < \kappa\left(\alpha\pi_{t-1} + (1-\alpha)\,\pi_{t+1|t} + I_{t|t}\right).$$

The bracket term equals $\alpha\pi_{t-1} + (1-\alpha)\,\pi_{t+1|t} + q_t - \pi_t = q_t\,(1-\kappa)$, which is negative if $\kappa < 1$ and $q_t < 0$. The realistic order of magnitude for κ is evidently $\kappa < 1$ (one cannot expect a one percentage point immediate decrease in inflation from a real exchange rate being one percentage point stronger than its equilibrium level). If we further assume that the central bank wants to maintain a positive real interest rate all the time (which looks plausible during a disinflation), then the real exchange rate shows an initial appreciation, then a gradual return to zero – thus it is always negative.

This implies that $\kappa_t > \kappa$ is equivalent to $\pi_t > \pi_{t|t}$, and $\kappa_t < \kappa$ to $\pi_t < \pi_{t|t}$. One can easily show that a similar result applies to the $i_t = \tau\pi_t$ choice of the reaction function: the only difference is that $\kappa_t > \kappa$ is equivalent to $|\pi_t| > \left|\pi_{t|t}\right|$, and vice versa. Using the explicit solutions of Section 4.2, one can also give a formal proof for the $i_t = \tau\pi_t$ case. In summary, it is true that parameter-pessimism is also inflation-pessimism: a too low expected κ_t implies a too high expected inflation number. Therefore, a lower than expected inflation number leads to an update of κ_t upwards.

4 The behavior of the realized exchange rate

4.1 Realized exchange rate movements

Let us turn now to realized exchange rate movements:

$$s_t = q_{\infty|t} + p_{t-1} + \left(\pi_t + \pi_{t+1|t} + ...\right) - \left(\left(i_t - \phi_t\right) + \left(i_{t+1|t} - \phi_{t+1|t}\right) + ...\right)$$

$$s_{t+1} = q_{\infty|t+1} + p_t + \left(\pi_{t+1} + \pi_{t+2|t+1} + ...\right) - \left(\left(i_{t+1} - \phi_{t+1}\right) + \left(i_{t+2|t+1} - \phi_{t+2|t+1}\right) + ...\right)$$

$$s_{t+1} - s_t = \underbrace{i_t - \phi_t}_{s_{t+1|t} - s_t} + \underbrace{q_{\infty|t+1} - q_{\infty|t}}_{\substack{\text{change in the "percieved"} \\ \text{equilibrium real exchange rate}}} + \underbrace{\pi_{t+1} - \pi_{t+1|t}}_{\substack{\text{inflation surprise having} \\ \text{no effect on interest rates}}} + \underbrace{\sum_{i=0}^{\infty}\left(\phi_{t+1+i|t+1} - \phi_{t+1+i|t}\right)}_{\text{change in risk premia}}$$
$$- \underbrace{\sum_{i=0}^{\infty}\left(i_{t+1+i|t+1} - i_{t+1+i|t}\right)}_{\substack{\text{change in the entire} \\ \text{time profile of interest rates}}} + \underbrace{\sum_{i=1}^{\infty}\left(\pi_{t+1+i|t+1} - \pi_{t+1+i|t}\right)}_{\substack{\text{change in the long-run} \\ \text{nominal exchange rate}}}.$$

Suppose first that there is no risk premia, or at least, it is constant (see Section 5 for a detailed investigation of the risk premium term). Then the nominal exchange rate changes by the (riskless) interest rate, the change in the expected (or perceived) equilibrium value of the real exchange rate, plus the cumulative effect of inflation and interest rate surprises.

We see that a surprise in the equilibrium real exchange rate moves the nominal exchange rate one in one, unless it also influences the inflation and interest rate process. If it is only the deviation from the equilibrium real exchange rate that matters for inflation and the output gap, and interest rates depend only on inflation and output, than there is no indirect effect of a surprise to $q_{\infty|t}$: looking back at the full macromodel (6), we find that wherever q_t appears, it corresponds to the deviation from $q_{\infty|t}$. The real interest parity condition in fact also contains a hidden $q_{\infty|t}$ on the right hand side, which means that a change in $q_{\infty|t}$ has no effect on $q_t - q_{\infty|t}$, π_t, and all other included variables. Consequently, a change in $q_{\infty|t}$ moves the absolute level of the real exchange rate, hence also the nominal exchange rate, one in one, and there is no indirect effect. For this reason, I will concentrate on the more complex, hence more interesting case of inflation surprises, and assume that $q_{\infty|t} \equiv 0$.

In the pessimistic case, the sum of inflation differentials is negative. If we assume that the central bank reacts more than one in one to future inflation, then the interest rate sum is also negative, and its absolute value is larger than that of the inflation sum. Altogether, these two terms act as a surprise monetary easing, leading to an even larger depreciation of the currency than implied by i_t.

There is a "free" inflation surprise term as well, corresponding to period $t+1$. This surprise, due to the assumption on the reaction function, does not imply any interest rate change. Should this term dominate the total effect of the two infinite sums, then the nominal depreciation will be smaller than i_t. If, however, we further assume that the time t real interest rate is at least as large as the risk premium, then $i_t - \phi_t - \pi_{t+1|t} > 0$. This implies that there is still a nominal depreciation, not less than realized inflation and the total (positive) effect of the two infinite sums.

If we assume that i_t depends on π_t (and not on $\pi_{t+1|t}$), then we do not have this extra "free" term, because $i_{t+1} - i_{t+1|t}$ is exactly proportional to this surprise. Therefore, in the pessimistic case, we always have a steady weakening of the currency after the initial appreciation, usually even on top of i_t.

A similar argument shows that everything is reversed in the optimistic case: inflation is higher than expected (predicted), so the two infinite sums are positive, and their joint effect is negative. Due to negative inflation surprises, there is an overall restrictive monetary surprise, so the nominal depreciation is smaller than implied by the nominal interest rate (assuming that the "free" term does not dominate the infinite sums).

At a first glance, the first year of the current Hungarian disinflation episode seems to correspond to the pessimistic case: inflation was decreasing faster than expected by market participants (based on Reuters poll observations), maybe even faster than predicted by the central bank. This would offer no immediate remedy for the persistently strong nominal exchange rate.

There are, however, signs of the optimistic case as well: initial bank forecasts were using a high parameter of exchange rate pass-through, which, according to the forecasting framework, implied a fast disinflation. By the end of 2001, these estimates were under a downward revision, in spite of better than expected inflation data for 2001, thus attributing a large part of the success to favorable exogenous shocks. The "deterministic" part of inflation – the one affected by monetary policy – may actually have shown the symptoms of the optimistic case.

4.2 Operationalizing the small macromodel framework

Given an initial π_{t-1}, inflation is determined by the conditional-expectations Phillips curve:

$$\pi_t = \alpha\pi_{t-1} + (1-\alpha)\,\pi_{t+1|t} + \kappa q_t.$$

Working with the $i_t = \tau\pi_t$ reaction function, the real interest parity condition becomes

$$q_t = q_{t+1|t} - \left(i_t - \pi_{t+1|t}\right) = q_\infty - \tau\pi_t - (\tau - 1)\sum_{s=t+1}^{\infty}\pi_{s|t} = -\tau\pi_t + (1-\tau)\sum_{s=t+1}^{\infty}\pi_{s|t},$$

so

$$\pi_t = \alpha\pi_{t-1} + (1 - \alpha - \kappa(\tau - 1))\,\pi_{t+1|t} - \kappa\tau\pi_t - \kappa(\tau - 1)\sum_{s=t+2}^{\infty}\pi_{s|t}.$$

Market expectations (predictions) are formed through a similar equation, but with κ replaced by κ_t:

$$\pi_{t|t} = \alpha\pi_{t-1} + (1 - \alpha - \kappa_t(\tau - 1))\,\pi_{t+1|t} - \kappa_t\tau\pi_{t|t} - \kappa_t(\tau - 1)\sum_{s=t+2}^{\infty}\pi_{s|t}.$$

The process $\pi_{s|t}$ thus follows a perfect foresight small macromodel with parameter κ_t. The characteristic equation of this system is

$$\begin{aligned}\lambda(1 + \kappa_t\tau) &= \alpha + (1 - \alpha - \kappa_t(\tau - 1))\,\lambda^2 - \kappa_t(\tau - 1)\frac{\lambda^3}{1-\lambda}\\ 0 &= \alpha - (1 + \alpha + \kappa_t\tau)\,\lambda + (2 - \alpha + \kappa_t)\,\lambda^2 - (1-\alpha)\,\lambda^3.\end{aligned}$$

As explained earlier for the general case, if $\alpha > 0.5$ (the backward-looking term dominates), $\tau > 1$ (active monetary policy), and $\kappa_t \approx 0$ (small magnitude of exchange rate effect), then this equation has two divergent and one convergent roots.[13] Denote the convergent root by $1 - \lambda(\kappa_t)$, which can be computed easily for any given value of α and τ. Therefore, $\pi_{t-1+j|t} = \pi_{t-1}(1 - \lambda(\kappa_t))^j$. We also have

$$q_{t+j|t} = -\tau\pi_{t+j|t} + (1-\tau)\sum_{s=t+1+j}^{\infty}\pi_{s|t}.$$

[13] Due to the extra term $-\tau\pi_t$ in q_t, this remains true even for values of τ slightly below one.

Plug this back to the original Phillips curve:

$$\pi_t = \alpha\pi_{t-1} + (1-\alpha-\kappa(\tau-1))\,\pi_{t+1|t} - \kappa\tau\pi_t - \kappa(\tau-1)\sum_{s=t+2}^{\infty}\pi_{s|t} =$$

$$\alpha\pi_{t-1} + (1-\alpha-\kappa(\tau-1))\,\pi_{t-1}(1-\lambda(\kappa_t))^2 - \kappa\tau\pi_t - \kappa(\tau-1)\sum_{s=t+2}^{\infty}\pi_{t-1}(1-\lambda(\kappa_t))^{s-t+1}$$

$$(1+\kappa\tau)\,\pi_t = \pi_{t-1}\left(\alpha + (1-\alpha-\kappa(\tau-1))(1-\lambda(\kappa_t))^2 + \kappa(\tau-1)\frac{(1-\lambda(\kappa_t))^3}{\lambda(\kappa_t)}\right)$$

$$\pi_t = \pi_{t-1}\left(1-\mu_{t-1}\right).$$

Again, this can be computed easily, thus we can get π_t, q_t given π_{t-1}. Based on the previous discussion, and explicitly shown in the Appendix, the values of π_t, i_t are indeed equal to the conditional expectation of π_t and i_t, where expectation is taken with respect to period t noise (ε_t), but conditional on the true value of κ. Also, q_t and s_t are equal to the real (and nominal) exchange rate coming from interest parity using the correct expectations – but all the other time t and future (expected) variables are biased predictions.

One would then specify various choices of α, τ, and "learning scenarios" $\kappa_t \to \kappa$ (either from above, or from below). That leads to a numerical path of q_t, π_t over time, from which we can infer $p_t = p_{t-1} + \pi_t$ ($p_0 = 0$ from normalization), and finally, the evolution of the nominal exchange rate s_t. The object of interest is its behavior: the size of the initial appreciation, and how much reversal follows.

4.3 Numerical results for the optimistic case

Accepting first the optimistic case, the question is whether one can find reasonable parameter values at which the nominal exchange rate can stay nearly constant. For this, we need large monetary surprises every period, which requires relatively slow disinflation (high persistence – α, and a weak exchange rate channel – κ), large inflation surprises (substantial drops in κ_t each period), and an aggressive reaction function (high τ).

The following choice leads to a particularly good-looking exchange rate behavior: $\alpha = 0.8$, $\tau = 2$, $\kappa_1 = 0.019$, $\kappa_2 = 0.011$, $\kappa_3 = 0.007$, $\kappa_4 = 0.005$, $\kappa_5 = 0.004$, $\kappa_6 = \kappa^{true} = 0.003$. The choice of $\alpha = 0.8$ is in line with the calibration of Svensson (2000), and it implies a large persistence of the inflation process. The Taylor parameter τ is around the "standard" choice of $1.5 - 2$. It is hard to access the values of κ directly, since it is an extremely reduced form parameter. Its quantitative meaning is that a 5% real overvaluation leads to a quarterly disinflation of $10-1.5$ basis points. The best guide is to look at the implied speed of disinflation: the halving time of inflation shocks is around three years, which is not unreasonable.[14] The initial condition of quarterly excess inflation is 1.5%, roughly matching the corresponding Hungarian number of early 2001. The time unit is a quarter of a year. All data is in percentage points, at a quarterly level. The results are depicted on Figure 4.

Before discussing the results, let me emphasize once more that all the future (expected) variables are in fact forecasts, and it is only the one-period ahead forecast of the nominal exchange rate which is constructed to be unbiased. The true expected values would show a similar time profile (decreasing inflation, a depreciating nominal and real exchange rate), but the actual numbers would slightly differ. Those expectations would satisfy the "expected value" version of the interest parity conditions (linking expected exchange rate movements and expected interest rates). Nevertheless, the forecasted variables also satisfy interest parity, but in a "prediction" version, linking predicted exchange rate movements and predicted interest rates.

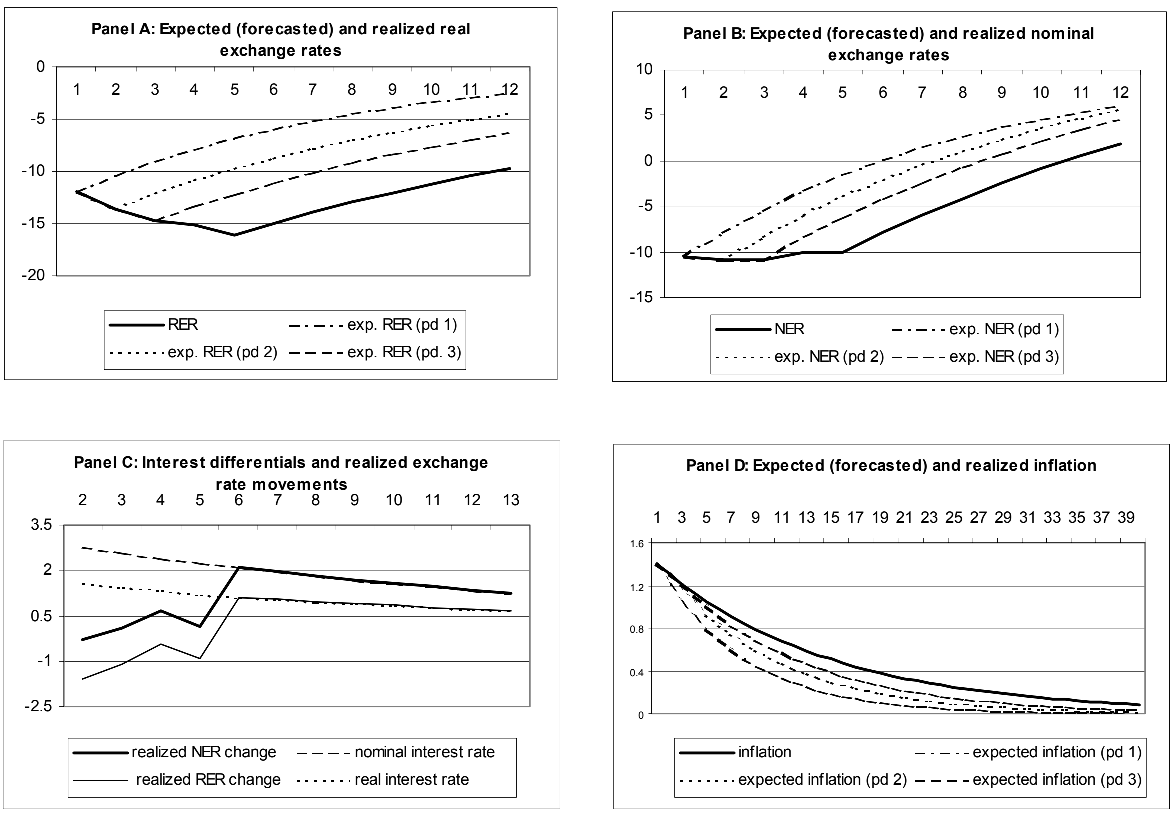

Figure 4: Numerical results for an optimistic learning scenario

Panels A and B show the behavior of the real and nominal exchange rate: after a large

[14] Benczúr, Simon and Várpalotai (2002) discusses the adaptation of a similar macromodel to Hungary.

initial appreciation, though there is an expected nominal and real depreciation every period, realizations show a further real appreciation, and a near steady nominal exchange rate. By construction, both exchange rates switch to the perfect foresight depreciation path after period six, when parameter learning ceases.

Panel C illustrates the deviation from interest parity. From period 2, both exchange rates are expected to depreciate. In periods 2-6, however, there is an inflation surprise every period, thus a shock to the expected (predicted) inflation and interest rate path, and the long run expected nominal exchange rate. Consequently, realized exchange rate movements are systematically lower than interest differentials.

Panel D depicts the speed of disinflation and size of the inflation surprises. Every period, there is an expected (predicted) inflation path starting from the current realization. One period later, the realization is higher than originally expected. This leads to an upward revision of the inflation forecast (based on a lower point estimate of κ). Changes in inflation forecasts lead to changes in interest rate forecasts and $s_{\infty|t}$. This process continues until the true value of κ is learned. In theory, this would require infinite periods, but practically, we can assume that all agents learn κ accurately in a couple of periods, and further updates are negligible.

5 Changing the risk premium

5.1 "Endogenizing" the risk premium

The results so far suggest that in the optimistic case, inflation surprises alone can account for the behavior of the exchange rate, keeping the risk premium fixed. Next I want to explore the implications of a changing risk premium term, particularly for the pessimistic case. The starting point is to clarify the meaning and interpretation of risk premium in the model.

Strictly speaking, the only model-consistent source of uncertainty of domestic bond returns comes from the parameter uncertainty and the noise ε, but the implied exchange rate volatility should not matter for risk neutral investors. One could still think about the premium as a correction term, reflecting some risk aversion. There can also be many further, not modeled sources of risk: liquidity, or default risk, for example. These factors can be taken as exogenous from the viewpoint of the model, so they can be considered as fixed.

A major part of the risk premium, however, is likely to come directly from the disinflation process itself: if its evolution (in terms of speed or costs) substantially differs from predictions, then the central bank may decide to change the regime (or, within the model, it may change the

Taylor coefficients). This means less than perfect credibility, but not necessarily a distrust in the central bank itself: based on new information, an update of model parameters should imply an update of the optimal reaction function, even for the same central bank objective function.

This premium, therefore, represents expected losses from a hypothetical monetary realignment, which takes the economy "out of the model". This event never occurs along the equilibrium path, but its risk is incorporated into the exchange rate every period. It looks plausible that the possibility of such a realignment means an expected loss for investors.

Since the probability of such a realignment decreases as the disinflation process matures, it is reasonable to set the risk premium proportional to inflation. As a starting point, assume that this ratio is a constant: $\phi_{t+i|t} = \phi\pi_{t+i|t}$. Keeping the same reaction function $i_t = \tau\pi_t$ as before, the total effect of inflation, interest rate and risk premium surprises becomes

$$-(\tau - 1 - \phi)\sum\left(\pi_{t+1+i|t+1} - \pi_{t+1+i|t}\right).$$

For a fixed ϕ, it is identical to a less aggressive reaction function, therefore, all previous considerations equally apply here. We saw that, in the optimistic case, a larger τ is more likely to give constant exchange rates, so we do not get any help from the risk premium term.

Note that $\tau - 1 - \phi$ and $\tau - 1$ may have opposite signs, meaning that in the pessimistic case, the no-premium behavior shows a continuous depreciation, while the risk premium behavior implies further appreciation. The problem with this argument is that $\tau - 1 - \phi < 0$ would mean that the riskless real interest rate is *negative* for positive inflation levels, but then the central bank is in fact not following a restrictive policy. This is not very sensible, moreover, if the backward looking term dominates in the Phillips curve, then such a reaction function does not lead to asymptotic disinflation.

Another problem here is that the assumption of $\tau - 1 - \phi \approx 0$ indeed implies near constant exchange rates (*for a fixed inflation path*), but it also eliminates the initial appreciation. Besides, a too mild reaction function leads to slow disinflation, increasing the expected inflation path $\pi_{t+j|t}$, and the effect on the real interest rate $(\tau - 1 - \phi)\pi$ is ambiguous. A less aggressive reaction function might even imply a bigger initial appreciation, and then a severe reversal, if the increase in π dwarfs the decrease in $\tau - 1 - \phi$.

Figure 5 illustrates the implications of a nonzero but fixed risk proportion ϕ, without any parameter learning. Formally, it is equivalent to working with different τ Taylor coefficients, since it is only $\tau - \phi - 1$ that matters. For the parameter choice of $\alpha = 0.8$, $\kappa = 0.003$, the figure contains the results of four different $\tau - \phi$ values: 1.5, 1.3, 1.1 and 1.[15] As transparent from Panel A, there is a reasonable tradeoff between the initial appreciation and the speed of the following depreciation for the *real* exchange rate: decreasing τ leads to a flatter real exchange rate path, but still allows for some initial appreciation.

Panel B, however, shows that this is not the case for the nominal exchange rate: a decrease in τ leaves the slope nearly unchanged, while it decreases the initial appreciation. The assumption of nonzero but fixed ϕ offers no explanation for a persistently strong nominal exchange rate.

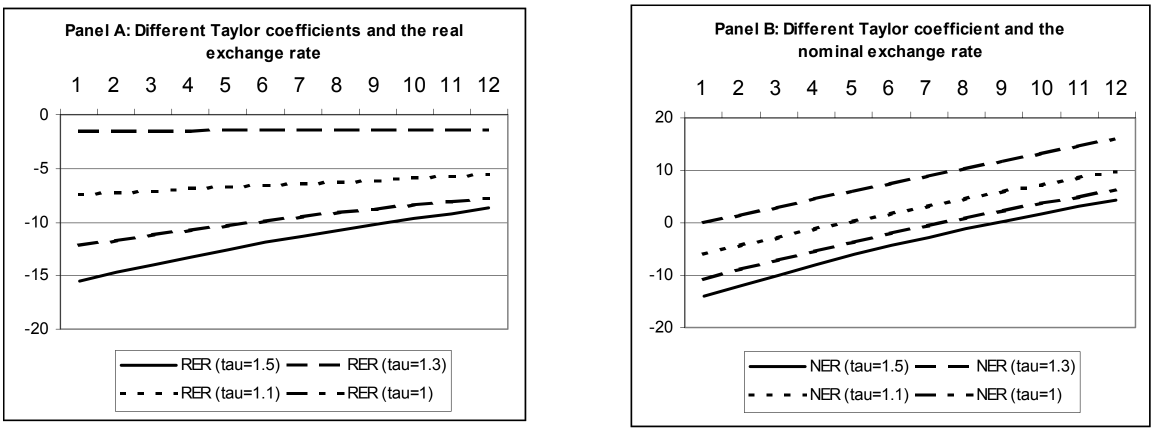

Figure 5: Varying the risk proportion

One can relax the constant proportion assumption in at least two ways. One is that even for a given set of current information, this proportion changes through time: $\frac{\phi_{t+i|t}}{\pi_{t+i|t}} \neq \frac{\phi_{t+j|t}}{\pi_{t+j|t}}$. Put differently, the risk premium content of the current interest rate is not the same as that of tomorrow's. This ratio may be declining (as disinflation continues, inflation gradually declines, decreasing the probability of a monetary correction), increasing (due to parameter uncertainty, there is a "probationary" period at the beginning, when the central bank may tolerate relatively large initial costs, and engage in a correction only later, once it is clear that the large costs are not due to bad luck); so a combination of these effects may imply any arbitrary pattern (for example, an initial increase, reflecting the probationary period, and then a gradual decline).

Another approach could be to postulate $\frac{\phi_{t+i|t}}{\pi_{t+i|t}} = \phi_t$: for given information, the ratio is fixed, but new information brings about a change in the risk proportion. In the optimistic case, it seems plausible that the arrival of negative surprises increases ϕ, and the opposite for the pessimistic case. In a credibility buildup story, we do not even need the extra parameter uncertainty: as

[15] Because of the $i_t = \tau\pi_t$ assumption, $q_t = -\tau\pi_t + (1-\tau)\sum \pi_{s|t}$, which is nonzero even for $\tau = 1$. With this reaction function, one can get a disinflation even for $\tau = 1$, and the critical level is approximately $\tau = 1 - \kappa$.

time goes on, markets subtract less and less from nominal interest rates, since they believe more and more that the bank would indeed stick to that particular reaction function. This again corresponds to a gradual decline of ϕ. The next section explores this possibility.

5.2 The proportional risk premium case

The optimistic scenario comes with a per period increase in the risk content of interest rates, which points to a further weakening of the exchange rate: the effect of the surprise interest rate increase is partly offset by a higher risk content. Even if the overall effect is contractionary, this would still not provide us with a new explanation for the observed exchange rate behavior: negative inflation surprises alone would remain the driving force, and changes in the risk premium would only work against it.

In the pessimistic case, however, the effect of the risk premium is in the desired direction: as a result of better than expected data, the predicted interest rate path is revised downwards, but its risk content also decreases. So the total change in the "riskless" interest rate may be positive. This would constitute an alternative explanation: the inflation surprise itself would point to further depreciation, but the drop in the risk content counters this effect, and keeps the nominal exchange rate constant. Notice that the same argument applies to the credibility interpretation, but without the offsetting effect of the surprise in κ.

Finding realistic structural parameters, learning and risk premium scenarios proved to be a much less fruitful experiment than in the optimistic case with fixed risk content. The major reason is the following. One needs a relatively large drop in the risk content (ϕ_t), which in turn requires a high Taylor coefficient (τ), thus high initial interest rates. This either gives too large inflation surprises (which work against the desired appreciation), or too low inflation (which weakens the effect of the decreasing risk premium). One may produce higher inflation by weakening the real exchange rate channel, but that also leads to a higher sensitivity of inflation to estimates of κ, hence, larger inflation surprises again. It is still possible to balance these effects, and achieve an initial appreciation, followed by 2-3 periods of nearly flat nominal exchange rate,[16] but then it quickly reverts to depreciation.

Figure 6 reports numerical results for the following set of parameters: $\alpha = 0.7$, $\tau = 1.5$, $\phi_1 = 0.4$, $\phi_2 = 0.3$, $\phi_3 = 0.2$, $\phi_4 = 0.1$, $\phi_5 = 0.1$, $\phi_6 = 0.05$, $\phi_7 = \ldots = 0$, $\kappa_1 = 0.003$, $\kappa_2 = 0.0032$, $\kappa_3 = 0.0035$, $\kappa_4 = 0.004$, $\kappa_5 = 0.0045$, $\kappa_6 = \kappa^{true} = 0.005$.

[16] Having substantial drops in ϕ_t for many periods requires a very high τ, thus unrealistically high risky interest rates.

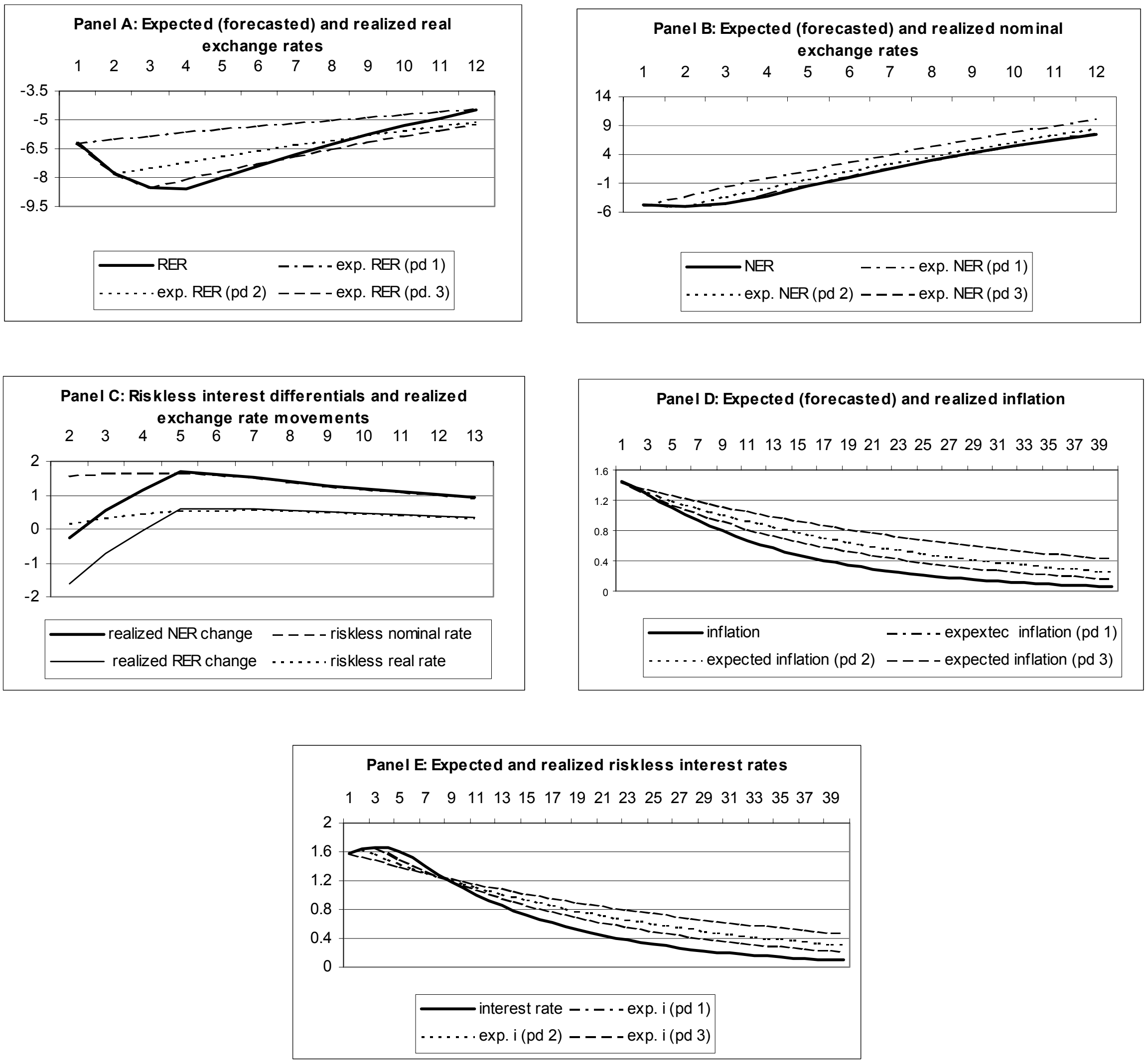

Figure 6: Numerical results for a pessimistic learning scenario with declining risk content

Panels A and B show the evolution of realized and expected (forecasted) exchange rates. The real exchange rate initially appreciates, and then is expected to depreciate slowly (since riskless real interest rates are very low: $\tau - \phi_1 - 1 = 0.1$). In period 2, because of the inflation surprise, realized risky rates are lower than expected, but realized riskless rates are higher, due to our assumption of the surprise drop in the risk premium. This leads to an even stronger real exchange rate, and a slightly appreciating nominal exchange rate. The same applies to period 3, but after that, the change in ϕ_t is too small to reverse the depreciation.

Panel C depicts the violation of uncovered interest parity ex post: instead of the heavy depreciation implied by excess yields, realized exchange rates exhibit some further appreciation, and then return to the perfect foresight path.

Panel D shows the path of realized inflation, and inflation surprises. Risky rates are proportional to inflation (both expected and realized), but due to changes in ϕ_t, the evolution of riskless rates is more complicated. One can see in Panel E that the overall change in riskless rates is initially restrictive (higher than expected), and then it becomes expansionary (lower than expected).

Finally, Figure 7 shows the results for a scenario where there is a per period (unexpected) drop in the risk premium content (ϕ_t), *without* a corresponding downward revision of κ. Its interpretation is that the credibility of the central bank's behavior (reaction function) is building up only gradually.

The figure corresponds to the following parameter values: $\alpha = 0.7$, $\tau = 2$, $\phi_1 = 0.9$, $\phi_2 = 0.7$, $\phi_3 = 0.5$, $\phi_4 = 0.3$, $\phi_5 = 0.1$, $\phi_6 = \phi_7 = \ldots = 0$, $\kappa = 0.005$. The results show no qualitative difference relative to Figure 6, though the nominal exchange rate is even flatter. The major point of this credibility interpretation is that it makes the method applicable for policy simulation: the original explanation with parameter uncertainty is hard to operationalize, since one would need to use the true parameter value in the simulation, and still assume that market participants learn it only gradually. With a gradually increasing credibility, it is more acceptable to "program in" these surprises.

6 Concluding remarks

There are many potential qualifications of the analysis. From the viewpoint of theoretical attractiveness, one could treat the learning effect precisely (either by Monte Carlo simulations for the average realized variables, or proving the missing pieces in the construction of the "certainty equivalent"), or model the source of the risk premium. The first is likely to leave most of the results unaltered; while the second may involve nontrivial interactions between the risk premium and the rest of the model.

An unpleasant feature of my model is the long-term behavior of the nominal exchange rate: plugging back all the "structural" terms, the long-term nominal exchange rate is determined by the long-term equilibrium real exchange rate (equilibrium real appreciation, plus a potential initial undervaluation), and the cumulative total inflation. If we are above structural inflation, then the long run nominal exchange rate can at most reflect the initial undervaluation of the real exchange rate – and markets seem to expect a larger long-run appreciation.

One can of course drive inflation below the structural level (thus get a long-run appreciation), in fact, the EMU criteria is likely to involve such a decision. This requires some modification

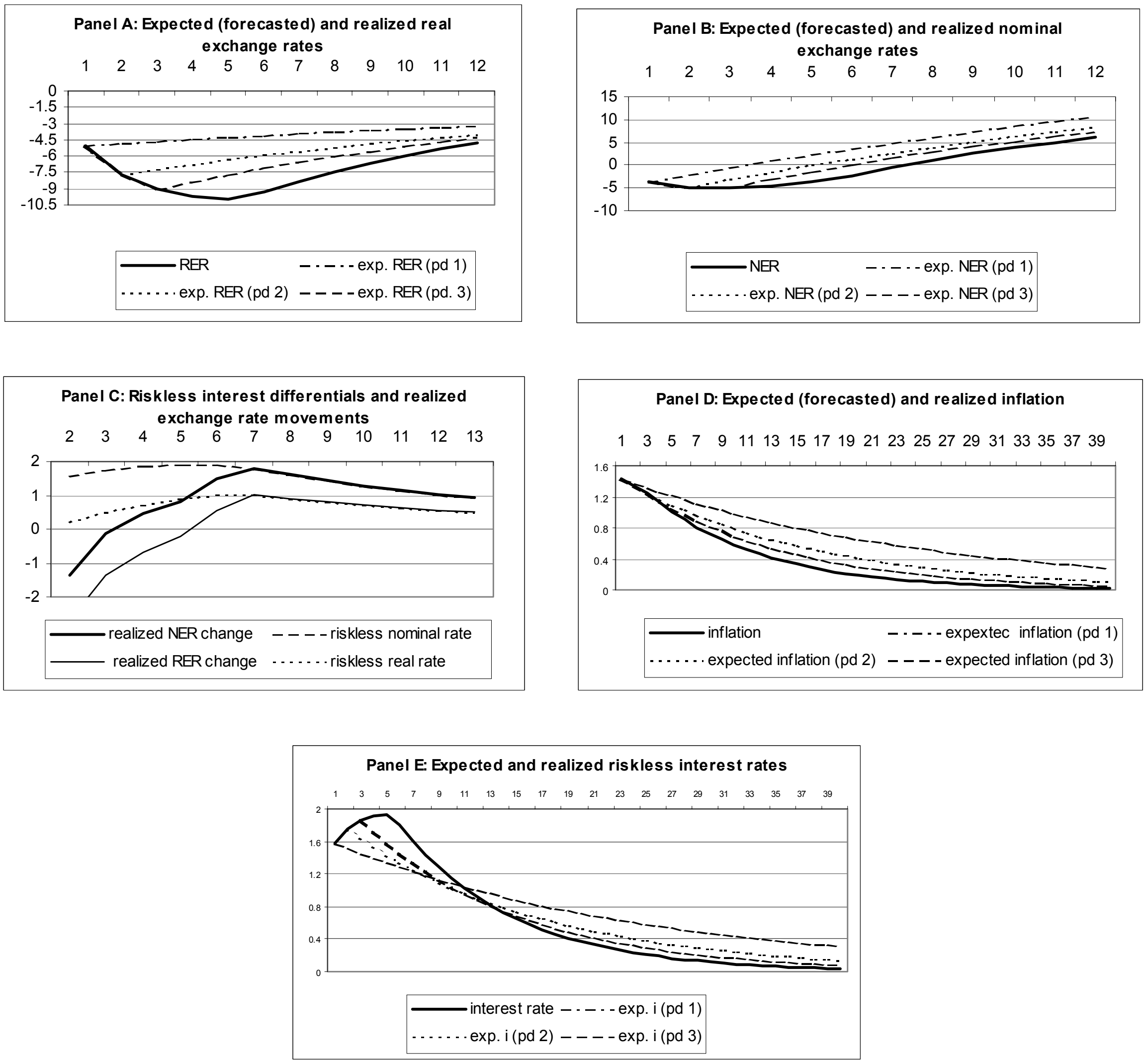

Figure 7: Increasing credibility: unexpected drops in the risk premium content

of the reaction function: $i = \tau\pi$ would make little sense, because a negative π would imply a negative (below equilibrium) level of the nominal exchange rate. Instead, one can use $i = \tau\left(\pi - \pi^{tar}\right)$, with $\pi^{tar} < 0$.

Besides this point about parameter uncertainty and learning, the actual behavior of nominal exchange rates around disinflations are likely to be influenced by a large number of additional factors. Some of them might not be restricted to disinflations: for example, Benczúr (2002a) estimates a bond pricing version of interest parity in a developing country sample, of the form

$$i_t = \alpha + \beta i_t^* + \lambda \triangle e_{t+1}^e,$$

and finds a significantly less than one coefficient of the benchmark interest rate ($\beta \approx 0$), but a near one coefficient of the exchange rate risk ($\lambda \approx 1$). Since here we keep foreign rates roughly fixed, such a modification can give at most a "level shift" of the exchange rate profile, but not a "slope shift".

Can we extract market expectations directly – from Reuters polls, for example? Then one could check whether UIP holds with measured expectations. If yes, then one should try to explain why it makes sense to have biased forecasts and trade based on them, or whether the bias is only an "ex post bias".[17] If it fails, then we need even further explanations, like slow capital inflows, or EMU speculation (see below).

If capital inflows respond slowly to extra gains (which is in fact related to a changing risk premium explanation) – one might get exactly the desired outcome. This is already more specific to a disinflation: there is a foreseen medium-term excess yield on the currency, and still, it is arbitraged away only slowly. It would be very interesting to make such a story consistent and rational, for example, with costly portfolio rebalancing.

Finally, in the case of Hungary and Poland (and other former or current EU accession countries), the level at which the country might join the monetary union offers an extra scope for exchange rate dynamics: at entry, there will be a "forever fixed" exchange rate, the level of which is uncertain. Moreover, the evolution of the "free market" exchange rate might influence this entry parity. Understanding and modeling this interplay should be a key research topic. Though this explanation may even be a dominant determinant of exchange rate behavior in EU accession countries – it cannot apply to Chile.

7 Appendix: the "certainty equivalent" of the realized exchange rate and inflation

I will illustrate the procedure for a simplified Phillips curve specification:

$$\pi_t = \pi_{t-1} (1 + \varepsilon_t) + \kappa q_{t-1.}$$

Inflation equals last period's inflation, a random shock proportional to past inflation, and the disinflating effect of an overvalued real exchange rate (of the previous period). The assumptions of full inflation inertia and a lag in the effect of the real exchange rate simplify the derivations and the signal extraction problem, while the proportionality of the noise makes the entire system

[17] Gourinchas and Tornell (2001) address the issue of ex ante misperceptions about future interest rates, which can be translated into misperceptions about future exchange rates.

scale-invariant.

The real exchange rate is determined by real interest parity:

$$q_{t-1} = E_{t-1}[q_t] - (i_{t-1} - E_{t-1}\pi_t) = E_{t-1}[q_t] - \tau\pi_{t-1} + E_{t-1}[\pi_t]. \tag{8}$$

Assume that κ can take two values, $\kappa_H > \kappa_L$.[18] As of time $t-1$, the current (posterior) distribution of κ is binary, with $P_{t-1}(\kappa = \kappa_L) = \lambda_{t-1}$, $P_{t-1}(\kappa = \kappa_H) = 1-\lambda_{t-1}$. After observing π_t, all agents (market participants and the central bank as well) update their distribution for κ, which can be summarized in the probability $\lambda_t(\lambda_{t-1}, \pi_{t-1}, \pi_t, q_{t-1})$.

Let us look for a solution of the form $q_{t-1} = f(\lambda_{t-1})\pi_{t-1}$. Then $\pi_t = \pi_{t-1}(1+\varepsilon_t) + \kappa f(\lambda_{t-1})\pi_{t-1}$, so $E_{t-1}[\pi_t] = \pi_{t-1} + E_{t-1}[\kappa] f(\lambda_{t-1})\pi_{t-1}$. Plugging everything into (8):

$$\begin{aligned}
f(\lambda_{t-1})\pi_{t-1} &= E_{t-1}[f(\lambda_t(\lambda_{t-1}, \pi_{t-1}, \pi_t))(\pi_{t-1} + \pi_{t-1}\varepsilon_t + \kappa f(\lambda_{t-1})\pi_{t-1})] \\
&\quad - \tau\pi_{t-1} + \pi_{t-1} + E_{t-1}[\kappa] f(\lambda_{t-1})\pi_{t-1} \\
f(\lambda_{t-1}) &= E_{t-1}[f(\lambda_t(\lambda_{t-1}, \pi_{t-1}, \pi_t))(1 + \varepsilon_t + \kappa f(\lambda_{t-1}))] - \tau + 1 + E_{t-1}[\kappa] f(\lambda_{t-1}). \qquad (9)
\end{aligned}$$

I will show that $\lambda_t(\lambda_{t-1}, \pi_{t-1}, \pi_t) = \lambda_t(\lambda_{t-1}, \varepsilon_t, \kappa)$ – that is, the level of inflation cancels from the inference problem. This is the consequence of the proportional noise assumption. Then (9) gives an equation for the function f :

$$\begin{aligned}
f(\lambda_{t-1}) &= \lambda_{t-1} E_\varepsilon[f(\lambda_t(\lambda_{t-1}, \varepsilon, \kappa_L))(1 + \varepsilon + \kappa_L f(\lambda_{t-1}))] \\
&\quad + (1-\lambda_{t-1}) E_\varepsilon[f(\lambda_t(\lambda_{t-1}, \varepsilon, \kappa_H))(1 + \varepsilon + \kappa_H f(\lambda_{t-1}))] \\
&\quad - \tau + 1 + (\lambda_{t-1}\kappa_L + (1-\lambda_{t-1})\kappa_H) f(\lambda_{t-1}). \qquad (10)
\end{aligned}$$

Specifying the distribution of ε, one can form the two expected values. For this, we also need the learning rules. Using Bayes' rule:

$$\begin{aligned}
&\lambda_t(\lambda_{t-1}, \pi_t, \pi_{t-1}) \\
&= P(\kappa = \kappa_L | \lambda_{t-1}, \pi_t, \pi_{t-1}) = \frac{P_{t-1}(\kappa = \kappa_L, \pi_t | \pi_{t-1})}{P_{t-1}(\pi_t | \pi_{t-1})} \\
&= \frac{P_{t-1}(\pi_t | \pi_{t-1}, \kappa = \kappa_L) P_{t-1}(\kappa = \kappa_L | \pi_{t-1})}{P_{t-1}(\kappa = \kappa_L | \pi_{t-1}) P_{t-1}(\pi_t | \pi_{t-1}, \kappa = \kappa_L) + P_{t-1}(\kappa = \kappa_H | \pi_{t-1}) P_{t-1}(\pi_t | \pi_{t-1}, \kappa = \kappa_H)}.
\end{aligned}$$

By definition, $P_{t-1}(\kappa = \kappa_L | \pi_{t-1}) = \lambda_{t-1}$ and $P_{t-1}(\kappa = \kappa_H | \pi_{t-1}) = 1 - \lambda_{t-1}$. Plugging this

[18] The general case could be treated in a similar though more complicated way. One should then derive the posterior density of κ, and all functions of the probability λ_{t-1} would become functionals of the pdf $\lambda_{t-1}(x)$.

back:

$$\lambda_t = \frac{\lambda_{t-1} P_{t-1}(\pi_t|\pi_{t-1}, \kappa = \kappa_L)}{\lambda_{t-1} P_{t-1}(\pi_t|\pi_{t-1}, \kappa = \kappa_L) + (1-\lambda_{t-1}) P_{t-1}(\pi_t|\pi_{t-1}, \kappa = \kappa_H)}$$
$$= \frac{\lambda_{t-1}}{\lambda_{t-1} + (1-\lambda_{t-1}) \frac{P_{t-1}(\pi_t|\pi_{t-1}, \kappa=\kappa_H)}{P_{t-1}(\pi_t|\pi_{t-1}, \kappa=\kappa_L)}}. \quad (11)$$

The two probabilities can be calculated by recalling $\pi_t = \pi_{t-1} + \pi_{t-1}\varepsilon_t + \kappa f(\lambda_{t-1}) \pi_{t-1}$: then $P_{t-1}(\pi_t|\pi_{t-1}, \kappa = \kappa_x) = P\left(\varepsilon_t = \frac{\pi_t - \pi_{t-1} - \kappa_x f(\lambda_{t-1})\pi_{t-1}}{\pi_{t-1}}\right) = P\left(\varepsilon_t = \frac{\pi_t}{\pi_{t-1}} - 1 - \kappa_x f(\lambda_{t-1})\right)$. If the density function of ε is f_ε, then this probability becomes $f_\varepsilon\left(\frac{\pi_t}{\pi_{t-1}} - 1 - \kappa_x f(\lambda_{t-1})\right)$.

As of time $t-1$, $\pi_t = \pi_{t-1}(1 + \varepsilon_t + \kappa f(\lambda_{t-1}))$. For a given κ, λ_t is the following function of ε_t (thus a random variable):

$$\lambda_t(\lambda_{t-1}, \pi_{t-1}, \varepsilon_t, \kappa) = \frac{\lambda_{t-1}}{\lambda_{t-1} + (1-\lambda_{t-1}) \frac{f_\varepsilon(\varepsilon_t + (\kappa - \kappa_H) f(\lambda_{t-1}))}{f_\varepsilon(\varepsilon_t + (\kappa - \kappa_L) f(\lambda_{t-1}))}} = \lambda_t(\lambda_{t-1}, \varepsilon_t, \kappa). \quad (12)$$

Then the unconditional distribution of λ_t (without the assumption of a fixed κ) is the following: it is equal to $\lambda_t(\lambda_{t-1}, \varepsilon, \kappa_L)$ with probability $f_\varepsilon(\varepsilon)\lambda_{t-1}$, and $\lambda_t(\lambda_{t-1}, \varepsilon, \kappa_H)$ with $(1-\lambda_{t-1}) f_\varepsilon(\varepsilon)$. This establishes the properties of the learning rule which were necessary for deriving (10).

Instead of trying to derive f for a given distribution of ε (say, a mean zero normal), let us proceed by defining the certainty equivalent of the learning process. The number $f(\lambda_{t-1})$ is equal to $\frac{q_{t-1}}{\pi_{t-1}}$. Iterating interest parity yields

$$q_{t-1} = q_{\infty|t} - \tau\pi_{t-1} + (1-\tau)\sum_{s=t}^{\infty} \pi_{s|t-1} = -\tau\pi_{t-1} + (1-\tau)\sum_{s=t}^{\infty} \pi_{s|t-1}.$$

It looks plausible that a higher λ_{t-1}, which means a weaker real exchange rate effect, leads to a slower disinflation, hence, a stronger (more negative, so smaller) initial q_{t-1}. It implies that the function f should be decreasing in λ. One would expect that this feature could be proven in general, but I have not succeeded. The intuitive logic, however, I find clear and convincing.

From this monotonicity, we get that $f(\lambda)$ is between $f(1)$ and $f(0)$. Let us look at these two extreme values: both correspond to a degenerate distribution of κ, being equal to κ_H or κ_L, without further learning. It is then easy to derive the corresponding values of f: if $\kappa = \kappa^*$ for sure, than we are back to a perfect foresight equilibrium with mean zero noise, which cancels from any expected value calculation. For a given level of κ^*, let us denote the corresponding value of f by $g(\kappa^*) = g$. Then

$$q_{t-1} = g\pi_{t-1} = gE_{t-1}[\pi_t] - \tau\pi_{t-1} + E_{t-1}[\pi_t] = g(\pi_{t-1} + \kappa^* g\pi_{t-1}) - \tau\pi_{t-1} + \pi_{t-1} + \kappa^* g\pi_{t-1}$$
$$0 = k^* g^2 + 1 - \tau + \kappa^* g$$
$$g_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{\tau - 1}{\kappa^*}}.$$

From the two roots, we must pick the one which is consistent with $E_{t-1}\pi_\infty = 0$. Substituting $q_{t-1} = g\pi_{t-1}$ back into the Phillips curve gives us

$$E_{t-1}\pi_t = \pi_{t-1}(1 + \kappa^* g).$$

So we must pick a root between $\frac{-2}{\kappa^*}$ and 0. It is clear that the bigger root is positive (using $\tau > 1$), while the smaller root is less than -1. It is straightforward to check that it stays above the lower bound as long as $\kappa^* < \frac{4}{1+\tau}$, and this holds for any reasonable choice of τ and κ^*.

All this implies that $g(\kappa^*) = -\frac{1}{2} - \sqrt{\frac{1}{4} + \frac{\tau-1}{\kappa^*}}$, and $f(0) = g(\kappa_H)$, $f(1) = g(\kappa_L)$. Since $f(\lambda)$ is decreasing, and g is increasing, there is a unique value of κ_{t-1} for every λ_{t-1} such that $f(\lambda_{t-1}) = g(\kappa_{t-1})$. This means that one can calculate the realized real exchange rate based on κ_{t-1}, instead of calculating all the complicated expected values for λ_{t-1}. Then κ_{t-1} is a certainty equivalent of λ_{t-1}: picking κ_{t-1} as a point estimate of κ, and forecasting all future variables with $\kappa = \kappa_{t-1}$ and no uncertainty, leads to $q_{t-1} = g(\kappa_{t-1})\pi_{t-1} = f(\lambda_{t-1})\pi_{t-1}$. Though all the predicted future real exchange rates, interest rates and inflation figures are biased predictors, κ_{t-1} is picked in such a way that the resulting q_{t-1} equals the correct value.

To define our final version of the certainty equivalent, we need to do one extra step. Our model starts at time $t-1$, with π_{t-1} and a prior distribution for κ, summarized by λ_{t-1}, and an assumption about the true value of $\kappa = \kappa^*$. The time t values of π_t and q_t are random variables: they depend on ε_t. What we want to calculate is the average value of π_t and q_t for different realizations of ε. Formally, these averages are equal to

$$\widetilde{\pi}_t = E_\varepsilon[\pi_{t-1}(1 + \varepsilon_t + \kappa^* q_{t-1})] = \pi_{t-1} + \kappa^* q_{t-1}$$

and

$$\widetilde{q}_t = E_\varepsilon[f(\lambda_t(\lambda_{t-1}, \varepsilon_t, \kappa^*))\pi_{t-1}(1 + \varepsilon_t + \kappa^* q_{t-1})]$$
$$= E_\varepsilon[f(\lambda_t(\lambda_{t-1}, \varepsilon_t, \kappa^*))](\pi_{t-1} + \kappa^* q_{t-1}) + E_\varepsilon[f(\lambda_t(\lambda_{t-1}, \varepsilon_t, \kappa^*))\varepsilon_t]\pi_{t-1}.$$

Now define κ_t such that $\widetilde{q_t} = g(\kappa_t)\widetilde{\pi}_t$. It means that if we have the *average realization of* π_t – which is easy to get, given π_{t-1}, κ^* and λ_{t-1}–, then we can calculate *the average realization of* q_t *by using* κ_t. Having calculated the average realization as of time t, we restart the system from this average point.

Finally, we need that if $\kappa^* = \kappa_H$ (which corresponds to our pessimistic learning case: the true effect is larger, so true disinflation is faster than originally thought), then $\kappa_{t-1} < \kappa_t$ holds. This looks plausible: when the true real exchange rate effect is stronger than currently expected, then new data dominantly move the distribution of κ upwards. This higher distribution leads to a faster expected disinflation path, therefore a weaker real exchange rate (relative to inflation). This argument suggests that the average of $\frac{q_t}{\pi_t}$ should be smaller than $f(\lambda_{t-1})$.

What we need, however, is not the average of $\frac{q_t}{\pi_t}$, but the ratio of the two averages. The difference between the two can be seen by noting that $\kappa_{t-1} < \kappa_t$ is equivalent to $g(\kappa_{t-1}) = f(\lambda_{t-1}) < g(\kappa_t) = \widetilde{q_t}/\widetilde{\pi}_t$. Rearranging:

$$
\begin{aligned}
f(\lambda_{t-1})(\pi_{t-1} + \kappa_H q_{t-1}) &< E_\varepsilon[f(\lambda_t(\lambda_{t-1}, \varepsilon_t, \kappa_H))\,\pi_{t-1}(1 + \varepsilon_t + \kappa_H f(\lambda_{t-1}))] \\
f(\lambda_{t-1})(1 + \kappa_H f(\lambda_{t-1})) &< E_\varepsilon[f(\lambda_t(\lambda_{t-1}, \varepsilon_t, \kappa_H))](1 + \kappa_H f(\lambda_{t-1})) \\
&\quad + E_\varepsilon[f(\lambda_t(\lambda_{t-1}, \varepsilon_t, \kappa_H))\,\varepsilon_t].
\end{aligned}
$$

The number $1 + \kappa_H f(\lambda_{t-1})$ is the average realized speed of disinflation – which is less than one, but since it is a quarterly speed, by not too much. All the $f(\lambda)$ terms are in absolute values around or greater than one, while the expected value of the product term $(\varepsilon f(\lambda_t))$ is in general much smaller (though can be shown to be negative). Note that $E_\varepsilon[f(\lambda_t)|\kappa_H] = E_\varepsilon[\frac{q_t}{\pi_t}|\kappa_H]$ – the average of the ratio of q_t and π_t. The certainty equivalent κ_t was defined to match the ratio of the averages. We can see that the two are quite close to each other, and the comparison $\kappa_{t-1} < \kappa_t$ is closely related to $f(\lambda_{t-1}) < E_\varepsilon[f(\lambda_t)|\kappa_H]$. Again, though I could not prove formally that $\kappa^* = \kappa_H$ implies that κ_t is increasing, it looks plausible that it holds, at least if the starting distribution is not too far from the truth.

This property would imply that the average evolution of our variables in the pessimistic learning case is equivalent to some increasing time path of $\kappa_t \to \kappa_H$. In principle, the distribution of ε and the starting value κ_0 fully fixes the time path of κ_t – but for any specified values of $\kappa_1 < \kappa_2 < \ldots < \kappa_T$, there is in general a distribution of ε that leads to this evolution of κ_t (until time T): select a family of distributions with at least T free parameters (say, moments), then we get T equations with T unknowns. Unless the distribution family is degenerate, these T equations should have full rank, so there should be a set of parameters yielding exactly the prescribed evolution of κ_t.

There is an alternative, less general but more manageable and fully precise way to define the series $\{\kappa_t\}$. Starting from a given level of inflation, the average realized inflation next period coincides with the realized inflation corresponding to $\varepsilon_t = 0$. So let us consider this model realization: though the noise in inflation is incorporated into parameter estimates, the realization of this noise is always zero. Define κ_t as the certainty equivalent of $\lambda_t = \lambda_t(\lambda_{t-1}, \pi_{t-1}, \varepsilon_t = 0)$ following the procedure described earlier. In this case, it is quite simple to establish the desired monotonicity property of κ_t: all we need is that zero is the most likely realization of the noise ($f_\varepsilon(0) > f_\varepsilon(x)$ for all $x \neq 0$ – which holds for a normally distributed noise, for example).

For the proof, suppose that $\kappa^{true} = \kappa_L < \kappa_{t-1}$. Then from (12), we have

$$\lambda_t = \frac{\lambda_{t-1}}{\lambda_{t-1} + (1-\lambda_{t-1})\frac{f_\varepsilon((\kappa_L-\kappa_H)f(\lambda_{t-1}))}{f_\varepsilon(0)}}.$$

This is greater than λ_{t-1} exactly when

$$f_\varepsilon(0) > f_\varepsilon((\kappa_L - \kappa_H) f(\lambda_{t-1})). \tag{13}$$

Since $f(\lambda_{t-1})$ is nonzero (negative), the argument of the right hand side is nonzero (positive), so (13) is valid. This means that $\lambda_t > \lambda_{t-1}$, thus $\kappa_t < \kappa_{t-1}$. A straightforward modification of this argument shows that if $\kappa^{true} = \kappa_H > \kappa_{t-1}$, then $\kappa_t > \kappa_{t-1}$. This establishes the monotonicity of $\{\kappa_t\}$, though in a more restricted interpretation of the certainty equivalent.

Let us now check the implications of this certainty equivalent approach for the validity of interest parity. We do know that the average observed real exchange rate follows the path that we determine with the certainty equivalent. Average realized real interest rates, on the other hand, are not precisely equal to the values obtained with κ_t. Denote the average realization of any variable x by $\widetilde{x}$, and the certainty equivalent (predicted) value, which is calculated by using κ_t, of x by x^f. Then

$$\begin{aligned}
\widetilde{r_t} &= \widetilde{i_t} - \widetilde{\pi_{t+1|t}} = \tau\widetilde{\pi}_t - \pi_t \widetilde{(1 + E_t[\kappa]q_t)} \\
&= \tau\pi_t^f - E_\varepsilon[\pi_t(1 + \lambda_t(\varepsilon)\kappa_L q_t + (1-\lambda_t(\varepsilon))\kappa_H q_t)] \\
&= i_t^f - \pi_t(1 + \kappa_H q_t + (\kappa_L - \kappa_H) q_t E_\varepsilon[\lambda_t(\varepsilon)]) \\
&\neq i_t^f - \pi_t(1 + \kappa_t q_t) = i_t^f - \pi_{t+1|t}^f = r_t^f.
\end{aligned}$$

However, if we get a flat initial real exchange rate, or a further real appreciation following the initial strengthening, we can be sure that interest parity is violated ex post, since $\widetilde{r_t} > 0$ remains true, so the real exchange rate was expected to depreciate. The numerical difference between r_t^f and the realized real exchange rate movement, however, is not equal to the average ex post deviation from real interest parity.

For the nominal interest rate, we are on safe ground: $\widetilde{i_t} = \tau\widetilde{\pi_t} = \tau\pi_t^f = i_t^f$. One also needs to check the certainty equivalent values of the nominal exchange rate, s :

$$\widetilde{s_t} = \widetilde{q_t + p_t} = \widetilde{q_t} + p_0 + \sum_{s=1}^{t} \widetilde{\pi_s} = q_t^f + p_0 + \sum_{s=1}^{t} \pi_s^f = s_t^f.$$

This means that the difference between realized nominal exchange rate movements and i_t^f is equal to the average deviation from nominal interest parity.

Finally, let me illustrate the extra difficulties this approach would be facing if there was any simultaneity in the Phillips curve. Suppose that the system is given by

$$\begin{aligned} \pi_t &= \pi_{t-1}(1+\varepsilon_t) + \kappa q_t \\ q_t &= q_{t+1|t} - \tau\pi_t + \pi_{t+1|t}. \end{aligned}$$

Again, we want to look for a solution of the form $q_t = f(\lambda_t)\pi_t$: since q_t depends directly on π_t, it will be determined using the updated distribution for κ. Then

$$\begin{aligned} \pi_t &= \pi_{t-1}(1+\varepsilon_t + \kappa f(\lambda_t)\pi_t) \\ \pi_t &= \frac{\pi_{t-1}(1+\varepsilon_t)}{1-\kappa f(\lambda_t(\lambda_{t-1},\varepsilon_t))}. \end{aligned}$$

The signal extraction problem leads to the same expression as in (11). The conditional probability $P = P(\pi_t|\pi_{t-1}, \lambda_{t-1}, \kappa^*)$, however, will turn out to be problematic:

$$P = P\left(\frac{\pi_{t-1}(1+\varepsilon_t)}{1-\kappa f(\lambda_t)} = \pi_t\right).$$

The issue is that λ_t also contains ε_t, so the realization π_t is a complicated function of ε_t. For a given π_t, there should be a unique ε_t being compatible with a fixed value of κ^*, but this one to one mapping depends on the updating rule $\lambda_t(\lambda_{t-1}, \varepsilon_t, \kappa^*)$. So in the expression for the learning rule, $\lambda_t(\lambda_{t-1}, \varepsilon_t, \kappa^*)$, both sides will contain this function.

In fact, we were having a similar issue for the $\pi_t = \pi_{t-1}(1+\varepsilon_t) + \kappa q_{t-1}$ Phillips curve as well: $\lambda_t(\lambda_{t-1}, \varepsilon_t, \kappa^*)$ contained the real exchange rate function $f(\lambda_{t-1})$, and the equation for $f(\lambda_{t-1})$

contained $\lambda_t(\lambda_{t-1}, \varepsilon_t, \kappa^*)$. There, however, the system was recursive: we could substitute the learning rule into the interest parity equation and get a single equation defining $f(\lambda_{t-1})$, and then plug the result back into the learning rule. Here, the two equations are not recursive.

All this means that it would be quite hard to prove all the necessary properties of the certainty equivalent approach for a forward looking and highly simultaneous Phillips curve, like $\pi_t = (\alpha + \varepsilon_t)\pi_{t-1} + (1-\alpha)\pi_{t+1|t} + \kappa q_t$. Still, it looks plausible that the average behavior of this system under the true learning process is closely resembled by the certainty equivalent results.

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