Evaluating the Performance of Dynamic and Tobit Models in Predicting Credit Default

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Abstract

In this paper we analyse the performance of various credit default models in predicting non-performing borrowers and transitions to default. In addition to conventional binary classifiers, which are typically used in practice, we evaluate the performance of two novel methodologies - dynamic and tobit credit risk models. Moreover, we introduce an approach for modelling credit default on quarterly frequency using mixed frequency data. We show that tobit model, where overdue in loan repayment is modelled explicitly, outperforms all the other models. The choice between dynamic and static version of the model depends whether one is interested in predicting state of non-performing borrowers or new defaulters. For the former, the persistence is of a key importance and therefore the dynamic model is the advantageous modelling methodology. For predicting new defaulters, however, static tobit model is shown to outperform all other models in terms of true positive rate by a large margin. Our results show that the prevailing credit risk methodologies can be significantly improved by including the dynamics and choosing the tobit functional form. This is especially pronounced for conventional default probability model that is typically used by banks and regulators and is shown to have very low classification accuracy. A number of robustness checks confirm the validity of the results.

JEL-Codes: C24, C25, G21, G32, G33

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[1]The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Bank of Slovenia.

1. Introduction

Credit default models are extensively used by banks and regulators. IRB regulation requires from banks to provide their own estimates of probability of default, which is one of the key parameters that determines capital requirements (BCBS, 2001, 2006). Identifying non-performing borrowers also enables banks and regulators to project expected losses and to assess potential capital needs to cover these losses. In addition, default probability models can also be used for stress testing purposes to simulate the effect of different scenarios.

In this paper we propose and test the performance of two novel methodologies for modelling credit risk. Credit default is typically modelled using discrete choice methodology as was first proposed by Altman (1968). The binary dependent variable is usually defined following BCBS (2006) default definition, which is based on number of days past due. The default event occurs when borrower is more than 90 days overdue. By transforming an overdue into a dichotomous variable, a lot of potentially useful information is lost. In addition, overdue is already a risk measure and therefore it seems reasonable to model it directly, without any transformations. Since it is censored at value zero, we apply tobit modelling approach. Our first set of tests is aimed to evaluate and compare the performance of classical binary probit model versus tobit model.

Credit default indicators show a lot of persistence. Once a borrower defaults (becomes more than 90 days overdue), it is not very likely that he will become performing again. Moreover, an overdue, once being positive, is expected to increase in time. Estimating default probability model, which includes autoregressive dynamics can thus significantly improve predicting performance. Our second proposed novelty is thus to estimate dynamic probit and dynamic tobit model using Wooldridge (2005) methodology and compare their performance with static version of the models.

We evaluate the performance of the models by looking at their ability to discriminate between performing and non-performing borrowers. Conventional default probability models, however, usually follow the discrete time hazard rate modelling approach, which gives the probability that borrower defaults in current period under the condition the default event did not occur before (see for instance Bonfim, 2009 and Carling et al., 2007). As described by Hamerle et al. (2003) this in an underlying methodology of IRB regulation. We therefore also estimate classical default probability model, where only transitions to default are taken into account, and compare its performance in predicting new defaulters with other proposed models. Our goal is not to find the best performing model specification, but rather to use the same explanatory variables in all the estimates and see how different functional form (probit vs. tobit) and different information set (static vs. dynamic) affects the performance in explaining state of default and transition to default. The performance of the models is evaluated using the data of Slovenian non-financial firms.

We find that tobit modelling methodology outperforms all other models. In predicting non-performing borrowers, where persistence is of a key importance, dynamic tobit correctly identifies more than 70% of defaulters and issues less than 1% of false alarms. High performance - 66% true positive rate - is also achieved by dynamic probit model, which outperforms the static version by more than 30 percentage points. An important advantage of tobit model, however, is that its prediction is number of days past due, which enables to form different classes of overdue. One can for instance predict defaulters using any overdue threshold, not only 90 days as is standard in binary models. We show that dynamic tobit has high classification accuracy across different classes of overdue, from 30 to 360 days. For predicting new defaulters, however, we find that the static tobit model is the advantageous modelling approach. It correctly identifies more than 50% of new defaulters and outperforms all the other models by a large margin. It also issues more false alarms comparing to other methodologies, but given the gain in identifying defaulters, this loss is relatively small and acceptable. This is especially true if one is more concerned in missing defaulters (type I error) than issuing false alarms (type II error), like is typically assumed in early warning literature (see Alessi & Detken, 2011 and Sarlin, 2013). Even though classical binary default probability model is estimated explicitly on transitions to default, it is able to correctly identify only 5% of new defaulters. Three sets of robustness checks confirm the validity of our results.

Our paper is related to a recent study performed by Jones et al. (2015). They test the performance of various binary classifiers in predicting credit rating changes. In addition to conventional techniques such as probit/logit, they also evaluate the performance of more advanced approaches like non-linear classifiers, neural networks, support vector machines and others. They find that newer classifiers significantly outperform all other modelling approaches. Although the goal of our paper is very similar, it provides two new pieces of evidence. First, we show the performance of the models can be significantly improved if, instead of conventional binary model, tobit modelling methodology is applied. Second, we provide evidence that the dynamic specification of the model significantly improves the performance in predicting non-performing borrowers. To our knowledge both, tobit and dynamic methodologies, have not yet been applied to credit risk modelling. Moreover, we propose an approach for modelling credit default on quarterly frequency using mixed frequency data. This enables to monitor the changes in credit portfolio on higher frequency and also more accurately since the information set is updated each quarter.

The findings of this paper have important implications for banks and banking regulation. We show that the conventional default probability model that is typically used by IRB banks achieves very low classification accuracy. This poses a question whether this modelling approach, which at the end determines banks capitalisation, is an appropriate methodology. A simple upgrade of the model with dummies indicating overdue in previous period significantly improves the classification accuracy. The performance can be further improved by using the tobit modelling approach. Although the prediction of the tobit model, which is days past due, can not be directly used in IRB formula for capital requirements, this approach is far more accurate in identifying new defaulters, and therefore it seems reasonable to use it in practice.

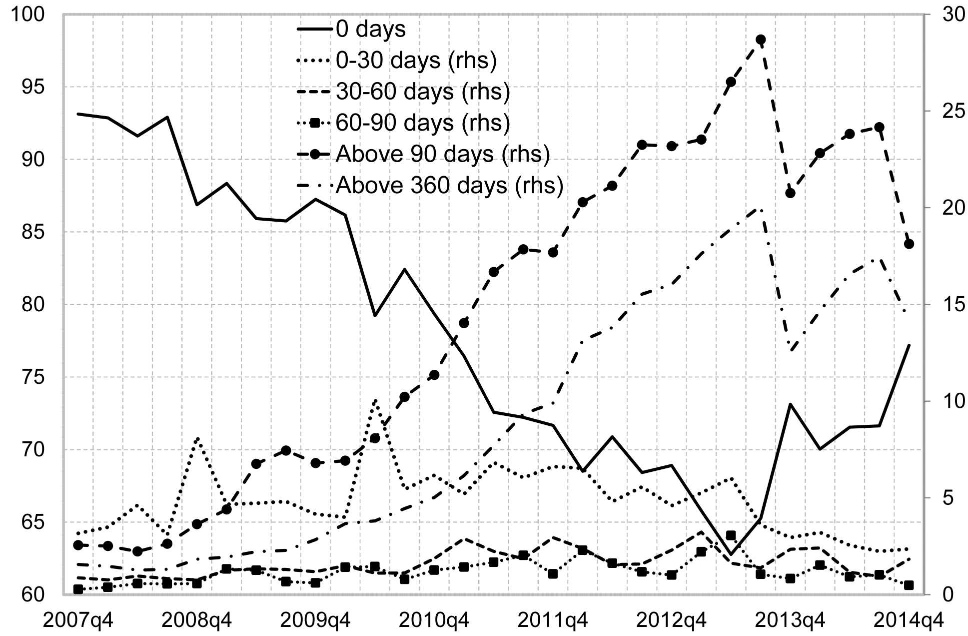
The rest of the paper is structured as follows. Section 2 provides descriptive analysis of the dynamics of different credit risk measures. In Section 3 we present the methodology for estimating and evaluating different credit default models. Estimation and evaluation results are presented in Section 4. Section 5 presents three sets of robustness checks, while Section 6 concludes the paper and discusses implications.

2. The dynamics of credit default measures

The key data source for our analysis is Credit register of Bank of Slovenia, which is exceptionally rich database with many information that are not publicly available. The variable we are most interested in is overdue in loan repayment, which signals financial problems of firms and is also a key credit risk measure under Basel regulation (see BCBS, 2006). It is first available in 2007q4, which limits our analysis to 29 quarterly cross sections from 2007q4 to 2014q4. Restricting the analysis to non-financial firms, which were during the crisis shown to be the most problematic segment, results in large sample of more than 1 million observations represented by a triple firm-bank-time.

Figure 1 shows the evolution of loans broken down to different classes of days of overdue in loan repayment. It can be seen that after the start of the crisis in 2008q4, the share of non-performing loans started rising rapidly and reached very high levels. The share of loans with more than 90 days overdue, which is a standard measure of non-performing loans (BCBS, 2006), rose by more than 25 percentage points until the third quarter of 2013. In 2013q4 it dropped by 8 percentage points, which is the result of transfer of bad loans from two largest banks to Bank Assets Management Company (BAMC). It should thus not be understand as natural improvement of banks' credit portfolio, but rather as an institutional measure that reduced the pressing burden of non-performing loans. Second tranche of transfer was carried out at the end of 2014. Contrary to non-performing loans, the share of loans with 0 days overdue dropped considerably in times of financial stress.

Figure 1: Share of loans across different classes of overdue (in %)



Source: Bank of Slovenia, own calculations.

Other classes between 0 and 90 days overdue represent only a small share of total loans, since these are in many cases only transition classes to higher days past due. The only exception is class between 0 and 30 days, which represents around 3 to 10 percentage share of total loans. There are many borrowers who occasionally have small delays in loan repayment, but whose overdue does not necessarily increase from one period to another.

Figure 1 reveals that overdue is highly autoregressive process. It can be best seen by increasing share of loans with overdue above 360 days. Once an overdue bridges a certain threshold, it is expected to increase in time and reach higher number of days past due. Since these borrowers are financially very weak and are not able to pay back their debt to banks, they are sooner or latter expected to bankrupt. In 83% of cases when an overdue changed between two consecutive quarters, this change was positive. This finding is partly the result of the fact that overdue is censored at zero, which means that by the nature of the variable the increases could be much more frequent. However, even when we look only at the cases when overdue> 0, we get a similar result: 80% increases and only 20% decreases. This dynamic is, however, very heterogeneous across different classes of overdue. As can be seen in Table 1, an overdue is more likely to decrease between two consecutive quarters when it is lower than 30 days. This is the result of already mentioned occasional delayers who are in majority of cases able to repay the debt and their overdue thus typically returns to zero in the next quarter. In other classes positive dynamic prevails and the higher is the overdue, more likely it is, that it will further increase. This is to be expected, since once an overdue exceeds a certain threshold, it is not very likely that a firm will ever be able to repay the debt.

Table 1: Share of increases and decreases of overdue over different classes, in %

Overdue class	One quarter horizon % of increases	One quarter horizon % of decreases	One year horizon % of increases	One year horizon % of decreases
0 days	4.4	-	8.7	-
0-5 days	27.4	57.7	34.4	56.5
5-10 days	36.2	58.7	43.3	52.9
10-20 days	41.0	52.9	48.3	47.5
20-30 days	46.6	47.6	50.1	45.0
30-60 days	53.4	43.5	57.5	40.2
60-90 days	62.9	35.5	66.0	32.6
90-180 days	75.6	23.5	74.1	25.2
180-360 days	88.8	10.9	84.3	15.4
>360 days	95.3	4.6	91.5	8.4

Source: Bank of Slovenia, own calculations.
Note: The table reports the percentage of increases and decrease of overdue over different classes of overdue and two horizons.

Looking at changes in one year period in Table 1 reveals similar dynamic, but decreases prevail only until overdue is below 10 days. In addition, with exception of last three classes, the increases of overdue are more frequent on yearly basis than quarterly. This means that also borrowers with fewer days past due can be more problematic on a long run. Although they were in majority of cases able to repay their debt on a short run, this signals that they might not be able to do so on a long run. Overall, Table 1 clearly reveals that overdue has strong positive autoregressive component, especially when it is higher than 30 days.

Default rate and its projection, probability of default, is typically of a main interest in banks, since it is one of the key factors that determines projected expected losses and capital requirements for IRB banks. In addition, PD is also an important factor in loan approval and pricing. Table 2 shows the default rate over different classes of overdue. It is calculated as a share of borrowers that had been performing in time $t-1$ and became more than 90 days overdue in time t. As expected, the share of transitions to non-performing status is higher, the higher was the overdue in previous period and it further increases when calculated on one year horizon. Lower levels of overdue can thus be used as an early warning signal for potential defaulters in future periods. Classical PD model, where the transition to default is typically explained with borrower-specific factors, is unable to fully capture this information. It only captures some part of it when problems in loan repayment are reflected also in firm financial ratios. These, however, are usually available only once a year, which disable updating the estimated probabilities of default on the same frequency as overdue is refreshed.

Our analysis thus far reveals three potential upgrades of current prevailing credit risk modelling techniques. First, overdue by itself is already a risk measure and thus it seems natural to model it directly. A lot of useful and valuable information is lost, when it is transformed to dichotomous variable and estimated with discrete choice model. An overdue, even if it is low, signals financial problems of a firm and it is thus important to monitor the whole spectrum of delays in loan repayment. Second, autoregressive component seems to be an important factor in modelling credit risk. As shown, an overdue is expected to increase in time, whereas default

Table 2: Default rate over different overdue classes, in %

Overdue class	One quarter horizon	One year horizon
0 days	0.3	3.7
0-5 days	6.1	15.7
5-10 days	12.3	25.1
10-20 days	16.2	31.2
20-30 days	23.3	35.5
30-60 days	40.3	49.1
60-90 days	59.6	64.1

Source: Bank of Slovenia, own calculations.
Note: The table reports the default rate - share of borrowers that were less than 90 days overdue in time $t-1$ and became more than 90 days overdue in time t - over different classes of overdue and two horizons.

status shows a lot of persistence. Past information on days past due can also significantly contribute to explaining transition to default. It is therefore sensible to estimate dynamic credit risk model and see if it adds valuable information comparing to static one. Third, credit risk should be monitored on higher frequency. One year horizon for modelling probability of default that is typically used in the literature and also proposed by BCBS (2001) to IRB banks, is a very long period, since a lot can change over such a long horizon. In extreme case, an overdue may increase from 0 to over 360 days. Standard PD model, which is usually estimated using firm financial ratios is not able to capture such severe deterioration, since its information set is not updated during the year.

3. Methodology

This section presents the methodology for estimating and comparing credit default models. We are interested in three sorts of comparison. First, does it matter if we change the functional form of the model? More specifically, we compare the performance of probit model, where the default is modelled as a binary variable, and model where overdue in loan repayment is modelled explicitly, without any transformations. Since overdue is censored at zero, standard OLS estimator would result in biased estimates. We therefore apply tobit estimator, which captures this source of non-linearity. Second, does the dynamic specification of the model improve performance? We estimate both probit and tobit model including autoregressive term and compare the resulting performance with static specification of the models. Third, we compare the performance of the models in explaining the state of default and transition to default. For the latter, the accuracy of classical PD model is compared with the dynamic version of PD model and with prediction ability of aforementioned models.

Overall, we estimate and compare performance of six models, which can be divided into three groups. They differ in the definition of the dependent variable and in functional form of the model. The first group includes the models where the dependent variable is state of default: *static probit* and *dynamic probit*. In the second group we model overdue in loan repayment and apply censored regression: *static tobit* and *dynamic tobit*. Lastly, the transition to default is modelled with *static PD model* and *dynamic PD model*. Our goal is not to find the best performing model specification, but rather to use the same explanatory variables in all the estimates and see how different functional form (probit vs. tobit) and different information set (static vs. dynamic) affects the performance in explaining state of default and transition to default.

To our knowledge this is the first attempt to model credit default in a dynamic setting. There are some analysis, like for instance Costeiu and Neagu (2013), where past information are included in the model, but not explicitly as lagged dependent variable. Hence, we first present some theory and solutions on how to estimate dynamic non-linear panel data models. Next, we present the specification of all the models and describe how we evaluate their performance.

3.1. Dynamic non-linear panel data models

The key issue in estimating dynamic panel data models is the initial conditions problem, which is the result of correlation between unobserved heterogeneity and past values of the dependent variable. In linear models this problem can be easily solved with appropriate transformation, like first differencing, which eliminates the unobserved effects. Although the transformed error term is correlated with transformed lagged dependent variable, instrumental variables can be used to achieve a consistent estimator. Anderson and Hsiao (1982) propose using y_{it-2} as an instrument in first-differenced equation. Arellano and Bond (1991) upgrade this approach by using a GMM-type of model with all possible instruments in each time period, whereas Blundell and Bond (1998) propose a system estimator, where also level equation with instruments in differences is estimated.

The problem with initial conditions is even more complicated in non-linear models. There are no transformations that would eliminate the unobserved effects. Suppose we are interested in modelling the process:

$$y^*_{it} = \alpha y_{it-1} + x'_{it}\beta + \eta_i + \varepsilon_{it} \tag{1}$$

where y^*_{it} is latent index, y_{it-1} is first lag of the dependent variable, x_{it} is a vector of strictly exogenous variables, η_i is unobserved individual effect and ε_{it} is error term, which is assumed to be distributed with mean 0 and variance σ^2_ε. As described by Akay (2012) the type of the model depends on how the dependent variable is observed. If y_{it} is observed as an indicator

$$y_{it} = \begin{cases} 1 & \text{if} \quad y^*_{it} > 0 \\ 0 & \text{if} \quad y^*_{it} \leq 0 \end{cases} \tag{2}$$

the model to be estimated is dynamic probit or logit model. If, on the other hand, y_{it} is observed as the variable that is censored at zero

$$y_{it} = \begin{cases} y^*_{it} & \text{if} \quad y^*_{it} > 0 \\ 0 & \text{if} \quad y^*_{it} \leq 0 \end{cases} \tag{3}$$

this leads to tobit model specification. Referring to our case, binary credit default models - state probit and transition probit - fit into equation 2, whereas overdue is censored at zero and can thus be represented with equation 3.

In estimating these models one needs to deal with unobserved individual-specific effect η_i, which is correlated with initial values y_{i0}, unless the start of the observed panel data set coincides with the start of the stohastic process. In this case initial values are non-stohastic constants and there is no need to deal with the initial conditions problem. In practice, however, we usually observe data after the start of the stohastic process and the conditional distribution of initial values must be specified. One option is to assume that initial values are not affected by past developments, i.e. to treat them as exogenous variables independent of all other regressors including unobserved individual effects. As described by Akay (2012) this is a very naive assumption, which typically leads to serious bias.

Another way of dealing with initial values is to use the fixed effect approach. Although explicit modelling of individual effects seems attractive, the results can be biased due to incidental parameters problem (Neyman & Scott, 1948). Honoré and Kyriazidou (2000) and Arellano and Carrasco (2003) propose a method for fixed effects logit model, which solves the initial condition problem by eliminating the unobserved heterogeneity. These models, however, can only be estimated for individuals that in the observed period switch between both observed states. If the states are persistent, like in our case, the number of observations would be considerably reduced.

The random effects solutions are much more common and attractive in practice [2]. Wooldridge (2005) proposes to use the density $(\eta_i | y_{i0}, x_{it})$ that specifies the functional form of unobserved heterogeneity:

$$\eta_i = \xi_0 + \xi_1 y_{i0} + x_i' \xi_2 + \psi_i \tag{4}$$

where x_i is $(x_{i1}, x_{i2}...x_{iT})$. The basic logic of this procedure is that correlation between unobserved heterogeneity η_i and initial value y_{i0} is captured by equation 4, which gives another unobserved individual effect ψ_i that is not correlated with initial value y_{i0}. This follows the logic of Chamberlain (1984) who proposes to model conditional expectation of the unobserved effect as a linear function of the exogenous variables and initial conditions. All that needs to be done is to replace η_i in equation 1 with functional form 4, which results in:

$$y_{it}^* = \alpha y_{it-1} + x_{it}' \beta + \xi_0 + \xi_1 y_{i0} + x_i' \xi_2 + \psi_i + \varepsilon_{it}. \tag{5}$$

The main advantage of this methodology is that it is computationally very simple and can be implemented using standard random effects software. Additionally, the same methodology can be used for estimating dynamic probit and dynamic tobit model. Since we are interested in comparing the performance of different functional forms of credit default models, it is very important that it is not affected by different methodology for estimating probit and tobit model. A strong support for using this estimator in our analysis is also study by Akay (2012), who finds that it performs especially well in panels that are longer than 5-8 periods, which is also the case in our models.

3.2. Model specification

In order to estimate the credit default models we link Credit register data with firm balance sheet and income statement data, which are for all Slovenian firms collected by the Agency of the Republic of Slovenia for Public Legal Records and Related Services (AJPES) at yearly basis. To do so, we aggregate Credit register data to firm-time level by taking the highest overdue a particular firm has to any bank in quarter t. Note that our final dataset is of a mixed frequency. Whereas Credit register data are on quarterly basis, balance sheet and income statement data vary only yearly. As is presented below, we select a model specification that takes this into account.

General specification of our models can be characterised with the following non-linear function:

$$y_{it} = f(y_{it-1},\ x_{it-1}^q,\ d_j x_{it-1}^y,\ \eta_i), \quad i = 1, ..., N,\ t = 1, ..., T_i,\ j = 1, ..., 4 \tag{6}$$

[2]Another random effects estimator is suggested by Heckman (1981a,b) who proposes approximating the conditional distribution of initial values using reduced form equation, estimated on the pre-sample information. As discussed by Akay (2012), the main problem with this method is that it requires simultaneous estimation of reduced form and structural model, which is computationally very difficult. In addition, it is not that often applied in empirical work.

where y_{it} is the dependent variable, which is defined as presented in Table 3. In both, probit and PD models, we apply the 90-days threshold, which is very common in the literature (see for instance Bonfim, 2009) and also in line with the recommendations of Basel Committee (BCBS, 2006). For static and dynamic probit we define the default indicator that is equal one if firm i is more than 90 days overdue in quarter t. Similarly also for the PD model where the indicator is equal one if firm became a defaulter in time t, but had still been performing in $t-1$. For the tobit models, we keep overdue as it is, without any transformations and thus use all the information content in it. y_{it-1} is lagged value of the dependent variable, i.e. lagged default indicator in dynamic probit case and lagged overdue in dynamic tobit case. In PD model lagged dependent variable can not be included explicitly since we are modelling the transition to default and thus it is equal to zero for all the firms. Similarly as Costeiu and Neagu (2013), we introduce the dynamics in the PD model by including dummies for different classes of overdue in previous period.

Table 3: Dependent variables in the models

Static & dynamic probit	state of default: $I(>90)_{it}$
Static & dynamic tobit	overdue$_{it}$
Static & dynamic PD	transition to default: $I(>90)_{it}/(\leq 90)_{it-1}$

Note: The table reports the dependent variables for probit, tobit and PD models.

Due to mixed frequency data, the distinction needs to be made between regressors that are available quarterly (x^q_{it-1}) and those that vary only yearly (x^y_{it-1}). Since the latter can have different effect across quarters, we multiply them with d_j, which are simply the dummy variables for each quarter. In this way we get a quarter-specific effect of yearly varying regressors on our dependent variables, which are observed quarterly. All the regressors are included with one period lag [3]. There are mainly two reasons for this. First, given current information, this will enable us to predict credit default at least one period ahead. Second, by including past values of regressors we avoid possible simultaneous causality problems.

In selecting the explanatory factors we follow the model specification by Volk (2012), who models the probability of default as a function of firm size, age, liquidity, indebtedness, cash flow, efficiency, number of days with blocked account and number of relations a particular borrower has with banks [4]. The last two variables are observed quarterly, while others that are calculated on a basis of firm balance sheet and income statement data, are available only once per year. Hence, we interact them with quarterly dummies.

η_i term in equation 6 captures the functional form for unobserved heterogeneity. As can be seen in equation 4, Wooldridge's (2005) original proposal is to include initial value of the dependent variable and the realizations of other regressors in each time period. This procedure would in our case lead to approx. 100 additional parameters to estimate. Given that we work with a large panel of data, this might not be so problematic. However, increasing the number of parameters to be estimated significantly extends the optimization procedure when the dataset is large and given that the model is already complex, this might also lead to problems with convergence. To avoid these problems we rely on evidence provided by Rabe-Hesketh and Skrondal (2013) who show that including only within means and initial values of each regressor does not lead to

[3]For variables that are observed at yearly frequency this means including its values form previous year not previous quarter, since this would result in contemporaneous values for quarters 2, 3 and 4.

[4]We also ran a stepwise selection procedure, which resulted in a model with very similar performance. The results are available upon request.

any bias comparing to Wooldridge's (2005) original specification. Therefore, our functional form for individual specific effects in dynamic probit and tobit model is the following:

$$\eta_i = \xi_0 + \xi_1 y_{i0} + x'_{i0}\xi_2 + \bar{x}'_i\xi_3 + z'_i\xi_4 \tag{7}$$

where y_{i0} is initial value of the dependent variable for each firm, which is the initial value of default indicator in case of dynamic probit model and the initial overdue in dynamic tobit case. The majority of initial values is taken from 2007q4 when our dataset starts. However, for those that enter subsequently, their first observation is taken as an inital value. x_{i0} is a vector of inital values for all the regressors, whereas $\bar{x}_i$ are within means of the regressors, defined as $\frac{1}{T_i}\sum_{t=0}^{T_i} x_{it}$ [5]. As explained by Wooldridge (2005), functional form for individual specific effects may include also other time invariant regressors. We add z_i, which is a set of industry dummies that controls for specificity of each industry.

We control for unobserved heterogeneity also in static and PD models. There are mainly two reasons for this. First, we capture the correlation between error term and firm specific effect and thus achieve consistent estimates (Chamberlain, 1984). Second, in this way the dynamic models do not have any advantage in terms of performance stemming from this additional terms. We use the same functional form as presented in equation 7 for dynamic models, with the only difference that we exclude initial values of the dependent variable. The same approach is used also for the dynamic PD model, which does not explicitly include lagged dependent variables and is thus not subject to initial conditions problem presented in section 3.1.

3.3. Model evaluation

Basic goal of this paper is to compare the performance of different functional forms and specifications of presented credit default models. We do this by looking at several measures that can be calculated from the contingency matrix presented in Table 4. The columns represent the actual observed state, whereas the rows are predicted state by the model. For the latter we take the in-sample fit that is actually the prediction one quarter ahead. The prediction accuracy measures that we use are shown under the Table 4. The most important measure is the true positive rate, which shows the share of correctly predicted defaults. Banks and regulators are mostly concerned in identifying problematic loans, but of course, not on the cost of issuing too many false alarms [6]. For this reason, we show also other measures that will help us to assess model performance. Accuracy, as an overall classification accuracy measure, is also important, but is largely driven by the classification of non-defaulters, which represent a large majority in our data.

We use several criteria that places the observations in the contingency matrix. First, we compare probit and tobit models in terms of their ability to predict non-performing borrowers - more than 90 days past due. Second, the main advantage of tobit model is that its outcome is the whole distribution of overdue, which enables to test the performance also on other overdue classes, like 30, 60, 90, 180 and 360 days past due. Lastly, we compare the models' ability to predict the transition to default - $\leq$90 days overdue in $t-1$, >90 days overdue in time t. In all the cases the predicted indicator is equal one if the predicted probability of state or transition probit models bridges the 0.5 cut-off, whereas for the tobit models it is equal one if its predicted overdue is above a certain threshold, like 90 days.

[5] For yearly varying regressors the mean is calculated by taking into account only one observation per year. In this way we avoid possible miscalculations for those firms that enter the dataset in the middle of the year.

[6] An alternative way of defining this is to use the loss function proposed by Alessi and Detken (2011) and Sarlin (2013), where different weights are placed on type I and type II error.

Table 4: Contingency matrix

	Actual ($I_{it}=1$)	Actual ($I_{it}=0$)
Predicted ($P_{it}=1$)	True positive (TP)	False positive (FP)
Predicted ($P_{it}=0$)	False negative (FN)	True negative (TN)

$$\text{True positive rate} = \frac{TP}{TP+FN} \qquad \text{True negative rate} = \frac{TN}{FP+TN}$$
$$\text{False positive rate} = \frac{FP}{FP+TN} \qquad \text{False negative rate} = \frac{FN}{TP+FN}$$
$$\text{Accuracy} = \frac{TP+TN}{TP+FP+FN+TN}$$

4. Results

Table 5 presents the estimated coefficients of all the models. In addition to the variables that are shown in the table, all the models also include controls for unobserved heterogeneity as presented in section 3.2. Most of the coefficients for these controls are statistically significant, which indicates that it is indeed important to control for these effects in order to achieve consistent estimates.

Lagged default indicator in dynamic probit model has, as expected, highly statistically significant positive effect on current value of indicator. This indicates that the default status, 0 or 1, is highly persistent. Being zero in previous quarter, it is very likely it stays zero also in current period. On the other hand, once a firm is more than 90 days overdue it is not likely to become performing in the next quarter. Similarly, the positive effect of the dependent variable is also found in dynamic tobit model, which shows that the overdue is expected to increase in time. Past information on overdue is also included in dynamic PD model in the form of dummies for different classes of days past due (dummy for 0 days past due is excluded). It can be seen that higher overdue in previous quarter adds more to the default probability. All these results are in line with the findings presented in section 2.

Table 5 also reveals the importance of using the model specification that takes into account the mixed frequency structure of the data. Most of the interaction terms between quarterly dummies and firm specific variables are statistically significant, especially so for static version of the models. This indicates that the effect of yearly-observed variables on default probability or days past due is indeed heterogeneous across quarters. It is expected that the shorter the information lag, the more informative are the variables about credit default indicators. It is exactly what we find in our estimates. The majority of statistically significant coefficients can be found for the first quarter (the terms that are not pre-multiplied with quarterly dummy in Table 5), where the information lag to the observed firm-specific variables is only one quarter.

We now turn our attention to prediction accuracy of the models. Table 6 presents the classification accuracy of probit and tobit models in predicting non-performing borrowers. It can be seen that the dynamic specification of the models significantly improves the performance, especially for the probit model where the true positive rate increases by more than 30 percentage points comparing to static version of the model. Tobit model has even better performance. Static tobit achieves more than 33 percentage points higher true positive rate than static probit model, whereas dynamic tobit adds additional 3 percentage points to the classification accuracy

Table 5: Estimated coefficients

	Static probit	Dynamic probit	Static tobit	Dynamic tobit	Static PD	Dynamic PD
Dependent variable	$I(>90)_{it}$	$I(>90)_{it}$	Overdue_{it}	Overdue_{it}	$I(>90)_{it}/ (\leq 90)_{it-1}$	$I(>90)_{it}/ (\leq 90)_{it-1}$
Dependent var.$_{it-1}$		2.096***		1.067***		
log(Total sales)$_{it-1}$	-0.182***	-0.056***	-81.820***	1.369*	0.033**	0.004
Age$_{it-1}$	0.267***	0.133***	60.193***	11.249***	0.107***	0.038***
Quick ratio$_{it-1}$	-0.023***	-0.014***	-1.368***	-1.249***	-0.018***	-0.006
Debt-to-assets$_{it-1}$	0.005*	0.002	2.954***	0.802***	-0.003	-0.002
Cash flow ratio$_{it-1}$	-0.011	-0.018	-8.654***	-7.447***	-0.045**	-0.025
Asset t. ratio$_{it-1}$	-0.263***	-0.149***	-26.766***	-22.156***	-0.209***	-0.076***
No. of days bl. ac.$_{it-1}$	0.017***	0.010***	2.805***	1.053***	0.012***	0.006***
No. of relations$_{it-1}$	0.345***	0.198***	69.505***	29.457***	0.241***	0.050***
d2*log(Total sales)$_{it-1}$	0.026***	0.019***	2.980***	-0.449	0.026***	-0.001
d2*Age$_{it-1}$	-0.003	-0.002	-0.040	-0.524***	-0.002	0.003
d2*Quick ratio$_{it-1}$	0.019***	0.011*	0.123	0.269	0.016**	0.007
d2*Debt-to-assets$_{it-1}$	0.006*	0.006	0.750	-0.072	0.008	0.003
d2*Cash flow ratio$_{it-1}$	-0.050***	-0.027	-1.355	1.759	0.003	-0.001
d2*Asset t. ratio$_{it-1}$	-0.007	-0.010	5.392**	7.079***	-0.023	-0.005
d3*log(Total sales)$_{it-1}$	0.053***	0.037***	2.529***	-0.793*	0.045***	0.032***
d3*Age$_{it-1}$	-0.005***	-0.003	0.743**	-0.238	-0.004*	-0.001
d3*Quick ratio$_{it-1}$	0.019***	0.008	1.381***	1.294***	-0.000	-0.001
d3*Debt-to-assets$_{it-1}$	0.008**	0.008*	1.332**	-0.113	0.009	0.004
d3*Cash flow ratio$_{it-1}$	-0.054***	-0.021	-10.257***	0.531	-0.011	-0.008
d3*Asset t. ratio$_{it-1}$	0.002	0.001	1.306	5.449***	-0.005	-0.023
d4*log(Total sales)$_{it-1}$	0.057***	0.022***	5.843***	-0.651	0.029***	0.026***
d4*Age$_{it-1}$	-0.007***	-0.003	0.099	-0.783***	-0.003	-0.000
d4*Quick ratio$_{it-1}$	0.019***	0.007	1.360***	1.256***	-0.020*	-0.015
d4*Debt-to-assets$_{it-1}$	0.007**	0.005	1.634***	-0.226	0.009	0.009
d4*Cash flow ratio$_{it-1}$	-0.056***	-0.020	-12.949***	1.365	-0.029	-0.033
d4*Asset t. ratio$_{it-1}$	0.028**	0.031**	4.097*	9.355***	0.038**	0.012
Overdue 0-5$_{it-1}$						1.095***
Overdue 5-10$_{it-1}$						1.352***
Overdue 10-20$_{it-1}$						1.533***
Overdue 20-30$_{it-1}$						1.840***
Overdue 30-60$_{it-1}$						2.290***
Overdue 60-90$_{it-1}$						2.716***
Constant	-10.629***	-7.002***	-824.385***	-190.530***	-2.164***	-2.546***
Observations	517964	517964	517964	517964	487969	487969

Source: Bank of Slovenia, AJPES, own calculations.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Notes: The table reports the coefficients for all the estimated models. The dependent variable for static and dynamic probit is an indicator $I(>90)_{it}$ that is equal one if firm i is more than 90 days past due in time t and zero otherwise. For both PD models, the dependent variable is defined as transition to default (≤90 days overdue in time $t-1$, >90 days overdue in time t). No. of days bl. ac. measures number of days a firm has blocked account. No. of relations is number of relationships between each firm and banks. d2 to d4 are dummy variables from second to fourth quarter. Overdue 0-5 to Overdue 60-90 are dummy variables for number of days a firm is past due. In addition to the variables that are shown in the table, the models also include controls for unobserved heterogeneity as described in section 3.2.

of defaulters. Importantly, this high prediction accuracy of defaulters is not on a cost of issuing too many false alarms. Tobit model has slightly higher false positive rate, but these are still very low values, especially in the case of dynamic model. Comparing to the gain in true positive rate, the loss in terms of false alarms is relatively minor.

Table 6: Performance of probit and tobit model in predicting performing and non-performing borrowers

	Probit		Tobit	
	Static	Dynamic	Static	Dynamic
True positive rate	0.356	0.663	0.688	0.714
True negative rate	0.990	0.993	0.947	0.991
False positive rate	0.010	0.007	0.053	0.009
False negative rate	0.644	0.337	0.312	0.286
Accuracy	0.949	0.972	0.930	0.973

Source: Bank of Slovenia, AJPES, own calculations.
Notes: The table reports the classification performance of probit and tobit models in predicting performing and non-performing borrowers (more than 90 days past due). See section 3.3 for the description of classification accuracy measures.

Table 7 shows the classification accuracy of dynamic probit and dynamic tobit model in predicting non-performing firms where we let the autoregressive process to proceed four quarters ahead. These are still the in-sample predictions, with the only difference that instead of actually observed values of lagged dependent variable its predictions are taken, which are obtained by recursively running the predictions four times. The results show that tobit is the superior model also on a longer horizon. Its true positive rate is expectedly decreasing on longer forecast horizon, but it stays above the performance of the probit model. Dynamic probit achieves slightly higher overall accuracy, but this is only due to better prediction of non-defaulters. False positive rate still stays very low for both models.

Table 7: Performance of dynamic probit and tobit model in predicting performing and non-performing borrowers from one to four quarters ahead

	Dynamic probit				Dynamic tobit			
	1q	2q	3q	4q	1q	2q	3q	4q
True positive rate	0.663	0.571	0.505	0.454	0.714	0.603	0.544	0.508
True negative rate	0.993	0.992	0.993	0.993	0.991	0.987	0.986	0.986
False positive rate	0.007	0.008	0.007	0.007	0.009	0.013	0.014	0.014
False negative rate	0.337	0.429	0.495	0.546	0.286	0.397	0.456	0.492
Accuracy	0.972	0.964	0.959	0.954	0.973	0.962	0.956	0.952

Source: Bank of Slovenia, AJPES, own calculations.
Notes: The table reports the classification performance of dynamic probit and dynamic tobit model in predicting performing and non-performing borrowers (more than 90 days past due) one to four quarters ahead. See section 3.3 for the description of classification accuracy measures.

Tobit model enables to form the predictions for different overdue classes. Table 8 shows the classification accuracy results of static and dynamic tobit model for five thresholds of days past due. All the classes are defined in the same way: when overdue or prediction is above a certain threshold the indicator is equal one, otherwise it is zero. It can be seen that in the case of static model, true positive rate is decreasing with higher overdue threshold. The model correctly classifies 85% of firms with overdue above 30 days, but only 39% of firms with overdue higher than 360 days. The performance of the dynamic model is much more stable and its true positive rate is fluctuating around 75%. Static model outperforms the dynamic one in terms of true positive rate for 30 and 60 days class. It, however, also has significantly higher false positive rate, which is for the 30-days class equal to 20%, comparing to only 3% of the dynamic model. The results presented in Table 8 thus reveal, that both, static and dynamic tobit model, are quite successful in classifying borrowers to different classes of overdue, but the dynamic version of the model is shown to be the superior one.

Table 8: Classification accuracy of static and dynamic tobit model across different groups of overdue

	Static tobit					Dynamic tobit				
Overdue threshold	30	60	90	180	360	30	60	90	180	360
True positive rate	0.854	0.752	0.688	0.561	0.390	0.746	0.720	0.714	0.733	0.774
True negative rate	0.800	0.918	0.947	0.973	0.987	0.972	0.986	0.991	0.997	0.998
False positive rate	0.200	0.082	0.053	0.027	0.013	0.028	0.014	0.009	0.003	0.002
False negative rate	0.146	0.248	0.312	0.439	0.610	0.254	0.280	0.286	0.267	0.226
Accuracy	0.805	0.906	0.930	0.953	0.967	0.952	0.967	0.973	0.984	0.991

Source: Bank of Slovenia, AJPES, own calculations.
Notes: The table reports the performance of static and dynamic tobit model in classifying borrowers into different groups of days past due. In all the cases an indicator is equal one if overdue is above certain threshold (30, 60, 90, 180 or 360 days past due) and zero if it is equal or below that threshold. See section 3.3 for the description of classification accuracy measures.

Banks and regulators are mostly concerned about predicting new non-performing borrowers. PD as a measure of likelihood that a borrower will default on a certain horizon is also a key credit risk parameter under the IRB capital regulation. Table 9 presents the performance of the models in predicting the transition to default ($\leq$90 days overdue in time $t-1$, >90 days overdue in time t). Classical static PD model that is most frequently used in practice and where only firm specific variables are used as regressors, is shown to have very low performance. It correctly identifies only 5% of new defaults. Extending the model with dummies for different classes of overdue in $t-1$ significantly improves the performance to 27% true positive rate. This is to be expected since, as we already presented in section 2, past information on days past due is very informative about current default status. The higher the overdue in previous quarter, more likely it is that the firm defaults in current period. A minor change in the model can thus lead to much more accurate estimates of the default probability.

Table 9: Models' performance in predicting transition to default

	Static PD	Dynamic PD	Static probit	Dynamic probit	Static tobit	Dynamic tobit
True positive rate	0.047	0.274	0.150	0.045	0.501	0.139
True negative rate	0.997	0.997	0.991	0.997	0.950	0.995
False positive rate	0.003	0.003	0.009	0.003	0.050	0.005
False negative rate	0.953	0.726	0.850	0.955	0.499	0.861
Accuracy	0.983	0.986	0.979	0.983	0.943	0.983

Source: Bank of Slovenia, AJPES, own calculations.
Notes: The table reports the performance of the estimated models in predicting the transition to default ($\leq$90 days overdue in time $t-1$, >90 days overdue in time t).

The performance in predicting new defaulters can also be calculated for the models where either state of default or overdue is used as the dependent variable. It is interesting to find that any other version of the model outperforms the classical PD model. The only exception is the dynamic probit model, where the autoregressive term leads to persistence of states and there is thus not a lot of switching between performing and non-performing states. [7] As expected, the dynamic tobit with lagged information about days past due is better able to capture the transition to default, but however, is still performing worse than some other models. It seems that the autoregressive component is not strong enough to lead to a sufficient increase of overdue between two consecutive periods.

The best performing model for predicting transition to default is found to be the static tobit. It achieves 50% classification accuracy of new defaulters and outperforms all other models by a large margin. It also has the highest false positive rate (5%), which also explains lower overall accuracy. This measure, however, is typically not of a primary interest in evaluating the performance of default probability models and comparing to the gain in correctly identifying defaulters, the loss of over-signalling is relatively small. We formally compare the performance of the two best performing models, dynamic PD and static tobit, using the methodology proposed by Alessi and Detken (2011). Applying equal weights on type 1 and type 2 error results in a loss of 0.365 for dynamic PD and only 0.275 for static tobit. Given that regulators and banks are typically more concerned about missing the defaulters than issuing false alarms, which would be reflected in higher weight on type 1 error, places the static tobit model to even more superior position.

Let us summarize our main results. We find two strong peace of evidence that the tobit modelling technique of credit risk is the advantageous one. Dynamic tobit model is shown to achieve the best classification accuracy of non-performing borrowers, whereas static tobit outperforms all the other models in predicting new defaulters. The advantage of tobit model is also that it enables classifying borrowers to different groups of overdue and thus get the whole spectrum of riskiness of credit portfolio. We also show that the static PD model, that is widely used by banks and regulators, actually has the worst classifying performance. Given the evidence in our paper, it thus seems reasonable to upgrade credit risk modelling techniques, since these lead to much more accurate predictions. We now check the robustness of our results.

5. Robustness checks

This section presents three sets of robustness checks. First, we show the out-of-sample performance results. Second, we extend the horizon in PD models from one quarter to one year. Third, we show the dynamic model predictions on a sub-sample of firms that are present at the beginning of the sample.

5.1. Out-of-sample performance

The classification accuracy results presented thus far are in-sample predictions. Models are typically used to forecast credit default on a certain horizon. We therefore check the validity of our results by also predicting out-of-sample. We do this by recursively estimating the models and predicting the state of default or overdue one quarter ahead. For instance, we estimate the models until 2010q4 and forecast 2011q1. We start the estimating process in 2008q4, such that we get the estimates for all the coefficients, including the interactions between quarterly dummies and firm specific variables. The applied estimating methodology, however, needs to be simplified due to a large computational burden. Using random effects estimator, it took the

[7] As shown in Table 6, dynamic probit achieves a high accuracy in predicting non-performing borrowers, where the persistence of both states is of a key importance.

computer approximately 12 days to estimate all the models presented in Table 5. Given that now all the estimates would need to be replicated 24-times, it would take a very long time to estimate all the models. We therefore use pooled estimators, which proceed much faster. The only difference comparing to random effects estimator is that the pooled version is less efficient, since it does not take into account the autoregressive structure of the variance-covariance matrix. Since we use an alternative methodology, the prediction accuracy of this procedure should not be directly compared to the results presented in previous section.

Table 10 presents the out-of-sample performance in predicting non-performing borrowers. Even though the estimation methodology is now different, the prediction accuracy is similar as presented in Table 6 for in-sample predictions. Similarly, we also find that the dynamic version of the models outperform the static ones. Dynamic probit achieves the highest classification accuracy of defaulters (78%) with low false positive rate below 1%. This model, however, is not able to break down firms to different overdue classes. Table 11 displays these results for static and dynamic tobit. Similar as before, we find that the predictions of the dynamic model are much more stable and accurate.

Table 10: Out-of-sample performance of probit and tobit model in predicting performing and non-performing borrowers

	Probit		Tobit	
	Static	Dynamic	Static	Dynamic
True positive rate	0.396	0.783	0.710	0.721
True negative rate	0.990	0.992	0.951	0.992
False positive rate	0.010	0.008	0.049	0.008
False negative rate	0.604	0.217	0.290	0.279
Accuracy	0.949	0.978	0.935	0.974

Source: Bank of Slovenia, AJPES, own calculations.
Notes: The table reports the out-of-sample classification performance of probit and tobit models in predicting performing and non-performing borrowers (more than 90 days past due). See section 3.3 for the description of classification accuracy measures.

Table 11: Out-of-sample classification accuracy of static and dynamic tobit model across different groups of overdue

	Static tobit					Dynamic tobit				
Overdue threshold	30	60	90	180	360	30	60	90	180	360
True positive rate	0.887	0.777	0.710	0.592	0.427	0.751	0.725	0.721	0.739	0.780
True negative rate	0.776	0.923	0.951	0.975	0.988	0.975	0.988	0.992	0.997	0.998
False positive rate	0.224	0.077	0.049	0.025	0.012	0.025	0.012	0.008	0.003	0.002
False negative rate	0.113	0.223	0.290	0.408	0.573	0.249	0.275	0.279	0.261	0.220
Accuracy	0.786	0.911	0.935	0.955	0.969	0.955	0.968	0.974	0.983	0.991

Source: Bank of Slovenia, AJPES, own calculations.
Notes: The table reports the out-of-sample performance of static and dynamic tobit model in classifying borrowers into different groups of days past due. In all the cases an indicator is equal one if overdue is above certain threshold (30, 60, 90, 180 or 360 days past due) and zero if it is equal or below that threshold. See section 3.3 for the description of classification accuracy measures.

We now turn to out-of-sample prediction accuracy of new defaulters. The results presented in Table 12 reveal that the prevailing modelling methodology, static PD model, performs very badly with true positive rate below 1%. The model is basically uninformative in identifying transitions to default. The performance can be significantly improved by moving to dynamic PD model and even more by using static tobit, which correctly classifies 53% of new defaulters. Overall, we can conclude that the out-of-sample prediction results are totally in line with the results obtained using in-sample predictions.

Table 12: Out-of-sample performance in predicting transition to default

	Static PD	Dynamic PD	Static probit	Dynamic probit	Static tobit	Dynamic tobit
True positive rate	0.008	0.233	0.157	0.005	0.528	0.114
True negative rate	0.999	0.997	0.991	1.000	0.955	0.997
False positive rate	0.001	0.003	0.009	0.000	0.045	0.003
False negative rate	0.992	0.767	0.843	0.995	0.472	0.886
Accuracy	0.984	0.986	0.978	0.985	0.948	0.983

Source: Bank of Slovenia, AJPES, own calculations.
Notes: The table reports the out-of-sample performance of the estimated models in predicting the transition to default ($\leq$90 days overdue in time $t-1$, >90 days overdue in time t).

5.2. Yearly horizon of default probability

Low prediction accuracy of static PD model could be the result of modelling the default probability on quarterly horizon. The underlying default rate is a very volatile series and the model may not be able to sufficiently capture all these dynamics. In addition, probabilities of default are usually estimated on a one year horizon as is also suggested by BCBS (2001) to IRB banks. Hence, to check the robustness of presented results, we re-estimate all our models using only end-of-year data. Models' specification is similar as before, with the only difference that the interactions between quarterly dummies and firm specific variables are now dropped and instead of one quarter lags, yearly lags of the dependent variables are used. Similarly, the dependent variable for PD models is now defined as transitions to default on one year horizon.

The results of yearly estimates are presented in Table 13. As expected, the prediction accuracy of static PD model is now improved. However, with 16% true positive rate it is still among the worst performing. We again find that static tobit outperforms all the other models' predictions by a large margin. It correctly classifies 56% transitions to default, which is even slightly improved comparing to quarterly estimates. Similar as we find before, static tobit issues more false alarms, but the evaluation of a loss function is still considerably in favour of this modelling approach.

Table 13: Models' performance in predicting yearly transition to default

	Static PD	Dynamic PD	Static probit	Dynamic probit	Static tobit	Dynamic tobit
True positive rate	0.159	0.235	0.248	0.145	0.559	0.354
True negative rate	0.994	0.993	0.988	0.994	0.955	0.982
False positive rate	0.006	0.007	0.012	0.006	0.045	0.018
False negative rate	0.841	0.765	0.752	0.855	0.441	0.646
Accuracy	0.963	0.966	0.961	0.963	0.941	0.959

Source: Bank of Slovenia, AJPES, own calculations.
Notes: The table reports the performance of the estimated models in predicting the yearly transition to default ($\leq$90 days overdue in time $t-1$, >90 days overdue in time t).

5.3. Dynamic model estimates on a sub-sample of firms

Wooldridge's (2005) methodology, which we use to estimate the dynamic models, requires that the estimates are performed on a balanced panel data.[8] Due to the nature of the modelling problem, our panel is unbalanced. Firms that become overdue on their credit obligation sooner or later bankrupt and disappear from the sample. In addition, new firms are entering into the sample. In applied work researchers usually estimate the dynamic models only on balanced part of the data set (see for instance O'Neill and Hanrahan, 2011). In our case, however, this would lead to serious sample selection bias. We would be mostly left with the firms that defaulted during the last periods of the sample. We rely on the evidence provided by Akay (2009), who shows by simulations that using Wooldridge's (2005) methodology on unbalanced panel does not lead to any serious bias. In addition, we also estimate our models on a sub-sample of firms that are represented at the beginning of the sample. We therefore exclude all the firms that subsequently enter the dataset. In this way we achieve that the initial values for all the firms are taken from the same time period (2007q4).

Table 14 presents the performance results of dynamic probit and tobit models estimated on a sample of firms present in 2007q4. As it can be seen, this only marginally changes the classification accuracy results. We can still find that the dynamic models outperform the static ones and that the dynamic tobit is the advantageous methodology for identifying non-performing borrowers. Overall, the results are in line with the findings presented in section 4.

Table 14: Performance of the dynamic models estimated on a sub-sample of firms

	Dynamic	Dynamic tobit				
	probit	30	60	90	180	360
True positive rate	0.678	0.766	0.738	0.731	0.746	0.781
True negative rate	0.993	0.969	0.985	0.991	0.996	0.998
False positive rate	0.007	0.031	0.015	0.009	0.004	0.002
False negative rate	0.322	0.234	0.262	0.269	0.254	0.219
Accuracy	0.972	0.951	0.967	0.974	0.983	0.991

Source: Bank of Slovenia, AJPES, own calculations.
Notes: The table reports the performance of dynamic probit and tobit model estimated on a sample of firms present at the beginning of the sample (2007q4). Dynamic probit performance is shown for the 90 days overdue threshold, whereas dynamic tobit results are for different thresholds from 30 to 360 days.

6. Conclusion

In this paper we evaluate the performance of several credit default models, which are compared in their ability to correctly predict non-performing borrowers and transitions to default. In addition to conventional static binary models, we also evaluate the performance of two novel methodologies that, to our knowledge, have not yet been applied in modelling credit risk. Overdue in loan repayment is already a risk measure and therefore it seems reasonable to estimate it directly, using the tobit model methodology. In addition, state of default and overdue are highly autoregressive processes. Overdue is expected to increase in time, whereas state of default shows a lot of persistence. Estimating the dynamic probit and tobit model, where lagged dependent variable is included among regressors, can significantly improve the performance of the model.

[8] Honoré (2002) shows that the initial conditions problem is especially problematic in unbalanced panels.

Same inputs are used in all the models, which means that the differences in classification accuracy can be fully attributed to different functional forms (probit vs. tobit) and additional information that enter the model in the form of lagged dependent variable.

We show that tobit modelling methodology outperforms all other methodologies. Dynamic tobit model is shown to achieve the highest classification accuracy in predicting non-performing borrowers. It correctly identifies more than 70% of defaulters and issues less than 1% of false alarms. In addition, its prediction is number of days past due, which enables to form different classes of overdue. This is a very valuable information, since it gives direct and easily interpretable information on expected portfolio riskiness. We show that tobit model performance is very high and stable across different overdue classes, from 30 to 360 days. High performance (66% true positive rate) is also achieved by dynamic probit, which outperforms the static version by more than 30 percentage points. This shows that the dynamic modelling methodology can significantly improve the performance of credit default models.

Tobit model also has the highest prediction ability for explaining transitions to default. In classifying firms into performing and non-performing class the dynamic structure of the model plays a crucial role. When a certain overdue threshold is bridged, it is not very likely that the borrower will become performing again. In explaining transitions to default, however, this autoregressive process is too slow to sufficiently capture the increase of overdue from one quarter to another. We therefore find the static tobit model to perform the best in terms of true positive rate. It correctly classifies 50% of new defaulters and outperforms all other models by a large margin. It also issues more false alarms, but as we show, the evaluation of loss function, which takes into account type I and type II error, is much in favour of this model. On the other hand, conventional PD model, that is typically used by banks and regulators, has very low performance. It is able to correctly identify only 5% of transitions to default. A number of robustness checks confirm the validity of our results.

The findings in this paper have several important implications for banks, banking regulation and credit risk modelling practitioners. We show that the prevailing credit risk modelling methodology, which is based on binary classifiers, can be significantly improved by including the dynamics and choosing the tobit functional form of the model. In addition, we propose a model specification to estimate credit risk on a quarterly basis, which enables much more frequent and accurate monitoring of expected changes in credit portfolio.

A more important finding of our empirical analysis is very low prediction performance of conventional static PD model. This type of model is usually used by banks to assess riskiness of their portfolio and to determine one of the crucial parameters for calculating capital requirements under IRB regulation - the probability of default. A simple upgrade of the model with dummies indicating overdue in previous period significantly improves the performance. Even higher prediction ability is achieved by static tobit model. Although the prediction of this model is not in the form of default probability and can thus not be directly used in IRB formula, it seems very useful for identifying new defaulters more accurately. This is important information for banks and regulators, since knowing which borrowers are expected to default in next period they are able to assess in advance the required loan loss provisions and capital to cover the losses. In addition, IRB regulation (BCBS, 2001) requires from banks to form classes of default probability and apply the same PD to all the firms within the class. Tobit predictions enable to form similar riskiness classes based on days past due. Combining this predictions with the information about default rate for each overdue class, one can, similarly as under IRB regulation, also attach default probability to each class.

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