

Exchange Rates in a Behavioural Finance Framework

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Abstract

We develop a simple model of the foreign exchange market in which agents optimize their portfolio and use different forecasting rules. They check the profitability of these rules ex post and select the more profitable one. This model produces two kinds of equilibria, a fundamental and a bubble one. In a stochastic environment the model generates a complex dynamics in which bubbles and crashes occur at unpredictable moments. We also analyse the empirical relevance of the model

1 Introduction

The rational expectations paradigm has been important in our understanding of financial markets. However, it has also shown its limitations. This is especially the case in the foreign exchange market where much of the observed short-term dynamics cannot easily be understood in the context of rational expectations models. As will be argued in this paper the difficulties of rational expectations models (RE-models) in correctly describing the short-term dynamics of the exchange market is related to the strong informational burden imposed on individual agents by these models.

In this paper we use an approach which is influenced by the behavioural finance literature. In this literature agents use relatively simple behavioural rules. These rules do not employ all available information, mainly because agents find it difficult to process and to evaluate the wealth and complexity of the available information (Shleifer(2000), Shiller(2001)). Agents are rational, however, in that they use these rules only as long as they are profitable. We will add to this the notion that agents compare the rule they currently use to alternative rules and decide to switch to the alternative if it turns out that the it is more profitable. This is tantamount to introducing a fitness criterion in the selection of behavioural rules. In this sense the model is also in the tradition of the literature on evolutionary dynamics (Kirman(1993), Brock and Hommes(1997), Lux(1998)).

The paper proceeds in different steps. We start by presenting the simplest possible version of the model. In this model there is no feedback from the goods market and there are no transactions costs. We show how this leads to endemic bubbles and crashes. In the next step we introduce a feedback from the goods market, first by endogenizing the current account, and then by introducing the assumption that it is costly to trade goods and services internationally. It will be shown that these complications alter substantially the nature of the dynamics in the foreign exchange markets. Finally, we discuss the empirical implications of the model.

2 The model

In this section we develop the simple version of the exchange rate model. The model consists of three building blocks. First, agents select their optimal portfolio using a mean-variance utility framework. Here we follow the tradition of mainstream economics in that agents are utility maximizing individuals. Second, these agents make forecasts about the future exchange rate based on simple but different rules. In this second building block we introduce concepts borrowed from the behavioural finance literature. Third, agents evaluate these rules ex-post by comparing their risk-adjusted profitability. Thus, the third building block relies on an evolutionary dynamics.

2.1 The optimal portfolio

We assume agents of different types i depending on their beliefs about the future exchange rate. Each agent can invest in two assets, a domestic and a foreign one. The agents' utility function can be represented by the following equation:

$$U(W_{t+1}^i) = E_t(W_{t+1}^i) - \frac{1}{2}\mu V^i(W_{t+1}^i) \quad (1)$$

where W_{t+1}^i is the wealth of agent of type i at time $t+1$, E_t is the expectation operator, μ is the coefficient of risk aversion and $V^i(W_{t+1}^i)$ represents the conditional variance of the wealth of agent i . The wealth is specified as follows:

$$W_{t+1}^i = (1 + r^*) s_{t+1} d_t^i + 1 + r (W_t^i - s_t d_t^i) \quad (2)$$

where r and r^* are respectively the domestic and the foreign interest rates, s_{t+1} is the exchange rate at time $t+1$, $d_{i,t}$ represents the holdings of the foreign assets by agent of type i at time t . Thus, the first term on the right-hand side of 2 represents the value of the foreign portfolio in domestic currency at time $t+1$, while the second term represents the value of the domestic portfolio at time $t+1$.

Substituting equation 2 in 1 and maximising the utility with respect to $d_{i,t}$ allows us to derive the standard optimal holding of foreign assets by agents of type i :

$$d_{i,t} = \frac{(1 + r^*) E_t(s_{t+1}) - (1 + r) s_t}{\mu \sigma_{i,t}^2} \quad (3)$$

The optimal holding of the foreign asset depends on the expected excess return corrected for risk. The market demand for foreign assets at time t is the sum of the individual demands, i.e.:

$$\sum_{i=1}^N n_{i,t} d_{i,t} = D_t \quad (4)$$

where $n_{i,t}$ is the number of agents of type i .

Market equilibrium implies that the market demand is equal to the market supply Z_t which we assume to be exogenous¹. Thus,

$$Z_t = D_t \quad (5)$$

Substituting the optimal holdings into the market demand and then into the market equilibrium equation and solving for the exchange rate s_t yields the equilibrium exchange rate:

¹ The market supply is determined by the net current account and by the sales or purchases of foreign exchange of the central bank. We assume both to be exogenous here. In section we will endogenize the current account.

$$s_t = \left(\frac{1+r^*}{1+r} \right) \frac{1}{\sum_{i=1}^N \frac{w_{i,t}}{\sigma_{i,t}^2}} \left[\sum_{i=1}^N w_{i,t} \frac{E_t^i(s_{t+1})}{\sigma_{i,t}^2} - \mu \frac{Z_t}{1+r} \right] \quad (6)$$

where $w_{i,t} = n_{i,t}/N$ is the weight (share) of agent i .

Thus the exchange rate is determined by the expectations of the agents, E_t^i , about the future exchange rate. These forecasts are weighted by their respective variances $\sigma_{i,t}^2$. When agent's i forecasts have a high variance the weight of this agent in the determination of the market exchange rate is reduced.

2.2 The forecasting rules

We now specify how agents form their expectations of the future exchange rate and how they evaluate the risk of their portfolio.

We start with an analysis of the rules agents use in forecasting the exchange rate. We take the view that individual agents are overwhelmed by the complexity of the informational environment, and therefore use simple rules to make forecasts. Here we describe these rules. In the next section we discuss how agents select the rules.

We assume that two types of forecasting rules are used. One is called a "fundamentalist" rule, the other a "chartist" (technical trading) rule². The agents using a fundamentalist rule, the "fundamentalists", base their forecast on a comparison between the market and the fundamental exchange rate, i.e. they forecast the market rate to return to the fundamental rate in the future. In this sense they use a negative feedback rule that introduces a mean reverting dynamics in the exchange rate. The speed with which the market exchange rate returns to the fundamental is assumed to be determined by the speed of adjustment in the goods market which is assumed to be in the information set of the fundamentalists (together with the fundamental exchange rate itself). Thus, the forecasting rule for the fundamentalists is :

$$E_t^f(\Delta s_{t+1}) = -\psi(s_{t-1} - s_{t-1}^*) \quad (7)$$

where s_t^* is the fundamental exchange rate at time t , which is assumed to follow a random walk and $0 < \psi < \infty$. The fundamental exchange rate is the value of the exchange rate that equilibrates the current account. At this stage, however, we do not model the goods market. As a result, the fundamental exchange rate is exogenous. We return to this issue in a later section.

The agents using technical analysis, the "chartists", forecast the future exchange rate by extrapolating past exchange rate movements. Their forecasting rule can be specified as :

$$E_t^c(\Delta s_{t+1}) = \beta \sum_{i=1}^T \alpha_i \Delta s_{t-i} \quad (8)$$

² This idea of distinguishing between fundamentalist and chartist rules was first introduced by Frankel and Froot().

Thus, the chartists compute a moving average of the past exchange rate changes and they extrapolate this into the future exchange rate change. The degree of extrapolation is given by the parameter β . Note that chartists take into account information concerning the fundamental exchange rate *indirectly*, i.e. through the exchange rate itself. In addition, chartist rules can be interpreted as rules that attempt to detect "market sentiments". In this sense the chartist rule can be seen as reflecting herding behaviour³.

It should be stressed that both types of agents, chartists and fundamentalists, use partial information. Thus our approach differs from the approach in the tradition of rational expectations models in which it is assumed an asymmetry in the information processing capacity of agents. In the latter approach some agents, the "rational" agents, are assumed to use all available information, while other agents, "noise traders", do not use all available information. Such an asymmetry is interesting in order to facilitate the mathematical analysis of the models. However, the basis on which such an asymmetry can be invoked remains unclear. In contrast with this tradition, we take the view that the informational complexity is similar for all agents, and that none of them can be considered to be superior on that count.

We now analyse how fundamentalists and chartists evaluate the risk of their portfolio. The risk is measured by the variance terms in equation 6, which we define as the weighted average of the squared (one period ahead) forecasting errors made by chartists and fundamentalists, respectively. Thus,

$$\sigma_{i,t} = \sum_{k=1}^{\infty} \gamma_k [E_{t-k}^i (s_{t-k+1}) - s_{t-k+1}]^2 \quad (9)$$

where γ_k are geometrically declining weights, and $i = f, c$.

2.3 Fitness of the rules

The next step in our analysis is to specify how agents evaluate the fitness of these two forecasting rules. The general idea that we will follow is that agents use one of the two rules, compare their (risk adjusted) profitability *ex post* and then decide whether to keep the rule or switch to the other one. Thus, our model is in the logic of evolutionary dynamics, in which simple decision rules are selected. These rules will continue to be followed if they pass some "fitness" test (profitability test). Another way to interpret this is as follows. When great uncertainty exists about how the complex world functions, agents use a trial and error strategy. They try a particular forecasting rule until they find out that other rules work better. Such a trial and error strategy can be considered to be a rational strategy when agents cannot understand the full complexity of the underlying model.

³There is a large literature on the use of chartist analysis (technical analysis). This literature makes clear that chartism is widely used in the foreign exchange markets. See Cheung and Chinn(1989), Taylor and Allen(1992), Cheung et al(1999), Mentkhoff(1997) and (1998).

In order to implement this idea we use an approach proposed by Brock and Hommes(1997) which consists in making the weights of the forecasting rules a function of the relative profitability of these rules, i.e. ⁴:

$$w_{c,t} = \frac{\exp [\gamma \pi'_{c,t-1}]}{\exp [\gamma \pi'_{c,t-1}] + \exp [\gamma \pi'_{f,t-1}]} \quad (10)$$

$$w_{f,t} = \frac{\exp [\gamma \pi'_{f,t-1}]}{\exp [\gamma \pi'_{c,t-1}] + \exp [\gamma \pi'_{f,t-1}]} \quad (11)$$

where $\pi'_{c,t-1}$ and $\pi'_{f,t-1}$ are the risk adjusted net profits made by chartists' and fundamentalists' forecasting the exchange rate in period $t - 1$, i.e. $\pi'_{c,t-1} = \pi_{c,t-1} - \mu \sigma_{c,t-1}^2$ and $\pi'_{f,t-1} = \pi_{f,t-1} - C - \mu \sigma_{f,t-1}^2$. We assume that the fundamentalists make a fixed cost C for the collection and processing of fundamental information, while the collection of information by chartists is assumed to be costless⁵.

Equations 10 and 11 can be interpreted as follows. When the risk adjusted profits of the chartists' rule increases relative to the risk adjusted net profits of the fundamentalists rule, then the share of agents who use chartist rules increases in period t increases, and vice versa. This parameter γ measures the rate with which the chartists and fundamentalists revise their forecasting rules. With an increasing γ agents revise their forecasts very frequently. In the limit when γ goes to infinity agents revise the forecasting rules instantaneously. When γ is low, chartists and fundamentalists revise their forecasts relatively slowly. When γ is equal to zero they do not revise their rules. In the latter case the fraction of chartists and fundamentalists is constant and equal to 0.5. Thus, γ is a measure of inertia in the decision to switch to the more profitable rule. As will be seen, this parameter is of great importance in generating bubbles.

Chartists and fundamentalists make a profit when they correctly forecast the direction of the exchange rate movement. They make a loss if they wrongly predict the direction of its movements. The profit (the loss) they make equals the one-period return of investing \$1.

3 Solution of the model

In this section we investigate the properties of the solution of the model. We first study the deterministic solution of the model. This will allow us to analyse the characteristics of the solution that are not clouded by exogenous noise. Then, we analyse the stochastic version of the model, i.e. we introduce stochastics in

⁴This specification of the decision rule is often used in discrete choice models. For an application in the market for differentiated products see Anderson, de Palma, and Thisse(1992). The idea has also been applied in financial markets, by Brock and Hommes (1997) and by Lux(1998).

⁵This asymmetry in the treatment of the cost of information for fundamentalists and chartists is not crucial for our results.

the exogeneous variables. We use simulation techniques since the non-linearities do not allow for a simple analytical solution. We select "reasonable" values of the parameters, i.e. those that come close to empirically observed values. As we will show later these are also parameter values for which the model replicates the observed statistical properties of exchange rate movements. We will also subject such results to an extensive sensitivity analysis.

We start with an analysis of the deterministic model. In figure 1 we show the solutions of the exchange rate for different initial conditions. These are fixed-point solutions (attractors). We plot such solutions as a function of the different initial conditions⁶. On the horizontal axis we set out the different initial conditions. These are initial shocks to the deterministic system. The vertical axis shows the solutions corresponding to these different initial conditions. The fundamental exchange rate was normalized to 0. We find two types of fixed point solutions. First, for small disturbances in the initial conditions the fixed point solutions coincide with the fundamental exchange rate. We call these solutions the *fundamental solutions*. Second, for large disturbances in the initial conditions, the fixed point solutions diverge from the fundamental. We will call these attractors, *bubble attractors*. It will become clear why we label these attractors in this way. The larger is the initial shock (the noise) the farther the fixed points are removed from the fundamental exchange rate. The border between these two types of fixed points is characterised by discontinuities. This has the implication that in the neighborhood of the border a small change in the initial condition (the noise) can have a large effect on the solution.

The different nature of these two types of fixed point attractors can also be seen from an analysis of the chartists' weights that correspond to these different fixed point attractors. We show these chartists weight as a function of the initial conditions in figure ??.

We find, first, that for small initial disturbances the chartists' weight converges to 50% of the market. Thus when the exchange rate converges to the fundamental rate, the weight of the chartists and the fundamentalists are equal to 50%. For large initial disturbances, however, the chartists' weight converges to 1. Thus, when the chartists take over the whole market, the exchange rate converges to a bubble attractor. The meaning of a bubble attractor can now be understood better. It is an exchange rate equilibrium that is reached when the number of fundamentalists has become sufficiently small (the number of chartists has become sufficiently large) so as to eliminate the effect of the mean reversion dynamics. It will be made clearer in the next section why fundamentalists drop out of the market. Here it suffices to understand that such equilibria exist. It is important to see that these bubble attractors are fixed point solutions. Once we reach them, the exchange rate is constant. The chartists' expectations are then model consistent, i.e. chartists who extrapolate the past movements, forecast no change. At the same time, since the fundamentalists have left the market, there is no force acting to bring back the exchange rate to its fundamental value. Thus

⁶These fixed point solutions of the exchange rate were obtained by running simulations of 100,000 periods. Each time the exchange rate converged to a fixed point.

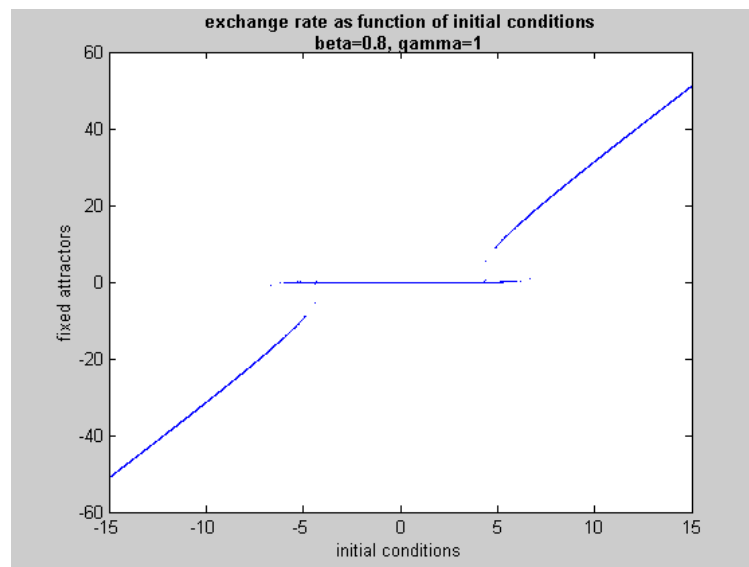


Figure 1:

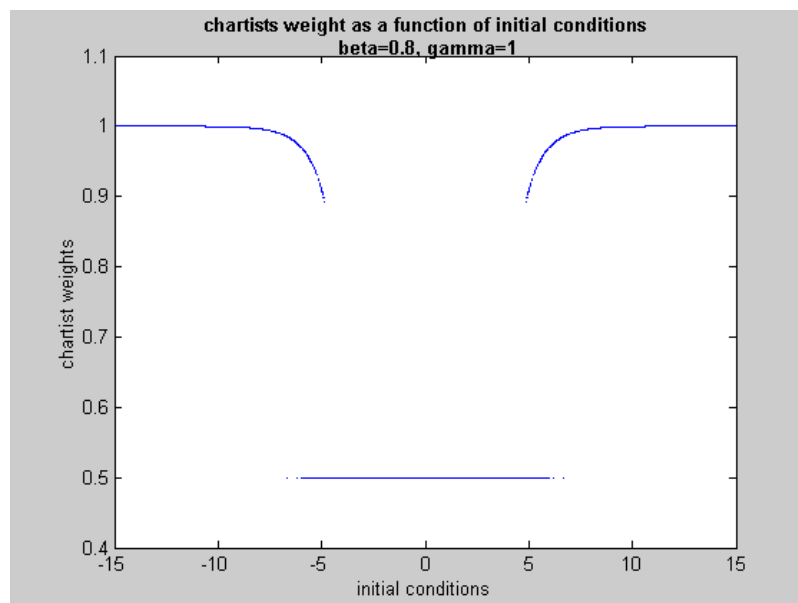


Figure 2:

two types of equilibria exist: a fundamental equilibrium where chartists and fundamentalists co-exist, and a bubble equilibrium where the chartists have crowded out the fundamentalists⁷. In both cases, the expectations of the agents in the model are consistent with the model's outcome.

These two types of equilibria differ in another respect. The *fundamental* equilibrium can be reached from many different initial conditions. It is locally stable, i.e. after small disturbances the system returns to the same (fundamental) attractor. In contrast there is one and only one initial condition that will lead to a particular *bubble* equilibrium. This implies that a small disturbance leads to a displacement of the bubble solution. Note again that the border between these two types of equilibria is characterized by discontinuities and complexity, i.e. small disturbances can lead to either a fundamental or a bubble equilibrium.

It is useful to compute the attractors for different values of the fundamental exchange rate while keeping initial conditions constant. We show such an exercise in figure 3. We now present different fundamental values of the exchange rate on the horizontal axis while keeping the initial condition unchanged. We have set the initial condition for the exchange rate equal to 4⁸. We obtain the following results. First when the fundamental shock and the initial condition are opposite in sign, the exchange rate converges to its fundamental value. This can be seen by the fact that for negative values of the fundamental shocks, the attractors are on a 45° line so that the equilibrium exchange rate equals its fundamental value. In the range of fundamental shocks between 0 and 4 we obtain bubble equilibria. This is the range in which the initial shock (noise) has the same sign as the fundamental shock. When the positive fundamental shock becomes large relative to the positive initial shock the system returns to a fundamental equilibrium. Thus, bubble equilibria arise when the fundamental shock and the noise have the same sign, and when the noise is relatively large relative to the fundamental shock. With sufficiently large fundamental shocks (relative to the noise) the equilibrium exchange rate is forced back to its fundamental value. In appendix 1 we show some additional simulations for smaller and larger initial conditions. These simulations confirm that as the noise increases relative to the fundamental shocks, the range of bubble equilibria increases and vice versa.

The previous results allow us to understand not only why bubbles can arise. They also shed light on why bubbles tend to crash. The noise that triggers a bubble is temporary. Fundamental shocks, however, typically have a large permanent component⁹. Thus, in a stochastic environment small fundamental shocks accumulate to large cumulative fundamental changes. These cumulative changes in the fundamental exchange rate at some point become overwhelming leading to a crash. We will return to this result when we present the stochastic

⁷Note that the intermediate points, i.e. when chartists' weight is less than 1 the solution has not converged yet to fixed points. Fundamentalists hold a very small share in the market which exerts some mean reverting force. However their influence is offset by the chartists pressure. In figure xxx the simulation results are for T=100000.

⁸Our results, however, are not affected qualitatively by the choice of this number.

⁹In the simulations reported here a fundamental shock is permanent.

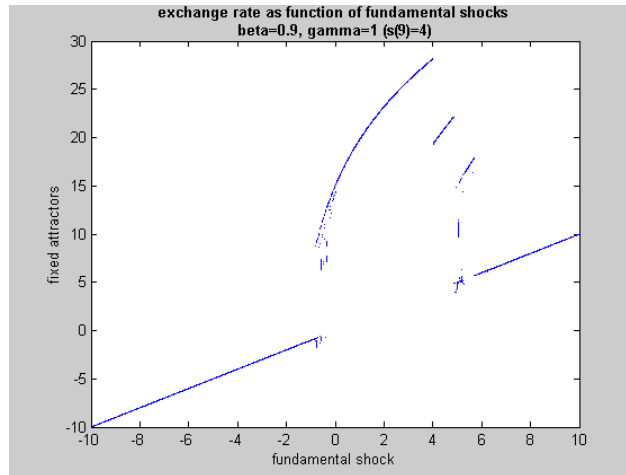


Figure 3:

simulations of the model.

4 The anatomy of bubbles and crashes

In the previous section we identified the existence of two different types of fixed point solutions, i.e. a fundamental solution characterised by the fact that the exchange rate converges to its fundamental value while chartists and fundamentalists "co-habitate", and a bubble solution in which the exchange rate deviates from its fundamental value and in which chartists dominate the market. In this section we show that in combination with stochastic shocks in the fundamental exchange rate these features of the model lead to the emergence of bubbles and crashes.

The way we proceed is to calibrate the model in such a way that it replicates the statistical properties of observed exchange rate movements. We describe this procedure in section ???. Here we present the results of simulations in the time domain using this calibrated model. We start by presenting a case study of a typical bubble and crash scenario as produced by the stochastic version of the model. In the next section we will analyse more systematically the factors that determine the frequency with which such bubbles and crashes occur. Figure 4 top panel shows the exchange rate and its fundamental value in the time domain; the bottom panel shows the weight of the chartists in the same time domain. These two pictures allow us to analyse a number of common features of a typical endogenously generated bubble and crashes in a stochastic environment.

First, once a bubble emerges, it sets in motion bandwagon effects. As the exchange rate moves steadily in one direction, the use of extrapolative forecasting rules becomes more profitable, thereby attracting more chartists in the market.

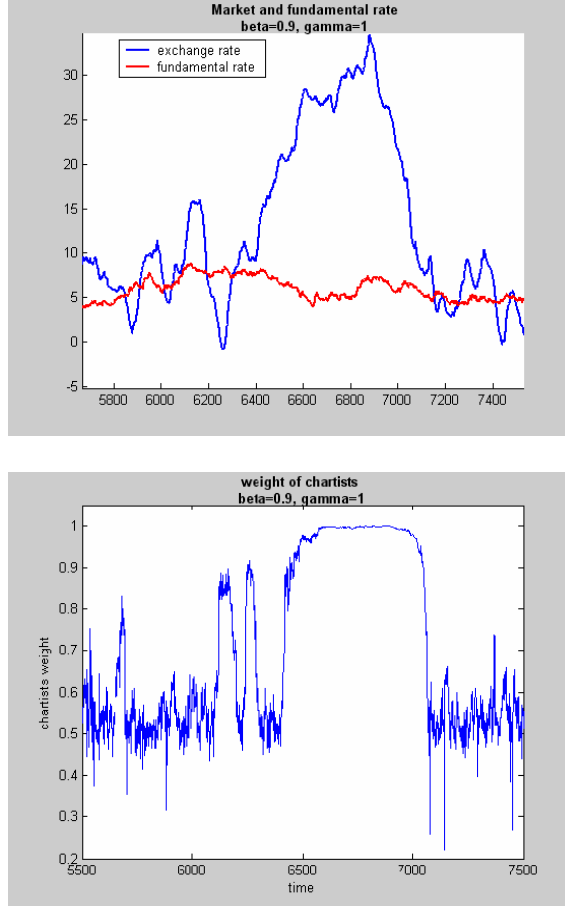


Figure 4:

This is clearly visible from a comparison of the bottom panel with the top panel of figure 4. We observe that the upward movement in the exchange rate coincides with an increase in the weight of chartists in the market. We have checked this feature in many bubbles produced by the model. In appendix 2 we show another example of a bubble, and we present the results of a causality test which shows that the exchange rate leads the weight of chartists during a bubble and the subsequent crash. Thus, typically a bubble starts after the exchange rate has moved in one direction, thereby attracting extrapolating chartists which in turn reinforces the exchange rate movement.

Second, a sustained upward (downward) movement of the exchange rate will not develop into a full scale bubble if at some point the market does not get sufficiently dominated by the chartists. As can be seen figure 4 at the height

of the bubble the chartists have almost 100% of the market. Put differently, an essential characteristic of a bubble is that at some point almost nobody is willing to take a contrarian fundamentalist view. The market is then dominated by agents who extrapolate the bubble into the future. This raises the question of why fundamentalists do not take an opposite position thereby preventing the bubble from developing. After all, the larger the deviation of the exchange rate from the fundamental the more the fundamentalists expect to make profit from selling the foreign currency. Yet they do not, and massively leave the market-place to the chartists. The reason why they do so, is that during the bubble phase the profitability of chartism increases dramatically precisely because so many chartists enter the market thereby pushing the exchange rate up and making chartism more profitable. In addition, during the bubbles phase fundamentalists make large forecasting errors, reducing their "appetite" for using fundamentalists forecasting rules. As a result, investors who are continuously acting against the trend will make losses. There is therefore a self-fulfilling dynamics in the profitability of chartism and losses for the fundamentalists.

The limit of this dynamics is reached when chartists have crowded out the fundamentalists. We arrive at our next characteristics of the bubble-crash dynamics. When the chartists' share is close to 100% the self-reinforcing upward movement in the exchange rate and in profitability slows down, increasing the expected relative profitability of fundamentalists. This is so because while the bubble developed, the *expected* profits from fundamentalism also increased. However, these were overwhelmed by the self-fulfilling profitability of chartism. When the latter tends to slow down, fundamentalism becomes attractive again. A small movement of the exchange rate can then trigger a fast decline in the share of chartism, back to its normal level of a tranquil market. A crash is set in motion.. .

The dynamics of bubbles and crashes we obtain in our simulated data is asymmetric, i.e. bubbles are relatively slow and crashes relatively rapid. An intuitive explanation of this result is that during a bubble chartists and fundamentalists rules push the exchange rate in two different directions, i.e. the positive feedback from chartists and the negative feedback from fundamentalists have the effect of slowing down the build-up of a bubble. In a crash the fundamentalists' mean reverting force is reinforced by the chartists' behaviour. As a consequence, the speed of a crash is higher than the speed with which a bubble arises.

This asymmetry between bubbles and crashes is a well-known empirical phenomenon in financial markets (see Sornette(2003)). In figure 5 we present the DEM-USD for the period 1980-1987, which is a remarkable example of a bubble in foreign exchange markets. As it can be seen from figure 5 the upward movement in the DEM-USD exchange rate is gradual and builds up momentum until a sudden and much faster crash occurs which brings the exchange rate back to its value of tranquil periods.

Our model provides a simple explanation for this empirical phenomenon. Note the contrast with RE-models of bubbles and crashes. These predict that

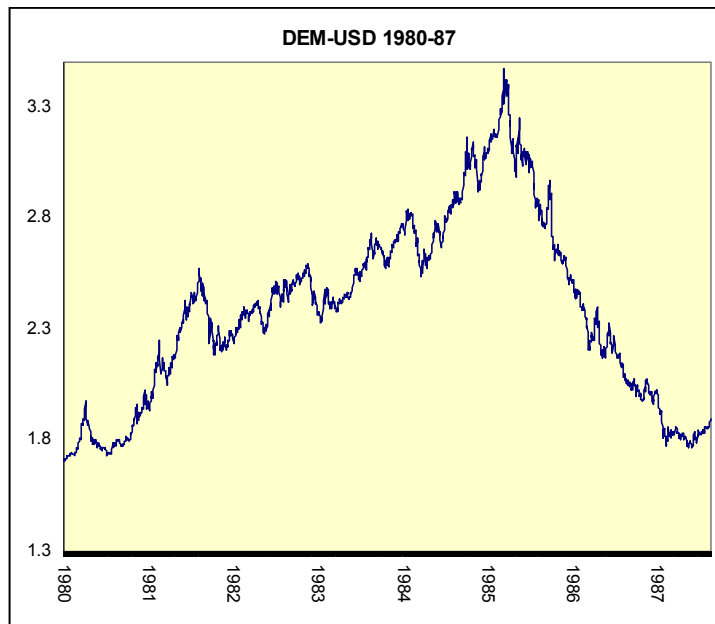


Figure 5:

bubbles and crashes are symmetric (Blanchard(1979) and Blanchard&Watson(1982))¹⁰.

5 The frequency of bubbles

In the previous sections we showed that a very simple model is capable of generating bubbles and crashes that have the basic features of bubbles and crashes observed in financial markets. All we need is the existence of agents who maximize the utility of their portfolio, make forecasts based on the use of different forecasting rules and switch to the more profitable of these rules. An important issue here concerns the frequency with which bubbles occur in our model. We analyse this issue by simulating the stochastic version of the model and by counting the number of periods the exchange rate is involved in a bubble. We define a bubble here to be a deviation of the exchange rate from its fundamental value by more than three times the standard deviation of the fundamental variable for a significant interval of time. We have set this interval equal to 20 periods. We show the result of such an exercise in figure 6 for different values of the chartists' extrapolation parameter β . It shows the percentage of time

¹⁰Moreover, the symmetry of bubbles and crashes neglects the time scale dynamics in which a long term change is an accumulation of short term changes. Thus, the symmetry property in foreign exchange markets is an approximation which holds only in the (very) short-run (see Johansen and Sornette (1999)).

the exchange rate is involved in a bubble dynamics. We observe that when β is smaller than 0.9 the frequency of the occurrence of bubbles is small. For values of β larger than 0.9 this frequency increases exponentially. Thus the extrapolation by chartists is an important parameter affecting the frequency with which bubbles occur. The results obtained in figure 6 are determined by the existence

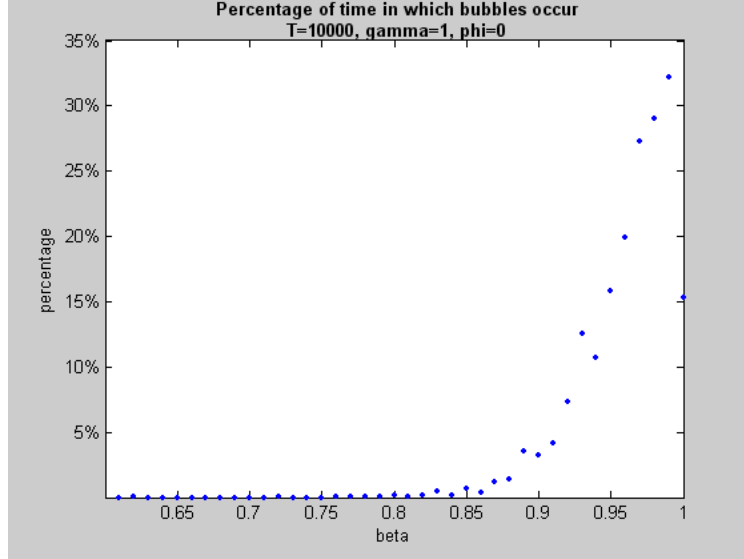


Figure 6:

of bubble equilibria in the deterministic version of the model. Therefore, it is useful to connect figure 6 with a figure that plots the exchange rate solutions obtained in the deterministic version of the model. We show this in figure 7 where we set out the equilibrium exchange rate on the vertical axis as a function of β (horizontal axis). We see that for values of $\beta < 0.85$, the exchange rate converges to its fundamental value (normalized to 0). When $\beta > 0.85$ we obtain bubble equilibria that increasingly deviate from the fundamental value. Note that when $0.88 < \beta < 0.9$ we have a complex structure. The equilibrium jumps back and forth between the fundamental and a bubble. Thus, in a way figure 7 predicts what should happen in a stochastic environment. When $\beta < 0.85$ the equilibrium exchange rate converges to its fundamental value. Around this fundamental value a basin of attraction exists which pulls the exchange rate. Only when the noise is sufficiently high will the exchange rate be attracted to a bubble equilibrium (see figure 1 where we showed that with $\beta = 0.8$ a sufficiently high initial shock will pull the exchange rate towards a bubble equilibrium). Thus, when $\beta < 0.85$ bubbles will be relatively infrequent events. When β increases above 0.85, however, bubble equilibria appear, increasing the probability of bubbles in a stochastic environment. Note however that even when β is large enough (e.g.

0.9) to produce only bubble equilibria in the deterministic version of the model, the probability of a bubble is not 1 in the stochastic version. The reason is that the noise can lead the exchange rate within the basin of attraction around the fundamental or, more importantly, that the shocks in the fundamentals displace the basin of attraction leading to a crash in the bubble. The frequency of

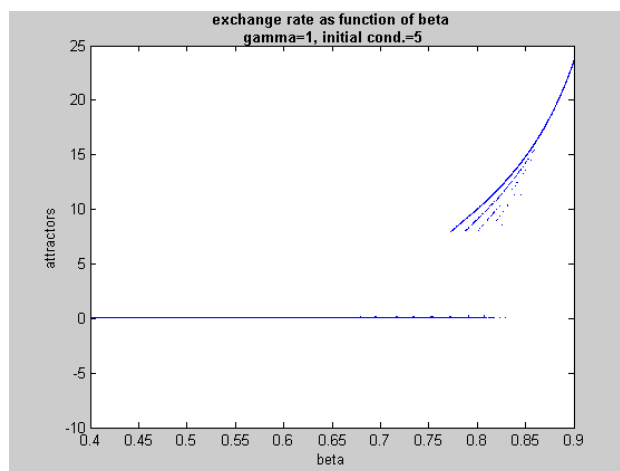


Figure 7:

the occurrence of bubbles also depends on the parameter γ which measures the rate with which chartists and fundamentalists revise their forecasting rules. We have called this parameter *rate of revision*. In a way, γ also measures the speed with which agents learn about the profitability of the other rule and revise their forecasts. The lower is this parameter the less frequently agents will revise their forecasting rules. In the limit when $\gamma = 0$ the agents never revise their forecasts which could be interpreted as a world which agents perceive to be stationary.

In order to illustrate the importance of this parameter, we first show the results of the deterministic simulations in figures 8. We observe that for values of γ lower than (approximately) 1.2 the exchange rate converges to its fundamental value. For higher values we obtain bubble equilibria¹¹. Note also a zone of complexity where the location of the bubble equilibria is very sensitive to small changes in the parameter γ . In figure 9 we show the results of the stochastic simulation under the same parameter configuration. We observe that for low values of γ the occurrence of bubbles is very infrequent. As γ increases the frequency of bubbles increases significantly.

The previous results allow us to shed some additional light on the nature of bubbles and crashes. As we have seen before, bubbles arise because agents are attracted by the profitability of the extrapolating (chartist) rule, and this

¹¹In appendix 3 we show a similar figure where we have set $\beta = 0.9$. In that case the critical value of γ which produces bubble equilibria is lowered.

attraction in turn makes this forecasting rule more profitable, leading to a self-fulfilling increase in profitability. For this dynamics to work, agents' decision to switch must be sufficiently sensitive to the relative profitabilities of the rules. If it is not, no bubble equilibria can arise, as is the case when γ does not exceed 1. The larger is γ the more likely it is that these self-fulfilling bubble equilibria arise. The interesting aspect of this result is that in a world where agents quickly react to changing profit opportunities, bubbles become more likely than in a world where agents do not react quickly to these new profit opportunities.

The policy implication of this result is that by increasing the inertia in the system so that agents react less quickly to changes in relative profitabilities of forecasting rules, the authorities could reduce the probability of the occurrence of bubbles. How this can be done and whether some form of taxation of exchange transactions can do this, is a question we want to analyse in future research.

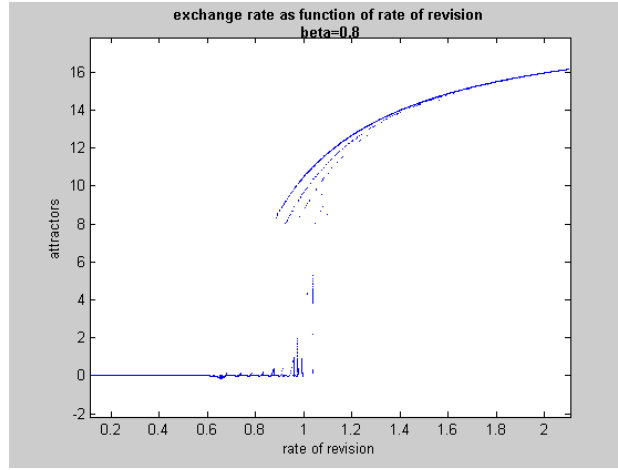


Figure 8:

6 The model with an endogenous current account

In the previous sections we used a model in which the exchange rate is often disconnected from its fundamentals. This happens despite the fact that the fundamentalists know the fundamental value of the exchange rate and use that information to forecast the future exchange rate. The selffulfilling expectations of the chartists regularly crowds out the fundamentalists from the market thereby weakening the mean reversion forces in the model.

One drawback of the model is that when the fundamentalists leave the market, there is no mean reverting force present anymore because the goods market

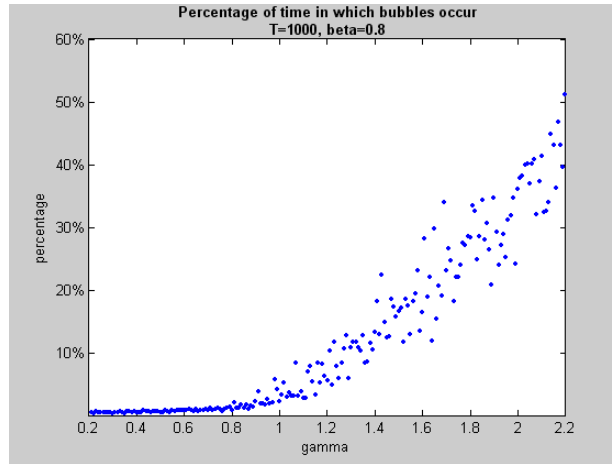


Figure 9:

is kept outside the model. Incorporating the goods market provides an independent channel through which the exchange rate can be forced back to its equilibrium even if the fundamentalists are temporarily absent. In this section we introduce the goods market into the model and allow for an interaction between the goods market and the foreign exchange market.

We use the same model as in the previous sections, except that we now endogenize the current account. It will be remembered that the current account determines the net supply of foreign assets in the model. Thus we define the current account as

$$\Delta Z_t = X_t.$$

The current account consists of the trade balance and the net income from net foreign assets. At this stage of the analysis we will concentrate on the role of the trade balance and we disregard the role of the net income from foreign assets. Thus, we will set the current account equal to the trade balance.

There is a large literature on the determinants of the trade balance. Here we focus on the role of the exchange rate. We postulate that an increase (decline) of the exchange rate leads to an improvement (deterioration) of the trade balance, and thus of the current account, *ceteris paribus*. The reason is that an increase in the exchange rate (a depreciation) stimulates exports and discourages imports. The opposite holds for a decline in the exchange rate (an appreciation). This relationship between the trade balance and the exchange rate holds as a *ceteris paribus* proposition. In particular, it holds for a given domestic and foreign price level. Put differently, it is the real exchange rate that matters for exporters and importers. In the context of our model it is the difference between the nominal exchange rate and the fundamental exchange rate that matters for the decisions of exporters and importers. The fundamental exchange rate can then be considered to be the difference between the domestic and the foreign price

levels. This leads us to postulate the following relationship between the trade account (the current account) and the exchange rate:

$$X_t = \rho_x X_{t-1} + (1 - \rho_x) \epsilon (s_{t-1} - s_{t-1}^*) \quad (12)$$

that is, when the exchange rate, s_{t-1} , exceeds its fundamental value, s_{t-1}^* , the current account improves and vice versa. The sensitivity of the current account with respect to the exchange rate is given by the parameter $\epsilon \geq 0$. This parameter synthesises the reactions of exporters and importers to changes in the exchange rate. It can easily be derived from a model of the export and import markets. We will call this parameter, the trade elasticity, or "elasticity" for short. We also assume that the reactions of the exporters and importers is not instantenous. The speed with which the trade account adjusts to the exchange rate is given by ρ_x . Note that $0 \leq \rho_x \leq 1$. When ρ_x is close to 1, there is a lot of inertia in the trade account and the adjustment to the exchange rate is very slow. The opposite holds when ρ_x is close to 0. Equation 12 can also be rewritten as follows:

$$X_t = \epsilon (1 - \rho_x) \sum_{i=1}^{\infty} \rho_x^{i-1} (s_{t-i} - s_{t-i}^*) \quad (13)$$

i.e., the trade account reacts to all past exchange rates with geometrically declining weights. From 13 it can be seen that when $s_t = s_t^*$ for all t , $X_t = 0$. Thus, when the exchange rate equals its fundamental value, the current account is in equilibrium. Put differently, we have an equivalence between current account equilibrium and fundamental equilibrium of the exchange rate.

We will now assume that the fundamentalists are aware of this. It will be remembered that the forecasting rule of the fundamentalists was assumed to be

$$E_t^f (\Delta s_{t+1}) = -\psi (s_{t-1} - s_{t-1}^*)$$

Taking the equivalence between current account and fundamental equilibrium, the fundamentalists set

$$\psi = -\epsilon (1 - \rho_x) \quad (14)$$

i.e., when the exchange rate deviates from its fundamental value (e.g. its PPP-value), the fundamentalists know that the current account dynamics will ensure that the speed of adjustment of the exchange rate towards equilibrium is given by equation 14.

We now have all the elements to solve the model. We proceed in the same way as before, i.e. we first analyse the deterministic solution, and then we add stochastics to the model.

6.1 Solution of the model

We first set out the solutions for the exchange rate (attactors) as a function of the initial conditions for a particular parameter configuration. (We will perform a sensitivity analysis later). We show the results in figure 10. The contrast with

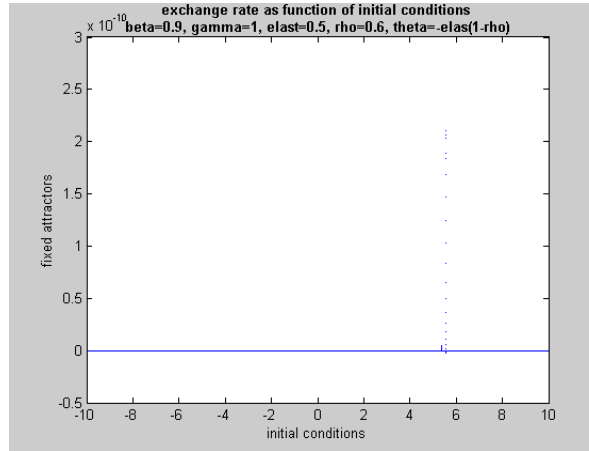


Figure 10:

the results obtained when the current account was exogenous is important. We observe that in the model with exogeneous current account (see equation ??) we obtain bubble equilibria for sufficiently large initial shocks. This is no more the case when the current account is endogenous. Whatever the initial shock the exchange rate converges to the fundamental exchange rate (which as will be remembered was normalised to 0). Thus the endogeneity of the current account adds an important mean reverting process preventing bubble equilibria from arising. Note that this result hinges on a particular value of the elasticity that was selected here. We will return to this when we perform a sensitivity analysis. Other types of equilibria are possible when the current account is endogenous. We show a simulation in which we increase the sensitivity of the chartists and fundamentalists to the profitability of the forecasting rules (the parameter γ). We present the results both for the cases of an endogenous and an exogenous current account (see figures 11 and 12). The most striking feature is that we now obtain chaotic attractors when the current account is endogenous. Note, however, that these chaotic attractors are centered around the fundamental exchange rate. The intuition of this result is that in the absence of endogeneity of the current account, the bubble equilibria are far removed from the fundamental equilibrium when γ is high. Making the current account endogenous introduces a strong mean reverting process ensuring that the exchange rate stays close to the fundamental one, but without preventing erratic movements around the fundamental exchange rate.

We have also analysed the behaviour of the attractors when shocks occur in the fundamental exchange rate, s_t^* . We show the results in figure 13 and 14 for the case with and without endogeneity in the current account. We observe a similar contrast between the two regimes. When the current account is exogenous (figure 14), we obtain bubble equilibria. These disappear when the current

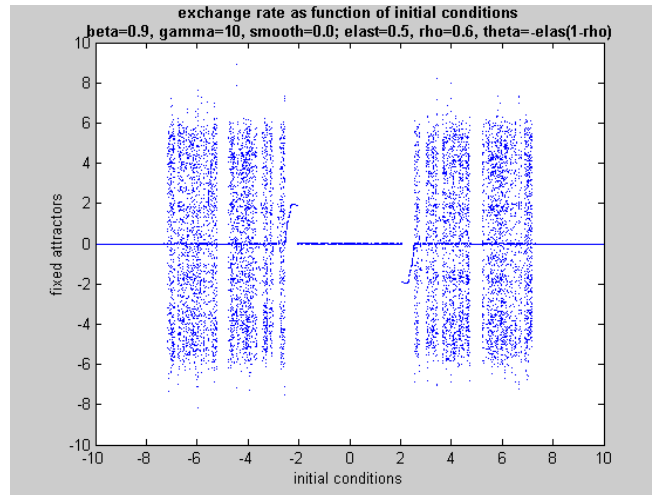


Figure 11:

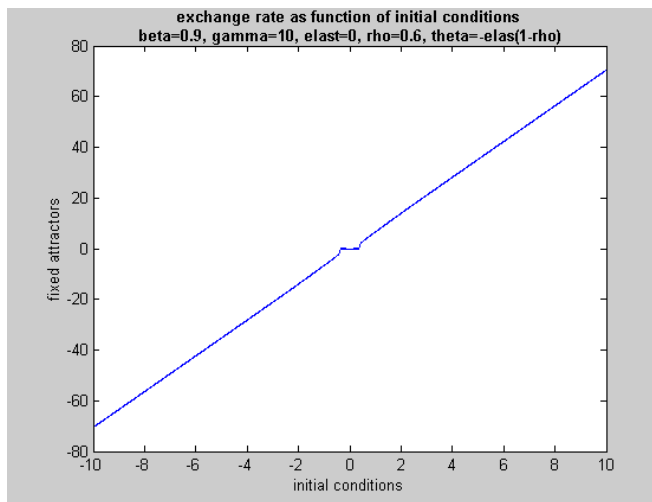


Figure 12:

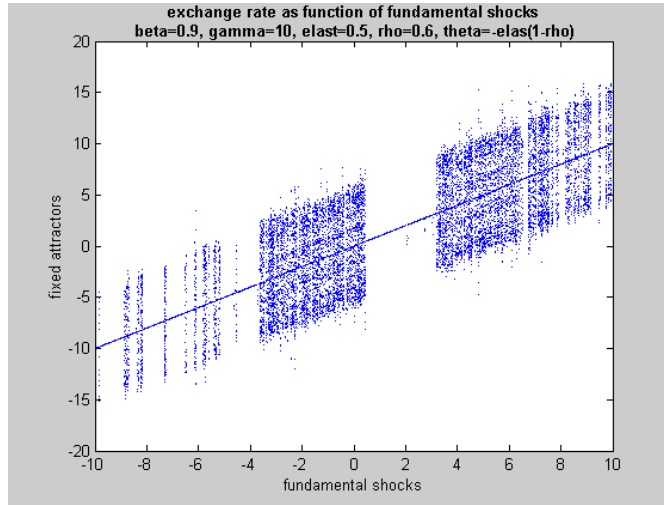


Figure 13:

account is endogenous. Instead bubble equilibria are transformed into chaotic attractors (at least if γ is sufficiently high).

6.2 Sensitivity analysis: the importance of the elasticity

In this section we present the result of applying an extensive sensitivity analysis. We start with an analysis of the importance of the sensitivity of the current account to the exchange rate (parameter ε). As will be remembered, we call this parameter the "elasticity". We show the results in figure 15. As before this figure is comparable to a "bifurcation" diagram. It plots the exchange rate solutions (attractors), obtained after simulating the model over 1000 periods as a function of the elasticity. When the elasticity is zero, the current account does not react to the exchange rate changes. We observe that in that case we obtain a bubble attractor. Note that the exact value of this attractor depends on the initial conditions, as was shown above. When the elasticity exceeds zero the current account reacts endogenously to exchange rate changes. For small values of the elasticity we continue to obtain bubble attractors. These, however, come closer to the fundamental value (which, as before is normalized to 0) as the elasticity increases. For sufficiently high values of the elasticity, we obtain a fundamental equilibrium. There is a range of elasticities for which the exchange rate moves within the confines of a chaotic attractor centered around the fundamental value of the exchange rate. It is also noteworthy that zones of chaotic attractors and fixed point attractors alternate. This phenomenon is sometimes called "intermittency".

In order to shed more light on the importance of the elasticity parameter for

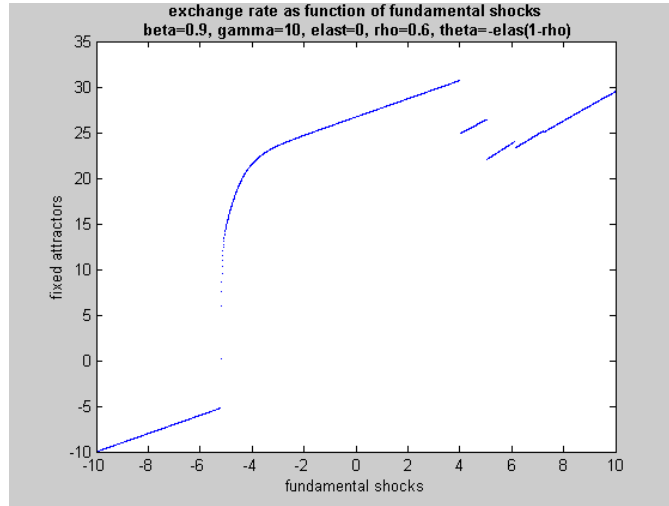


Figure 14:

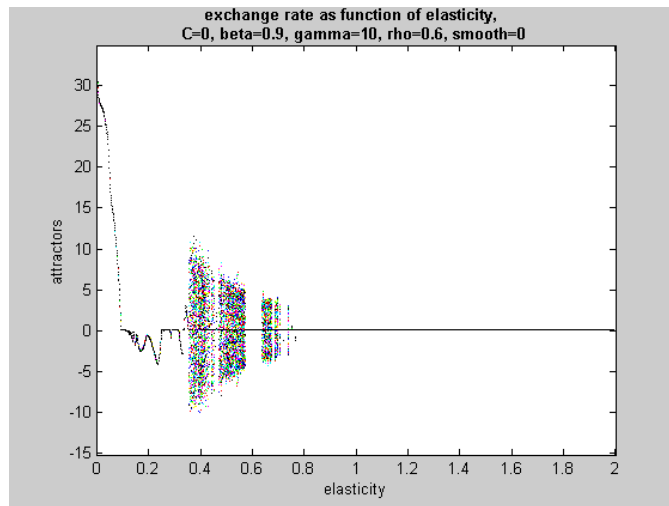


Figure 15:

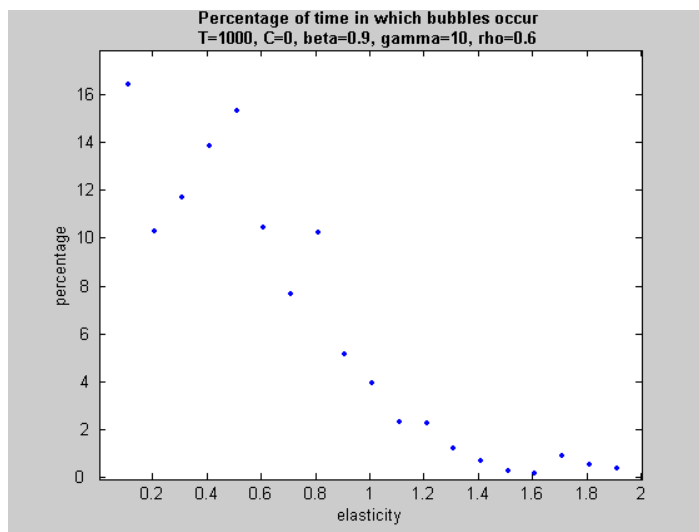


Figure 16:

the occurrence of bubbles we simulated the model in a stochastic environment. We then computed the frequency of the occurrence of bubbles. As before, we define a bubble as a situation in which the exchange rate deviates from its fundamental value by three times the standard deviation of the fundamental during at least 20 consecutive periods. We show the results in figure 16. Each point represents the average frequency of the occurrence of such a bubble for a particular value of the elasticity. These frequencies were obtained by making 100 simulations of 1000 periods for each elasticity. We observe that for low elasticities we obtain a significant frequency of bubbles. For elasticities above 1 the frequency of bubbles drops close to zero.

6.3 Sensitivity analysis: the importance of other parameters

We performed similar sensitivity analysis with respect to other parameters of the model. Figure 17 shows such a sensitivity analysis of the importance of the chartists' extrapolation parameter, β . It can be seen that with an increasing β , we move from fixed point attractors into a zone of chaotic attractors. This contrasts with the case of an exogenous current account. In that case, increases in β lead to bubble equilibria. A similar phenomenon is observed when we allow the parameter γ to increase. As will be remembered this parameter measures the sensitivity of the chartists and fundamentalists to changes in profitability of the forecasting rules. With an exogenous current account increases in γ lead to bubble equilibria, while they lead to chaotic attractors when the current account is endogenous(see figure18).

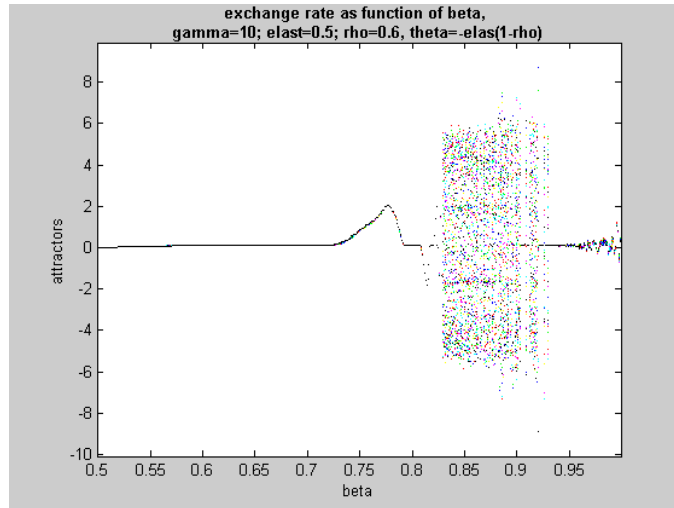


Figure 17:

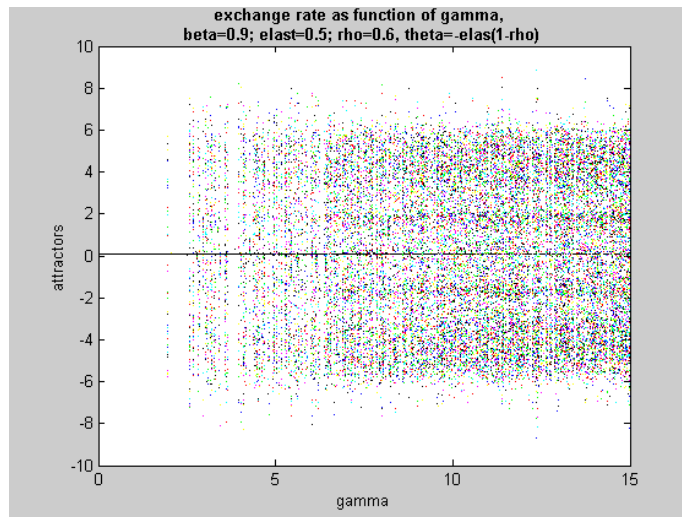


Figure 18:

We conclude from this sensitivity analysis that the endogeneity of the current account changes the dynamics of the exchange rate profoundly. In general, we find that in a model without a feedback from the current account, bubbles and crashes are an endemic feature of the exchange rate dynamics. When, however, the current account reacts to exchange rate changes, this dynamics is changed. The bubbles and crashes dynamics is weakened. Instead a complex, sometimes, chaotic dynamics takes the place of the bubbles dynamics.

7 The model with transactions costs

In this section we add an additional complication, i.e. the existence of transaction costs. There is an increasing body of theoretical literature stressing the importance of transactions costs in the goods markets as a source of non-linearity in the determination of the exchange rate (Dumas(1992), Sercu, Uppal and Van Hulle(1995), Obstfeld and Rogoff(2000)). The importance of transaction costs in the goods markets has also been confirmed empirically (Taylor, Peel, and Sarno(2001), Kilian and Taylor(2001)). In addition, several recent empirical studies report the continued existence of large price differentials for the same traded goods across borders (see Haskel and Wolf(2001) and Engel and Rogers(1995)). These authors document price differentials of up to 40% for identical products in different countries. This indicates that producers apply ‘pricing to market’. Such pricing strategies, however, can only be applied successfully if transaction costs prevent arbitrage. Thus, the large observed price differentials suggest that transactions costs for traded goods are large. In addition, for many services, which are non-traded goods, transactions costs are even higher. (See Obstfeld and Rogoff(2001) who argue that transactions costs are key to understanding the major puzzles in international economics).

We assume that the existence of transactions costs creates a band of inaction, i.e. when the deviation of the exchange rate from its fundamental is lower than the transactions costs, goods arbitrage is not profitable. Put differently, when the exchange rate is within this band, export and import decisions are not sensitive to changes in the exchange rates (as long as these changes do not move the exchange rate outside the band). We, therefore, write that

$$X_t = \rho_x X_{t-1} + (1 - \rho_x) \epsilon (s_{t-1} - s_{t-1}^*) \quad (15)$$

holds if $|s_{t-1} - s_{t-1}^*| \geq C$, where C is the transaction cost in the goods market assumed to be of the iceberg type. Otherwise $X_t = 0$. (In the stochastic version of the model we will then set $X_t = \varepsilon_t$, where ε_t is white noise).

In addition, we assume that fundamentalists are aware of the existence of this band of inaction. As a result, when the exchange rate deviation from its fundamental value is larger than the transaction costs C , then the fundamentalists follow the forecasting rule as in equation 7. More formally,

$$\text{when } |s_{t-1} - s_{t-1}^*| \geq C \quad \text{holds,} \quad \text{then equation 7 applies}^{12}.$$

¹²Note that since $\psi < \infty$ market inefficiencies other than transaction costs continue to play

However when the exchange rate deviations from the fundamental value are smaller than the transaction costs in the goods markets, there is no mechanism that drives the exchange rate towards its equilibrium value. As a result, fundamentalists expect the changes in the exchange rate to follow a white noise process and the best they can do is to forecast no change. More formally, when $|s_t - s_t^*| < C$, then $E_t^f(\Delta s_{t+1}) = 0$. or $E_t^f(\Delta s_{t+1}) = \eta_t$ and η_t is white noise in the stochastic version of the model

7.1 Solution of the model

We proceed as before, i.e. we solve the deterministic part of the model, and present the attractors as a function of the initial conditions. We have selected a value of the elasticity of the current account such that bubbles equilibria do not occur. We show the solution for the same parameter combinations as in the basic model (see figure 1). The results are now affected in a fundamental way. First, all the fixed point attractors are located within the transaction cost band. Second the attractors are arranged in a complex manner. Some attractors are located on a continuous line, others are spread around in a discontinuous way. As a result, small changes in the initial conditions can lead to large displacements in the attractors. As will be shown, this feature of the attractors creates a complex dynamics in a stochastic environment characterised by sensitivity to initial conditions. Note that in the simulations reported here there is no chaotic dynamics, as each attractor is a fixed point. These fixed point attractors are separated by basins of attraction with complex features. These are responsible for the sensitivity to initial conditions.

Before analysing the stochastic solutions it is useful to contrast the attractors obtained in figure 19 with those that one obtains when the current account is exogenous (elasticity = 0). We show these in the following figure 20. We find, as before, that in the absence of a feedback from the current account, the model produces bubble equilibria. Thus the existence of transactions costs does not affect our previous result.

7.2 The model in a stochastic environment

We now proceed in solving the model in a stochastic environment. We show the results in the time domain for a particular combinations of parameters that do not produce bubbles and crashes, i.e. with an elasticity of the current account that is sufficiently large ($\epsilon = 0.5$). We show the results in figure 21. Figure 21 shows the simulated exchange rate together with the fundamental exchange rate. We observe that the exchange rate is disconnected most of the time from its underlying fundamental. This feature is present despite the absence of bubble equilibria. Note that sometimes the exchange rate dynamics

a role when the exchange rate moves outside the transaction costs band. As a result, these inefficiencies prevent the exchange rate from adjusting instantaneously.

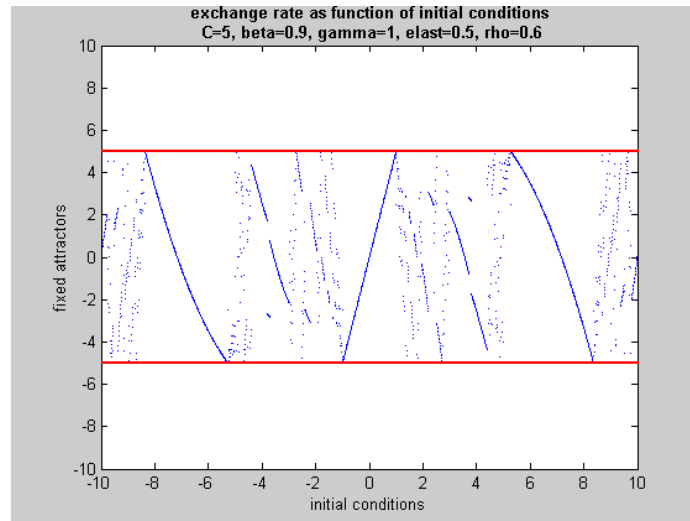


Figure 19:

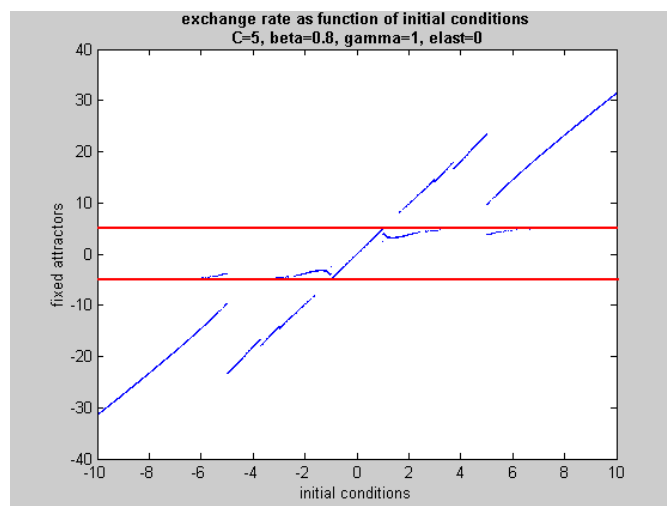


Figure 20:

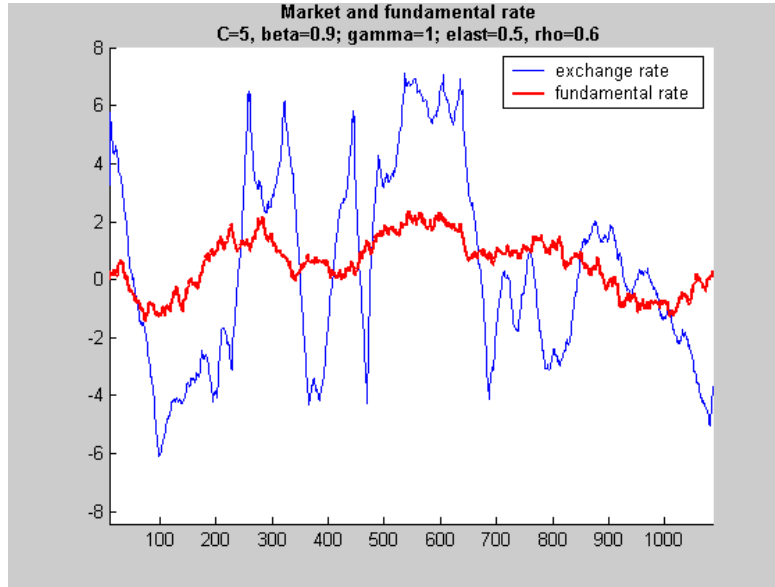


Figure 21:

has the appearance of bubbles and crashes although there is no such underlying dynamics. We will analyse the nature of this "disconnect puzzle" in greater detail when we discuss the empirical relevance of the model. We also observe that the short-term volatility of the simulated exchange rate appears to be significantly higher than the volatility of the underlying fundamental. This feature is related to the sensitivity to initial conditions dynamics: small changes in the underlying stochastics can lead to large displacements of the attractor. This sensitivity to initial conditions can also be illustrated in another way. We simulated the model with two different initial conditions in the exchange rate. In one simulation we set the initial exchange rate equal to 4, in the other to 4.01. All the rest is kept identical in the two simulations (the parameters, the underlying stochastics driving the exogenous variables). The results of figure 22 illustrate the power of the sensitivity to initial conditions. A small disturbance in the initial exchange rate leads to sustained deviations between the two exchange rates, despite the fact that the underlying fundamentals are identical. It appears that the two exchange rates follow a different "history".

One can conclude this section as follows. Without a feedback from the current account the model produces bubbles and crashes. These bubbles and crashes tend to disappear when we allow the current account to react to exchange rate changes. The introduction of transactions costs does not change this result. However, transactions costs create a new, and complex dynamics whereby the exchange rate appears to be disconnected most of the time from the underlying fundamental creating a resemblance with bubbles and crashes. In addition, by

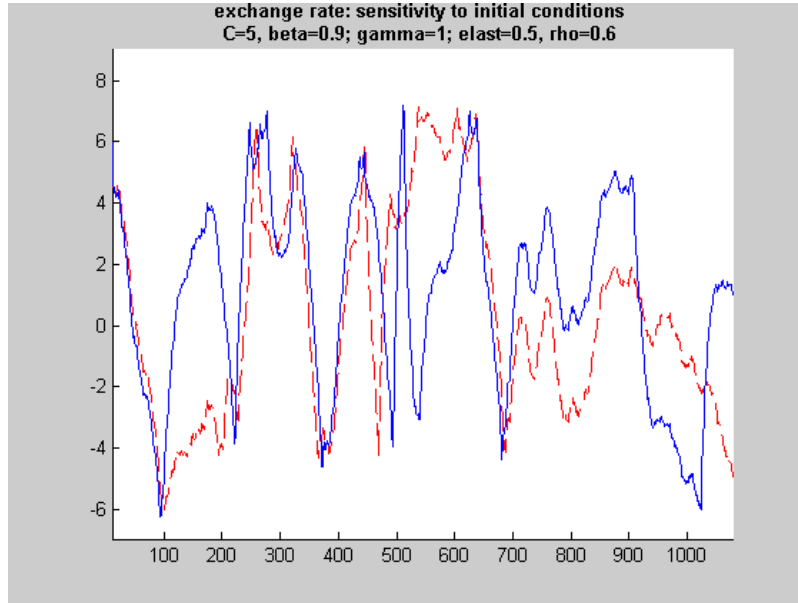


Figure 22:

creating a band of inaction transactions costs are also responsible for the appearance of sensitivity to initial conditions that make it possible for small changes in the initial conditions to have profound effects on the future movements of the exchange rate. Thus, in such a world, history seems to matter.

8 Empirical relevance of the model

In this section we analyse how well our model mimics the empirical anomalies and puzzles that have been uncovered by the flourishing empirical literature. We calibrate the model such that it replicates the observed statistical properties of exchange rate movements. In order to do so we selected a parameter configuration that mimics these properties most closely. We discuss these different statistical properties in the following sections.

8.1 The disconnect puzzle

In the previous section we showed how the model was capable of replicating a widely observed phenomenon, i.e. that the exchange rate is often disconnected from its underlying fundamentals. In that analysis we stressed one dimension of the disconnect puzzle, i.e. the causality (or the lack of it) from the fundamental to the exchange rate. There is, however, another dimension to the disconnect puzzle, which relates to the causality running from the exchange rate to the

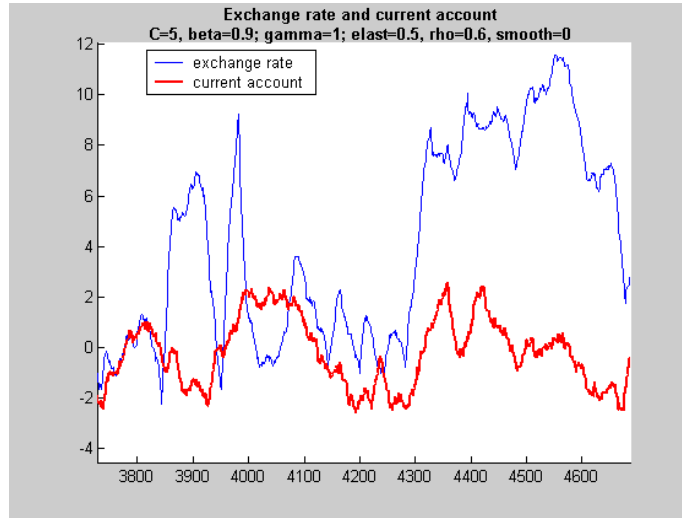


Figure 23:

current account. From the empirical evidence it appears that the exchange rate has a weak and unpredictable influence on the fundamentals, in particular on the current account. (see Obstfeld and Rogoff for a formulation of this puzzle). The version of the model in which the current account reacts endogenously to the exchange rate can be used to shed light on this puzzle. In order to do so we simulated the model assuming an elasticity of 0.5. Thus, we assume that there is a causality running from the exchange rate to the current account. We show two examples of such a simulation for different parameter configurations in figures 23 and 24. The interesting aspect of this simulation is that there are periods in which the current account is influenced by the exchange rate movements. These periods, however, alternate with other periods during which this influence is almost totally absent, making the effect of exchange rate changes on the current account very unpredictable. The intuition of this result is that during periods of turbulence, the effect of exchange rate changes on the current account is weakened. Since turbulent and tranquil periods alternate in unpredictable fashion, the effects of exchange rate changes on the current account also alternate from being predictable to becoming unpredictable.

In order to be more precise about the nature of the disconnect puzzle we analyzed the simulated exchange rate and current account econometrically. We first tested for cointegration of the two series, and found that in general the two series are cointegrated. We then specified a vector error correction (VEC) model in the following way:

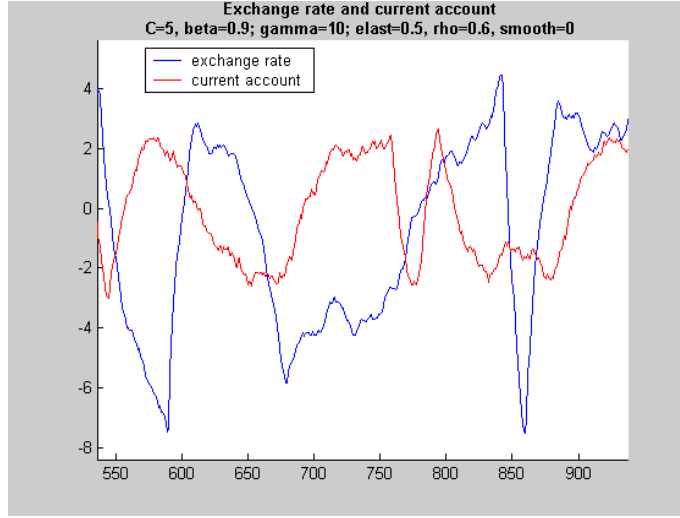


Figure 24:

$$\Delta s_t = \mu(s_{t-1} - \gamma X_{t-1}) + \sum_{i=1}^n \lambda_i \Delta s_{t-i} + \sum_{i=1}^n \phi_i \Delta X_{t-i} \quad (16)$$

$$\Delta X_t = \mu'(s_{t-1} - \gamma' X_{t-1}) + \sum_{i=1}^n \lambda'_i \Delta s_{t-i} + \sum_{i=1}^n \phi'_i \Delta X_{t-i} \quad (17)$$

The first term on the right hand side in both equations are the error correction terms. We have estimated this model for a broad range of parameter values. The result of estimating equation 16 for selected parameter values is presented in table 1 where we have set $C = 5$, $\beta = 0.9$, $\gamma = 1$, $\epsilon = 0.5$ and $\rho_x = 0.6$ and number of lags $n = 5$.

We find that the error correction coefficients (μ and μ') are low in both equations. This suggests that the mean reversion towards the equilibrium exchange rate and current account takes a long time. In particular, only 0.7% and 0.6% of the adjustments take place each period. It should be noted that in the simulations we have assumed a speed of adjustment in the current account equation of 0.2 ($\rho = 0.6$ and $\epsilon = 0.5$), implying that structurally the 20% of the disequilibrium in the current account should be corrected. Instead, our model generates a much slower adjustment in the current account. This slow adjustment in the current account is due to the chartists' extrapolation behaviour which adds a lot of noise in the exchange rate movements. Thus, our model helps to explain the disconnect puzzle. As stressed by Obstfeld and Rogoff(2000) the disconnect phenomenon runs in both directions: the exchange rate is disconnected from the fundamentals (in this case the current account) and the fundamental (the

Table 1: error correction model

Error correction		$\Delta \mathbf{s}_{t-i}$					$\Delta \mathbf{X}_{t-i}$				
μ	γ	λ_1	λ_2	λ_3	λ_4	λ_5	φ_1	φ_2	φ_3	φ_4	φ_5
0.007	-3.5	0.38	0.17	0.13	0.03	-0.03	0.02	0.06	0.04	0.05	0.08
2.2	-3.2	8.4	3.5	2.7	0.5	-0.7	0.4	1.2	0.7	1.1	1.6
μ'	γ'	λ'_1	λ'_2	λ'_3	λ'_4	λ'_5	φ'_1	φ'_2	φ'_3	φ'_4	φ'_5
0.006	-3.5	0.24	0.04	0.08	0.09	0.13	-0.10	-0.07	-0.07	-0.02	0.01
2.5	-3.2	6.2	0.9	1.9	2.0	3.0	-2.3	-1.5	-1.6	-0.4	0.2

current account) is disconnected from the exchange rate most of the time.

From table 1, we also note an asymmetry in the disconnect puzzle. We observe that the past changes in the current account have a weak and insignificant effect on the current exchange rate changes. The converse is not true. Past changes in the exchange rate have a significant effect on the current changes in the current account.

The picture that emerges from this analysis can be summarized as follows. There is a long run cointegration relationship between the exchange rate and the current account. However, the speed of adjustment of both the exchange rate and the current account towards this long run equilibrium relationship is very weak despite the fact that we have built into the structure of the model relatively strong speeds of adjustment. It is in this sense that there is a disconnect puzzle that runs in both directions. There is an asymmetry though. Past exchange rate changes have a significant effect on today's changes in the current account. The reverse does not seem to be the case.

We have estimated similar error correction models on simulated exchange rates and current accounts for other parameter values of the model. We show the estimated μ 's and γ 's in table 5. We find similarly low speeds of adjustment, and an asymmetry in the coefficients of the past changes in the exchange rate and the current account (not shown).

8.2 Fat tails and excess kurtosis

It is well known that the exchange rate changes do not follow a normal distribution. Instead it has been observed that the distribution of exchange rate changes has more density around the mean than the normal and exhibits fatter tails than the normal (see de Vries(2001)). This phenomenon was first discovered by Mandelbrot (1963), in commodity markets. Since then, fat tails and excess kurtosis have been discovered in many other asset markets including the exchange market. In particular, in the latter the returns have a kurtosis typically exceeding 3¹³ and a measure of fat tails (Hill index) ranging between 2 and 5 (see Koedijk, Stork and de Vries (1992), Huisman, et al.(2002)). It implies that most of the time the exchange rate movements are relatively small

¹³The normal distribution has a kurtosis index equal to 3.

Table 2: ecm for different parameter values

	EC-coefficient		Error correction
parameter values	μ	μ'	γ
$C = 5, \beta = 0.8, \gamma = 1, \epsilon = 0.5$	-0.003	0.01	-1.66
	-0.5	1.6	-3.8
$C = 5, \beta = 0.8, \gamma = 10, \epsilon = 0.5$	-0.01	0.01	-1.45
	-1.7	1.9	-4.7
$C = 5, \beta = 0.8, \gamma = 10, \epsilon = 1$	-0.01	0.01	-0.64
	-1.6	2.5	-4.7
$C = 5, \beta = 0.9, \gamma = 10, \epsilon = 1$	-0.02	0.1	-0.65
	-2.8	0.9	-5.7
$C = 5, \beta = 0.8, \gamma = 1, \epsilon = 0.5$	-0.02	0.01	-0.05
	-4.2	1.4	-0.04

Table 3: Kurtosis and Hill index USD-DEM and JPY-DEM 1975-98

Exchange rate	kurtosis	median Hill index		
		2.5% tail	5% tail	10% tail
USD-DEM	12.1	4.0	3.6	3.1
JPY-DEM	19.6	3.7	3.6	2.9

but that occasionally periods of turbulence occur with relatively large exchange rate changes.

In table 3 we show the kurtosis and the Hill index of the USD-DEM and the JPY-DEM exchange rate returns for the period 1975-1998. We computed the Hill index for different cut-off points of the tails (2.5%, 5%, 10%) and for 4 different subsamples of the original series. We find that these exchange rates exhibited excess kurtosis and fat tails during the sample period.

Another empirical finding that has been observed is that the kurtosis is reduced under time aggregation (see Lux(1998), Calvet and Fisher(2002)). We checked this finding for the same exchange rates. In table 4 we show the results for USD-DEM and JPY-DEM exchange rates, and we confirm that the kurtosis declines under time aggregation.

The next step in the analysis was to check whether these empirical features are also shared by the simulated exchange rate changes in our model.

The model was simulated using normally distributed random disturbances (with mean = 0 and standard deviation = 1). We computed the kurtosis and

Table 4: Kurtosis and time aggregation USD-DEM and JPY-DEM 1975-98

Parameter values	5 period returns	10 period returns	50 period returns
USD-DEM	7.4	5.3	3.4
JPY-DEM	14.9	5.7	2.7

the Hill index of the simulated exchange rate returns. We computed the Hill index for 4 different samples of 2000 observations. In addition, as before, we considered three different cut-off points of the tails (2.5%, 5%, 10%). We show the results of the kurtosis and of the Hill index in table 5. We find that for a broad range of parameter values the kurtosis exceeds 3 and the Hill index indicates the presence of fat tails.

Table 5: Kurtosis and Hill index

Parameter values	kurtosis	median Hill index		
		2.5% tail	5% tail	10% tail
$C=5, \beta=0.9, \gamma=1; \epsilon=0$	21.5	6.1	5.3	4.3
$C=5, \beta=0.9, \gamma=0.5; \epsilon=0.5$	7.9	3.5	3.4	3.2
$C=5, \beta=0.9, \gamma=1; \epsilon=0.5$	20.5	3.1	2.8	3.0
$C=5, \beta=0.9, \gamma=5; \epsilon=0.5$	35.9	3.4	3.6	3.5
$C=5, \beta=0.8, \gamma=1; \epsilon=0$	15.5	5.6	5.0	4.0
$C=5, \beta=0.8, \gamma=0.5; \epsilon=0.5$	6.0	3.3	3.8	3.5
$C=5, \beta=0.8, \gamma=1; \epsilon=0.5$	7.3	3.5	3.7	3.6
$C=5, \beta=0.8, \gamma=0.5; \epsilon=0.8$	11.9	2.9	3.3	3.1

In figure 25 we show the probability density of the USD-DEM exchange rate and of our simulated exchange rates, up-left and down-left panel respectively. For the sake of comparison, we plot the probability density of normally distributed returns on the right panel. We observe that the empirical distribution differs from the normal distribution and that it strikingly resembles the distribution of our simulated exchange rate returns.

Finally we check if the kurtosis of our simulated exchange rate returns declines under time aggregation. In order to do so, we chose different time aggregation periods and we computed the kurtosis of the time-aggregated exchange rate returns. We found that the kurtosis declines under time aggregation. In table 6 we show the results for some sets of parameter values¹⁴.

The previous results suggest that the speculative dynamics of the model transforms normally distributed noise in the exchange rate into exchange rate movements with tails that are significantly fatter than the normal distribution and with more density around the mean. Thus, our model mimics an important empirical regularity, i.e. that exchange rate movements are characterised by tranquil periods (occurring most of the time) and turbulent periods (occurring infrequently). This phenomenon has been also called *intermittency phenomenon* (see Lux(1998)).

8.3 The "excess volatility" puzzle

In this section we analyse another important empirical regularity, which has been called the "excess volatility" puzzle, i.e. the volatility of the exchange rate

¹⁴ Another empirical regularity of the distribution of exchange returns is its symmetry. We computed the skewness, and we could not reject that the distribution is symmetric.

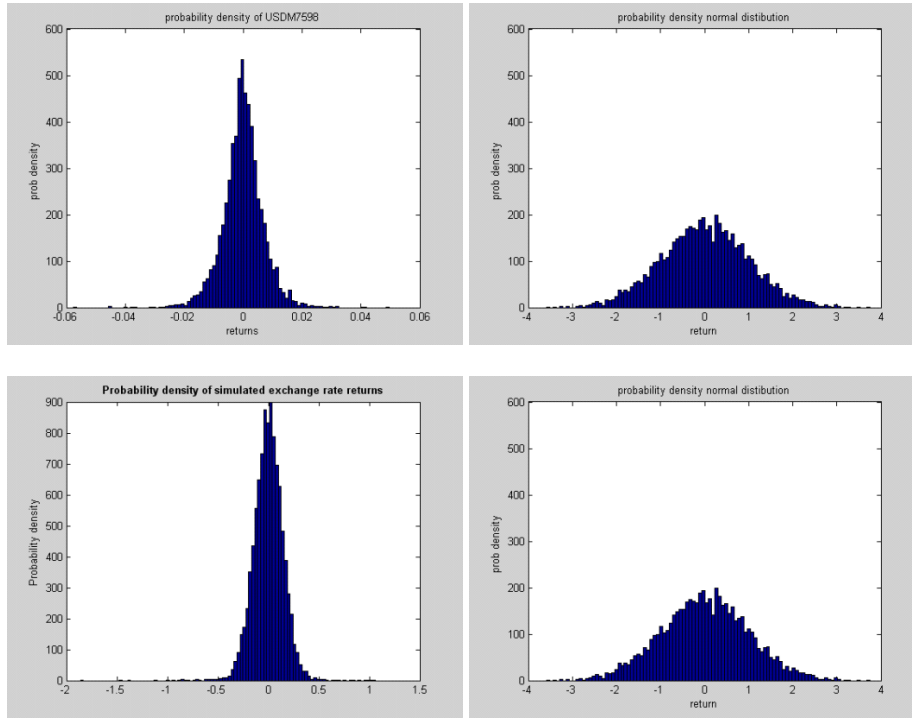


Figure 25:

by far exceeds the volatility of the underlying economic variables. Baxter and Stockman (1989) and Flood and Rose (1995) found that while the movements from fixed to flexible exchange rates led to a dramatic increase in the volatility of the exchange rate no such increase could be detected in the volatility of the underlying economic variables. This contradicted the 'news' models that predicted that the volatility of the exchange rate can only increase when the variability of the underlying fundamental variables increases (see Obstfeld and Rogoff (1996) for a recent formulation of this model)¹⁵.

In order to deal with this puzzle we compute the noise to signal ratio in the simulated exchange rate. We derive this noise to signal ratio as follows:

$$var(s) = var(f) + var(n) \quad (18)$$

where $var(s)$ is the variance of the simulated exchange rate, $var(f)$ is the variance of the fundamental and $var(n)$ is the residual variance (noise) pro-

¹⁵In addition, Goodhart (1989) and Goodhart and Figlioli (1991) found that most of the changes in the exchange rates occur when there is no observable news in the fundamental economic variables. This finding contradicted the theoretical models (based on the efficient market hypothesis), which imply that the exchange rate can only move when there is news in the fundamentals.

Table 6: Kurtosis and time aggregation

Parameter values	5 period returns	10 period returns	25 period returns	50 period returns
$C=5, \beta=0.9, \gamma=1, \epsilon=0$	44.9	32.3	4.9	3.8
$C=5, \beta=0.9, \gamma=0.5, \epsilon=0.5$	9.7	10.7	3.2	3.0
$C=5, \beta=0.9, \gamma=1, \epsilon=0.5$	10.8	10.9	3.4	2.4
$C=5, \beta=0.9, \gamma=5, \epsilon=0.5$	17.3	6.4	3.4	3.6
$C=5, \beta=0.8, \gamma=1, \epsilon=0$	85.3	30.3	3.9	2.5
$C=5, \beta=0.8, \gamma=0.5, \epsilon=0.5$	5.4	3.4	3.8	2.9
$C=5, \beta=0.8, \gamma=1, \epsilon=0.5$	6.2	3.9	3.8	2.9
$C=5, \beta=0.8, \gamma=0.5, \epsilon=0.8$	5.3	4.0	4.5	3.0

duced by the non-linear speculative dynamics which is uncorrelated with $\text{var}(f)$. Rewriting (18) we obtain

$$\frac{\text{var}(n)}{\text{var}(f)} = \frac{\text{var}(s)}{\text{var}(f)} - 1 \quad (19)$$

The ratio $\text{var}(n)/\text{var}(f)$ can be interpreted as the noise to signal ratio. It gives a measure of how large the noise produced by the speculative dynamics is with respect to the exogenous volatility of the fundamental exchange rate. We simulate this noise to signal ratio for different values of the extrapolation parameter β (see figure ??). In addition, since this ratio is sensitive to the time interval over which it is computed we checked how it changes depending on the length of the time interval. In particular, we expect that the noise-to-signal ratio is larger when it is computed on a short than on a long time horizon. We show the results in figure ?? which assumes the same parameter configuration as ??.

First, we find that with increasing β the noise to signal ratio increases. This implies that when the chartists increase the degree with which they extrapolate the past exchange rate movements, the noise in the exchange rate, which is unrelated to fundamentals, increases. Thus, the signal about the fundamentals that we can extract from the exchange rate becomes more clouded when the chartists extrapolate more. Second, we find that when the time horizon increases the noise-to-signal ratio declines. This is so because over long time horizons most of the volatility of the exchange rate is due to the fundamentals' volatility and very little to the endogenous noise. In contrast, over short time horizons the endogenous volatility is predominant and the signal that comes from the fundamentals is weak. This is consistent with the empirical findings following Meese and Rogoff(1983) celebrated studies. This literature tells us that when the forecasting horizon increases the performance of forecasting based on fundamentals tends to improve relative to random walk forecasting (see Mark(1995), Faust, et al. (2002)).

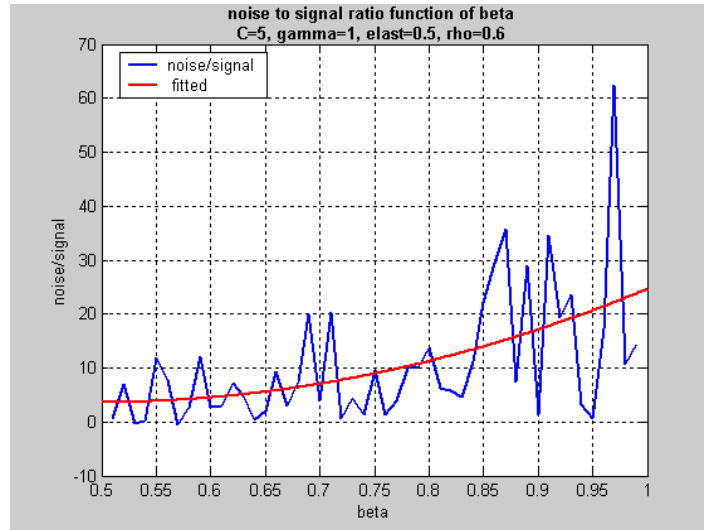


Figure 26:

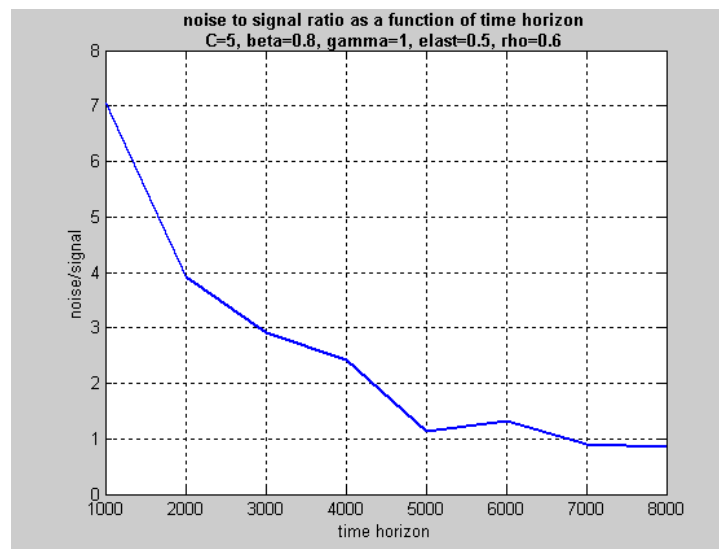


Figure 27:

9 Conclusion

In this paper we provide a framework for analysing the dynamics of exchange rate movements. The special feature of our model is that individual agents recognize that they are not capable of understanding and processing the complex information structure of the underlying model. As a result, they use simple rules to forecast the exchange rates. None of these rules is rational in the technical sense. Yet we claim that these agents act rationally within the context of the uncertainty they face. That is, agents check the 'fitness' (profitability) of the forecasting rule at each point in time and decide to reject the rule if it is less profitable (in a risk adjusted sense) than competing rules. Our model is in the tradition of evolutionary dynamics where agents use trial and error strategies. We assume that some of the forecasting rules are based on extrapolating past exchange rate movements (chartism) and others are based on mean reversion towards the fundamental rate (fundamentalism).

We analysed this model first within a framework where the current account (the fundamental) is exogenous. The model then generates two types of equilibria. The first one, which we called a fundamental equilibrium, is one in which the exchange rate converges to its fundamental value. The exchange rate, however, can also converge to a second type of equilibrium, which we called a bubble equilibrium, and which is reached in a self-fulfilling manner. An important feature of the bubble equilibrium is that chartism (extrapolative forecasting) takes over most of the market. We simulated the model in a stochastic environment and generated complex scenarios of bubbles and crashes. One interesting aspect of the model is that it explains both the emergence of the bubble and its subsequent crash. The model also predicts that bubbles and crashes are asymmetric, i.e. the bubble phase is slower than the subsequent crash. This asymmetry has been widely observed in financial markets. It cannot be explained by RE-models of bubbles and crashes which predict symmetry (Blanchard and Watson(1982)).

We also analysed under what conditions bubbles and crashes occur. We find that when agents react quickly to changing relative profitabilities of the different forecasting rules, the frequency of bubbles increases. In such an environment chartists will make large profits and will tend to dominate the market, crowding out fundamentalists who have a poor forecasting record and make losses. It will then be quite rational to be a chartist.

In a second stage we extended the model to allow for a feedback from the current account. We found that when the elasticity of the current account with respect to exchange rate changes is sufficiently high the bubble equilibria disappear.

In a third stage, we introduced transactions costs in the goods market. These transactions costs create a band of inaction within which exports and imports do not react to exchange rate changes. The implications of introducing such transactions costs for the dynamics of the exchange rate are far-reaching. We found that this version of the model is capable of generating the disconnect phenomenon (misalignment), whereby the exchange rate is disconnected most of the time from the underlying fundamentals. Interestingly, these misalignments

resemble the bubbles and crashes dynamics, although the deterministic part of the model does not produce bubble equilibria.

We also found that the model produces a "sensitivity to initial conditions", i.e. small stochastic changes have permanent effects on the future movements of the exchange rate. This implies that exchange rates are influenced by trivial shocks in a permanent way.

Finally we tested our model in the sense that we reproduced the statistical properties of exchange rate changes observed in reality, i.e. excess volatility, excess kurtosis and fat tails.

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A Fixed attractors and fundamental shocks: additional results

In this appendix we present additional simulations of the effect of shocks in the fundamental on the exchange rate. We assume different values of the initial conditions. The results are shown in figures 28, 29 and 30. When the initial condition (noise) is small (figure A1) no bubble equilibria exist and the exchange rate always coincides with its fundamental value. When the initial condition is gradually increased (figures A2 and A3) the range of bubble equilibria progressively increases.

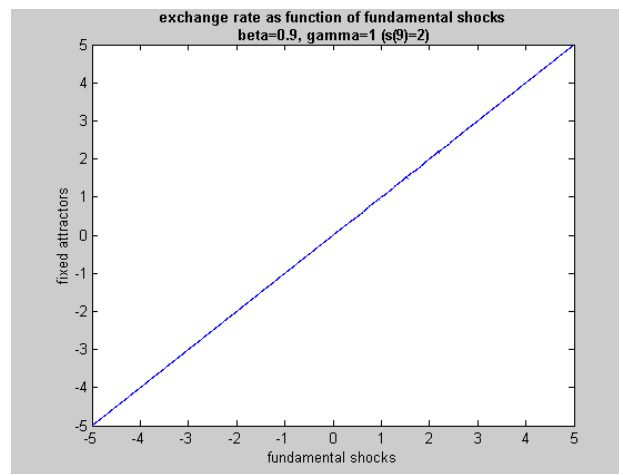


Figure 28:

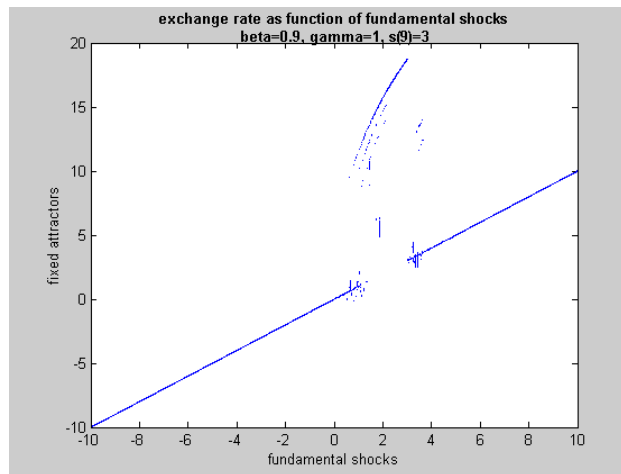


Figure 29:

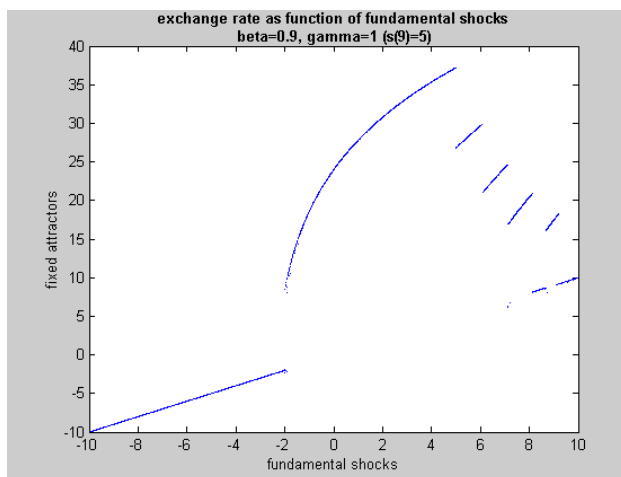
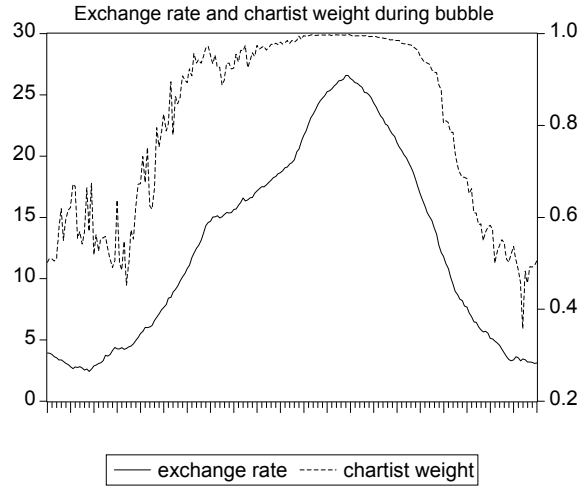


Figure 30:

B Causality tests between exchange rate and chartist weight

In this appendix we present the results of causality tests between the exchange rate and the weight of chartists during a bubble and crash episode. We simulated the model using the standard set of parameters, and we selected an episode during which a bubble and crash occurred. We show such an episode in figure A2. A visual inspection of the graph reveals that the exchange rate appears to lead the chartist weight, at least when the bubble starts and later when the bubble bursts. Note also that the crash occurs faster than the bubble phase, a feature we often find in our simulated bubbles and crashes. This has also been found in empirical data (see Sornette(2003))



Next we performed a Granger causality test on the exchange rate and the chartist weight during the bubble and crash episode represented in figure A2¹⁶. The result of this causality test is presented in table A1. We observe that we cannot reject the hypothesis that the exchange rate leads the chartists' weight during the bubble and crash episode, while we can reject the reverse. We find this feature in most bubble and crash episodes.

¹⁶We checked for stationarity and could not reject that the two series are stationary during the sample period.

Table 7: Granger causality tests

Null Hypothesis:	F-statistic	Probability
cw not Granger cause exchange rate	0.377	0.865
exchange rate not Granger cause cw	6.85	6.4E-06

Note: obs=211, lags=5.

C Stylised Facts of JPY-DEM Exchange Rate

The up left panel shows the distribution of the JPY-DEM returns. The bottom left panel represents the distribution of our simulated returns. The right panels show the distribution of normally distributed exchange rate returns.

