An I(2) cointegration analysis of purchasing power parity between the euro area and the United States*

Chiara Osbat†

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Abstract

The so-called purchasing power parity (PPP) puzzle takes center stage in the empirical debate in the international economics literature. The empirically controversial question is whether the speed of adjustment of deviations from PPP is compatible with the assumption that PPP holds. This paper uses I(2) inference to account appropriately for the statistical properties of the price data, and to investigate the dynamic structure of the relationship between the nominal exchange rate and the domestic and foreign price indices, but it also analyses the effect of "augmenting" the PPP relation with a measure of relative productivity. In this respect, a caveat is in order, as there are clear practical difficulties related to measuring appropriately both the relative prices and the relative productivity. This study shows that these difficulties affect the empirical analysis quite strongly, and proposes a tentative interpretation of the results in terms of the dynamic adjustment structure in the proposed model.

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†Department of Economics, European University Institute, 50016 San Domenico di Fiesole and European Central Bank, Kaiserstrasse 29, 60311 Frankfurt am Main. Contact address: chiara.osbat@ecb.int.

1 Introduction

The so-called purchasing power parity (PPP) puzzle takes center stage in the empirical debate in the international economics literature. The main empirical question can be formulated as follows: is the speed of adjustment of deviations from PPP compatible with the assumption that PPP holds? Depending on the definition of PPP used, the underlying economic model and the econometric methods employed, the answer to this question is still largely a controversial issue.

The theory of PPP in its absolute form states that due to arbitrage, the domestic and foreign prices of a given basket of goods should be equal when they are denominated in a common currency.[1] For this reason, PPP is an obvious benchmark in assessing whether two currencies can be considered to be in equilibrium. Since the real exchange rate between two currencies is a constant when PPP holds, this is not a useful empirical concept. In fact, there is little reason to believe that PPP would hold continuously; rather, PPP is considered as a long-run phenomenon, meaning that deviations of real exchange rates from their PPP-implied equilibrium level should only be temporary. Empirically, this amounts to assuming that real exchange rates should be stationary.

PPP is a fundamental building block for both the classic and the new open economy macroeconomic models (see, for example, Obstfeld and Rogoff (1995), Cavallo and Ghironi (forthcoming)).

Beside its theoretical relevance, PPP has a direct practical policy relevance, as estimated PPP-based equilibrium levels are often used to assess the existence and the extent of exchange rate misalignments. Although the debate on equilibrium exchange rates is much wider, PPP still forms the basic starting point. Given the centrality of the PPP hypothesis both in theoretical open economy models and in the debate on exchange rates equilibria, it is somewhat disquieting to find that empirical evidence on PPP can be characterized as very weak at best. As a consequence, in the literature there have been numerous attempts at empirically resurrecting PPP in some form.

For the purposes of the analysis presented in this paper, it is useful to classify the empirical literature on PPP and equilibrium exchange rates in two main categories. The first approach starts from the viewpoint that the inability of empirical studies to validate the PPP hypothesis is mostly due to the lack of power of the statistical procedures used, and attempts to increase the power of statistical tests by either enlarging the available sample span of data, or using panel data methods by pooling time series information from many different currencies. The second category, which is more concerned with the policy perspectives outlined above, endeavors to estimate equilibrium exchange rates by "augmenting" PPP with real economic fundamental variables, which may explain deviations from it. Among the latter approaches, the most widely discussed is probably the so-called "Balassa-Samuelson" effect, which was first proposed by Balassa (1964) and Samuelson (1964), and states that when the relative productivity in the tradable vs non-tradable goods sectors of a country rises compared to that of another country, its real exchange rate will appreciate.

The empirical analysis conducted in this paper stands somewhat between

[1] The term "purchasing power parity" was proposed originally by Gustav Cassel in 1918, but the idea of purchasing power parity is a very old one in economics: for a brief history and references on the origin of the concept, see for example Sarno and Taylor (2002).

these two empirical approaches, as it uses I(2) inference to account appropriately for the statistical properties of the data and to investigate the dynamic structure of the relationship between the nominal exchange rate and the domestic and foreign price indices, but it also analyses the effect of "augmenting" the PPP relation with a measure of relative productivity. In this respect, a caveat is in order, as there are clear practical difficulties related to measuring both relative prices and relative productivity. This study shows that these difficulties affect the empirical analysis quite strongly, and proposes a tentative interpretation of the results in terms of the dynamic adjustment structure in the proposed model.

The rest of the paper is organised as follows: section 2 defines purchasing power parity, section 3 presents and motivates the statistical model, section 4 describes the data and discusses the problems related to the measurement of relative productivity, section 5 presents the empirical results, and section 6 concludes.

2 Purchasing power parity and real exchange rates

The PPP hypothesis is based on the Law of One Price (LOP), which states that for every tradable good, arbitrage causes the domestic and foreign price to equalise when converted to a common currency:

$$P_{it}^{*} = P_{it}E_t, \qquad i = 1, 2, \ldots N$$

where N is the number of goods, P_{it} is the domestic price, P_{it}^{*} is the foreign price and E_t is the bilateral exchange rate, expressed as the number of foreign currency units per unit of domestic currency. However, even for relatively homogeneous goods, there are various reasons why departures from the LOP could be expected at any point in time: imperfect goods market integration, transaction and transportation costs, international differences in consumption preferences, price-setting behaviour of firms in segmented markets, can all be expected to drive a wedge between domestic and foreign prices.[2]

If the LOP holds for all goods on average, the absolute PPP hypothesis states that the domestic and foreign prices of the same basket of tradable goods should equalise when prices are expressed in a common currency:

$$P_t^{*} = P_t E_t$$

or, in logarithms,

$$p_t^{*} = p_t + e_t$$

PPP is a fundamental parity condition used in open economy macroeconomic models. Textbook models of the balance of payments, for example, rely on complete exchange rate pass-through to derive the response of the trade balance to exchange rate movements via the Marshall-Lerner condition. The amount of pass-through to import prices also affects the international transmission of inflation. In this context, the literature on partial pass-through and international

[2] See Goldberg and Knetter (1997) for a discussion of the sources of price divergence in international goods markets and for a review of the empirical evidence on the LOP.

price discrimination in segmented markets contributes to explaining why large and persistent deviations from the LOP and PPP can be rationalized.

There are various reasons why absolute PPP is unlikely to hold. First, there is quite strong empirical evidence that pass-through from exchange rates to prices is incomplete. This observation can be reconciled with pricing-to-market strategies of firms in segmented markets, which amount to maintaining differences in the foreign and domestic mark-ups, in order to preserve market shares. Incomplete pass-through can also be explained by the perception of the nominal exchange rate appreciation as a temporary phenomenon, which makes it unprofitable to change export prices instantaneously, due to menu costs etc. A further reason for incomplete pass-through can be related to international outsourcing of production, so that exports from a given country are only partially affected by domestic production costs in the home currency. Also other factors, such as trade barriers, transportation costs, goods heterogeneity, and the need to use price indices, make it unlikely for the absolute form of PPP to hold in practice. If, however, the price differential is stable, so that the home and foreign prices change proportionally over time, relative PPP holds:

$$P_t^* = \theta P_t E_t$$

in logarithms,

$$p_t^* = \ln\theta + p_t + e_t \tag{1}$$

In empirical applications, the PPP relation (1) is typically tested as

$$e_t = \lambda_0 + \lambda_1 p_t^* + \lambda_2 p_t + \nu_t. \tag{2}$$

It is said that PPP holds in weak form if the parameters in (2) are unconstrained, while it holds in strong form if the price-homogeneity restriction holds, i.e. if $\lambda_0 = 0$ and $\lambda_1 = -\lambda_2 = 1$:

$$e_t = p_t^* - p_t + \nu_t. \tag{3}$$

Price homogeneity may be violated due to the different consumption baskets included in the domestic and foreign price indices, to imperfect tradeability of the two consumption bundles, to measurement errors. If PPP held continuously, ν_t in (3), i.e. the real exchange rate, would be constant. Due to the rigidities mentioned above, however, PPP is more reasonably seen as a long-run phenomenon, implying that ν_t is mean-reverting. As a consequence, a natural empirical test for PPP hinges on whether ν_t is $I(0)$ in either (2)or (3), depending on whether the weak or strong form is investigated. A common test for strong-form PPP is thus based on testing for unit roots in real exchange rates. Alternatively, weak-form PPP can be tested by cointegration methods, as weak PPP implies that the nominal exchange rate and prices in (2) are cointegrated with unrestricted cointegration coefficients. Possible explanations for heterogeneous coefficients in the PPP relationship may be related to differences in the definition of the domestic and foreign price indices, hence with corresponding different sources of measurement error, and also to market frictions or different real shocks affecting each economy.

Both empirical strategies mentioned above usually lead to either a non-rejection of the non-stationarity hypothesis, or to the finding of a very slow adjustment to deviations from PPP. This result is discussed by Rogoff (1996), who christens it the "purchasing power parity puzzle": nominal shocks coming from shifts in portfolio preferences, monetary shocks, or price bubbles could explain the high volatility that characterises real exchange rates in the long run; however, the effects of such shocks should dampen relatively fast, in 1-2 years at most, so they cannot explain the persistence of deviations from PPP. On the other hand, the very slow speed of mean reversion found in the literature could be explained by real shocks, which however could not explain the high short-term volatility. The paper by Rogoff also provides an extensive review of the empirical attempts to refute the hypothesis of a unit root in real exchange rates, and concludes that the persistent deviations from PPP are probably due to price sluggishness in goods markets.

An important strand of empirical literature is based on fundamental-based models of equilibrium exchange rates. The main idea behind these studies is to "resurrect" PPP by conditioning on a measure of real fundamentals, such as relative productivity, relative current accounts or net foreign asset positions, relative government spending, and measures of the terms of trade.

Most studies include a variety of measures of real fundamentals in equilibrium real exchange rate equations, although some researchers concentrate more on specific fundamental determinants, e.g. relative productivity (Chinn (2000), Chinn and Alquist (2002), Chinn (1997), Maeso-Fernandez, Osbat, and Schnatz (2002)), net foreign assets and trade balances (Lane and Milesi-Ferretti (2000) and Lane and Milesi-Ferretti (2002)), or terms of trade (Rautava (2002), Amano and van Norden (1995)).

3 The I(2) cointegration model and its interpretation

In empirical analyses of PPP and real exchange rate determination, it is important to take into consideration appropriately the stochastic properties of the data. In particular, price indices are often found to be second-order integrated in the sample corresponding to the post-Bretton Woods period.[3] Some of the I(2) literature, in modelling cointegration taking into account the order of integration of the data, also attempts to explain why it is reasonable to augment PPP relationships with rates of inflation. In general, while in an I(1) setting relative prices are stationary and inflation has no long-run effect, in an I(2) setting relative prices are integrated, but less persistent than price levels, and inflation rates can have a long-run effect. Accordingly, Banerjee, Cockerell, and Russell (2001) discuss and estimate an imperfect competition model where price-setting firms face inflation costs, giving rise to a negative long-run relationship between inflation and the mark-up, which is independent of wage pressures but depends on competitive pressure. Productivity has a unit coefficient in this long-run relationship, if the income shares of labour and firms are constant in the long run, irrespective of trend productivity. Looking at PPP in terms of domestic adjustment to import prices, which affect both the cost structure and the competitive environment of domestic price-setting firms, this model may explain why inflation rates are needed to explain deviations from purchasing power parity. This

[3] See, for example, Juselius (1995), Kongsted (1998), Banerjee, Cockerell, and Russell (2001), Banerjee and Russell (2001) and Banerjee and Russell (forthcoming), Nielsen (2001).

view is supported in Bacchiocchi and Fanelli (2002), who propose an interpretation of the need for polynomial cointegration in PPP analyses along the lines of Banerjee, Cockerell, and Russell (2001), Banerjee and Russell (2001) and Banerjee and Russell (forthcoming). Other I(2) analyses of price determination that provide an interpretation of polynomial cointegration include Kongsted (1998) and Nielsen (2001). Kongsted (1998) discusses pricing-to-market behaviour in the context of small-country import price determination, and includes domestic inflation among the variables characterizing domestic cyclical conditions, and thus affecting the pricing decisions of firms. Nielsen (2001) analyses Danish exports of manufactured goods, and also finds an effect of long-run inflation on the mark-up. Haldrup (1998) and Engsted and Haldrup (1999a) present a general argument of the reason why polynomially cointegrating relations may arise in economics, based on linear quadratic adjustment cost models. This argument is reviewed in some detail in section 3.2.

3.1 The statistical model

The empirical analysis in this paper builds on a vector autoregressive (VAR) model, which allows for I(2) variables and linear deterministic trends in the I(2), I(1) and stationary directions, as described in Rahbek, Kongsted, and Jørgensen (1999). The choice of this restriction on the deterministic components is motivated by the fact that in this model, the test for determination of the cointegrating ranks is asymptotically similar with respect to the deterministic components. Furthermore, the model naturally excludes the possibility of having quadratic trends in any direction. The model in error correction form is given by

$$\Delta^2 X_t = \Pi X_{t-1} - \Gamma \Delta X_{t-1} + \sum_{i=1}^{k-2} \Psi_i \Delta^2 X_{t-i} + \mu_0 + \mu_1 t + \Phi D_t + \varepsilon_t \qquad (4)$$

where Π, Γ and Ψ_i are $p \times p$ matrices, μ_0 and μ_1 are $p-$dimensional vectors, D_t is a vector of deterministic terms, such as seasonal or intervention dummies, and ε_t are i.i.d. gaussian random variables with zero mean and variance Ω. The model is I(2) if the roots of the characteristic polynomial of (4) are either on or outside the unit circle and the following two reduced-rank restrictions hold:

$$\Pi = \alpha\beta' \qquad (5)$$

$$\alpha'_\perp \Gamma \beta_\perp = \xi\eta' \qquad (6)$$

where α and β are $p \times r$ matrices, with rank $r < p$, while ξ and η have dimension $(p-r) \times s$, with rank $s < p - r$. A further rank restriction must hold to exclude I(3) components, see Johansen (1992). The matrices $\alpha_\perp$ and $\beta_\perp$ span the nullspace of α and β respectively, and have dimension $p \times (p-r)$. The restrictions on μ_0 and μ_1 necessary to insure at most linear deterministic trends in each direction are discussed in Rahbek, Kongsted, and Jørgensen (1999). For ease of notation, define $\overline{\alpha} = \alpha\left(\alpha'\alpha\right)^{-1}$, $\overline{\beta}$ is similarly defined, and $\beta_1 = \overline{\beta}_\perp \eta$ and $\beta_2 = \beta_\perp \eta_\perp$. The process X_t can then be partitioned into the $p - r - s$ I(2) variables $\beta'_2 X_t$, the s I(1) variables $\beta'_1 X_t$, which do not cointegrate to I(0), and finally the r I(1) variables $\beta' X_t$, which cointegrate to I(0) with the I(1) linear combinations $\beta'_2 \Delta X_t$, giving rise to the r polynomially cointegrating relations

$$\beta' X_t + \delta\beta'_2 \Delta X_t \qquad (7)$$

which in general are trend-stationary. If $r > p - s$, $\beta' X_t$ cointegrates directly to I(0), without involving the combinations $\beta_2' \Delta X_t$. The idea of polynomial cointegration has been developed in Engle and Yoo (1991) and Granger and Lee (1990); for an application of the concept of multicointegration within an I(2) model see Engsted and Haldrup (1999b).

The I(2) model can be estimated by the two-step algorithm suggested in Johansen (1995), see also Paruolo (1996) and **?** or by maximum likelihood, as described in **?** The latter method is preferred, as it allows to perform statistical tests on the significance of some of the coefficients. The analysis in this paper is based on the MLE algorithm implemented by P. Omtzigt (2003).

The analysis of weak exogeneity in model (4) is more involved than in the I(1) cointegration setting, and has been developed in **?** A test which corresponds to the test for weak exogeneity in the I(1) model can be performed, but its interpretation differs. To discuss weak exogeneity, it is useful to re-write (4) as

$$\begin{aligned}\Delta^2 X_t = {} & \alpha \left(\beta' X_{t-1} + \delta \beta_2' \Delta X_{t-1}\right) + (\zeta_1, \zeta_2)\,(\beta, \beta_1)' \,\Delta X_{t-1} \\ & + \sum_{i=1}^{k-2} \Psi_i \Delta^2 X_{t-i} + \mu_0 + \mu_1 t + \Phi D_t + \varepsilon_t.\end{aligned} \tag{8}$$

Paruolo and Rahbek (1999) give the conditions for weak exogeneity in I(2) systems, which are equivalent to those in the I(1) model if $r > 0$ and $s = 0$, i.e. if the system is driven by $p - r$ common $I(2)$ trends and no I(1) trends. For one variable to be weakly exogenous in an I(2) system, the corresponding rows in Π and Γ, as well as in the adjustment coefficients α and (ζ_1, ζ_2) must be zero. Looking at the moving average representation, it turns out that, similar to the I(1) model, a variable is weakly exogenous for the long-run parameters when the corresponding cumulated innovations correspond to one of the I(2) trends driving the system.

3.2 Multicointegration

Engsted and Haldrup (1999b) discuss how multicointegration can arise in stock-flow models, and show how an equilibrium correction mechanism with multicointegration can arise as a solution to a particular optimal control problem, with proportional, integral and derivative control within a linear quadratic adjustment model. In this framework, the adjustment coefficient α in (8) corresponds to the integral control term, the coefficients ζ_1 and ζ_2 correspond to the proportional control terms, and finally the Ψ_i represent the derivative control terms. Slow dynamic adjustment to disequilibrium can be explained as a result of adjustment costs (see references in **?** who also propose a forward-looking error correction specification). If the decision variable and at least one of the forcing variables are I(2), a forward-looking equilibrium correction formulation rationalizes the inclusion of first differences in the long-run relations, hence polynomial cointegration arises.

In the PPP setting, following Engsted and Haldrup (1999a), define p_t, p_t^* and s_t as the logarithms of domestic and foreign prices, and of the nominal exchange rate defined in terms of one unit of domestic currency. If we take p_t as the control variable, which has to adjust to the target variable, i.e. foreign prices in domestic currency, $p_t^* - s_t$, a linear quadratic adjustment cost model implies the following minimization problem:

$$\min E_t \sum_{1=0}^{\infty} \beta^i \left[\theta \left(p_{t+i} - \left(p^*_{t+i} - s_{t+i} \right) \right)^2 + \left(\Delta p_{t+i} \right)^2 \right] \tag{9}$$

where E_t is the operator indicating the expectation conditional on information available at time t, β is a discount rate, θ measures the cost of disequilibrium relative to the cost of changing the control variable, which, in this case, corresponds to the cost of domestic inflation. Define the forcing variable $x_t = (p^*_t \quad s_t)'$; then the first order condition of this minimization problem is given by the Euler equation

$$\Delta p_t = \beta E_t \Delta p_{t+1} - \theta \left(p_t - x_t \right). \tag{10}$$

The characteristic polynomial for (10) is $\beta z^2 - (1 + \beta + \theta) z + 1$, and its stable root is $\lambda < 1$. The forward-looking solution to (10), is then

$$p_t = \lambda p_{t-1} + (1 - \lambda)(1 - \lambda\beta) \sum_{i=0}^{\infty} (\lambda\beta)^i E_t x_{t+i}$$

which, following Engsted and Haldrup (1999a) and assuming that s_t is I(1) and p^*_t is I(2), can also be written in equilibrium-correction form:

$$\begin{aligned} \Delta^2 p_t = (\lambda - 1) \left[p_{t-1} - \gamma x_{t-1} - \frac{1}{\lambda - 1} \Delta p_{t-1} - \frac{1}{1 - \lambda\beta} \Delta p^*_{t-1} \right] \\ + (1 - \lambda) \sum_{i=0}^{\infty} (\lambda\beta)^i E_t \left[\Delta s_{t+i} + \frac{1}{1 - \lambda\beta} \Delta^2 p^*_{t+i} \right] + error \end{aligned} \tag{11}$$

which shows that the change in domestic price inflation depends on a polynomial cointegration relation (the first term in (11)), and on expected future values of exchange rate depreciation and the change in foreign inflation. If the discount factor $\beta = 1$, the multicointegrating relation depends on the differential between domestic and foreign inflation. As noted in Engsted and Haldrup (1999a), only one of Δp_t and Δp^*_t needs to be included in the cointegrating regression, as p_t and p^*_t are $C(2,1)$ and, if entered unrestrictedly, the two differenced variables would contribute with a stationary term.

Pedersen (2002), following Gregory, Pagan, and Smith (1993), also interprets the polynomial cointegrating relation in the context of PPP in the framework of a linear quadratic adjustment cost model similar to the one presented above.

4 Data description and measurement issues

The empirical analysis is based on quarterly data spanning the period from the first quarter of 1974 to the last quarter of 2001. The dollar-euro nominal exchange rates are analysed jointly, and PPP is investigated both on the basis of CPI and WPI. Univariate ADF tests indicate that the nominal exchange rates are I(1). Assessing the stochastic properties of the price series is more difficult (see Figure 1). While there is quite strong evidence of the presence of a unit root in US and euro area CPI inflation, evidence on US and euro area WPI inflation is more mixed (See Figure 2).

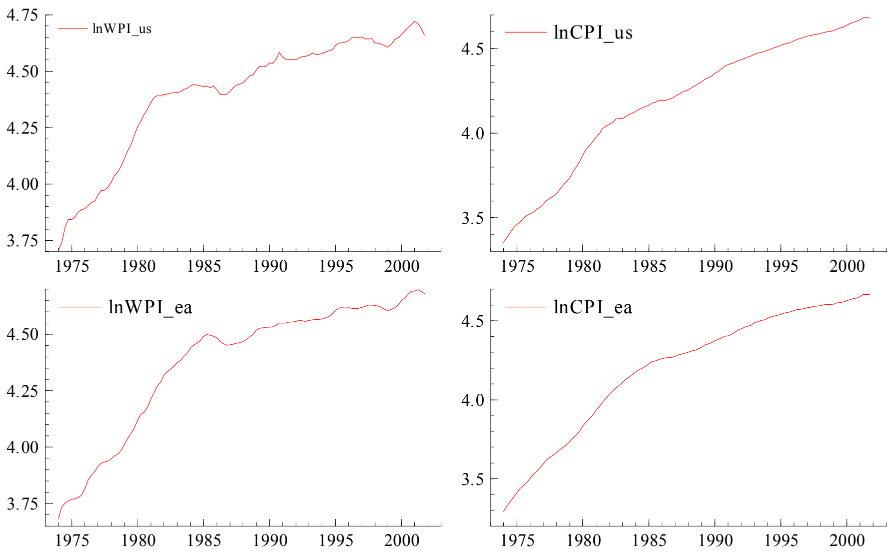

Figure 1: Logarithms of price variables in the euro area and in the USA

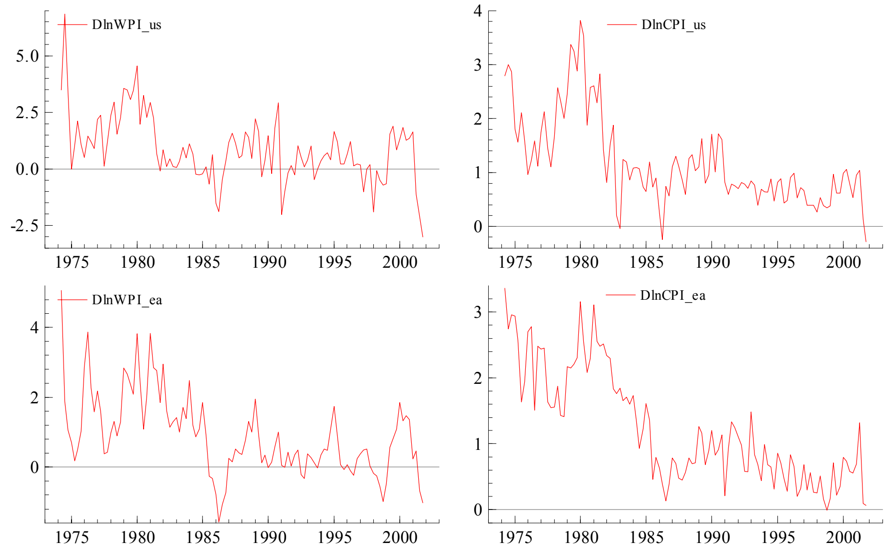

Figure 2: Logarithm of $\Delta prices$ in the euro area and in the USA

data comparison0.emf

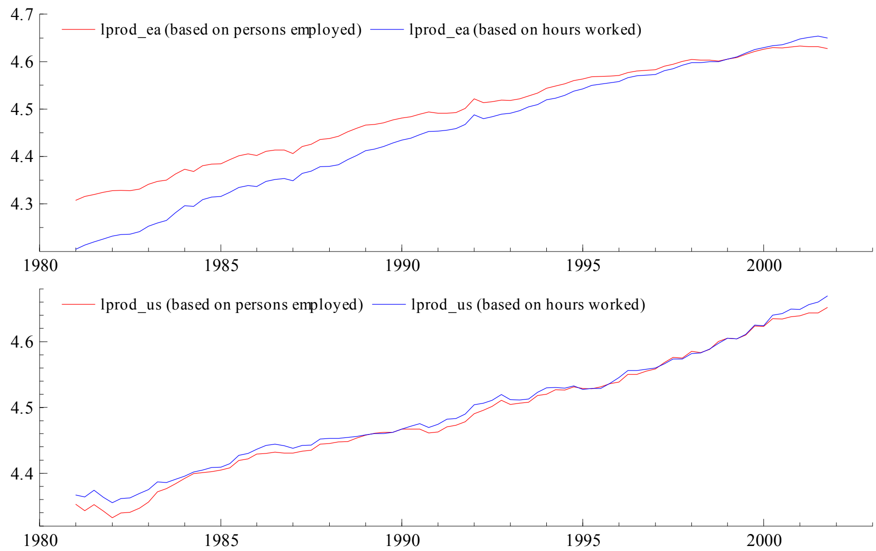

Figure 3: Alternative produtivity measures in the euro area and in the USA

4.1 Productivity measures

Two productivity measures are available, one defined as output per person employed, the other in terms of hours worked[4]. The data are shown in Figure 3. Even though the two measures seem to exhibit a similar trending behaviour, they do not cointegrate, either in the US or in the euro area, as demonstrated in this section. Any cointegration study which uses productivity as a fundamental determinant of the real exchange rate should take this finding into account, because the results will depend very strongly on the productivity measure that is chosen.

For the US productivity measures, a VAR(2) estimated over the period 1981q3 to 2001q4 yields no evidence of misspecification, and the trace test for the null hypothesis of a rank of 0 depends strongly on the deterministic specification. If a restricted constant is included, to allow for a non-zero long-run mean of the difference between the two measures, the trace test for $r \leq 0$ is 37.062 (corresponding to a zero tail probability). The test for homogeneity in the cointegration vector is equal to 0.73727, which in a $\chi^2(1)$ distribution has a tail probability of 0.39. This may seem to indicate that the difference between the two measures is stationary. However, the roots of the restricted model suggest otherwise, as the second root of the companion matrix becomes 0.999, indicating that a trend is left unaccounted for. If the VAR(2) is re-estimated with an unrestricted constant, allowing for the linear trend in the data, the trace test suggests no evidence of cointegration: the test statistic for $r \leq 0$ is 7.3052, with a tail probability of 0.549. The two largest roots are in this case 1.005

[4]This productivity measure was kindly supplied by Focco Vijselaar; details on how it is constructed can be found in Korteweg and Vijselaar (2002).

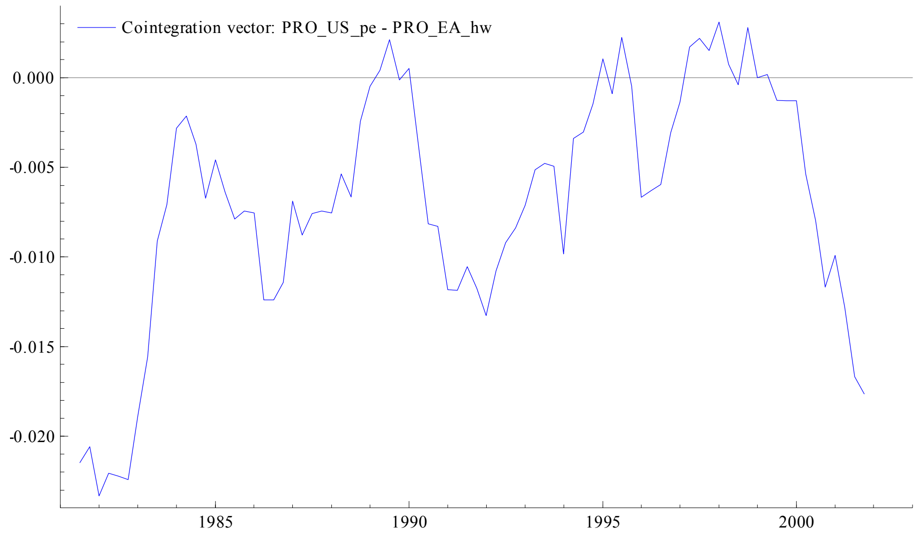

Figure 4: Cointegration relationship between different productivity measures in the USA

and 0.9149. If this (quite strong) evidence is disregarded and the restricted VAR(2) is re-estimated imposing $r = 1$, the test for homogeneity yields a test statistic of 1.1387, with a tail probability of 0.2859. In this case, the second root is equal to 0.8833. Summing up, the evidence in favour of cointegration and homogeneity of the two measures, implying that the difference between the two is non-systematic, is mixed. Standard ADF unit root tests also provide mixed results, with little evidence against the null hypothesis of a unit root in the difference between the two variables. The estimated cointegration vector is displayed in Figure 4, and exhibits long persistent swings.

For the euro area productivity measures, a VAR(3) with impulse dummies for the first quarter of 1987 and 1992, which also includes a restricted constant, yields a trace test of 17.819, corresponding to a tail probability of 0.020 for $r \leq 0$ and of 4.8898 (corresponding to a p-value 0.027) for $r \leq 1$. The largest eigenvalues of the companion matrix are 0.9924 and a complex pair of 0.8210. If the VAR is re-estimated under the restriction $r = 1$ the cointegrating vector exhibits a clear trend. Hence, the model was re-estimated with a linear trend restricted to the cointegrating space. In this case, the trace test for $r \leq 0$ is 22.443 (corresponding to a p-value 0.127), indicating no cointegration, and the largest roots are 0.9605 and a complex pair with modulus 0.8202. When the restricted model with $r = 1$ is estimated despite the trace test result, there are clear signs of instability at the very end of the sample, hence the model was re-estimated limiting the sample to 2001q3. The homogeneity restriction yields a test statistic of 3.1719, which is distributed as a $\chi^2(1)$ and corresponds to a tail probability of 0.0749. The second largest root is 0.9363. If the estimation is conducted only up to 2000q4, the tail probability for the homogeneity test increases and the largest unrestricted root drops to 0.8961. The restricted

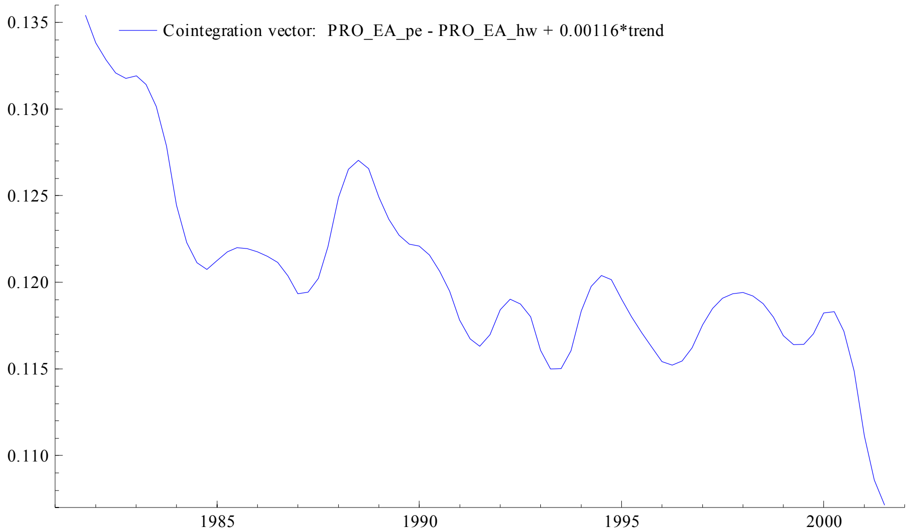

Figure 5: Cointegration relationship between different productivity measures in the euro area

cointegration vector shows clear evidence of non-stationary behaviour and of changes in mean at the beginning of the sample and in the course of 2001, as shown in Figure 5.

Summing up, there is only limited evidence of cointegration between the two different measures of productivity both in the USA and in the euro area. Under the hypothesis of cointegration, the estimated equilibrium relations exhibit some instability after 2000: in both cases, the long-run relations show that productivity measured in terms of persons employed has dropped quite sharply with respect to productivity measured in terms of hours worked. In the case of the euro area, this development is an acceleration of a general decline that characterizes the whole sample.

Another interesting question to ask is whether productivity developments in the United States and in the euro area exhibit common trends, both in terms of hours worked and in terms of persons employed. Figure 6 shows relative productivity in the USA and in the euro area according to both measures. This graph indicates very clearly that the catch-up in productivity experienced by the euro area since 1980 has been faster in terms of hours worked than in terms of persons employed. This can be attributed to the rise of part-time work and to the reduction of the average working week in the euro area, a phenomenon which was not matched in the USA. Another interesting development which is clearly evidenced in the chart is the inversion of the catch-up trend which took place since 1995, when productivity in the USA started rising faster than in the euro area again. It is interesting to remark that this period also corresponded to the start of a long spell of dollar appreciation. To illustrate this point, Figure 7 shows the EUR-USD nominal exchange rate and the US-EA productivity differential in terms of hours worked and of person employed. The plot clearly

US-EA comparison0.emf

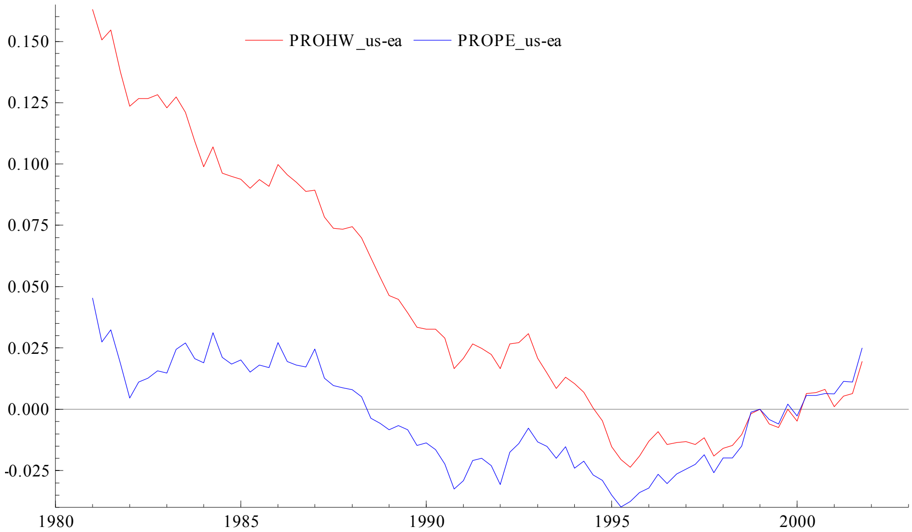

Figure 6: Comparison of relative productivity measures

$H_0 : r \leq$	Trace test [p-value]
0	10.444 [0.252]
1	0.077730 [0.780]
* indicates rejection at 5%, ** at 1% level	

Table 1: Test for cointegration rank, relative output per hour worked in the EA and the USA

shows a comovement of these variables, especially when productivity is defined in terms of persons employed.

When modelling USA- euro area relative productivity developments in terms of hours worked, the specification of the deterministic component turns out to be very important. When a VAR(3) is estimated with an unrestricted constant, which allows for the linear trend observed in the levels of the variables, and a dummy for an outlier in 1992q1, which is needed to obtain well-behaved residuals, the trace test, reported in Table 1, gives no sign of cointegration. If the restriction $r = 1$ is imposed anyway, the second largest root of the system is equal to 0.9863, clearly indicating that the imposition of a cointegration vector is invalid. The estimated cointegration vector exhibits a clear downward trending behaviour.

As a consequence, the model was re-estimated with a different deterministic specification, which takes into account the apparent broken trend in the data, i.e with an unrestricted constant and a restricted linear trend, along with a restricted broken trend, which is set to zero before 1995q4. With this specification there is clear evidence of cointegration, and the hypothesis of homogeneity in the cointegration coefficients between the productivity variables is not rejected, as it yields a LR test statistic of 0.25596, which under a $\chi^2(1)$ distribution

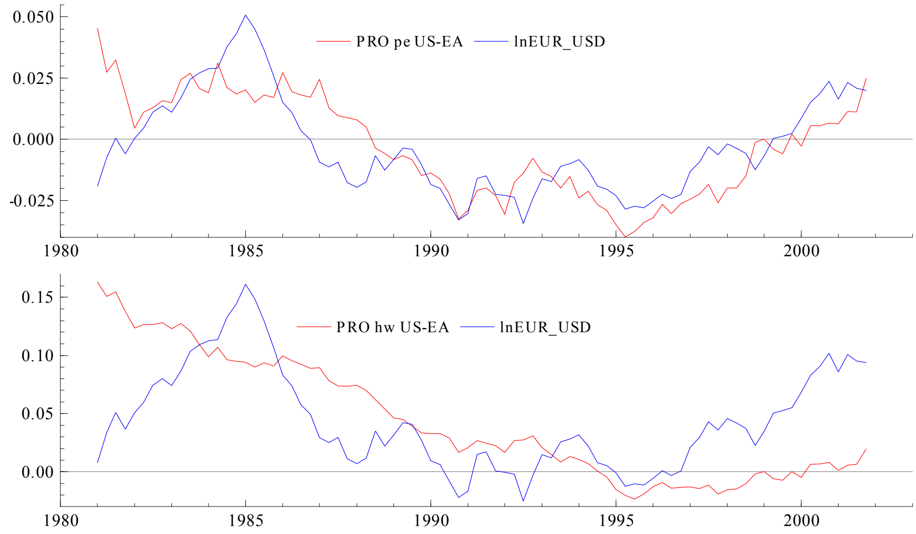

Figure 7: Different productivity measures and USD/EUR nominal exchange rate

$H_0 : r \leq$	Trace test [p-value]
0	8.2603 [0.445]
1	0.40914 [0.522]
* indicates rejection at 5%, ** at 1% level	

Table 2: Test for cointegration rank, relative output per person employed in the EA and the USA

has a tail probability of 0.6129. The test for weak exogeneity of the euro area productivity variable is not rejected at the 5% level, as it yields a p-value of 0.0713.

Using productivity measured in terms of persons employed, the same pattern appears: there is no evidence of cointegration in the model with an unrestricted constant, as the trace test, reported in Table 2, fails to reject the hypothesis that $r \leq 0$. If a cointegration vector is imposed regardless of the test outcome, the second largest root becomes 0.9907, clearly indicating that there is no cointegration. When the VAR(3) is re-estimated allowing for a linear trend both in the I(1) and I(0) directions, and with a restricted broken trend after 1995Q4, the trace test indicates clear evidence of cointegration, and imposing $r = 1$ the hypothesis of homogeneous coefficients on US and euro area productivity is not rejected. In this case, however, the test for weak exogeneity of euro area productivity is rejected, as it yields a p-value of 0.0028. The estimated coefficients for the deterministic components, as well as the adjustment coefficients, are reported in Tables 3 and 4 for both productivity measures.

The analysis of cointegration between US and euro area productivity shows that the productivity differential between the two areas has been stationary around a broken trend, with an overall downward trend of US productivity

	Productivity (hours worked)	
	β coeff. (*t-value*)	α coeff. (*t-value*)
$PROhw_t^{US}$	1	-0.291 (-5.053)
$PROhw_t^{EA}$	-1	0
broken trend	-0.004 (-11.022)	$-$
trend	0.003 (24.863)	-
	LR test: $\chi^2(2) = 5.2830$ [0.0713]	

Table 3: Estimated cointegration vector, relative output per hour worked in the EA and the USA

	Productivity (persons employed)	
	β coeff. (*t-value*)	α coeff. (*t-value*)
$PROhw_t^{US}$	1	-0.110 (-2.117)
$PROhw_t^{EA}$	-1	0.158 (3.302)
broken trend	-0.004 (-7.513)	-
trend	0.001 (7.340)	-
	LR test: $\chi^2(1) = 1.2981$ [0.2546]	

Table 4: Estimated cointegration vector, relative output per person employed in the EA and the USA

with respect to the euro area. The linear trend slope is steeper for the measure involving hours worked, as could be expected, as hours worked displayed a relative decline in the euro area with respect to the USA. However, after the last quarter of 1995 this tendency was reversed, as US productivity started rising according to both measures, but to a larger extent in terms of persons employed.

The analysis in this section also indicates that a simple productivity differential, which does not consider the presence of a broken deterministic trend, behaves like an I(1) variable. This affects any attempt at modelling the relationship between exchange rates and productivity between the US and the euro area, irrespective of which productivity measure is used.

5 The empirical results

In empirical analyses, it is common to use data transformations that reduce the order of integration of the variables. When analysing a system that may contain I(2) variables, it is important to preserve the full cointegration structure when applying the nominal to real transformation, in order to completely eliminate the I(2) components. Kongsted (1999) presents a testing procedure to test the validity of the nominal-to-real transformation. In particular, in the case of PPP, long-run homogeneity should satisfy not only the relation that cointegrates from I(2) to I(0) (with or without the difference terms involved in the polynomially cointegrating relations), but also the set of linear combinations that cointegrate from I(2) to I(1). If the latter are not taken into consideration adequately, the resulting VECM will still contain I(2) components. This problem is particularly relevant for analyses that include domestic and foreign price indices, as it is not obvious what the appropriate transformation is, when more than one I(2) trend is potentially involved. The empirical analysis below is based on an I(2) model to establish the appropriate nominal-to-real transformation, which completely removes the I(2) trends present in the data. The I(2) model also allows appropriate testing for weak exogeneity, which amounts to establishing what shocks contribute to the I(2) trends driving the system.

The lag length for each estimated model was selected on the basis of the minimum lag length that yielded white-noise residuals, as evaluated on the basis of a battery of standard misspecification tests.

Before looking at the empirical results, and in order to interpret them more clearly, it is useful to spell out the rank structure that would be theoretically expected (see Bacchiocchi and Fanelli (2002) for a more detailed analysis of the different possible rank configurations). If the domestic and foreign price indices contain a common I(2) trend, the expected ranks are $r = 1$, $s = 1$, implying one I(2) trend and one I(1) trend, hence three unit roots. If, however, the two price indices have different I(2) trends, a rank configuration compatible with PPP is p-r-s=2, s=0 and r=2, implying that the nominal exchange rate and a linear combination of the prices are I(1) and cointegrate to I(0) with some combination of the exchange rate depreciation and inflation rates. A linear combination of the two multicointegration vectors is in this case compatible with PPP in weak or strong form, depending on whether the homogeneity restriction holds.

The I(2) model was estimated twice, using consumer price indices and wholesale price indices, as it is often found in the literature that deviations from PPP are less persistent when WPIs are used. This is justified from a theoretical point of view, as WPIs typically include a larger share of tradable goods.

5.1 Empirical results using CPI

A VAR(4) including US and euro area CPI and the nominal exchange rate defined in terms of domestic currency, so that an increase indicates an appreciation of the euro, yields a well-specified model on the basis of standard misspecification tests.[5] The deterministic specification allows for at most a linear trend in all directions, including the polynomial cointegrating relation. The *a priori* exclusion of quadratic trends seems a natural restriction from both a theoretical and an empirical point of view. Another advantage of this choice of deterministic specification is that in this setting the rank test is independent of the deterministic parameters. The significance of the linear trend can be tested and its coefficient can accordingly be restricted to zero. The model also includes a set of centred seasonal dummies and an impulse dummy for the first quarter of 1991, which corresponds to the Gulf War and is needed to achieve normality of the residuals in the price equations. All variables are in natural logarithms, and the sample spans the period from the first quarter of 1976 to the last quarter of 2001, hence includes 104 data points, accounting for the observations lost due to lags. The maximum likelihood rank test is reported in Table 5 below; the sequential testing procedure involves testing from the upper left hand corner, moving right along the first row, and then proceeding similarly along the second row, until the critical value exceeds the test statistic. In this case, this procedure suggests the rank combination $r = 1$, $p - r - s = 2$, $s = 0$ or, more comfortably, the combination $r = 1$, $p - r - s = 1$, $s = 1$, which would be theoretically straightforward to interpret, as it would allow for a common I(2) price trend and an I(1) trend. In order to better discriminate between the two rank configurations, one should look at the implied number of unit roots, and of the corresponding roots of the companion matrix. Under the first rank configuration with $s = 0$, there are four unit roots, corresponding to the two I(2) trends, while under the second configuration there would be only three, one corresponding to the I(1) trend and two corresponding to the I(2) trend. The fourth largest unrestricted roots of the companion matrix are 0.99, a complex pair with modulus 0.96 and 0.82, supporting the hypothesis that $r = 1$ and $s = 1$. This is the working hypothesis for the subsequent analysis. Once the three unit roots are imposed, the fourth remains 0.82, further corroborating that this is an appropriate choice for the cointegration ranks.

In order to estimate and test the appropriate nominal-to-real transformation, a hypothesis test on the $\tau = (\beta \quad \beta_1)$ can be conducted. This test is distributed as a χ^2 with degrees of freedom equal to the number of unrestricted parameters. As τ includes the variables that cointegrate from I(2) to I(1), it forms the basis for the C(2,1) transformation. The *a priori* interesting restriction is that the exchange rate should include an I(1) trend, and the two CPIs should share a common I(2) trend, with or without homogeneous coefficients. The test on τ also allows testing for the significance of the linear trend in the I(1) direction. The test for the theoretically expected structure for τ,and for weak exogeneity of the US CPI, gives a test statistic of 5.14, which is distributed as a $\chi^2(4)$, yielding

[5] Computations were performed using PcGive10 and P. Omtzigt's ME2 program, implemented in MATLAB.

Maximum Likelihood Inference					
$p-r$	r				
3	0	122.1 (87.6)	93.4 (68.2)	75.5 (53.2)	73.3 (42.7)
2	1		45.8 (47.6)	26.7 (34.4)	24.2 (25.4)
1	2			14.0 (19.9)	5.3 (12.5)
$p-r-s$		3	2	1	0

Table 5: Test for I(2) cointegration ranks using CPI price indices

a tail probability of 0.4. The test for exclusion of the linear trend in the CPI differential yields a statistic of 37.97, corresponding to a zero tail probability. The I(2) analysis hence indicates that the rank structure is as expected, but the common I(2) trend enters the two CPIs with different coefficients. This finding can be rationalized considering that the euro area and US CPIs are based on different consumption baskets, and have different proportions of tradeable and non-tradeable goods. A nominal-to-real transformation based on homogeneous coefficients would leave part of the I(2) trend in the data, thus invalidating inference in the I(1) model. The nominal-to-real transformation that emerges from the I(2) analysis is given by $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -0.9182 \\ 0 & -0.0021 \end{pmatrix}' \begin{pmatrix} s_t \\ p_t^{US} \\ p_t^{EA} \\ t \end{pmatrix}$.

After applying this transformation to the data,[6] an I(1) system is estimated, with the nominal exchange rate, the C(2,1) linear combination of CPIs and trend, and the euro area CPI inflation. A VAR(4) is estimated, including a restricted constant, the impulse dummy for the first quarter of 1991 and centred seasonal dummies. This model yields no substantial evidence of misspecification. There is clear evidence of one cointegrating vector, with a trace statistic for $r < 1$ of 46.505, corresponding to a p-value of 0.0016, while the hypothesis of $r < 2$ corresponds to a p-value of 0.91. When the rank is restricted to one, the third largest root of the companion matrix drops to 0.8164, corroborating the choice of one cointegrating vector. The resulting cointegrating vector, however, is difficult to interpret in terms of PPP, as the nominal exchange rate is insignificant in the long-run relationship, and weakly exogenous, as shown in Tables 6 and 7.

The recursive parameter estimates, displayed in Figure 8, show that the model is very stable across the whole sample.

The cointegrating vector is remarkably stable, but it is difficult to interpret, as it represents an equilibrium relationship between the relative CPIs and euro area inflation, which does not involve the nominal exchange rate.

5.1.1 Introducing productivity in the CPI-based PPP analysis

When relative productivity is brought into the picture, by re-estimating the VAR(4) augmented with the USA-euro area productivity differential, measured in terms of hours worked, a relationship between the nominal exchange rate, rel-

[6] See Froot and Rogoff (1995) for a word of caution on the interpretation of heterogeneous coefficients in PPP relations.

	unrestricted	
	β coeff. (t-*value*)	α coeff. (t-*value*)
s_t	-0.002 (-0.138)	-0.778 -1.915
$p_t^{US} - p_t^{EA}$	1	-0.085 -2.507
Δp_t^{EA}	-0.062 (-25.00)	12.720 5.791
constant	-0.131 (-2.387)	-

Table 6: Estimated unrestricted cointegration vector using CPI price indices

	restricted: LR test: $\chi^2(2) = 4.2858$ [0.1173]	
	β coeff. (t-*value*)	α coeff. (t-*value*)
s_t	0	0
$p_t^{US} - p_t^{EA}$	1	-0.079 -2.356
Δp_t^{EA}	-0.063 -25.508	12.426 5.751
constant	-0.138 -42.026	-

Table 7: Estimated restricted cointegration vector using CPI price indices

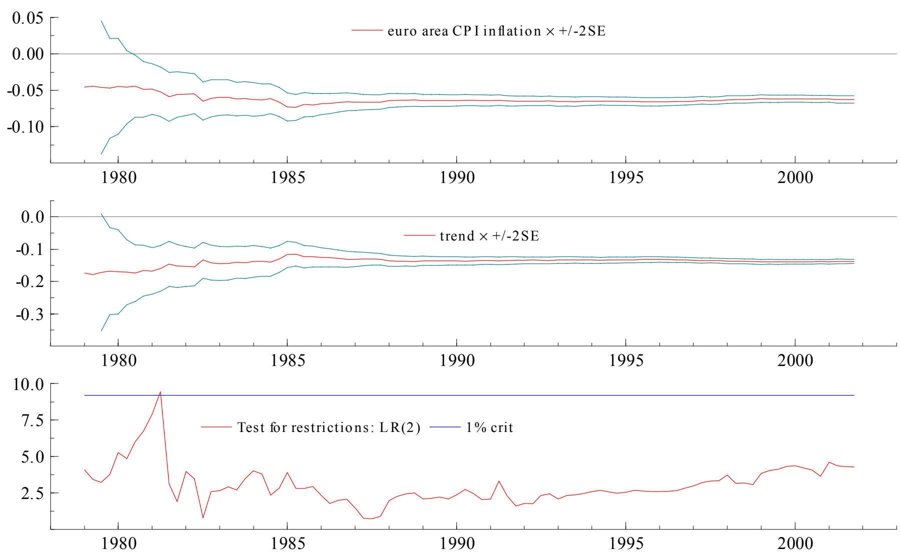

Figure 8: Recursive parameter estimates of cointegration coefficients

	β coeff. (t-*value*)		α coeff. (t-*value*)	
s_t	1	0	-0.069 (-2.882)	0
$p_t^{US} - p_t^{EA}$	0	1	-0.005 (-2.973)	0
Δp_t^{EA}	0	-0.069 (-10.385)	0	12.556 (5.050)
$PROus - ea$	9.988 (4.188)	0	-0.010 (-3.384)	0
$Step1991Q1$	0.500 (2.441)	0	-	-
$constant$	-5.067 (-26.271)	-0.134 (-29.212)	-	-

Table 8: Estimated restricted cointegration vector using CPI price indices and relative productivity

ative prices and inflation emerges again. The estimated model includes the same variables as above, i.e. the nominal exchange rate, the I(1) linear combination of CPIs suggested by the I(2) analysis, an impulse dummy for 1991 q1, centred seasonal dummies and a restricted constant, plus the differential between US and euro area productivity, measured in terms of hours worked. A step dummy was also included, restricted to the cointegration space, following the analysis of productivity in the previous section. The trace test indicates that there are two cointegration vectors, of which one corresponds almost exactly to the equilibrium relationship that emerged from the model without productivity, the other is a relationship between the nominal exchange rate and the productivity differential. The step dummy is significant in the latter vector, meaning that the mean level of the equilibrium relationship between the exchange rate and the productivity differential has changed since 1991. The results are summarised in Table 8. The LR test for the restrictions in this model is 10.667, which is distributed as a $\chi^2(7)$, and corresponds to a tail probability of 15%.

The test on α indicates that no variable is weakly exogenous, and that the inflation rate adjusts to the nominal relationship, while the exchange rate, the price differential and relative productivity adjust to the relationship between the nominal exchange rate and productivity.

5.2 Empirical results using WPI

A VAR(4) was estimated using WPIs, with the same specification used for the case of CPI, ie.e allowing for a linear trend in all directions, and including centred seasonal dummies and an impulse dummy for the first quarter of 1991, to allow for the sharp rise in prices due to the Gulf war. The estimated residuals exhibit no signs of misspecification, and the sequential testing procedure indicates that the rank structure can be either $r = 1$, $s = 0$, implying two different I(2) trends, or $r = 1$ and $s = 1$, implying a common I(2) trend. In the first hypothesis, the system contains four unit roots, while in the second case it has only three. The sequential test suggests that $s = 0$, as indicated in Table 9. However, the theoretical prior and the observation that the four largest unre-

Maximum Likelihood Inference					
$p-r$	r				
3	0	103.1 (87.6)	72.4 (68.2)	52.4 (53.2)	46.9 (42.7)
2	1		44.5 (47.6)	24.4 (34.4)	20.9 (25.4)
1	2			13.7 (19.9)	8.9 (12.5)
$p-r-s$		3	2	1	0

Table 9: Test for I(2) cointegration ranks using WPI price indices

$H_0 : r \leq$	Trace test [p-value]
0	44.667 [0.003] $**$
1	121.648 [0.030]$*$
2	26.6800 [0.149]
* indicates rejection at 5%, ** at 1% level	

Table 10: Test for I(1) cointegration ranks using WPI price indices

stricted eigenvalues of the companion matrix are 0.98, a complex pair of 0.96 and 0.77, indicate that it may be more appropriate to impose $r = 1$ and $s = 1$. When this restriction on the rank is imposed, the fourth largest root becomes 0.79, supporting this rank choice.

The expected structure of τ, that one vector corresponds to the nominal exchange rate and the other to the price differential, is strongly supported by the data: the homogeneity restriction on τ yields a p-value of 90%, while it was strongly rejected when CPI were used. The trend coefficient can be restricted to zero for the nominal exchange rate (p-value = 97%),but not for the WPI differential, as the latter restriction yields a p-value of 1%. When the two vectors in τ are completely identified as corresponding respectively to the nominal exchange rate and to the price differential around a linear trend, the test for weak exogeneity of the US WPI yields a statistic of 8.32, which is distributed as a $\chi^2(6)$ and corresponds to a p-value of 22%.

The expected nominal-to-real transformation can thus be based on homogeneous coefficients, but it also involves a trend in the WPI differential, and is given by $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \\ 0 & -0.0013 \end{pmatrix}' \begin{pmatrix} s_t \\ p_t^{US} \\ p_t^{EA} \\ t \end{pmatrix}$.

After applying this transformation to the data, an I(1) system is estimated, with the nominal exchange rate, the WPI differential around the trend, and the euro area WPI inflation. A VAR(4) is estimated, including a restricted constant, the impulse dummy for the first quarter of 1991 and centred seasonal dummies. This model yields no evidence of misspecification, and the lag length can be reduced to three, introducing an impulse dummy variable for the third quarter of 1980. The trace test, as reported in Table 10, indicates evidence for one or two cointegrating vectors.

Imposing $r = 1$ yields a third root of 0.8051, supporting the hypothesis of one cointegrating vector. The coefficient on the nominal exchange rate in the

	β coeff. (*t-value*)	α coeff. (*t-value*)
s_t	-0.204 (-4.666)	0
$p_t^{US} - p_t^{EA}$	1	-0.095 (-3.824)
Δp_t^{EA}	-0.033 (-4.592)	4.146 (2.044)
constant	0.952 (4.740)	-

Table 11: Estimated restricted polynomial cointegration vector using WPI price indices

cointegrating vector is unstable at the beginning of the sample, hence the vector is normalized on the relative prices variable. This yields the coefficients reported below in Table 11. The coefficients have the expected signs, but the hypothesis of homogeneity between the nominal exchange rate and the relative prices is rejected, as it yields a test statistic of 7.6336, which corresponds to a tail probability of 0.0057 under a $\chi(1)$ distribution. The tail probability rises to 2% if the test is conducted jointly with a test of weak exogeneity of the nominal exchange rate, but overall it seems inadequate to impose the homogeneity restriction between the nominal exchange rate and the price differential. The estimated long-run parameters, reported in Table 11, are quite unstable in the first part of the sample, as shown in Figure 9.

The results reported above indicate that strong form PPP does not hold even if the common I(2) trend is completely removed by taking the WPI differential in the nominal-to-real transformation. Also, the weak exogeneity results indicate that in the long run the nominal exchange rate is determined outside of the system, and the adjustment to (weak-form) PPP is done by prices and inflation, not by the nominal exchange rate.

In the case of WPI, given that the "natural" nominal-to-real transformation holds, it is interesting to ascertain whether the deviation from PPP can be partly explained by real determinants, and in particular by the US-euro area productivity differential. It is also interesting to see if the result is sensitive to the way productivity is measured, as is expected on the basis of the results of section 4.1. Starting with productivity measured in terms of hours worked, a VAR(3) with a linear trend both in the I(1) and I(0) direction and one impulse dummy for 1991Q1, with the addition of a restricted broken trend after 1995q4, as found in the section above on productivity measures, yields well-behaved residuals according to standard misspecification tests. The rank test, reported in Table 12, indicates the presence of two or three cointegrating vectors, and the three largest eigenvalues of the companion matrix are a complex pair of modulus 0.8824 and 0.7773. Imposing $r = 2$ leaves the third largest eigenvalue at 0.7393, confirming that the choice of $r = 2$ is appropriate.

The two identified cointegrating relationships are reported in table 13. The LR test corresponding to these overidentifying restrictions is equal to 12.529, yielding a tail probability of 0.1851 under a $\chi^2(9)$ distribution. The second vector is the same that arises from the productivity analysis reported in section 4.1, with a slightly higher parameter for the broken trend. The first vector

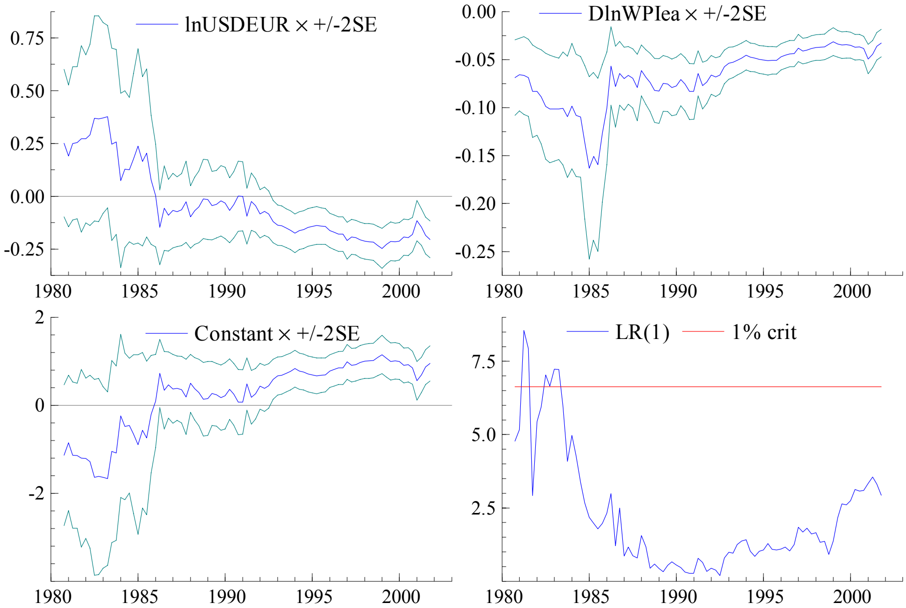

Figure 9: Recursive estimates of cointegration coefficients

$H_0 : r \leq$	Trace test [Prob]
0	89.751 [0.000] **
1	52.035 [0.004] **
2	29.156 [0.017] *
3	11.249 [0.080]
* indicates rejection at 5%, ** at 1% level	

Table 12: Test for I(1) cointegration ranks using WPI price indices and productivity

	Productivity (hours worked)			
	β coeff. (t-value)		α coeff. (t-value)	
s_t	1	0	0	0
$p_t^{US} - p_t^{EA}$	-1	0	0.004 (1.628)	0
Δp_t^{EA}	-0.520 (-4.144)	0	0.472 (3.059)	0
$PROus - ea$	0	1	0.004 (2.170)	-0.294 (-5.373)
broken trend	0	-0.005 (-10.97)	-	-
trend	0	0.003 (19.84)	-	-
	$\chi^2(9) = 12.529$ [0.1851]			

Table 13: Estimated restricted cointegration vector using WPI price indices and relative productivity (hours worked)

corresponds to the real exchange rate and inflation, as expected from the I(2) analysis. It is interesting to see that bringing the productivity differential into the picture, the homogeneity between the nominal exchage rate and relative price coefficients holds. As seen above, the nominal exchange rate is weakly exogenous, meaning that the adjustment towards this modified PPP equilibrium is done by the inflation rate and by relative prices. Recursive estimates show that the parameters are unstable in 2001, and in fact the adjustment coefficient for relative WPI becomes less significant when the model is estimated in the full sample, but it is significant until the end of 2000. The puzzling finding is that the nominal exchange rate is weakly exogenous with respect to the long-run parameters; this implies that any adjustment in the real exchange rate is done by relative prices and inflation, while the nominal exchange rate is determined by forces that are not captured by this model.

The recursive parameter estimates, displayed in Figure 10, show signs of instability especially at the end of the sample. However, the identifying restrictions are comfortably accepted throughout the sample.

When productivity is measured in terms of persons employed, a VAR(3) with a restricted linear trend and a restricted broken trend after 1995q4 and one impulse dummy for 1991Q1 yields well-behaved residuals according to standard misspecification tests. The rank test, reported in Table 15, indicates the presence of two or three cointegrating vectors, and the largest eigenvalues of the companion matrix are a complex pair of modulus 0.8980 and a comlex pair of modulus 0.7311. Imposing $r = 2$ leaves the largest unrestricted eigenvalues at 0.7243, confirming that the choice of $r = 2$ is appropriate.

The two identified cointegrating relationships are reported in Table 13. The LR test corresponding to these overidentifying restrictions is equal to 14.767, yielding a tail probability of 0.0975 under a χ^2 (9) distribution. The linear trend coefficient is not significant in the first cointegrating vector, and restricting it to zero gives a test statistic of 18.594, which under a χ^2 (10) distribution gives a p-value of 0.0457. The second vector is the same that arises from the productivity analysis reported in section 4.1, with a slightly lower parameter for the broken

	Productivity (persons employed)			
	β coeff. (t-value)		α coeff. (t-value)	
s_t	1	0	-0.09 (-3.896)	0
$p_t^{US} - p_t^{EA}$	-1	0	0	0
Δp_t^{EA}	0.273 (5.844)	0	0	0
$PROus - ea$	0	1	-0.02 (-4.113)	-0.458 (-6.032)
$broken\ trend$	0	-0.003 (-10.074)	-	-
$trend$	0.004 (1.921)	0.001 (7.438)	-	-
	$\chi^2(9) = 14.767$ [0.0975]			

Table 14: Estimated restricted cointegration vector using WPI price indices and relative productivity (persons employed)

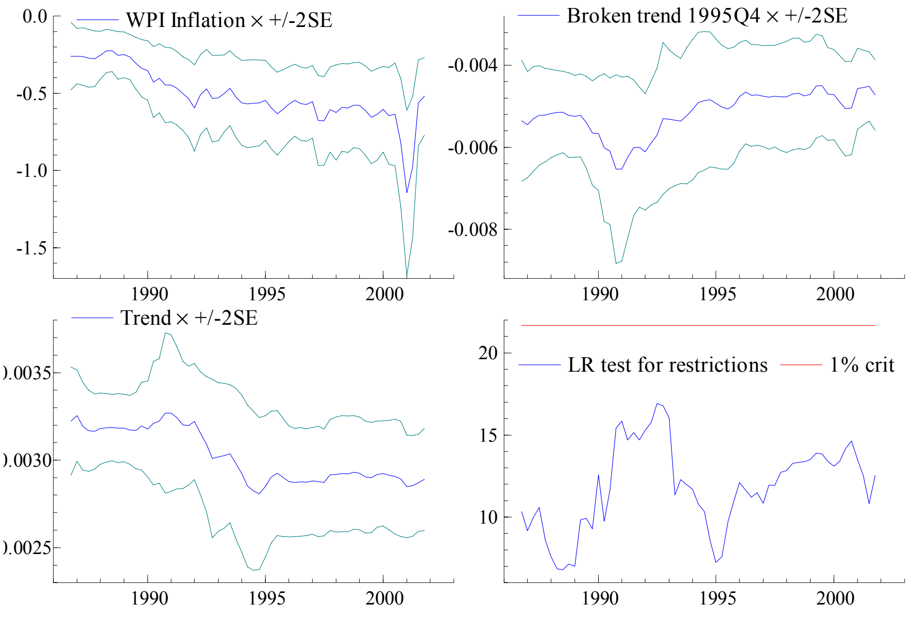

Figure 10: Recursive parameter estimates of cointegration vector

$H_0 : r \leq$	Trace test [Prob]
0	97.814 [0.000] **
1	55.589 [0.001] **
2	30.564 [0.010] *
3	11.316 [0.078]
* indicates rejection at 5%, ** at 1% level	

Table 15: Test for cointegration rank using WPI price indices and relative productivity (persons employed)

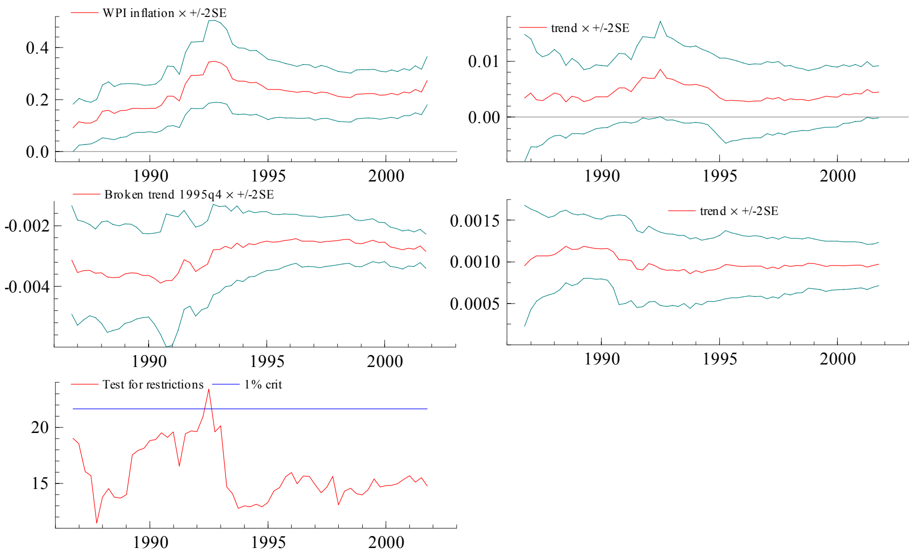

Figure 11: Recursive estimates of cointegration coefficients

trend. The first vector corresponds to the real exchange rate and inflation, as expected from the I(2) analysis, but the coefficient for inflation has a different sign than in the case when the other measure of productivity is used. Also in this case, bringing the productivity differential into the picture, the homogeneity between the nominal exchage rate and relative price coefficients holds. Contrary to the case where productivity in terms of hours worked is used, the nominal exchange rate is not weakly exogenous, while the relative WPI and inflation are. Recursive estimates, reported in Figure 11, show little evidence of instability, and they also show that the linear trend is insignificant in the first cointegrating vector over most of the sample.

6 Conclusions

This paper suggests that mean-reversion of deviations from PPP can only be found once real shocks, represented by relative productivity, are properly accounted for. The findings of this paper about the dynamic structure of adjustment of deviations from PPP seems to confirm earlier findings in the literature.

For instance, Engel and Morley (2001), using an unobserved components model to analyse the dynamic adjustment of nominal exchange rates and prices, find that the former converge much more slowly to PPP. The authors, in reference to Rogoff's formulation of the "PPP puzzle", note that "the real puzzle is why nominal exchange rates converge so slowly", and reject an explanation based on the persistence of real shocks. To the extent that the analysis in the present paper attempts to account also for real shocks, this explanation for the common finding seems indeed insufficient. **?**uggest that goods market segmentation is still predominant, leading to a "band of inaction" where the exchange rate does not adjust. An alternative tentative explanation, which they also hint at, could be given on the basis of the fact that nominal exchange rates are asset prices, and as such they are traded in the markets on the basis of considerations that are related to fundamentals, such as relative prices and productivity differentials. In fact, the finding that the dollar-euro nominal exchange rate seems to adjust to deviations from its long-run relationship with respect to the measure of relative productivity which is most widely available to the markets, may support this interpretation. In a recent study, **?**ind that in the context of forecasting asset prices, unrevised, "real-time" data on fundamentals have better properties than the revised data, despite the fact that the latter are better measures of the underlying economic phenomena. As a consequence, the authors advocate the use of raw data, despite the larger measurement error, for empirical analyses of asset prices. This reasoning may partly explain the findings in this paper, as the productivity measure based on hours worked, although preferable from a theoretical point of view, is not as widely and promptly available as the one based on persons employed.

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