Sovereign risk and asset market dynamics in the euro area*

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December 19, 2017

Abstract

This paper studies the behaviour of euro area asset market comovements during the period 2010-2014, through the lens of a DSGE model. The economy is a two-country model with a banking sector, asset markets featuring home bias in bond holdings, and default. The periphery is buffeted by a sovereign risk shock, whose process is estimated from the data. The model successfully accounts for the divergence in core-periphery correlations between stock and bond's returns. Finally, the sovereign risk shock explains 50% of the increase in sovereign and loan-deposit spreads and 7% of the decrease in global output during the sovereign debt crisis.

JEL: F41, F44, G15

Keywords: Currency union, international financial markets, sovereign risk, general equilibrium

*We thank Vincent Bodart, Hafedh Bouakez, Antonio Cosma, Paul Hubert, Luca Pensieroso, Olivier Pierrard, Cèline Poilly, Henri Sneessens and Fabien Tripier for useful comments.

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1 Introduction

The pattern of stock-bond correlations is typically time-varying as it mirrors the change in agent's decisions according to the business cycle and the different shocks that impact the economy. Among highly integrated economies, this relation should vary in the same way over time. In the euro area, that was the case until 2009. As Figure 1 displays, in the period 2010-2014 the evolution of stock-bond correlations diverged within the union, as shown by an increase in the difference between periphery and core comovements (black dotted line). This behaviour is consistent with the flight-to-quality in sovereign bond markets -from risky periphery bonds to core safer ones- and, conversely, a stable high correlation in stock markets in the union.[1] Table 1 summarizes these stylized facts. The understanding of the evolution of the relation between stock and sovereign bond's returns is of primary importance for investors as well as for policy makers for three main reasons. First, from a finance point of view, this correlation implies the presence of risk and calls for a diversification of investors' portfolio. Second, from an economic point of view, asset prices are the mechanism by which consumption and investment are allocated across time and states of nature. Finally, in a currency union, the correlation in bond and stock markets among member countries reflects the degree of integration of the different economies.

In this paper, we study the behaviour of core and periphery asset markets in the euro area through the lens of a quantitative model, explaining the divergence in the comovements during the years 2010-2014 and the underlying propagation mechanism. In order to do so, we build a two-country DSGE model with a banking sector and asset markets. This framework is well suited to account for the multiple linkages between core and periphery allowing a comprehensive study of asset market's dynamics from a macroeconomic perspective. The model combines features from the models developed by Enders et al. (2011), Corsetti et al. (2013) and Coeurdacier et al. (2007).

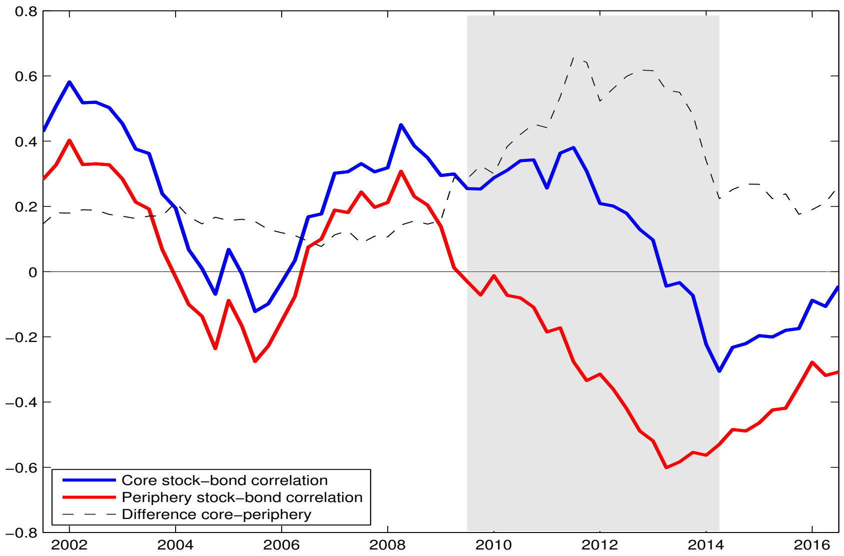
The literature has separately studied stock-bond correlations and sovereign risk both from a theoretical and empirical perspective. On the theoretical side, the literature on open economy financial macroeconomics focuses on the international dimension of asset markets as Coeurdacier and Rey (2013) exhaustively review. Closer to our approach is the branch of this literature dealing with the hedging properties of bond and equities. However, this set of models does not focus on the euro area specificity and does not consider the risk of default on sovereign bonds. On the other hand, DSGE models studying the impact of sovereign default look at the pricing of debt (Falko et al., 2016), the role of debt maturity (Auray and Eyquem, 2017) and the transmission of sovereign default through the banking sector (van der Kwaak and van Wijnbergen, 2014; Faia, 2017) in close economies. There is a scant of papers analysing the impact of sovereign default at the euro area level and these focus on explaining the transmission through the banking sector, Guerrieri et al. (2012), or the stabilizing effect of monetary policy, Auray et al. (2014). In this respect we are the first examining the impact of sovereign risk also on equity markets.[2]

On the empirical side, the literature has extensively analysed the macroeconomic determinants of the euro area stock-bond comovements. Among the others, Kim et al. (2006) and Andersson et al. (2008) explain the role of variables such as inflation, GDP growth and market uncertainty as main drivers of the correlations. Perego and Vermeulen (2016) document the heterogenous behaviour of the stock-bond correlation in the euro area during the sovereign debt crisis and highlight the additional role of relative imbalances and diverging fundamentals among the determinants. On the other hand, there is a wide empirical literature on sovereign risk that looks at i) the pricing of risk and contagion (Beirne and Fratzcher, 2013); ii) the impact of sovereign CDS on bank CDS (Alter and Beyer, 2014) and borrowing costs (Delatte et al., 2012); and iii) on stock market prices (Grammatikos and Vermeulen, 2012). At

[1] Appendix A and B explain more in detail the data.

[2] Additionally, general equilibrium models in the macro-finance literature have looked at the term structure of bond interest rates and on the asset pricing of stock and bonds in closed economies. A good review of the stock-bond asset pricing literature is provided by Campbell et al. (2014) and Swanson (2015).

Figure 1.1: Core and Periphery stock-bond correlations



Notes. Stock market series are total return indexes on non-financial firms; bond series are DS benchmark 10 years index of yields to redemption. Countries belonging to the core are: Austria, Belgium, Finland, France and Germany. Whereas countries belonging to the periphery are: Greece, Ireland, Italy, Portugal and Spain. The series are centered 3 year moving averages on quarterly correlations contructed over daily returns. The series are aggregated at the core and periphery level as a weighted average based on the value of GDP for the year 2004. The shaded areas highlight the period of the sovereign debt crisis.

Data source: Datastream and author's calculations.

the euro area level and closer to our approach, Neri and Ropele (2013) quantify the transmission of a sovereign risk shock onto periphery and core real economies. However, they disregard equity markets. This paper aims at filling this gap from a theoretical point of view.

The contribution of this paper is twofold. First, we build a comprehensive framework able to reconcile the economic and finance perspectives explaining the effect of a sovereign risk shock on euro area asset markets (bond and stock) and on the underlying macroeconomic dynamics. Second, we estimate the sovereign risk shock process from the data, following the approach of deGrauwe and Ji (2013), and we feed it to the model. We show that the model is able to reproduce the divergence in the stock-bond returns' correlations in the core and in the periphery and that the sovereign risk shock can explain 50% of the increase in sovereign and loan-deposit spreads and 7% of the decrease in global output during the sovereign debt crisis.

Table 1: Correlation data

	2000-2009	2010-2014	2000-2017
corr($R^{b,p}$,$R^{b,c}$)	0.93	0.27	0.70
corr($R^{S,p}$,$R^{S,c}$)	0.79	0.70	0.76
corr($R^{b,c}$,$R^{S,c}$)	0.25	0.12	0.16
corr($R^{b,p}$,$R^{S,p}$)	0.10	-0.38	-0.09
corr(Sb, c) - corr(Sb, p)	0.14	0.50	0.26

$R^{b,j}$ is the gross return on sovereign bonds and $R^{S,j}$ the gross return on equity in country $j \in \{c, p\}$. Corr(Sb, c)-corr(Sb, p) is the difference between the stock-bond correlation in the core (corr($R^{b,c}$,$R^{S,c}$)) and the periphery (corr($R^{b,p}$,$R^{S,p}$)).

The remainder of the paper is structured as follows. Section 2 details the model, Section 3 explains the calibration and Section 4 shows the dynamic simulations. Section 5 concludes.

2 Model

We develop an international business cycle model for the euro area. It consists of two regions, we call the first country/region as *core* and we denote it by c hereafter. We call the second country/region as *periphery* and we denote it by p hereafter. The model features an international banking sector, an equity market and a probability of default on sovereign debt as in Corsetti et al. (2013). We assume that the two regions are perfectly symmetric except for a higher level of debt to output in the periphery country. For a detailed description of the model and the list of equilibrium equations, see Appendix D.

2.1 Households

Households, in each country $j \in \{c, p\}$, maximize their lifetime utility subject to a budget constraint. The representative household may consume C_t^j, invest D_t^j in one-period bank deposits or b_t^j in one-period sovereign debt. Moreover households can invest in financial markets both in domestic and foreign equity $S_{i,t}^j$ issued by the respective firms-capital producers $i \in \{c, p\}$.[3] By investing in deposits the households obtain $R_{t-1}^{d,j}$, the predetermined gross return on deposits. The expected gross return on sovereign bond is $R_{t-1}^{b,j}$ while the actual net return is $R_{t-1}^{b,j} - \epsilon_t^j$, where $\epsilon_t^j \geq 0$ captures the share of outstanding sovereign debt lost by households because of – partial – sovereign default. The expected return on equity holdings is given by the price at which households can sell the share $\rho_{s,t}^j$ bought in the previous period and the dividend payout div_t^j coming from the capital producers. $Q_{j,t}^i$ is the real exchange rate of country i when country j is taken as the numeraire. The household also supplies h_t^j hours to the firms and receives wages w_t^j. Moreover, it owns the firms located in j and receives their profits Υ_t^j. Finally, the household receives a lump-sum transfer $H_t^{h,j}$ from the government and must pay taxes T_t^j as well as a quadratic portfolio adjustment cost on sovereign debt represented by the parameter $\phi_b > 0$. This cost makes the households' portfolio choices less sensitive to interest rate differentials. Additionally, the households pay a cost related to their equity holdings represented by the parameter $\phi_s > 0$. [4]

The households first order conditions (FOCs) read:

$$\psi_n (h_t^j)^\eta = w_t^j \ , \tag{2.1}$$

$$\lambda_t^j = \frac{\psi_d}{D_t^j} + E_t \beta \lambda_{t+1}^j R_t^{d,j} \ , \tag{2.2}$$

$$\lambda_t^j \left(1 + \phi_b (b_t^j - \bar{b^j})\right) = E_t \beta \lambda_{t+1}^j (R_t^{b,j} - \epsilon_{t+1}^j) \ , \tag{2.3}$$

$$Q_{j,t}^i \lambda_t^j (1 + \phi_s S_{i,t}^j) = E_t \beta \lambda_{t+1}^j (R_{i,t+1}^S) Q_{j,t+1}^i \ , \tag{2.4}$$

where λ_t^j is the marginal utility of consumption and β is the subjective discount factor.

Equation (2.1) shows that the wage is equal to the marginal disutility of hours worked. Equations (2.2), (2.3) and (2.4) state that, at equilibrium, marginal costs are equal to expected marginal income

[3] Households are assumed to only invest in domestic bonds in order to reproduce the existence of home bias in sovereign debt holdings. The cross-country holdings of sovereign bonds are instead mainly held by banks.

[4] The households, in order to minimize the sum of the squared costs associated to equity holdings, optimally choose to hold the same amount of c and p shares in equilibrium. For this reason we can interpret this cost as a way to mimic preferences for a diversified portfolio. If the shares' holdings deviate from the optimal reference value, the households bear an additional cost.

from, respectively, deposits, sovereign bonds and equity. Equation (2.4) represents the FOCs for equity holding for country j households with respect to country $i \in \{c, p\}$, $i \neq j$ issuer.

The expected real return on equity is:

$$E_t[R^S_{i,t+1}] = \frac{E_t[\rho^s_{i,t+1}] + E_t[div_{i,t+1}]}{\rho^s_{i,t}}. \tag{2.5}$$

These returns are defined as the change in price plus dividend payouts.

Comparing equations (2.3) and (2.4) we can analyse the relation between sovereign bond and equity rates in the households' portfolio. If we define the net return on sovereign bonds $R^{nb,j}_t$ as the actual return net of default, - abstaining from adjustment costs and price dynamics -, the relation between the sovereign bond and equity rates is the following:

$$E_t[R^S_{i,t+1}] \quad = \quad R^{nb,j}_t + \lambda^j_t \phi_s S^j_{i,t}. \tag{2.6}$$

Equation (2.6) shows that the two assets are not perfect substitute. There are two sources of differentiation: sovereign debt default and the cost associated to equity holdings. Changes in the amount of shares bought reduces the correlation between equity and sovereign returns. The more the shares held, the higher the return demanded by the households in order to hold such an asset. Analogously, periphery default on sovereign debt determines a wedge between the return on equity and periphery sovereign bonds.

2.2 Capital producers

The capital producers in country $j \in \{c, p\}$ have the choice of financing either via one-period loans from the bank or through asset markets in the form of equity. They may payout dividends, div^j_t, to the households or invest I^j_t in domestic firms. In turn, investment increases firms' capital stock K^j_t according to the following law of motion:

$$K^j_t = (1 - \delta)K^j_{t-1} + I^j_t, \tag{2.7}$$

where $0 < \delta < 1$ is the capital depreciation rate. Capital provides a net real return r^j_t and capital producers pay a gross nominal interest rate $R^{l,j}_{t-1}$ on loans, as well as an adjustment cost on investment represented by the parameter $\phi_i > 0$.[5] If the capital producers decide to pay out dividends they face an adjustment cost represented by the parameter κ_d. As in Jermann and Quadrini (2012) the equity payout cost can be interpreted as a pecuniary cost as well as a way to model the speed of fund's adjustment when financial conditions change. In this model's specification of the cost, when κ_d is infinitely large capital producers have access to only one source of funds: bank loans. For smaller values, the capital producers can be financed both via (negative) dividend payouts and bank's loans. High values of κ_d oblige the capital producers to pay a high cost when they want to adjust dividend payouts from their steady state value. Lower values allow more flexibility in the payout policy.

The first order conditions for this problem read:

$$\lambda^{e,j}_t = E_t \beta^{e,j} \lambda^{e,j}_{t+1} R^{l,j}_t \ , \tag{2.8}$$

$$\lambda^{e,j}_t q^j_t = E_t \beta^{e,j} \lambda^{e,j}_{t+1} \left(r^j_{t+1} + (1 - \delta) q^j_{t+1} \right) \ , \tag{2.9}$$

$$I^j_t = \bar{I}^j + \frac{1}{\phi_I}(q^j_t - 1) \ , \tag{2.10}$$

[5] A convex adjustment cost on investment is common in the literature as it helps to match empirical behaviour of aggregate investment and prevents the investment demand curve to be perfectly elastic. For the early literature that assumes this cost see Gould (1968) and Lucas (1967) among others.

where $\lambda_t^{e,j}$ is the Lagrangian multiplier associated to the capital producers' budget constraint and $\beta^{e,j}$ is the entrepreneur's discount factor.

Equation (2.8) says that, at equilibrium, the marginal income from loans is equal to the expected marginal cost weighted by the households' discount factor. Equation (2.9) defines the shadow value of capital, q_t^j, as the expected discounted value of the marginal profits of having one additional unit of capital. If $q_t^j < 1$, meaning that the shadow value of capital is smaller than the price of capital, equation (2.10) states that investments should decline, if $q_t^j > 1$ that investments should increase.[6] Furthermore, from equation (2.9), we see that the shadow value of capital increases when the expected future dividend payouts are lower than the actual ones.

2.3 Nonfinancial firms

In each country $j \in \{c, p\}$ firms are perfectly competitive. The intermediate j firm produces a good that is sold in the domestic country as well as in the foreign one. A final firm in each country combines the intermediate goods from the j and $-j$ countries into a final one.

The production function for the intermediate tradable good is a Cobb-Douglass whose inputs are capital and labour rented respectively from capital producers and households:

$$Y_t^j = Z_t^j \left(K_t^j\right)^{\mu} \left(h_t^j\right)^{1-\mu} , \tag{2.11}$$

where Z_t^j represents total factor productivity (assumed to be constant at $\bar{Z}^j = 1$) and $0 < \mu < 1$ is the elasticity of output to capital. The first order conditions for this maximization problem equate the marginal productivity of factors with their marginal cost:

$$r_t^j = \mu \frac{\phi_t^j Y_t^j}{K_{t-1}^j} , \tag{2.12}$$

$$w_t^j = (1-\mu) \frac{\phi_t^j Y_t^j}{h_t^j} . \tag{2.13}$$

2.4 Banking sector

The banking sector is represented by an international and perfectly competitive bank *à la* Enders et al. (2011). The bank is located in the core but trades with all countries $j \in \{c, p\}$. It collects deposits D_t^j from households and can invest in sovereign bonds s_t^j as well as provide loans L_t^j to the firms in both regions. The bank maximizes its consumption, its profits, over the two regions. The bank faces a capital requirement having to set aside a fraction $0 < \gamma < 1$ of loans as own capital. The bank can deviate from legal requirements ($x_t = 0$) but this is costly.

The bank's balance sheet constraint is:

$$(1-\gamma)\sum_j Q_{c,t}^j L_t^j + \sum_j Q_{c,t}^j s_t^j = \sum_j Q_{c,t}^j D_t^j + x_t . \tag{2.14}$$

The bank pays a real return $R_{t-1}^{d,j}$ on deposits, it receives $R_{t-1}^{l,j}$ on loans and $R_{t-1}^{b,j}$ on sovereign bonds. Sovereign bonds are risky assets as government can default on them with a probability ϵ_t^j. The bank might receive a lump-sum transfer $H_t^{b,j}$ from the government. Moreover, the bank faces different types of costs: operational costs on deposits as in Enders et al. (2011), captured by Γ_d; adjustments costs on loans, Γ_l, as in Guerrieri et al. (2012); and the cost of deviating from the legal requirement that, following Enders et al. (2011), we capture by $\Gamma_x > 0$.

The first order conditions are:

[6] This formulation of the investment equation follows Tobin's Q theory of investment (Tobin, 1969).

$$Q^p_{c,t} = \frac{1-\vartheta}{\vartheta}\frac{C^{b,c}_t}{C^{b,p}_t} \ , \tag{2.15}$$

$$\lambda^b_t\left(Q^j_{c,t} - \Gamma_d + Q^j_{c,t}\Gamma_x x_t\right) = \beta E_t \lambda^b_{t+1} Q^j_{c,t+1} R^{d,j}_t \ , \tag{2.16}$$

$$\lambda^b_t\left(Q^j_{c,t} + \Gamma_l(L^j_t - \bar{L}^j) + (1-\gamma)Q^j_{c,t}\Gamma_x x_t\right) = \beta E_t \lambda^b_{t+1} Q^j_{c,t+1} R^{l,j}_t \ , \tag{2.17}$$

$$\lambda^b_t\left(Q^j_{c,t} + Q^j_{c,t}\Gamma_x x_t\right) = \beta E_t \lambda^b_{t+1} Q^j_{c,t+1}(R^{b,j}_t - \epsilon^j_{t+1}) \ , \tag{2.18}$$

where $\lambda^{b,j}_t$ is the Lagrangian multiplier associated to the bank budget constraint and $\beta^{b,j}$ is the bank's discount factor.

Equation (2.15) shows that the ratio of consumption of the bank for the two regions depends on the ratio of relative price indexes. Equations (2.16), (2.17), (2.18) represent respectively the Euler equation for deposits, loans and sovereign bonds.

2.5 Government

The government consumption in each region $j \in \{c, p\}$, G^j, is financed via lump-sum taxes, T^j_t, from the households, as well as via public debt, B^j_t, according to:

$$\begin{aligned} G^j + H^{h,j}_t + H^{b,j}_t + (R^{b,j}_{t-1} - \epsilon^j_t)B^j_{t-1} &= B^j_t + T^j_t \ , & (2.19)\\ T^j_t &= \bar{T} + \tau(B^j_t - \bar{B}) \ . & (2.20) \end{aligned}$$

Moreover, the government may transfer $H^{h,j}_t$ to the households and $H^{b,j}_t$ to the bank. Both for the tax rule and the transfer specification we follow Corsetti et al. (2013). As estimated by Bohn (1998), taxes react positively to the increase in debt such as to stabilize it. This implies that the government cannot finance public expenditure only via debt.[7] Equation (2.19) also shows that sovereign default may happen through the term $0 \leq \epsilon^j_t \leq 1$. Everything else equal, a strictly positive ϵ^j_t reduces the stock of sovereign debt in the next period. Finally, we define public expenditures as a fixed fraction, G^j, of debt at any period.

Sovereign risk shock

To determine the default rate ϵ^j_t we tightly refer to the methodology used by Corsetti et al. (2013) and van der Kwaak and van Wijnbergen (2014) by introducing an exogenous fiscal limit for the economy. [8] Behind this limit there is the intuition that there exists a maximum level of taxes that can be raised before the economy becomes politically unstable. This translates, through equation (2.20), into a maximum level of sovereign debt-to-output ratio BY^{max}_t that the government is able to service. We moreover assume that this maximum *sustainable* level is stochastic and follows:

$$BY^{max}_t = \bar{BY}^{max} + \gamma_b(BY^{max}_{t-1} - \bar{BY}^{max}) + u^b_t \ , \tag{2.21}$$

where $0 < \gamma_b < 1$ is the autoregressive component, and u^b_t is an i.i.d. shock. This stochastic behaviour captures the uncertainty around political instability in the context of sovereign debt and taxation.[9]

[7] As the focus of the paper is not on the fiscal dimension we use debt-smoothing lump-sum taxes rather than more complicated distortionary tax schemes.

[8] The concept of exgenous fiscal limit was firstly introduced by Bi (2012).

[9] In reality, the maximum sustainable government debt level is not exogenous but depends on expected growth rates, on expected growth volatility or on the expected government ability to raise taxes (see for instance Collard et al. (2015)). But this is beyond the scope of this paper.

Let us define $\tilde{B}_t^j$ as the level of debt in the economy when no default occurs. If this level of debt-to-output $\tilde{B}_t^j/(4Y_t^j)$ is lower (resp. higher) than the maximum sustainable level BY_t^{max}, the government does not (resp. does) default. We approximate this default process with the continuous normal cdf:

$$\epsilon_t^j = F\left(\frac{\tilde{B}_t^j}{4Y_t^j} - BY_t^{max} \; ; \; 0 \; , \; \sigma^2\right) \tag{2.22}$$

where $\sigma > 0$ represents the variance and $\Phi(.)$ is the standard normal cdf. We see that when $\sigma \to 0$, then $\epsilon_t^j \to \Delta_t$. A reduction (resp. increase) in the maximum sustainable level of debt-to-output, through the stochastic shock u_t^b in equation (2.21), increases (reduces) the default rate in the economy. Similarly, a higher (resp. lower) debt-to-output ratio $\tilde{B}_t^j/(4Y_t^j)$ increases (resp. reduces) the default rate in the economy. This shock impacts both on prices (interest rates changes) as on quantities (partial default on the amount of sovereign debt held by agents). To deal with the risk dimension of the shock we isolate the price effect from the quantity effect by assuming that the government makes transfers to the households ($H_t^{h,j} = \epsilon_t^j b_{t-1}^j$) and the bank ($H_t^{b,j} = \epsilon_t^j s_{t-1}^j$) to compensate the loss. In this way we capture the effect of a change in the interest rate on bonds, as agents in the economy observe the current economic conditions and form expectations on default according to equation (2.22), but we abstract from the consequences of the direct wealth loss.[10]

2.6 Closing the model

Asset market clearing conditions

The sovereign bond market clearing condition for country $j \in \{c, p\}$ is:

$$B_t^j = b_t^j + s_t^j \tag{2.23}$$

where b_t^j and s_t^j is the amount of bonds held respectively by the households and the bank.

The equity market clearing condition for country $i \in \{c, p\}$ issuing and country $j \in \{c, p\}$ holding is:

$$1 = S_{i,t}^j + S_{i,t}^{-j} \tag{2.24}$$

implying that there is a fixed amount of shares traded in the economy normalized to 1.

Good market clearing condition

Let's define the domestic demand for country j as:

$$A_t^j = C_t^j + C_t^{b,j} + I_t^j + G_t^j + \texttt{costs}_t^j \tag{2.25}$$

where $\texttt{costs}_t^j$ collects all adjustment and operative costs beared by households, capital producers and firms in country j. Moreover, $\texttt{costs}_t^c$ also includes the costs related to the bank.

The good market clearing condition for each region j reads:

$$Y_t^j = A_{j,t}^j + A_{j,t}^{-j} \tag{2.26}$$

By summing them up we obtain the resource constraint for the two-country economy:

$$\sum_j \phi_t^j Q_{c,t}^j Y_t^j = \sum_j Q_{c,t}^j A_t^j \tag{2.27}$$

stating that the total production has to be equal to the demand in the whole currency area.

[10]The same specification has been used by Corsetti et al. (2013). This procedure is helpful to reproduce the sovereign debt crisis' dynamics in the euro area where only Greece effectively, partially, defaulted.

3 Calibration

Table 2 presents an overview of the parameters of the model. Most of the values are widely used in the DSGE and sovereign default literature. The calibration refers to euro area stylized facts over the period 1992Q1-2017Q2. Time is discrete and one period represents one quarter. We specify the two country model for the euro area distinguishing between the core and the periphery in terms of debt-to-output ratios. The periphery refers to the GIIPS (Greece, Ireland, Italy, Portugal and Spain) for which we assume a higher debt to GDP ratio with respect to that of the core. We assume the same size for the core and periphery area in order to focus on the main asymmetry brought by differentials in debt levels.[11] Unless otherwise specified we opt for the same parameter choice in the two country blocs.

3.1 Parameters governing the steady state

At steady state all the agents in the economy discount the future via the same discount factor β as $\bar{\beta}^e = \beta$. We assume no default at steady state, $\bar{\epsilon}^j = 0$, both for the periphery and the core, and that $\bar{x} = 0$ implying no excess bank capital at steady state. We set $\Gamma_D = 0.005$ and $\beta = 0.99$ in order to have the annualized returns on loans and bonds of 4% and on deposits of 2%.[12] Additionally, we set ϕ_s equal to 0.01 in order to obtain an annualized return on equity of 6%[13] given a steady state holdings of domestic as well as foreign shares of $\bar{S}_i^j = 1 - \bar{S}_i^{-j} = 0.5$. This value is consistent with the studies of ECB (2012b) and Jochem and Volz (2011) on the intra-EA home bias in equity holdings assessing a degree of cross-border holdings around 40-60%. For what concerns the sovereign bond holdings in the euro zone, we follow Guerrieri et al. (2012) and we assume that 33% of sovereign debt is held by domestic household and the rest by the bank.[14]

We set the required bank capital ratio at $\gamma = 0.08$ consistently with the minimum capital requirement of Basel II for Tier 2 capital as this is the regulation prevailing during the euro area period and in particular when the sovereign debt crisis started. Finally, the size of the bank balance sheet is of 111% of yearly total output ($\bar{Y}^c + \bar{Y}^p$). This number is in line with the euro area data on bank balance sheet for loans and holdings of securities issued by euro area residents.

The loans to physical capital ratio is set at around 1/3 and it pins down the households weight on deposits Ψ_d as in Enders et al. (2011). Ψ_n, the disutility of the labour parameter, is pined down by setting $\bar{h^j} = 0.2$ following the RBC literature implying that households work 20% of their time. We calibrate η, the parameter governing the shape of the labour disutility, in order to have a Frisch elasticity of 0.25 as it is in line with micro-based measures.[15] The production function is Cobb-Douglass with the capital share at 0.3; setting the depreciation rate at $\delta = 0.025$ implies $\bar{K}^j/\bar{Y}^j = 8.54$ and $\bar{I}/\bar{Y} = 0.21$ which is in line with the RBC literature and empirical observations. The consumption of households in total output is of 55% while the one of the bank of 2.3%. The consumption of the bank falls equally in the two regions as we impose $\vartheta = 0.5$. Finally, following the NOEM literature as in Gali and Monacelli (2008), we assume a bias for domestic goods and we calibrate $\alpha = 0.3 < 0.5$.[16]

On the fiscal side we distinguish between the core and the periphery in terms of debt-to-output ratios:

[11] Data on the euro area period suggest that the core output accounts for about 63-67% of EA output and the periphery for the remaining 33-37%. Calibrating the model accounting for different sizes for the core and the periphery would change the steady state of the model but not the dynamics.

[12] These steady state values are set to match the empirical evidence for the 10 years bond's returns and the 5 years maturity loans' rate. This choice is made in order to have comparable maturities on the two assets, given the data availability.

[13] This value is in line with the annualized returns from the non-financial corporation equity index for EA countries.

[14] We do not have data on non-resident holdings of sovereign debt so we assume that it is mainly held by banks rather than foreign households. A different assumption would not change the implications of the model as long as the majority of the debt held by household is domestic.

[15] MaCurdy (1981) and Altonji (1986) estimate the Frisch elasticity - determined from hours and wage fluctuations on an individual basis - to be in the range of 0 to 0.54. More recently, Chetty et al. (2011) reconcile micro-macro measures of the Frisch elasticity suggesting a value of 0.25 for the labour supply extensive margin.

[16] The value selected is in the range of those used in recent macro-finance model. See Coeurdacier et al. (2007).

Table 2: Parameter values

Parameter	Value	Description
Households		
β	0.99	Discount factor
ϕ_b	0.001	Bond adjustment cost
ϕ_s	0.01	Stock adjustment cost
ψ_n	219	Weight of labour in (dis-)utility
ψ_d^c	0.068	Weight of deposits in utility
ψ_d^p	0.048	Weight of deposits in utility
η	4	Inverse of the intertemporal elasticity of labour supply
Global bank		
ϑ	0.5	Elasticity of substitution between c and p consumption goods
γ	0.08	Bank capital ratio requirement
Γ_d	0.005	Deposit operating cost
Γ_l	0.001	Loan adjustment cost
Γ_x	0.105	Capital requirement cost
Production		
δ	0.025	Capital depreciation rate
ϕ_i	0.1	Investment adjustment cost
κ_d	0.1	Dividend adjustment cost
α	0.3	Index of openess
μ	0.3	Elasticity of production w.r.t. capital
Authorities		
τ	0.13	Elasticity of taxes w.r.t. debt
$\bar{G}^j/\bar{Y}^j$	0.20	Public consumption-output ratio objective
$\bar{B}^c/(4\bar{Y}^c)$	0.60	Debt-output ratio objective in the *core* country
$\bar{B}^p/(4\bar{Y}^p)$	0.85	Debt-output ratio objective in the *periphery* country
$\bar{BY}^{max}$	0.92	Maximum sustainable debt-output ratio
σ	0.015	Standard deviation of default pdf
Shock		
γ_b	0.8354	Autoregressive parameter for the maximum sustainable debt level

we set the one of the periphery at 85%, at steady state, while the one of the core at 60%.[17] Public expenditures are set to 20% of GDP as in line with EA data.[18] We set the maximum level of debt $\bar{BY}^{max}$ and the standard deviation of default, σ, in order to obtain an elasticity of default risk to debt of 0.1 around the steady state for the periphery country.[19] Finally, we assume that only the periphery can default on its debt.

3.2 Parameters governing the dynamics

This set of parameters does not affect the steady state but rather the dynamics of the model.[20] Regarding consumption, we assume a logarithmic utility function for households and a linear one for the bank in order to account for the different degrees of risk aversion (higher for the households and null for the bank).[21] Adjustment costs on bonds, loans, and investments (ϕ_b, Γ_l and ϕ_i) are standard in the literature of DSGE and their values are reported in Table 2.[22]

In the bank specification we set $\Gamma_x = 0.105$, the capital cost parameter, in order to match the impact of a 1% increase in regulation on the loan-deposit spread.[23] On the capital producers' side, the adjustment cost on dividend payout is associated to the parameter κ_d that, in the benchmark calibration, is set such as to match the standard deviation of EA dividend payout (0.08 in the core).

For the elasticity of the fiscal rule we follow Corsetti et al. (2013) and set the value of $\tau = 0.13$ that is a sufficiently high value to ensure that the debt remains bounded during simulations.

In order to calibrate the maximum sustainable level of debt shock we identify the part of riskiness of periphery debt that is not explained by fundamentals, as discussed in Appendix C. We calibrate consequently γ_b in equation (2.21) to 0.8354 consistent with the autocorrelation of the maximum sustainable level of debt with respect to output.

4 Dynamic simulation

In this section we first explain how we identify the sovereign risk shock in the data. We then study the propagation of this shock and the economic mechanism at play, adding a sensitivity analysis on the key parameters. Finally, we simulate the model using the estimated shock and we compare the corresponding stock-bond markets' correlations with their empirical counterparts.

To simulate the model we take a first order approximation of the model's equations.[24]

[17] These values are in line with the IMF economic outlook for 2010.

[18] This implies that taxes-to-output are 22% of GDP for the core and 23% for the periphery where the difference is due to the different debt burden in the two regions that forces a higher taxation in the periphery.

[19] The elasticity of – yearly – default wrt. debt to output implied by the model is:

$$\texttt{elasticity}^j = \frac{4\Delta\epsilon_t^j}{\Delta X_{j,t}^b} = \frac{4}{\sigma}\,\phi\left(\frac{\bar{X}_j^b}{\sigma}\right),$$

where $\phi(.)$ is the standard normal pdf and $X_{j,t}^b = \frac{\tilde{B}_t^j}{4Y_t^j} - BY_t^{max}$ with $\bar{X}_j^b$ its steady state. The methodology used to calibrate the default process follows the one of Corsetti et al. (2013) using 10 year sovereign CDS spreads and debt-to-gdp data. The value of the elasticity of 0.1 is consistent with the empirical stylized facts for the EA for the year 2011.

[20] In each country the sum of all the dynamic costs accounts for 1% of domestic output.

[21] As a sensitivity analysis we substitute the utility function used in the baseline model with a KPR formulation as of King et al. (1988). Correlation results are unchanged to the new utility specification. Moreover the results are also qualitatively consistent to changes in the value of the intertemporal substitution of consumption. However, the baseline formulation helps to better match empirical regularities, given the absence of wealth effect on the labour supply, and to have correlation results closer to the data estimates.

[22] ϕ_i is set to a higher value in order to better control for the volatility of investments. As its choice, together with κ_d has an impact on firm choices and equity dynamics, we provide a sensitivity analysis in section 4.2.

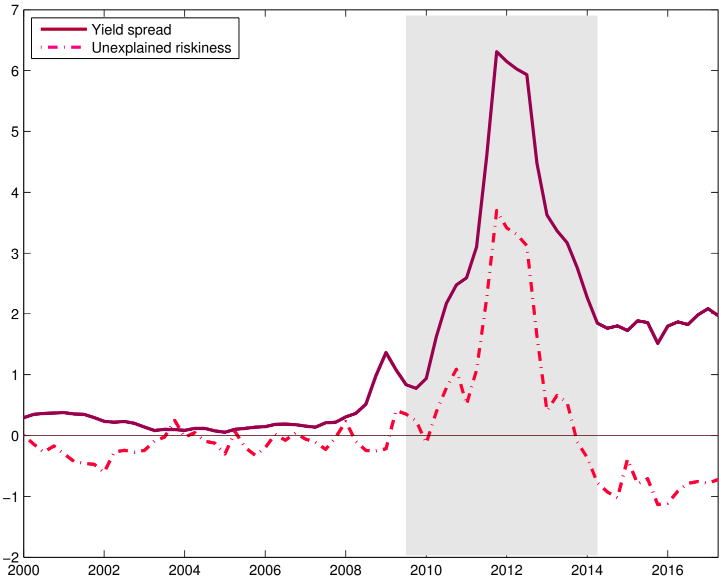
[23] We set a value of Γ_x to match a Loan-Deposit spread increase in the euro area of 15 basis points. Empirical estimates of the reaction of lending spreads to a 1% regulation shock are in the range of 14.3-15.5 basis points. See for instance Slovik and Cournède (2011) and Roger and Vitek (2011).

[24] The model is solved using Dynare 4.4.3. For more information refer to Adjemian et al. (2011).

4.1 Sovereign risk shock in the data

The period 2010-2014 was characterised by the sovereign debt crisis of the euro zone. Periphery countries experienced increasing troubles on their sovereign debt reflected in high interest rates.[25] Prior to this crisis[26], measures of debt sustainability, competitiveness and growth have been the main determinants of the interest rate demanded on sovereign bonds.

Figure 4.1: Yields spread and sovereign risk shock for the periphery of the euro area



Notes. The figure plots the yields of periphery countries with rispect to Germany (in percentage points) and the residuals from the panel estimation at the aggregated periphery level. The sereies are weighted averages based on GDP measure as of 2004 Q4. The residuals capture the part of sovereign yields spreads that is unexplained by fundamentals' changes and that identifies the sovereign risk shock. The shaded areas highlight the period of the sovereign debt crisis.

Data sources: Author's calculations.

However, during the sovereign debt crisis a big part of the riskiness was not explained by these macroeconomic dynamics anymore. In order to identify this unexplained part of risk we regress a set of macroeconomic variables onto periphery sovereign bond yields' spreads -vis-à-vis Germany- following deGrauwe and Ji (2013) and as explained in Appendix C. As we can see in Figure 4.1, until 2010 the yields (or riskiness vis-à-vis Germany) were well explained by macroeconomic fundamentals as the residuals of the regression were centred around zero. From 2010, the model does not fit the data as well as before suggesting that something else was the driver of the risk. This is the shock we identify and estimate from the data, and that has its major impact between 2010 and 2014.[27] In the model, this shock is introduced by computing the value of u_t^b consistent with the behaviour of the unexplained-riskiness part of sovereign spreads as shown in Figure 4.1.

4.2 Theoretical responses

A sovereign risk shock in the periphery is mimicked by a shock to the maximum level of sustainable debt that determines an increase in the expected default rate on the periphery sovereign bond as shown in Figure 4.2.

[25] See for instance Figure A.2 in Appendix A.

[26] The main issues shaping the crisis were weak actual and potential growth, low competitiveness and large (and growing) debt-to-output ratios. See for instance Petrakis et al. (2013).

[27] If we run the panel regression by splitting the sample before and after 2010 and compare the R^2, we see that it is much lower in the second sample. This provides additional evidence of a change in the explanatory power of the model.

An increase in the maximum sustainable level of debt brings to a wealth loss for the periphery households via taxation. They decrease consumption of both domestic as well as foreign goods. Given the home biased composition of their consumption basket, the decrease in consumption in the periphery leads to a reallocation of relative prices with the appreciation of the core currency. The marginal productivity of factors of production falls, determining a decrease in labour and output. Furthermore, as the shadow value of capital falls, investments decrease which reduces capital. Households decrease the supply of deposits and increase the holdings of equity and of domestic sovereign bonds.

The global bank is also hit by the sovereign shock and substitutes risky debt with other assets (riskless debt and loans). This happens because the bank is financially constrained and cannot be too leveraged (positive, or negative, excess capital). The shift drives a decrease in the interest rate of the risk-less asset as well as a short-lived decrease of loans' rates.[28]

A change in the bank balance sheet composition traslates into spreads' adjustment. Loan-deposit spread increase in both the core and the periphery determining the trasmission of the shock also to the core country. As a consequence, the expected value of capital decreases also in the core reducing investments. The consequent fall in capital in the two regions brings to a fall in output, wages and labour supply in the periphery, and a short lasting increase in core output and employment. Real exchange rate dynamics drive the different behaviour. Nevertheless, as also core investments fall, output in the core is forced to decrease after the initial boost and consumption falls.

At the same time, as the bank demands more core bonds than periphery ones, this determines an increase in the spread between the two reflecting the flight to quality between risky sovereigns (periphery bonds) and the riskless ones (core bonds).[29]

For what concerns equity, the owners of the firms in both countries demand more equity payouts in order to sustain their wealth. Regardless of the increase in dividend payouts, the price of equity falls in both regions as the expected discounted value of future dividends decreases due to the decrease in investments. This, in turn, leads to a decrease in the return on equity in both regions, though more pronounced in the periphery country.

The disruption of the sovereign bond market and the high correlation on the stock one ultimately impact in a different way on the core and periphery stock-bond correlations. In the core the correlation is positive. After a sovereign risk shock, on the one hand the interest rate on bonds is lowered by the flight-to-quality towards this asset; on the other hand, the equity returns decrease as firms are affected by a credit crunch that impacts negatively on investment and output. In the periphery the correlation turns instead to be negative as the sovereign returns spike whereas the stock returns decrease.

Sensitivity analysis

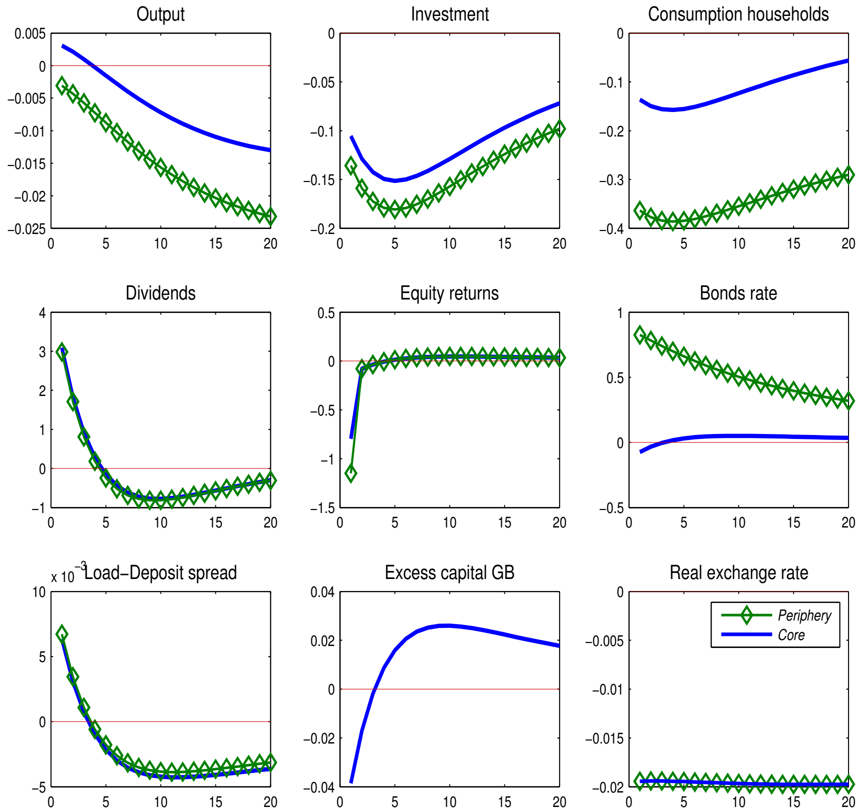
In this section we highlight the main mechanism for the behaviour of the stock-bond correlations by making a sensitivity analysis on the parameters that are important for the firm and bank's choices.

[28] A complete description of the reaction of the bank to a sovereign risk shock should include the additional role of sovereign debt as a source of collateral, as highlighted in Faia (2017). If banks are constrained in the amount of credit they can provide to the economy to the value and riskiness of their collateral, a sovereign risk shock determines a decrease in credit and an increase in loan rates. As in Perego and Pierrard (2016), addiing a collateral constraint helps to discipline the -short term- behaviour of loans and loans' rates amplifying the negative impact of the shock onto the real economy. In this paper we do not include such a mechanism in order to keep the model tractable and simple enough to be able to clearly explain the transmission of the shock from sovereign debt to equities.

In the same vein, a more realistic bank representation would allow for some degree of bank home bias mitigating the effect of the global bank with complete integration of bank activities between the core and the periphery. Relaxing the complete integration assumption would entail more asymmetric responses between regions (more negative for the periphery and less so for the core) but not qualitatively different results as long as core and periphery banks are still exposed one-another consistently with data (ECB, 2012a).

[29] We refer to flight-to-quality in bond markets by comparing the level of the correlation before 2010, that was very high, to the one during the sovereign debt crisis that fell to approximately 0.2. The flight-to-quality in the sovereign bond market has been widely documented by media and scholars in the recent years. Among the others see for instance Barrios et al. (2009) for an analysis of core-periphery sovereign bond spreads.

Figure 4.2: IRFs after a negative maximum sustainable debt-output ratio shock in the periphery.

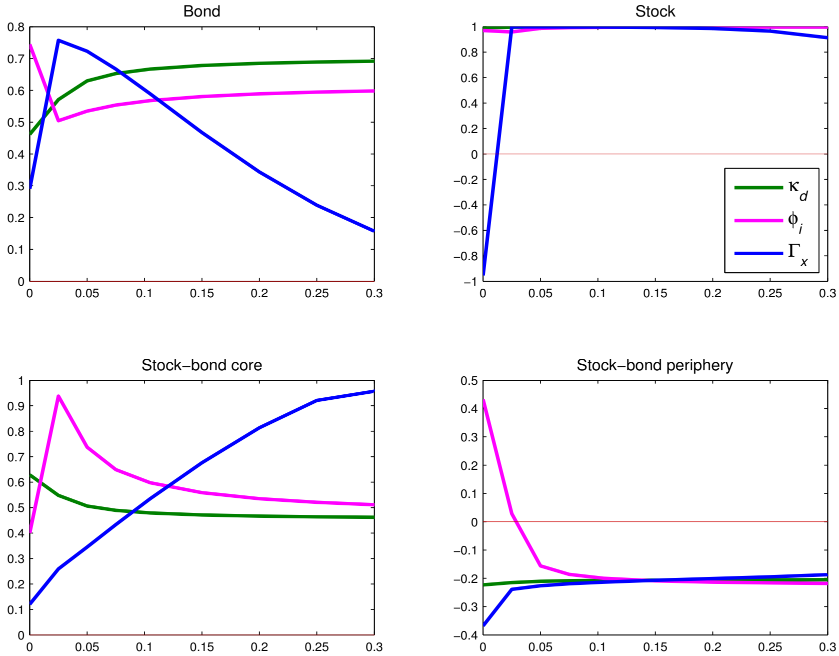


Notes. IRFs show the benchmark model responses for the periphery (diamond line) and the core (solid line). 'Bank excess capital' and 'exchange rate' are common variables to the two regions. Results, in deviation from the steady state, are expressed respectively in percentage points for rates and in percent for the remaining variables.

On the firm side, the key parameters are κ_d and ϕ_i. The first controls the dividend payout. When this parameter increases, for the firm it is more costly to adjust dividends that become less volatile, less dividend are paid to households and as a consequence the firm needs less loans to sustain its activity. R^l decreases less, and eventually it increases along with a decrease in loans. Qualitatively, different values of κ_d do not change the stock-bond correlation behaviour. The parameter ϕ_i instead controls the volatility of investments. With lower values of this parameter the marginal return on capital in the core country temporarily increases driving higher equity prices, more investments and lower dividend payouts. As a consequence the returns on equity increase in the core. In the periphery, as adjusting investments is less costly, entrepreneurs substitute investments for loans that are used to fulfill the demand of dividend payouts. In this case, the increase of dividend payouts overcomes the decrease in equity prices and the return on equity also increases determining a positive correlation of stock and bonds in the periphery.

On the bank side, the key parameter is Γ_x governing the costs the bank has to face when adjusting its balance sheet exposure. In the absence of this friction (Γ_x=0.0001), asymmetric shocks imply that the core and periphery country face two opposite cycles and an asymmetric impact on asset markets. Without constraints, the bank freely makes an arbitrage across assets determining an increase in the loan-deposit spread in the periphery country and a decrease in the core that boosts core investment, equity returns and output. This behaviour determines a negative correlation of stocks between the core and the periphery. For higher values of Γ_x the bank substitutes more risky for risk free bonds. As the flight to quality between bonds becomes more important, it determines lower values of the bond correlation and

Figure 4.3: Correlation sensitivity to key parameters



Notes. The graph shows the evolution of correlations (y axis) to the change in the value of key parameters (x axis) for the different asset markets (bond, stock and regional stock-bond markets). κ_d and ϕ_i affect the choices of the entrepreneurs and monitor, respectively, the behaviour of dividend payout and investment adjustment. Γ_x affects the bank choices as it changes the cost of leverage and the balance sheet composition.

higher values of the core stock-bond correlation. Figure 4.3 shows these behaviours.

4.3 Empirical simulations

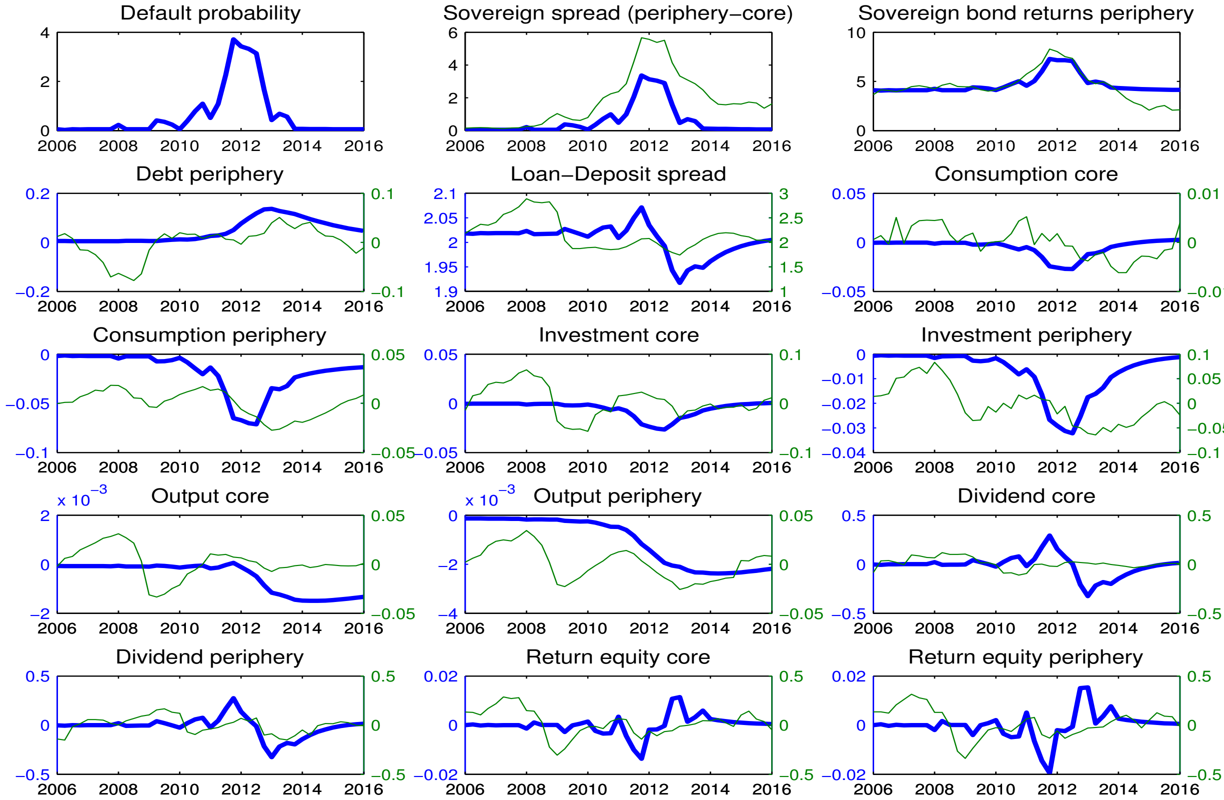
Figure 4.4 shows the responses of the model to simulations using the sovereign risk process estimated from the data as the shock. The model matches the behaviour of sovereign interest rates and the dynamics of debt during the sovereign debt crisis.[30] Additionally, it explains a part of the dynamics at the level of financial intermediaries (around 50% of loan-deposit spread increase) and the corresponding transmission to i) the real economy and ii) the equity market.

We observe that a sovereign risk shock determines a decrease in investments both in the core and in the periphery that can explain respectively the 10% and 50% of the decrease in investments in the data. For output the decrease explains around 7% of the one in the data. On the asset market side, we see that a sovereign risk shock can explain around the 8% and 10% of equity returns' behaviour, respectively in the core and the periphery, during the sovereign debt crisis.

Table 3 compares theoretical and simulated moments with the correlations in the data for the different asset markets. As we can see, the model can reproduce the behaviour of the periphery and core stock and bond correlations and the increase in the divergence in core-periphery correlations between stock and bond's returns as of 2010, during the sovereign debt crisis.

[30]The quantified impacts are the reaction of the model with respect to data counted from peak to trough in the period 2010-2014.

Figure 4.4: Empirical simulation



Notes. The graph shows the simulated series (left axes) compared with actual data (right axes). Interest rates are expressed in percent. All the remaining variables are expressed in percentage points deviation from the steady state for the simulated data and actual data (logged and HP filtered).

Data source: Author's calculations

Table 3: Correlation data versus the model

	Model			Data	
	Theoretical	Simulations			
		2010-2014	2000-2016	2010-2014	2000-2016
corr($R^{b,p}$,$R^{b,c}$)	0.64	-0.23	0.11	0.27	0.70
corr($R^{S,p}$,$R^{S,c}$)	0.99	0.99	0.99	0.70	0.76
corr($R^{b,c}$,$R^{S,c}$)	0.54	0.87	0.78	0.12	0.16
corr($R^{b,p}$,$R^{S,p}$)	-0.19	-0.40	-0.31	-0.38	-0.09
corr(Sb, c) - corr(Sb, p)	0.73	1.27	1.10	0.50	0.26

$R^{b,j}$ is the gross return on sovereign bonds in country $j \in \{c, p\}$ and $R^{S,j}$ the gross return on equity. Corr(Sb, c)-corr(Sb, p) is the difference between the stock-bond correlation in the core (corr($R^{b,c}$,$R^{S,c}$)) and the periphery (corr($R^{b,p}$,$R^{S,p}$)) The column "Theoretical" presents the theoretical moments after a 1% sovereign risk shock, the column "Simulations" presents the results of our empirical simulations with the identified sovereign risk shock for the sovereign debt period and the whole sample, in line with the "Data" column. A sovereign risk shock is a negative shock to the maximum sustainable level of debt in the periphery country.

5 Conclusion

This paper studies, from a macroeconomic perspective, the behaviour of euro area asset market comovements and, in particular, the divergence in the core and periphery stock-bond correlations during the period 2010-2014. We estimate sovereign risk from the data and show that it is able to reconcile the empirical evidence.

A sovereign risk shock determines a shift in holdings from risky to risk-free sovereign bonds, the so called flight-to-quality, that determines an increase in periphery bond returns and a decrease in core ones. On the equity market, stock returns decrease as the expected value of future dividends falls due to a decrease in credit and investments. The disruption of the sovereign bond market and the high correlation on the stock one ultimately impact in a different way on the core and periphery stock-bond correlations. In the core the correlation is positive. After a sovereign risk shock, on the one hand the interest rate on bonds is lowered by the flight-to-quality towards this asset; on the other hand, the equity returns decrease as firms are affected by a credit crunch that impacts negatively on investment and output. In the periphery the correlation turns instead to be negative as sovereign returns spike -given the higher risk- whereas stock returns decrease as explained above.

Quantitatively, we find that feeding a sovereign risk shock to the model not only can reproduce the stylized facts on asset markets but also account for 50% of the increase in sovereign and loan-deposit spreads, 10% in the decrease in core investments, 50% of the decrease in periphery investments and 7% of the decrease in core and periphery output during the period of the sovereign debt crisis.

This paper is a first attempt to study the impact of a sovereign risk shock on euro area asset markets together with the underlying macroeconomic dynamics. Future research should focus more on the bank representation as well as on its regulation. Moreover, a closer look at the central bank's dimension and monetary policy would be worthy. Finally, it could be of interest to add to the dimension of asset markets by including bank and corporate debt in the model. On the firm side, introducing corporate debt and allowing for new issuing of equity would refine the capital choice of firms and add new insights to the corporate finance literature.

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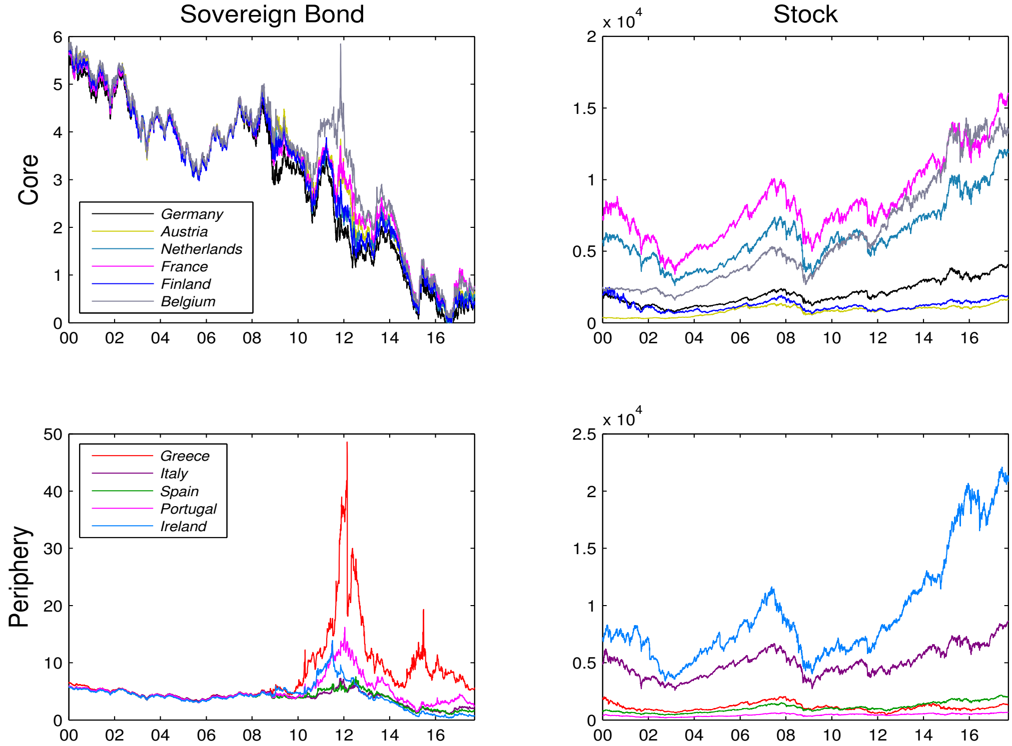
A Individual and aggregated return series

Figure A.1 presents the disaggregated behaviour of sovereign bond and stock markets in the euro zone for the period 2000Q1 to 2017Q3. Legends for the individual countries refer to the rows.

Figure A.1: Behaviour of core and periphery bond and stock returns

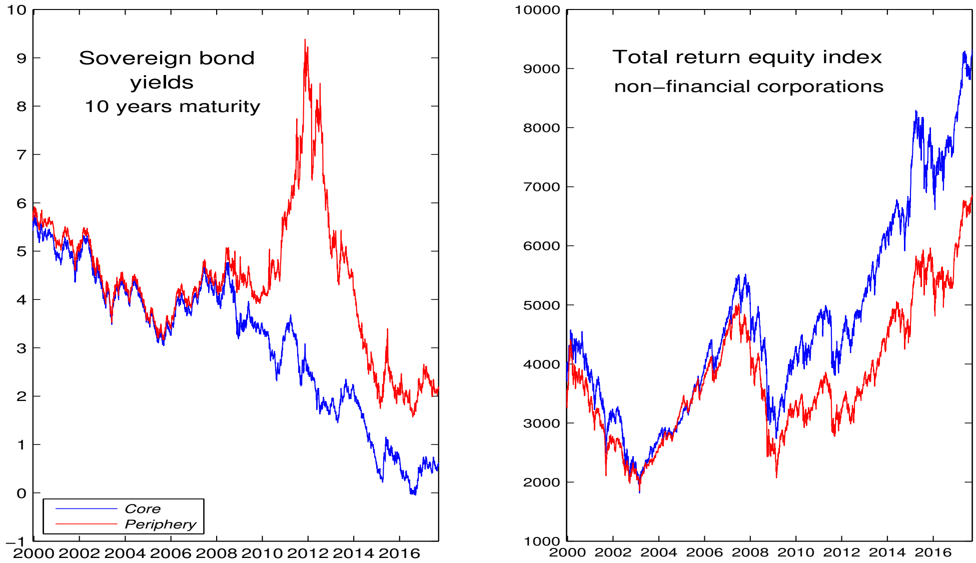


Notes. Stock market series are total return indexes on non-financial firms; bond series are DS benchmark 10 years index of yields to redemption expressed in percentage points.

Data Source: Datastream.

Figure A.2 presents the aggregated behaviour of these series. The series are aggregated at the core and periphery level by weighted average based GDP values of Q4 2004.

Figure A.2: Returns in the euro zone stock-bond markets



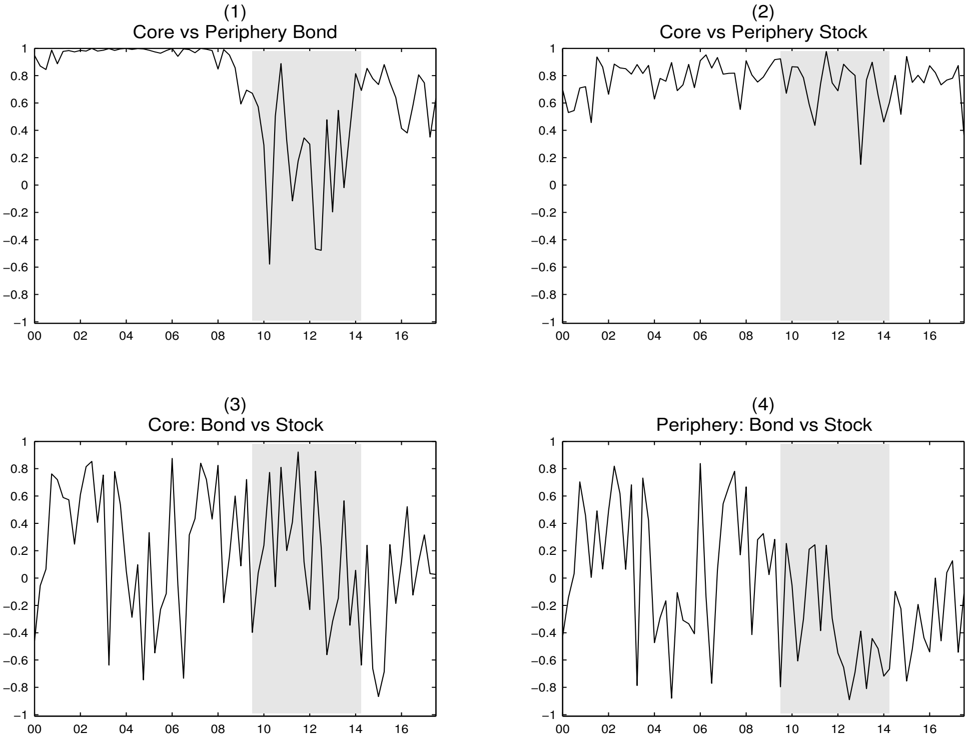
Notes. Stock market series are total return indexes on non-financial firms; bond series are DS benchmark 10 years index of yields to redemption expressed in percentage points. Countries belonging to the core are: Austria, Belgium, Finland, France and Germany. Whereas countries belonging to the periphery are: Greece, Ireland, Italy, Portugal and Spain. The series are aggregated at the core and periphery level as a weighted average based on the value of GDP for 2004 Q4.

Data source: Datastream and author's calculations.

B Quarterly correlations

Figure B.1 presents the aggregated behaviour of the different correlations. The series are aggregated using GDP values of Q4 2004. The realized correlations are computed on quarterly windows and show the dynamic relations between the returns on stock and the yields on sovereign bonds within the euro zone over the period 2000Q1 to 2017Q3.

Figure B.1: Realized correlations in the euro zone stock-bond markets



Notes. Stock market series are total return indexes on non-financial firms; bond series are DS benchmark 10 years index of yields to redemption. Countries belonging to the core are: Austria, Belgium, Finland, France and Germany. Whereas countries belonging to the periphery are: Greece, Ireland, Italy, Portugal and Spain. The series are aggregated at the core and periphery level as a weighted average based on the value of GDP for Q4 2004. The realized correlations are computed on quarterly windows and show the dynamic relations between the returns on stock and the yields on sovereign bond within the euro zone over the period 2000Q1 to 2017Q3. The shaded areas highlight the period of the sovereign debt crisis.

Data source: Datastream and author's calculations.

C Sovereign risk shock

In this appendix we present a detailed explanation of the methodology used to estimate the sovereign risk shock. The shock we identify is the part of periphery sovereign bond yield' spreads that can not be explained by macroeconomic fundamentals. This part of risk is what we call the sovereign risk shock and it is what we introduce in the model as a change in the maximum sustainable level of debt (see section 2.5). Data for the yield series are taken from Datastream for the 10 years central government bonds. The $Yield$ series used in the estimations are computed as the difference of periphery country yields from the one of Germany[31]. For what concerns the independent variables, $Debt$ data are expressed as percentage of GDP and are taken from the quarterly national account statistics from Eurostat; current account (CA), real effective exchange rate ($REER$), $Growth$ and total tax revenues (used to construct the ratio of government debt to total tax revenues ($Fiscalstance$)) are taken from the OECD database. Periphery countries considered are the GIIPS: Greece, Ireland, Italy, Portugal and Spain over the period 2000Q1-2017Q2.[32]

C.1 Panel estimation

For the panel estimation we follow tightly the procedure used in deGrauwe and Ji (2013). We estimate the following linear panel regression:

$$Yield_{i,t} = \alpha + \beta_1 \, CA_{i,t} + \beta_2 \, Debt_{i,t} + \beta_3 \, REER_{i,t} + \beta_4 \, Growth_{i,t} + \alpha_i + \tau_i + u^b_{i,t} \, , \tag{C.1}$$

and the non-linear regression:

$$\begin{aligned} Yield_{i,t} = \alpha + \beta_1 \, CA_{i,t} + \beta_2 \, Debt_{i,t} + \beta_3 \, REER_{i,t} + \beta_4 \, Growth_{i,t} + \beta_5 \, (Debt_{i,t})^2 + \alpha_i + \tau_i + u^b_{i,t} \, , \\ \text{for } i = \text{Greece, Ireland, Italy, Portugal, Spain;} \\ t = 2000q1, \ldots, 2017q2. \end{aligned} \tag{C.2}$$

Differently from deGrauwe and Ji (2013) we are not interested in assessing the determinants of the yield spread and in attesting whether there was a non-linear relation of the macroeconomic fundamentals before and after the crisis.[33] We are instead interested in the (non-linear) unexplained part of risk, that we isolate in the residuals, after having controlled for the macroeconomic dynamics. The choice of regressors follows deGrauwe and Ji (2013), equation (1) and (2), and additionally controls for quarter fixed effects τ_i. CA is the current account deficit to gdp ratio, $Debt$ stands either for debt-to-GDP ratio or for the fiscal space (the ratio of government debt to total tax revenues), $REER$ is the real effective exchange rate and $Growth$ is the quarter-on-quarter GDP growth rate, α is a constant term and α_i is a country fixed effect.

Table C.1 presents the estimation results for equations (C.1) and (C.2).[34] As in deGrauwe and Ji (2013) current account has a negative sign as it can be interpreted as an increase in the net foreign debt of a country that increases the riskiness. The real effective exchange rate is a measure of competitiveness

[31] An alternative measure is the CDS on the underlying sovereign bonds. Unfortunately the data availability for this variable is limited to the period 2007-2017 that is much shorter than the period considered in this study. For this reason we use yields in deviation from the benchmark risk-free asset (Germany's bonds) on the sample period 2000Q1-2017Q2 as a proxy for the CDS informations.

[32] Data on total tax revenues are at annual frequency and only available up to 2015Q4; Current account data for Greece and Ireland are available as of 2002Q1.

[33] Another difference with respect to deGrauwe and Ji (2013) is the focus only on a subsample of countries they consider as we are interested in calibrating the shock for the periphery of the eurozone.

[34] The data sample has been cleaned from outliers by removing the top and bottom 1 percentile. The data removed are the negative yields for Ireland in the period 2004Q2-2005Q1 and for the very high (positive) values of Greece in the period 2011Q4-2012Q3.

and it can be viewed as an early warning of future troubles of a country turning into a real appreciation. Growth has an impact on the ease of a country to service its debt. As expected the sign is negative although not significant as the quarter fixed effects capture all the volatility of the variable. Debt, in both forms, is positive and significant when taken alone, it becomes negative when interacted with $Debt^2$ as the latter captures the positive but non-linear relation. An increase in debt increases the burden of servicing that higher amount of liabilities and thus it entails an increasing probability of default. As in deGrauwe and Ji (2013) the best specification of the model, in terms of R^2, is column (4) from which we take the residuals to compute the sovereign risk shock.[35]

Table C.1: Alternative panel estimations

Variables	(1) Yield	(2) Yield	(3) Yield	(4) Yield
CA	-0.0686	-0.1043**	0.0692	0.0227
	(0.03540)	(0.0239)	(0.08111)	(0.0554)
REER	-2.3068	-3.0184	-5.900	-6.4211
	(5.9467)	(3.5291)	(5.9180)	(3.3614)
Growth	-0.5005	-0.4490	-0.5463	-0.4084
	(0.3759)	(0.3377)	(0.4040)	(0.2695)
Debt/GDP	0.0712**	-0.0191*		
	(0.0162)	(0.0072)		
Debt/GDP squared		0.0004**		
		(0.0000)		
Fiscal space			1.7847**	-4.3725**
			(0.3988)	(1.0026)
Fiscal space squared				1.1583**
				(0.1768)
Country FE	yes	yes	yes	yes
Quarter FE	yes	yes	yes	yes
Observations	296	296	296	296
R-squared	0.3613	0.4320	0.4075	0.5401
Number of i	5	5	5	5

Errors are cluster at country level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

C.2 Sovereign risk shock construction

Figure C.1 shows the disaggregated residuals. As the panel is not balanced, the length of the series of residuals differ from country to country. We account for this by using a changing composition of the periphery shock measure weighted by GDP.[36] To pass from the aggregated series of residuals to the model shock we need to reconstruct the series u_t^b in equation (2.21). The residuals of the panel can be interpreted as the percentage of yields' spread that is not explained by fundamentals, as shown in Figure 4.1. To introduce the shock in the model we look for the shock magnitude that can reproduce such a spread's behaviour. First, we need to reconstruct the series of maximum debt level by using equation (2.21) rearranged in the following way:

[35] As the sign and magnitude of coefficients remains quite stable across the different estimations, we use the specification that explains $Yields$ the most in terms of R^2.

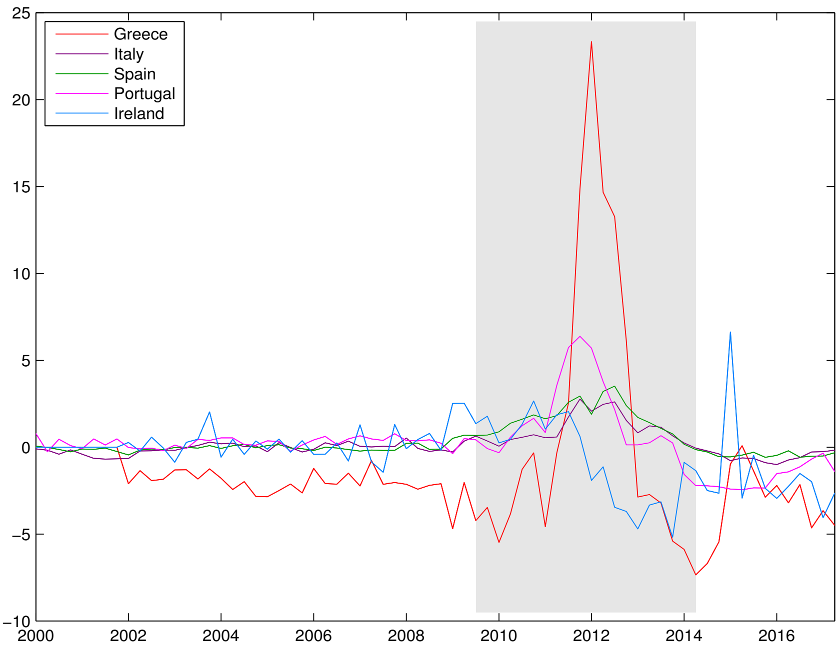
[36] As for the rest of the calibration, the measure used for aggregation is Q4 2004 GDP values.

$$BY_t^{max} = \frac{B_t}{4Y_t} - norminv(\epsilon_t^p, 0, \sigma^2) \qquad \text{(C.3)}$$

where $\frac{B_t}{4Y_t}$ is aggregated periphery debt to gdp data, ϵ_t^p is the series of aggregated residuals and BY_t^{max} is the maximum sustainable level of debt. We estimate this equation with data for $\frac{B_t}{4Y_t}$ in order to reconstruct the maximum sustainable level of debt series as implied by the probability of default with a changing debt composition. In the data the distance between $\frac{B_t}{4Y_t}$ and BY_t^{max} reduces during the sovereign debt crisis due to a slower increase in the maximum sustainable level of debt with respect to the actual burden of debt to gdp. Once we have reconstructed the series of BY_t^{max} we compute γ_b as the autocorrelation of BY_t^{max} before the beginnig of the sovereign debt crisis.

We then simulate the entire general equilibrium model introducing the value of ϵ_t^p as exogenous in order to find the corresponding value of u_t^b, the shock to the maximum sustainable level of shock (equation (2.22)). Figure C.2 shows the corresponding shock behaviour.

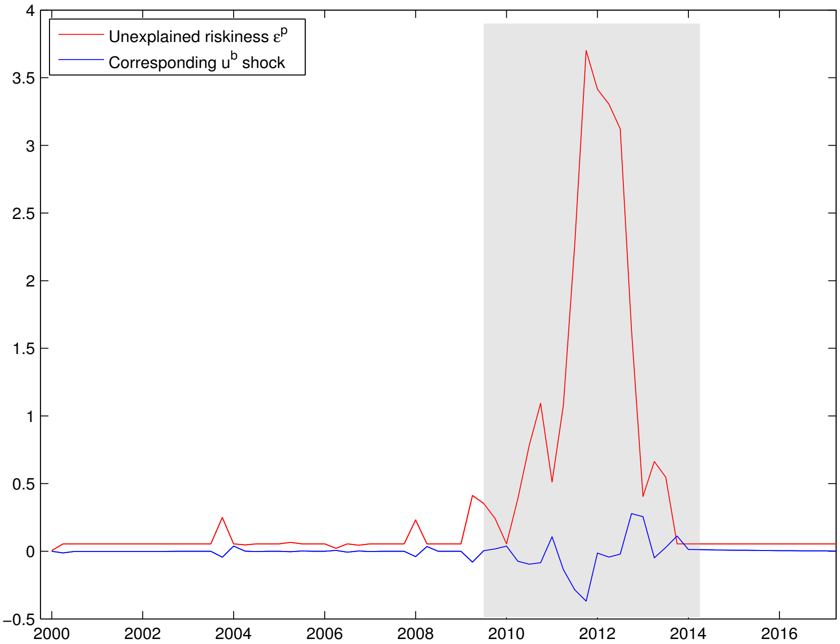
Figure C.1: Residuals of the panel estimation - unexplained risk -



Notes. The graph shows the unexplained part of sovereign yield spreads (with respect to Germany) expressed in percentage points after the panel regression explained in section C.1. Greece and Ireland data start in 2002 q1 (the missing part is set at zero). The shaded areas highlight the period of the sovereign debt crisis.

Data source: Author's calculations

Figure C.2: Aggregated unexplained riskiness and corresponding shock



Notes. The graph shows the aggregated series of unexplained riskiness expressed in percent and computed as explained in section C.1, and the corresponding maximum sustainable debt level shock. The shaded areas highlight the period of the sovereign debt crisis.

Data source: Author's calculations

D Model details

The model is an international business cycle model for the euro area. It consists of two regions: the *core* and the *periphery* that we denote, respectively, by c and p hereafter. The model features an international banking sector as in Enders et al. (2011), an equity market and a probability of default on sovereign debt as in Corsetti et al. (2013). The two regions are perfectly symmetric except for a higher level of debt to output in the periphery country.

D.1 Households

In each country $j \in \{c, p\}$, the representative household may consume C_t^j, invest D_t^j in one-period bank deposits or b_t^j in one-period sovereign debt. Moreover households can invest in financial markets both in domestic and foreign equity $S_{i,t}^j$ issued by the respective firms-capital producers $i \in \{c, p\}$.[37] By investing in deposits the households obtain $R_{t-1}^{d,j}$, the predetermined gross return on deposits. The expected gross return on sovereign bond is $R_{t-1}^{b,j}$ while the actual net return is $R_{t-1}^{b,j} - \epsilon_t^j$, where $\epsilon_t^j \geq 0$ captures the share of outstanding sovereign debt lost by households because of – partial – sovereign default.

The expected return on equity holdings is given by the price at which households can sell the share $\rho_{s,t}^j$ bought in the previous period and the dividend payout div_t^j coming from the capital producers. $Q_{j,t}^i$ is the real exchange rate of country i when country j is taken as the numeraire. The household also supplies h_t^j hours to the firms and receives wages w_t^j. Moreover, it owns the firms located in j and receives their profits Υ_t^j. Finally, the household receives a lump-sum transfer $H_t^{h,j}$ from the government and must pay taxes T_t^j as well as a quadratic portfolio adjustment cost on sovereign debt represented by the parameter $\phi_b > 0$ in equation (D.1). Additionally, the households pay a cost related to their equity holdings represented by the parameter $\phi_s > 0$. The household's budget constraint is:

$$\begin{aligned} C_t^j + D_t^j + b_t^j + \sum_i Q_{j,t}^i \rho_{i,t}^s S_{i,t}^j + \frac{\phi_s}{2} \sum_i Q_{j,t}^i \rho_{i,t}^s \left(S_{i,t}^j\right)^2 + \frac{\phi_b}{2}(b_t^j - \bar{b}^j)^2 \\ = w_t^j h_t^j + R_{t-1}^{d,j} D_{t-1}^j + (R_{t-1}^{b,j} - \epsilon_t^j) b_{t-1}^j + \sum_i Q_{j,t}^i (\rho_{i,t}^s + div_{i,t}) S_{i,t-1}^j \\ + \Upsilon_t^j + H_t^{h,j} - T_t^j. \end{aligned} \tag{D.1}$$

Throughout the paper, $\bar{z}$ represents the steady state of any variable z_t. The household's expected lifetime utility at date s is:

$$\max E_s \sum_{t=s}^{\infty} \beta^{t-s} \left(\ln \left(C_t^j - \psi_n \frac{(h_t^j)^{\eta+1}}{\eta + 1} \right) + \psi_d \ln D_t^j \right) \tag{D.2}$$

$0 < \beta < 1$ is the subjective discount factor, η is the inverse of the intertemporal elasticity of labour supply and $\psi_n, \psi_d > 0$ are parameters.[38] The household maximizes (D.2) subject to (D.1). It gives the following first order conditions (FOCs):

$$\psi_n (h_t^j)^\eta = w_t^j \ , \tag{D.3}$$

[37] Households are assumed to only invest in domestic bonds in order to reproduce the existence of home bias in sovereign debt holdings. The cross-country holdings of sovereign bonds are instead mainly held by banks.

[38] The choice of a GHH utility function is motivated by the international framework. The absence of wealth effect on the labour supply helps to match a series of empirical regularities as explained by Garcia-Cicco et al. (2010), Raffo (2008) and Schmitt-Grohe and Uribe (2012). Moreover, the results of the paper are not changed by using a different utility formulation (KPR for instance).

We introduce deposits in the utility function in order to be able to pin down the core and periphery deposit's Euler equations as in Enders et al. (2011). Nevertheless with this formulation deposits play the role of real value for cash. An increase in deposits increases the means of payments of the households and (ceteris paribus) increases their consumption.

$$\lambda_t^j = \frac{\psi_d}{D_t^j} + E_t \beta \lambda_{t+1}^j R_t^{d,j} \ , \tag{D.4}$$

$$\lambda_t^j \left(1 + \phi_b(b_t^j - \bar{b^j})\right) = E_t \beta \lambda_{t+1}^j (R_t^{b,j} - \epsilon_{t+1}^j) \ , \tag{D.5}$$

$$Q_{j,t}^i \lambda_t^j (1 + \phi_s S_{i,t}^j) = E_t \beta \lambda_{t+1}^j (R_{i,t+1}^S) Q_{j,t+1}^i \ , \tag{D.6}$$

$$\lambda_t^j = \left(C_t^j - \psi_n \frac{(h_t^j)^{\eta+1}}{\eta+1} \right)^{-1} . \tag{D.7}$$

Equation (D.3) shows that the wage is equal to the marginal disutility of hours worked. Equations (D.4), (D.5) and (D.6) state that, at equilibrium, marginal costs are equal to expected marginal income from, respectively, deposits, sovereign bonds and equity. Equation (D.6) represents the FOCs for equity holding for country j households with respect to country $i \in \{c, p\}$, $i \neq j$ issuer. The expected real return on equity is:

$$E_t[R_{i,t+1}^S] = \frac{E_t[\rho_{i,t+1}^s] + E_t[div_{i,t+1}] + u_t^s}{\rho_{i,t}^s}. \tag{D.8}$$

These returns are defined as the change in price plus the dividend payout. u_t^s is an i.i.d. shock to the expected returns. An increase (decrease) of u_t^s mimics overly optimistic (pessimistic) expectations on equity returns. It can be interpreted as a noise component, a subjective belief, that makes expectations on future returns detach from their fundamental values.[39]

Comparing equations (D.5) and (D.6) we can analyse the relation between the sovereign bond and equity rates in the households' portfolio. Let us define $R_t^{nb,j}$ as the net return on sovereign bonds. As we assume that only the periphery country can default, net returns are respectively given by

$$R_t^{nb,p} = R_t^{b,p} - E_t[\epsilon_{t+1}^p] \ , \tag{D.9}$$
$$R_t^{nb,c} = R_t^{b,c}. \tag{D.10}$$

Abstaining from adjustment costs and price dynamics, the relation between the sovereign bond and equity rates is the following:

$$E_t[R_{i,t+1}^S] = R_t^{nb,j} + \lambda_t^j \phi_s S_{i,t}^j. \tag{D.11}$$

Equation (D.11) shows that the two assets are not perfect substitute. There are two sources of differentiation: sovereign debt default and the cost associated to equity holdings. Changes in the amount of shares bought reduces the correlation between equity and sovereign returns. The more the shares held, the higher the return demanded by the households in order to hold such an asset. Analogously, periphery default on sovereign debt determines a wedge between the return on equity and periphery sovereign bonds.

D.2 Capital producers

The capital producers in country $j \in \{c, p\}$ have the choice of financing either via one-period loans from the bank or through asset markets in the form of equity. They may payout dividends, div_t^j, to the households or invest I_t^j in domestic firms. In turn, investment increases firms' capital stock K_t^j according to the following law of motion:

[39] For a detailed description of the financial expectation shock see Section 4.

$$K_t^j = (1-\delta)K_{t-1}^j + I_t^j, \tag{D.12}$$

where $0 < \delta < 1$ is the capital depreciation rate. Capital provides a net real return r_t^j and capital producers pay a gross nominal interest rate $R_{t-1}^{l,j}$ on loans, as well as an adjustment cost on investment represented by the parameter $\phi_i > 0$.[40] If the capital producers decide to pay out dividends they face an adjustment cost represented by the parameter κ_d. As in Jermann and Quadrini (2012) the equity payout cost can be interpreted as a pecuniary cost as well as a way to model the speed of fund's adjustment when financial conditions change. The capital producers' budget constraint is:

$$div_t^j + I_t^j + \frac{\phi_i}{2}(I_t^j - \bar{I}^j)^2 + \frac{\kappa_d}{2}(div_t^j - \bar{div}^j)^2 + R_{t-1}^{l,j}L_{t-1}^j = L_t^j + r_t^j K_{t-1}^j. \tag{D.13}$$

As equity shares are held internationally, the capital producers are owned by the households of both the core and the periphery country. They maximize:

$$\max E_s \sum_{t=s}^{\infty} (\beta^{e,j})^{t-s}\; div_t^j$$

subject to (D.12) and (D.13). With β^e being the time varying weighted average of the discount factors of the core and periphery households, expressed in terms of the capital producers' domestic price index:

$$\begin{aligned}
\beta^{e,c} &= \beta\left(S_{c,t}^c\left(\frac{\lambda_{t+1}^c}{\lambda_t^c}\right) + S_{c,t}^p\left(\frac{Q_{c,t+1}^p}{Q_{c,t}^p}\frac{\lambda_{t+1}^p}{\lambda_t^p}\right)\right), \\
\beta^{e,p} &= \beta\left(S_{p,t}^p\left(\frac{\lambda_{t+1}^p}{\lambda_t^p}\right) + S_{p,t}^c\left(\frac{Q_{c,t}^p}{Q_{c,t+1}^p}\frac{\lambda_{t+1}^c}{\lambda_t^c}\right)\right).
\end{aligned}$$

As equity shares are held internationally the discount factor of capital producers accounts for the relative importance of each owner's marginal utility. The weights are set according to the time-varying amount of shares each household holds of one country's capital producers. The first order conditions for this problem read:

$$\lambda_t^{e,j} = E_t \beta^{e,j} \lambda_{t+1}^{e,j} R_t^{l,j}\;, \tag{D.14}$$

$$\lambda_t^{e,j} q_t^j = E_t \beta^{e,j} \lambda_{t+1}^{e,j}\left(r_{t+1}^j + (1-\delta)q_{t+1}^j\right)\;, \tag{D.15}$$

$$I_t^j = \bar{I}^j + \frac{1}{\phi_I}(q_t^j - 1)\;, \tag{D.16}$$

$$\lambda_t^{e,j} = \frac{1}{1 + \kappa_d(div_t^j - \bar{div}^j)}\;, \tag{D.17}$$

where $\lambda_t^{e,j}$ is the Lagrangian multiplier associated to the capital producers' budget constraint.

Equation (D.14) says that, at equilibrium, the marginal income from loans is equal to the expected marginal cost weighted by the households discount factor. Equation (D.15) defines the shadow value of capital, q_t^j, as the expected discounted value of the marginal profits of having one additional unit of capital. If $q_t^j < 1$, meaning that the shadow value of capital is smaller than the price of capital, equation (D.16) states that investments should decline, if $q_t^j > 1$ that investments should increase.[41] Furthermore, from equation (D.15), we see that the shadow value of capital increases when the expected future dividend payouts are lower than the actual ones.

[40] A convex adjustment cost on investment is common in the literature as it helps to match empirical behaviour of aggregate investment and prevents the investment demand curve to be perfectly elastic. For the early literature that assumes this cost see Gould (1968) and Lucas (1967) among others.

[41] This formulation of the investment equation follows Tobin's Q theory of investment (Tobin, 1969).

D.3 Nonfinancial firms

In each country $j \in \{c, p\}$ firms are perfectly competitive. The intermediate j firm produces a good that is sold in the domestic country as well as in the foreign one. A final firm in each country combines the intermediate goods from the j and $-j$ countries into a final one.

Final firms

In each region the demand for goods is a composite of the home and foreign intermediate goods. The aggregate demand for country j is:

$$A^j = \left(\frac{A^j_j}{1-\alpha}\right)^{(1-\alpha)} \left(\frac{A^j_{-j}}{\alpha}\right)^{\alpha} , \tag{D.18}$$

where A^j_j and A^j_{-j} are respectively the demands of the final firm j for goods j and $-j$. $0 < 1-\alpha < 1$ is the degree of home bias or, alternatively, it can be interpreted as the index of country openness. We set this parameter to be $0 < \alpha < 0.5$ implying a certain degree of home bias. The composite final good can be used for consumption and investment by all the agents in the economy.[42] The optimal demand for each variety of the final good is given by the following first order conditions:[43]

$$\begin{aligned} A^c_{c,t} &= (1-\alpha)\frac{1}{\phi^c_t}A^c_t\ , \qquad & A^p_{p,t} &= (1-\alpha)\frac{1}{\phi^p_t}A^p_t\ , \\ A^c_{p,t} &= \alpha\frac{1}{Q^p_{c,t}\phi^p_t}A^c_t\ , \qquad & A^p_{c,t} &= \alpha\frac{Q^p_{c,t}}{\phi^c_t}A^p_t\ . \end{aligned}$$

The welfare based price index (for both regions) corresponding to these preferences is:

$$P^j_t = (p^j_{j,t})^{(1-\alpha)}(p^j_{-j,t})^{\alpha}. \tag{D.19}$$

Dividing by P^j, and by the law of one price, the price index can be simplified as:

$$\begin{aligned} 1 &= \phi^j_t\phi^{-j}_t\ , \\ Q^{-j}_{j,t} &= (\phi^j_t)^{\frac{2\alpha-1}{\alpha}}\ , \end{aligned}$$

with $\phi^j_t = \frac{p^j_{j,t}}{P^j_t}$ being the share of domestic produced goods' prices in the domestic price index and $Q^p_{c,t} = \frac{e_t P^p_t}{P^c_t}$ being the real exchange rate for the core country. The nominal exchange rate e_t is set to 1 as the two economies belong to the same currency union.

Intermediate firms

There is a competitive non financial sector in the economy which produces a tradable good under a Cobb-Douglass production function. The inputs are capital and labour rented respectively from capital producers and households. The maximization problem of the firms reads:

$$\max\ \Upsilon^j_t$$

[42] We assume that the same Cobb-Douglas CES aggregator applies to the consumption bundles of all the agents as well as for investment. As a consequence the price index for consumption and investment is the same. For the choice of the CES function we follow Gali and Monacelli (2008).

[43] Optimal demands are the solution of the final firm maximization problem: $\{A^j_{j,t}, A^j_{-j,t}\}^{\infty}_{t=0}$ to maximize $P^j_t A^j_t - p^j_{j,t}A^j_{j,t} - p^j_{-j,t}A^j_{-j,t}$.

$$\text{s.t. } \Upsilon_t^j = \phi_t^j Y_t^j - w_t^j h_t^j - r_t^j K_{t-1}^j \,, \tag{D.20}$$
$$Y_t^j = Z_t^j \left(K_t^j\right)^{\mu} \left(h_t^j\right)^{1-\mu} \,, \tag{D.21}$$

where Z_t^j represents total factor productivity and $0 < \mu < 1$ is the elasticity of output to capital. The first order conditions for this maximization problem equate the marginal productivity of factors with their marginal cost:

$$r_t^j = \mu \frac{\phi_t^j Y_t^j}{K_{t-1}^j} \,, \tag{D.22}$$
$$w_t^j = (1-\mu) \frac{\phi_t^j Y_t^j}{h_t^j} \,. \tag{D.23}$$

One source of aggregate risk in this model comes from the total factor productivity Z_t^j:

$$Z_t^j = \left(Z_{t-1}^j\right)^{\gamma_z} exp(u_t^z) \,, \tag{D.24}$$

that is represented as a stochastic autoregressive process with $0 < \gamma_z < 1$, and u_t^z i.i.d.

D.4 Banking sector

The banking sector is represented by an international and perfectly competitive bank *à la* Enders et al. (2011). The bank is located in the core but trades with all countries $j \in \{c, p\}$. It collects deposits D_t^j from households and can invest in sovereign bonds s_t^j as well as provide loans L_t^j to the firms in both regions. The bank maximizes its consumption, its profits, over the two regions. The bank faces a capital requirement having to set aside a fraction $0 < \gamma < 1$ of loans as own capital. The bank can deviate from legal requirements ($x_t = 0$) but this is costly. The bank's balance sheet constraint is:

$$(1-\gamma) \sum_j Q_{c,t}^j L_t^j + \sum_j Q_{c,t}^j s_t^j = \sum_j Q_{c,t}^j D_t^j + x_t \,. \tag{D.25}$$

The bank budget constraint is:

$$\begin{aligned} &\sum_j Q_{c,t}^j C_t^{b,j} + \sum_j Q_{c,t}^j R_{t-1}^{d,j} D_{t-1}^j + \sum_j Q_{c,t}^j L_t^j + \sum_j Q_{c,t}^j s_t^j + \Gamma_d \sum_j (D_t^j - \bar{D}^j) \\ &\quad + \frac{\Gamma_l}{2} \sum_j (L_t^j - \bar{L}^j)^2 + \frac{\Gamma_x}{2} (x)^2 \\ &= \sum_j Q_{c,t}^j D_t^j + \sum_j Q_{c,t}^j R_{t-1}^{l,j} L_{t-1}^j + \sum_j Q_{c,t}^j (R_{t-1}^{b,j} - \epsilon_t^j) s_{t-1}^j + \sum_j Q_{c,t}^j H_t^{b,j} \,. \end{aligned} \tag{D.26}$$

The bank pays a real return $R_{t-1}^{d,j}$ on deposits, it receives $R_{t-1}^{l,j}$ on loans and $R_{t-1}^{b,j}$ on sovereign bonds. Sovereign bonds are risky assets as government can default on them with a probability ϵ_t^j. The bank might receive a lump-sum transfer $H_t^{b,j}$ from the government. Moreover, the bank faces different types of costs: operational costs on deposits as in Enders et al. (2011), captured by Γ_d; adjustments costs on loans, Γ_l, as in Guerrieri et al. (2012); and the cost of deviating from the legal requirement that, following Enders et al. (2011), we capture by $\Gamma_x > 0$.

The bank utility is:

$$\max E_s \sum_{t=s}^{\infty} \beta^{t-s} (C_t^{b,c})^{\vartheta} (C_t^{b,p})^{1-\vartheta} \tag{D.27}$$

where ϑ is the share of consumption goods from country c in the utility that we set to 0.5 such that the bank consumes its profits equally in the two regions. The bank maximizes (D.27) with respect to (D.25) and (D.26). The first order conditions are:

$$\lambda_t^b = \vartheta \frac{(C_t^{b,c})^{\vartheta}(C_t^{b,p})^{1-\vartheta}}{C_t^{b,c}} \ , \tag{D.28}$$

$$Q_{c,t}^p = \frac{1-\vartheta}{\vartheta}\frac{C_t^{b,c}}{C_t^{b,p}} \ , \tag{D.29}$$

$$\lambda_t^b \left(Q_{c,t}^j - \Gamma_d + Q_{c,t}^j \Gamma_x x_t \right) = \beta E_t \lambda_{t+1}^b Q_{c,t+1}^j R_t^{d,j} \ , \tag{D.30}$$

$$\lambda_t^b \left(Q_{c,t}^j + \Gamma_l (L_t^j - \bar{L}^j) + (1-\gamma) Q_{c,t}^j \Gamma_x x_t \right) = \beta E_t \lambda_{t+1}^b Q_{c,t+1}^j R_t^{l,j} \ , \tag{D.31}$$

$$\lambda_t^b \left(Q_{c,t}^j + Q_{c,t}^j \Gamma_x x_t \right) = \beta E_t \lambda_{t+1}^b Q_{c,t+1}^j (R_t^{b,j} - \epsilon_{t+1}^j) \ . \tag{D.32}$$

Equation (D.29) shows that the ratio of consumption of the bank for the two regions depends on the ratio of relative price indexes. Equations (D.30), (D.31), (D.32) represent respectively the Euler equation for deposits, loans and sovereign bonds.

D.5 Government

The government consumption in each region $j \in \{c, p\}$, G^j, is financed via lump-sum taxes, T_t^j, from the households, as well as via public debt, B_t^j, according to:

$$\begin{aligned} G^j + H_t^{h,j} + H_t^{b,j} + (R_{t-1}^{b,j} - \epsilon_t^j) B_{t-1}^j &= B_t^j + T_t^j \ , && \text{(D.33)} \\ T_t^j &= \bar{T} + \tau (B_t^j - \bar{B}) \ . && \text{(D.34)} \end{aligned}$$

Moreover, the government may transfer $H_t^{h,j}$ to the households and $H_t^{b,j}$ to the bank. Both for the tax rule and the transfer specification we follow Corsetti et al. (2013). As estimated by Bohn (1998), taxes react positively to the increase in debt such as to stabilize it. This implies that the government cannot finance public expenditure only via debt.[44] Equation (D.33) also shows that sovereign default may happen through the term $0 \leq \epsilon_t^j \leq 1$. Everything else equal, a strictly positive ϵ_t^j reduces the stock of sovereign debt in the next period. Finally we define public expenditures as a fixed fraction, G^j, of debt at any period.

Default

To determine the default rate ϵ_t^j we tightly refer to the methodology used by van der Kwaak and van Wijnbergen (2014) by introducing an exogenous fiscal limit for the economy. Behind this limit there is the intuition that there exists a maximum level of taxes that can be raised before the economy becomes politically unstable. This translates, through equation (D.34), into a maximum level of sovereign debt-to-output ratio BY_t^{max} that the government is able to service. We moreover assume that this maximum *sustainable* level is stochastic and follows:

$$BY_t^{max} = \bar{BY}^{max} + \gamma_b (BY_{t-1}^{max} - \bar{BY}^{max}) + u_t^b \ , \tag{D.35}$$

[44] As the focus of the paper is not on the fiscal dimension we use debt-smoothing lump-sum taxes rather than more complicated distortionary tax schemes.

where $0 < \gamma_b < 1$ is the autoregressive component, and u_t^b is a i.i.d. shock. This stochastic behaviour aims at capturing the uncertainty around political instability in the context of sovereign debt and taxation.[45]

Let us define $\tilde{B}_t^j$ as the level of debt in the economy when no default occurs:

$$G_t^j + R_{t-1}^{b,j}\, B_{t-1}^j = T_t^j + \tilde{B}_t^j\,. \tag{D.36}$$

If this level of debt-to-output $\tilde{B}_t^j/(4Y_t^j)$ is lower (resp. higher) than the maximum sustainable level BY_t^{max}, the government does not (resp. does) default. In other words, we define the default decision Δ_t as:

$$\Delta_t = \begin{cases} 0 & \text{if } \frac{\tilde{B}_t^j}{4Y_t^j} < BY_t^{max} \\ 1 & \text{otherwise} \end{cases} \tag{D.37}$$

This default process Δ_t is a step function that we approximate with the continuous normal cdf:

$$\begin{aligned} \epsilon_t^j &= F\left(\frac{\tilde{B}_t^j}{4Y_t^j} - BY_t^{max}\ ;\ 0\ ,\ \sigma^2\right) \\ &= \Phi\left(\frac{\frac{\tilde{B}_t^j}{4Y_t^j} - BY_t^{max}}{\sigma}\right)\,, \end{aligned} \tag{D.38}$$

where $\sigma > 0$ represents the variance and $\Phi(.)$ is the standard normal cdf. We see that when $\sigma \to 0$, then $\epsilon_t^j \to \Delta_t$. A reduction (resp. increase) in the maximum sustainable level of debt-to-output, through the stochastic shock u_t^b in equation (D.35), increases (reduces) the default rate in the economy. Similarly, a higher (resp. lower) debt-to-output ratio $\tilde{B}_t^j/(4Y_t^j)$ increases (resp. reduces) the default rate in the economy. Agents in the economy observe the current economic conditions and, as a consequence, they form expectations on default according to equation (D.38). If we assume that only the periphery country can default, the difference between the core and the periphery sovereign interest rate -abstracting from other general equilibrium dynamics- is driven by default expectations reflecting low economic growth and high levels of debt (with respect to the fiscal limit) in the periphery country.

Default risk A stochastic shock (negative for instance) to the maximum sustainable level of debt increases default implying a change in the interest rate on bonds as well as a direct loss on the households and bank's portfolio. This shock impacts both on prices (interest rates changes) as on quantities (partial default on the amount of sovereign debt held by agents). To deal with the risk dimension of the shock we want to isolate the price effect from the quantity effect. In order to do so we assume that the government makes transfers to the households ($H_t^{h,j} = \epsilon_t^j b_{t-1}^j$) and the bank ($H_t^{b,j} = \epsilon_t^j s_{t-1}^j$) to compensate the loss. In this way we capture the effect of a change in the interest rate on bonds and abstract from the consequences of the direct wealth loss. The same specification has been used by Corsetti et al. (2013). This procedure is helpful to reproduce the sovereign debt crisis' dynamics in the euro area where only Greece effectively, partially, defaulted.

D.6 Closing the model

Asset market clearing conditions

The sovereign bond market clearing condition for country $j \in \{c, p\}$ is:

$$B_t^j = b_t^j + s_t^j \tag{D.39}$$

[45]In reality, the maximum sustainable government debt level is not exogenous but depends on expected growth rates, on expected growth volatility or on the expected government ability to raise taxes (see for instance Collard et al. (2015)). But this is beyond the scope of this paper.

where b_t^j and s_t^j is the amount of bonds held respectively by the households and the bank.

The equity market clearing condition for country $i \in \{c,p\}$ issuing and country $j \in \{c,p\}$ holding is:

$$1 = S_{i,t}^{j} + S_{i,t}^{-j} \tag{D.40}$$

implying that there is a fixed amount of shares traded in the economy normalized to 1.

Good market clearing condition

Let's define the domestic demand for country j as:

$$A_t^j = C_t^j + C_t^{b,j} + I_t^j + G_t^j + \texttt{costs}_t^j \tag{D.41}$$

where $\texttt{costs}_t^j$ collects all adjustment and operative costs beared by households, capital producers and firms in country j. Moreover, $\texttt{costs}_t^c$ also includes the costs related to the bank.

The good market clearing condition for each region j reads:

$$Y_t^j = A_{j,t}^{j} + A_{j,t}^{-j} \tag{D.42}$$

By summing them up we obtain the resource constraint for the two-country economy:

$$\sum_j \phi_t^j Q_{c,t}^j Y_t^j = \sum_j Q_{c,t}^j A_t^j \tag{D.43}$$

stating that the total production has to be equal to the demand in the whole currency area.