Non-technical summary

Research Question

In Germany industrial production has a remarkable share in the overall economic activity and thus plays an important role in the current macroeconomic analysis. Given its monthly availability with a publication delay of about five weeks, industrial production is also considered as a crucial indicator for the gross domestic product. Against this background, reliable forecasts of industrial production are one of the key elements in the short-term macroeconomic analysis and projections at the Bundesbank. This paper introduces a parsimonious forecast model for industrial production in Germany. The aim of the model is to feature a transparent approach and to improve the forecast performance, especially for horizons longer than three months.

Contribution

Our approach introduces the idea of multicointegration to forecasting industrial production. While standard vector error correction models are based only on the theoretical relation between industrial production and new orders received, the multicointegration framework allows for a second long-run relation in the model: the relation between industrial production and inventories and/or stock of orders. Moreover, we augment all model classes with survey-based indicators specifically targeted at the industrial sector. We consider both linear and threshold-type nonlinear specifications, and estimate the models with Bayesian and frequentist methods. For the Bayesian method we extend the long-run priors of Giannone, Lenza, and Primiceri (2016) to the multicointegrated case.

Results

Our results based on a real-time forecast evaluation between 2006 and mid-2017 point out that the parsimonious forecast routine with around ten models can reduce the root mean squared forecast errors by up to 30% over a 12-month forecast horizon against the random-walk benchmark.

Nicht-technische Zusammenfassung

Fragestellung

In Deutschland hat die Industrieproduktion einen bemerkenswerten Anteil an der gesamtwirtschaftlichen Aktivität und spielt damit eine wichtige Rolle bei der aktuellen makroökonomischen Analyse. Angesichts seiner monatlichen Verfügbarkeit mit einer Publikationsverzögerung von etwa fünf Wochen gilt die industrielle Produktion auch als entscheidender Indikator für das Bruttoinlandsprodukt. Vor diesem Hintergrund sind verlässliche Prognosen der Industrieproduktion eines der Schlüsselelemente der kurzfristigen makroökonomischen Analyse und Projektionen bei der Bundesbank. In diesem Beitrag wird ein sparsames Prognosemodell für die industrielle Produktion in Deutschland vorgestellt. Ziel des Modells ist es, einen transparenten Ansatz zu verfolgen und die Prognosegüte vor allem für Horizonte länger als drei Monate zu verbessern.

Beitrag

Unser Ansatz stellt die Idee der Multikointegration zur Prognose der Industrieproduktion vor. Während die Standardvektorfehlerkorrekturmodelle nur auf der theoretischen Beziehung zwischen der Industrieproduktion und den neuen Aufträgen basieren, ermöglicht der Multikointegrationsansatz eine zweite Langzeitbeziehung im Modell: das Verhältnis zwischen Industrieproduktion und Lagerbestand und/oder Bestandsaufträgen. Darüber hinaus ergänzen wir alle Modellklassen mit umfragebasierten Indikatoren, die speziell auf den industriellen Sektor ausgerichtet sind. Wir betrachten sowohl lineare als auch nichtlineare Spezifikationen und schätzen die Modelle mit Bayesianischen und frequentistischen Methoden. Für die Bayesianische Methode erweitern wir die Langzeitprioren von Giannone et al. (2016) auf den multikointegrierten Fall.

Ergebnisse

Unsere Ergebnisse, die auf einer Echtzeit-Prognosebewertung zwischen 2006 und Mitte 2017 basieren, weisen darauf hin, dass die sparsame Prognoseroutine mit rund zehn Modellen die durchschnittlichen quadratischen Prognosefehler um bis zu 30% über einen 12-monatigen Prognosehorizont gegenüber der random-walk Benchmark reduziert.

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Forecasting Industrial Production in Germany*

- work in progress, please do not circulate -

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Abstract

This paper introduces a parsimonious forecast model for industrial production in Germany. Our framework is based on the combination of point and density forecasts obtained from a pool with a relatively low number of models capturing, in particular, the long-run dynamics of industrial production. The model pool consists of three model classes: vector autoregressive and error correction as well as multicointegration models. While standard vector error correction models are based on the relation between industrial production and new orders received, the multicointegration framework allows for a second long-run relation in the model: the relation between industrial production and inventories and/or stock of orders. Moreover, we augment all model classes with survey-based indicators specifically targeted at the industrial sector. We consider both linear and threshold-type nonlinear specifications, and estimate the models with Bayesian and frequentist methods. At a final step we combine the point and density forecasts in various ways. Our results based on a real-time forecast evaluation between 2006 and mid-2017 reveal that the parsimonious forecast routine with around ten models can reduce the root mean squared forecast errors by up to 30% over a 12-month forecast horizon against the random walk benchmark.

Keywords: Bayesian estimation, Forecasting, Industrial production, Multicointegration, Nonlinear models

JEL classification: C11, C53, E37.

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1 Introduction

In Germany industrial production has a remarkable share in the overall economic activity and thus plays an important role in the current macroeconomic analysis. Given its monthly availability with a publication delay of about five weeks, industrial production is also considered a crucial indicator for gross domestic product. Against this background, reliable forecasts of industrial production are of paramount interest and one of the key elements in the short-term macroeconomic analysis and projections in Germany.

However, the literature on forecasting industrial production is quite sparse despite its importance in understanding and predicting German business cycles. A first set of papers like Osborn, Heravi, and Birchenhall (1999), Heravi, Osborn, and Birchenhall (2004) and Hassani, Heravi, and Zhigljavsky (2009) study the seasonality properties and focus on forecasting the major components of industrial production in Germany, France and the UK. While Osborn et al. (1999) attribute seasonality a significant share in monthly growth rates of the selected series, Heravi et al. (2004) and Hassani et al. (2009) suggest choosing the model depending of the type of the forecast, such as monthly growth rates or directional change. Another set of papers, Fritsche and Stephan (2002), Hüfner and Schröder (2008) and Robinzonov and Wohlrabe (2008), among others, are in search of leading indicators for the economic activity which is measured by industrial production in Germany. Consequently, these papers commonly indicate that the forecasting ability of selected indicators may depend on the forecast setting. In a more recent paper, Ulbricht, Kholodilin, and Thomas (2017) study the predictive power of media data in forecasting German industrial production and show that it may improve forecast accuracy in the 10- to 12-month horizons. While these studies provide forecasters with a valuable insight on the characteristics of potential leading indicators for industrial production, they do not make an attempt to build a forecast model for industrial production. Against this backdrop, we aim to fill this gap in the related literature and propose a model for predicting German industrial production which can also be extended with various leading indicators.

This paper introduces a parsimonious forecast model for industrial production in Germany. The aim of the model is to feature a transparent approach and to improve the forecast performance for horizons up to twelve months. Our framework is based on the combination of point and density forecasts obtained from a pool with a relatively low number of models capturing, in particular, the long-run dynamics of industrial production. The model pool consists of three model classes: two of them, vector AR and error correction models, are fairly standard and one is novel. Our novel approach introduces the idea of multicointegration to forecasting industrial production. While standard vector error correction models are based "only" on the theoretical relation between industrial production and new orders received, the multicointegration framework allows for a second long-run relation in the model: the relation between industrial production and inventories and/or stock of orders. Moreover, we augment all model classes with survey-based indicators specifically targeted at the industrial sector.

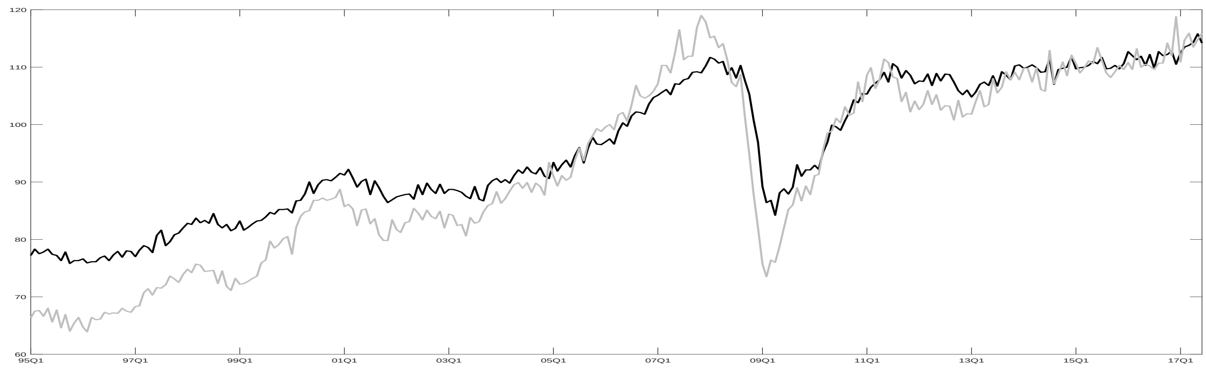
Furthermore, we consider both linear and threshold-type nonlinear specifications, and estimate the models with Bayesian and frequentist methods. For the Bayesian method we extend the long-run priors of Giannone et al. (2016) to the multicointegrated case. At a final step we combine the point and density forecasts in various ways. Our results based on a real-time forecast evaluation between 2006 and mid-2017 point out that the parsimonious forecast routine with less ten models can reduce the root mean squared forecast errors by up to 30% over a 12-month forecast horizon against the random-walk benchmark.

The remainder of this paper is set out as follows. Section 2 describes the data and provides a preliminary analysis. Section 3 introduces the forecasting framework, while Section 4 discusses the empirical findings. Section 5 concludes.

2 Data and preliminary analysis

We employ monthly real-time data to forecast industrial production in Germany over the period from January 1995 to June 2017, while the forecast evaluation sample spans from July 2006 to June 2017 due to data availability. We use calender and seasonally adjusted time series for industrial production and new orders received in the manufacturing sector. Moreover, we adjust industrial production with ice-, bridge- and vacation days in order to account for the impact of non-working days on the production. The industrial production and new orders received volume indices are given in logarithms and denoted by variables ip_t and or_t, respectively. Figure 1 plots the time series data over the period considered in our study. Moreover, log growth rates of industrial production and new orders received are given in percentage points and calculated as $\Delta ip_t = ip_t - ip_{t-1}$ and $\Delta or_t = or_t - or_{t-1}$, respectively. All time series have been obtained from the Bundesbank statistical database.

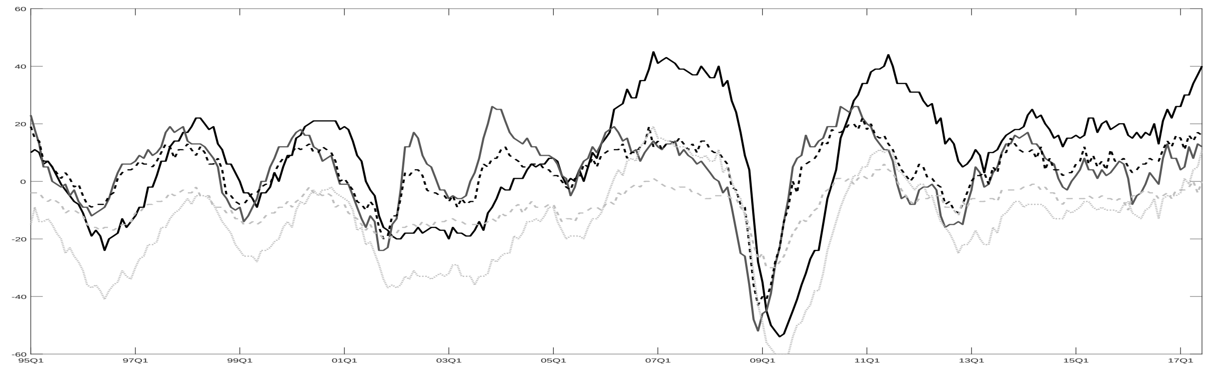
Figure 1: Industrial production and new orders received



Notes: The graph plots monthly industrial production (solid black line) and new orders received (solid grey line) in logarithms over the period January 1995 − June 2017.

Moreover, we also make use of sentiment indicators with focus on industrial sector in Germany. Specifically, we use various survey-based ifo indicators, such as the assessment of the current business situation, expectations with regard to business development in the next 6 months, assessment of finished goods inventory, assessment of orders on hand and expectations with regard to production activity in the next three months. Related time series are rescaled and given in logs and denoted by variables ifo_c, ifo_e, ifo_{lb}, ifo_{ab} and ifo_{pp}, respectively. Figure 2 plots the ifo indicators used in our study.

Figure 2: Ifo survey indicators



Notes: The graph plots ifo current situation (ifo_c, solid black line), ifo business expectations (ifo_e, solid dark grey line), the assessment of the finished goods inventory (ifo_{lb}, dashed black line), and that of the orders in hand (ifo_{ab}, dashed grey line) and production plans (ifo_{pp}, dotted grey line), respectively over the period January 1995 $-$ June 2017.

3 Econometric methodology

Our econometric framework is based on the combination of point (and density) forecasts obtained from a pool with a relatively low number of models capturing, in particular, the long-run dynamics of industrial production. The model pool consists of three model classes, vector AR, vector error correction and multicointegration models. First, we use simple AR models and vector autoregressions (VAR) with up to four variables: industrial production, new orders received as well as the assessment of the current situation and/or business expectations according to ifo surveys. Moreover, we employ vector error correction models (VECM) in order to capture the long-run relationship between industrial production and new orders received. Finally, we consider multicointegration models allowing for a second long-run relationship between industrial production and inventory/stock of orders.

We consider both linear and threshold-type nonlinear specifications, and estimate the models with Bayesian and frequentist methods. For the Bayesian method we extend the long-run priors of Giannone et al. (2016) to the multicointegrated case. At a final step we combine the point and density forecasts in various ways.

3.1 VARs and VECMs

Our first model class is a simple vector autoregression. Accordingly, the conditional mean equation can be written as[1]

$$\Delta \boldsymbol{y_t} = \boldsymbol{\mu} + \sum_{i=1}^{p} \boldsymbol{\Psi_i} \Delta \boldsymbol{y_{t-i}} + \sum_{j=1}^{q} \boldsymbol{\Phi_j} \Delta \boldsymbol{x_{t-j}} + \epsilon_{\mathbf{t}} \tag{1}$$

with $\boldsymbol{y_t}$ and $\boldsymbol{x_t}$ being $n \times 1$ and $m \times 1$ vectors of endogenous and exogenous variables,

[1] The reader is referred to Hamilton (1994) and Lütkepohl (2005) for a more detailed overview of VAR and VEC models.

respectively. The model reduces to a simple AR for $\boldsymbol{y_t} = ip_t$ and all elements of $\boldsymbol{\Phi_j}$ being equal to zero. Moreover, $\boldsymbol{\mu}$ is a vector of intercept terms and $\boldsymbol{\Psi_i}$ and $\boldsymbol{\Phi_j}$ are the $n \times n$ and $n \times m$ coefficient matrices for the i^{th} and j^{th} lagged values of endogenous and exogenous variables, respectively. Finally, $\epsilon_t \sim N(0, \Sigma_\epsilon)$ is the n-dimensional innovation vector with zero mean and covariance matrix Σ_ϵ.

As a second model class we include vector error correction models, based on Engle and Granger (1987), in our model pool in order to model the long-run relationship between industrial production and new orders received. Accordingly, the VECM takes following form:

$$\Delta \boldsymbol{y_t} = \boldsymbol{\mu} + \boldsymbol{\alpha} z_{t-1} + \sum_{i=1}^{p} \boldsymbol{\Psi_i} \Delta \boldsymbol{y_{t-i}} + \sum_{j=1}^{q} \boldsymbol{\Phi_j} \Delta \boldsymbol{x_{t-j}} + \epsilon_{\mathbf{t}} \tag{2}$$

where $\boldsymbol{\alpha}$ is the vector consisting of adjustment coefficients toward the long-run equilibrium which is denoted by the error correction term z_{t-1}. Note that we model the cointegration relationship in Eq. (2) as $z_t = ip_t - \beta or_t$ without any deterministic term and further endogenous variables in order to capture the long-run equilibrium between industrial production and new orders received only. However, we estimate the β-coefficient in the cointegrating equation to account for cancellations in new orders received as similarly done in related studies.

3.2 Multicointegration models

One core element of the industrial production model proposed here that distinguishes it from many other forecasting models is the idea of multicointegration. In comparison with conventional VECMs, multicointegration is a more elaborate concept: it allows for a second long-run relationship between stock and flow variables in the model. The development of the multicointegration framework has been twofold: (i) Granger and Lee (1989) and Lee (1992, 1996) extend Engle and Granger (1987)'s two-step least square estimation method into multicointegration framework, (ii) Engsted and Haldrup (1999) and Paruolo (2000) build on the Johansen (1995) methodology. While we estimate the multicointegration models based on Granger and Lee (1989) framework with the frequentist approach, we extend the long-run priors of Giannone et al. (2016) to multicointegration models based on Engsted and Haldrup (1999) for Bayesian estimations.

3.2.1 Multicointegration: Frequentist approach

The underlying economic theory behind the idea of multicointegration suggests that firms may want to maintain two equilibria at the same time: one between flow variables (e.g., production and orders) and another between stock and flow variables (e.g., inventory/backlog of orders and production/orders). While conventional cointegration, denoted as Eq. (3), captures the former, the latter equilibrium is captured by the multicointegration relation. That is why, from a theoretical point of view, the multicointegration model should draw a more complete picture of the entire production process. The hope is that this may translate into a better forecasting model.

According to Engle and Granger (1987) two $I(1)$ variables, production and orders, are cointegrated if their linear combination

$$z_t = ip_t - \beta or_t \tag{3}$$

is $I(0)$. Suppose that ip_t and or_t are $I(1)$ and z_t is $I(0)$, then it follows that the cumulative sum of z_t written as

$$S_t = \sum_{i=0}^{t} z_{t-i} \tag{4}$$

will also be an $I(1)$ variable. Moreover, if S_t and ip_t are cointegrated ip_t and or_t are called multicointegrated as introduced by Granger and Lee (1989). Hence, second cointegration relationship in the model, also referred to as multicointegration relationship, takes the following form

$$u_t = S_t - \omega_0 - \delta ip_t \tag{5}$$

with ω_0 being the intercept and when it follows that $u_t \sim I(0)$. Note that, as opposed to Eq. (3), we include a constant in Eq. (5) in order to capture firms' inventory behaviour. Against this backdrop, the multicointegration model can be specified as

$$\Delta \boldsymbol{y_t} = \boldsymbol{\mu} + \boldsymbol{\alpha} z_{t-1} + \boldsymbol{\gamma} u_{t-1} + \sum_{i=1}^{p} \boldsymbol{\psi_i} \Delta \boldsymbol{y_{t-i}} + \sum_{j=1}^{q} \boldsymbol{\Phi_j} \Delta \boldsymbol{x_{t-j}} + \epsilon_{\mathbf{t}} \tag{6}$$

where $\boldsymbol{y_t}$ and $\boldsymbol{x_t}$ are the vectors of endogenous and exogenous variables and z_t and u_t are specified as in Eq. (3) and (5), respectively. The multicointegration model can easily be estimated stepwise in a manner similar to Engle and Granger (1987) two-step least square method. First, the cointegration and multicointegration relationships are determined. Then, the Eq. (6) can be estimated with a classical maximum likelihood or ordinary least square method.

Economically, z_t represents excess production or excess orders per month depending on its sign. A positive z_t would indicate that in a given period production is more than required by the orders received. In contrast, a negative sign of z_t points at orders that are not processed in the same month. Accordingly, S_t, the cumulative sum of z_t, can be considered, if positive, as the stock of inventory, and, if negative, as the backlog of orders which has to be processed in the future. This suggests that production may also depend on the state of new orders received providing economic support for the inclusion of a second long-run equilibrium in the model and hence for the multicointegration framework.

However, the implementation of multicointegration models for forecasting industrial production in Germany is not as straightforward as economic theory might suggest. First, the time series for industrial production and new orders received need to be provided in levels and in nominal terms. However, industrial production and news orders received are both indexes, set to 100 in a specific base year (currently 2010) regardless of their initial nominal levels. Therefore, above (below) equilibrium values of z_t should be interpreted as changes in the stock of inventory (backlog of orders) rather than positive (negative) values, because the long-run equilibrium between industrial production and new orders received must not necessarily hover around zero due to data construction and firms' inventory behaviour.

Moreover, the time series for industrial production and new orders received are also constructed in slightly different ways with respect to their definition and weighting schemes. While the index for industrial output focuses merely on the industrial production, the time series for new orders received takes also order-related services into account. In addition to that these indices do not necessarily cover same industrial sectors with same weights. In fact, industrial sectors are aggregated according to their shares of value added in base-year to compose the industrial production index, whereas new orders are weighted on the basis of the sector-specific order volumes to form the overall new orders received index. Last but not least, weights of different sectors in both indices change with the base-year over time as these are calculated with the nominal data in the base-year. Consequently, the differences in data construction, sectoral coverage and weighting may lead to different trend profiles in both indices.[2]

Finally, a preliminary analysis shows that, since the early 1990s until the mid-2000s, industrial production in Germany was much higher than the level that new orders received would imply. However, industrial output did not grow at the same pace as new orders received during the same period. By contrast, the two time series started to move much closer toward each other since the mid-2000s which is also in line with the economic theory. As a result, the discrepancy between industrial production and new orders received disappeared gradually during the first half of our sample period. Indeed, Seiler, Wohlrabe, and Wojciechowski (2014) show that both, intermediate products as well as finished goods inventory, have decreased since the early 1980s, whereas the range of the stock of orders in the German manufacturing sector has increased during the 2000s. On top of that, the authors explain the negative correlation between inventory behaviour (of both intermediate products as well as finished goods) and the range of the stock of orders with a transition to "*just-in-time*" production the manufacturing sector exhibited around the turn of the millennium. Against this backdrop, on-demand production is supposed to make firms less dependent on the inventory and may thus indicate that the relationship between production and inventory has been changed during this transition period.

Econometrically, this may be consistent with structural breaks in the long-run equilibria, i.e., in the cointegration relationships of industrial production with new orders received as well as with inventory/backlog of orders. Note that proper modelling of the cointegration relationship between production and new orders received is of paramount interest due to its direct impact on the multicointegation relationship. A straightforward way to account for such changes in the long-run equilibria would be to allow for structural breaks in related cointegration relationships. While this approach may be easily implemented in an ex-post perspective, the real-time detection of such breaks in the long-run equilibria of the model may not be so simple. Therefore, we extend the initial multicointegration framework considering the specific requirements of forecasting industrial production. First, we propose to approximate the multicointegration relationship with survey-based ifo indicators focusing on the assessment of the finished goods inventory and of the orders in hand as well as on production plans instead of building it as in equations (4) and (5). Second, we estimate the multicointegration models with Bayesian estimation methods in order to implement our prior beliefs in long-run equilibria of the model. Therefore, we extend the long-run priors of Giannone et al. (2016) to the multicointegrated case. The

[2]See the Deutsche Bundesbank (2007) Monthly Report February p. 52-53 for more details on this matter.

two extensions are briefly introduced in the next subsections.

3.2.2 Multicointegration: Approximation with ifo indicators

Instead of specifying the multicointegration relationship as in equations (4) and (5), we propose to approximate it with survey-based ifo indicators focusing on the assessment of the finished goods inventory and of the orders in hand as well as on production plans. Specifically, we replace the accumulated error term of the first cointegration equation, formerly defined as S_t, in Eq. (5) with ifo indicators on the assessment of the finished goods inventory (ifo_{lb}) and of the orders in hand (ifo_{ab}) as well as on production plans (ifo_{pp}), while we keep the remainder of the cointegration equation unchanged as built in Eq. (3). Hence, the multicointegration equation takes the following form:

$$u_{i,t} = ifo_{i,t} - \omega_0 - \delta ip_t \tag{7}$$

for $i = lb, ab, pp$. The first specification $u_{lb,t} = ifo_{lb,t} - \omega_0 - \delta ip_t$ aims at capturing the long-run relation between industrial production and inventories, while the second one $u_{ab,t} = ifo_{ab,t} - \omega_0 - \delta ip_t$ models the interaction between ip_t and the stock of orders in hand. These specifications may be able to model the second equilibrium which firms may want to maintain in the long-run in addition to the first one between production and new orders received. Moreover, we also consider the linkage between actual production and production plans in the third specification $u_{pp,t} = ifo_{pp,t} - \omega_0 - \delta ip_t$ in order to incorporate forward-looking production adjustments.

3.2.3 A Bayesian perspective on the long run

While in the two previous subsection we were explicitly estimating the long-run relationships, we now fix the multi-cointegrating vectors a-priori based on our theoretical considerations. Specifically, in the "cumulated" version of the orders/production VAR,

$$\Delta x_t = c_t + \Pi \begin{bmatrix} X_{t-1} \\ x_{t-1} \end{bmatrix} + \sum_{i=1}^{p-2} \Psi_i \Delta x_{t-i} + u_t, \tag{8}$$

with $X_t = \left[\sum_{j=0}^{t} or_j \; \sum_{j=0}^{t} ip_j\right]'$ and $x_t = [or_t \; ip_t]'$ we elicit a prior for Π guided by economic theory.

Following Giannone et al. (2016), and extending their ides to the multicointegation casre, we introduce an additional matrix H whose rows contain what theory tells us about the long-run relationship between production and orders. The VAR above can then be rewritten as

$$\Delta x_t = c_t + \Lambda \bar{H}' H \begin{bmatrix} X_{t-1} \\ x_{t-1} \end{bmatrix} + \sum_{i=1}^{p-2} \Psi_i \Delta x_{t-i} + u_t, \tag{9}$$

in which $\bar{H} = (HH')^{-1}H$. Eliciting a prior for Π in (8) has now transformed into choosing a prior for the "adjustment" coefficient Λ, conditional on H. The idea behind the approach of Giannone et al. (2016) is that rows in H representing some sort of error correction mechanisms are a-priori more likely to have adjustment coefficients differenct from one (i.e. less shrinkage). Likewise, rows in H containing non-stationary linear combinations of the variables call for more shrinkage toward zero on the elements of the corresponding columns of Λ.

In the orders/production example our choice of the long-run matrix consistent with multi-cointegration is

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & -\delta & -\delta \end{bmatrix}, \tag{10}$$

in which the first row contains the $I(2)$ trend and the second one the multi-cointegration relationship. For the multi-cointegrating parameter δ, theory would tell us to set it such that the linear combination in the second row will be close to zero. How we pick a specific value for δ will become clear momentarily.

Giannone et al. (2016) suggest to operationalize the idea of a prior for the long run in the following way:

$$\text{vec}(\Lambda)|H,\Sigma \sim N\left(0, \text{diag}\left(\left[\frac{\lambda_i^2}{\left(H_{1.}\left[\bar{X}_0' \quad \bar{x}_0'\right]'\right)^2}, \cdots \frac{\lambda_i^2}{\left(H_{n.}\left[\bar{X}_0' \quad \bar{x}_0'\right]'\right)^2}\right]\right) \otimes \Sigma\right), \tag{11}$$

in which $H_{n.}$ denotes the n-th row of H. The column vectors $\bar{X}_0$ and $\bar{x}_0$ contain the average of the initial p observations. In our specific example with $n = 2$ and H as in (10) we then choose δ such that $H_{2.}\left[\bar{X}_0' \quad \bar{x}_0'\right]' = \begin{bmatrix}1 & -1\end{bmatrix}\bar{x}_0$. Admittedly, this formulation is fairly ad-hoc, but all we want to ensure a-priori is that there is less shrinkage on the loadings in Λ associated with the stationary multi-cointegrating relation than with the non-stationary $I(2)$ trend.

Since the prior is conjugate it can be implemented with dummy observations in a cumulated-level VAR and it is easy to combine with other priors, such as the Minnesota prior.[3] Because of conjugacy the marginal likelihood can be compute in closed form which, in turn, implies that the hyperparameters λ can be estimated as in Giannone, Lenza, and Primiceri (2015).

In the forecast exercise below we also compare our multi-cointegrating prior with the original long-run prior of Giannone et al. (2016) using only the "simple" cointegrating relationship between orders and production to elicit the prior (the VAR is then specified in levels) and the standard sum-of-coefficients prior (VAR specieifed in first differences).

3.3 Asymmetric adjustment

Macroeconomic time series may exhibit asymmetric characteristics over the business cycles. In an earlier paper Neftçi (1984) points out that the US unemployment shows asymmetric behaviour over the course of the business cycles. Moreover, Teräsvirta and Anderson (1992) find evidence for nonlinearities in industrial production in various developed countries. Against this background, Sichel (1993) characterises nonlinearities of business cycles with deep (troughs are further below the long-term average than peaks are above it) and sharp (downturns are steeper than expansions) cycles and finds evidence for deepness for industrial production, while Ramsey and Rothman (1996) show

[3]Given we specify the VAR in cumulated levels, the $I(2)$ nature of the cumlated variables suggests a prior of 2 on the $AR(1)$ coefficients and -1 on the $AR(2)$ coefficients.

that industrial production exhibits sharpness-type of asymmetries.

Against this background, linear cointegration models implicitly assuming a symmetric adjustment toward the long-run equilibrium may fail to capture nonlinear dynamics of the macroeconomic time series under consideration. For this reason, we also consider error correction models allowing for an asymmetric adjustment toward the long-run equilibrium. Specifically, we adopt the threshold cointegration framework of Balke and Fomby (1997) with asymmetric error correction such as in Granger and Lee (1989). Accordingly, our threshold vector error correction and multicointegration models with asymmetric error correction are constructed as

$$\Delta \boldsymbol{y_t} = \boldsymbol{I_{z,t}} \boldsymbol{\alpha}^+ z_{t-1} + (\boldsymbol{1} - \boldsymbol{I_{z,t}})\boldsymbol{\alpha}^- z_{t-1} + \boldsymbol{\mu} + \sum_{i=1}^{p} \boldsymbol{\Psi_i} \Delta \boldsymbol{y_{t-i}} + \sum_{j=1}^{q} \boldsymbol{\Phi_j} \Delta \boldsymbol{x_{t-j}} + \epsilon_{\mathbf{t}} \tag{12}$$

and

$$\begin{aligned}\Delta \boldsymbol{y_t} = \boldsymbol{I_{z,t}} \boldsymbol{\alpha}^+ z_{t-1} + (\boldsymbol{1} - \boldsymbol{I_{z,t}})\boldsymbol{\alpha}^- z_{t-1} + \boldsymbol{I_{u,t}} \boldsymbol{\gamma}^+ u_{t-1} + (\boldsymbol{1} - \boldsymbol{I_{u,t}})\boldsymbol{\gamma}^- u_{t-1} + \\ \boldsymbol{\mu} + \sum_{i=1}^{p} \boldsymbol{\Psi_i} \Delta \boldsymbol{y_{t-i}} + \sum_{j=1}^{q} \boldsymbol{\Phi_j} \Delta \boldsymbol{x_{t-j}} + \epsilon_{\mathbf{t}}\end{aligned} \tag{13}$$

respectively with the indicator functions being

$$\boldsymbol{I_{z,t}} = \begin{cases} 1 & \text{if } z_{t-1} \geq \tau_z \\ 0 & \text{if } z_{t-1} < \tau_z \end{cases} \qquad \boldsymbol{I_{u,t}} = \begin{cases} 1 & \text{if } u_{t-1} \geq \tau_u \\ 0 & \text{if } u_{t-1} < \tau_u \end{cases} \tag{14}$$

where $\boldsymbol{\alpha}^+, \boldsymbol{\gamma}^+$ and $\boldsymbol{\alpha}^-, \boldsymbol{\gamma}^-$ are vectors of adjustment coefficients for above- and below-average error correction terms, respectively. Moreover, τ_z and τ_u are the threshold values for the first and second cointegration relations. As far as estimation is converned, we follow Tsay (1998)'s stepwise modelling procedure for threshold models. First, we set the deviations from the long-run equilibria, i.e., z_t and u_t, as threshold series. Then, we estimate both threshold values, τ_z and τ_u, with Chan (1993)'s grid search method minimising the sum of squared residuals in both regimes; see also Enders and Granger (1998) and Enders and Siklos (2001). Finally, threshold-VEC and -multicointegration models are estimated with classical methods such as maximum likelihood and ordinary least square estimators.

3.4 Forecast combination

As mentioned before, our parsimonious forecast combination approach is based on a relatively small pool of models which consists of three model classes: (vector) AR and error correction as well as multicointegration models. We augment selected models with survey-based ifo indicators targeted at the industrial sector and also consider linear and threshold-type nonlinear specifications of related models. We estimate the models with Bayesian and frequentist methods. At a final step we combine the point and density forecasts using equal and/or performance based weighting schemes. Table 1 presents selected model specifications in the model pool.

Table 1: Forecast models

#	Model	Variables	Lags	Adjustment		
				Vacation	Bridge	Weather
1	AR	ip	1	off	off	off
2	AR	Δip	1	off	off	off
3	VAR	Δip, Δor	1	off	off	off
4	VAR	Δip, Δor	4	off	off	off
5	MCOINT	Δip, Δor	4	on	on	on
6	VECM	Δip, Δor, Δifo_c, Δifo_e	4	on	on	on
7	TVECM	Δip, Δor	4	off	off	off
8	TVECM	Δip, Δor	4	on	on	on
9	V2ECM	Δip, Δor, Δifo_c, Δifo_{pp}	4	on	on	on
10	V2ECM	Δip, Δor, Δifo_e, Δifo_{ab}	4	on	on	on
11	TV2ECM	Δip, Δor, Δifo_c, Δifo_{pp}	4	on	on	on
12	TV2ECM	Δip, Δor, Δifo_e, Δifo_{ab}	4	on	on	on
13	BVAR (pfr)	ip, or, ifo_c, ifo_e	12	on	on	on
14	BVAR (psf)	$\sum ip$, $\sum or$, $\sum ifo_c$, $\sum ifo_e$	12	on	on	on

Table 1 provides an overview of the selected model specifications in our model pool. While (V)AR and VECM stand for (vector) autoregressive model and vector error correction model, respectively, MCOINT denotes the baseline multicointegration model. Moreover, V2ECM represents the approximation of MCOINT with various ifo indicators, while T denotes the threshold-type nonlinear specifications of the respective models. Finally, multivariate models have either two or four variables. The variable selection is based on the theoretical relationship between industrial production and new orders received as well as on the forward-looking properties of the survey-based ifo indicators focusing on the industrial sector in Germany. Note that we do not include any exogenous variables and hence the term $\sum_{j=1}^{q} \boldsymbol{\Phi_j \Delta x_{t-j}}$ drops from our multivariate model specifications. Moreover, except in the (V)AR models and one TVECM specification, the industrial production series is adjusted for bridge-, ice- and vacation days. The latter variables are calculated as monthly deviations from their long-run averages in respective months. Focusing on such a parsimonious model pool, coupled with a small number of variables in multivariate specifications, hopefully improves the transparency of our projections.

Considering survey-based ifo indicators introduces the well-known ragged-edge feature to our dataset. While these indicators provide us with timely information on the industrial sector, industrial production and new orders received both have a publication delay of about five weeks. Consequently, ifo indicators have a one- or two-observation lead, depending on the time of the forecast, vis-à-vis industrial production and new orders received. In order to make use of this information we follow the conditional forecasting approach developed by Waggoner and Zha (1999) assuming that these indicators only have an indirect simultaneous impact on the industrial production.

Last but not least, we consider two weighting schemes in order to pool individual forecasts: (i) equal weighting and (ii) performance based weighting scheme. While the former is a simple mean of point forecasts, the latter assigns more weight to models which performed relatively better for a given period of time in the recent past. Specifically, the weight for model i in period t is inversely proportional to its RMSFEs for a given horizon h computed over a specific period in the past, i.e. three months. Note that one needs to normalise the weights in order for them to sum up to one. Accordingly, it can be written as

$$\hat{\omega}_{i,t}^{(h)} = \frac{(1/RMSFE_{i,t}^{(h)})}{\sum_{j=1}^{N}(1/RMSFE_{j,t}^{(h)})} \tag{15}$$

where N stands for the number of models.[4] For now, however, we mainly emphasise results based on the equal weighting scheme, as performance-based weights were found to perform rather slightly poorly for industrial production in preliminary analyses.[5]

4 Empirical results

For the evaluation of our parsimonious forecast combination approach we use relative RMSFEs as the main measure for comparison. We report RMSFEs of selected models and our forecast exercise relative to the random-walk benchmark. We consider two periods: (i) July 2006 - December 2010, covering the pre-crisis period and the Great Recession, and (ii) January 2011 - June 2017, representing the post-crisis period.

Table 2 presents relative RMSFEs of various model specifications for horizons 1, 3, 6 and 12 months for both sub-samples with an information lead of the sentiment indicators of 1 month.[6] We refer to point forecasts of our approach as F_{comb} with *mean*, *median* and *msfe* indicating the simple average, median forecast and performance based weighting scheme as in the Eq. (15), respectively. The Bayesian VARs are denoted by BVAR with the letters d, l and c referring to specifications estimated in first differences, levels and cumulative levels. While MCOINT stands for the multicointegration model, V2ECM represents the approximation of multicointegration models with ifo indicators as introduced in Subsection 3.2.2. Moreover, the letter T stands for threshold-type nonlinear specification of related models. While numbers in parenthesis show the lag structure of each model, on/off shows whether the data is adjusted for bridge-, ice- and vacation days. Furthermore, ifo means both ifo_c and ifo_e are included in the model, whereas in case of models with ifo_i for $i = lb, pp, ab$ ifo_c or ifo_e is replaced with related ifo indicators. Considering the relative RMSFEs values less (greater) than one imply that the selected forecast model makes, on average, less (more) error than the benchmark. Moreover, p-values for the Diebold and Mariano (1995) tests are given in brackets. Finally, bottom line of Table 2 provides absolute values of the RMSFEs for the random walk benchmark as scale reference.

The top line in Table 2 presents that our parsimonious forecast combination approach with an equal-weighting scheme is able to reduce the RMSFEs by up to 30% over a 12-months forecast horizon in the post crisis period. However, its forecast accuracy deteriorates at the long-end of the forecast horizon when the first sub-period is considered. While F_{comb} (median) and F_{comb} (msfe) show similar patterns as the simple average in first sub-sample, the performance of the three combination schemes diverge from each other in

[4]See Stock and Watson (2006) and Timmermann (2006) for more details on forecast combination.

[5]See Claeskens, Magnus, Vasnev, and Wang (2016), Genre, Kenny, Meyler, and Timmermann (2013) and Smith and Wallis (2009) for possible explanations on this forecast combination puzzle.

[6]Results of the forecast comparison based on a 2-months information lead of the ifo indicators remain mainly unchanged. Hence, they will not be presented here for brevity. All results are available from the authors on request.

Table 2: Forecast comparison

Relative RMSFEs	(Pre) Crisis[1], $h = \ldots$				Post Crisis[1], $h = \ldots$			
	1	3	6	12	1	3	6	12
A: Information lead of the ifo indicators – 1 month								
RMSFEs relative to RW								
F_{comb} (mean)	0.78	0.75	0.89	0.97	0.75	0.80	0.80	0.70
	(0.03)	(0.09)	(0.24)	(0.70)	(0.00)	(0.01)	(0.00)	(0.00)
F_{comb} (median)	0.77	0.73	0.90	0.97	0.75	0.83	0.82	0.74
	(0.06)	(0.12)	(0.29)	(0.62)	(0.00)	(0.02)	(0.01)	(0.01)
F_{comb} (msfe)	0.76	0.74	0.91	0.99	0.75	0.78	0.85	0.80
	(0.04)	(0.12)	(0.33)	(0.82)	(0.00)	(0.00)	(0.04)	(0.03)
BVAR(12) / d / soc / on / ifo	0.72	0.69	0.77	0.80	0.96	0.97	1.09	1.22
	(0.06)	(0.15)	(0.18)	(0.03)	(0.53)	(0.78)	(0.55)	(0.11)
BVAR(12) / l / plr / on / ifo	0.83	0.77	0.87	0.93	0.77	0.87	0.90	0.99
	(0.08)	(0.15)	(0.31)	(0.39)	(0.01)	(0.17)	(0.27)	(0.99)
BVAR(12) / c / psf / on / ifo	0.75	0.71	0.91	1.05	0.84	0.87	0.92	1.14
	(0.09)	(0.16)	(0.56)	(0.77)	(0.07)	(0.23)	(0.41)	(0.28)
TV2ECM(4) / on / ifoLB	0.75	0.71	0.92	1.03	0.85	0.92	0.93	0.98
	(0.07)	(0.15)	(0.57)	(0.84)	(0.09)	(0.37)	(0.42)	(0.85)
TV2ECM(4) / on / ifoPP	0.76	0.66	0.82	0.99	0.82	0.89	0.89	1.07
	(0.09)	(0.14)	(0.25)	(0.98)	(0.05)	(0.30)	(0.23)	(0.48)
TV2ECM(4) / on / ifoAB	0.75	0.72	0.88	0.97	0.81	0.88	0.97	0.93
	(0.07)	(0.13)	(0.27)	(0.58)	(0.04)	(0.21)	(0.78)	(0.56)
V2ECM(4) / on / ifoLB	0.76	0.70	0.89	1.03	0.84	0.92	0.94	0.95
	(0.07)	(0.14)	(0.48)	(0.86)	(0.07)	(0.31)	(0.44)	(0.56)
V2ECM(4) / on / ifoPP	0.76	0.65	0.79	0.92	0.81	0.88	0.88	0.86
	(0.08)	(0.11)	(0.15)	(0.35)	(0.04)	(0.29)	(0.17)	(0.10)
V2ECM(4) / on / ifoAB	0.74	0.68	0.87	1.01	0.80	0.88	0.98	0.92
	(0.07)	(0.13)	(0.31)	(0.93)	(0.04)	(0.21)	(0.81)	(0.40)
MCOINT(4) / on / ifo	0.74	0.68	0.83	0.92	0.80	0.93	1.05	1.49
	(0.08)	(0.15)	(0.27)	(0.13)	(0.03)	(0.42)	(0.58)	(0.00)
VECM(4) / on / ifo	0.78	0.73	0.87	0.92	0.82	0.93	0.98	0.93
	(0.14)	(0.15)	(0.29)	(0.25)	(0.04)	(0.40)	(0.85)	(0.38)
VAR(4) / on / ifo	0.70	0.72	0.87	0.87	0.96	0.98	1.09	1.15
	(0.04)	(0.16)	(0.27)	(0.13)	(0.55)	(0.88)	(0.54)	(0.10)
RW (abs.)	0.0248	0.0517	0.0841	0.1322	0.0175	0.0193	0.0248	0.0251

[1] (Pre) Crisis = 2006 July to 2010 December; Post Crisis = 2011 January to 2017 June.

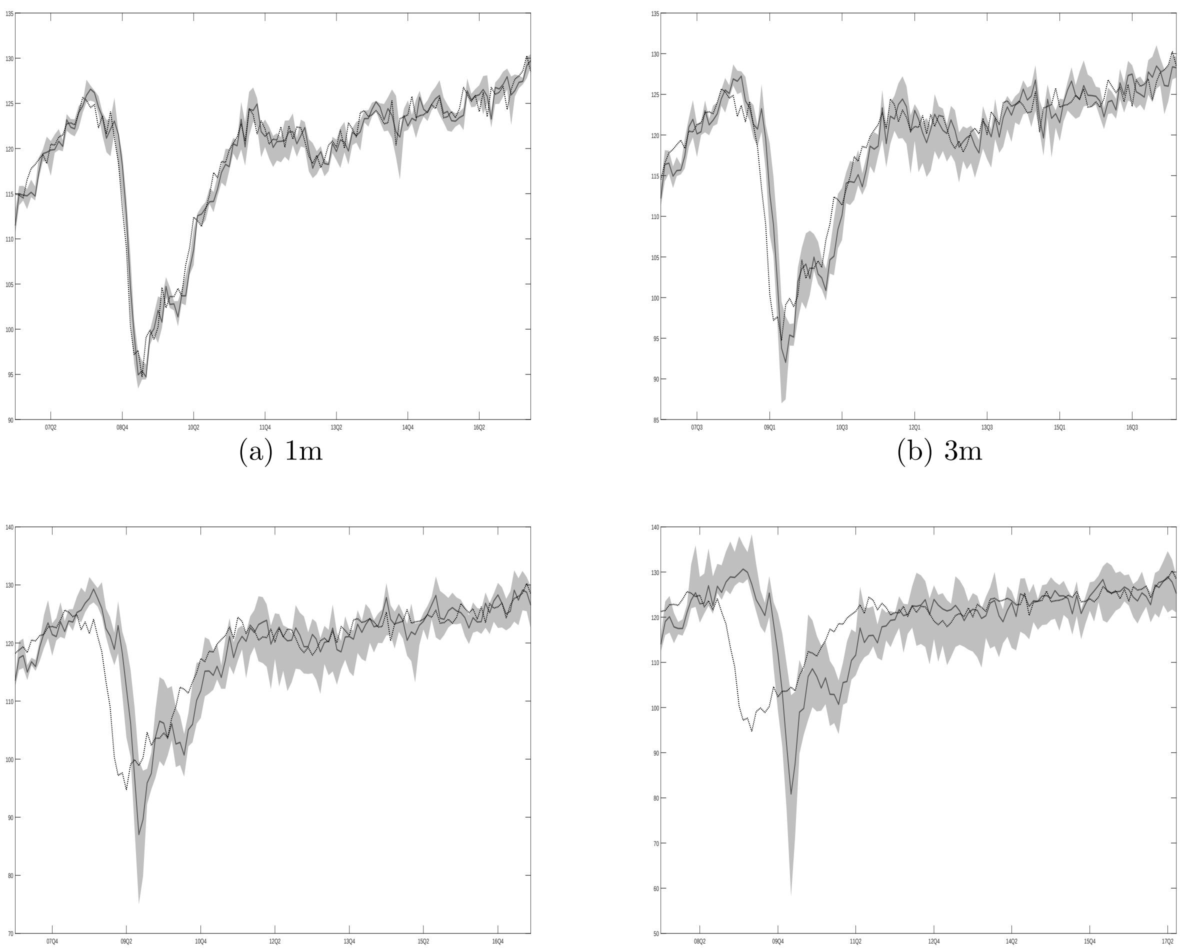
the aftermath of the Great Recession. While the performance-based combination scheme (msfe) is slightly better at the short-end, both F_{comb} (median) and (msfe) perform rather poorly at forecast horizons longer than three months.

The middle panel shows the forecast performance of models proposed in this paper relative to the random-walk. Moreover, we also include forecast performance of the conventional multicointegration framework, denoted as MCOINT(4), as this model serves as a reference for the Bayesian VARs and (T)V2ECMs. Overall, all proposed models with some exceptions perform fairly well at the long-end of the forecast horizon compared to the benchmark as well as to the conventional multicointegration framework. The improvement in forecast accuracy is even more pronounced for forecast horizons up to 3 months.

Considering the BVARs the results show that taking the long-run relation between industrial production and new orders received into account reduces the forecast error in the aftermath of the recent financial crisis, the BVAR in first differences is superior to the other two Bayesian specifications in the first sub-sample. Moreover, the BVAR in cumulative levels which extends the prior for the long-run of Giannone et al. (2016) to the multicointegration framework performs slightly better than the BVAR in levels at the short-end of the forecast horizon before and during the crisis, its forecast accuracy deteriorates for longer horizons in both sub-samples.

The linear approximations of the multicointegration model with various sentiment indicators, denoted as V2ECMs, seem to reduce the forecast error compared to the benchmark as well as MCOINT remarkably, whereas the nonlinear specifications of these models perform rather slightly poorer as often found in the related literature. The results indicate that the improvement in forecast accuracy by including the second long-run relation between production and inventory, stock of orders on hand or production plans into the model is more pronounced at the short-end of the forecast horizon in the first sub-sample, whereas the same holds for the long-end in the aftermath of the most recent global financial crisis.

Figure 3: Industrial production: realised vs. forecast



(a) 1m (b) 3m

(c) 6m (d) 12m

Notes: The graphs plot realised industrial production (dotted black line) and the corresponding h-month-ahead mean forecasts (solid grey line) over the period from July 2006 to June 2017. The shaded area covers the min-max band of individual model forecasts.

While Table 2 provides the reader with a valuable insight on the relative forecast performance of the new approach, it is not possible to trace if forecasts are biased, and if they are, in which direction. In addition, nothing can be derived about the uncertainty associated with the new forecast combination approach. Against this background Figure 3 plots the time series for the realised industrial production together with the 1-, 3-, 6- and 12-month-ahead point forecasts of the RFM as well as the min-max interval of the individual model forecasts. This should provide us with an idea about the direction of the forecast errors and the uncertainty around the point forecasts.

Overall, Figure 3 shows that forecast errors - with the exception of the period covering the Great Recession - exhibit no clear pattern preventing our framework from being systemically biased toward one direction.

5 Concluding remarks

In this paper we introduce multicointegration models to forecasting industrial production in Germany. Moreover, considering country-specific characteristics we proposed two extension of this model class: (i) Approximation of multicointegration equation with ifo survey-based indicators, (ii) Extension of Giannone et al. (2016) to multicointegration models. Furthermore, we include both linear and nonlinear specifications of selected models and estimate them with frequentist and Bayesian methods. Overall, real-time forecast evaluation results revealed that our parsimonious forecast combination with around ten models is able to reduce the forecast error by up to 30% compared to the random-walk benchmark. Especially, for long forecast horizons, the forecast performance seems to improve markedly when relying on our model.

While we focused on predicting the German industrial production, our framework can be easily extended and implemented to other countries for which a similar dataset is available.

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