Relative Demand Shocks: An Exploration*

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Abstract

This paper proposes a multi-sector model of stochastic relative demand shifts among different commodities or services. There are no technology (productivity) shocks in the model. The model proposes an economic mechanism, complementary to the standard Real Business Cycle theory. Relative demand shocks change the desired composition of consumption expenditure on a period by period basis, thereby inducing an intra-sectoral and inter-temporal resource reallocation. A consequence of this variation is that consumers' subjective discount factor changes in tandem with the current compositions of consumption expenditure. The model is effectively able to reproduce the main stylized facts of the US economy. The labor market regularities, and the observed correlations between aggregate output with the aggregate consumption and investment, with price index and with the inflation rate are also matched (at all lags and leads). Finally, the model generates a *false* "Solow Residual", and we explore its properties.

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1 Introduction

Standard Real Business Cycle (RBC) models driven by productivity shocks (e.g. Kydland and Prescott, 1982; Long and Plosser, 1983) have difficulty explaining several important stylized facts of the U.S. economy, such as the substantial volatility of consumption relative to output, the cross correlation (at all leads and lags) of consumption and investment with output, or the high volatility of hours.[1] In addition, a recent body of literature questions the very foundations of RBC theory, by suggesting that positive technological shocks lead to declines in input use and that selected productivity measures are essentially uncorrelated with output, and negatively correlated with input growth (e.g. Basu, Kimball and Fernald, 2003; Basu, 1998; Shea, 1998; Gali, 1999; Francis and Ramey, 2003).[2] Several other contributions suggest that demand shocks bear significant responsibility for business cycles in the U.S. and in major European countries (France, Germany, and the United Kingdom) (see e.g. the seminal papers by Blanchard and Quah, 1989; Karras, 1994; Hartley and Whitt, 2003, Gali, 1996; Gali, 1999).

This paper introduces the concept of a *relative demand shocks*. Under relative demand shocks consumer tastes randomly shift across different commodities, as manifested by unexpected relative increases or decreases in the marginal utility of the various goods. This formulation is complementary to that of Baxter and King (1991), and Bencivenga (1992), whose models rely on aggregate demand disturbances; i.e. shocks to the marginal utility of the single composite consumption good. More recent contributions (e.g. Wen, 2003; Wen 2002; Benhabib and Wen, 2002) rely on the Baxter and King (1991) definition.

We analyze the consequences of these relative demand shocks in the context of a dynamic equilibrium two-sector two-good model with labor-leisure choice, and where changes in relative demand are driven by autonomous shifts in preferences. While labor services can be reallocated across sectors, consumption and capital goods are sector specific. Aggregate uncertainty here originates from the demand side, and it is modelled using a state dependent utility function. The benchmark economy is then extended to incorporate intra-sector and/or inter-temporal labor adjustment costs, and endogenous capacity utilization. Finally, the model's performance is compared with that of the analogous economy, where fluctuations are driven by relative technology shocks only.

The paper focuses on **six** major issues. **First**, the model proposes an economic mechanism, complementary to the standard Real Business Cycle theory. Relative demand shocks change the desired composition of consumption expenditure on a period by period basis, thereby inducing an intra-sectoral and inter-temporal resource reallocation. A consequence of this variation is that consumers' subjective discount factor changes in tandem with the current compositions of consumption expenditure. **Second**, the model performs quite well in replicating most regularities of the U.S. business cycle. It performs particularly well with respect the aggregate consumption volatility and its cross-correlation with output, the main labor market stylized facts, the price index and the inflation rate

[1]Some of these counterfactual prediction are so robust to model specification that are addressed in the literature as *puzzles*. Examples in point are *consumption volatility puzzle* (e.g. Cochrane, 2001), the *employment variability puzzle* and the *productivity puzzle* (e.g. Stadler, 1994).

[2]It is fair to mention, however, that there is no a final answer on this debate. For example, Christiano, Eichenbaum and Vigfusson (2003) present evidence supporting standar RBC theory, and Fisher (2002) suggest that technology, investment-specific, shocks are an important source for business cycles.

volatilities and their correlations with aggregate output. It is worth to mention that the model generates a negative correlation between average productivity of labor and hours worked, which is a stylized fact not explainable by a technology driven model. **Third**, the introduction of relative demand shocks improves the internal propagation of stochastic disturbances. In particular, the model is capable of replicating several business cycle facts using shocks that are *smaller* relative to standard productivity shocks. **Fourth**, under relative demand shocks the strongest correlations between consumption and output, and between consumption and consumption occur at zero lags(leads), consistently with the data for the US economy (Stock and Watson, 1998). This a significant improvement upon the standard business cycle model where consumption's strongest correlation with output occurs at lag -1; in this sense consumption is said to lag output. That happens because the exogenous increase of income leads to an increase in consumption and investment. Instead, in a demand driven model, the causality order is inverted, since the increase in consumption desire pulls income up, via labor market channel. This also suggests that the model is not subject to the crowding out effect between consumption and investment, as described by Baxter and King (1991) and typical of several one-sector formulations.[3] **Fifth**, the stochastic properties of sectorial business cycles are consistent with the U.S. economy. Capital stocks, labor flows, production outputs, investments and consumptions move together, and, more importantly, all sectoral quantities are procyclical with aggregate GDP. This the most important of the regularities common to all business cycles (Lucas, 1977). **Sixth**, the model generates a *false* Solow residual, whose stochastic properties are consistent with the U.S. Solow residual data. In this context, this quantity measures something completely different from technology or productivity.

Before proceeding, it is important to stress that the goal of this research is **not** to argue that either aggregate shocks of any kind or sectoral technology shocks are irrelevant to the study of macroeconomic fluctuations. It is simply to reduce economists' reliance on them by identifying a role for relative demand shocks in generating sectorial and aggregate co-movements.

The paper is organized as follows. Section 2 briefly presents the literature on which the model draws, Section 3 presents select arguments supporting a theory of relative demand shocks, and offers selected empirical evidences supporting the concept of relative demand shifts. Section 4 details the benchmark economy and three generalizations, while Section 5 presents numerical results. Finally Section 6 concludes, and Section 7 includes all proofs and derivations.

2 Background

The paper draws on three background literatures. The first concerns the failure of standard business cycle models to account for selected empirical regularities, and the claim that demand shocks bear a significant part of business cycle fluctuations (Section 2.1). The second evaluates the possibility of explicitly incorporating different definitions of demand

[3] Baxter and King notice that when a demand shock impinges the economy, in a one-sector model, people increase consumption, while reducing investment, and, by this end, capital accumulation. Output, being a a monotone transform of capital stock, subsequently falls, depicting a significant crowding out effect.

shocks into a neoclassical scheme (Section 2.2), while the third concerns multi-sector models (Section 2.3). Our paper differs from each of them in some important dimensions.

2.1 Macroeconomic Fluctuations: Demand or Supply?

The debate whether business cycles are driven by demand and/or supply shocks have been re-opened by the seminal paper of Blanchard and Quah (1989). To provide an answer, current literature is testing the theoretical implications of demand an supply shocks, mainly focusing on the labor market. It should be noted at the very beginning, however, that this debate remains unresolved.

Several contributions (e.g. Basu, Kimball and Fernald, 2003; Gali, 1999; Basu, 1998) reject a *technology driven business cycle hypothesis*. In particular, Gali (1999) concludes that for several countries the movement in hours worked seem consistent with demand shocks, and not with technology shocks.[4] Francis and Ramey (2003) confirm Gali's (1999) results. Basu, Kimball and Fernald (2003) remeasure technology's (productivity) impact over the business cycle, controlling for variable capacity utilization of capital and labor, variable workers' effort, imperfect competition, and different characteristics across industries. This complementary measure turns out to be essentially uncorrelated with output, and negatively correlated with with inputs' growth, again, rejecting the technology driven business cycle hypothesis.

Other contributions, however, support a *technology driven business cycle hypothesis*. Christiano, Eichenbaum and Vigfusson (2003) provide empirical evidence that a positive shock to technology increases per capita hours worked, consumption, investment, average productivity and output. Karras (1994) and Hartley and Whitt (2003) argue that aggregate demand shocks (e.g. changes in demographics, fiscal policy or export demand) bear significant responsibility for business cycles in the major European countries (France, Germany, and the United Kingdom), but that aggregate supply shocks are also found to be important for short-run fluctuations.

2.2 If the Technology Driven Business Cycle is Dead, What's Next?

At this point, a legitimate question is "*How should we think about these findings?*"[5] With the exception of Christiano, Eichenbaum and Vigfusson (2003), all these contributions suggest a potential paradigm shift, since the idea of the technology-driven real business cycle loses much of its appeal. This set of new econometric results has generated a theoretical debate within which it is possible to distinguish between two different bodies of research.

One supports the *New-Keynesian* view of the economy, concluding that the empirical evidence is clearly at odds with the predictions of RBC theory, but is largely consistent

[4] In particular, he shows that for the US economy the estimated conditional correlations of hours with productivity are negative for technology shocks, and positive for demand shocks, that impulse response functions show a persistent decline of hours in response to a positive productivity shock and measured components of productivity increase after a demand shock.

[5] This question is adapted from Cooley, 1998.

with a class of Neo-Keynesian monetary model with sticky prices, monopolistic competition and variable effort.[6]

A second body of research supports a *reformulated Neo-Classical* approach, which can be further distinguished from two perspectives.

Several contributions reformulate the standard technology driven business cycle model for replicating the negative correlation between labor input and technology shocks (e.g. Francis and Ramey, 2003; Campbell, 1998). Others specialize the analysis, suggesting that investment-specific technological change, in the spirit of Greenwood, Hercowitz and Krusell, 1997) account for a large part of business cycle fluctuations (e.g. Fisher, 2002).[7]

Other contributions, instead, abandon the technology driven hypothesis, and directly focus on demand shocks as the main source of aggregate fluctuations. The definition itself of a demand shock is vague, and the identification of demand and supply shocks accounts for a whole field of econometric literature.[8] For example, aggregate demand shocks can be caused by a change in government spending, taxes, planned investment, autonomous consumption, money supply, money demand, and other international factors, or can alter savings relative to consumption, or the composition of consumption expenditure.

Within the context of neoclassical dynamic general equilibrium models we are aware of only two definitions of demand shocks that have been implemented. The first one is proposed in a seminal paper by Baxter and King (1991), where demand shocks are modelled as random disturbances to the marginal utility of consumption (or to the habitual consumption level), which generate the urge to consume. Alternatively, Bencivenga (1992) proposes a specification in which households can be interpreted as experiencing shocks to their ability to transform time and consumption purchases into consumption services. Bencivenga argues that these shocks may also be interpreted as a change in relative prices. More recently, Wen (2003) casts a Baxter and King (1991) preference's shock into an open economy business cycle model, while Benhabib and Wen (2002) incorporate it into a one sector model with indeterminacy.[9] Wen (2002) casts Bencivenga's (1992) consumption shocks into a model with factor hoarding and labor adjustment costs, and suggests that dynamic labor adjustment costs are important for generating procyclical labor productivity.

[6]In this context, some prices do not adjust immediately after a positive technology shock, while worked hours decline since the actual output level can be produced with lesser labor input, because of the technological change.

[7]This suggests that supply shocks, as a principal driving mechanism, cannot be ruled out.

[8]Most empirical studies on RBC modelling use multivariate Vector Autoregressive (VAR) models or the common trends-cointegration approach in order to disentangle supply and demand shocks (see e.g. Blanchard and Quah, 1989, King et al., 1991, Mellander et al. , 1992, Christiano and Eichenbaum, 1992, Fisher, Ingram et al., 1994, Karras, 1994, Bergman, 1996). This methodology requires, as identifying restrictions, that two disturbances which are associated with demand and supply shocks, are uncorrelated at all leads and lags (Blanchard and Quah, 1989; Gamber and Joutz, 1993; and Gali', 1996). In other words, the identifying restrictions associate supply shocks with permanent effects (on output) and demand shocks with temporary effects.

[9]In particular, Wen (2003), using simplified version of the standard two-country general equilibrium model of Backus, Kehoe and Kydland (1992), shows that preference shocks explain both domestic and international business cycles. Benhabib and Wen (2002) show that under indeterminacy aggregate demand shocks are able to explain not only the aspects of actual fluctuations that standard RBC models predict fairly well, but also other aspects of actual fluctuations that standard RBC models fail to explain

Notice that the Baxter and King (1991)'s shock defines a truly *intertemporal* and *aggregate* demand shock. In particular, it urges consumers to substitute aggregate consumption tomorrow (that is saving) with aggregate consumption today. Bencivenga (1992)'s preference shocks directly affect marginal utility of consumption and of leisure.[10] The leisure's shock increases the disutility of labor, generating an inward shift of labor supply schedule.

In other words, both the Baxter and King (1991) and the Bencivenga (1992) shocks implicitly assume that all consumers suddenly want to consumer more of all commodities. On the contrary, this paper introduce an *intra-sectoral relative demand shock*. This shock makes a set of commodities relatively more desirable for the share of the population directly affected by that shock (λ_i, using model's notation, see Section 4). In terms of economic intuition, it is easier to come up with a story where a different share of the population changes tastes every quarter, and it sticks with them for several quarters.

2.3 Propagation and Comovements in Multi-Sector Economies

A defining characteristic of the business cycle is the comovement in the pace of economic activities in different sectors of the economy (e.g. Lucas, 1977; Burns and Mitchell, 1947).[11]. But investment and employment in various sectors are not perfectly correlated, which suggests that there may be some sector specific driving forces (Huffman and Wynne, 1999; Hornstein, 2000).

Several contributions suggest that multi-sector versions of the neoclassical growth model are consistent with the observed positive comovement across sectors if one accounts for the input-output structure of the economy (i.e. see the seminal paper by Long and Plosser, 1983). More recent contributions are that of Hornstein (2000), Huffman and Wynne (1999), Hornstein and Praschnik (1997), Horvath (2000). All these contributions assume that technology shocks (aggregate and/or sector specific) are the driving force of the economy.

Beyond a different source of uncertainty, our model (see Section 4) departs from the standard literature on multi-sector economies, where it is usually assumed that one sector produces a consumption good, while the other(s) produces capital goods. Our model considers two sectors producing two goods, which can be used as consumption and as investment goods in each sector. This formulation allows for a more general preference representation, since they can be defined over two different consumption profiles, which is a necessary assumption to analyze relative demand shifts across commodities. The next section presents a first set of evidence on relative preference shifts, while additional results are discussed in the Calibration (Section 4.2.1).

[10] More formally, she specifies a model characterized by the following utility function $u(c_t, \ell_t) = \theta_t^c \log c_t + \theta_t^\ell \log \ell_t$, where θ_t^c and θ_t^ℓ are shocks to preferences, and from a Cobb-Douglas production function $\phi_t A k_t^\gamma (1-\ell)^{1-\gamma}$ where ϕ_t is a shock to technology. Here the notation closely follows Bencivenga (1992).

[11] Lucas (1977) notes that the comovements of economic activities across different sectors of the economy is the most important of the regularities common to all business cycles. This evidence is the prerequisite for a theory of aggregate business cycle. We are aware of only two sectors which employment is countercyclical: the home production sector as documented by Benhabib, Rogerson and Wright (1991), and the underground sector , as documented by Busato and Chiarini (2003).

3 Why *Relative Demand Shocks*

The debate "*demand vs technology shocks*" goes beyond the contribution of this paper, and selected conclusions of this research body have been briefly summarized in Section 2. However, these results do not comment about relative demand shocks.

Demand shocks, as well as technology shocks, are unobservable. However, they can be estimated using a model's first order conditions, as in Stockman and Tesar (1995). An alternative approach is to use measures for consumer sentiment as a proxy for preference.[12] The University of Michigan Index of Consumer Sentiment (ICS) has proven to be an accurate indicator of the future course of the national economy.[13] Household sentiment has been cited as one of the leading causes of the 1990-91 recession (Carroll, Fuhrer and Wilcox, 1994), and unexpected shifts in consumer confidence have also been used to explain swings in financial markets.[14] In particular, Carroll, Fuhrer and Wilcox (1994) present evidence suggesting that lagged consumer sentiment has some explanatory power for current changes in household spending even after controlling for the information sentiment contains about income growth.

In order to comment about relative demand shocks, however, it is necessary to distinguish among different the domestic consumption components. For this reason we focus on the aggregate ICS, and on the three constituent components capturing market conditions for large household goods (BC_{LHG}), vehicles (BC_C), and houses (BC_H).[15] **Table 1** presents the contemporaneous correlation among these four quantities.

These correlations support a *relative demand shock* hypothesis. Indeed, if there were only one aggregate demand shock, and no relative disturbances, the three sectoral indexes (BC_{LHG}, BC_C, BC_H) should comove with the aggregate counterpart (ICS), and, most importantly, they should be perfectly correlated among each other. Table 1 does not support these facts. On the contrary, it shows that the contemporaneous correlations

[12] See for example, Baxter and King (1991), and Stockman and Tesar (1995); Guo and Sturzenegger (1998), Wen (2003) and Wei (2003)

[13] The University of Michigan Index of Consumer Expectations focuses on three areas: how consumers view prospects for their own financial situation, how they view prospects for the general economy over the near term, and their view of prospects for the economy over the long term. The core questions cover three broad areas of consumer sentiment: personal finances, business conditions, and buying conditions. Finally, several questions probe for the respondent's appraisal of present market conditions for large household durables, vehicles, and houses. In each area, consumers are not only asked to give their overall opinions, but are also asked to describe in their own words their reasons for holding these views. These follow-up questions reflect a growing interest in not only projecting what consumers will do, but also understanding why consumers make certain spending and saving decisions.

[14] Early investigators of the explanatory power of consumer confidence include Fair (1971), and Mishkin (1978), who argues that the Michigan index may be a good proxy for the consumers subjective assessment of the probability of future financial distress. More recent work analyzing the Michigan index can be found in Carroll and Dunn (1997), Carroll, Fuhrer, and the references quoted there..

[15] Consider, for example, the index relative to the buying conditions for cars. Consumers are asked: "*Speaking now of the automobile market do you think the next 12 months or so will be a good time or a bad time to buy a car?*" The correspondence between car buying attitudes and subsequent vehicle sales is quite high. On average, changes in buying attitudes preceded changes in sales by two quarters. A time series correlation of 0.73 with actual sales series was achieved when the attitude series was led two quarters. Consumers generally anticipated changes in vehicle sales 6 months in advance of the actual change. The Survey of Consumers (http://www.sca.isr.umich.edu/documents.php?c=i) reports several other examples proving the accuracy of the Index as a leading indicator.

Table 1: Indices of Consumer Sentiment: Household Goods, Vehicles, Houses

	BC_C	BC_H	BC_{LHG}	ICS
BC_C	1.00			
BC_H	0.79	1.00		
BC_{LHG}	0.70	0.55	1.00	
ICS	0.69	0.45	0.82	1.00

Table 1. BC_C denotes Buying conditions for Vehicles, BC_H denotes Buying conditions for Houses, and BC_{LHG} denotes Buying conditions for Large Household Goods. Source: University of Michigan Surveys of Consumers (1978:01-2003:01), and Author's calculations.

between BC_{LHG}, BC_C and BC_H, while positive, are different from 1. We are aware that, *a priori*, these evidence could be consistent with a theory where changes in consumption are driven by productivity improvements. However, the model presented in the following section suggests that relative demand shocks plays a critical role in generating fluctuations.

4 A Multi-Sector Model with Relative Demand Shifts

This section presents the baseline dynamic equilibrium model with relative demand shocks, together with several extensions. Subsequently, the model is generalized introducing labor adjustment costs and variable capacity utilization.

4.1 The Preference Structure

Imagine an economy with several commodities, say N $(0 < N < \infty)$. Suppose, in addition, that consumer tastes randomly change, as reflected in unexpected relative increases in the marginal utility of one or of an other consumption goods.[16] Denote with $s_t^i \in \mathbb{R}_+^N$ a desirability index associated with i-th consumption flow c_t^i.[17]

In a discrete time setting each s_t^i will be jumping on a period by period basis. Consider, without loss of generality, the first commodity, c_t^1. At time $t+1$ its preference index may increases (decreases) to s_{t+1}^1, where $s_{t+1}^1 > s_t^1$ $(s_{t+1}^1 < s_t^1)$, making first commodity relatively more (less) desirable with respect to other ones, *ceteris paribus*.

This generates a reallocation of resources across sectors and over time. The intertemporal resouce reallocation is triggered by the modified composition of *desired* consumption expenditure. In particular, changes in composition of consumption expenditures modifies consumers' intertemporal marginal rates of substitution.[18]

[16]The question "*why preferences change*" it is hard to answer with accuracy. There are so many factors affecting individual preferences, that the choice of a specific one would make the analysis too peculiar and tied to that choice. For this reason we abstract from choosing a specific *preference-shift-factor*, while assuming that they are exogenous to the model.

[17]Often time demand shocks are indicated as having a Keynesian flavor. Indeed, in the General Theory Keynes often mentions the existence of "*subjective factors of consumption*" affecting consumption decisions. In this sense it is more difficult to justify an aggregate demand shock, as it has been modelled in the actual literature. Keynes' perspective might be better captured with idiosyncratic disturbance. Thus a legitimate *Keynesian business cycle model* should incorporate some kind of heterogeneity, we are not aware of any model explicitly incorporating this issue.

[18]This formulation may represent, for example, a situation where consumers increase their relative desirability for T-shirts, relative to music CDs. It should be noticed that this kind of shock does not

4.2 Models with Relative Demand Shocks and Labor Transfers.

4.2.1 The Benchmark Economy.

The benchmark model is structured as two-sector, two-good economy, with labor-leisure choice, and where changes in relative demand are driven by autonomous changes in preferences. Aggregate uncertainty originates from the demand side, and it is modelled using a state dependent utility function. Consumption and capital goods are sector specific, while labor services can be reallocated across sectors, without bearing any cost of adjustment.[19] Since there are no restrictions to trade, optimal allocation may be derived from a planning problem.[20]

Preferences. Preferences over consumption flows ($\mathbf{c}_t = \left(c_t^1, c_t^2\right)$) and hours worked ($\mathbf{n}_t = \left(n_t^1, n_t^2\right)$) are described by a state dependent return function $u(\mathbf{c}_t, \mathbf{n}_t; \tilde{\mathbf{s}}_t) : \mathbb{R}_+^2 \times \mathcal{S}^2 \times [0,1]^2 \to \mathbb{R}$, where $\tilde{\mathbf{s}} = \left(\tilde{s}_t^1, \tilde{s}_t^2\right)$ denotes a vector of realizations of sectorial (idiosyncratic) relative preference shocks (defined below):

$$u(\mathbf{c}_t, \mathbf{n}_t; \mathbf{s}_t) = \lambda_1 u_{(1)}(c_t^1; \tilde{s}_t^1) + \lambda_2 u_{(2)}(c_t^2; \tilde{s}_t^2) + \upsilon\left(\mathbf{n}_t; B\right), \tag{1}$$

where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ denotes preference weights, $u_{(1)}(c_1, \tilde{s}_1) = \tilde{s}_t^1 \frac{(c_t^1)^{1-q_1}}{1-q_1}$, $u_{(2)}(c_2, \tilde{s}_2) = \tilde{s}_t^2 \frac{(c_t^2)^{1-q_2}}{1-q_2}$, and q_1 and q_2 denote the relative risk aversion coefficients over consumption. Next, $\upsilon\left(\mathbf{n}_t; B\right)$ is a well behaved (continuous, twice continuously differentiable) function of $\mathbf{n}_t$, representing the utility of leisure, and $B(B > 0)$ is a scaling parameter.

Production Technologies. Each good is produced with physical capital and labor, using a sector-specific Cobb-Douglas technology:.

$$y_t^1 = \left(k_t^1\right)^{\alpha_1} \left(n_t^1\right)^{1-\alpha_1} \text{ and } y_t^2 = \left(k_t^2\right)^{\alpha_2} \left(n_t^2\right)^{1-\alpha_2}, \tag{2}$$

where n_t^i denotes labor demand in sector i, for $i = 1, 2$; notice that there are no random quantities measuring exogenous productivity disturbances.

Feasibility and Capital Accumulation Constraints. Feasibility of the optimal program is ensured by the following two customary constraints where production technologies have been substituted for y_t^1 and y_t^2.

$$c_t^1 + i_t^1 = \left(k_t^1\right)^{\alpha_1} \left(n_t^1\right)^{1-\alpha_1} \qquad \text{and} \qquad c_t^2 + i_t^2 = \left(k_t^2\right)^{\alpha_2} \left(n_t^2\right)^{1-\alpha_2}, \tag{3}$$

completely cancel the desire for a certain commodity, it only makes one good relative more desirable than others. I argue that is an every-day life experience. Imagine, for example, to be in a two good world, the two goods being T-shirt and CDs. If, for some reasons, relative desirability for T-shirt increases, we will still purchase some music CDs, but it is likely the relative spending share of T-shirts will increase relative to that of CDs .

[19] Just notice that Section 4.2.2 extends the analysis, while investigating the role of inter-temporal and intra-sectoral adjustment costs.

[20] However, Proposition 1 proves that the Planner allocation coincides with a competitive equilibrium.

where i_t^i denotes investment flows, for $i = 1, 2$. Capital accumulation constraints are defined as follows:

$$k_{t+1}^1 = (1 - \Omega_1)k_t^1 + i_t^1 \quad \text{and} \quad k_{t+1}^2 = (1 - \Omega_2)k_t^2 + i_t^2, \tag{4}$$

where the $\Omega's$ denote quarterly depreciation rates. This formulation implicitly assumes that capital is not mobile across sector. The idea here is that the capital used in the production of food and drinks cannot easily be used to produce cloths and shoes. Consumers allocate hours to each sector; leisure is the residual according to the following constraint:

$$\ell_t = 1 - n_t^1 - n_t^2, \tag{5}$$

where available hours are normalized to 1. Notice that we are assuming perfect substitutability between labor services in the two sectors. We are expecting, therefore, a rapid movement of labor to where the planner marginal utility of consumption is higher. Notice that this is an argument distinctive of a demand-driven model. In a model with technology shocks only, labor services shift to the sector where the marginal productivity of labor (wage) is relatively higher.

Demand and Technology Shocks Structure The relative (idiosyncratic) demand shocks $\left\{\tilde{s}_t^1, \tilde{s}_t^2\right\}_{t=1}^{\infty}$ have transitory, but persistent effects. Shocks may be (or may be not) positively correlated. Demand shocks follow autoregressive processes in logs:

$$\log \tilde{s}_{t+1}^i = \omega \log \bar{s}^i + (1 - \omega) \log \tilde{s}_t^i + \epsilon_t^i,$$

where $\epsilon_t^i \sim \mathcal{N}\left(0, \sigma_{\epsilon^i}^2\right)$, for $i = 1, 2$.

Equilibrium Characterization. Since there are no restriction to trade, a Planner maximizes the expected present discounted value of the return function (1), $\mathcal{V}_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(\mathbf{c}_t, \mathbf{n}_t; \mathbf{s}_t)$, subject to the feasibility constraints (3), the capital accumulation constraints, (4), and the constraint on total hours (5). The state of the economy at time t is represented by a vector $\boldsymbol{\chi}_t = \left\langle k_t^1, k_t^2, s_t^1, s_t^2 \right\rangle$. Controls of the problem are consumption flows $\mathbf{c}$, investment flows $\mathbf{i}$, and the labor services $\mathbf{n}$.

Function $v(\mathbf{n}_t; B)$ is then specified as $v(\mathbf{n}_t; B) = B\frac{(1-n_t^1-n_t^2)^{1-\gamma}}{1-\gamma}$, where $\gamma \geqq 0$ measures the inverse labor supply elasticity. The Planner problem belongs to the class of stationary decision problem analyzed by Blackwell (1961) or for the convex case by Lucas and Prescott (1971). Introducing dynamic multipliers ϕ_t^1 and ϕ_t^2, forming the Hamiltonian $\mathcal{H}$ yields:

$$\begin{aligned}
\max_{c_t^1, c_t^2, n_t^1, n_t^2} \mathcal{H} \quad = \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \lambda_1 \left[\frac{\tilde{s}_t^1 (c_t^1)^{1-q_1}}{1 - q_1} \right] + \lambda_2 \left[\frac{\tilde{s}_t^2 (c_t^2)^{1-q_2}}{1 - q_2} \right] + B \frac{(T - n_t^1 - n_t^2)^{1-\gamma}}{1 - \gamma} \right. \\
& + \phi_t^1 \left[\left(k_t^1\right)^{\alpha_1} \left(n_t^1\right)^{1-\alpha_1} - c_t^1 + (1 - \Omega_1) k_t^1 - k_{t+1}^1 \right] + \\
& \left. + \phi_t^2 \left[\left(k_t^2\right)^{\alpha_2} \left(n_t^2\right)^{1-\alpha_2} - c_t^2 + (1 - \Omega_2) k_t^2 - k_{t+1}^2 \right] \right\},
\end{aligned}$$

where $\mathbb{E}_0$ is the expectation operator, conditional on time 0 information. Optimality conditions (6) and (7) equate the marginal disutility of working (on the LHS) with the marginal productivity of labor (the wage), weighted with the marginal utility of consumption (on the RHS).

$$\begin{aligned} B(1-n_t^1-n_t^2)^{-\gamma} &= \lambda_1 \tilde{s}_t^1 \left(c_t^1\right)^{-q_1} (1-\alpha_1) \left(k_t^1/n_t^1\right)^{-\alpha_1}, && (6) \\ B(1-n_t^1-n_t^2)^{-\gamma} &= \lambda_2 \tilde{s}_t^2 \left(c_t^2\right)^{-q_2} (1-\alpha_2) \left(k_t^2/n_t^2\right)^{-\alpha_2}. && (7) \end{aligned}$$

Investment dynamics is determined by the following two Euler Equations:

$$\begin{aligned} \tilde{s}_t^1 \left(c_t^1\right)^{-q_1} &= \mathbb{E}_t \beta \tilde{s}_{t+1}^1 \left(c_{t+1}^1\right)^{-q_1} \left\{\alpha_1 \left(k_{t+1}^1/n_{t+1}^1\right)^{\alpha_1-1} + 1 - \Omega_1\right\}, && (8) \\ \tilde{s}_t^2 \left(c_t^2\right)^{-q_2} &= \mathbb{E}_t \beta \tilde{s}_{t+1}^2 \left(c_{t+1}^2\right)^{-q_2} \left\{\alpha_2 \left(k_{t+1}^2/n_{t+1}^2\right)^{\alpha_2-1} + 1 - \Omega_2\right\}, && (9) \end{aligned}$$

where $\mathbb{E}_t$ denotes the expectations operator, conditional on information available at time t. Finally, equilibrium is characterized by feasibility and constraints.

$$\begin{aligned} c_t^1 - k_{t+1}^1 + (1-\Omega_1)\, k_t^1 &= \left(k_t^1\right)^{\alpha_1} \left(n_t^1\right)^{1-\alpha_1} && (10) \\ c_t^2 + k_{t+1}^2 - (1-\Omega_2)\, k_t^2 &= \left(k_t^2\right)^{\alpha_2} \left(n_t^2\right)^{1-\alpha_2} && (11) \\ \ell_t &= T - n_t^1 - n_t^2 && (12) \end{aligned}$$

Next, it is convenient to decentralize the economy, and to show that the Planner allocation coincides with that of a competitive economy (Proposition 1).

Proposition 1 (Decentralization) *The Planner allocation coincides with that of a Recursive Competitive Equilibrium of the Prescott and Mehra (1980) type. In addition, under linear disutility of labor (Hansen, 1985), the same allocation can be supported with a more decentralized setting, with two types of firms, and two groups of consumers.*

Proof. Appendix B. ■

In particular, a Recursive Competitive Equilibrium for this economy consists of a set of continuous price functions, $\mathbf{p}$, a value function, and optimal policy functions for consumption, investment, such that market clearing conditions hold. Finally, notice that the economy satisfies conditions for the existence and the uniqueness of the Equilibrium as detailed in Prescott and Mehra (1980), to which we refer for details.

Deterministic Steady State. The first order conditions can be used to describe this stationary state in a recursive manner. Equations below describe the deterministic steady state when leisure enters linearly into the utility function (Hansen, 1985). In all other cases it is not possible to derive a closed form solution, and a numerical solution is used.

$$
\begin{aligned}
\bar{c}_1 &= \left(\frac{\lambda_1(1-\alpha_1)\bar{s}_1}{B}\right)^{\frac{1}{q_1}}\left(\frac{\alpha_1}{\beta^{-1}-1+\Omega_1}\right)^{\frac{\alpha_1}{q_1(1-\alpha_1)}}, \\
\bar{c}_2 &= \left(\frac{(\lambda_2)(1-\alpha_2)\bar{s}_2}{B}\right)^{\frac{1}{q_2}}\left(\frac{\alpha_2}{\beta^{-1}-1+\Omega_2}\right)^{\frac{\alpha_2}{q_2(1-\alpha_2)}}, \\
\bar{k}_1 &= \left(\frac{\alpha_1}{\beta^{-1}+1+(1-\alpha_1)\Omega_1}\right)\left(\frac{\lambda_1(1-\alpha_1)\bar{s}_1}{B}\right)^{\frac{1}{q_1}}\left(\frac{\alpha_1}{\beta^{-1}-1+\Omega_1}\right)^{\frac{\alpha_1}{q_1(1-\alpha_1)}}, \\
\bar{k}_2 &= \left(\frac{\alpha_2}{\beta^{-1}+1+(1-\alpha_2)\Omega_2}\right)\left(\frac{\lambda_2(1-\alpha_2)\bar{s}_2}{B}\right)^{\frac{1}{q_2}}\left(\frac{\alpha_2}{\beta^{-1}-1+\Omega_2}\right)^{\frac{\alpha_2}{q_2(1-\alpha_2)}}, \\
\bar{n}_1 &= \left(\frac{\beta^{-1}-1+\Omega_1}{\alpha_1\lambda_1}\right)^{\frac{1}{1-a_1}}\bar{k}_1;\ \bar{n}_2 = \left(\frac{\beta^{-1}-1+\Omega_2}{\alpha_2\lambda_2}\right)^{\frac{1}{1-\alpha_2}}\bar{k}_2, \\
\bar{y}_1 &= \bar{k}_1^{\alpha_1}\bar{n}_1^{1-\alpha_1};\ y_2 = \bar{k}_2^{\alpha_2}\bar{n}_2^{1-\alpha_2}, \\
i_1 &= \Omega_1\bar{k}_1;\ i_2 = \Omega_2\bar{k}_2.
\end{aligned}
$$

Once we have the equilibrium quantities for each sector, it is possible to derive aggregate variables: k_t, i_t, y_t, n_t, c_t.

Aggregation. Proposition 2 derives relative price vector. Since labor is perfectly mobile across sectors, its relative price equals one.

Proposition 2 (Relative Prices) *Let the first commodity be the numeraire of the system, and let* $\mathbf{p}_t = \left(1, p_t, p_t^{k^1}, p_t^{k^2}, p_t^{n^1}, p_t^{n^2}\right)$ *be the price vector. Then, denote the relative price vector as* $\hat{\mathbf{p}}_t = \left(\hat{p}_t, \hat{p}_t^k, \hat{p}_t^n\right)$ *where* $\hat{p}_t$ *denotes relative price for consumption and investment goods,* $\hat{p}_t^k = \frac{\hat{p}_t^{k^1}}{\hat{p}_t^{k^2}}$ *and* $\hat{p}_t^n = \frac{\hat{p}_t^{n^1}}{\hat{p}_t^{n^2}} = 1$ *are relative prices of capital stocks and labor services, respectively.*

$$
\begin{aligned}
&\hat{p}_t = \frac{MU_t^2}{MU_t^1} \\
&\hat{p}_t^{k^1} = \alpha_1\left(k_t^1\right)^{\alpha_1-1}\left(n_t^1\right)^{1-\alpha_1};\quad \hat{p}_t^{k^2} = \hat{p}_t\alpha_2\left(k_t^2\right)^{\alpha_2-1}\left(n_t^2\right)^{1-\alpha_2} \\
&\hat{p}_t^{n^1} = (1-\alpha_1)\left(k_t^1\right)^{\alpha_1}\left(n_t^1\right)^{-\alpha_1};\quad \hat{p}_t^{n^2} = \hat{p}_t(1-\alpha_2)\left(k_t^2\right)^{\alpha_2}\left(n_t^2\right)^{-\alpha_2},
\end{aligned}
$$

where MU_t^i *denotes marginal utility from consuming* c_t^i, $i = 1, 2$.

Proof. Appendix B. ■

Since investment goods, consumption goods and outputs have the same price, in each sector, aggregate counterparts are defined as:

$$
\begin{aligned}
c_t &\equiv c_t^1 + \hat{p}_t c_t^2 \\
i_t &\equiv i_t^1 + \hat{p}_t i_t^2 \\
y_t &\equiv y_t^1 + \hat{p}_t y_t^2,
\end{aligned}
$$

where $\hat{p}_t$ is defined in Proposition 2. Then, labor services are aggregated as follows:

$$n_t \equiv n_t^1 + n_t^2.$$

To calculate aggregate capital stock we use the relative prices for capital stock in each sector. The aggregate capital stock is:

$$k_t \equiv k_t^1 + \hat{p}_t^k k_t^2,$$

where $\hat{p}_t^k$ is defined in Proposition 2. Finally, the consumer price index is defined as:

$$CPI_t \equiv \frac{c_t^1}{y_t} + \hat{p}_t \frac{c_t^2}{y_t}.$$

Calibration. The model is calibrated for the US economy, over the sample 1947:Q1-1996:Q4. This sample choice allows to compare our results with the benchmark simulations presented in King and Rebelo (1999a), and the data analyzed by Stock and Watson (1998). Given its nature, the model could be calibrated using data on consumption of nondurables, of services, and/or on data from wholesale and retail trade.

However, it is appropriate to restrict the analysis to different constituent components for nondurables, or using wholesale trade and/or retail trade data. Changes in services' consumption are more associated with technological improvement. In other words, it may be hard to tell a story where consumer preferences shift between "*cheese-burgers*" and "*online banking*". Generalizing the argument, it would be more plausible to argue services' consumption (e.g. online banking) increases with improvement in (communications) technology (e.g. broad-band internet connections).

The model is thus calibrated using data on expenditures on Food and on Clothing and Shoes.[21] Food sales and Clothing-Shoes sales accounts for 53% and for 18% of personal consumption expenditures, respectively. HP-filtered Food sales are less volatile than Clothing and Shoes sales ($\sigma^F = 0.96$, while $\sigma^{C\&S} = 1.28$), but are more persistent ($\rho^F = 0.81$, while $\rho^{C\&S} = 0.69$). The sales of the two different nondurables components move together ($\rho^{F,C\&S} = 0.52$), and are positively correlated with aggregate nondurable expenditure ($\rho^{F,ND} = 0.89$, and $\rho^{F,C\&S} = 0.70$).

Proposing a generalized methodology for assessing preference shifts from the data goes beyond the goal of this paper. It is interesting, however, to explore briefly how one might asses whether there have been *relative preference shifts* between Food and Clothing-Shoes sales. A possible formal approach to an empirical examination of our hypothesis would be to estimate a demand function for Food (or for Clothing and Shoes). A log-linear demand equation derived from a very general class of preferences over Food and Clothing & Shoes can be written as $c_t^F = \zeta_0 + \delta_1 \frac{y_t}{p_t^F} + \delta_2 \frac{p_t^{C\&S}}{p_t^F}$,where p_t^F and $p_t^{C\&S}$ denote, respectively, the prices for Food and Clothing & Shoes, and y_t represents total expenditures on Food and

[21]Food and Clothing & Shoes Series. Sales: NIPA Tables 2.2; Price Indexes: NIPA Tables 7.2. Personal Disposable Income: NIPA Tables 2.1. All series are seasonally adjusted.

on Clothing & Shoes. To allow for heterogeneity one can assume that $\zeta_0 \sim \mathcal{N}\left(\bar{\zeta}_0, \sigma_\zeta^2\right)$, and rewrite the demand function as

$$c_t^F = \bar{\zeta}_0 + \delta_1 \frac{y_t}{p_t^F} + \delta_2 \frac{p_t^{C\&S}}{p_t^F} + \varepsilon_t,$$

where $\varepsilon_t \sim \mathcal{N}\left(0, \sigma^2\right)$. The corresponding marginal rate of substitution of Food for Clothing & Shoes $\frac{u'_F}{u'_{C\&S}}$ can be written as a function of parameters and consumption flows, $\frac{u'_F}{u'_{C\&S}} = g\left(\overset{(-)}{\zeta_0}, \overset{(\pm)}{\delta_1}, \overset{(+)}{\delta_2}, \overset{(-)}{\frac{c^F}{c^{C\&S}}}\right)$. Since a relative preference shift affects the marginal rate of substitution, it can be modelled by parameterizing the mean of ζ_0 as a function of time (i.e. den Butter, Delifotis and Koning, 1997). Hence we could estimate the following equation:

$$c_t^F = \zeta_0 + \zeta_1 t + \delta_1 \frac{y_t}{p_t^F} + \delta_2 \frac{p_t^{C\&S}}{p_t^F} + \varepsilon_t, \tag{13}$$

where the sign and the magnited of ζ_1 indicates whether the demand for Food has increased or decreased, if we control for changes in relative price and changes of income.

Notice, however, that a parametric regression would not be helpful for understanding relative preference shifts, since its outcome would the point estimate for ζ_1, denoted as $\hat{\zeta}_1$. This quantity can be interpreted as an average demand shock, which is not what, ideally, we would like to have. A nonparametric regression, on the other hand, allows us to fully exploit the presence of relative demand shifts. The consumption for food can be written as an unknow function $\Delta(\cdot)$

$$c^F = \Delta\left(\tilde{s}, \frac{y_t}{p_t^F}, \frac{p_t^{C\&S}}{p_t^F}\right), \tag{14}$$

where $\tilde{s}$ denotes a proxies for preference shifts. In this case, the impact of relative demand shocks is represented by the estimated partial derivative $\frac{\partial c^F}{\partial \tilde{s}}$. Demand equation (14) is estimated using the Nadaraya-Watson (1964) estimator (see Appendix C for more details). **Table 2** reports the results.

Then, **Figure 1** presents the estimated coefficient $\frac{\partial c^F}{\partial \tilde{s}}$. The picture is perfectly consistent with the concept of relative demand shocks. This is seen from the fact the impact of preference shocks on sales changes over time, After controlling for income, technology and relative prices.

Notice however, that this analysis does not address the issue of causality between preference shocks and sales's increases. Further research should be useful, but the results presented here can be certainly interpreted as a preliminar result in support of existence of relative preference shocks.

The system of equations we use to compute the dynamic equilibria of our benchmark model depends on a set of 10 parameters. Five pertain to the supply side $(\alpha_i, \Omega_i)_{i=1}^2$ and

Goodness of fit	$\frac{\partial \hat{c}^F}{\partial \tilde{s}}$	$\frac{\partial \hat{c}^F}{\partial y^F}$	$\frac{\partial \hat{c}^F}{\partial p^{F,C\&S}}$	MSE	MAE	MAPE
99.9613	0.3513	0.1431	0.000	2.5663E^{-5}	0.0038E^{-5}	0.0016E^{-5}
99.9622	0.3330	0.1593	0.000	2.5064E^{-5}	0.0038E^{-5}	0.0016E^{-5}

Table 2: Nadaraya-Watson Regression

Table 2. The first (second) row presents results when the proxy for preference shock is the HP-trend of food sales (personal disposable income); $\frac{\partial \hat{c}^F}{\partial \tilde{s}}$, $\frac{\partial \hat{c}^F}{\partial y^F}$, and $\frac{\partial \hat{c}^F}{\partial p^{F,C\&S}}$ denote average coefficients on $\tilde{s}$, y^F, and on $p^{F,C\&S}$, respectively. Bandwith are chosen using Least-Squares Cross-Validation. MAE: Mean Absolute Error; MSE: Mean Squared Error; MAPE: Mean Absolute Percentage Error. Source: NIPA Tables (1947:01-1996:01), and Author calculations.

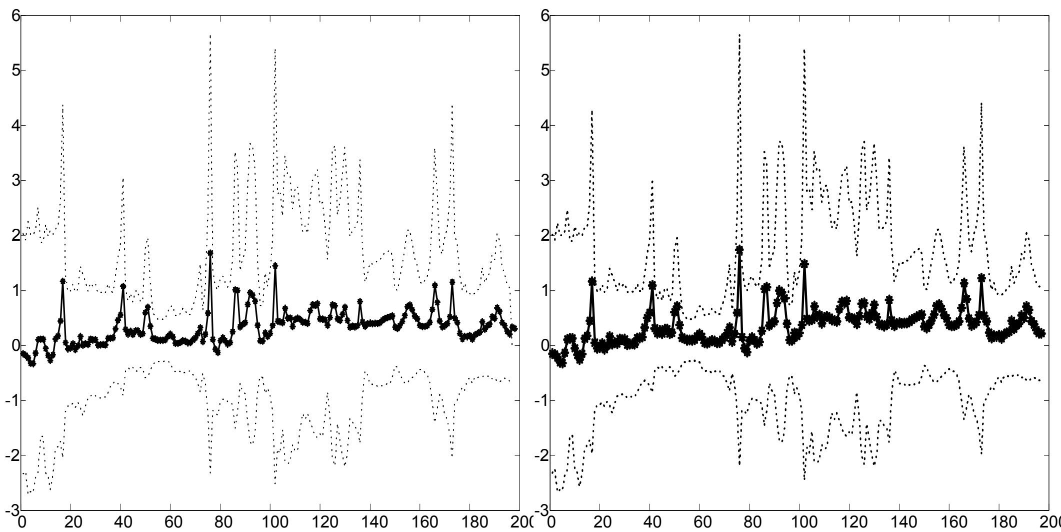

Figure 1: Relative Demand Shock on Food. The left (right) window presents the $\frac{\partial \hat{c}^F}{\partial \tilde{s}}$ coefficient when the proxy for demand shock is the HP-trend of Food sales (personal disposable income). The solid line with ball marker denotes $\frac{\partial \hat{c}^F}{\partial \tilde{s}}$, while the dotted lines represent 2-standard deviation intervals.

B, and five belong to demand side (q_1, q_2, λ_1, λ_2, β). The model is calibrated for the US economy, over the sample 1947:Q1- 1996:Q4.[22]

1. **Supply side parameters**. Both consumption goods we consider belong to the categories of nondurable goods, and therefore we assume that the technology structure is symmetric. We set $\alpha_1 = \alpha_2 = 0.33$, the standard value for the US economy (see King and Rebelo, 1999a, and Stock and Watson, 1998), and rates of capital depreciation are chosen to be $\Omega_1 = \Omega_1 = .025$ on a quarterly basis, assuming the same capital depreciation rate for both production technology. Notice that a symmetric parameterization allows a direct comparison with alternative formulations, at least along the supply side of the model.

[22]This sample choice allows to compare our results with the benchmark simulations presented in King and Rebelo (1999a) and the data analyzed by Stock and Watson (1998)

2. **Demand side parameters**. The instantaneous utility functions over both consumption goods are assumed to be logarithmic ($q_i = q = 1$), and the subjective discount factor β is set to 0.984, a standard value for the US economy. Since $\lambda_2 = 1-\lambda_1$, the parameter λ_1 is calibrated to match the ratio of steady state consumption sales. Manipulating the FOCs, it can be showed that:

$$\frac{\bar{c}^1}{\bar{c}^2} = \frac{\bar{c}^F}{\bar{c}^{C\&S}} = \left(\frac{\lambda_1}{1-\lambda_1}\right)^{\frac{1}{q}}, \tag{15}$$

   where $\frac{\bar{c}^F}{\bar{c}^{C\&S}}$ denotes the steady state ratio between Food Sales ($\bar{c}^F$) and Clothing and Shoes sales ($\bar{c}^{C\&S}$). Since in equilibrium $\frac{\bar{c}^F}{\bar{c}^{C\&S}} = 3.00$, $q = 1$, then λ_1 is calibrated to 0.75.

3. **Demand Shocks**. The demand shock process is modelled as AR(1) process in logs in order to facilitate the comparison with standard RBC models. The autocorrelation coefficients and the standard deviation of $\tilde{s}_t^1$ and $\tilde{s}_t^2$ are chosen to match the autocorrelation and the volatility of Food and Clothing-Shoes sales, respectively. In particular autocorrelation $\rho_F = 0.98$, and $\rho_{C\&S} = .94$, and the cross-correlation coefficient of relative demand shocks equal $corr(\epsilon_t^1, \epsilon_t^2) = 0.01$. Standard deviation for the innovation process equals $\sigma_F = 0.212$ and $\sigma_{C\&S} = 0.512$ (in percentage units).[23] This parameter choice is in line with Wen (2003), Wei (2003) and Guo and Sturzenegger (1998). They find that estimated persistence parameters range from 0.50 to 0.90 for the US economy.[24]

The parameterization of the model is otherwise standard, as from King and Rebelo (1999). This allows to carry out a meaningful comparison with standard RBC formulation.

Finally, notice there are two main aggregation methodologies: a fixed-weight aggregation method and chain-weighted type procedure. Until 1995 (included) the Bureau of Economics Analysis (BEA) has adopted the traditional fixed-weight approach, while since 1996 BEA has adopted a "*chain-index*" methods, which uses continually updated relative price weights. This paper use the fixed-weight approach since our model is calibrated over the sample 1947:1996, over which national account aggregated were computed with the fixed-index approach.

4.2.2 Extensions of the Benchmark Model

The model presented in the previous section is fairly simple, but anticipating some results, it performs quite well in generating fluctuations consistent with actual data. It is, however, natural to ask whether the model would deliver the same qualitative and quantitative results if relative demand shocks were replaced with relative technology shocks, or if labor adjustment costs were added to the model. Moreover, there is one essential element that gives to demand shocks a primary role for explaining business cycles and fluctuations,

[23] It should be acknowledged, however, that the two sectors produce capital goods, as well, while equation (**??**) refers only to consumption flow.

[24] Their correlation between innovations are not comparable since they propose open economy models where the cross-correlation refers to demand shocks in different countries.

that is the existence of some idle capacities in the system. These resources will be put in use when demand increases.[25] This suggests that a third natural extension consists in endogenizing the capacity utilization of capital, and to allow for variable labor effort. We consider each of this possible variations in turn.

Relative Technology Shocks. The structure of the economy is perfectly symmetric to that presented in the previous page. Specifically, the instantaneous utility function becomes state independent,

$$u(\mathbf{c}_t, \mathbf{n}_t) = \lambda_1 u_{(1)}(c_t^1) + \lambda_2 u_{(2)}(c_t^2) + \upsilon\left(\mathbf{n}_t; B\right)$$

where notation is as in equation (1). Production technologies are augmented with relative (sector-specific) technology shocks, denoted as ξ_t^i, $i = 1, 2$.

$$y_t^i = \xi_t^i \left(k_t^i\right)^{\alpha_i} \left(n_t^i\right)^{1-\alpha_i}, \;\; i = 1, 2,$$

where ξ_t^i are assumed to follow customary autoregressive processes in logs:

$$\log \xi_{t+1}^i = \omega \log \bar{\xi}^i + (1 - \omega) \log \xi_t^i + \epsilon_t^i,$$

where $\epsilon_t^i \sim \mathcal{N}\left(0, \sigma_{\epsilon^i}^2\right)$, for $i = 1, 2$.

The Case of Inter-temporal Adjustment Costs. Suppose that due to the technological and organizational specificity of labor services firms incur hiring costs because they need to inform and instruct newly hired workers before they are as productive as the incumbent workers. The creation and destruction of jobs (turnover) also entails costs for the workers, not only because they may need to learn to perform new tasks, but also in terms of the opportunity cost of unemployment and the costs of moving. The fact that mobility is costly for workers affects the equilibrium dynamics of wages and employment.

Adjustment costs may be strictly convex. In that case, the unit costs of turnover would be an increasing function of the actual variation in the employment level. This would slow down the optimal response to changes in the exogenous variables. There are also good reasons to suppose, however, that adjustment costs are concave.[26]

We consider quadratic adjustment costs. In particular, real income is reduced, in each sector, by a positive quantity $\frac{\delta_i}{2}\left(n_t^i - b_t^i n_{t-1}^i\right)^2 k_t^i$, where b_t^i $\left(0 \leqq b_t^i \leqq 1\right)$ is a scaling parameter, and $\delta_i \geqq 0$. Assume that b_t^i is defined as follows:

[25] Examples in cases might be leisure, variable capacity utilization, variable effort, (in this model and Wen, 2002), consumption inventories (Busato, 2003b), or energy. Alternatively it can be introduced a small and empirically plausible externality in production (Benhabib and Wen, 2002). This will magnify the marginal product of labor, enhancing the propagation mechanism in the short run.

[26] For instance, a single instructor can train more than one recruit, and the administrative costs of a firing procedure may well be at least partially independent of the number of workers involved. A case of linear adjustment costs lies in be-tween these extremes. The simple proportionality between the cost and the amount of turnover simplifies the characterization of the optimal labor demand policies

$$b_t^i = \begin{cases} 0 < b_t^i \leqq 1 \text{ if } n_t^i \neq n_{t-1}^i \text{ for } i = 1,2 \\ b_t^i = 0 \text{ if } n_t^i = n_{t-1}^i \text{ for } i = 1,2, \end{cases}$$

This formulation implies that labor adjustment costs do not affect the stationary state (when $n_t^i = n_{t+j}^i$ for all i and j), while they affects the transitional dynamics. In this model b_t^i is set to unity when $\tilde{s}_t^i \neq \bar{s}^i$.

Hence feasibility constraints may be rewritten as

$$c_t^i + i_t^i = \left(k_t^i\right)^{\alpha_i} \left(n_t^i\right)^{1-\alpha_i} - \frac{\delta_i}{2} \left(n_t^i - b_t^i n_{t-1}^i\right)^2 k_t^i \text{ for } i = 1, 2.$$

Introduction of a labor adjustment cost impacts the first order conditions for the optimal choice of labor services, which are then modified as follows, for $i, j = 1, 2$ and $j \neq i$

$$\begin{aligned} B\left(1 - n_t^i - n_t^j\right)^{-\gamma} &= \phi_t^i \left[(1-\alpha_i)\left(k_t^i/n_t^i\right)^{\alpha_i} - \delta_i \left(n_t^i - b_t^i n_{t-1}^i\right) k_t^i\right] + \\ &\quad + \mathbb{E}_t \beta \phi_{t+1}^i \delta_i \left(n_{t+1}^i - b_{t+1}^i n_t^i\right) k_{t+1}^i \end{aligned}$$

where ϕ_t^i denotes the marginal utility of consumption flows c_t^i at time t. Euler Equations are, for $i = 1, 2$.

$$\tilde{s}_t^i \left(c_t^i\right)^{-q_i} = \mathbb{E}_t \beta \tilde{s}_{t+1}^i \left(c_{t+1}^i\right)^{-q_i} \left\{\alpha_1 \left(k_{t+1}^i/n_{t+1}^i\right)^{\alpha_i - 1} + 1 - \Omega_i - \frac{\delta_i}{2} \left(n_{t+1}^i - b_{t+1}^i n_t^i\right)^2 k_{t+1}^i\right\},$$

Equilibrium is characterized by the following set of first order conditions, reported after algebraic manipulations.

$$\begin{aligned} B\left(1 - n_t^i - n_t^j\right)^{-\gamma} &= \lambda_i \tilde{s}_t^i \left(c_t^i\right)^{-q_i} \left[(1-\alpha_i)\left(k_t^i/n_t^i\right)^{\alpha_i} - \delta_i \left(n_t^i - b_t^i n_{t-1}^i\right) k_t^i\right] + \\ &\quad + \mathbb{E}_t \beta \lambda_i \tilde{s}_{t+1}^i \left(c_{t+1}^i\right)^{-q_1} \delta_i \left(n_{t+1}^i - b_{t+1}^i n_t^i\right) k_{t+1}^i \end{aligned} \quad (16)$$

Equation (16) equates the marginal disutility of working (on the LHS) with the marginal utility of consumption, weighted with marginal productivity of labor. The difference with respect to (6) and (7), is that the current marginal productivity of labor is diminished by the adjustment cost ($\delta_i \left(n_t^i - b_t^i n_{t-1}^i\right) k_t^i$), and $\mathbb{E}_t \beta \lambda_i \tilde{s}_{t+1}^i \left(c_{t+1}^i\right)^{-q_1} \delta_i \left(n_{t+1}^i - b_{t+1}^i n_t^i\right) k_{t+1}^i$ represents the expected saving due to having adjusted labor demand at time $t+1$. This is more evident rewriting the RHS of equation ((16)) as $B\left(1 - n_t^i - n_t^i\right)^{-\gamma} = \phi_t^i (1-\alpha_i) \left(k_t^i/n_t^i\right)^{\alpha_i} +$ $[\mathbb{E}_t \beta \phi_{t+1}^i \delta_i \left(n_{t+1}^i - b_{t+1}^i n_t^i\right) k_{t+1}^i - \phi_t^i \delta_i \left(n_t^i - b_t^i n_{t-1}^i\right) k_t^i]$. Proposition 3 shows that in a non-explosive solution, it should be null.

Proposition 3 *A recursive formulation of the problem implicity implies that the quantity* $\mathbb{E}_t \beta \lambda_i \tilde{s}_{t+1}^i \left(c_{t+1}^i\right)^{-q_i} \delta_i \left(n_{t+1}^i - b_{t+1}^i n_t^i\right) k_{t+1}^i = 0$ *for* $i = 1, 2.$

Proof. Appendix B. ■

A recursive formulation is *myopic* relative to the more general optimal control solution. The latter incorporates the fact, that once a labor adjustment has been made at time t, next period labor expected cost will be lower, since the expected labor change is smaller. In the context of a recursive formulation, the cost of adjustment hits more sharply the economy. Next, relying on the results of proposition 3, equilibrium in the labor market of our model is characterized by the following necessary and sufficient conditions.

$$B\left(1-n_t^i-n_t^j\right)^{-\gamma}=\lambda_i\tilde{s}_t^i\left(c_t^i\right)^{-q_1}\left[\left(1-\alpha_i\right)\left(k_t^i/n_t^i\right)^{\alpha_i}-\delta_i\left(n_t^i-b_t^in_{t-1}^i\right)k_t^i\right]\text{ for } i,j=1,2\text{ and } j\neq i$$

The Case of Intra-sector Adjustment Costs. The simplest way to introduce such adjustment costs into the basic model is to rewrite the constraint on the allocation of time as follows:

$$\ell_t=T-\left\{\left(n_t^1\right)^{-\nu}+\left(n_t^2\right)^{-\nu}\right\}^{-1/\nu},$$

where where ℓ_t denotes leisure at date t, and $\nu(\nu\leqq-1)$ denotes the elasticity of substitution between labor services. This specification of the time allocation constraint captures the idea that it is costly to reallocate labor from one sector to the other. The quantity $\left\{\left(n_t^1\right)^{-\nu}+\left(n_t^2\right)^{-\nu}\right\}^{-1/\nu}$ may be interpreted as a reverse CES technology. A reverse formulation ensures the optimization problem to be concave, since isoquants are concave toward the origin. Figure 2 illustrates this relationship by plotting the graph of $\left\{\left(n_t^1\right)^{-\nu}+\left(n_t^2\right)^{-\nu}\right\}^{-1/\nu}=n_t$ for $\nu=-1.0,-1.1,-1.5,-3.0$. Now, when $\nu=-1$, the transformation frontier is linear, and the transformation rate between hours equals 1. In other words, there are no adjustment cost in reallocating hours worked across sectors.

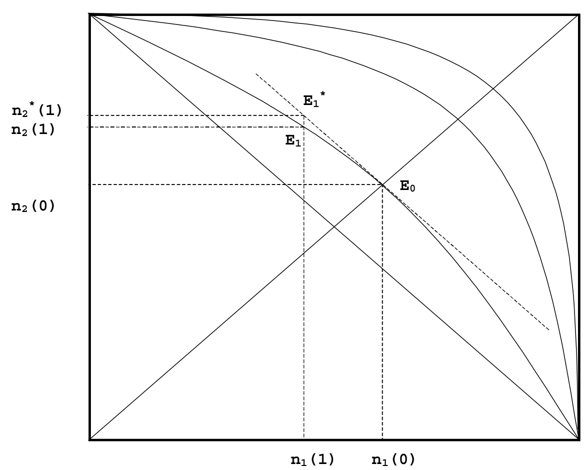

Figure 2: Intra-Sector Adjustment Costs

Next, let $\nu < -1$, and consider equilibrium point $E_0 = (n_2(0), n_1(0))$, where the representative agent supplies $n_1(0)$ and $n_2(0)$ hours to various sectors. Then assume that after a shock the consumer increases the total amount of hours worked. Notice that this is not what happens in the model, but it allows us to illustrate how this mechanism works. Now, suppose that a positive demand shock affects the second good, and that hours are reallocated from the first to the second sector. If the transformation function were linear, the new equilibrium would be $E_1^* = (n_2^*(1), n_1(1))$, where $|n_2^*(1) - n_2(0)| = |n_1(1) - n_1(0)|$. If, instead, the hours reallocation is costly, like it is assumed in this model, the new equilibrium is $E_1 = (n_2(1), n_1(1))$. Notice that in this case $|n_2(1) - n_2(0)| < |n_1(1) - n_1(0)|$, reflecting the fact that some time is lost while reallocating hours. It can be imagined that there exist transportation costs, or that it takes some time to reorganize ideas before starting a different activity. Figure 2 shows that, as the absolute value of ν gets bigger, it becomes more difficult to alter the composition of n_t. As $\nu \to -\infty$, it becomes impossible to alter the composition of labor supply.

The introduction of intra-sector adjustment costs, impact only on the first order conditions for labor services. After some algebraic manipulations, equations (6) and (7) are rewritten as follows:

$$B(1-n_t)^{-\gamma} n_t^{\frac{-1-\nu}{\nu}} \left(n_t^i\right)^{-\nu-1} = \lambda_i \tilde{s}_t^i \left(c_t^i\right)^{-q_i} \lambda_i \left(1-\alpha_i\right) \left(k_t^i / n_t^i\right)^{-\alpha_i}, \ i = 1, 2$$

where $n_t = \left\{ \left(n_t^1\right)^{-\nu} + \left(n_t^2\right)^{-\nu} \right\}$.

Variable Capacity Utilization. Under variable capacity utilization, production technologies are specified as follows:

$$y_t^1 = \left(u_t^i k_t^i\right)^{\alpha_i} \left(n_t^i\right)^{1-\alpha_i}, \ \ i = 1, 2, \tag{17}$$

where u_t^i denote the capital utilization rates, and n_t^i denotes labor demand in sector i, for $i = 1, 2$. To have an interior solution for u_t^i in the steady state, assume that the capital stock depreciates faster if it used more intensively, following Greenwood, Hercowitz and Huffman (1988):

$$\Omega_t^i = \frac{1}{\theta^i} (u_t^i)^{\theta^i - 1}, \ \ \theta^i > 1, \ \ i = 1, 2,$$

where Ω_t^i is the rate of depreciation. This structure endogenizes capacity utilization, and, at the same time, convexifies capital utilization. In the steady state θ^i is calibrated to that $\Omega_t^i = 0.025$, the customary depreciation rate for the US economy (on a quarterly basis). Then, capital accumulation constraints are defined as follows:

$$k_{t+1}^1 = (1 - \Omega_t^1) k_t^1 + i_t^1 \quad i = 1, 2 \tag{18}$$

where the $\Omega_t^i s$ denote quarterly endogenous depreciation rates.

5 Numerical Results

Being highly non-linear, the system has no closed form solution. To study its stochastic properties we apply the well known procedure developed by King Plosser and Rebelo (1988a, b); certainty equivalence is assumed, the system is linearized around its non stochastic steady state, and is solved by applying linear approximations (e.g. Campbell 1994; Uhlig 1999).

A multi-sector model offers several dimensions along which it can be compared to the actual data. We focus first on the aggregate series, presenting volatility measures, and contemporaneous correlations. The propagation mechanism and the cross correlation of output with consumption and investment at different leads and lags are considered. The price side of the model economy and volatility measures and correlations of sectoral variables are analyzed as well.

5.1 Aggregate Real Variables

This section describes how well the model accounts for aggregate and sectoral fluctuations.

5.1.1 Volatility Measures and Comovements

Table 3 reports the relative volatility with respect to aggregate output for the model series, and compares them with their counterparts for the US economy (sample period 1953:Q1-1996:Q4). Also present in the Table are the corresponding statistics for standard benchmark Real Business Cycle model of Hansen (1985), for other demand-driven models (e.g. Wen, 2003; Bencivenga, 1992; Baxter and King, 1991), and for selected multi-sector general equilibrium models (Huffman and Wynne, 1999; Horvath, 2000).

In all five versions of the model, consumption is less volatile than output, and investment is more volatile than output and than consumption. Both series are highly positively correlated with output. These positive comovements and the relative volatility order among these three variables are two of the most celebrated stylized business cycle facts. It should be noted that the model is not subject to the *crowding out* between consumption and investment, and between consumption and output, typical of several one-sector demand-driven formulations. Baxter and King (1991), notice that when an aggregate demand shock impinges on a one-sector model, people increase consumption, while reducing investment, and, by this end, capital accumulation. Output, being a a monotone transform of capital stock, subsequently falls, depicting a significant crowding out effect.

When relative demand shocks are the driving source for the economy, consumption becomes much more volatile than in the standard business cycle models. In this sense our model is not subject to the so called *consumption volatility puzzle*.[27] More importantly, notice these results are obtained under "*small*" preference shocks (see next section for more details). In this respect, it is an interesting improvement upon the *indivisible labor* version of Hansen (1985), and on all other technology-driven business cycle model. We argue that this improvement is due to the fact that the consumption is the first variable

[27] Finally, the *consumption volatility puzzle* (Cochrane, 2001) refers to the fact that consumption volatility generated by stochastic growth models is often too small relative to the data.

Table 3: Selected Moments, Aggregate Real Series

	σ_X/σ_Y			$\rho(X,Y)$				$\rho(APN,N)$
	N	C	I	N	C	I	APN	
US Economy	0.99	0.76	2.99	0.81	0.83	0.89	0.12	-0.25
Relative Demand Shocks (Benchmark Model)	**1.42**	**0.74**	**2.05**	**0.99**	**0.98**	**0.97**	**-0.19**	**-0.82**
+ Intra-Sec. Adj	0.89	0.71	2.07	0.97	0.71	0.97	0.18	-0.48
+ Inter-Temp. Adj	1.36	0.61	2.50	0.99	0.99	0.98	-0.08	-0.83
+ Variable Cap. Util.	1.16	0.89	1.42	0.99	0.99	0.99	-0.02	-0.93
+ Variable Cap. Util and Inter-Temp. Adj.	0.89	0.69	2.15	0.97	0.97	0.98	0.09	-0.85
Relative Tech. Shocks	0.78	0.45	3.24	0.99	0.96	0.98	0.77	0.81
Hansen (1985)	0.67	0.61	4.09	0.97	0.94	0.99	0.98	0.87
Wen (2003)	1.38	0.65	3.68	0.99	0.65	0.90	-	-
Bencivenga (1992)	1.19	1.25	-	0.94	0.98	-	-0.30	-0.60
Baxter-King (1991)								
Huffman-Winnye (1999)	0.80	0.43	2.81	0.87	0.97	0.97	0.74	0.57
Horvath (a)	0.57	0.58	3.94	0.88	0.86	0.72	0.87	0.49
Horvath (b)	0.54	0.51	3.18	0.95	0.94	0.85	0.94	0.79

Table 3. *Relative Demand Shock* refers to the baseline model where there are only relative demand shocks; *+Intra-Sec. Adj*, *+Inter-Temp. Adj*, and *+Variable Cap. Util.* refer to the introduction of labor adjustment costs, and of endogenous capacity utilization; *Relative Tech. Shock* indicates the model with only sectorial technology shocks, and no demand shocks. The letters N, C, I, denote respectively aggregate employment, consumption, investment, and total factor productivity; σ_X/σ_Y denotes relative volatility between a variable X and aggregate output Y, $\rho(X,Y)$ is the contemporaneous correlation with aggregate output, and $\rho(APN,N)$ represents the contemporaneous correlation between hours worked and the average productivity of labor. All statistics are computed based on 1000 simulations of 200 periods length. Sources: Stock and Watson (1998) for US data.

affected by the shocks, and it responds much more compared to a standard RBC model where it responds to what is, first, an increase in income.

It is also interesting to compare our model's performance along selected labor market dimensions, focusing especially on the so called *productivity puzzle.* The *productivity puzzle* looks at the correlation between labor productivity and GDP, and between labor productivity and employment. If productivity shocks drive the cycle, the productivity will be, by construction, highly correlated with GDP and aggregate employment. The puzzle is that average labor productivity and employment are negatively correlated for most economies ($\rho\,(APN,N) < 0$), while average labor productivity and GDP presents a weak (or null) correlation ($\rho\,(APN,Y) \geqq 0$) .[28] The Stock and Watson (1998)'s estimates for the U.S. economy, in particular, are $\hat{\rho}\,(APN,N) = -0.25$ and $\hat{\rho}\,(APN,Y) = 0.12$ respectively.

Consider, first, the correlation between average productivity of labor and employment $\rho\,(APN,N)$. This statistics is negative in all formulations of the relative-demand model, ranging between -0.48 and -0.83. On the contrary, technology driven models induce a large positive correlation. The economic mechanism of our models improves upon this failure, as the first order conditions suggest. In particular, the first order conditions for hours worked (equation (6)) can be written as $MU_{\ell_t}/MU_{c_t^i} = APN_t^i$, where MU_{ℓ_t} and MU_{c_t} denote marginal utilities of leisure and of $i-th$ consumption flow, while APN_t^i is the average productivity of $i-th$ sector labor services. The LHS represents the labor supply,

[28]As reported by Stadler (1994) this correlation is negative or zero for almost all the countries.

while the RHS the labor demand. Now, after a demand shock, $MU_{c_t^i}$ and MU_{ℓ_t} increase since consumption and total hours increase, but MU_{ℓ_t} responds relatively less than $MU_{c_t^i}$. This shifts out the labor supply schedule, along the labor demand. Demand does not shift, inducing a negative correlation between hours worked and wage. In a technology driven model, the mechanism is exactly the opposite. The APN_t^i increases after a positive productivity shocks, and labor demand shifts out, along labor supply. This results in a positive correlation between wage and hours, which is, however, absent in the data.

The correlation between wage rate and GDP also deserves mention. It is convenient to analyze this fact in conjunction with volatility of hours worked. The baseline version of the model overpredicts the relative volatility of hours worked $\frac{\sigma_n}{\sigma_y}$, because of diminishing returns to labor services. This seems, however, a feature peculiar of demand driven models (see Bencivenga, 1992; Wen, 2003). Moreover, this fact has the unfortunate implications of inducing a negative comovement between aggregate GDP (Y) and APN. That happens because over the business cycle N fluctuates relatively more that Y, inducing a negative correlation between $APN = (1-\alpha)\frac{Y}{N}$ and N. The introduction of intra-sector adjustment costs strengthens comovements between labor flows, thereby incresing the volatility of aggregate hours. Endogenizing the capacity utilization helps to reduce hours' volatility, because the variable capital capacity utilization offers additional flexility to the model. Also the introduction of intra-temporal adjustment costs reduces the ratio $\frac{\sigma_n}{\sigma_y}$ to 0.89. As a consequence the correlation between APN and Y becomes negative. This model, however, induces a negative correlation between consumption and investment flows in each sector (statistics are not presented here). That happens because costs of adjustment make it more difficult to increase labor supply after a demand shock, and thus it is more convenient to substitute investment with consumption. On the contrary, a model with intra-temporal adjustment costs *and* variable capacity utilization is capable to generate comovement between both sectors, as well as a procyclical average productivity of labor. The variable capacity utilization gives back to the system some of the flexibility lost because the cost of adjustment.[29]

The model with relative technology shocks performs better than some demand driven formulations only along the correlation between output and productivity. The corresponding consumption flow is much less volatile than output, and the wage is highly procyclical. In this sense, the stochastic properties of the relative technology model are qualitative analogous to corresponding one-sector formulations.

With a different kind of demand shocks, our model represents an improvement upon the Baxter and King (1991), and Bencivenga (1992) models especially along the consumption volatility dimension, while it performs as well as Wen (2003).[30] Next, compared to multi-sector models driven by technology shocks, our model performs quite well in predicting labor market behavior, and aggregate consumption volatility. Unfortunately, the

[29]Wen (2002) obtains analogous results in a model with Baxter an King (1991) type of aggregate demand shock.

[30]In particular, Bencivenga (1992)'s model has several undesirable properties, like a a negative correlation between hours and output. Consumption too is very volatile, even more than output (relative variability is 1.25). In summary, the model falls short under several dimensions, and, its results are, in some sense, weakened in the light of Gali (1999)'s contribution. Infact, Bencinvenga presents results only for the unconditional moments, still using multiple sources of fluctuations. It would be very interesting to have more information concerning the conditional moments.

comparison with alternative formulation is often time far from complete, since a detailed set of statistics for all models is usually not available.

5.1.2 Shocks' Propagation under Demand Uncertainty

In their well known survey on Real Business Cycle models King and Rebelo (1999) discuss extensively the central role of productivity shocks in driving the business cycle. They also stress how their benchmark model's performance relies on large and highly persistent technology shocks. To generate macroeconomic series consistent with the US and European data, their RBC models require a considerable variability in productivity, and a serial correlation parameter of the stochastic component of productivity near one.

The propagation mechanism of our model differs from the standard one, and it is distinctive of a two-sector model driven by relative demand shocks. In this context household smooth consumption *over time* (exchanging consumption and investment within each sector), and *across sectors*. To hedge consumption profiles against idiosyncratic shocks, consumers allocate labor supply to both sectors. On the other hand, firms face uncertainty about next period consumption goods' demand; to insure against that idiosyncratic risk, firms symmetrically increase (reduce) labor demand in both sectors. The intra-temporal shock transmission channel is quite important in this model, since relative demand shocks are exactly about substitution among commodities.

It is interesting to notice that the intra-sector risk hedging acts in this context as a propagation enhancer. To smooth across sectors aggregate consumption, consumers can only reallocate labor services, because consumption and capital are sector specific.[31] This makes risk hedging more valuable than it would be if consumption flows could be reallocated. Section 5.3 argues that this values is reflected into a *false* Solow residual.

It is particularly welcome that we obtain these results even if we use a logarithmic utility function for consumption, and very small shocks. We define an improvement in the propagation mechanism of a stochastic growth model in the sense of necessitating a lower autocorrelation coefficient for the process of stochastic disturbances, and a smaller standard deviation of the innovations for replicating business cycle facts. To highlight this feature, **Table 4** compares the model's parameterization with the one used in the standard benchmark model (Hansen, 1985), with the parameterization used in the so-called *high substitution class* models (see King and Rebelo, 1999),with a home production model (Benhabib, Greenwood and Wright, 1996)

It is interesting to note that while the standard RBC models need significantly larger shocks (standard deviation of the innovation process, $\sigma = 0.712$) and a high autocorrelation coefficient for the shock ($\rho \simeq 0.99$), the high substitution economies reduce the first of these magnitudes to about, $\sigma = 0.12$ but still need a high persistence coefficient ($\rho \simeq 0.99$). The latter class of models also requires a high risk aversion parameter ($q = 3$). Finally, our model require fairly small shocks, and a relatively lower autocorrelation coefficients for innovation process. The last column of Table 5 present the asymptotic variance of the stochastic shock $\tilde{\sigma}^2$, as generated by the different models. This quantity is defined as $\tilde{\sigma}^2 = \frac{\sigma^2}{1-\rho}$, and it represents a volatility measure reflecting both persistence and standard deviation of the innovation processes.

[31] Notice, that if consumer could directly reallocate consumption across sector, this would induce a lot of smoothing.

Table 4 Stochastic Disturbance Parameterizations: Various Model Formulations

Parameters	q	α	β	δ	ρ	σ	$\tilde{\sigma}^2$
Indivisible Labor[i]	1.00	0.36	0.984	0.025	0.979	0.72	**0.247**
High Substitution[ii]	3.00	0.36	0.984	0.025	0.989	0.12	**0.131**
Cho and Cooley[iii]	1.00	0.36	0.990	0.025	0.950	1.02	**0.208**
Home Production[iv]	1.00	0.29	0.989	0.023	0.950	0.70	**0.098**
Relative Demand Shocks	**1.00**	**0.36**	**0.984**	**0.025**	**0.96**	**0.36**	**0.181**

Table 4. Parameter q denotes relative risk aversion parameter, α capital share in a Cobb-Douglas production function, β the subjective discount factor, ρ and σ (unit:percent) the autocorrelation coefficient and the standard deviation of innovation process, respectively. Finally, $\tilde{\sigma}^2$ (unit:percent) denotes variance of the stochastic shocks, and it is computed as $\tilde{\sigma}^2 = \sigma/(1-\rho)$. References (i): Hansen (1985), (ii): Burnside, Eichenbaum and Rebelo (1995). (iii): Cho and Cooley (1994). (iv): Greenwood, Rogerson and Wright (1995). For multi-sector model we present the simple mean of the parameters.

5.2 Cross Correlation Analysis

5.2.1 Prices and Price Index

Table 5 reports the cross correlation between consumer price index (CPI) and aggregate output. The upper part of the table presents data on the CPI level, while the bottom part displays corresponding statistics for CPI growth rate; that is the inflation rate. Interestingly, a relative demand-driven dynamic general equilibrium model generates a negative correlation between CPI and aggregate output, and a positive correlation between inflation and aggregate output.[32]

Table 5: Consumer Price Index Cross Correlation with Output (Benchmark Model)

lead/lag	-4	-3	-2	-1	0	1	2	3	4
$\rho^*(P, Y_{t+k})$	0.43	0.39	0.28	0.07	-0.28	-0.33	-0.28	-0.29	-0.33
$\hat{\rho}(P, Y_{t+k})$	0.12	-0.04	-0.21	-0.38	-0.51	-0.62	-0.68	-0.67	-0.59
$\rho^*(\pi, Y_{t+k})$	0.18	0.26	0.35	0.47	0.25	0.19	0.23	0.25	0.25
$\hat{\rho}(\pi, Y_{t+k})$	0.58	0.64	0.62	0.52	0.35	0.14	-0.08	-0.27	-0.40

Table 5. $\rho^*(P, Y_{t+k})$ and $\hat{\rho}(P, Y_{t+k})$ denotes the simulated and the actual correlations between price index level at time t with aggregate output at time $t+k$, respectively. All statistics are computed based on 1000 simulations of 200 periods length.

This is consistent with the actual US data. One interpretation for this regularity is that supply shocks plays a dominant role in driving the cycle. For example, Barro (1993) interprets these results as evidence in favor of real business cycle models where productivity generates countercyclical price movements, and against new-keynesian models. But, such evidence should be interpreted with caution as a number of studies have shown that standard sticky-price models with only demand shocks can generate negative correlation

[32]Recent studies by Kydland and Prescott (1990), Cooley and Ohanian (1991), and Backus and (1992) present evidence of negative correlation between prices and output.

Table 6: Consumption and Investment Cross Correlation with Output									
lead/lag	-4	-3	-2	-1	0	1	2	3	4
$\rho^*(C, Y_{t+k})$	0.11	0.21	0.38	0.66	**0.98**	0.72	0.45	0.31	0.22
$\rho^*(I, Y_{t+k})$	0.20	0.10	0.27	0.56	**0.91**	0.69	0.45	0.33	0.27
$\hat{\rho}(C, Y_{t+k})$	0.29	0.53	0.75	0.89	**0.90**	0.76	0.51	0.21	-0.07
$\hat{\rho}(I, Y_{t+k})$	0.18	0.41	0.65	0.83	**0.89**	0.82	0.61	0.32	0.04
$\rho^*(C, Y_{t+k})$ (KPR)	-0.33	-0.07	0.23	0.53	0.77	**0.88**	0.87	0.75	0.56
$\rho^*(I, Y_{t+k})$ (KPR)	-0.46	-0.22	0.09	0.40	0.67	0.81	**0.84**	0.75	0.68

Table 6. $\rho(C, Y_{t+k})$ denotes the correlation between aggregate consumption at time t with aggregate output at time $t + k$, and $\rho(I, Y_{t+k})$ denotes the correlation between aggregate investment at time t with aggregate output at time $t + k$. The star denotes a simulated moments, while the hat an estimated one. All statistics are computed based on 1000 simulations of 200 periods length. Sources: Stock and Watson (1998) for the US economy, KPR denotes King, Plosser and Rebelo (1988).

coefficients between prices and output (e.g. Chada and Prasad (1993), Ball and Mankiw (1994), Judd and Trehan (1995)).[33]

Our results contribute to this debate, showing that in a neoclassical model driven by relative demand shocks prices are low in expansions. We argue it happens because of the Permanent Income Hypothesis (PIH).

The CPI is defined as $\frac{c_t^1}{y_t} + \hat{p}_t \frac{c_t^2}{y_t}$. After a relative demand shock, both c_t^1 and c_t^2 respond less that aggregate output because of PIH (Table 8). The relative price $\hat{p}_t$ responds positively to a demand shock on c_t^2 and negatively to one on c_t^1. This mechanism implies that during an expansion y_t increases, and $\frac{c_t^1}{y_t} + \hat{p}_t \frac{c_t^2}{y_t}$ decreases, inducing a negative correlation between CPI and aggregate output.

5.2.2 Consumption and Output

Standard RBC models cannot explain that the largest cross-correlations between consumption and output, and between investment and output occur at lead/lag equal zero (e.g. Stock and Watson, 1998).[34] The introduction of demand shocks improves upon the standard RBC model also along this dimension, as **Table 6** shows.

Causality order is inverted with respect to a standard RBC scheme. In the latter an increase in income leads to an increase in consumption and investment, while in the

[33] In a classical sticky price model, indeed, a demand shock raises output in the impact period, but it leave price unchanged. In the long run, output returns to its pre-shock level (this is usually defined as *long-run neutrality*) but prices are permanently higher. During the adjustment process, prices are below trend for some periods while output is above trend. This can generate a negative correlation between detrended prices and output.

[34] Table 14 reports for convenience these data. In particular, the strongest correlation between consumption and output in the King, Plosser and Rebelo model occurs at $k = +1$ lag, while the largest correlation between consumption and investment occurs at $k = +2$. This suggests that consumption lags output, and investment in their model.

former model the increase in consumption pulls income up, via the labor market channel. Notice the difference with a technology driven business cycle model. Transitory technology shocks incentivates consumers to increase capital stock. The permanent income hypothesis implies that consumption comoves with capital stock, in order to smooth consumption over time. Capital stocks, however, follows a quite sluggish dynamics because of the depreciation rate, and it lags investment, which, on the contrary comoves with output. As a consequence, consumption lags output.

5.3 A *False* Solow Residual

Prescott (1986) suggests that one way of measuring technological change within the context of real business cycle models is to follow Solow (1957).[35] But, as Prescott (1986) stresses, there may be errors in measuring the labor and the capital inputs, and the Solow Residual has been directly or indirectly at the center of many discussions. In calculating it, full and constant utilization of both capital and labor inputs is often assumed.[36] Hall (1988) challenged the assumption that movements in Solow Residual represent exogenous technology shocks. He argues

> "[...] that under competition and constant returns to scale the Solow residual is uncorrelated with all variables known to be neither cause by productivity shifts, nor the causes of productivity shifts [...]"

The Solow residual seems, indeed, to be correlated with government expenditure (Hall, 1988), with various monetary aggregates (Evans, 1992), and government consumption (Burnside, Eichembaum and Rebelo, 1993). Jovanovic (1991) argues that secular changes in organization might explain a large portion of the change in the Solow residual in one country over time. Burnside, Eichembaum and Rebelo (1993) investigate the sensitivity of Solow Residual to the presence of labor hoarding behavior. Quite interestingly, their results are supportive of the view that a large part of fluctuations in Solow residual depends on labor hoarding type behavior. They eventually conclude that the existing real business cycle models substantially overstate the role of technology shocks that accounts for the volatility in the GDP postwar series. Hoover and Salyer (1998) demonstrate that the Solow residual does not carry useful information about technology shocks.

Our paper also contributes to this literature. It shows that under relative demand shocks model is capable of generating a "*false*" Solow residual whose statistical properties are consistent with the analogous computation using US data (**Table 7**).[37] The false "Solow Residual" is computed following original Solow (1957) definition. It is denoted with Greek letter ψ, since in ancient Greek $\psi\varepsilon\upsilon\delta\acute{\eta}s$ (*pseudes*) means *false, untrue*.

[35] In this case, Solow growth accounting suggests that the process of the technology parameter is highly persistent. Its volatility, measured with Solow residual's standard deviation, is approximate 0.763 for the US economy.

[36] Since the utilization of capital is likely to be highly procyclical, it can be argued that this assumption could have important implications for the interpretation of the procyclical behavior and exogeneity of productivity shocks,as well as the degree of increasing returns to scale and market power in the economy.

[37] In an economy without distortions, the Solow residual measures aggregate technology change. The assumptions made by Solow (1957) are: perfect competition in both the markets of product and labor, constant return to scale, zero transaction costs, full utilization of all inputs and their instantaneous adjustment to the desired demand levels. Under these hypotheses, the Solow residual measures technology change (i.e. productivity and technology coincide).

Table 7: *False Solow Residual* Properties (Benchmark Model)

	σ_{FSR}/σ_Y	ρ_{FSR}/σ_Y
US Economy	0.54	0.78
Relative Demand Shocks	**0.13**	**0.66**
+ Intra-Sector Adj	0.22	0.45
+ Inter-Temporal Adj	0.17	0.41
+ Variable Cap. Util.	0.22	0.90
Relative Tech. Shocks	0.51	0.99
Hansen (1985)	-	-

Table 7. *Relative Demand Shock* refers to the baseline model where there are only relative demand shocks, *+Intra-Sector Adj*, *+Inter-Temporal Adj*, and *+Variable Cap. Util.* refer to introduction of labor adjustment costs, and of endogenous capacity utilization; *Relative Tech. Shock* indicates the model with only sectorial technology shocks, and no demand shocks; σ_{FSR}/σ_Y and $\rho(FSR, Y)$ respectively denote relative volatility and contemporaneous correlation of *False Solow Residual* with the aggregate output Y. All statistics are computed based on 1000 simulations of 200 periods length.

$$\log \psi_t = \log(y_t) - \alpha \log(k_t) - (1 - \alpha) \log(n_t). \tag{19}$$

The difference is that this quantity measures something completely different from technology or productivity. It captures pure sectorial demand effects over aggregate factor productivity. It is interesting to notice, however, that actual and simulated Solow Residual present an analogous contemporaneous correlation with output ($\hat{\rho} = 0.78$, and $\rho^* = 0.66$), while the simulated False Solow residual is less volatile that the actual one, relative to GDP ($\hat{\sigma} = 0.53$ and $\sigma^* = 0.13$) (Table 7). The False Solow Residual generated by our model does not reflect, by its very construction, any change in technology and productivity.

5.4 Sectoral Business Cycle

Table 8 presents volatility measures (relative to GDP's) for disaggregated series for production, consumption, investment, and labor services. The first row reports the volatility of each series relative to corresponding sectoral GDP, while the second one presents the volatility of each series, relative to the corresponding aggregate variable. Consider c_1 for example. From the first row of Table 8, we know that $\frac{\sigma_{c_1}}{\sigma_{y_1}} = 0.93$, and from the second one we realize that $\frac{\sigma_{c_1}}{\sigma_c} = 0.40$.

Notice that consumption of FCM (c_1) is more volatile than ME (c_2), relative to the total consumption. In particular, $\frac{\sigma_{c_1}}{\sigma_c} = 0.40$ and $\frac{\sigma_{c_2}}{\sigma_c} = 0.74$, where σ_c denotes the volatility of aggregate consumption. This is consistent with the actual data (Table 9), which reports that the series of Motor and Energy sales is relative more volatile than that of Food, Cloth and Music.

Table 9, then, presents the contemporaneous correlations among disaggregated variables, and aggregate variables. Two features of the model deserve more attention. First,

Table 8 Volatility Measures: Disaggregated Series (Benchmark Model)

	n_1	c_1	i_1	r_1	y_1	n_2	c_2	i_2	r_2	y_2
$\sigma_{x_i}/\sigma_{y_i}$	1.49	0.93	1.69	0.04	1.00	1.48	0.72	2.14	0.04	1.00
σ_{x_i}/σ_x	0.34	0.40	0.26	0.00	0.32	0.80	0.74	0.80	0.00	0.76

Table 8. $\sigma_{x_i}/\sigma_{y_i}$ denotes the standard deviation of variable x_i relative to corresponding output y_i, while σ_{x_i}/σ_x denotes the standard deviation of variable x_i relative to corresponding aggregate counterpart x; all statistics are computed based on 1000 simulations of 200 periods length.

all sectors comove, since contemporaneous correlation between capital stocks, labor flows, production output, investments and consumptions are positively correlated. Second, both disaggregated series comove with the corresponding aggregate series.

Table 9. Comovement across Sectors (Benchmark Model)

	k_1	k_2	k
k_1	1.00		
k_2	0.98	1.00	
k	0.40	0.43	1.00

	n_1	n_2	n
n_1	1.00		
n_2	0.67	1.00	
n	0.89	0.93	1.00

	y_1	y_2	y
y_1	1.00		
y_2	0.67	1.00	
y	0.84	0.97	1.00

	c_1	c_2	c
c_1	1.00		
c_2	0.95	1.00	
c	0.79	0.91	1.00

	i_1	i_2	i
i_1	1.00		
i_2	0.97	1.00	
i	0.99	0.99	1.00

Table 9.all statistics are computed based on 1000 simulations of 200 periods length.

It is also interesting to notice the positive comovements do not depends on the positive (but very tiny) correlation between demand innovations (the off diagonal elements of the correlation matrix between innovation are set equal to 0.01). There are two key elements that explain the positive comovements across sectors. First, the relative demand shocks change not only the intra-sector desirability between consumption goods, but also modify the inter-temporal preferences of consumers (Proposition 1).

Notice, that a relative demand shocks increases the desirability of one consumption bundle relative to the other, but, at the same time, it changes the inter-temporal discount factor of the representative agent.

Second, labor reallocation ensures that wage rates are equated across sector, on a period by period basis. Notice that labor services increase in the sector directly affected by the shock, triggering in increase in capital stocks. Because marginal productiveness of labor services are equated across sectors, the increase in wage in both sectors induce consumers to work more.

5.5 Extensions and Developments

There exist several interesting developments originating from this theoretical scheme, like, for example, the analysis of asset pricing under relative demand shocks, and the study of fiscal policy under demand uncertainty.

The former issue, is quite actual. Consider, for example, the stock market performances in the last couple of years. Often time it has been argued that they have been significantly damaged by *lower-than-expected* demand for telecommunication services. A decentralization with financial markets of the model presented in this paper (e.g. following Danthine and Donaldson, 2002) would be a valid benchmark for analyzing such an issue. The argument, then, can be extended, investigating which are the more general asset pricing implications of relative demand shocks.

Consider, then, the fiscal policy extension. A typical classification would attribute fiscal policy three main tasks: allocative, redistributive, and stabilizing. Within the context of our model a natural question to ask is whether fiscal policy should or should not stabilize the economy after a relative demand shock. More in general, it is important to understand whether stabilization policy differs from a standard model driven by technology (supply-side) shocks.

6 Conclusions

This model proposes an economic mechanism complementary to the standard RBC theory, where fluctuations are generated by autonomous changes in preferences over different commodities. The driving force of this model is represented by *relative demand shocks*.

The model is able to reproduce effectively the main stylized facts of the US economy, such labor market regularities as the weak correlation between hours and factor productivity. The observed correlations between aggregate output with the aggregate consumption and investment, with price index and with the inflation rate are also matched (at all lags and leads). In this sense, the model can be proposed as a benchmark for the US economy.

Finally, the model generates a *false* "Solow Residual", even though there are no technology shocks. It is interpret as representing the value of intra-sector risk sharing (if we allow for agents' heterogeneity), or the value of hedging against idiosyncratic demand shocks (for a representative agent economy).

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7 Appendix

7.1 Appendix A: Utility Function in More Details

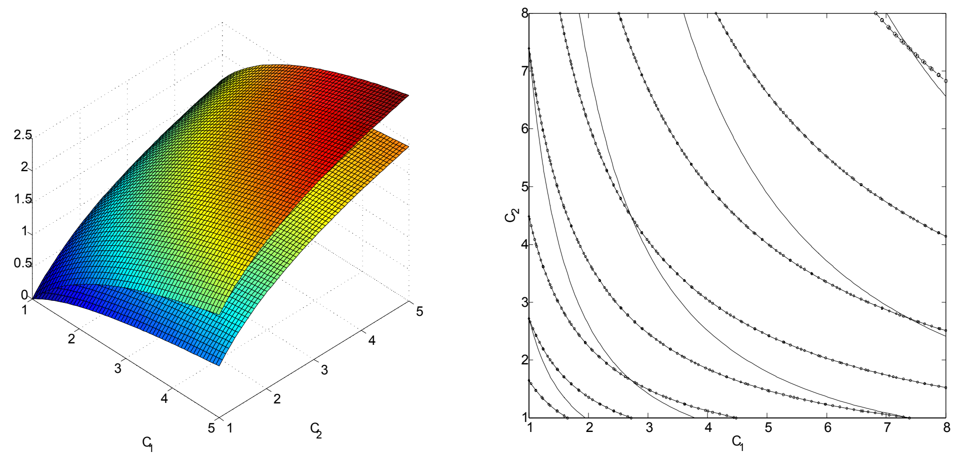

Figure 3: Utility Function Indifference Curves

Figure 3 plots the graph of utility function (1), abstracting from the leisure component when $\rho = 1$, $u_{(i)}(c_t^i; \tilde{s}_t^i) = \tilde{s}_t^i \log(c_t^i)$, and for two values of $s_t^1 = \left\{1, \frac{2}{3}\right\}$, while $s_t^2 = \{1, 1\}$. This scenario illustrates how a positive and relative demand shock on c_t^1 impacts on consumer preferences. The left window present the graph of the function, where the upper surface represents graph of $\log(c_t^1) + \log(c_t^2)$, and the lowers surface denotes that of $\frac{2}{3}\log(c_t^1) + \log(c_t^2)$. The right window presents corresponding indifference curves, where lines with the ball marker denote indifference curve map for $\log(c_t^1) + \log(c_t^2) = \bar{u}$, and the solid lines represents indifference map for $\frac{2}{3}\log(c_t^1) + \log(c_t^2) = \bar{u}$. After a relative demand shock occurs, the indifference map shifts (in or out, depending on the shift, and it rotates. In particular, the left window of Figure 3 shows that when relative desirability for the first commodity increases, indifference curves shift out, and rotate left, giving incentive to consumers to substitute consumption from c_t^2 to c_t^1. It is important to notice that this is only the static part of the argument. In fact, in a dynamic models the relative preference shift generates also an intertemporal resource reallocation, and an expansion. Next sections describe in more details this mechanism.

7.2 Appendix B: Proofs and Derivations

7.2.1 Derivations

7.2.2 Proofs

Proof of Proposition 1. Let $\mathbf{p} = \left(p_t^{c^i}, p_t^{i^i}, p_t^{n^i}, p_t^{k^i}\right)_{i=1,2}$ be a price system, where $p_t^{c^i}$ is consumption i time-t price, $p_t^{i^i}$ denotes $i-th$ investment flow price, $p_t^{n^i}$ represents labor services price, and $p_t^{k^i}$ is the price of $i-th$ capital stock. The allocation that solves the planning problem can be supported by a recursive competitive equilibrium of the Prescott and Mehra (1980) notion (**Part 1**). In addition, it is here showed that under linear disutility of labor (Hansen, 1985), and when disutilities of labor are proportional to the population relative shares, the same allocation can be supported with a more decentralized setting, with two firms and two groups of consumers (**Part 2**).

Part 1. Recursive Competitive Equilibrium.

Lemma 1 (Firms) *There exist two types of firms: Type I firms produce the first commodity c_t^1, while Type II firms produce the second one c_t^2. The choice is without loss of generality. Firms face a sequence of static problems. Each firm maximizes its profits on a period by period basis, given market prices $\mathbf{p}_t$. A Type i firms ($i = I, II$) maximized its profits π_t^i:*

$$\begin{aligned} \max \pi_t^i &\equiv p_t^{c^i} c_t^i + p_t^{i^i} i_t^i - p_t^{k^i} k_t^i - p_t^{n^i} n_t^i \\ s.to &: c_t^i + i_t^i = \left(k_t^i\right)^{\alpha_i} \left(n_t^i\right)^{1-\alpha_i}. \end{aligned}$$

Introduce multiplier μ_t^i, and form the lagreangean $\mathcal{L}^i$

$$\mathcal{L}^i = p_t^{c^i} c_t^i + p_t^{i^i} i_t^i - p_t^{k^i} k_t^i - p_t^{n^i} n_t^i + \mu_t^i \left(-c_t^i - i_t^i + \left(k_t^i\right)^{\alpha_i} \left(n_t^i\right)^{1-\alpha_i}\right).$$

After algebraic manipulations first order conditions can be written as:

$$\begin{aligned} p_t^{c^i} &= p_t^{i^i} && (1) \\ p_t^{k^i} &= p_t^{c^i} \alpha_i \left(k_t^i\right)^{\alpha_i - 1} \left(n_t^i\right)^{1-\alpha_i} \equiv p_t^{c^i} MPK_i && (2) \\ p_t^{n^i} &= p_t^{c^i} (1-\alpha_i) \left(k_t^i\right)^{\alpha_i} \left(n_t^i\right)^{-\alpha_i} \equiv p_t^{c^i} MPN_i, && (3) \end{aligned}$$

for $i = 1, 2$. Just notice that $p_t^{c^i} = p_t^{i^i} = \mu_t^i > 0$, since constraint holds with equality.

Lemma 2 (Consumers) *Suppose there exist a continuum of consumers, uniformly distributed over a unit interval, supplying labor to both sectors. Consumer $\gamma \in [0, 1]$ has preference over sequences of consumption and labor, and maximizes expected utility as summarized by the lifetime utility function (time separable between consumption (c_t^1, c_t^2) and leisure (ℓ_t)) $U_0^\gamma = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{u\left(c_t^1, c_t^2; \tilde{s}_t^1, \tilde{s}_t^2\right) + \upsilon(\ell_t)\right\}$, where $\mathbb{E}_t$ is the mathematical expectations operator conditional on information available at time t, $\upsilon(\ell_t)$ is a well behaved (continuous, twice continuously differentiable) function of ℓ_t, representing the disutility of* *working, and β is a subjective discount factor. Consumer γ solves the following dynamic optimization problem:*

$$\max_{c_t^1, n_t^1, c_t^2, n_t^2} \left\{ u\left(c_t^1, c_t^2; \tilde{s}_t^1, \tilde{s}_t^2\right) + v\left(\ell_t\right) \right\},$$

$$\begin{aligned}
s.to \quad &: \quad p_t^{c^1}\left(c_t^1 + i_t^i\right) = p_t^{k^1} k_t^1 + p_t^{n^1} n_t^1 \\
&: \quad p_t^{c^2}\left(c_t^2 + i_t^2\right) = p_t^{k^2} k_t^2 + p_t^{n^2} n_t^2 \\
&: \quad k_{t+1}^i = \left(1 - \Omega_1\right) k_t^i + i_t^i,\ i = 1,2 \\
&: \quad \ell_t = T_t - n_t^1 - n_t^2, \\
&: \quad \tilde{s}_{t+1}^i = \varrho \tilde{s}_t^i + \epsilon_t^i,\ i = 1,2 \\
&: \quad k_0^i > 0,\ i = 1,2
\end{aligned}$$

where T_t^i denotes total hours available. Introduce multiplier θ_t^i, and form the Hamiltonian $\mathcal{H}_t$

$$\begin{aligned}
\max_{c_t^i, n_t^i, k_{t+1}^i} \mathcal{H}_0 \quad = \quad & \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \left\{ u\left(c_t^1, c_t^2; \tilde{s}_t^1, \tilde{s}_t^2\right) + v\left(T_t - n_t^1 - n_t^2\right) + \right. \\
& + \theta_t^1 \left(p_t^{k^1} k_t^1 + p_t^{n^1} n_t^1 - p_t^{c^1} c_t^1 - p_t^{c^1} k_{t+1}^1 + p_t^{c^1} \left(1 - \Omega_1\right) k_t^1 \right) + \\
& \left. + \theta_t^2 \left(p_t^{k^2} k_t^2 + p_t^{n^2} n_t^2 - p_t^{c^2} c_t^2 - p_t^{c^2} k_{t+1}^2 + p_t^{c^2} \left(1 - \Omega_2\right) k_t^2 \right) \right\}
\end{aligned}$$

First order conditions are, for $i = 1,2$

$$u_i\left(c_t^1, c_t^2; \tilde{s}_t^1, \tilde{s}_t^2\right) = \theta_t^i p_t^{c^i} \tag{4}$$

$$v_i\left(T_t - n_t^1 - n_t^2\right) = \theta_t^i p_t^{n^i} \tag{5}$$

$$\theta_t^i p_t^{c^i} = \beta \mathbb{E}_t \theta_{t+1}^i \left(p_{t+1}^{k^i} + \left(1 - \Omega_i\right) p_{t+1}^{c^i} \right), \tag{6}$$

$$\lim_{t \to \infty} \beta^t \mathbb{E}_t \theta_t^i k_t^i = 0$$

Lemma 3 (Equilibrium Characterization) *From (4) and (6) we have*

$$u'\left(c_t^i; s_t^i\right) p_t^{c^i} = \beta \mathbb{E}_t u'\left(c_{t+1}^i; s_{t+1}^i\right) \left(p_{t+1}^{k^i} + \left(1 - \Omega_i\right) p_{t+1}^{c^i} \right), \tag{7}$$

while from (1) and (1):

$$v_i\left(T_t - n_t^1 - n_t^2\right) = u_i\left(c_t^1, c_t^2; \tilde{s}_t^1, \tilde{s}_t^2\right) \frac{p_t^{n^i}}{p_t^{c^i}} \tag{8}$$

Substituting the firm's optimality conditions (2) and (3) into (7) and (8) we have:

$$u'\left(c_t^i; \tilde{s}_t^i\right) p_t^{c^i} = \beta \mathbb{E}_t u'\left(c_{t+1}^i; \tilde{s}_{t+1}^i\right) p_{t+1}^{c^i} \left(\alpha_i \left(k_t^i\right)^{\alpha_i - 1} \left(n_t^i\right)^{1 - \alpha_i} + 1 - \Omega_1 \right) \quad i = 1,2 \tag{9}$$

since $p_{t+1}^{k^i} = p_t^{c^i} \left(1 - \alpha_i\right) \left(k_t^i\right)^{\alpha_i} \left(n_t^i\right)^{-\alpha_i}$, and

$$v_i\left(T_t - n_t^1 - n_t^2\right) = u'\left(c_t^1; \tilde{s}_t^1\right) \left(1 - \alpha_i\right) \left(k_t^i\right)^{\alpha_i} \left(n_t^i\right)^{-\alpha_i},\ i = 1,2 \tag{10}$$

Conditions (9) and (10) characterize optimal choice under a competitive equilibrium, together with the feasibility constraint (11)

$$\begin{aligned} c_t^i + k_{t+1}^i - (1-\Omega_i)\, k_t^i &= \frac{p_t^{k^i}}{p_t^{c^i}} k_t^i + \frac{p_t^{n^i}}{p_t^{c^i}} n_t^i, \quad i=1,2 \\ &= MPK_i k_t^i + MPN_i n_t^i \\ &= y_i, \end{aligned} \tag{11}$$

where the latter inequality follows from homegeneity of degree 1 of production technologies ($f\left(k_t^1, n_t^1\right) = f_1\left(k_t^i, n_t^i\right) k_t^i + f_2\left(k_t^i, n_t^i\right) n_t^i$). *Now specialize* $u_{(1)}(c_1, \tilde{s}_1) = \tilde{s}_t^1 \frac{(c_t^1)^{1-q_1}}{1-q_1}$, $u_{(2)}(c_2, \tilde{s}_2) = \tilde{s}_t^2 \frac{(c_t^2)^{1-q_2}}{1-q_2}$, $v\left(\ell_t\right) = B\frac{(T-n_t^1-n_t^2)^{1-\gamma}}{1-\gamma}$. *Hence optimality condition for the aggregate economy are:*

$$B(1-n_t^1-n_t^2)^{-\gamma} = \lambda_i \tilde{s}_t^i \left(c_t^i\right)^{-q_i}(1-\alpha_i)\left(k_t^i/n_t^i\right)^{-\alpha_i} \tag{12}$$

$$\tilde{s}_t^i\left(c_t^i\right)^{-q_i} = \mathbb{E}_t \beta \tilde{s}_{t+1}^i \left(c_{t+1}^i\right)^{-q_i}\left\{\alpha_i\left(k_{t+1}^i/n_{t+1}^i\right)^{\alpha_i-1} + 1 - \Omega_i\right\} \tag{13}$$

$$c_t^i + k_{t+1}^i - (1-\Omega_i)\, k_t^i = y_i \tag{14}$$

Notice that in equilibrium individual quantities equal aggregate counterparts. To see this, just notice that any aggregate variable $X = \int_0^1 x^\gamma d\xi = x$*, where* x *is the individual variable. Hence all individual optimality conditions holds at aggregate level, too. Notice that* (6) = (12), (7) = (13). *Capital accumulation constraints do not change, and feasibility constraints* (3) = (14). *Hence, Planner Equilibrium presented in Section 4 is equivalent to a RCE of the Prescott and Mehra (1980) notion, in the sense that optimality conditions and constraints have been proven to be identical. Finally, notice that since a Pareto Optimal equilibrium exists, so does a RCE. Since the former is unique, so is the latter. Primitives of the problem satisfy all nice conditions for existence and uniqueness of the equilibrium.*

Part 2. Recursive Competitive Equilibrium with Weak Heterogeneity

Lemma 4 (Firms) *see Lemma 1.*

Lemma 5 (Consumers, type I and II) *Suppose, next, that consumers of group i are uniformly distributed over the support* $[0, \lambda_i]$. *Households belonging to i-th class maximize:*

$$\begin{aligned} &\max_{c_t^i, n_t^i} u\left(c_t^i; \tilde{s}_t^i\right) + B_i n_t^i, \\ s.to \ &: \ p_t^{c^i}\left(c_t^i + i_t^i\right) = p_t^{k^i} k_t^i + p_t^{n^i} n_t^i \\ &: \ k_{t+1}^i = (1-\Omega_1)\, k_t^i + i_t^i \\ &: \ \ell_t^i = T_t^i - n_t^i, \end{aligned}$$

where $B_i < 0$ *is a scaling parameter, and* T_t^i *denotes total hours available. Introduce multiplier* θ_t^i*, and form the Hamiltonian* $\mathcal{H}_t^i$*, for* $i = 1,2$.

$$\begin{aligned} \max_{c_t^i, n_t^i, k_{t+1}^i} \mathcal{H}_0^i = \ & \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \left\{ u\left(c_t^i; \tilde{s}_t^i\right) + B_i n_t^i + \right. \\ & \left. + \theta_t^i \left(p_t^{k^i} k_t^i + p_t^{n^i} n_t^i - p_t^{c^i} c_t^i - p_t^{c^i} k_{t+1}^i + p_t^{c^i}(1-\Omega_i)\, k_t^i \right) \right\}. \end{aligned}$$

First order conditions are, for $i = 1, 2$

$$
\begin{aligned}
u'\left(c_t^i; s_t^i\right) &= \theta_t^i p_t^{c^i} && (15)\\
B_i &= \theta_t^i p_t^{n^i} && (16)\\
\theta_t^i p_t^{c^i} &= \beta \mathbb{E}_t \theta_{t+1}^i \left(p_{t+1}^{k^i} + (1-\Omega_i)\, p_{t+1}^{c^i}\right), && (17)
\end{aligned}
$$

where $\mathbb{E}_t$ denotes a conditional (on time t) expectation operator.

Lemma 6 (Equilibrium Characterization) *From (15) and (17) we have*

$$
u'\left(c_t^i; s_t^i\right) p_t^{c^i} = \beta \mathbb{E}_t u'\left(c_{t+1}^i; s_{t+1}^i\right) \left(p_{t+1}^{k^i} + (1-\Omega_i)\, p_{t+1}^{c^i}\right), \tag{18}
$$

while from (15) and (16):

$$
B_i = u'\left(c_t^i; s_t^i\right) \frac{p_t^{n^i}}{p_t^{c^i}} \tag{19}
$$

Substituting the firm's optimality conditions (2) and (3) into (18) and (19) we have:

$$
u'\left(c_t^i; \tilde{s}_t^i\right) p_t^{c^i} = \beta \mathbb{E}_t u'\left(c_{t+1}^i; \tilde{s}_{t+1}^i\right) p_{t+1}^{c^i} \left(\alpha_i \left(k_t^i\right)^{\alpha_i - 1} \left(n_t^i\right)^{1-\alpha_i} + 1 - \Omega_1\right) \; i = 1, 2 \tag{20}
$$

since $p_{t+1}^{k^i} = p_t^{c^i} (1-\alpha_i) \left(k_t^i\right)^{\alpha_i} \left(n_t^i\right)^{-\alpha_i}$, and

$$
B_i = u'\left(c_t^1; \tilde{s}_t^1\right) (1-\alpha_i) \left(k_t^i\right)^{\alpha_i} \left(n_t^i\right)^{-\alpha_i}, \; i = 1, 2 \tag{21}
$$

Conditions (20) and (21) characterize optimal choice under a competitive equilibrium, together with the feasibility constraint (22)

$$
\begin{aligned}
c_t^i + k_{t+1}^i - (1-\Omega_i)\, k_t^i &= \frac{p_t^{k^i}}{p_t^{c^i}} k_t^i + \frac{p_t^{n^i}}{p_t^{c^i}} n_t^i\\
c_t^i + k_{t+1}^i - (1-\Omega_i)\, k_t^i &= y_i && (22)
\end{aligned}
$$

Suppose, next, that consumers of group 1 are uniformly distributed over the support $[0, \lambda_i]$. Individual quantities x_t^i can be aggregate as follows $X = \int_0^{\lambda_1} x^{\psi} d\psi = \lambda_1 x$. Substituting into (9), (10) and (11), λ_i simplifies out. Hence (9), (10) and (11) characterize optimal choice of type 1 consumers under a competitive equilibrium.

Lemma 7 (Planner Economy, Varian (1978)) *From Varian (1978) corresponding to every Pareto Optimal (PO) allocation, there exist a set of non negative numbers $(\lambda_i)_{i=1}^N$ such that the same allocation can be achieved by a social planner maximizing a linear combination of individuals' utility function, using $(\lambda_i)_{i=1}^N$ as weights. Since $N = 2$ in our model, Planner solves the following problem:*

$$
\begin{aligned}
& \max_{c_t^1, c_t^2, n_t^1, n_t^2} \left\{\lambda_1 \left\{u\left(c_t^1; \tilde{s}_t^1\right) + B_1 n_t^1\right\} + \lambda_2 \left\{u\left(c_t^2; \tilde{s}_t^2\right) + B_2 n_t^2\right\}\right.\\
s.to \; &: \; \left(k_t^i\right)^{\alpha_i} \left(n_t^i\right)^{1-\alpha_i} - c_t^i - \overset{i}{i_t}, \; i = 1, 2\\
&: \; k_{t+1}^i = (1-\Omega_i)\, k_t^i + \overset{i}{i_t}, \; i = 1, 2\\
&: \; \ell_t^i = T^i - n_t^i, \; i = 1, 2.
\end{aligned}
$$

Introduce multipliers ϕ_t^1, and ϕ_t^2, and form the Hamiltonian. After customary algebraic manipulations, first order conditions are:

$$
\begin{aligned}
B_1\lambda_1 &= u'(c_t;\tilde{s}_t^1)\,(1-\alpha_1)\left(k_t^1/n_t^1\right)^{-\alpha_1}, && (23)\\
B_2\lambda_2 &= u'\left(c_t^2;\tilde{s}_t^2\right)(1-\alpha_2)\left(k_t^2/n_t^2\right)^{-\alpha_2}. && (24)
\end{aligned}
$$

Investment dynamics of the model is determined by the following two Euler Equations:

$$
\begin{aligned}
u'(c_t;\tilde{s}_t^1) &= \mathbb{E}_t\beta u'\left(c_{t+1}^1;\tilde{s}_{t+1}^1\right)\left\{\alpha_1\left(k_{t+1}^1/n_{t+1}^1\right)^{\alpha_1-1}+1-\Omega_1\right\}, && (25)\\
u'\left(c_t^2;\tilde{s}_t^2\right) &= \mathbb{E}_t\beta u'\left(c_{t+1}^2;\tilde{s}_{t+1}^2\right)\left\{\alpha_2\left(k_{t+1}^2/n_{t+1}^2\right)^{\alpha_2-1}+1-\Omega_2\right\}, && (26)
\end{aligned}
$$

Finally, equilibrium is characterized by feasibility and constraints.

$$
\begin{aligned}
c_t^1-k_{t+1}^1+(1-\Omega_1)\,k_t^1 &= \left(k_t^1\right)^{\alpha_1}\left(n_t^1\right)^{1-\alpha_1} && (27)\\
c_t^2+k_{t+1}^2-(1-\Omega_2)\,k_t^2 &= \left(k_t^2\right)^{\alpha_2}\left(n_t^2\right)^{1-\alpha_2} && (28)\\
\ell_t &= T-n_t^1-n_t^2. && (29)
\end{aligned}
$$

Lemma 8 (Equivalence Conditions) *Consider optimality conditions under a competitive equilibrium, and under a planning equilibrium. In order to equate (23) with (19), it is necessary to set*

$$\lambda_i=\frac{B_2}{B_1}.$$

In words: the larger the disutility from work is, the larger the weight of the consumer's type is in equilibrium. In the market equilibrium (9) and (10) are satisfied. Now, set $\lambda_i=\frac{\phi_t^i}{\theta_t^i p_t^{c^i}}$ for $i=1,2$. It follows that an allocation in a competitive equilibrium sastisfies (23), (24), (25), and (26) and this is Pareto optimal.

■

Proof of Proposition 2. Optimality conditions (2) and (3) in Proposition 1 define prices for consumption flows c_t^i, for investment flows i_t^i, for labor services $c_{t,}^i$, and for capital stocks k_t^i.

$$
\begin{aligned}
p_t^{c^i} &= p_t^{i^i}=\mu_t^i\\
p_t^{k^i} &= p_t^{c^i}\alpha_i\left(k_t^i\right)^{\alpha_i-1}\left(n_t^i\right)^{1-\alpha_i}\\
p_t^{n^i} &= p_t^{c^i}(1-\alpha_i)\left(k_t^i\right)^{\alpha_i}\left(n_t^i\right)^{-\alpha_i}.
\end{aligned}
$$

Let the first commodity be the numeraire of the system, and denote with $\hat{\mathbf{p}}_t=\left(1,\hat{p}_t,\hat{p}_t^{k^1},\hat{p}_t^{k^2},\hat{p}_t^{n^1},\hat{p}_t^{n^2}\right)$ the relative price vector. In particular, from the previous set of equation it is as follows:

$$
\begin{aligned}
\hat{p}_t &= \frac{\mu_t^2}{\mu_t^1} && (30)\\
\hat{p}_t^{k^1} &= \alpha_1\left(k_t^1\right)^{\alpha_1-1}\left(n_t^1\right)^{1-\alpha_1} && (31)\\
\hat{p}_t^{k^2} &= \hat{p}_t\alpha_2\left(k_t^2\right)^{\alpha_2-1}\left(n_t^2\right)^{1-\alpha_2} && (32)\\
\hat{p}_t^{n^1} &= (1-\alpha_1)\left(k_t^1\right)^{\alpha_1}\left(n_t^1\right)^{-\alpha_1} && (33)\\
\hat{p}_t^{n^2} &= \hat{p}_t(1-\alpha_2)\left(k_t^2\right)^{\alpha_2}\left(n_t^2\right)^{-\alpha_2}, && (34)
\end{aligned}
$$

where μ_t^i denotes marginal utility from consuming c_t^i, and it is derived from first order condition with respect to c_t^i. Relative prices supporting the Pareto Optimal equilibrium are obtained substituting equilibrium quantities derived from the planning problem into equations (30)-(34). ■

Proof of Proposition 3. This claim is proved in a constructive way. A recursive formulation rules out, by construction, non-stationary equilibrium patters. It is not necessary to formally prove that the problem have a recursive formulation. With the period utility function defined as in equation (1), the value function $\mathcal{V}_t(\mathbf{k}_t, \mathbf{s}_t)$ satisfies:

$$\begin{aligned}
\mathcal{V}_t(\mathbf{k}_t, \mathbf{s}_t) &= \max\left\{\lambda_1 u\left(c_t^1; s_t^1\right) + \lambda_2 u\left(c_t^2; s_t^2\right) + B\left(n_t^1 + n_t^2\right)\right\} + \\
&\quad +\beta\mathbb{E}_t\mathcal{V}_{t+1}(\mathbf{k}_{t+1}, \mathbf{s}_{t+1}) \\
s.to &: \quad c_t^i + i_t^i = f_i(k_t^i, n_t^i) - \frac{\delta}{2}\left(n_t^i - b_t^i n_{t-1}^i\right)^2, \text{ for } i = 1, 2 \\
&: \quad k_{t+1}^i = (1 - \Omega_i)\, k_t^i + i_t^i, \text{ for } i = 1, 2.
\end{aligned}$$

Next, substituting for c_t^i into $u\left(c_t^1; s_t^1\right)$, and for k_{t+1}^i into $\mathcal{V}_{t+1}(\mathbf{k}_{t+1}, \mathbf{s}_{t+1})$, first order conditions are:

$$\begin{aligned}
\lambda_i u'\left(c_t^i; s_t^i\right) &= \beta\mathbb{E}_t\mathcal{V}_{t+1}(\mathbf{k}_{t+1}, \mathbf{s}_{t+1}) \\
B &= \lambda_i u'\left(c_t^i; s_t^i\right)\left(f_2(k_t^i, n_t^i) - \delta\left(n_t^i - b_t^i n_{t-1}^i\right)\right),
\end{aligned}$$

where $f_2(k_t^i, n_t^i)$ denotes partial derivative with respect to n_t^i. Envelope conditions are:

$$\mathcal{V}_t'(\mathbf{k}_t, \mathbf{s}_t) = \lambda_i u'\left(c_t^i; s_t^i\right) f_1(k_t^i, n_t^i) + (1 - \Omega_i)\,\beta\mathbb{E}_t\mathcal{V}_{t+1}(\mathbf{k}_{t+1}, \mathbf{s}_{t+1}), \text{ for } i = 1, 2.$$

Then, manipulating first order conditions, the following two Euler Equations are derived:

$$u'(c_t; \tilde{s}_t^1) = \mathbb{E}_t\beta u'\left(c_{t+1}^1; \tilde{s}_{t+1}^1\right)\left\{\alpha_1\left(k_{t+1}^1/n_{t+1}^1\right)^{\alpha_1 - 1} + 1 - \Omega_1\right\}, \text{for } i = 1, 2$$

■

7.3 Appendix C: the Nadaraya-Watson Regression

The Nadaraya-Watson kernel estimator of conditional mean is defined as (p. 25 Hardle, 1990):

$$\hat{m}_h(x) = \frac{n^{-1}\sum_{i=1}^{n} K_h(x - X_i)\, Y_i}{n^{-1}\sum_{i=1}^{n} K_h(x - X_i)}$$

Where h is a vector of bandwidths that determine the size of the kernel weights in the regression; the shape of the weights is determined by the kernel which is a continuous, bounded and symmetric real function that integrates to one and x is the point at which the conditional mean is estimated. The denominator of the above expression is an empirical density that, according to the way in which is defined, allows to take into account the fraction of observations that have a distance smaller than a certain value from the point where the conditional mean is estimated. This empirical density is used to normalized the weights, it makes possible to adapt the local density of the X-variables (Hardle, 1990) and guarantees the weights sum to 1.The approximate variance of this estimator (Pagan-Ullah, 1999 pg 103)

$$V(\hat{m}(x)) \approx \frac{\sigma^2(x)}{n\prod_{i=1}^{p} h_i f(x)} \int K^2(z)\, dz$$

The non parametric estimator of the partial derivatives of the Nadarya Watson estimator at point x is given by the formula

$$\hat{\beta}(x) = \frac{\partial \hat{m}(x)}{\partial x} = \sum_{i=1}^{n} y_i \frac{\partial W(x)}{\partial x}$$

where $W(x) = \frac{K_h(x-X_i)}{\sum_{i=1}^{n} K_h(x-X_i)}$. The approximate variance of the estimated partial derivatives is:

$$V\left(\hat{\beta}(x)\right) \approx \frac{2\sigma^2(x)}{nh^3\prod_i h_i f(x)} \left(\int_{\Re^p} K^2(z)\, dz - \int_{\Re^p} K(z + \iota_j h/2)\, K(z - \iota_j h/2)\, dz\right)$$

where ι_j is a vector of zeros except for j-th element, which equals one.

The optimal bandwidth for the Nadaraya Watson estimator is of the form:

$$h_j^* = \psi_j \sigma_j n^{-1/(2l+p)}$$

where ψ_j is a scaling sector, σ_j is the variance of x_j, l is the order of the Kernel, and p is the number of variables in the joint density being estimated.