Non-performing Loans, Fiscal Costs and Credit Expansion in China*

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December 30, 2017

Abstract

This paper studies how credit expansion policy in the post-crisis episode distorts bank-firm loan contracts and examines its economic impacts. We build a structural model with financial friction similar as Bernanke et al. (1999), in which the optimal loan contract reflects the tradeoff between higher leverage and higher default probability facing by firms. In the context of China's economic stimulus plan responding to the recent financial crisis, credit expansion is introduced in the form of government's partial guarantee on bank loans when firms default. A more generous bailout policy encourages banks to lend. On one hand, higher leverage helps to alleviate firms' working capital constraint, raising their production; on the other hand, a higher break-even threshold raises the non-performing loan ratio, reducing firms' net worth. We show that when the credit expansion is highly persistent, the first channel dominates, leading to positive credit multipliers at all horizons. When credit expansion is short lived, however, the second channel dominates, driving negative credit multipliers at all horizons.

*The views expressed in this paper are those of the authors and do not represent those of the Bank of Canada, the Federal Reserve Bank of Kansas City or the Federal Reserve System.

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1 Introduction

Financial intermediaries play an important role in shaping the real economy through channeling credit from savers to borrowers. Government policy can affect, either facilitate or hinder, this credit allocation channel. Chinese government, for instance, actively influenced the credit market to stimulate its economy in the wake of the recent global financial crisis, while advanced economies used conventional instruments of government spending and taxes. The investment-based stimulus package adopted by Chinese government was supported by the central bank's credit expansion policy, featuring tools like "window guidance", pledge supplementary lending etc.[1] Figure 1 shows that the Chinese banks' lending to the so-called 'risky' sectors of manufacturing, wholesale and retail, mining, and construction rose steadily from 39% in 2009 to 47% in 2012. In response, the non-performing loan ratios have been on the rise, increasing steadily since 2011.[2] It highlights that credit expansion that may boost economic growth in the short run can sow seeds for financial challenges in the medium and long run, as non-performing loans increase.

With these observations in mind, we construct a structural model with financial frictions to study the mechanism through which credit expansion can distort bank-firms loan contract, and to what extent such distortion can undermine the effectiveness of credit expansion. In our model, before production takes place, firms have to pay working capital through their own net worth and external loans. Some firms may have to default on their loans if their idiosyncratic productivity is lower than the break-even threshold. Even though firms would like to borrow enough to alleviate the working capital constraint, banks are willing to lend more only at a higher lending rate, which would raise the break-even threshold and therefore default probability. The optimal contract, therefore, reflects the tradeoffs between higher leverage and higher default probability facing by firms.

We then consider credit expansion in the form of partial government guarantee on bank loans, similar as what Chinese government adopted following the global financial crisis. In case of firm default, banks can partially recover their loss through government bailout funds. A more generous bailout policy encourages banks to lend, alleviating the tradeoffs facing by

[1] Window guidance refers to the dialogue between the central bank and commercial banks on the general orientation of monetary policy and pace of lending activities. Pledge supplementary lending is a lending facility under which the central bank directly provides loans with favorable interest rates to policy banks for their re-lending.

[2] Chinese commercial banks classify loans into five categories: pass, special-mention, substandard, doubtful, and loss, with only the last three being considered NPLs according to China's official reports. When deciding whether a loan is a NPL, Chinese banks are allowed to consider whether they expect to suffer a loss if the borrower defaults. A loan that is more than 90 days past due in the U.S. is classified as a NPL, even if its collateral value is estimated to be greater than the loan; but in China this loan is categorized as a special-mention loan but not a NPL. However, in this paper we follow the worldwide standard approach and include the category of special-mentioned loans.

Figure 1: NPL Ratios and Loan Share to Risky Sectors

Source: Bank annual reports for BoC and ICBC; and Bloomberg

firms. As a result, firms are willing to accept more loans at a higher break-even threshold: the former helps to alleviate the working capital constraint, raising firms' production; while the latter raises the non-performing loan ratio and reduces firms' net worth, which could tighten future working capital constraint. The overall impact on the aggregate economy will depend on which channel dominates.

Our quantitative analysis shows that the overall impact on economy depends on the persistence of credit expansion. With a short-lived credit expansion, the negative channel of non-performing loan dominates. Lower, instead of higher, expected return on capital prompts households to cut investment. Even though relaxing working capital constraint raises output initially, higher bailout costs as a result of credit easing wipe out the initial gain, lowering GDP even in the short term. On the other hand, if credit expansion is persistent, the positive channel of relaxing working capital constraint dominates. Higher expected return on capital raises investment persistently, boosting GDP over the long term, despite part of the gain in output is wiped out by higher bailout costs.

The contrast between the short-lived and the persistent credit expansions can be quantified through credit multipliers. A highly persistent credit shock leads to positive accumulative multipliers across all horizons, while an i.i.d credit shock drives negative multipliers at all horizons. In addition, credit expansion is inferior to the conventional instrument of government spending in terms of stimulating the economy, unless the credit expansion is highly persistent. This intriguing result highlights that credit expansion can distort bank-firms loan contract, which can significantly undermine the effectiveness of credit expansion.

Our paper contributes to the recent debate on the effectiveness of China's stimulus package in 2008. Wen and Wu (2014) argue that the bold and decisive fiscal stimulus program was the key for China to recover from the Great Recession quickly. Ouyang and Peng (2015), on the other hand, estimate that the program had raised the annual real GDP growth in China by about 3.2 percent, but only temporarily. More recent papers, including Bai et al. (2016) and Cong et al. (2017), find that the stimulus-driven credit expansion has reduced the GDP growth in the medium and long run through resource misallocation. Our paper contributes to this debate by showing that the persistence of credit policy plays an important role in determining its impact on aggregate economy.

In addition, our paper is related to the literature on credit allocation and macroeconomics. Empirical studies suggest that banks' credit allocation can affect firms' production and employment. For instance, Chodorow-Reich (2013) finds that bank-firm relationship has a significant impact on employment, particularly for small or medium firms. Caballero et al. (2008) find that Japanese big banks engaged in bad loan restructuring and kept credit flowing to "zombie" firms, which depresses economic growth. Structural models, for instance those developed in Bernanke et al. (1999) and Kiyotaki and Moore (1997), are useful vehicles to explain those empirical findings. For instance, Chang et al. (2016) study reserve requirement policy in China by focusing on a two-sector model with both state- and private-owned firms. We differ from Chang et al. (2016) by focusing the interaction among credit policy, non-performing loans and fiscal costs.

The paper proceeds as follows. Section 2 first develops the baseline model with lump-sum taxes and balanced budget constraint. Section 3 introduces the calibration and illustrates the implications of credit shocks at various persistence. We extend the model setup to a distortionary tax setting in Section 4, where distortionary taxes are collected at each period to finance bailout expenses. Finally, Section 5 concludes.

2 Baseline Model

In this section, we lay out the baseline model with partial government guarantee on bank loans. Intermediate goods firms, which use capital and labor as input to produce, face idiosyncratic shocks on their productivity. In addition, they also face a working capital constraint: they have to pay wages and capital rents before production takes place. Those expenses are financed through firms' own net worth and external loans made by financial intermediaries. Depending on the realizations of the idiosyncratic shocks on productivity, some firms may be unable to repay their loans. Government, however, can provide partial guarantee on bank loans, which change the incentives for financial intermediaries to lend and firms to borrow. In this baseline model, government collects lump-sum taxes to pay for the bailout costs and government consumption. Later, we will extend the setup to allow distortionary income taxes. Monetary policy follows a Taylor rule.

2.1 HOUSEHOLDS The economy is populated with a continuum of identical households. Every household consumes, works, invests in physical capital, and deposits at financial intermediary. The preference is,

$$E_0 \sum_{t=0}^{\infty} \beta^t u \left(\ln C_t - \psi \frac{{h_t}^{1+\sigma}}{1+\sigma} \right), \tag{2.1}$$

where $\frac{1}{\sigma}$ is the Frisch elasticity of labor supply. The household maximizes the lifetime utility by choosing consumption, C_t, investment, I_t, labor supply, h_t , and deposits, D_t, subject to the budget constraint

$$C_t + I_t + \frac{D_t}{P_t} = W_t h_t + r_t^k K_{t-1} + \frac{R_{t-1} D_{t-1}}{P_t} + \Upsilon_t. \tag{2.2}$$

where Υ_t are lump-sum transfers from retailers' monopoly profits. Financial intermediary pays the risk-free interest rate R_t on deposits. W_t and r_t^k are the rental rates of household's labor and capital respectively. Equation (2.3) characterizes the evolution of capital with depreciation rate of δ.

$$K_t = (1-\delta) K_{t-1} + \left(1 - \frac{\Omega_k}{2} \left(\frac{I_t}{I_{t-1}} - g_I \right)^2 \right) I_t. \tag{2.3}$$

Investments incur an adjustment cost, $\frac{\Omega_k}{2} \left(\frac{I_t}{I_{t-1}} - g_I \right)^2 I_t$, as the investment growth rate deviates from its steady state rate, g_I. Ω_k characterizes the size of the cost.

Defining the Lagrangian multipliers associated with equation (2.2) and equation (2.3) as

Λ_t and $q_t^k \Lambda_t$ respectively, we have the following first-order conditions

$$
\begin{aligned}
W_t &= \psi h_t{}^{\sigma} C_t && (2.4)\\
\frac{1}{R_t} &= \beta E_t \frac{C_t}{C_{t+1}} \frac{1}{\pi_{t+1}} && (2.5)\\
q_t^k &= \beta E_t \frac{C_t}{C_{t+1}} \left(q_{t+1}^k (1-\delta) + r_{t+1}^k \right) && (2.6)\\
1 &= q_t^k \left(1 - \frac{\Omega_k}{2} \left(\frac{I_t}{I_{t-1}} - g_I \right)^2 - \Omega_k \left(\frac{I_t}{I_{t-1}} - g_I \right) \frac{I_t}{I_{t-1}} \right) + && (2.7)\\
& \quad \beta E_t q_{t+1}^k \frac{C_t}{C_{t+1}} \Omega_k \left(\frac{I_{t+1}}{I_t} - g_I \right) \left(\frac{I_{t+1}}{I_t} \right)^2 .
\end{aligned}
$$

2.2 Retailers There are a continuum of retail goods firms indexed by $i \in [0,1]$. The imperfectly competitive retail goods firms enjoy some monopoly power in producing a differentiated product. They face Rotemberg adjustment costs in changing prices of the form, $\frac{\Omega_p}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} \frac{1}{\pi} - 1 \right)^2 Y_t$, such that price changes in excess of steady-state inflation rates are costly. Final goods are CES aggregates of retail goods

$$
Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{2.8}
$$

where Y_t is the final goods and $Y_t(i)$ is the retail goods from sector i. ϵ is the elasticity of substitution among retail goods. The demand curve for each retail good $Y_t(i)$ is given by

$$
Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t. \tag{2.9}
$$

Retail goods firms need to buy intermediate inputs at price P_t^w from intermediate goods firms. The retail goods firm i maximizes

$$
E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} \left(\frac{P_{t+j}(i) - P_{t+j}^w}{P_{t+j}} Y_{t+j}(i) - \frac{\Omega_p}{2} \left(\frac{P_{t+j}(i)}{P_{t+j-1}(i)} \frac{1}{\pi} - 1 \right)^2 Y_{t+j} \right),
$$

subject to the demand function (2.9). The first-order condition is

$$
\frac{1}{x_t} = \frac{(\epsilon-1)}{\epsilon} + \frac{\Omega_p}{\epsilon} \left(\frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} - \beta \frac{\Omega_p}{\epsilon} E_t \frac{C_t}{C_{t+1}} \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} \frac{Y_{t+1}}{Y_t} \tag{2.10}
$$

where $x_t = \frac{P_t}{P_t^w}$. Equation (4.2) represents the New Keynesian Phillips curve (NKPC) under Rotemberg pricing, which would, upon linearization, correspond to the standard NKPC under Calvo pricing. Monopoly profits are given by

$$\Upsilon_t^R = Y_t - \frac{1}{x_t}Y_t - \frac{\Omega_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 Y_t \tag{2.11}$$

2.3 Intermediate Goods Firms There are a continuum of intermediate goods firms, which use capital and labor as inputs for production. Each firm i faces an idiosyncratic productivity shock ω_t^i that is an i.i.d drawn from a distribution $F(.)$ with a mean of 1. As a result, each firm's output can vary depending on the realization of ω_t^i,

$$Y_t^i = \omega_t^i (K_{t-1}^i)^{1-\alpha}\left(A_t\left(h_t^{e,i}\right)^{1-\theta}(h_t^i)^\theta\right)^\alpha \tag{2.12}$$

where $h_t^{e,i}$ is the demand for managerial labor and h_t^i is the demand for household labor.

Those firms, however, face a working capital constraint: they have to make allocation decisions on labor and capital, and pay wages and capital rents before the idiosyncratic shock is realized and production takes place. Following the literature, we assume each firm starts with the same level of beginning-of-period capital K_{t-1}. Therefore, all intermediate goods firms face the same ex ante cost minimization problem since the idiosyncratic productivity shock is i.i.d,

$$\min W_t h_t + r_{kt}k_{t-1} + W_t^e h_t^e \tag{2.13}$$

$$s.t. \qquad Y_t = \underbrace{E_i(\omega_t^i)}_{=1}(K_{t-1})^{1-\alpha}\left(A_t\,(h_t^e)^{1-\theta}\,h_t^\theta\right)^\alpha \tag{2.14}$$

A_t represents the aggregate productivity shock, which has a deterministic growth trend, g, and a stationary component, A_t^m, such that $A_t = g^t A_t^m$. A_t^m follows an AR(1) process. Assuming ν_t is the Lagrangian multiplier attached to production function, then the first-order conditions are

$$W_t = \nu_t \alpha\theta \frac{Y_t}{h_t} \tag{2.15}$$

$$W_t^e = \nu_t \alpha(1-\theta)\frac{Y_t}{h_t^e} \tag{2.16}$$

$$r_{kt} = \nu_t(1-\alpha)\frac{Y_t}{K_{t-1}} \tag{2.17}$$

In a model without the working capital constraint, the competitive market drives profits to zero for intermediate goods firm, and therefore $\nu_t = \frac{1}{x_t}$. In this model, however, the working capital constraint renders a wedge between ν_t and $\frac{1}{x_t}$. Combining the (2.15), (2.16), and

(2.17),

$$\frac{1}{\nu_t} = \left(\frac{\alpha\theta Y_t}{W_t h_t}\right)^{\alpha\theta} \left(\frac{\alpha(1-\theta)Y_t}{W_t^e h_t^e}\right)^{\alpha(1-\theta)} \left(\frac{(1-\alpha)Y_t}{r_t^k K_{t-1}}\right)^{1-\alpha} \tag{2.18}$$

$$= \left(\frac{1-\alpha}{r_t^k}\right)^{1-\alpha} \left(A_t \left(\frac{\alpha(1-\theta)}{W_t^e}\right)^{1-\theta} \left(\frac{\alpha\theta}{W_t}\right)^{\theta}\right)^{\alpha} \tag{2.19}$$

To finance those expenses on wage and capital rents, firms resort to their own beginning-of-period net worth N_{t-1}, which are assumed to the same across firms, and external debt. Again, all firms would borrow the same amount of debt B_t for the given state of economy, as the idiosyncratic productivity shock is i.i.d.

$$\frac{N_{t-1}+B_t}{P_t} = W_t h_t + W_t^e h_t^e + r_t^k K_{t-1} \tag{2.20}$$

Therefore,

$$Y_t \nu_t = \frac{N_{t-1}+B_t}{P_t} \tag{2.21}$$

$$\rightarrow \quad \frac{Y_t}{x_t} = \tilde{A}_t \frac{N_{t-1}+B_t}{P_t} \tag{2.22}$$

where $\tilde{A}_t$ is the overall return on working capital, given by

$$\tilde{A}_t = \frac{1}{x_t}\left(\frac{1-\alpha}{r_t^k}\right)^{1-\alpha} \left(A_t \left(\frac{\alpha(1-\theta)}{W_t^e}\right)^{1-\theta} \left(\frac{\alpha\theta}{W_t}\right)^{\theta}\right)^{\alpha} \tag{2.23}$$

2.4 Financial Intermediaries We adopt the BGG (Bernanke et al. (1999)) framework to model the financial frictions. At the beginning of each period, a risk-neutral financial intermediary (FI) obtains household deposit D_t at the interest rate of R_t. It lends to intermediate goods producers, which choose the level of debt prior to the realization of idiosyncratic firm-specific productivity shocks, and charges a rate of Z_t. The optimal contract is then characterized by a threshold on idiosyncratic productivity, $\bar{\omega}_t$, such that the intermediate goods producer with the cutoff productivity is just able to repay the external debt B_t.

$$\bar{Y}_t P_t^w = Z_t B_t$$

where $\bar{Y}_t$ is the firm production with the cutoff idiosyncratic productivity,

$$\begin{aligned}\bar{Y}_t &= \bar{\omega}_t (K_{t-1})^{1-\alpha} \left(A_t \left(h_t^e\right)^{1-\theta} (h)_t^{\theta}\right)^{\alpha} \\ &\equiv \bar{\omega}_t Y_t\end{aligned}$$

From intermediate goods firms' working capital constraint,

$$\tilde{A}_t \left(N_{t-1} + B_t\right) = Y_t P_t^w$$

therefore,

$$\bar{\omega}_t = \frac{Z_t B_t}{\tilde{A}_t \left(N_{t-1} + B_t\right)} \tag{2.24}$$

When $\omega_t \geq \bar{\omega}_t$, the firm repays the loan and FI receives the payoff of $Z_t B_t$. When $\omega_t < \bar{\omega}_t$, the firm cannot pay the contractual return and has to default. In this case, the FI pays a monitoring cost, defined as a fraction (m_t) of the firm's realized total revenue, to observe the realized idiosyncratic productivity shock and collect the firm's production. Government can partially guarantee bank loans, in which case the FI receives a fraction of the monitoring costs, $l_t m_t \tilde{A}_t \omega_t (N_{t-1}+B_t)$, from the government bailout funds. Overall, the expected nominal income for the lender is given by,

$$\begin{aligned}&(1 - F(\bar{\omega}_t)) Z_t B_t + \int_0^{\bar{\omega}_t} \Big((1-m_t)\tilde{A}_t \omega_t (N_{t-1} + B_t) + l_t m_t \tilde{A}_t \omega_t (N_{t-1} + B_t)\Big)\, dF(\omega) \\ = \quad & \tilde{A}_t (N_{t-1} + B_t) \underbrace{\left(\left[1 - F(\bar{\omega}_t)\right] \bar{\omega}_t + (1 - m_t + l_t m_t) \int_0^{\bar{\omega}_t} \omega dF(\omega) \right)}_{g(\bar{\omega}_t)}\end{aligned} \tag{2.25}$$

As a result, the FI would lend to firms if the following participation constraint holds,

$$\tilde{A}_t (N_{t-1} + B_t) g(\bar{\omega}_t) \geq R_t B_t \tag{2.26}$$

which illustrates the FI's supply of debt.

Given the participation constraint, firms choose $\bar{\omega}_t$ and B_t to maximize their expected income, which is

$$\tilde{A}_t (N_{t-1} + B_t) \underbrace{\left(\int_{\bar{\omega}_t}^{\infty} \omega dF(\omega) - (1 - F(\bar{\omega}_t)) \bar{\omega}_t \right)}_{f(\bar{\omega}_t)} \tag{2.27}$$

Given firms' total asset $N_{t-1} + B_t$, $\tilde{A}_t f(\bar{\omega}_t)$ can be interpreted as firms' expected return on their assets. The FOC characterizing the optimal contract is then the following,

$$\frac{N_{t-1}}{N_{t-1} + B_t} = -\frac{g'(\bar{\omega}_t)}{f'(\bar{\omega}_t)} \frac{\tilde{A}_t f(\bar{\omega}_t)}{R_t} \tag{2.28}$$

which illustrates firms' demand for external debt.

In addition, we follow the literature and assume that only a share (ζ) of intermediate goods firms survive at each period. This assumption ensures that firms won't accumulate enough net worth; instead they always need to resort to external debt for financing. As a result, the end-of-period aggregate net worth N_t depends on profits from surviving firms and managerial labor income, which an be described as follows:

$$N_t = \zeta \tilde{A}_t (N_{t-1} + B_t) f(\bar{\omega}_t) + P_t W_t^e h_t^e \tag{2.29}$$

2.5 Resource Constraint and Government Policy

Monetary Policy The central bank conducts monetary policy by following a standard Taylor rule,

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\psi_\pi} \left(\frac{GDP_t}{GDP_{t-1} g}\right)^{\psi_y} \tag{2.30}$$

where g is the target GDP growth rate, and parameters ψ_π and ψ_y capture the magnitude of responses.

Fiscal Policy The fiscal authority collects taxes to pay for the government consumption G_t and bailout costs. In this baseline model, we assume that the government collects lump-sum taxes and is unable to borrow. The government budget, therefore, is balanced at each period,

$$T_t = \underbrace{\tilde{A}_t (N_{t-1} + B_t) l_t m_t \int_0^{\bar{\omega}_t} \omega_t dF(\omega)}_{S_t} + G_t \tag{2.31}$$

where S_t is the bailout costs. The government can affect the economy using two instruments: government spending G_t and bailout ratio l_t. Both instruments are assumed to be exogenous and follow

$$\ln \frac{G_t}{G} = \rho_g \ln \frac{G_{t-1}}{G} + \epsilon_t^g \tag{2.32}$$

$$\ln \frac{l_t}{l} = \rho_l \ln \frac{l_{t-1}}{l} + \epsilon_t^l \tag{2.33}$$

where $\epsilon_t^g \sim N(0, \sigma^g), \epsilon_t^l \sim N(0, \sigma^l)$.

ARC The total GDP includes private consumption, investment, and government consumption. The total aggregate output, on the other hand, is split among the GDP, monitoring costs, and price adjustment costs.

$$\begin{aligned} GDP_t &= C_t + I_t + G_t && (2.34) \\ \frac{Y_t}{x_t} &= GDP_t + \frac{\Omega_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 Y_t + \tilde{A}_t \frac{N_{t-1} + B_t}{P_t} m_t \underbrace{\int_0^{\bar{\omega}_t} \omega dF(\omega)}_{d(\bar{\omega}_t)} && (2.35) \end{aligned}$$

3 Quantitative Analysis: Baseline Model

3.1 Calibration Our model is calibrated at quarterly frequency. In order to match empirical evidence in the Chinese economy, we follow the calibration in Chang et al. (2016) closely except for parameter values related to log normal distribution and government bailout policy.

We set the technology growth to be 1.0125, which implies the steady state annual growth rate is 5%. Steady state inflation is set to be .5% to match 2% annual inflation in the long run. We set subjective discount factor to be .996. The real interest rate at steady state is $R = g\bar{\pi}/\beta = 2.16\%$. The elasticity of substitution among intermediate goods ϵ is set to 10 to match the average markup 11%. G/GDP is set to be .13 to match the average of government spending over GDP. The price adjustment cost parameter Ω_p is set to be 22. The inverse of the Frisch elasticity is set to be 2. The depreciation rate is set to be 0.035 to match the average annual depreciation rate of 14%. The investment adjustment cost parameter Ω_I is set to be 1. The capital share is calibrated to $\alpha = 0.5$ according to (Brandt et al. (2008); Zhu (2012)). The share of household labor is set to .94.

To capture the standard deviation of the logarithm of TFP across firms in China, we set σ_ω to be 1 based on Hsieh et al. (2015). We set the fraction of output loss from bankruptcy m to be .3. The steady state bailout ratio $\bar{l}$ is set to be .2, which means the financial intermediary can recover 20% of the bankruptcy loss from the government at the steady state. We set the entrepreneur's survival rate to be .9.

The Taylor rule parameters ψ_π and ψ_y are set to be 1.5 and .5 respectively. TFP shock follows AR(1) process with persistence .95 and standard deviation .01. Government spending shock follows AR(1) process with .95 and standard deviation 0.045.

Table 1: Calibration

Parameter	Description	Values
	Households	
β	Discount factor	0.996
ψ	Weight of disutility of working	23
σ	Inverse elasticity of labor supply	2
Ω_k	Capital adjustment cost	1
g_I	Steady state growth rate of investment	1.0125
δ	Capital depreciation rate	0.035
π	Steady state inflation rate	1.005
	Retailers	
ϵ	Elasticity of substitution between retail goods	10
Ω_p	Price adjustment cost	22
	Firms	
g	Steady state growth rate	1.0125
σ_ω	Scale parameter for log-normal distribution	1
α	Capital income share	0.5
θ	Share of household labor	0.94
m	FI monitoring cost	0.3
ζ	Firm survival rate	0.9
	Government policy	
ψ_π	Response coefficient to inflation in Taylor rule	1.5
ψ_y	Response coefficient to GDP growth in Taylor rule	0.5
G/GDP	Steady state government spending ratio	0.13
$\bar{l}$	Steady state government bailout ratio	.2
	Government policy	
ρ_A	Persistence of TFP shock	0.95
ρ_g	Persistence of government spending shock	0.95
ρ_l	Persistence of government bailout shock	0/0.95/0.99
σ_A	Standard deviation of TFP shock	0.01
σ_g	Standard deviation of governemnt spending shock	0.045
σ_l	Standard deviation of governemnt bailout shock	2.5

3.2 Loan Demand and Supply with Credit Shocks In this section, we study how changes in government bailout policy – which we call credit shocks – affect incentives for the FI to lend and firms to borrow, thereby the overall borrowing and firm's default decision.

As explained in section 2.4, the zero profit condition for the FI implies the following constraint has to hold for any loan contract,

$$\tilde{A}_t(N_{t-1} + B_t)g(\bar{\omega}_t) = R_t B_t$$

Note that the risk-free rate on deposit depends on the average return on intermediate goods firms (as shown in the definition of $g(\bar{\omega}_t)$). This is a source of inefficiency, as a benevolent planner would prefer that the risk-free rate corresponds to the marginal return on firms. Let's define the leverage ratio as $lev_t = \frac{N_{t-1}+B_t}{N_{t-1}}$, then the loan supply constraint becomes,

$$lev_t = \frac{1}{1 - \frac{\tilde{A}_t}{R_t} g(\bar{\omega}_t)} \tag{3.1}$$

A higher return on working capital, relative to deposit rate, raises the FI's willingness to lend and therefore the leverage ratio. As $g(\bar{\omega}_t)$ is an increasing function of $\bar{\omega}_t$, a higher productivity cutoff also raises the leverage ratio.

Given the FI participation constraint, on the other hand, firms maximize their expected income given by,

$$\tilde{A}_t(N_{t-1} + B_t)f(\bar{\omega}_t) = N_{t-1} \underbrace{\frac{1}{1 - \frac{\tilde{A}_t}{R_t} g(\bar{\omega}_t)}}_{\text{leverage}} \underbrace{\tilde{A}_t f(\bar{\omega}_t)}_{\text{firms' expected return}} \tag{3.2}$$

For a given set of return on working capital $\tilde{A}_t$ and existing net worth N_{t-1}, firms' expected income depends on the leverage ratio and the expected return on their assets, both of which depends on the productivity cutoff $\bar{\omega}_t$. Moreover, $f'(.) < 0$ and $g'(.) > 0$, imply that a higher cutoff imposes tradeoffs for firms: it raises the leverage but reduces the expected return. Therefore, firms choose $\bar{\omega}_t$ optimally to balance the tradeoffs and maximize the overall income. Its first-order condition becomes,

$$\underbrace{\frac{\frac{\tilde{A}_t}{R_t} g'(\bar{\omega}_t)}{1 - \frac{\tilde{A}_t}{R_t} g(\bar{\omega}_t)}}_{\text{elasticity of leverage w.r.t } \bar{\omega}_t} = \underbrace{-\frac{f'(\bar{\omega}_t)}{f(\bar{\omega}_t)}}_{\text{elasticity of firms' expected return w.r.t } \bar{\omega}_t} \tag{3.3}$$

Now consider a scenario of the government raising the bailout ratio l_t. Since $f(.)$ is independent from l_t, this policy change doesn't affect the right-hand side of first-order condition, i.e. the the elasticity of firms' expected return w.r.t $\bar{\omega}_t$. However, $g(.)$ depends on l_t; so does therefore the elasticity of leverage w.r.t $\bar{\omega}_t$. A more generous bailout policy raises the elasticity of leverage w.r.t $\bar{\omega}_t$, as the FI is willing to lend more. As a result, firms are willing to accept a loan contract with a higher cutoff on productivity, which, according to the FI's loan supply constraint, implies higher loans.

3.3 Aggregate Economy and Credit Shocks

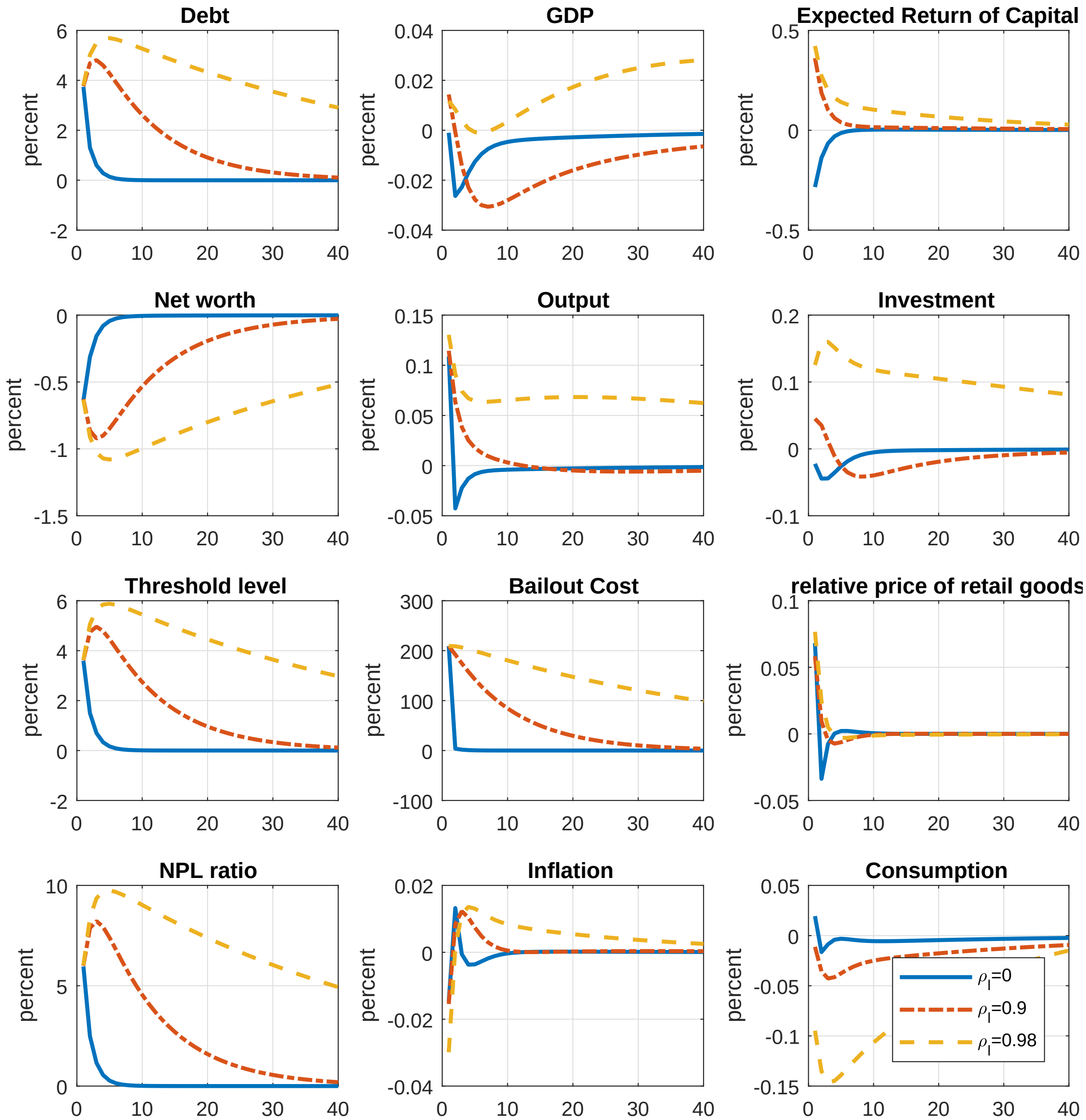
In this section, we study how changes in government bailout policy affect the aggregate economy at a general equilibrium environment.

We first consider a one-time increase in the bailout ratio from 0.1 to 0.3, whose impulse responses are shown as blue lines in figure 2. As explained in the previous section, credit easing leads firms to borrow more from the FI. Higher external debt relaxes the working capital constraint, raising wages and capital rent at period t = 1. Households work more, raising their consumption and output initially. On the other hand, credit easing also raises the productivity cutoff and forces more firms to default, significantly raising the bailout costs. In fact, the rise in bailout costs wipes out the initial gain in output, lowering GDP slightly at period $t = 1$. Higher default rate also lowers firms net worth. With an i.i.d credit shock, agents don't expect credit easing to persist. The expected return on capital drops together with firms net worth, prompting households to cut investment. Therefore, a temporary credit easing reduces, instead of boosting, investment.

Investment dynamics, however, change significantly if credit shocks are more persistent. The orange lines in figure 2 illustrate the impulse responses under a very persistent credit shock with $\rho_l = 0.98$. In this case, firms can borrow more from the FI persistently, leading to a hump-shaped path for external debt. Access to external debt relaxes the working capital constraint persistently, raising the expected return on capital. Despite firms' net worth are lower than its steady state, households increase their investment. The boost from investment raises the total output. Part of the gain in output, however, is wiped out by higher bailout costs, reducing the GDP over the short term (after the initial jump). Over the long run, higher investment persistently increases GDP.

The benign impact on GDP, however, requires credit shocks to be highly persistent. The red lines in figure 2 shows that if $\rho_l = 0.9$, the output jumps up following the shock but quickly returns to its steady state level after 10 quarters. Persistently high non-performing loans, however, lead to persistently high bailout costs. As a result, GDP drops below its steady state shortly after the initial jump and only converges back to its steady state over the long run.

Figure 2: Impulse Responses under Credit Shocks with Different Persistence



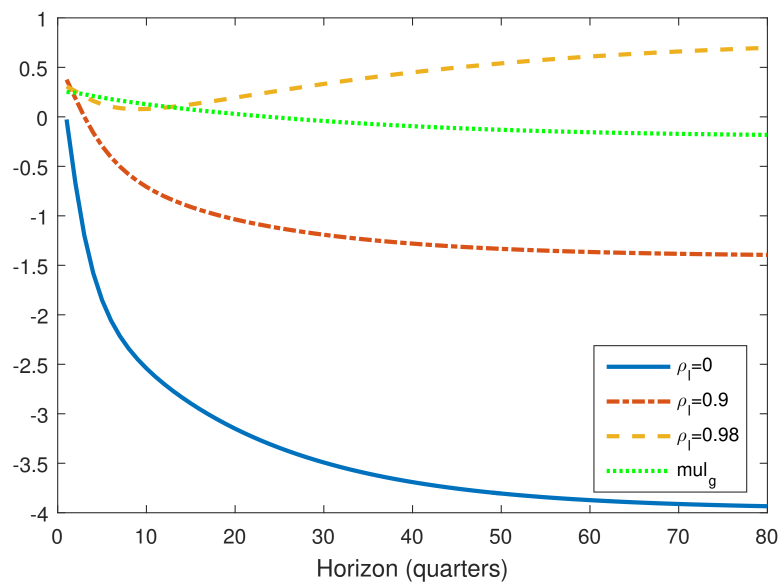
Even though highly persistent credit shocks can raise investment and GDP, consumption is persistently lower than its steady state. In this baseline model, more generous bailout policy doesn't improve welfare. In addition, credit shocks act like a supply shock initially: they relax the working capital constraint for firms, increasing the aggregate supply; therefore, inflation drops initially.

3.4 Fiscal Multipliers To quantify the impact of credit shocks on the aggregate economy, we compute credit multipliers defined in a similar way as conventional government spending multipliers. Following Uhlig (2010) and Leeper et al. (2017), present-value multipliers, which embody the full dynamics associated with exogenous policy actions and properly discount future macroeconomic effects, are adopted. The present value of additional GDP over a $k-$period horizon produced by an exogenous change in the present value of credit expansion is,

$$\text{mul}_t^l(k) = \frac{E_t \sum_{j=0}^{k} \prod_{i=0}^{j}(1+r_{t+i})^{-1}\Delta GDP_{t+j}}{E_t \sum_{j=0}^{k} \prod_{i=0}^{j}(1+r_{t+i})^{-1}\Delta S_{t+j}} \tag{3.4}$$

Figure 3 shows that credit multipliers crucially depend on the persistence of credit shocks. With an i.i.d credit shock (blue line), the present-value accumulative multiplier is negative at all horizons, reaching -3 after 5 years. A more persistent credit expansion ($\rho_l = 0.9$, red line) leads to a positive multiplier on impact, which is close to 0.4. Over the long run, however, the multiplier drops and eventually reach -1 after 5 years. If the credit expansion is highly persistent ($\rho_l = 0.98$, orange line), the accumulative multiplier is positive at all horizon.

Figure 3: Fiscal Multipliers: credit multipliers with different shock persistence vs. government spending multipliers



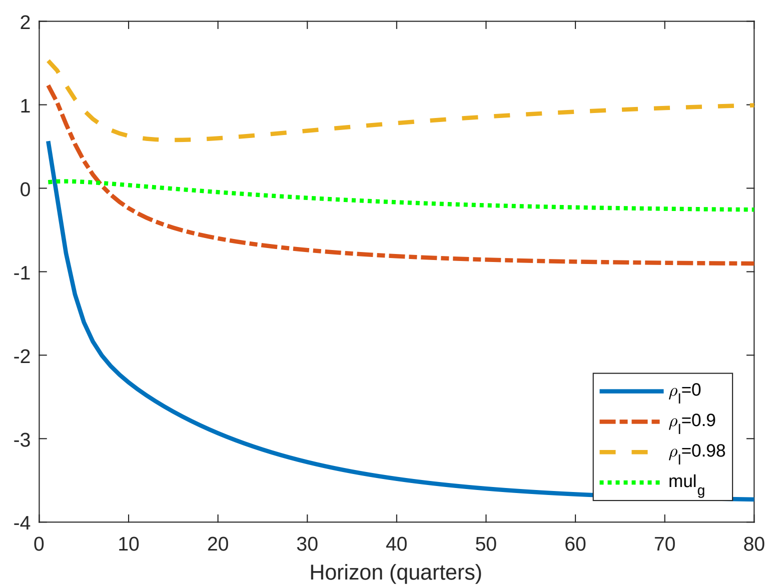
The distinct multiplier patterns are consistent with the impulse responses in figure 2. Credit easing can relax the working capital constraint for firms, potentially boosting production; on the other hand, it also increases the default rate and NPL ratio, cutting firms' net worth and raising bailout costs. If credit shocks are short lived, the second channel dominates, discouraging investments. With highly persistent credit expansion, the first channel dominates, raising investment.

For comparison, we also compute the present-value multipliers for government spending in a similar way,

$$\text{mul}_t^g(k) = \frac{E_t \sum_{j=0}^{k} \prod_{i=0}^{j} (1 + r_{t+i})^{-1} \Delta GDP_{t+j}}{E_t \sum_{j=0}^{k} \prod_{i=0}^{j} (1 + r_{t+i})^{-1} \Delta G_{t+j}} \tag{3.5}$$

The green lines in figure 3 shows the government spending multiplier over different time horizons. For each dollar the government spends, the conventional instrument of government spending dominates credit expansion in almost all horizons unless the credit expansion is highly persistent, in which case credit multiplier is higher in the long run. This is intriguing, as one might think credit expansion, which helps to alleviate the working capital constraint and therefore market inefficiency, should be more useful than government spending, which is entirely useless in this model (as in most fiscal multiplier papers). The analysis provided in figure 3 suggests that higher default rate and NPLs brought by credit expansion are quantitatively important.

Figure 4: Fiscal Multipliers: without investment adjustment cost



We further examine the sensitivity of credit multipliers with respect to the size of investment adjustment costs. Figure 4 shows the government spending multiplier and credit multipliers with credit shocks at various persistence when the investment adjustment cost is set to zero. Whether the credit shock is i.i.d, persistent, or highly persistent, zero friction in adjusting investment leads to positive credit multipliers upon impact, and higher multipliers than the counterparts in the baseline calibration at all horizons. This is consistent with the transmission mechanism. Since credit shocks boost the economy through the channel of investment, a frictionless environment helps to channel the credit expansion to the real economy, raising the multipliers.

4 Extended Model

The baseline model with lump-sum taxes serves as a good benchmark to illustrate the mechanism through which credit policy affects the economy. Taxation, however, is often distortionary. If government has to raise distortionary taxes to finance higher bailout costs, then credit easing may come with a higher price tag. In this section, we assume that retail firms have to pay a proportional sales tax τ_t and, therefore, their optimization problem becomes,

$$E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} \left(\frac{(1-\tau_{t+j})P_{t+j}(i) - P^w_{t+j}}{P_{t+j}} Y_{t+j}(i) - \frac{\Omega_p}{2} \left(\frac{P_{t+j}(i)}{P_{t+j-1}(i)} \frac{1}{\pi} - 1 \right)^2 Y_{t+j} \right) \tag{4.1}$$

Their first-order condition becomes,

$$\frac{1}{x_t} = \frac{(\epsilon-1)(1-\tau_t)}{\epsilon} + \frac{\Omega_p}{\epsilon} \left(\frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} - \beta \frac{\Omega_p}{\epsilon} E_t \frac{C_t}{C_{t+1}} \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} \frac{Y_{t+1}}{Y_t} \tag{4.2}$$

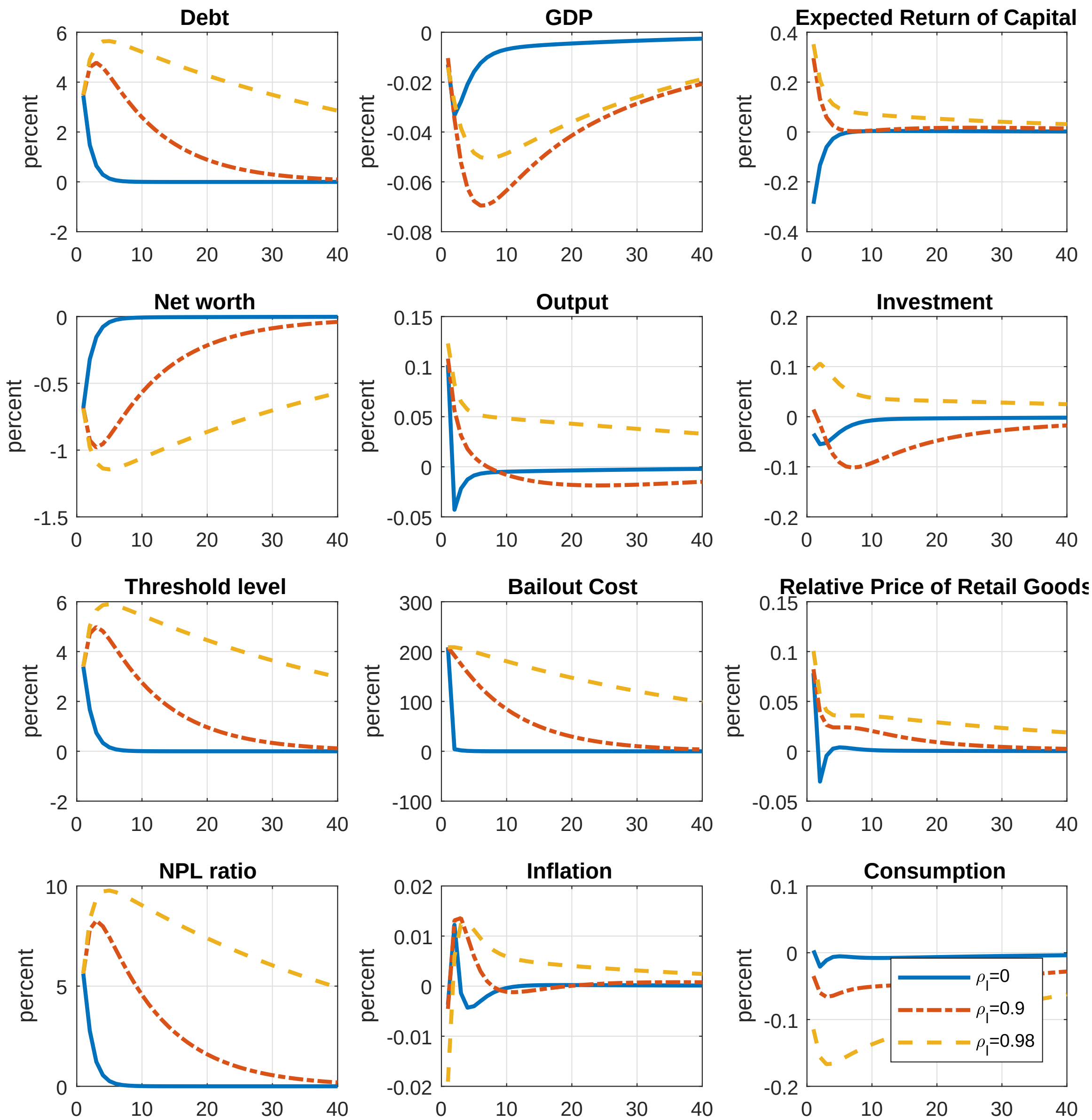
The government budget constraint becomes,

$$\frac{\tau_t Y_t}{x_t} = \tilde{A}_t (N_{t-1} + B_t) l_t m_t \int_0^{\bar{\omega}_t} \omega_t dF(\omega) + G_t \tag{4.3}$$

In the case of a distortionary tax, the impulse responses under a one-time increase in the bailout ratio are similar to the baseline case with lump-sum taxes: the rise in bailout costs wipes out the initial gain in output thus lowering GDP at period t = 1; over the medium and long run, the expected return on capital decreases and households cut investment. The responses to a highly persistent credit shock, however, is much less benign once the government can only collect distortionary taxes. The orange lines in figure 5 show that a highly persistent credit expansion reduces the GDP both in the short and medium term, despite it relaxes working capital constraint and leads to positive investment response across all horizons. This is because the sales tax drives up the relative price of retail goods to intermediate goods, thus tightening the working capital constraint for the intermediate firms.

As shown in figure 6, regardless the credit shock is persistent or not, credit expansion leads to a negative multiplier upon impact, which further drops in the long run. Government spending multiplier is also negative at all horizons. Roughly the same order remains in the sense that the conventional instrument of government spending dominates credit expansion at almost all horizons unless the credit expansion is highly persistent. In other words, the

Figure 5: Impulse Responses under Credit Shocks with Distortionary Taxes

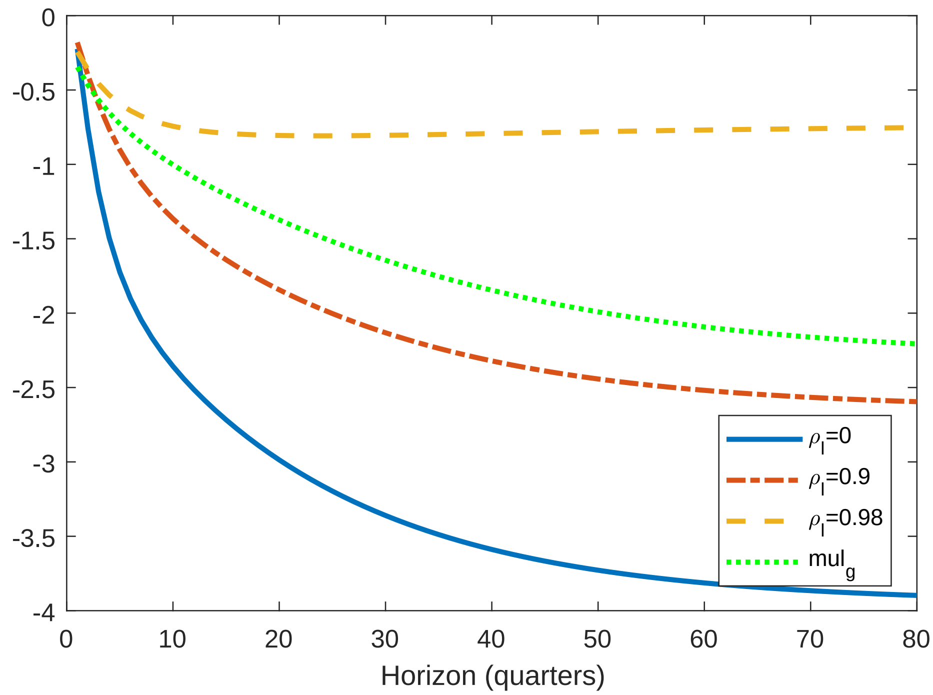


highly persistent credit expansion drives least negative accumulative multipliers at almost all horizons.

5 Conclusion

Our paper is related to the stream of research that studies the drivers and consequences of China's credit boom. Several recent papers have analyzed the unintended consequences of the financial liberalization in part due to the 2008 stimulus plan. We contribute to the discussion by taking into account of the coordinated fashion that fiscal and monetary policies

Figure 6: Fiscal Multipliers with Distortionary Taxes



are implemented in China. Through examining the impact of the government bailout shock (or credit expansion shock), we show that credit easing can, on one hand, boost production by relaxing the working capital constraint for firms; while can, on the other hand, increase bailout costs due to rising default rate. When the credit expansion shock is persistent, the first channel dominates, leading to positive credit multipliers in the short term, or even at all horizons in the case of a highly persistent credit shock. When credit shocks are i.i.d, however, the second channel dominates, driving negative credit multipliers at all horizons.

Our model abstracts from the lending friction between state-owned and private-owned firms. The trend that private firms in China were experiencing a relatively higher increase in borrowing relative to state-owned firms from 2000 to 2008 was reversed post the crisis and credit expansion policies, see Cong et al. (2017), Deng et al. (2014), and others.[3] We plan to introduce a dichotomy of state-owned enterprises versus privately owned enterprises to examine the allocation impacts of credit policies.

In addition, we exclude government debt in the model, assuming a balanced government rule instead. In China, government debt – in particular local government debt – has played a crucial role in facilitating the credit expansion policy. Between 2006 and 2013, Chinese local government debt rose from 5.8 percent of GDP to 22 percent. Huang et al. (2016) find that the massive bond issuance by local government, in conjunction direct bank loans

[3] Firth et al. (2009) finds state-owned firm gain more access to bank finance in China.

targeting at the state-owned companies, has crowded out credit resources and tightened (instead of relaxed) the credit constraint for private firms. In future work, we will incorporate government debt and possibility fiscal sustainability.

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A Model Summary with the lumpsum tax

On a balanced growth path, we make the following transformation $xx_t = \frac{XX_t}{g^t}$ where $XX = (C_t, Y_t, I_t, H_t, K_t, W_t, W_t^e, GDP_t, T_t)$; and $yy_t = \frac{YY_t}{g^t P_t}$ where $YY = (N_t, B_t)$. Assume $h_t^e = 1$.

Households

$$\begin{aligned}
k_t g &= (1-\delta)k_{t-1} + \left(1 - \frac{\Omega_k}{2}\left(\frac{i_t g}{i_{t-1}} - g_I\right)^2\right) i_t && \text{(A.1)}\\
w_t &= \psi {h_t}^{\sigma} c_t && \text{(A.2)}\\
\frac{1}{R_t} &= \beta E_t \frac{c_t}{c_{t+1} g}\frac{1}{\pi_{t+1}} && \text{(A.3)}\\
q_t^k &= \beta E_t \frac{c_t}{c_{t+1} g}\left(q_{t+1}^k(1-\delta) + r_{t+1}^k\right) && \text{(A.4)}\\
1 &= q_t^k\left(1 - \frac{\Omega_k}{2}\left(\frac{i_t g}{i_{t-1}} - g_I\right)^2 - \Omega_k\left(\frac{i_t g}{i_{t-1}} - g_I\right)\frac{i_t g}{i_{t-1}}\right) + && \text{(A.5)}\\
&\quad \beta E_t q_{t+1}^k \frac{c_t}{c_{t+1} g}\Omega_k\left(\frac{i_{t+1} g}{i_t} - g_I\right)\left(\frac{i_{t+1} g}{i_t}\right)^2
\end{aligned}$$

Firm

$$\begin{aligned}
y_t &= \int_0^\infty \omega_t k_{t-1}^{1-\alpha}\left(h_t^\theta A_t^m\right)^\alpha dF(\omega) = k_{t-1}^{1-\alpha}\left(h_t^\theta A_t^m\right)^\alpha && \text{(A.6)}\\
w_t h_t &= \alpha\theta\left(\frac{n_{t-1}}{\pi_t g} + b_t\right) && \text{(A.7)}\\
w_t^e &= \alpha(1-\theta)\left(\frac{n_{t-1}}{\pi_t g} + b_t\right) && \text{(A.8)}\\
k_{t-1} r_t^k &= (1-\alpha)\left(\frac{n_{t-1}}{\pi_t g} + b_t\right) && \text{(A.9)}\\
\tilde{A}_t &= \frac{1}{\frac{n_{t-1}}{\pi_t g} + b_t}\frac{y_t}{x_t} && \text{(A.10)}\\
R_t b_t &= \tilde{A}_t\left(\frac{n_{t-1}}{\pi_t g} + b_t\right) g(\bar{\omega}_t) && \text{(A.11)}\\
\frac{n_{t-1}}{n_{t-1} + b_t g \pi_t} &= -\frac{g'(\bar{\omega}_t)}{f'(\bar{\omega}_t)}\frac{\tilde{A}_t}{R_t} f(\bar{\omega}_t) && \text{(A.12)}\\
n_t &= \zeta \tilde{A}_t\left(\frac{n_{t-1}}{\pi_t g} + b_t\right) f(\bar{\omega}_t) + w_t^e && \text{(A.13)}\\
\frac{1}{x_t} &= \frac{(\epsilon-1)(1-\tau_t)}{\epsilon} + \frac{\Omega_p}{\epsilon}\left(\frac{\pi_t}{\pi} - 1\right)\frac{\pi_t}{\pi} - \beta\frac{\Omega_p}{\epsilon}E_t\frac{c_t}{c_{t+1}}\left(\frac{\pi_{t+1}}{\pi} - 1\right)\frac{\pi_{t+1}}{\pi}\frac{y_{t+1}}{y_t} && \text{(A.14)}
\end{aligned}$$

Policy and ARCs

$$
\begin{aligned}
\ln \frac{R_t}{R} &= \psi_\pi \ln \frac{\pi_t}{\pi} + \psi_y \ln \frac{gdp_t}{gdp_{t-1}} && \text{(A.15)}\\
t_t + \tau_t y_t &= l_t \left(\frac{n_{t-1}}{\pi_t g} + b_t \right) \tilde{A}_t s(\bar{\omega}_t) + g_t && \text{(A.16)}\\
\ln \frac{g_t}{\bar{g}} &= \rho_g \ln \frac{g_{t-1}}{\bar{g}} + \epsilon_t^g && \text{(A.17)}\\
gdp_t &= g_t + c_t + i_t && \text{(A.18)}\\
y_t &= c_t + i_t + g_t + \frac{\Omega_p}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 y_t + \tilde{A}_t \left(\frac{n_{t-1}}{\pi_t g} + b_t \right) m_t d(\bar{\omega}_t) && \text{(A.19)}
\end{aligned}
$$

Assume ω_t is drawn from a log-normal distribution distribution, $ln(\omega_t) \sim N(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2)$. $\Phi(\cdot)$ and $\phi(\cdot)$ are the standard normal cdf and pdf respectively and $z \equiv (ln(\bar{\omega}) + 0.5\sigma_\omega^2)/\sigma_\omega$ Then,

$$
\begin{aligned}
f(\bar{\omega}_t) &= \int_{\bar{\omega}_t}^{\infty} \omega dF(\omega) - (1 - F(\bar{\omega}_t))\bar{\omega}_t \\
&= 1 - \Phi(z_t - \sigma_\omega) - \bar{\omega}[1 - \Phi(z)] && \text{(A.20)}\\
f'(\bar{\omega}_t) &= -\frac{\phi(z_t - \sigma_\omega)}{\sigma_\omega \bar{\omega}} - (1 - \Phi(z_t)) + \phi(z_t)/\sigma_\omega && \text{(A.21)}\\
g(\bar{\omega}_t) &= [1 - F(\bar{\omega}_t)]\, \bar{\omega}_t + (1 - (1 - l_t)m_t) \int_0^{\bar{\omega}_t} \omega dF(\omega) \\
&= (1 - \Phi(z_t))\, \bar{\omega}_t + (1 - (1 - l_t)m_t)\Phi(z_t - \sigma_\omega) && \text{(A.22)}\\
g'(\bar{\omega}_t) &= 1 - \Phi(z_t) - \phi(z_t)/\sigma_\omega + \frac{(1 - (1 - l_t)m_t)\phi(z_t - \sigma_\omega)}{\sigma_\omega \bar{\omega}_t} && \text{(A.23)}\\
s(\bar{\omega}_t) &= l_t m_t \int_0^{\bar{\omega}_t} \omega dF(\omega) = l_t m_t \Phi(z_t - \sigma_\omega) && \text{(A.24)}\\
d(\bar{\omega}_t) &= \int_0^{\bar{\omega}_t} \omega dF(\omega) = \Phi(z_t - \sigma_\omega) && \text{(A.25)}
\end{aligned}
$$

The (20x1) vector of endogenous variable is
$[y_t, c_t, i_t, g_t, gdp_t, k_t, h_t, n_t, b_t, \tilde{A}_t, \bar{\omega}_t, w_t, w_t^e, r_t^k, R_t, \pi_t, x_t, g_t^c, t_t, q_t^k]$

Log-linearized System g is growth rate for the whole economy, $\bar{g}$ is the steady state of government spending.

$$g k \hat{k}_t = (1-\delta) k \hat{k}_{t-1} + \left(1 - \frac{\Omega_k}{2}(g-g_I)^2 - \Omega_k (g-g_I) g\right) i g \hat{i}_t + \Omega_k (g-g_I) g^2 i \hat{i}_{t-1} \tag{A.26}$$

$$\hat{w}_t = \sigma \hat{h}_t + \hat{c}_t \tag{A.27}$$

$$-\hat{R}_t = \hat{c}_t - E_t \hat{c}_{t+1} - E_t \pi_{t+1} \tag{A.28}$$

$$\hat{q}_t^k - \hat{c}_t + E_t \hat{c}_{t+1} = \frac{(1-\delta) q^k}{q^k(1-\delta) + r^k} E_t \hat{q}_{t+1}^k + \frac{r^k}{q^k(1-\delta)+r^k} E_t \hat{r}_{t+1}^k \tag{A.29}$$

$$\begin{aligned} 0 = &\left(1 - \frac{\Omega_k}{2}(g-g_I)^2 - \Omega_k (g-g_I) g\right) \hat{q}_t^k + \beta g \Omega_k (g-g_I) E_t \hat{q}_{t+1}^k \\ &+ \Omega_k g (3g - 2g_I)(\hat{i}_{t-1} - (\beta+1)\hat{i}_t + \beta E_t \hat{i}_{t+1}) + \beta g \Omega_k (g-g_I)(\hat{c}_t - E_t c_{t+1}) \end{aligned} \tag{A.30}$$

$$\hat{y}_t = (1-\alpha)\hat{k}_{t-1} + \alpha\theta \hat{h}_t + \alpha \hat{a}_t^m \tag{A.31}$$

$$wh(\hat{w}_t + \hat{h}_t) = \alpha\theta\left(\frac{n}{\pi g}(\hat{n}_{t-1} - \hat{\pi}_t) + b\hat{b}_t\right) \tag{A.32}$$

$$w^e \hat{w}^e = \alpha(1-\theta)\left(\frac{n}{\pi g}(\hat{n}_{t-1} - \hat{\pi}_t) + b\hat{b}_t\right) \tag{A.33}$$

$$k r^k (\hat{k}_{t-1} + \hat{r}_t^k) = (1-\alpha)\left(\frac{n}{\pi g}(\hat{n}_{t-1} - \hat{\pi}_t) + b\hat{b}_t\right) \tag{A.34}$$

$$\tilde{A}\hat{\tilde{A}}_t = \frac{1}{\frac{n}{\pi g} + b} \frac{y}{x}(\hat{y}_t - \hat{x}_t) - \frac{1}{(\frac{n}{\pi g} + b)^2} \frac{y}{x}\left(\frac{n}{\pi g}(\hat{n}_{t-1} - \hat{\pi}_t) + b\hat{b}_t\right) \tag{A.35}$$

$$\begin{aligned} Rb(\hat{R}_t + \hat{b}_t) = &\left(\frac{n}{\pi g} + b\right) g(\bar{\omega}) \tilde{A}\hat{\tilde{A}}_t + \left(\frac{n}{\pi g} + b\right) g'(\bar{\omega}) \tilde{A} \bar{\omega} \hat{\bar{\omega}}_t \\ &+ \tilde{A} g(\bar{\omega}) \left(\frac{n}{\pi g}(\hat{n}_{t-1} - \hat{\pi}_t) + b\hat{b}_t\right) \end{aligned} \tag{A.36}$$

$$\begin{aligned} &\frac{nbg\pi}{(n+bg\pi)^2}(\hat{n}_{t-1} - \hat{b}_t - \hat{\pi}_t) = \frac{g'(\bar{\omega})\tilde{A} f(\bar{\omega})}{f'(\bar{\omega}) R}(\hat{R}_t - \hat{\tilde{A}}_t) \\ &- \frac{\tilde{A}}{R}\left(\frac{g''(\bar{\omega}) f(\bar{\omega})}{f'(\bar{\omega})} + g'(\bar{\omega}) - \frac{g'(\bar{\omega}) f(\bar{\omega}) f''(\bar{\omega})}{[f'(\bar{\omega})]^2}\right) \bar{\omega} \hat{\bar{\omega}}_t \end{aligned} \tag{A.37}$$

$$n\hat{n}_t = \zeta\tilde{A}\left(\frac{n}{\pi g} + b\right)(f(\bar{\omega})\hat{\tilde{A}}_t + f'(\bar{\omega})\bar{\omega}\hat{\bar{\omega}}_t) + \zeta\tilde{A}f(\bar{\omega})\left(\frac{n}{\pi g}(\hat{n}_{t-1} - \hat{\pi}_t) + b\hat{b}_t\right) + w^e\hat{w}_t^e \quad \text{(A.38)}$$

$$\frac{-\hat{x}_t}{\bar{x}} = -\frac{(\epsilon-1)\tau}{\epsilon}\hat{\tau}_t + \frac{\Omega_p}{\epsilon}\hat{\pi}_t - \beta\frac{\Omega_p}{\epsilon}E_t\hat{\pi}_{t+1} \quad \text{(A.39)}$$

$$\hat{R}_t = \psi_\pi\hat{\pi}_t + \psi_y(g\hat{d}p_t - g\hat{d}p_{t-1}) \quad \text{(A.40)}$$

$$t\hat{t}_t + \tau y(\hat{\tau}_t + \hat{y}_t) = l_t\left(\frac{n}{\pi g} + b\right)\tilde{A}(s(\bar{\omega})\hat{\tilde{A}}_t + s'(\bar{\omega})\bar{\omega}\hat{\bar{\omega}}_t) + l_t\tilde{A}s(\bar{\omega})\left(\frac{n}{\pi g}(\hat{n}_{t-1} - \hat{\pi}_t) + b\hat{b}_t\right) + \bar{g}\hat{g}_t \quad \text{(A.41)}$$

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + \varepsilon_t^g \quad \text{(A.42)}$$

$$gdp\ g\hat{d}p_t = \bar{g}\hat{g}_t + c\hat{c}_t + i\hat{i}_t \quad \text{(A.43)}$$

$$y\hat{y}_t = c\hat{c}_t + i\hat{i}_t + \bar{g}\hat{g}_t + (\frac{n}{\pi g} + b)\tilde{A}(md(\bar{\omega}))\hat{\tilde{A}}_t + (\frac{n}{\pi g} + b)\tilde{A}(md'(\bar{\omega}))\bar{\omega}\hat{\bar{\omega}}_t - (\frac{n}{\pi g} + b)\tilde{A}(md(\bar{\omega}))\hat{\pi}_t$$
$$+ (\frac{n}{\pi g} + b)\tilde{A}(md(\bar{\omega}))\hat{n}_{t-1} + \tilde{A}(md(\bar{\omega}))b\hat{b}_t \quad \text{(A.44)}$$

where,

$$f''(\bar{\omega}) = -\frac{\phi'(z_t - \sigma_\omega) - \sigma_\omega\phi(z_t - \sigma_\omega)}{\sigma_\omega^2\bar{\omega}^2} + \frac{\phi'(z_t)}{\sigma_\omega^2\bar{\omega}} + \frac{\phi(z_t)}{\sigma_\omega\bar{\omega}}$$

$$g''(\bar{\omega}) = -\phi(z_t) - \phi'(z_t)/(\sigma_\omega^2\bar{\omega})$$
$$+ (1 - (1 - l_t)m_t)\phi'(z_t - \sigma_\omega)/(\sigma_\omega^2\bar{\omega}^2) - (1 - l_t)(1 - m_t)\phi(z_t - \sigma_\omega)/(\sigma_\omega\bar{\omega}^2)$$

$$d'(\bar{\omega}) = \phi(z_t - \sigma_\omega)/(\sigma_\omega\bar{\omega})$$