Limited Asset Market Participation and Optimal Monetary Policy*

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Abstract

We study the design of optimal monetary policy in an economy characterized by staggered wage and price contracts together with limited asset market participation (LAMP). We provide analytical conditions for the determinacy of the REE in a fourth order dynamic system. We find that once nominal wage stickiness, an incontrovertible empirical fact, is considered optimal simple interest rate rules are active for any plausible degree of LAMP. Further we show that, in response to shocks, LAMP has just minor effects on the dynamics of welfare relevant variables under Ramsey optimal monetary policy. For these reasons we argue that LAMP does not fundamentally affect the design of optimal monetary policy.

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1 Introduction

This paper studies the implications of limited asset market participation for the conduct of monetary policy. In particular, it investigates the dynamic properties of interest rate rules and optimal monetary policy in an economy characterized by staggered wage and price contracts together with limited asset market participation (LAMP henceforth).

Wage and price stickiness arises from the Calvo-type mechanism. As in Galì et al. (2004, 2007), Bilbiee (2008) and in a number of recent studies, we model LAMP assuming that a portion of agents face a liquidity constraint such that they spend their current labor income in each period.

The remaining households hold assets and smooth consumption. This heterogeneity between households breaks the Ricardian Equivalence. For this reason in the remainder of the paper we refer to liquidity constrained agents as to non-Ricardian consumers and symmetrically we define other agents as Ricardian consumers.[1]

The resulting framework nests two popular environments in the optimal monetary policy literature: Bilbiie (2008) and Erceg et al. (2000). The former studies determinacy properties of simple interest rate rules and optimal monetary policy in a NK economy with LAMP and a frictionless labor market. The latter develop a full participation NK model characterized by both staggered prices and wages which features an endogenous trade-off between the stabilization of the output gap, price inflation and wage inflation. Hence, Bilbiie (2008) features sticky prices and LAMP, but not sticky wages; Erceg et al. (2000), instead, features sticky prices and wages, but not LAMP. We find that the results in Erceg et al. (2000) are robust to the introduction of LAMP, while Bilbiie's (2008) findings are sensitive to the introduction of wage stickiness. As a consequence, our analysis suggests that once wage stickiness is taking into account LAMP do not substantially affect the functioning of monetary policy.

The tractable nature of our encompassing framework allows us to establish a series of analytical results, thus providing a clear economic intuition for our main findings. We take the reader to subsequent analytical steps that shed lights, first, on the transmission mechanism of monetary policy and then on the featurs of optimal monetary policy in this setting.

As a first step, we show that in a model with staggered wages and LAMP, the real wage gap, defined as the gap between the actual and the efficient equilibrium real wage, affects the IS curve.[2] As it will be clear in the text, Non-Ricardian households' consumption is determined

[1] This modelling choice was originally adopted by Mankiw (2000) to account for the empirical relationship between consumption and disposable income, which seems to be stronger than suggested by the permanent income hypothesis.

[2] We follow the common practice in the optimal policy literature and linearize our model equations around the efficient steady state. Our linearized economy reduces to a system of four equations in terms of aggregate variables only.

by labor income only. As a result, deviations of the real wage from its efficient counterpart lead to deviations of aggregate demand from the efficient level. For this reason LAMP implies that the wage gap affects the IS curve. Thus, while the supply side of the model is isomorphic to that of a fully Ricardian economy with sticky prices and wages, the demand side, represented by the IS equation, is affected by the degree of asset market participation.

Second, we analyze the implications of the degree of LAMP on the IS equation. This is important since monetary policy affects the model economy through the IS equation which contains the monetary policy instrument, i.e., the interest rate. We thus identify the factors that lead to the inversion of the slope of IS curve. In this respect we can compare our results to those in Bilbiie (2008), who considers a model with flexible wages. Bilbiie (2008) finds that, when asset market participation is restricted beyond a certain threshold, the slope of the IS curve may turn positive leading to what he calls and Inverted Aggregate Demand Logic (IADL). In the parameter space where the IADL holds, aggregate demand increases with the real interest rate. The inversion of the slope of the IS curve requires an inversion of the Taylor Principle for monetary policy to support a unique rational expectation equilibrium (REE, henceforth). We show that when the wage gap appears into the IS curve, the degree of wage stickiness affects the slope of the IS curve, severely restricting the parameter space where the IADL holds.

As a third step, we recover analytical conditions for the determinacy of the REE in a fourth order dynamic system characterized by both predetermined and forward looking variables. As far as we know, this is a novel analytical approach and we regard our analytical solution as a methodological contribution. We show how to use Hurwitz polynomials to transform the polynomial derived from the characteristic equation, as in Felippa and Park (2004), to deal analytically with the conditions for the determinacy. We then prove analytically that LAMP does not invalidate the Taylor Principle: for any plausible share of non-Ricardian agents an active interest rate rule ensures the uniqueness of the rational expectation equilibrium.

Finally, and most importantly, to the best of our knowledge, this paper is the first to derive analytically the Central Bank welfare-loss function by taking a second order approximation to a weighted average of households' lifetime utilities in a model with LAMP and staggered price and wages and to study optimal monetary policy. We find that the central bank loss function is characterized by the presence of the real wage-gap besides the terms identified by Erceg et al. (2000), i.e. the output-gap and price and wage inflation. The wage gap enters the loss function for the same reasons it appears into the IS equation (20), namely because deviations of the real wage from its efficient counterpart lead to deviations of aggregate demand from the efficient level.

With all these analytical insights, we then can proceed to study numerically the optimal

monetary policy under commitment and optimal simple rules. We find that none of them are affected by LAMP. Erceg et al (2000) argue that the trade-off faced by monetary policy when trying to minimize its welfare loss originates entirely from the supply side of the economy. More precisely in a social optimum the real wage should follow one-to-one the marginal productivity of labor, however this is not possible if the variance of both price and wage inflation is stabilized. Thus the Central Bank must trade-off between the competing distortions due to sticky prices and sticky wages. As described above, the supply side of the economy is not affected by LAMP. For this reason LAMP does not change the trade-offs faced by monetary policy when trying to minimize the welfare loss. It follows that the optimal response of welfare relevant variables to a shock is just marginally affected by LAMP. What LAMP does affect, through the IS curve, is the optimal path of the nominal interest rate required to implement the optimal path of welfare-relevat variables. Furthermore, differently from Bilbiie (2008), optimal inflation targeting rules, contemporaneous or forward looking, are restored to be strongly active if wages are sticky, as in the standard NK model. Finally, as in Erceg et al. (2000), we find that price inflation targeting may cause relevant welfare costs. Price inflation targeting leads to higher welfare with respect to wage inflation targeting just in the case in which asset market participation is restricted to an implausible extent.

The intuition for our results is the following. Variations in the real wage lead to variations in profits and hence in the dividend income of Ricardian agents. This has wealth effects that can overturn the standard impact of changes in the real interest rate on aggregate demand. Specifically, when asset market participation is restricted beyond a certain extent, the slope of the IS curve could turn positive leading to an inversion in the standard principles for the conduct of monetary policy. Wage stickiness dampens the changes in the real wage, and thus in profits, in response to variations in economic conditions. This prevents the reversal of the slope of the IS curve that could obtain under wage flexibility, restoring standard policy prescriptions for the monetary authority.

Several authors analyze the implications of LAMP for monetary policy in NK models. A paper closely related to ours is Colciago (2011). In particular, Colciago (2011) shows numerically that wage stickiness helps restoring the standard Taylor Principle as a necessary condition for determinacy in the presence of LAMP. We show analytically that the finding in Colciago (2011) is due to wage stickiness preventing the inversion of the slope of the IS curve in the relevant parameter space. This allows us to provide a neat economic intuition of the result and to find analytical conditions for determinacy of alternative interest rate rules for an arbitrary degree of asset market participation. Furthermore, we also study the determinacy properties of the optimal simple rules and the features of optimal monetary policy. The latter analysis is neglected by Colciago (2011), who deals with a positive analysis of the NK model

with LAMP and does not draw any optimal policy implications. Indeed, the focus is the main difference between Colciago (2011) and our paper. While the former focuses on fiscal policy issues, studying the response of private consumption to a government spending shock in the presence of LAMP and nominal rigidities, this paper analyses the implications of LAMP for the design of optimal monetary policy in a NK model with price and wage stickiness.

Galì et al. (2004) study determinacy properties of interest rate rules in a sticky-price economy with a fraction of non-Ricardian consumers and capital accumulation. They show that if the share of non-Ricardian agents is sufficiently large and prices are sticky enough, determinacy of the *REE* requires that the central bank adopts a Reinforced Taylor Principle, whereby the inflation coefficient response is considerably larger than unity. Amato and Laubach (2003) model non-Ricardian behavior as a consumption habit and show that the optimal interest rate becomes more inertial as the fraction of non-Ricardian consumers increases. Di Bartolomeo and Rossi (2007) show that monetary policy effectiveness increases with the degree of LAMP.

Most of the works mentioned so far are characterized by a frictionless labor market.[3] The few papers which consider the interactions between a non-Walrasian labor market and LAMP focus on fiscal policy issues. This is motivated by recent VAR evidence suggesting that an innovation in government spending causes a persistent rise in private consumption. This evidence cannot be easily addressed resorting to fully Ricardian business cycle models. For this reason, Galì et al. (2007) study the effect of government spending shocks in a model with LAMP. They show that an imperfectly competitive labor market is a fundamental ingredient to obtain the crowding-in of consumption in response to an expansionary government spending shock identified, *inter alia*, by Blanchard and Perotti (2002) and Fatàs and Mihov (2001). Colciago (2011) and Furlanetto (2007) extend the analysis in Galì et al. (2007) to the case of nominal wage stickiness.[4] Forni et al. (2009) build a medium-scale NK model with LAMP and a rich description of the fiscal side. They use Bayesian techniques to estimate the effects of innovations in fiscal policy variables in the Euro area, finding only mild Keynesian effects of public expenditure, but a large fraction of non-Ricardian agents, close to 40%.[5] Motta and Tirelli (2012 a,b) study the stabilizing properties of fiscal rules in a medium scale business cycle model characterized by LAMP.

This paper bridges these strands of the literature by providing an exhaustive analysis of

[3] Sveen and Weinke (2007) find that in the presence of firms specific capital a NK model with both staggered price and wages may generate multiple equilibria. Flaschel et al. (2008) analytically studies the determinacy properties of the model in the Erceg et al. (2000). However these papers consider a model with full asset market participation.

[4] In a similar framework, Furlanetto and Seneca (2012) concentrate their analysis on the dynamics of hours worked in response to a productivity shocks.

[5] Di Bartolomeo et al. (2010)estimate a NK model with external habits in consumption and LAMP for the G7 countries. They report an average fraction of non-Ricardian agents of about 26%.

the implications of LAMP for the design of optimal monetary policy in a NK model with price and wage stickiness.

2 The Model

2.1 Households

There is a continuum of households indexed by $i \in [0,1]$. Households in the interval $[0,\lambda]$ consume their available labor income in each period and do not hold assets. Households in the interval $(\lambda,1]$ hold assets and smooth consumption. The period utility function is common across households and it has the following separable form:

$$U_t = \Psi_t u\left[C_t\left(i\right)\right] - v\left[L_t\left(i\right)\right], \tag{1}$$

where $C_t(i)$ is agent *i's* consumption and $L_t(i)$ are hours worked. The functions u and v satisfy the usual properties,[6] while Ψ_t is a taste shock. We assume a continuum of differentiated labor inputs indexed by j $\in$ [0, 1]. As in Schmitt-Grohe and Uribe (2005), agent i supplies each possible type of labor input. Wage-setting decisions are made by labor type specific unions indexed by j $\in$ [0, 1]. Given the wage W_t^j fixed by union j, agents stand ready to supply as many hours to the labor market j, L_t^j, as required by firms, that is: $L_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} L_t^d$, where θ_w is the elasticity of substitution between labor inputs. Here L_t^d is aggregate labor demand and W_t is an index of the wages prevailing in the economy at time t. Formal definitions of labor demand and of the wage index can be found in the section devoted to firms. Agents are distributed uniformly across unions; hence aggregate demand for labor type j is spread uniformly across the households.[7] It follows that the individual quantity of hours worked, $L_t\left(i\right)$, is common across households, and we denote it as L_t. This must satisfy the time resource constraint $L_t = \int_0^1 L_t^j dj$. Combining the latter with labor demand we obtain $L_t = L_t^d \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} dj$. The labor market structure rules out differences in labor income between households without the need to resort to contingent markets for hours. The common labor income is given by $L_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} dj$.[8] Notice that each union pools the labor income of agents, leading

[6] The function u is increasing and concave and the function v is increasing and convex.

[7] Thus a share λ of the members of each union are non-Ricardian consumers, while the remaining portion is composed of Ricardian agents.

[8] Our assumption is similar to Woodford (2003) among others, but different from the one in Erceg et al. (2000). As in most of the literature on sticky wages, Erceg et al. (2000) assume that each agent is the monopolistic supplier of a single labor input. In this case, only households providing the same labor type will exhibit the same labor income. However, the assumption of complete markets and full insurance against the risk associated to labor income fluctuations, rule out differences in income between households. In our model this framework would imply a tractability problem, because non-Ricardian agents do not participates in the asset market, and thus cannot share the risk associated to labor income fluctuations.

Ricardian and non-Ricardian households to work for the same amount of time. This implies that under flexible wages the model does not fully nest Bilbiie (2008), where Ricardian and non-Ricardian agents are free to make different labor choices. In the on-line Appendix we investigate an alternative labor market arrangement where the wage depends solely on the preferences of Ricardian households, and agents are not forced to work for the same amount of time. In that case the model fully nests Bilbiie (2008), but our results are not affected, and, if anything, strengthened. So our findings do not depend on the chosen structure of the labor market.[9]

2.1.1 Ricardian Households

Ricardian agents face the following flow budget constraint in nominal terms :

$$E_t\Lambda_{t,t+1}X_{t+1} + \Omega_{S,t+1}V_t \leq X_t + L_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} dj + \Omega_{S,t}\left(V_t + P_tD_t\right) - P_tC_{S,t}. \quad (2)$$

In each period t, Ricardian agents (indicated with the subscript S) can purchase any desired state-contingent nominal payment X_{t+1} in period $t+1$ at the dollar cost $E_t\Lambda_{t,t+1}X_{t+1}$. The variable $\Lambda_{t,t+1}$ denotes the stochastic discount factor between period $t+1$ and t. A Ricardian agent has labor income $L_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} dj$ and holds a share $\Omega_{S,t}$ of the stock market value, V_t, of firms producing intermediate goods. Nominal dividends received for the ownership of firms are denoted by P_tD_t. Combining the FOCs with respect to $C_{S,t}, \Omega_{S,t}$ and X_{t+1} together with the arbitrage condition on asset markets, i.e. $E_t\Lambda_{t,t+1} \equiv (1+i_t)^{-1}$ we find the Euler equation for Ricardian agents:

$$\frac{1}{1+i_t} = E_t\left\{\beta \frac{\Psi_{t+1}u_c\left(C_{S,t+1}\right)}{\Psi_t u_c\left(C_{S,t}\right)} \frac{P_t}{P_{t+1}}\right\}. \quad (3)$$

2.1.2 Non-Ricardian Households

Non-Ricardian agents (indicated with the subscript H) do not enjoy firms' profits in the form of dividend income and cannot trade in the financial markets. The nominal budget constraint of a typical non-Ricardian household is thus simply given by:

$$P_tC_{H,t} = L_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} dj. \quad (4)$$

Agents belonging to this group consume disposable income in each period and delegate wage decisions to unions. For these reasons there are no first order conditions with respect to consumption and labor supply.

[9] The on-line Appendix is available at the web page of the authors. We take the framework spelled out in the text as the baseline because we regard it as a more rigorous microfoundation.

2.2 Wage Setting

Nominal wage rigidities are modeled according to the Calvo (1983) mechanism. In each period a union faces a constant probability $1-\xi_w$ of being able to reoptimize the nominal wage. As in Colciago (2011) the nominal wage newly reset at t, $\widetilde{W}_t$, is chosen to maximize a weighted average of agents' lifetime utilities. The weights attached to the utilities of Ricardian and non-Ricardian agents are $(1-\lambda)$ and λ, respectively. The union problem is

$$\max_{\widetilde{W}_t} E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \left\{ [(1-\lambda)\, u\left(C_{S,t+k}\right) + \lambda u\left(C_{H,t+k}\right)] - v\left(L_{t+k}\right) \right\}$$

subject to $L_t = \int_0^1 L_t^j dj$, (2) and (4). The FOC with respect to $\widetilde{W}_t$ is

$$E_t \sum_{s=0}^{\infty} (\beta \lambda_w)^{t+s} \Phi_{t,t+s} \left\{ \left[\lambda \frac{1}{MRS_{H,t+s}} + (1-\lambda) \frac{1}{MRS_{S,t+s}} \right] \frac{\widetilde{W}_t}{P_{t+s}} - \mu^w \right\} = 0 \qquad (5)$$

where $\Phi_{t,t+s} = v_L\left(L_{t+s}\right) L_{t+s}^d W_{t+s}^{\theta_w}$ and $\mu^w = \frac{\theta_w}{(\theta_w - 1)}$ is the, constant, wage mark-up in the case of wage flexibility. The variables $MRS_{H.t}$ and $MRS_{S,t}$ denote the marginal rates of substitution between labor and consumption of non-Ricardian and Ricardian agents respectively.

2.3 Firms

In each period t, a final good Y_t is produced by perfectly competitive firms combining a continuum of intermediate inputs $Y_t(z)$ according to the following standard *CES* production function: $Y_t = \left(\int_0^1 Y_t(z)^{\frac{\theta_p - 1}{\theta_p}} dz \right)^{\frac{\theta_p}{\theta_p - 1}}$, with $\theta_p > 1$. The competitive final good producers' demand of the intermediate good z and the price of the final good are thus equal to: $Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\theta_p} Y_t$ and $P_t = \left[\int_0^1 P_t(z)^{1-\theta_p} dz \right]^{\frac{1}{1-\theta_p}}$.

Intermediate inputs are produced by a continuum of monopolistic firms indexed by $z \in [0,1]$. The production technology is simply linear in labor services, $L_t(z)$:

$$Y_t(z) = A_t L_t(z), \qquad (6)$$

where A_t represents, exogenous, technology.

The labor input is defined as $L_t(z) = \left(\int_0^1 \left(L_t^j(z) \right)^{\frac{\theta_w - 1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w - 1}}$. Firm's z demand for labor type j and the aggregate wage index are then respectively: $L_t^j(z) = \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} L_t(z)$ and $W_t = \left(\int_0^1 \left(W_t^j \right)^{1-\theta_w} dj \right)^{1/(1-\theta_w)}$. Finally, given that the production function has constant return to scale, the nominal marginal cost, MC_t, is common across producers.

2.4 Price Setting

Intermediate producers set prices according to the same mechanism assumed for wage setting. Firms in each period have a fixed chance $1-\xi_p$ to re-optimize their price. A price setter takes

into account that the choice of its time t nominal price, $\widetilde{P}_t$, might affect not only current but also future profits. The FOC for price setting is:

$$E_t \sum_{k=0}^{\infty} \left(\beta \xi_p\right)^k \gamma_{t+k} P_{t+k}^{\theta_p} Y_{t+k} \left[\widetilde{P}_t - (1+\mu^p) MC_{t+k}\right] = 0, \tag{7}$$

which has the usual interpretation.[10] Notice that $\mu^p = (\theta_p - 1)^{-1}$ represents the net markup over the price which would prevail in the absence of nominal rigidities.

2.5 Aggregation and Market Clearing

Aggregate consumption is given by

$$C_t = \lambda C_{H,t} + (1-\lambda) C_{S,t}. \tag{8}$$

The variable $\Omega_t = (1-\lambda)\Omega_{S,t}$ represents aggregate asset holdings. In equilibrium $\Omega_t = 1$, thus each Ricardian agent has asset holdings equal to $\frac{1}{1-\lambda}$. The clearing of good and labor markets requires:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\theta_p} Y_t^d \quad \forall z \quad Y_t^d = Y_t; \tag{9}$$

$$L_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} L_t^d \quad \forall j \quad L_t = \int_0^1 L_t^j dj \tag{10}$$

where $Y_t^d = C_t$ represents aggregate demand, $L_t^j = \int_0^1 L_t^j(z)\, dz$ is total aggregate demand of labor input j and $L_t^d = \int_0^1 L_t(z)\, dz$ denotes firms' aggregate demand of the composite labor input L_t.

2.6 Pareto-efficient Equilibrium

For comparability with Bilbiie (2008) and Erceg et al. (2000), we follow the bulk of the literature (see Woodford, 2003) and impose an efficient steady state. To induce equality between the steady state marginal product of labor and the steady state marginal rate of transformation we assume that the Government subsidies firms by means of a constant employment subsidy, τ. Firms are also taxed through a constant lump-sum tax which leads to zero steady state profits. This device allows to study the welfare properties of the economy without resorting to a full second order approximation to the model equations. We assume a period utility of the form $\frac{\Psi_t C_{i,t}^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\phi}}{1+\phi}$ for $i = S, H$, where σ is the relative risk aversion (and the inverse intertemporal elasticity of substitution), while ϕ is the elasticity of marginal disutility of labor.

Next, we define the equilibrium of the model under flexible prices and wages. Appendix A.1 shows that the log-deviations from the efficient steady state of the efficient output, the

[10] The variable γ_t is the lagrange multiplier on Ricardian housheols nominal flow budget constraint. Thus γ_t represents the value of an additional dollar for ricardian households, who own the firm shares.

efficient real wage and the efficient real rate of interest are respectively given by:[11]

$$y_t^{Eff} = \frac{1+\phi}{\sigma+\phi} a_t + \frac{1}{(\sigma+\phi)} \psi_t, \tag{11}$$

$$\omega_t^{Eff} = a_t, \tag{12}$$

$$r_t^{Eff} = \sigma \left(\frac{1+\phi}{\sigma+\phi} \Delta a_{t+1} - \frac{\phi}{\sigma\,(\sigma+\phi)} \Delta \psi_{t+1} \right). \tag{13}$$

Assuming an AR(1) process for the logarithms of the exogenous state variables

$$a_t = \rho^a a_{t-1} + \varepsilon_t^a \tag{14}$$

$$\psi_t = \rho^\psi a_{t-1} + \varepsilon_t^\psi \tag{15}$$

fully specifies the dynamics of the log-deviations from the efficient equilibrium.

2.7 The Log-linear model

The following equations summarize log-linear equilibrium dynamics:

$(M1)$	$\pi_t = \beta E_t \pi_{t+1} + \kappa_p \tilde{\omega}_t$	$NKPC$
$(M2)$	$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_w [(\sigma+\phi) x_t - \tilde{\omega}_t]$	$Wage\ Inflation\ Curve$
$(M3)$	$\tilde{\omega}_t = \tilde{\omega}_{t-1} + \pi_t^w - \pi_t - \Delta \omega_t^{Eff}$	$Real\ Wage\ Gap$
$(M4)$	$x_t = E_t x_{t+1} - \frac{1}{\sigma} E_t \left(i_t - \pi_{t+1} - r_t^{Eff} \right) - \frac{\lambda}{(1-\lambda)} E_t \Delta \tilde{\omega}_{t+1}$	$IS\ curve$

Equation $(M1)$ is the NKPC obtained from the firms' price setting problem. The variable $\tilde{\omega}_t = \omega_t - \omega_t^{Eff}$ represents the *real wage gap,* which is defined as the gap between the current and the efficient equilibrium real wage. Given the linear in labor production function it follows that $mc_t = \omega_t - y_t + l_t = \omega_t - a_t = \tilde{\omega}_t$, i.e. the real wage gap is equal to the log-deviations of the real marginal cost from the efficient steady state. For this reason $\tilde{\omega}_t$ appears on the RHS of equation $(M1)$. The *real wage gap* in the NKPC identifies a *labor demand gap* being equal to the difference between the current wage and the marginal productivity of labor. The parameter $\kappa_p = \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}$ is the slope of the NKPC. Equation $(M2)$ is a wage inflation curve, similar to that in Erceg et al. (2000) with slope $\kappa_w = \frac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w}$. Symmetrically to the NKPC, the term $[(\sigma+\phi)x_t - \tilde{\omega}_t]$ in $(M2)$ identifies a *labor supply gap* being equal to the difference between the average (across agent types) marginal rate of substitution between labor and consumption and the real wage. Given the period utility, the production function, the market clearing and the definition of efficient output, it follows that:

$$\begin{aligned} & (1-\lambda)\, mrs_{S,t} + \lambda mrs_{H,t} - \omega_t \\ = \; & [(\sigma+\phi)\, y_t - \phi a_t - \psi_t] - \omega_t = (\sigma+\phi)\, x_t - \tilde{\omega}_t, \end{aligned} \tag{16}$$

[11] We denote log-deviations by lower case letters, and ω stands for the log-deviation of the real wage.

where $x_t = y_t - y_t^{Eff}$ denotes the *output gap*, i.e. the gap between actual output and the efficient output. The parameters ϕ and σ are respectively the elasticity of intertemporal substitution in labor supply and in consumption. Equation $(M3)$ simply provides the definition of the real wage gap in terms of wage and price inflation and $\Delta\omega_t^{Eff} = \omega_t^{Eff} - \omega_{t-1}^{Eff}$.

Equations $(M1) - (M3)$ are identical to those which would characterize a fully Ricardian NK model with price and wage stickiness, as in Erceg et al. (2000).[12] Notice that the heterogeneity between households does not affect wage inflation dynamics.[13]

Aggregating the Euler equation of Ricardian agents with the budget constraint of non-Ricardian agents delivers the IS curve, equation $(M4)$.[14] The latter differs from a standard IS equation because of the extra term $\frac{\lambda}{1-\lambda} E_t \Delta\tilde{\omega}_{t+1}$, which represents the expected growth of the real wage gap. The wage gap affects aggregate demand relative to the efficient allocation through the consumption of non-Ricardian consumers and for this reason appears in the IS curve.

Note that our framework encompasses the models in Erceg et al. (2000) and Bilbiie (2008). Indeed, the extra term in the IS disappears if the model is fully Ricardian (i.e. if $\lambda = 0$) as in Erceg et al. (2000). Further, under nominal wage flexibility the labor supply gap is nil and equation (16) implies a strict proportionality between the wage gap and the output gap given by:

$$\tilde{\omega}_t - (\sigma + \phi)\, x_t = 0. \tag{17}$$

By substituting the latter into equation $M4$ the IS curve can be rewritten solely in terms of the output gap, as in Bilbiie (2008).

It is worth stressing that the supply side of the model, constituted by equations $(M1) - (M3)$, is isomorphic to that of a fully Ricardian economy with sticky prices and wages. On the contrary the demand side of the model, represented by equation $(M4)$, is affected by the degree of asset market participation and hence characterizes a LAMP economy with sticky wages and prices.

To close the model the behavior of the nominal interest rate needs to be specified. To this end we will consider both interest rate rules and a welfare maximizing policy. We will show that, in both cases, the presence of non-Ricardian agents does not fundamentally alter the design of monetary policy once nominal wage stickiness is considered.

[12] The only minor difference with Erceg et al. (2000) is in the expression for κ_w. This is due to the different assumption regarding the labor market explained in footnote 8.

[13] As emphasized in Colciago (2010), the is due to the fact that the union maximizes a weighted average of agents utilities.

[14] Please see Appendix A.2 for analytical details

3 Slopes of the IS curve and the determinacy properties of simple interest rate rules

In this section we explore the role played by nominal wage stickiness for the dynamics of the model and for the determinacy properties of simple interest rate rules.

We will naturally compare our results to those in Bilbiie (2008), who considers a model with flexible wages. The aforementioned author shows that, when asset market participation is restricted beyond a certain threshold, the slope of the IS curve may turn positive leading to what he calls and *Inverted Aggregate Demand Logic* (IADL). In the parameter space where the IADL holds, aggregate demand increases with the real interest rate. Importantly, the inversion of the slope of the IS curve requires an inversion of the Taylor Principle for monetary policy to support a unique *REE*. Thus, given a simple policy rule as

$$i_t = \phi_\pi E_t \pi_{t+1}, \tag{18}$$

in the IADL region the inflation response coefficient, ϕ_π, must be less than 1 to induce equilibrium uniqueness. In Leeper's (1991) words, monetary policy should be passive. Moreover, Bilbiie (2008) claims that the IADL case is empirically relevant and proposes an intriguing reinterpretation of the great inflation versus great moderation debate on the basis of the Inverted Taylor Principle.

In the remainder of this section, we show that in a model with nominal wage rigidity most of these results are overturned. In particular wage stickiness confines the IADL to extreme parameterizations, re-establishing the relevance of the *Taylor Principle* for the conduct of monetary policy.

3.1 The slope of the IS curve

To make our point fully transparent we consider three alternative scenarios resulting from polar parameterization of the model: 1) flexible prices and sticky wages ; 2) flexible wages and sticky prices; 3) sticky prices and wages.

1. Flexible Prices and Sticky Wages. In this case, given that firms' are always on their labor demand schedule, $\omega_t = a_t$, the real wage-gap is zero. Hence, there is no NKPC, i.e. equation $(M1)$. As a result the IS curve, equation $(M4)$, coincides with the standard one. It does not depend on λ, ruling out the possibility of the inversion of the slope of the IS curve. The intuition for this result is as follows. Under flexible prices firms' price markup, real marginal costs and profits are constant. Thus, consumption of both agents deviates from the efficient steady state only because of fluctuations in labor income. Since the latter is common across households and consumption of Ricardian agents must obey an Euler equation, the

resulting setup is isomorphic to a fully Ricardian framework.[15]

2. Flexible Wages and Sticky Prices. This case amounts to that considered by Bilbiie (2008). Appendix A.2.1 shows that the IS equation can be expressed as:

$$x_t = E_t x_{t+1} - \frac{\left(\delta^{fw}\right)^{-1}}{\sigma} E_t\left(i_t - \pi_{t+1} - r_t^{Eff}\right) \tag{19}$$

where $\delta^{fw} = 1 - \frac{\lambda(\sigma+\phi)}{1-\lambda}$.[16] The slope of the IS becomes positive if $\delta^{fw} < 0$ which requires $\lambda > \bar{\lambda}^{fw} = \frac{1}{1+\sigma+\phi}$.

3. Sticky Prices and Sticky Wages. The counterpart of equation (19) is:

$$\begin{aligned} x_t &= E_t x_{t+1} - \frac{(\delta^{sw})^{-1}}{\sigma} E_t\left(i_t - \pi_{t+1} - r_t^{Eff}\right) + \\ &\frac{\lambda}{1-\lambda}\frac{(\delta^{sw})^{-1}}{1+\beta+\kappa_w} E_t\left[\Delta\pi_{t+1}^w - \Delta\tilde{\omega}_{t+1} - \beta\left(\Delta\tilde{\omega}_{t+1} + \Delta\pi_{t+2}^w\right)\right] \end{aligned} \tag{20}$$

where $\delta^{sw} = 1 - \frac{\lambda(\sigma+\phi)}{1-\lambda}\frac{\kappa_w}{1+\beta+\kappa_w}$. Note that under flexible wages, i.e., $\xi_w = 0$, then $\kappa_w \to \infty$, and $\delta^{sw} \to \delta^{fw}$. Equation (20) thus collapses to (19). We state the main finding of this section in Proposition 1.

Proposition 1: The slope of the IS curve. *Under sticky prices and wages the slope of the IS curve, i.e., δ^{sw}:(i) is always larger than the one under flexible wages, i.e. δ^{fw}; (ii) it increases with the degree of wage stickiness; (iii) it turns positive if $\lambda > \bar{\lambda}^{sw} = \frac{1}{1+(\sigma+\phi)\frac{\kappa_w}{1+\beta+\kappa_w}}$, where the threshold value $\bar{\lambda}^{sw}$ increases with the degree of wage stickiness.*

Proof See Appendix A.2.2.

As in the flexible wages case there exists a threshold value $\bar{\lambda}^{sw} \in [0,1]$ such that we can define a region where the IADL holds. Only in the limiting case in which wages are fixed (i.e., $\xi_w \to 1 \Longrightarrow \kappa_w \to 0$) the slope of the IS schedule never changes sign, regardless of the value of λ. Importantly, the value $\bar{\lambda}^{sw}$ increases as the average duration of wage contract gets longer. Proposition 1 leads to the following corollary.

Corollary 1: The IADL region. *Nominal wage stickiness severely restricts the IADL region, and confines it to extreme parameterizations.*

Our baseline calibration implies a threshold value $\bar{\lambda}^{sw} = 0.71$[17]. The IADL holds if the share of non-Ricardian agents is larger than 71%. Compared to some empirical evidence

[15] This result does not rely on the assumption of an efficient steady state. If steady state profits are non zero, agents have different steady state levels of consumption. In this case the IS curve would be affected by the share of non-Ricardian consumers, but it can be shown that the interest rate elasticity of aggregate demand cannot turn positve.

[16] The expression is slightly different from Bilbiie (2008) again because of our assumption on the labor market.

[17] Section 4.2 reports the baseline calibration. Throughout the current section, as in the coming Figures, we will use the parameters' values reported there.

by Campbell and Mankiw that places this at around 40–50% for the US economy for data running up to the mid eighties, or to data from the 1989 Survey of Consumer Finances which shows that 59% of US population had no interest-bearing financial assets, this value seems implausibly large.[18] Notice that under flexible wages $\bar{\lambda}^{fw} = 0.17$, i.e. 4 and half times smaller.

3.2 Intuition and the role of the labor market

Rearranging equation $(M2)$ delivers the wage schedule

$$WS^{sw} : \omega_t = \Gamma\kappa_w(\sigma + \phi)l_t + \Gamma\Upsilon_t + \Gamma\kappa_w \left[1 + (\sigma + \phi)\left(1 - \frac{1+\phi}{\sigma+\phi}\right)\right] a_t$$

where $\Gamma = \frac{\xi_w}{1+\beta\xi_w^2}$ and $\Upsilon_t = [\omega_{t-1} - \pi_t + \beta E_t\omega_{t+1} + \beta E_t\pi_{t+1}]$. Notice that the product $\Gamma\kappa_w = \frac{(1-\beta\xi_w)(1-\xi_w)}{1+\beta\xi_w^2}$ is decreasing in the degree of wage stickiness and equals 1 in the case of flexible wages. In the latter case, the wage schedule reduces to

$$WS^{fw} : \omega_t = (\sigma + \phi)l_t + \left[1 + (\sigma + \phi)\left(1 - \frac{1+\phi}{\sigma+\phi}\right)\right] a_t$$

As expected, wage stickiness dampens the sensitivity of the real wage to changes in hours. Also, the coefficient on hours increases linearly with the elasticity of marginal disutility of labor. Figure 1 depicts curves WS^{sw} and WS^{fw} in the space (L_t, ω_t), where we also represent labor demand. For the reasons mentioned above, the wage schedule is flatter under wage stickiness. To see how the interaction between non Ricardian agents and wage stickiness affects the slope of the IS curve consider the following mental experiment, which builds on Bilbiie (2008).

Consider an increase in the interest rate. Ricardian agents reduce their demand, while firms which cannot change price reduce labor demand. This is depicted in Figure 1, where the labor demand curve shifts inward in L_2^d (we assumed that inflation and expectations of future variables, i.e. the variable Υ_t, remain unchanged in the face of the change in the interest rate). Under flexible wages this translates into a large reduction in the real wage and to a modest change in hours, and the more so the higher the elasticity of the marginal disutility of working, ϕ, and the inverse intertemporal elasticity of substitution in consumption, σ. The decrease in the real wage depresses demand by non-Ricardian agents and reinforces the effects on aggregate demand due to the initial increase in the real interest rate. However, as emphasized by Bilbiie, this effect is not monotonic in λ.

The sizeable decrease in the real wage, and hence in marginal costs, together with the small change in hours, and hence in output and sales, imply a potential increase in profits.

[18] Note that we are choosing a parameterization against our argument, since we assume high values for $\sigma = 2$ and $\phi = 3$, and an average duration of wage contracts of 3 quarters. By choosing a rather standard alternative calibration, as log-utility in consumption and labor, and an average duration for wage contracts of 4 quarters, then $\bar{\lambda}^{sw}$ would have been equal to 0.92.

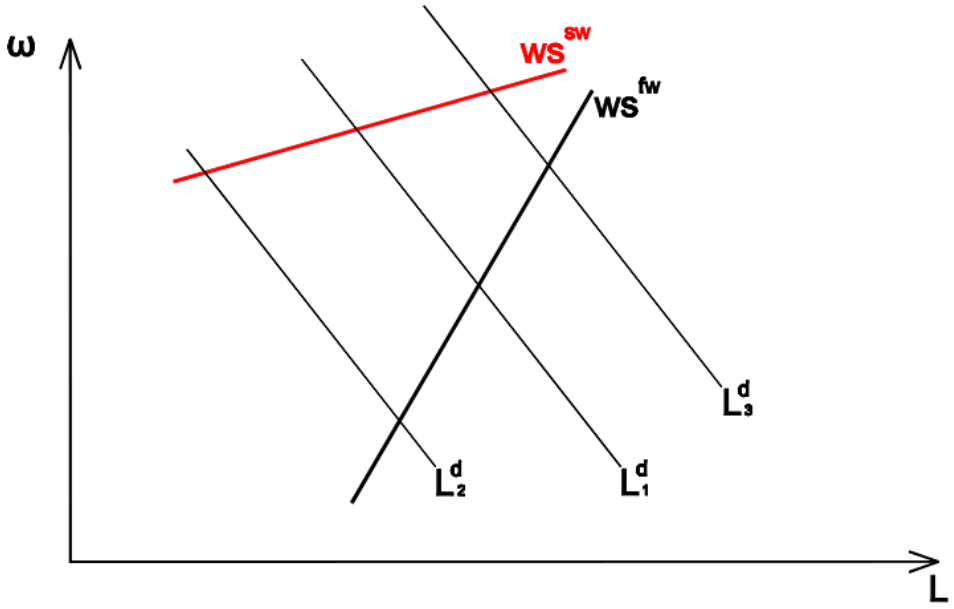

Figure 1: The wage schedule under sticky wages (WS^{sw}) and flexible wages (WS^{fw}) and the equilibrium in the labor market.

This leads, in turn, to a positive income effect on Ricardian agents. The latter is stronger the larger λ, since Ricardian agents would obtain a higher individual dividend income.

If asset market participation is restricted enough ($\lambda > \lambda^{fw}$), the positive income effect may counteract the substitution effect induced by the interest rate change and finally lead to an increase in aggregate demand.[19] As a result labor demand would shift rightward, in L_3^d, leading to an equilibrium with higher-that-initial real wage and output, i.e. where the initial interest rate increase is associated with higher aggregate demand.

Consider now the case of sticky wages. The inward shift in labor demand due to the reduction in consumption by Ricardian agents, results in a large response in hours worked together with a modest reduction in the real wage. The potential negative wealth effect is thus less likely and, however, dampened with respect to the case of flexible wages. For this reason, under wage stickiness the inversion of the IS curve requires a much larger share of non Ricardian agents ($\lambda > \lambda^{sw}$) to magnify the, eventual, wealth effect at the individual level. To see the link between the labor market and the likelihood of an inversion of the slope of the IS curve, notice that

$$\delta^{sw} = 1 - \frac{\lambda}{1-\lambda} \underbrace{\Gamma\kappa_w(\sigma+\phi)}_{\text{slope of wage schedule}}$$

For any given share of non Ricardian agents, the inversion of the slope of the IS curve becomes more likely as wages become more flexible, i.e. as $\Gamma\kappa_w$ approaches one. Notice that

[19] Notice that λ^{fw} is lower the steeper the wage schedule, i.e. the higher are σ and ϕ, as emphasized above.

our results do not depend on having a union which prevents Ricardian agents from substituting labor intertemporally. As mentioned earlier, in the on-line Appendix we propose an alternative labor market structure where the real wage depends solely on the preferences of asset holders and agents are free to make different labor choices. In this case changes in the interest rate or in dividend income affect the willingness to supply labor by Ricardian consumers, reinforcing the link between the asset and the labor market. Nevertheless, we show that the area where the IADL holds is further restricted with respect to that we obtain under the baseline design of the labor market.

3.3 Determinacy analysis

In this section, we prove analytically the condition for the determinacy of the *REE*, despite the dynamic system is 4th order.[20] As in Bilbiie (2008), sticky prices lead to the inversion of the Taylor principle in the IADL region of the parameter space. Similarly to the section above, with staggered wages the inversion of the Taylor principle is confined to implausible parameterizations.

3.3.1 Forward Looking Rule

It is instructive to start with same the polar case of sticky wages and flexible prices.

Proposition 2: Flexible prices and sticky wages. *Under flexible prices and sticky wages, and the policy rule* (18) *the rational expectation equilibrium is unique iff* $\phi_\pi \in \left(1, 1+\frac{2\sigma(1+\beta)}{\kappa_w(\sigma+\phi)}\right)$, *i.e. iff the Taylor Principle is satisfied.*

Proof See Appendix A.3.1.

As expected from the discussion in the previous section, this case is isomorphic to a fully Ricardian economy, there is no inversion of the IS slope and hence the standard Taylor principle applies.

The following proposition holds, instead, in the case of both sticky wages and sticky prices.

Propositon 3: Forward-looking price inflation targeting rule. *Let* $i_t = \phi_\pi \pi_{t+1}$. *The REE is determinate iff:*

1) either $\phi_\pi \in \left(1; \bar{\phi}_\pi^{FR}\right)$ if $\bar{\phi}_\pi^{FR} > 1$;
2) or $\phi_\pi \in \left(\bar{\phi}_\pi^{FR}; 1\right)$ if $\bar{\phi}_\pi^{FR} < 1$;
where $\bar{\phi}_\pi^{FR} = 1 + \frac{2\sigma(1+\beta)\left[2(1+\beta)+\kappa_p+\kappa_w-\frac{\lambda}{1-\lambda}\kappa_w(\sigma+\phi)\right]}{\kappa_w\kappa_p(\sigma+\phi)}$.

[20] We follow the strategy of transforming the polynomial derived from the characteristic equation (see Samuelson, 1941, and section 4 in Felippa and Park, 2004).

Proof. See Appendix A.3.3.

A necessary condition for the inversion of the Taylor principle it is thus given by $\bar{\phi}_\pi^{FR} < 1$, since otherwise the standard Taylor principle applies. The condition $\bar{\phi}_\pi^{FR} < 1$, however, defines a threshold value $\bar{\lambda}^{FR}$, since:

$$\bar{\phi}_\pi^{FR} \lesseqgtr 1 \Leftrightarrow \bar{\lambda}^{FR} \gtreqless \frac{2(1+\beta)+\kappa_w+\kappa_p}{\kappa_w(\sigma+\phi)+2(1+\beta)+\kappa_w+\kappa_p}. \tag{21}$$

Figure 1 depicts determinacy areas in the space (λ, ϕ_π). The solid curved line represents the threshold value $\bar{\phi}_\pi^{FR}$ described in Proposition 3 as a function of λ. Note that $\bar{\phi}_\pi^{FR}$ decreases with the degree of LAMP, λ.

If $\lambda = 0$, $\bar{\phi}_\pi^{FR} > 1$, and the standard Taylor principle holds. As λ increases, however, $\bar{\phi}_\pi^{FR}$ decreases, and the interval for ϕ_π described in case 1) of Proposition 3 shrinks and eventually becomes empty when $\lambda = \bar{\lambda}^{FR}$. As λ increases further, then, condition 2) applies and the interval for ϕ_π in the inverted Taylor principle case enlarges, becoming $\phi_\pi \in (-\infty; 1)$ at the limit when $\lambda \to 1$.

Moreover, wage stickiness shifts to the right the $\bar{\phi}_\pi^{FR}$ curve, because $\frac{\partial \bar{\phi}_\pi^{FR}}{\partial \kappa_w} < 0$.[21] Hence, the threshold value $\bar{\lambda}^{FR}$ increases with the degree of wage stickiness. As κ_w tends to 0, i.e. with fix wages, then $\bar{\phi}_\pi^{FR} \longrightarrow \infty$, and the Taylor principle is restored, because Proposition 3 guarantees determinacy if and only if $\phi_\pi \in (1; \infty)$. Indeed, in the limiting case of fix wages the slope of the IS schedule does not change sign for any value of $\lambda \leq 1$. In the case of flexible wages, instead, $(\kappa_w \longrightarrow \infty)$, the threshold value becomes $\bar{\lambda}^{FR,fw} = \frac{1}{\sigma+\phi+1}$, that is lower than $\bar{\lambda}^{FR}$ and coincides with $\bar{\lambda}^{fw}$, i.e., the threshold value for the inversion of the slope of the IS curve and, hence, for the definition of the IADL region.

Furthermore, since $\frac{\partial \bar{\lambda}^{FR}}{\partial \kappa_p} > 0$, the threshold value, $\bar{\lambda}^{FR}$ decreases with the degree of price stickiness (lower κ_p). In the limiting case of fully flexible prices $(\kappa_p \longrightarrow \infty)$, $\bar{\phi}_\pi^{FR} \longrightarrow 1 + \frac{2\sigma(1+\beta)}{\kappa_w(\sigma+\phi)}$ and Proposition 3 collapses to Proposition 2.

Corollary 2: Numerical results. Let $i_t = \phi_\pi \pi_{t+1}$. *Under sticky wages and sticky prices the Taylor Principle is a necessary condition for equilibrium determinacy for all the plausible parameterizations of the share of non-Ricardian agents.*

To give a quantitative flavour of Proposition 3, Figure 2 depicts indeterminacy regions in the parameter space (ϕ_π, λ), obtained by numerical simulations. Panel (i) displays the case of flexible wages. A share of non-Ricardian agents larger than 0.167 requires the inverted Taylor Principle to ensure equilibrium uniqueness. Thus, *"the inverted Taylor principle holds 'generically' (i.e., if we exclude some extreme values for some of the parameters)"*(Bilbiie, 2008, p. 180). Panel (ii) refers to the case of sticky wages, with an average duration of wage

[21] Recall that the higher the degree of stickiness in wages, the lower is κ_w.

contracts equal to three quarters.[22] Unless the share of non-Ricardian consumers assumes values which are not compatible with any possible estimate, the Taylor Principle leads to equilibrium determinacy. Thus, wage stickiness *"generically"* restores standard determinacy conditions. The intuition for this result is straightforward. When the relationship between aggregate demand and the real interest rate has the conventional sign, a real interest rate increase is required to rule out increase in aggregate demand generated by sunspot variations in output.

Finally, it is worth to notice that numerically the curve that defines $\bar{\phi}_\pi^{FR}$ in the space (ϕ_π, λ) is almost horizontal at $\bar{\lambda}^{FR}$, meaning that the Taylor principle (i.e., the condition $\phi_\pi \gtrless 1$) is what really matters to define the uniqueness of the REE, while $\bar{\phi}_\pi^{FR}$ numerically matters only in determining $\bar{\lambda}^{FR}$, i.e., the threshold value for λ where the inversion of the Taylor principle occurs.

3.3.2 Contemporaneous Rules

We now consider the contemporaneous rule $i_t = \phi_\pi \pi_t$.

Proposition 4: Current price inflation targeting rule. *Let $i_t = \phi_\pi \pi_t$. The REE is determinate iff:*

1) either $\phi_\pi > \max\left\{1; \bar{\phi}_\pi^{a,CR}; \bar{\phi}_\pi^{b,CR}\right\}$;
2) or $\phi_\pi < \min\left\{1; \bar{\phi}_\pi^{a,CR}; \bar{\phi}_\pi^{b,CR}\right\}$;
where $\bar{\phi}_\pi^{a,CR} = -1 - \frac{2\sigma(1+\beta)\left[2(1+\beta)+(\kappa_p+\kappa_w)-\frac{\lambda}{1-\lambda}(\sigma+\phi)\kappa_w\right]}{(\sigma+\phi)\kappa_p\kappa_w}$ and $\bar{\phi}_\pi^{b,CR} = \frac{\sigma(1-\beta)\left[\frac{\lambda}{1-\lambda}(\sigma+\phi)\kappa_w-(\kappa_p+\kappa_w)\right]}{(\sigma+\phi)\kappa_p\kappa_w}$.

Proof. See Appendix A.3.5.[23]

This case is different from the previous one. Figure 3 visualizes the determinacy regions in the (ϕ_π, λ) space. Note that the two curves defining $\bar{\phi}_\pi^{a,CR}$ and $\bar{\phi}_\pi^{b,CR}$ are now both increasing, rather than decreasing, in λ. The two cases 1) and 2) in proposition 4 characterize two frontiers: $\max\left\{1; \bar{\phi}_\pi^{a,CR}; \bar{\phi}_\pi^{b,CR}\right\}$ and $\min\left\{1; \bar{\phi}_\pi^{a,CR}; \bar{\phi}_\pi^{b,CR}\right\}$, respectively. Determinacy, thus, occurs below the lower frontier ($\max\left\{1; \bar{\phi}_\pi^{a,CR}; \bar{\phi}_\pi^{b,CR}\right\}$) and above the upper frontier ($\min\left\{1; \bar{\phi}_\pi^{a,CR}; \bar{\phi}_\pi^{b,CR}\right\}$). In this case, it is impossible to define an "inversion of the Taylor principle". On the one hand, for each value of λ, there exist two values of ϕ_π, such that the

[22] To understand that Figure 1 and panel (ii) of Figure 2 are equivalent, recall that Proposition 3 only focuses on the necessary and sufficient conditions for determinacy of the REE, and do not consider the difference between indeterminacy and instability whenever the REE is not unique. Moreover, given our calibration, the curve that defines $\bar{\phi}_\pi^{FR}$ in the space (ϕ_π, λ) is almost horizontal at $\bar{\lambda}^{FR}$ and it bends only for extreme values of λ (or ϕ_π).

[23] The Appendix shows that this Proposition assumes: $\frac{\sigma(1-\beta)\left[(\sigma+\phi)\kappa_p\kappa_w+4\sigma(1+\beta)^2\right]}{(\sigma+\phi)\kappa_p\kappa_w\sigma[1+3\beta]} < 1$, that holds for value of β sufficiently close to one and for our benchmark calibration.

REE is unique: one satisfies the Taylor principle, while the other does not. On the other hand, we can define threshold values for the share of non-Ricardian agents, such that: if $\lambda < \bar{\lambda}^{a,CR}$, then $\phi_\pi > 1$ is a sufficient (but not necessary) condition for the uniqueness of the REE; if $\lambda > \bar{\lambda}^{b,CR}$, then $\phi_\pi < 1$ is a sufficient (but not necessary) condition for the uniqueness of the REE. These threshold values are given by the intersection between $\phi_\pi = 1$, and $\bar{\phi}_\pi^{a,CR}$ and $\bar{\phi}_\pi^{b,CR}$, respectively (see Appendix A.3.5).[24]

Moreover, wage stickiness shifts to the left both the $\bar{\phi}_\pi^{a,CR}$ and the $\bar{\phi}_\pi^{b,CR}$ curves in Figure 3, because $\frac{\partial \bar{\phi}_\pi^{a,CR}}{\partial \kappa_w} < 0$ and $\frac{\partial \bar{\phi}_\pi^{b,CR}}{\partial \kappa_w} < 0$. Hence, both the threshold values $\bar{\lambda}^{a,CR}$ and $\bar{\lambda}^{b,CR}$ increase with the degree of wage stickiness. Again as κ_w tends to 0 (limiting case of fix wages), Proposition 4 collapses to the standard Taylor principle ($\phi_\pi > 1$), because ϕ_π^a and ϕ_π^b tend to $(-\infty)$.

Furthermore, both $\bar{\lambda}^{a,CR}$ and $\bar{\lambda}^{b,CR}$ are decreasing with the degree of price stickiness (i.e., increases with κ_p). In the limiting case of fully flexible prices ($\kappa_p \longrightarrow \infty$), Proposition 4 defines the following condition for determinacy: either $\phi_\pi > 1$ or $\phi_\pi < -1 - \frac{2\sigma(1+\beta)}{(\sigma+\phi)\kappa_w}$. Thus, as in the case of a forward rule, in an economy with flexible prices and sticky wages, the degree of LAMP has no effect on the shape of the determinacy regions, and the Taylor principle holds, at least for positive values of ϕ_π.

It is worth noting that if $\beta \cong 1$, then $\bar{\phi}_\pi^{b,CR} \cong 0$, so that the standard Taylor principle holds for positive values of ϕ_π, because the REE is always indeterminate for $0 < \phi_\pi < 1$, as in the standard case.

Finally, if there are no non-Ricardian consumers, i.e., $\lambda = 0$, then both $\bar{\phi}_\pi^{a,CR}$ and $\bar{\phi}_\pi^{b,CR}$ are negative, so that case 1) in Proposition 4 restores the standard Taylor principle for positive value of ϕ_π.

To sum up, our analysis shows that, given a contemporaneous inflation targeting rule, it is not appropriate to refer to an "inversion of the Taylor principle". First, for each value of λ, there exist two values of ϕ_π, such that the REE is unique: one satisfies the Taylor principle, while the other is negative. Second, if the share of non-Ricardian agents is lower than a certain threshold, i.e., $\bar{\lambda}^{a,CR}$, then $\phi_\pi > 1$ is a sufficient condition for the uniqueness of the REE.

From a numerical point of view, Figure 4 shows that the result in the Corollary 2 is confirmed also in the case of contemporaneous rule: under sticky wages, the Taylor principle is a necessary and sufficient condition for the uniqueness of the REE, for all the plausible values of λ (abstracting from highly negative values of ϕ_π). This is not the case instead when wages are flexible, since the $\bar{\phi}_\pi^{a,CR}$ curve shifts downward. Indeed, $\bar{\lambda}^{a,CR} = 0.831$, given our

[24] Depending on parameter values $\bar{\lambda}_\pi^{a,CR}$ can be larger or smaller than $\bar{\lambda}_\pi^{b,CR}$. In general, $\bar{\lambda}_\pi^{a,CR} \lesseqgtr \bar{\lambda}_\pi^{b,CR}$ iff $\frac{(1-\beta)(1+\beta)^2}{\beta} \lesseqgtr \frac{(\sigma+\phi)\kappa_p\kappa_w}{\sigma}$. Hence for values of β sufficiently close to 1, then $\bar{\lambda}_\pi^{a,CR} < \bar{\lambda}_\pi^{b,CR}$, as in Figure 3.

standard calibration, and it lowers to 0.197 in the case of flexible wages.[25]

In a model that features both sticky wages and sticky prices, it is natural to consider also a monetary policy rule that targets both price and wage inflation, as $i_t = \phi_\pi \pi_t + \phi_{\pi^w} \pi_t^w$. Both Erceg et al. (2000) and Galí (2008) numerically study the properties of such a rule. Galí (2008) numerically shows that, for $\phi_\pi, \phi_{\pi^w} \in (0, \infty)$, the condition $\phi_\pi + \phi_{\pi^w} > 1$ is necessary and sufficient (see also Flaschel et al., 2008) for the uniqueness of the REE. Proposition 5 shows analytically that such a condition is still crucial in a model with LAMP.

Proposition 5: Price Inflation and Wage Inflation Targeting Rule *Let* $i_t = \phi_\pi \pi_t + \phi_{\pi^w} \pi_t^w$. *A necessary condition for the REE to be determined is either* $(\phi_\pi + \phi_{\pi^w}) > \max\left\{1, \bar{\phi}_{\pi,\pi^w}\right\}$ *or* $(\phi_\pi + \phi_{\pi^w}) < \min\left\{1, \bar{\phi}_{\pi,\pi^w}\right\}$ *where* $\bar{\phi}_{\pi,\pi^w} = -1 - \frac{2\sigma(\beta+1)\left[2(\beta+1)+(\kappa_p+\kappa_w)-\frac{\lambda}{1-\lambda}(\sigma+\phi)\kappa_w+\frac{1}{\sigma}(\sigma+\phi)\kappa_w\phi_{\pi^w}\right]}{(\sigma+\phi)(1+\phi_\pi+\phi_{\pi^w})\kappa_p\kappa_w}$.

Proof. See Appendix A.3.6.

The conditions in Proposition 5 refers now to the sum $(\phi_\pi + \phi_{\pi^w})$. $\bar{\phi}_{\pi,\pi^w}$ is increasing in λ. Thus, it is possible to define a threshold for the share of non-Ricardian agents, such that if λ is lower than this threshold, then $(\phi_\pi + \phi_{\pi^w}) > 1$ is a necessary condition for the uniqueness of the REE for positive values of ϕ_π and ϕ_{π^w}. This is always true for either fix wages or flexible prices.

4 Optimal Monetary Policy

In this section we look at the optimal policy problem, cast in the standard linear quadratic framework (see Woodford, 2003). First, we derive the welfare loss function and describe the relevant trade-offs faced by the monetary authority. Next, we characterize optimal monetary policy under full commitment, which we take as a benchmark for the remainder of the analysis. Finally, as in Erceg et al. (2000), we consider strict targeting rules, i.e. rules which fully close one of the gaps in the welfare loss, and optimal simple interest rate rules *a là* Schmitt-Grohé and Uribe (2007).

4.1 The Welfare Loss Function

We assume that the central bank maximizes a convex combination of the utilities of two types of households, as in Bilbiie (2008). Weights correspond to the relative importance of agents' groups in the economy. In this case the period welfare function is given by:

$$W_t = \Psi_t \left[\lambda u\left(C_{H,t}\right) + (1-\lambda)\, u\left(C_{S,t}\right)\right] - v\left(L_t\right) \tag{22}$$

[25] Moreover, similarly to the case of the forward-looking rule, Figure 4 reveals that the curve $\bar{\phi}_\pi^{a,CR}$ is flat at $\bar{\lambda}^{a,CR}$, given our standard calibration.

Proposition 6: The aggregate welfare loss function. *The aggregate welfare loss function approximated at second-order around the efficient steady state is given by:*

$$L = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\frac{(\sigma-1)\,\lambda}{1-\lambda}\tilde{\omega}_t^2 + (\sigma+\phi)\,x_t^2 + \frac{\theta_w}{\kappa_w}\left(\pi_t^w\right)^2 + \frac{\theta_p}{\kappa_p}\pi_t^2\right) \tag{23}$$

Proof. See Appendix A.4.

The interaction between nominal wage stickiness and non-Ricardian agents implies that the loss function is characterized by the additional term $\frac{(\sigma-1)\lambda}{1-\lambda}\tilde{\omega}_t^2$ with respect to the loss function of a fully Ricardian model. The wage gap enters the loss function for the same reasons it appears into the IS equation (20): deviations of the real wage from its efficient counterpart lead to deviations of aggregate demand from the efficient level.[26]Note that when $\lambda = 0$ the welfare loss function reduces to that in Erceg et al. (2000).

When wages are flexible, wage inflation does not affect welfare. Moreover, the *labor supply gap* is nil, because the real wage equals the average marginal rate of substitution between consumption and labor. In this case equation (17) holds and by closing the output gap the central bank automatically closes the wage gap. Further note that substituting (17) into (23), the loss function reduces to (see Appendix A.4.1 for details)

$$L = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left((\sigma+\phi)\left(1+\frac{(\sigma-1)\,(\phi+\sigma)\,\lambda}{1-\lambda}\right)x_t^2 + \frac{\theta_p}{\kappa_p}\pi_t^2\right), \tag{24}$$

which has a form similar to that in Bilbiie (2008), and collapses to the standard text-book welfare-loss for $\lambda = 0$.

How monetary policy should be conducted in the LAMP economy with price and wage stickiness? Let us consider the trade-offs faced by monetary policy in the aftermath of a technology shock. We consider technology shocks since preference shocks do not imply any trade-off for the monetary authority. Price-wage stickiness induces an endogenous inflation-output trade-off for monetary policy. Given $(M1)$ and $(M2)$ monetary policy should contemporaneously close the wage and the output gap to fully stabilize wage and price inflation. However this is unfeasible since after a technology shock, that affects $\Delta\omega_t^{Eff}$, price and wage inflation should jointly move according to $(M3)$. The intuition is also straightforward: in the social optimum the real wage follows one-to-one the marginal productivity of labor (a_t), but this is simply not possible if the variance of both price and wage inflation is stabilized. Importantly,

[26]The wage gap terms disappears from the loss function also when $\sigma = 1$. This is for purely technical reasons. The term $(\sigma - 1)$ is due to two different approximations applied to $U(C) : 1)$ σ derives from the second-order approximation of the utility function; 2) 1 is instead the curvature of the logarithmic function used to transform C into log deviations from steady state.

this trade-off originates entirely from the supply side of the model and therefore it is not affected by LAMP. As a result LAMP does not change the trade-offs faced by the monetary authority. However , LAMP alters the IS curve and the welfare loss function, thus it may affect the optimal response to shocks. Nevertheless, in the next section we show that, once nominal wage stickiness is brought into the picture, LAMP has only marginal quantitative effects on the optimal path of the main macro-variables in response to a technology shock.

4.2 Commitment

Model Calibration. Given that our results are partly numerical, we detail the baseline calibration of the model. Time is measured in quarters. The discount factor β is set to 0.99, so that the annual interest rate amounts to 4%. The utility parameters σ and ϕ are equal to 2 and 3, respectively. According to the estimates in Basu and Fernald (1997) the value added mark-up of prices over marginal cost is around 20%, for this reason we set θ_p to 6. We assign an identical value to the elasticity of substitution between labor inputs, θ_w. We set $\xi_p = \xi_w = 0.75$, which implies an average duration of price and wage contracts of one year, a value which is in compatible with most available empirical estimates (see for example Smets and Wouters 2003 and Levine et al. 2005). However, we evaluate the dependence of our results on the average duration of wage contracts which is a fundamental magnitude in our analysis.

We draw the autoregressive coefficient and the standard deviation of the technology shock from Schmitt-Grohé and Uribe (2007), while for what concerns the preference shock we refer to the estimates by Galí and Rabanal (2004). Selected values are $\rho_a = 0.855$, $\sigma_a = 0.0064$, $\rho_\psi = 0.93$ and $\sigma_\psi = 0.025$. Notice that we assume that the technology and the preference shock are independent from each other.

Optimal Monetary policy in response to technology shocks. In the presence of a credible commitment, the central bank minimizes the welfare loss function (23) subject to $(M1) - (M3)$, taking $\tilde{\omega}_{t-1}$ as given. Then, the IS curve determines the optimal path of the nominal interest rate, while the resource constraint of non-Ricardian agents and the definition of aggregate consumption determine the sharing of resources between agents. Figure 5 depicts the optimal deviations from the efficient steady state of the main macroeconomic variables in response to a persistent technology shock. We consider alternative degrees of asset market participation. Consider the fully Ricardian case ($\lambda = 0$). Since the monetary policy is endowed with a single instrument, it must trade-off between the competing distortions due to sticky prices and sticky wages. The resulting optimal dynamics feature a persistent reduction in inflation and a prolonged adjustment of the output gap. Remarkably, in response to an increase in productivity, hours worked fall. The contraction in hours following a positive

productivity shock is in line with recent U. S. evidence (see, for example, Galì and Rabanal, 2004).

Restricting asset market participation has just quantitative implications on the optimal IRFs. This does not come as a surprise since, as discussed above, LAMP does not affect the trade-offs faced by the Central Bank in response to a technology shock. Specifically, restricting asset market participation (i.e. higher λ) amplifies the propagation of the technology shock to the economy. The intuition for this outcome is as follows. The rise in technology leads to lower marginal costs and higher output which translate into an increase in total profits. This has a positive income effect on Ricardian households. The latter gets stronger as the portion of non-Ricardian agents enlarges, resulting into a more pronounced reaction of Ricardian agents' consumption to the shock. To support such an outcome the Euler equation requires lower asset market participation to be associated with more aggressive cuts of the nominal interest rate. Because of price stickiness firms satisfy higher demand of the final good via an increase in labor demand. This ultimately affects the real wage and hours worked and thus consumption of non-Ricardian agents.

The main point, however, is that the effect of LAMP on welfare relevant variables such as gaps and inflation rates are minor also from a quantitative point of view. The optimal policy response of a NK model with price and wage stickiness is, therefore, *only marginally* affected by the LAMP assumption.[27]

Moreover, when $\sigma = 1$, the LAMP hypothesis has no effect at all on the optimal monetary policy response. In this case, neither the objective function (23) nor the constraints, $(M1) - (M3)$, depend on the share of non-Ricardian agents. Thus, in response to shocks, the optimal policy implements the same equilibrium path for the welfare relevant variables as in a full participation economy. In this case, society welfare will not be affected by the presence of non-Ricardian agents and just the interest rate will be affected by LAMP assumption through the IS curve.

To conclude this section we report in Table 1 the unconditional welfare loss under full commitment as a function of the share of non-Ricardian agents and the average durations of wage contracts (i.e., $(1-\xi_w)^{-1}$). The unconditional welfare loss is expressed as a percentage of aggregate consumption at the efficient steady state . As well known, in the case of flexible wages (i.e., $\xi_w = 0 => (1-\xi_w)^{-1} = 1$) the monetary authority faces no trade-off at stabilizing welfare relevant variables in response to a technology shock, for this reason the welfare loss in nil. As expected, the welfare loss increases with the magnitude of the two distortions considered.

[27] The response of the efficient level of output is somewhat in between the responses in the top left panel of Figure 5. Hence the output gap switches sign from negative to positive as λ changes, but this effect is quantitatively negligible.

4.3 Strict targeting rules

In this section we consider policy rules aimed at fully stabilizing, at each date and state, one of the welfare relevant variables, that is either one between $\tilde{\omega}_t, \pi_t, \pi_t^w$, or x_t. These rules are often defined as strict targeting rules. The next proposition provides a general result concerning LAMP and strict targeting policies.

Proposition 7: LAMP and strict targeting rules. *Under a strict targeting rule (whatever the target among $(\tilde{\omega}_t, \pi_t, \pi_t^w, x_t)$) the path $\{\tilde{\omega}_t, \pi_t, \pi_t^w, x_t\}_{t=0}^{\infty}$ is not affected by LAMP. As a consequence the unconditional variances of welfare relevant variables do not depend on λ. The path of the instrument, $\{i_t\}_{t=0}^{\infty}$, required to implement the allocation depends, instead, on the degree of asset market participation.*

Proof. This follows from the fact that the supply side of the model does not depend on the degree of asset market participation, λ. Once either one between $\tilde{\omega}_t, \pi_t, \pi_t^w, x_t$ is set equal to zero, equations $(M1) - (M3)$ are sufficient to generate the path of the other three variables. Since λ enters only in the IS equation, its value only matters for the behavior of i_t, but not for the allocation of welfare relevant variables.

We can further specialize the previous proposition showing that strict price inflation targeting and strict wage gap targeting amount to the same policy.

Proposition 8: LAMP, strict price inflation and real wage-gap targeting. *Strict price inflation targeting and strict real wage gap targeting are implemented by the same path of the policy instrument $\{i_t\}_{t=0}^{\infty}$. They also deliver the same welfare loss given by $W = \frac{\sigma_a^2}{2\kappa_w^2(\sigma+\phi)}\left[\frac{1}{1-\rho_a^2}\left((\rho_a-1)(1-\beta\rho_a)\right)^2 + (1-\beta(\rho_a-1))^2\right] + \frac{\theta_w}{\kappa_w(1+\rho_a)}\sigma_a^2$ which: (i) is independent of the degree of asset market participation, (ii) tends to zero in the case of flexible wages and (iii) increases with the average duration of wage contracts.*

Proof See Appendix A.4.2

The intuition for this result is straightforward. The NKPC implies that whenever $\pi_t = 0$ it has to be the case that $\tilde{\omega}_t = 0$ and vice-versa. In this case the extra term in the IS curve vanishes and the path of the interest rate needed to implement the allocation is the same, and it is independent of the share of non-Ricardian agents. Since both policies lead to the same path for welfare relevant variables, and in particular imply that $\tilde{\omega}_t = 0$ at all t, the welfare loss is also independent of λ. Price inflation targeting gets more costly as the mean duration of wage contracts gets longer. Finally, we consider the case of strict wage inflation targeting.

Proposition 9: LAMP and strict wage inflation targeting. *Under strict wage inflation targeting the wage gap is proportional to the output gap. The path $\{\pi_t, \tilde{\omega}_t\}_{t=0}^{\infty}$ is independent of the degree of asset market participation, while the path of the instrument needed*

to implement the equilibrium does depend on it. The unconditional welfare loss increases with the degree of LAMP.

Proof. Equation $(M2)$ implies that $(\sigma + \phi)x_t = \tilde{\omega}_t$. In this case equations $(M1)$ and $(M3)$ suffice to determine the path $\{\pi_t, \tilde{\omega}_t\}_{t=0}^{\infty}$. The latter is independent of the degree of asset market participation. Equation $(M4)$ suggests, instead, that the path of the instrument required to implement this policy depends on λ. Since the coefficient on the wage gap variable, $\frac{(\sigma-1)\lambda}{1-\lambda}$, in the welfare loss function (23) is increasing in the share of non-Ricardian agent, society's welfare loss get larger as asset market participation becomes more restricted.

Finally, we compare the welfare performance of strict targeting rules. As a Corollary to Proposition 8 and Proposition 9 we can state the following.

Corollary 3. *Under nominal wage stickiness, there exist a threshold value $\tilde{\lambda}$, such that for $\lambda > \tilde{\lambda}$ wage inflation targeting delivers a higher society's welfare loss with respect to price inflation targeting.*

LAMP could overturn the optimality of strict wage inflation targeting over strict price inflation targeting emphasized by Erceg et al. (2000) in a full participation framework. For any empirically relevant degree of asset market participation, however, the Erceg et al. (2000) result holds. This is evident from Figure 6 that depicts welfare losses under strict wage inflation targeting and strict price inflation (or wage gap) targeting. The latter is shown for two alternative mean durations of wage contracts.

4.4 Optimal simple rules

To conclude our analysis we evaluate how the interaction between LAMP and wage stickiness affects the design of optimal simple Taylor-type interest rate rules. Motivated by the analysis in Bilbiie (2008) we initially consider two pure inflation targeting rules, where the interest rate responds solely to current and expected inflation, respectively. Next, as in Erceg et al. (2000), we consider a hybrid rule where the interest rate reacts to current price inflation and to the deviations of output from the steady state. Finally, we consider a hybrid rule in which the interest rate is a function of current price and wage inflation. Following Schmitt-Grohé and Uribe (2007), we require the simple rules described above to be implementable and optimal. The implementability condition requires policies to deliver local uniqueness of the REE. Optimality requires, instead, selecting policy coefficients in order to minimize the unconditional expectation of the welfare loss function (23). We search for optimal policy coefficients numerically. In doing this we limit attention to the interval [-10, 10] for each

coefficient. Notice that larger coefficients response would fall out of any plausible estimate and would have little credibility. We evaluate the performance of each rule for a range of values of the two relevant parameters: λ and ξ_w.[28]

In the remainder we state two main results. Result 1 refers to pure inflation targeting rules, while Result 2 to hybrid rules.

Result 1. Pure inflation targeting rules *In the case of pure inflation targeting rules the optimal rule calls for a strong response of monetary policy. LAMP makes the optimal rule highly passive if wages are flexible. However, if wages are sticky, the optimal rule is restored to be highly active, as in the standard NK model.*

Table 2 reports optimal policy coefficients and the associated welfare loss for the contemporaneous and forward-looking inflation targeting rules. Consider the current pure inflation targeting rule (Panel A). In a fully Ricardian economy ($\lambda = 0$) with flexible wages ($(1-\xi_w)^{-1} = 1$) the optimal response coefficient implies a strong anti-inflationary stance. The reason is that in the absence of a trade-off between inflation and the output gap, stabilizing inflation also results in output stabilization.

In our exercise, thus, the inflation coefficient hits the upper bound (i.e. $\phi_\pi = 10$). Removing the upper bound on policy parameters would result in an unbounded inflation coefficient response and zero welfare loss. The optimal rule is extremely effective, as it delivers a welfare loss equal to 0.002 % of steady state consumption. These results resembles those in other studies such as Schmitt-Grohé and Uribe (2007).

Introducing LAMP in this environment has dramatic consequences for the design of optimal interest rate rules. The optimal contemporaneous rule turns passive and features a strongly negative inflation response, indeed ϕ_π hits the lower bound equal to -10. We are in the IADL region implying that the relationship between aggregate demand and the real interest rate is reversed with respect to the standard case. It is worth emphasizing that the negative inflation coefficient obtained under LAMP and flexible wages does not merely serve the purpose of ensuring the uniqueness of the REE. Under the contemporaneous rule also a very strong increase in the real interest rate in response to a positive change in inflation would, in fact, guarantee determinacy in the LAMP economy (See Figure 4). However, it would deliver a lower welfare with respect to the passive rule considered here.

Even a very low, and below estimates, degree of wage stickiness restores the optimality of an active rule for any empirically plausible share of non-Ricardian agents. Moreover, when the degree of wage stickiness assumes values compatible with the empirical evidence, the optimal

[28] To facilitate understanding we report in the tables below the average duration of wage contracts: $(1-\xi_w)^{-1}$ .The latter is expressed in quarters.

policy is highly active no matter the extent to which we limit asset market participation. Again, wage stickiness limits the likelihood of a reversal in the slope of the IS curve and it restores standard policy prescriptions. In other words, once wage stickiness is considered, LAMP has just minor quantitative implications for the design of optimal simple rules. In particular, the optimal policy calls for a stronger reaction to inflation as the share of non-Ricardian agents increases.

Similar considerations extend to the forward looking inflation targeting rule in Table 2 (Panel B). As in a standard economy, the simple rules considered here perform quite well in terms of welfare even in the presence of non-Ricardian agents. The welfare loss gets large just in the case where wage stickiness is coupled with an implausibly large share of non-Ricardian consumers. However, this is partly due to the fact that we restrict the interval of admissible values for ϕ_π. We next turn to the second result, concerning hybrid rules.

Result 2. Hybrid Rules *In the case of hybrid rules (i) Result 1 is confirmed: nominal wage stickiness makes the optimal rule active; (ii) a rule targeting both price and wage inflation delivers the best performance in terms of welfare; (iii) responding to output only marginally improves the performance of a pure inflation targeting rule.*

Table 3 reports the performance of the hybrid rules we consider. Results 1 is confirmed: nominal wage stickiness makes the optimal policy strongly active, no matter the degree of LAMP. In line with Erceg et al. (2000), a rule responding to both price and wage inflation substantially reduces the welfare loss with respect to a pure price inflation targeting rule.

The relative magnitude of the optimal coefficients on price and wage inflation depends on the relative degree of stickiness between prices and wages. The larger between the two coefficients is the one multiplying the inflation of the stickier variable. Further, both coefficients are increasing in the degree of LAMP and are generally very large (possibly unbounded in the case of wage inflation targeting for high degree of wage stickiness). It follows that for realistic values of the degree of wage stickiness, this rule calls for complete wage stabilization.

5 Conclusions

We design a model to study monetary policy in an economy characterized by staggered wage and price contracts and by an arbitrary degree of asset market participation. Our model nests two widely used framework for the analysis of monetary policy. The LAMP model by Bilbiie (2008) and the sticky prices-sticky wages model by Erceg et al. (2000).

We find that wage stickiness fundamentally affects results obtained by the first author, while LAMP leaves the main results in Erceg et al. (2000) unchanged. In particular, determinacy and welfare properties of simple interest rules and the design of optimal monetary

policy differ from those observed in a full participation model just in the case in which asset market participation is limited to an empirically implausible extent. For values of the share of non-Ricardian agents consistent with the empirical estimates, monetary policy prescriptions are isomorphic to those which characterize a standard NK model with no LAMP.

This suggests that reappraisals of the conduct of monetary policy in specific past periods, such as that of the Great Inflation, based on the presence of non-Ricardian agents cannot neglect nominal wage stickiness, which is, in fact, an incontrovertible empirical fact.

Our analysis is conducted in the context of a highly stylized economy. For instance, as in Bilbiie (2008), we assume that the government has access to a subsidy, financed with lump-sum taxes, which offset the distortions introduced by imperfect competition in the product and factor markets. Also we neglect the role of capital accumulation. These assumptions allow to obtain many of our results analytically, but they are empirically unrealistic. An extension of our analysis would be that of considering a larger scale business cycle model similarly to those in Christiano et al. (2005) or Smets and Wouters (2003). While this would add in terms of realism, we believe that it would not alter the main message of this paper.

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A Technical Appendix

A.1 The Efficient Steady State and the Efficient Equilibrium Output

As in Bilbiie (2008) society's welfare loss will be represented by a second order approximation to a weighted average of households lifetime utilities, where weights are given by the relative importance of agents's groups in the economy. In order to study the welfare properties of the economy without resorting to a full second order approximation to the model equations, we assume an efficient steady state of the economy. More precisely, we assume that the government imposes a lump sum tax, T, on firms such that steady state profits are zero. The tax proceeding are then used by the government to subsides steady state firms' labor demand at the constant rate τ.[29] In this case steady state profits read as

$$D = Y - \frac{(1-\tau)\,W}{P}L - T, \tag{25}$$

where $T = \tau\frac{W}{P}L$. Profit maximization implies

$$\frac{W}{P} = \frac{1}{(1+\mu_p)(1-\tau)}MPL$$

where MPL is the steady state marginal product of labor. Given steady state profits are zero it follows that $C_S = C_L = C$ and, thus, that agents have a common marginal rate of substitution between labor and consumption, denoted by MRS. As a consequence the steady state wage set by unions reads as

$$\frac{W}{P} = (1+\mu^w)\,MRS.$$

The steady state labor market equilibrium implies

$$\frac{1}{(1+\mu_p)(1-\tau)}MPL = (1+\mu^w)\,MRS. \tag{26}$$

Given the selected production function, and normalizing labor productivity to unity, at the efficient steady state it has to be the case that

$$MPL = MRS = 1, \tag{27}$$

From equation (26), the latter condition is satisfied if

$$\tau = 1 - \frac{1}{(1+\mu_p)(1+\mu^w)} \tag{28}$$

As argued above the implied value of τ leads to zero steady state profits

$$D = Y - \frac{(1-\tau)\,W}{P}L - T = Y - \frac{Y}{(1+\mu_p)^2(1+\mu^w)} - \left(1 - \frac{1}{(1+\mu_p)^2(1+\mu^w)}\right)Y = 0$$

[29] Out of steady state taxes are zero: $\tau = T = 0$. This allows to preserve inequality between agents out of the steady state and at the same time delivers steady state equality. Since the focus of the paper is not on the long-run differences across households, we view this devise as innocuous.

Next, we solve the Social Planner problem (SPP). The equilibrium output which solve the SPP corresponds to efficient equilibrium output. The SPP reads as

$$\begin{aligned}&\max_{\{C_{H,t},C_{S,t},L_t\}} \lambda\frac{\Psi_t C_{H,t}^{1-\sigma}}{1-\sigma}+(1-\lambda)\frac{\Psi_t C_{S,t}^{1-\sigma}}{1-\sigma}-\lambda\frac{L_{H,t}^{1+\phi}}{1+\phi}-(1-\lambda)\frac{L_{S,t}^{1+\phi}}{1+\phi} \\ &s.t.\ C_t=Y_t=A_tL_t=\lambda C_{H,t}+(1-\lambda)C_{S,t}=A_t\left(\lambda L_{H,t}+(1-\lambda)L_{S,t}\right)\end{aligned} \tag{29}$$

Writing the Lagrangian $\mathcal{L}$,and taking the first order condition with respect to $C_{H,t}, C_{S,t}, L_{H,t}$ and $L_{S,t}$ we find

$$C_{H,t} = C_{S,t}=C_t \tag{30}$$

$$L_{H,t} = L_{S,t}=L_t. \tag{31}$$

In short, at the efficient equilibrium the economy behaves as if there was a representative agent with marginal rate of substitution between consumption and hours given by $\Psi_t^{-1}C_t^{\sigma}L_t^{\phi}$. The social planner sets the latter equal to the marginal product of labor, A_t , which also represents the equilibrium real wage, $(W/P)_t^{Eff}$. Using the relationship just described, imposing the market clearing condition $Y_t = C_t$ and using the production function, delivers the efficient level of output as

$$Y_t^{Eff}=A_t^{\frac{1+\phi}{\sigma+\phi}}\Psi_t^{\frac{1}{\sigma+\phi}} \tag{32}$$

Log-linearizing and considering that $\Psi = 1$ delivers the log-deviations of the efficient level of output from the efficient steady state as in equation (11) in the main text. In the efficient equilibrium the Euler equation for Ricardian must be satisfied. Since the consumption is equal for the two class of agents, then the Euler equation must be satisfied by output. The natural rate of interest is equal to the one specified in equation (13) of the main text.

A.2 Derivation of the IS curve

Log-linearization of the Euler equation of Ricardian agents leads

$$c_{s,t}=E_tc_{s,t+1}-\frac{1}{\sigma}E_t\left(i_t-\pi_{t+1}\right)-\frac{1}{\sigma}\Delta\psi_{t+1} \tag{33}$$

while from the consumption function of non-Ricardian consumer we get:

$$c_{H,t}=l_t+\omega_t. \tag{34}$$

Aggregate consumption is

$$c_t=(1-\lambda)c_{s,t}+\lambda c_{H,t} \tag{35}$$

combined with the Euler equation:

$$c_t=E_t\left(c_{t+1}-\lambda\Delta c_{H,t+1}\right)-\frac{(1-\lambda)}{\sigma}E_t\left(i_t-\pi_{t+1}\right)-\frac{(1-\lambda)}{\sigma}\Delta\psi_{t+1}. \tag{36}$$

Substituting for $c_t = y_t$, for $E_t \Delta c_{H,t+1} = E_t (\Delta l_{t+1} + \Delta \omega_{t+1})$ and for $l_t = y_t - a_t$ we get

$$y_t = E_t y_{t+1} + \frac{\lambda}{1-\lambda} E_t \Delta a_{t+1} - \frac{\lambda}{1-\lambda} E_t \Delta \omega_{t+1} - \frac{1}{\sigma} E_t (i_t - \pi_{t+1}) - \frac{1}{\sigma} \Delta \psi_{t+1} \tag{37}$$

rewriting equation (37) in terms of output gap from the efficient equilibrium output ($x_t = y_t - y_t^{Eff}$), considering that $r_t^{Eff} = \sigma \Delta y_{t+1}^{Eff} - \Delta \psi_{t+1}$ and given the definition of the real wage gap $\tilde{\omega}_t = \omega_t - \omega_t^{Eff}$, we can finally write the IS as

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} E_t \left(i_t - \pi_{t+1} - r_t^{Eff} \right) - \frac{\lambda}{1-\lambda} E_t \Delta \tilde{\omega}_{t+1}. \tag{38}$$

A.2.1 The slope of the IS curve

Flexible wages In the case of flexible wages the real wage is given by

$$\omega_t = \sigma c_t + \phi l_t - \psi_t, \tag{39}$$

$$mc_t = \omega_t - (y_t - l_t) = \omega_t - a_t = (\sigma + \phi) x_t. \tag{40}$$

Hence $\Delta \tilde{\omega}_{t+1} = (\sigma + \phi) \Delta x_{t+1} + \Delta a_{t+1}$. Substitute in (38) to get

$$x_t = E_t x_{t+1} - \frac{\left(\delta^{fw}\right)^{-1}}{\sigma} E_t \left(i_t - \pi_{t+1} - r_t^{Eff} \right) \tag{41}$$

where $\delta^{fw} = 1 - \frac{\lambda}{1-\lambda} (\sigma + \phi)$.

Sticky wages In the case of sticky wages the real wage is given by

$$\omega_t = \frac{1}{1 + \beta + \kappa_w} [w_{t-1} - p_t] + \frac{\beta}{1 + \beta + \kappa_w} E_t (w_{t+1} - p_t) + \frac{\kappa_w}{1 + \beta + \kappa_w} ((\sigma + \phi) x_t + a_t) \tag{42}$$

This is a weighted average between: (i) the past nominal wage at current prices; (ii) the future nominal wage at current prices; (iii) the flexible wage ($mc + a$). Note that as $\xi_w \longrightarrow 0$, then $\kappa_w \longrightarrow \infty$, and this expression collapses to the usual flexible wage case. Then

$$\Delta \tilde{\omega}_{t+1} = \digamma + \frac{\kappa_w}{1 + \beta + \kappa_w} ((\sigma + \phi) \Delta x_{t+1} + \Delta a_{t+1}) \tag{43}$$

where $\digamma = \frac{1}{1+\beta+\kappa_w} [\Delta w_t - \Delta p_{t+1}] + \frac{\beta}{1+\beta+\kappa_w} E_t (\Delta w_{t+2} - \Delta p_{t+1})$

Substituting (43) into (38) we get

$$\begin{aligned} x_t \;=\; & E_t x_{t+1} - \frac{(\delta^{sw})^{-1}}{\sigma} E_t (i_t - \pi_{t+1}) - \frac{(\delta^{sw})^{-1}}{\sigma} \Delta \psi_{t+1} + (\delta^{sw})^{-1} \Delta y_{t+1}^{Eff} + \\ & \frac{\lambda}{1-\lambda} \frac{(\delta^{sw})^{-1}}{1 + \beta + \kappa_w} \left\{ (1+\beta) E_t \Delta a_{t+1} - E_t \left[(\pi_t^w - \pi_{t+1}) + \beta \left(\pi_{t+2}^w - \pi_{t+1} \right) \right] \right\} \end{aligned} \tag{44}$$

where $\delta^{sw} = 1 - \frac{\lambda}{1-\lambda} \frac{\kappa_w (\sigma + \phi)}{1+\beta+\kappa_w}$, which is equivalent to (20) in the main text.

A.2.2 Proof of Propositions 1

When wages are flexible $\xi_w = 0$,which implies that $\kappa_w \to \infty$, and $\delta^{sw} \to \delta^{fw} = 1 - \frac{\lambda(\sigma+\phi)}{1-\lambda}$, where δ^{fw} defines the slope of the IS curve in the case of flexible wages.[30] Notice that $\delta^{sw} - \delta^{fw} = \frac{\lambda(\sigma+\phi)}{1-\lambda}\left(1 - \frac{\kappa_w}{1+\beta+\kappa_w}\right) = \frac{\lambda(\sigma+\phi)}{1-\lambda}\frac{1+\beta}{1+\beta+\kappa_w} > 0$ which proves (i). Moreover $\frac{\partial \delta^{sw}}{\partial \xi_w} = \frac{\partial \delta^{sw}}{\partial \kappa_w}\frac{\partial \kappa_w}{\partial \xi_w}$. Since $\frac{\partial \delta^{sw}}{\partial \kappa_w} = -\frac{\lambda(\sigma+\phi)}{1-\lambda}\frac{1+\beta}{(1+\beta+\kappa_w)^2} < 0$ and $\frac{\partial \kappa_w}{\partial \xi_w} = -\frac{\kappa_w+\beta(1-\xi_w)+(1-\beta\xi_w)}{\xi_w} < 0$, it follows that $\frac{\partial \delta^{sw}}{\partial \xi_w} > 0$ which proves (ii). Since $\delta^{sw} < 0 \Leftrightarrow \lambda > \bar{\lambda}^{sw} = \frac{1}{1+(\sigma+\phi)\frac{\kappa_w}{1+\beta+\kappa_w}}$, and $\frac{\partial \bar{\lambda}^{sw}}{\partial \xi_w} = \frac{\partial \bar{\lambda}^{sw}}{\partial \kappa_w}\frac{\partial \kappa_w}{\partial \xi_w} = -\left[1 + (\sigma+\phi)\frac{\kappa_w}{1+\beta+\kappa_w}\right]^{-2}\frac{1+\beta}{(1+\beta+\kappa_w)^2}\frac{\partial \kappa_w}{\partial \xi_w} > 0$. It follows that (iii) is also proved.

A.3 Determinacy analysis

A.3.1 Proof of Propositions 2: Flexible prices and sticky wages

Define, as in Bilbiie (2008), $d_t = \frac{D_t}{Y}$ as the deviation of profits' share over output from its (zero) steady state level. Deviations of consumption of Ricardian agents from the efficient steady state can thus be written as $\frac{1}{Y}c_{S,t} = \frac{WL}{Y}(\omega_t + l_t) + \frac{1}{1-\lambda}d_t$, while consumption of non-Ricardian agents reads as $\frac{1}{Y}c_{H,t} = \frac{WL}{Y}(\omega_t + l_t)$. Also notice that $d_t = -mc_t$. Under flexible prices $mc_t = 0$, implying $d_t = 0$. In this case $c_{S,t} = c_{H,t} = c_t$, i.e. up to a log-linear approximation the economy behaves as if there was a representative agent. Price flexibility implies that the supply side of the model is defined solely by equation M2. Further, since the wage gap is nil, it follows that the extra term in the IS curve, $\frac{\lambda}{1-\lambda}E_t\Delta\tilde{\omega}_{t+1}$ vanishes. Equilibrium dynamics are thus given by the system

$$(M2^{fp}) \qquad \pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_w(\sigma+\phi)x_t$$
$$\left(M4^{fp}\right) \qquad x_t = E_t x_{t+1} - \tfrac{1}{\sigma}E_t\left(i_t - \pi_{t+1} - r_t^{Eff}\right)$$

where the superscript *fp* stands for flexible prices. The latter is independent of the share of non-Ricardian agents. As a consequence when monetary policy is conducted according to policy rule (18) the requirement for determinacy is also isomorphic to that to be imposed on a fully Ricardian model. To see this notice that $\pi_t^w = \pi_t + \Delta\omega_t^{eff} = \pi_t + \Delta a_t$ in this case the system $(M2^{fp}) - \left(M4^{fp}\right)$ in matrix form reads as

$$A_0\begin{bmatrix}\pi_t \\ x_t\end{bmatrix} = A_1\begin{bmatrix}\pi_{t+1} \\ x_{t+1}\end{bmatrix} \tag{45}$$

Where $\mathrm{A}_0 = \begin{bmatrix}1 & -\kappa_w(\sigma+\phi) \\ 0 & 1\end{bmatrix}$ and $\mathrm{A}_1 = \begin{bmatrix}\beta & 0 \\ \frac{1}{\sigma}(1-\phi_\pi) & 1\end{bmatrix}$. Defining $B = A_0^{-1}A_1$, conditions for having two roots within the unit circle are: 1) $\det B < 1$,2) $trB - \det B < 1$ and 3) $trB + \det B > -1$. Given $Det(B) = \beta$ condition 1 is always satisfied. $Trace(B) = \beta - \frac{1}{\sigma}\kappa_w(\phi_\pi - 1)(\sigma+\phi) + 1$, thus condition 2 is satisfied if $\phi_\pi > 1$. If condition 2 is satisfied, condition 3 imposes an upper bound on the value of ϕ_π, i.e. $\phi_\pi < 1 + \frac{2\sigma(1+\beta)}{(\sigma+\phi)\kappa_w}$. QED

[30] Notice also that under flexible wages the second lines in (20) vanishes.

A.3.2 Determinacy Analysis of the full 4X4 system

To obtain analytical results regarding the stability properties of the dynamic system resulting from the model equations, we follow the strategy of transforming the polynomial derived from the characteristic equation (see, Samuelson, 1941, and more recently section 4 in Felippa and Park, 2004). More formally, as explained in Felippa and Park (2004), given the characteristic polynomial

$$P_A(\gamma) = \gamma^4 + a_1\gamma^3 + a_2\gamma^2 + a_3\gamma + a_4, \tag{46}$$

the stability properties would depend on the location of the root inside the unit circle $|\gamma| < 1$ (such a polynomial is known in the literature as *amplification* polynomial). One can transform this polynomial in an Hurwitz polynomial, $P_H(s)$, whose stability properties would depend on the location of the roots in the left-hand plane $\Re(s) \leq 0$. To pass from $P_A(\gamma)$ to $P_H(s)$, one uses the conformal involuntary transformation

$$\gamma = \frac{1+s}{1-s}. \tag{47}$$

Given (47), it is easy to check that $|\gamma| \lesseqgtr 1 \Leftrightarrow s \lesseqgtr 0$.

In our case, the fourth order characteristic (amplification) polynomial can be transformed into the Hurwitz polynomial by using $\gamma = \frac{1+s}{1-s}$

$$\tilde{P}_H(s) = \left(\frac{1+s}{1-s}\right)^4 + a_1\left(\frac{1+s}{1-s}\right)^3 + a_2\left(\frac{1+s}{1-s}\right)^2 + a_3\frac{1+s}{1-s} + a_4. \tag{48}$$

Expanding the polynomial, one obtains a quotient of two polynomials: $\tilde{P}_H(s) = \frac{P_H(s)}{Q_H(s)}$ where the roots of $\tilde{P}_H(s)$ are the roots of $P_H(s)$. Hence one needs to study the stability properties of the following Hurwitz polynomial:

$$P_H(s) = \underbrace{\tilde{a}_4}_{\frac{a_1+a_2+a_3+a_4+1}{a_2-a_1-a_3+a_4+1}} + s\underbrace{\tilde{a}_3}_{\frac{2(2+a_1-a_3-2a_4)}{a_2-a_1-a_3+a_4+1}} + s^2\underbrace{\tilde{a}_2}_{\frac{2(3a_4-a_2+3)}{a_2-a_1-a_3+a_4+1}} + s^3\underbrace{\tilde{a}_1}_{\frac{2(a_3-a_1-2a_4+2)}{a_2-a_1-a_3+a_4+1}} + s^4 \tag{49}$$

A.3.3 Forward Rule: Proof of Propositions 3

We consider the following policy rule

$$i_t = \phi_\pi \pi_{t+1}. \tag{50}$$

So the system is in matrix formulation (where $\frac{\lambda}{1-\lambda} = \chi$) :

$$\begin{bmatrix} \pi^w_{t+1} \\ \pi_{t+1} \\ x_{t+1} \\ \tilde{\omega}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & 0 & -\frac{1}{\beta}\kappa_w(\sigma+\phi) & -\frac{1}{\beta}\kappa_w \\ 0 & \frac{1}{\beta} & 0 & -\frac{1}{\beta}\kappa_p \\ \frac{1}{\beta}\chi & \frac{1}{\sigma}\frac{1}{\beta}(\phi_\pi - 1) - \chi\frac{1}{\beta} & 1 - \chi\frac{1}{\beta}\kappa_w(\sigma+\phi) & \chi\frac{1}{\beta}(\kappa_w+\kappa_p) - \frac{1}{\sigma}\frac{1}{\beta}(\phi_\pi - 1)\kappa_p \\ \frac{1}{\beta} & -\frac{1}{\beta} & -\frac{1}{\beta}\kappa_w(\sigma+\phi) & 1 + \frac{1}{\beta}(\kappa_w+\kappa_p) \end{bmatrix} \begin{bmatrix} \pi^w_t \\ \pi_t \\ x_t \\ \tilde{\omega}_t \end{bmatrix} \tag{51}$$

The the coefficient of the characteristic polynomial are

$a_1 = -\gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 = -trace(J_0)(= -$ sum of the principal first-order minors of $J_o)$

$a_2 = \gamma_1\gamma_2 + \gamma_1\gamma_3 + \gamma_1\gamma_4 + \gamma_2\gamma_3 + \gamma_2\gamma_4 + \gamma_3\gamma_4 =$ sum of the principal second-order minors of J_o

$a_3 = -\gamma_1\gamma_2\gamma_3 - \gamma_1\gamma_2\gamma_4 - \gamma_1\gamma_3\gamma_4 - \gamma_2\gamma_3\gamma_4 = -$ (sum of the principal third-order minors of J_o)

$a_4 = \gamma_1\gamma_2\gamma_3\gamma_4 = \det(J_0)$ (= principal of fourth-order).

The characteristic polynomial is then equal to

$$\begin{aligned} p(\gamma) &= \gamma^4 + \left[\frac{1}{\beta}\left[-2-2\beta-(\kappa_w+\kappa_p)+\chi\kappa_w(\sigma+\phi)\right]\right]\gamma^3 \\ &+ \left[\frac{1}{\beta}(\kappa_p+\kappa_w+\beta+1) - \frac{1}{\beta}\kappa_w(\sigma+\phi)\left(\chi\left(1+\frac{1}{\beta}\right)+\frac{1}{\sigma\beta}\kappa_p(\phi_\pi-1)\right)\right. \\ &\left. + \frac{1}{\beta^2}(1+3\beta+\kappa_w+\kappa_p)\right]\gamma^2 \\ &+ \left[-\frac{1}{\beta^2}(2+2\beta+\kappa_w+\kappa_p-\chi\kappa_w(\sigma+\phi))\right]\gamma + \frac{1}{\beta^2} \end{aligned}$$

Applying the above transformation in (49) to get the Hurwitz polynomial, it yields

$$\begin{aligned} &\underbrace{\frac{-\frac{1}{\sigma}\kappa_w\kappa_p(\sigma+\phi)(\phi_\pi-1)}{D}}_{\tilde{a}_4} \\ &+s\underbrace{\frac{2(\beta-1)\left[-\kappa_w-\kappa_p+\chi\kappa_w(\sigma+\phi)\right]}{D}}_{\tilde{a}_3} \\ &+s^2\left[\underbrace{\frac{4\beta^2+4-8\beta+2(1+\beta)\left[-\kappa_p-\kappa_w+\kappa_w(\sigma+\phi)\chi\right]+\frac{2}{\sigma}\kappa_w(\sigma+\phi)\kappa_p(\phi_\pi-1)}{D}}_{\tilde{a}_2}\right] \\ &+s^3\left(\underbrace{\frac{-8+8\beta^2+2(1-\beta)\left[-(\kappa_w+\kappa_p)+\chi\kappa_w(\sigma+\phi)\right]}{D}}_{\tilde{a}_1}\right)+s^4 \end{aligned} \tag{52}$$

where

$$D = 4\beta^2+4+8\beta+2[\beta+1](\kappa_p+\kappa_w)-\frac{1}{\sigma}\kappa_w\kappa_p(\sigma+\phi)(\phi_\pi-1)-2(1+\beta)\chi\kappa_w(\sigma+\phi).$$

Note there should be 3 positive roots and 1 negative root for the REE to be unique. It follows that a necessary condition must be that $\tilde{a}_4 < 0$. Proof strategy: we look at the signs of the coefficients $\tilde{a}_i$, and we exploit the Decartes' rule of sign.

Look separately at the case when $\phi_\pi > 1$ and when $\phi_\pi < 1$.

Case $\phi_\pi > 1$.

$\tilde{a}_4$) In this case the numerator of $\tilde{a}_4$ (i.e., $N_{\tilde{a}_4}$)[31] is negative, hence the denominator must be positive. For D to be positive, the following restriction must hold:
$\phi_\pi < 1 + \frac{4\sigma\beta^2+4\sigma+8\sigma\beta+2\sigma(1+\beta)([\kappa_p+\kappa_w-\chi\kappa_w(\sigma+\phi)]}{\kappa_w\kappa_p(\sigma+\phi)}$.

$\tilde{a}_3$) Then, since $D > 0$, there are two cases:

i) $N_{\tilde{a}_3} > 0 \Rightarrow \tilde{a}_3 > 0$, that happens for low values of χ, more precisely when $\chi < \frac{\kappa_w+\kappa_p}{\kappa_w(\sigma+\phi)}$.

Note that in this case $\frac{4\sigma\beta^2+4\sigma+8\sigma\beta+2\sigma(1+\beta)([\kappa_p+\kappa_w-\chi\kappa_w(\sigma+\phi)]}{\kappa_w\kappa_p(\sigma+\phi)} + 1 > 1$ and so the set is non empty. Moreover $N_{\tilde{a}_1} = -8 + 8\beta^2 + 2(1-\beta)[-(\kappa_w+\kappa_p) + \chi\kappa_w(\sigma+\phi)] < 0 \Rightarrow \tilde{a}_1 < 0$.

Whatever the sign of $\tilde{a}_2$, the signs of the coefficients in (52) are: -,+,?,-,+. By Decartes' rule of sign, $P_H(s)$ then admits then 1 or 3 positive roots. However, $P_H(-s) =$ +,-,?,+,+, and hence there can be only one negative root. It follows that under the above conditions

$$\frac{2\sigma(1+\beta)[2(1+\beta)+\kappa_p+\kappa_w-\chi\kappa_w(\sigma+\phi)]}{\kappa_w\kappa_p(\sigma+\phi)} + 1 > \phi_\pi > 1$$
$$\chi < \frac{\kappa_w+\kappa_p}{\kappa_w(\sigma+\phi)}$$

the REE is determinate.

ii) $N_{\tilde{a}_3} < 0 \Rightarrow \tilde{a}_3 < 0$, that happens for high values of χ, more precisely when $\chi > \frac{\kappa_w+\kappa_p}{\kappa_w(\sigma+\phi)}$. In this case, however, the set $\frac{2\sigma(1+\beta)[2(1+\beta)+\kappa_p+\kappa_w-\chi\kappa_w(\sigma+\phi)]}{\kappa_w\kappa_p(\sigma+\phi)} + 1 > \phi_\pi > 1$ is non empty iff $\chi < \frac{\kappa_p+\kappa_w}{\kappa_w(\sigma+\phi)} + \frac{2(1+\beta)}{\kappa_w(\sigma+\phi)}$. Hence now we are looking at values of χ such that

$$\frac{\kappa_p+\kappa_w}{\kappa_w(\sigma+\phi)} + \frac{2(1+\beta)}{\kappa_w(\sigma+\phi)} > \chi > \frac{\kappa_w+\kappa_p}{\kappa_w(\sigma+\phi)} \tag{53}$$

Since the first two coefficients $(\tilde{a}_4, \tilde{a}_3)$ are negative and the last is positive, it must be that $\tilde{a}_2 > 0$ and $\tilde{a}_1 < 0$ to have three signs inversions. This is always true if $\phi_\pi > 1$ and (53) hold. It follows that under the above conditions

$$\frac{2\sigma(1+\beta)[2(1+\beta)+\kappa_p+\kappa_w-\chi\kappa_w(\sigma+\phi)]}{\kappa_w\kappa_p(\sigma+\phi)} + 1 > \phi_\pi > 1$$
$$\frac{\kappa_p+\kappa_w}{\kappa_w(\sigma+\phi)} + \frac{2(1+\beta)}{\kappa_w(\sigma+\phi)} > \chi > \frac{\kappa_w+\kappa_p}{\kappa_w(\sigma+\phi)}$$

the REE is determinate.

Putting together i) and ii), the equilibrium is determinate iff

$$\frac{2\sigma(1+\beta)[2(1+\beta)+\kappa_p+\kappa_w-\chi\kappa_w(\sigma+\phi)]}{\kappa_w\kappa_p(\sigma+\phi)} + 1 > \phi_\pi > 1 \tag{54}$$

and

$$\frac{\kappa_p+\kappa_w}{\kappa_w(\sigma+\phi)} + \frac{2(1+\beta)}{\kappa_w(\sigma+\phi)} > \chi > \frac{\kappa_w+\kappa_p}{\kappa_w(\sigma+\phi)}. \tag{55}$$

Case $\phi_\pi < 1$.

$\tilde{a}_4$) $N_{\tilde{a}_4} > 0$, hence it must be that $D < 0$. For D to be negative, the following restriction must hold: $\frac{2\sigma(1+\beta)[2(1+\beta)+\kappa_p+\kappa_w-\chi\kappa_w(\sigma+\phi)]}{\kappa_w\kappa_p(\sigma+\phi)} + 1 < \phi_\pi$. In this case, however, the set

$$1 > \phi_\pi > 1 + \frac{2\sigma(1+\beta)[2(1+\beta)+\kappa_p+\kappa_w-\chi\kappa_w(\sigma+\phi)]}{\kappa_w\kappa_p(\sigma+\phi)} \tag{56}$$

[31] N stands for numerator, D for denominator and the pedix for the correspondent coefficient $\tilde{a}_i$.

is non empty iff:

$$\chi > \frac{\kappa_p + \kappa_w}{\kappa_w(\sigma+\phi)} + \frac{2(1+\beta)}{\kappa_w(\sigma+\phi)}. \tag{57}$$

$\tilde{a}_3$) Given (57), $\Longrightarrow N_{\tilde{a}_3} < 0 => \tilde{a}_3 > 0$, since $D < 0$. In this case, since the first two coefficients: $\tilde{a}_4 < 0, \tilde{a}_3 > 0$, and the last is positive, the only way to have three signs inversions is that at least one between $\tilde{a}_2$ and $\tilde{a}_1$ is negative (in other words they cannot be both positive).

Condition for $\tilde{a}_2 < 0 => N_{\tilde{a}_2} > 0 =>$

$$\phi_\pi > 1 - \frac{\sigma\left(1+\beta\right)\left[\kappa_w\left(\sigma+\phi\right)\chi - \kappa_p - \kappa_w\right]}{\kappa_w\left(\sigma+\phi\right)\kappa_p} - \frac{2\sigma\left(1-\beta\right)^2}{\kappa_w\left(\sigma+\phi\right)\kappa_p}$$

which, if (56) holds, is satisfied iff:

$$\chi < \frac{\kappa_p + \kappa_w}{\kappa_w(\sigma+\phi)} + \frac{4\left(1+\beta\right)}{\kappa_w(\sigma+\phi)} + \frac{2\left(1-\beta\right)^2}{\kappa_w(\sigma+\phi)\left(1+\beta\right)}. \tag{58}$$

In other words, (58) guarantees that

$1 + \frac{2\sigma(1+\beta)[2(1+\beta)+\kappa_p+\kappa_w-\chi\kappa_w(\sigma+\phi)]}{\kappa_w\kappa_p(\sigma+\phi)} > 1 - \frac{\sigma(1+\beta)[\kappa_w(\sigma+\phi)\chi-\kappa_p-\kappa_w]}{\kappa_w(\sigma+\phi)\kappa_p} - \frac{2\sigma(1-\beta)^2}{\kappa_w(\sigma+\phi)\kappa_p}$.

Condition for $\tilde{a}_1 < 0 => N_{\tilde{a}_1} > 0 =>$

$$\chi > \frac{4(1+\beta) + \kappa_w + \kappa_p}{\kappa_w(\sigma+\phi)}. \tag{59}$$

Note that: if (57) holds, at least one between (58) and/or (59) is satisfied, since $\frac{\kappa_p+\kappa_w}{\kappa_w(\sigma+\phi)} + \frac{2(1+\beta)}{\kappa_w(\sigma+\phi)} < \frac{4(1+\beta)+\kappa_w+\kappa_p}{\kappa_w(\sigma+\phi)} < \frac{\kappa_p+\kappa_w}{\kappa_w(\sigma+\phi)} + \frac{4(1+\beta)}{\kappa_w(\sigma+\phi)} + \frac{2(1-\beta)^2}{\kappa_w(\sigma+\phi)(1+\beta)}$. Hence (57) guarantees that at least one between $\tilde{a}_2$ and $\tilde{a}_1$ is negative. Decartes' rule of signs then implies 3 positive roots.

To conclude, in the case $\phi_\pi < 1$, the equilibrium is determinate iff

$$1 > \phi_\pi > 1 + \frac{2\sigma\left(1+\beta\right)\left[2\left(1+\beta\right) + \kappa_p + \kappa_w - \chi\kappa_w(\sigma+\phi)\right]}{\kappa_w\kappa_p\left(\sigma+\phi\right)} \tag{60}$$

and

$$\chi > \frac{\kappa_p + \kappa_w}{\kappa_w(\sigma+\phi)} + \frac{2(1+\beta)}{\kappa_w(\sigma+\phi)}. \tag{61}$$

Putting together the two cases $\phi_\pi > 1$ and $\phi_\pi < 1$., it yields Proposition 3. QED

A.3.4 Contemporaneous Rule

We consider the following policy rule:

$$i_t = \phi_\pi \pi_t + \phi_{\pi^w} \pi_t^w \tag{62}$$

The corresponding matrix formulation of our dynamic system is:

$$\begin{bmatrix} \pi^w_{t+1} \\ \pi_{t+1} \\ x_{t+1} \\ \tilde{\omega}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & 0 & -\frac{1}{\beta}\kappa_w(\sigma+\phi) & +\frac{1}{\beta}\kappa_w \\ 0 & \frac{1}{\beta} & 0 & -\frac{1}{\beta}\kappa_p \\ \frac{1}{\sigma}\phi_{\pi^w} + \frac{1}{\beta}\chi & \frac{1}{\sigma}\phi_\pi - \frac{1}{\sigma}\frac{1}{\beta} - \chi\frac{1}{\beta} & 1 - \chi\frac{1}{\beta}\kappa_w(\sigma+\phi) & \chi\frac{1}{\beta}\left(\kappa_w+\kappa_p\right) + \frac{1}{\sigma}\frac{1}{\beta}\kappa_p \\ \frac{1}{\beta} & -\frac{1}{\beta} & -\frac{1}{\beta}\kappa_w(\sigma+\phi) & 1 + \frac{1}{\beta}\left(\kappa_w+\kappa_p\right) \end{bmatrix} \begin{bmatrix} \pi^w_t \\ \pi_t \\ x_t \\ \tilde{\omega}_t \end{bmatrix}.$$

The coefficients of the characteristic polynomial are:

$$a_1 = -trace(J_0) = -\frac{1}{\beta}\left[2\left(1+\beta\right)+\left(\kappa_p+\kappa_w\right)-\chi\kappa_w\left(\sigma+\phi\right)\right]$$

$$a_2 = 1+\frac{4}{\beta}+\frac{1}{\beta^2}+\frac{1}{\beta}\left(1+\frac{1}{\beta}\right)\left(\kappa_p+\kappa_w\right)-\frac{1}{\beta}\left(1+\frac{1}{\beta}\right)\chi\kappa_w\left(\sigma+\phi\right)+\frac{1}{\sigma\beta}\left(\phi_{\pi^w}+\frac{1}{\beta}\kappa_p\right)\left(\phi+\sigma\right)\kappa_w$$

$$a_3 = -\frac{2}{\beta}\left(\frac{1}{\beta}+1\right)-\frac{1}{\beta^2}\left(\kappa_p+\kappa_w\right)+\left(\frac{\chi}{\beta^2}-\frac{1}{\sigma\beta^2}\left(\sigma+\phi\right)\kappa_p\kappa_w\left(\phi_\pi+\phi_{\pi^w}\right)-\frac{1}{\sigma\beta}\left(1+\frac{1}{\beta}\right)\phi_{\pi^w}\right)\left(\phi+\sigma\right)\kappa_w$$

$$a_4 = \frac{1}{\beta^2}\left(1+\frac{1}{\sigma}\left(\sigma+\phi\right)\kappa_w\phi_{\pi^w}\right).$$

Repeating the steps above in (49), the Hurwitz polynomial is given by:

$$\underbrace{\frac{\frac{1}{\sigma}\left(1-\left(\phi_\pi+\phi_{\pi^w}\right)\right)\left(\sigma+\phi\right)\kappa_p\kappa_w}{D}}_{\tilde{a}_4}$$
$$+s\underbrace{\frac{2\left(1-\beta\right)\left[\kappa_p+\kappa_w-\kappa_w(\phi_{\pi^w}+\left(\sigma+\phi\right)\chi)\right]+\frac{2}{\sigma}\kappa_w\left[\left(\sigma+\phi\right)\left(\phi_\pi+\phi_{\pi^w}\right)\kappa_p+\left(\beta-1\right)\phi\phi_{\pi^w}\right]}{D}}_{\tilde{a}_3}$$
$$+s^2\underbrace{\frac{4\left(1-\beta\right)^2-2\left(1+\beta\right)\left[\kappa_p+\kappa_w-\chi\kappa_w\left(\sigma+\phi\right)\right]-\frac{2}{\sigma}\kappa_w\left(\sigma+\phi\right)\left[\kappa_p-\left(\beta-3\right)\phi_{\pi^w}\right]}{D}}_{\tilde{a}_2}$$
$$+s^3\underbrace{\frac{8\left(\beta^2-1\right)+2\left(\beta-1\right)\left[\kappa_p+\kappa_w-\left(\sigma+\phi\right)\chi\kappa_w\right]-\frac{2}{\sigma}\left(\phi+\sigma\right)\kappa_w\left[\left(\beta+3\right)\phi_{\pi^w}-\kappa_p\left(\phi_\pi+\phi_{\pi^w}\right)\right]}{D}}_{\tilde{a}_1}$$
$$+s^4 \tag{63}$$

where

$$\begin{aligned} D &= 2\left(1+\beta\right)\left[2\left(1+\beta\right)+\left(\kappa_p+\kappa_w\right)-\left(\sigma+\phi\right)\chi\kappa_w\right] \\ &\quad +\frac{2}{\sigma}\left(\beta+1\right)\left(\sigma+\phi\right)\kappa_w\phi_{\pi^w}+\frac{1}{\sigma}\left(\sigma+\phi\right)\left(1+\phi_\pi+\phi_{\pi^w}\right)\kappa_p\kappa_w \end{aligned}$$

Note there should be 3 positive roots and 1 negative root for the REE to be unique. It follows that a necessary condition must be that $\tilde{a}_4 < 0$. As for the proof above in A.3.3, we look at the signs of the coefficients $\tilde{a}_i$, and we exploit the Decartes' rule of sign.

A.3.5 Proof of Propositions 4: Case $\phi_{\pi^w} = 0$

If $\phi_{\pi^w} = 0$, the Hurwitz polynomial is:

$$\underbrace{\frac{\frac{1}{\sigma}\left(1-\phi_\pi\right)\left(\sigma+\phi\right)\kappa_p\kappa_w}{den}}_{\tilde{a}_4}+$$

$$s\underbrace{\frac{2\left(1-\beta\right)\left[\left(\kappa_p+\kappa_w\right)-\left(\sigma+\phi\right)\chi\kappa_w\right]+\frac{2}{\sigma}\left(\sigma+\phi\right)\phi_\pi\kappa_p\kappa_w}{den}}_{\tilde{a}_3}+$$

$$s^2\underbrace{\frac{4\left(1-\beta\right)^2-2\left(1+\beta\right)\left[\left(\kappa_p+\kappa_w\right)-\chi\kappa_w\left(\sigma+\phi\right)\right]-\frac{2}{\sigma}\left(\sigma+\phi\right)\kappa_p\kappa_w}{den}}_{\tilde{a}_2}+$$

$$s^3\underbrace{\frac{8\beta^2-8+2\left(\beta-1\right)\left[\left(\kappa_p+\kappa_w\right)-\left(\sigma+\phi\right)\chi\kappa_w\right]-\frac{2}{\sigma}\kappa_p\kappa_w\left(\phi+\sigma\right)\phi_\pi}{den}}_{\tilde{a}_1}$$

$$+s^4$$

where

$$den=2\left(1+\beta\right)\left[2\left(1+\beta\right)+\left(\kappa_p+\kappa_w\right)-\left(\sigma+\phi\right)\chi\kappa_w\right]+\frac{1}{\sigma}\left(\sigma+\phi\right)\left(1+\phi_\pi\right)\kappa_p\kappa_w$$

Look separately at the case when $\phi_\pi>1$ and when $\phi_\pi<1$.

Case $\phi_\pi>1$.

$\tilde{a}_4$) In this case $N_{\tilde{a}_4}<0$, hence D must be positive. For D to be positive, the following restriction must hold:

$$\phi_\pi>-1-\frac{2\sigma\left(1+\beta\right)\left[2\left(1+\beta\right)+\left(\kappa_p+\kappa_w\right)-\left(\sigma+\phi\right)\chi\kappa_w\right]}{\left(\sigma+\phi\right)\kappa_p\kappa_w}=\bar{\phi}_\pi^{a,CR}. \tag{64}$$

$\tilde{a}_3$) Then, since $D>0$, there are two cases:

i) $N_{\tilde{a}_3}>0 \Rightarrow \tilde{a}_3>0$, that happens for:

$$\phi_\pi>\frac{\sigma\left(1-\beta\right)\left[\left(\sigma+\phi\right)\chi\kappa_w-\left(\kappa_p+\kappa_w\right)\right]}{\left(\sigma+\phi\right)\kappa_p\kappa_w}=\bar{\phi}_\pi^{b,CR}. \tag{65}$$

Note that in this case $\tilde{a}_1<0$, since $\tilde{a}_1=-\tilde{a}_3-8(1-\beta^2)$. It follows that, whatever the sign of $\tilde{a}_2$, $P_H(s)$ exhibits three sign changes, while $P_H(-s)$ only one. So there will be 3 positive roots and 1 negative root. This proves that if is $\phi_\pi>\max\left\{1;\bar{\phi}_\pi^{a,CR};\bar{\phi}_\pi^{b,CR}\right\}$, the REE is determinate.

ii) $N_{\tilde{a}_3}<0 \Rightarrow \tilde{a}_3<0$, that happens for: $\phi_\pi<\frac{\sigma(1-\beta)[(\sigma+\phi)\chi\kappa_w-(\kappa_p+\kappa_w)]}{(\sigma+\phi)\kappa_p\kappa_w}=\bar{\phi}_\pi^{b,CR}$. Since anyway, it should be $\phi_\pi>1$ and $\phi_\pi>\bar{\phi}_\pi^{a,CR}$, the condition $\phi_\pi<\bar{\phi}_\pi^{b,CR}$ then requires $\bar{\phi}_\pi^{b,CR}>1$ and $\bar{\phi}_\pi^{b,CR}>\bar{\phi}_\pi^{a,CR}$ to have an interval where the 3 conditions are all jointly satisfied. This implies the following conditions:

$$\chi>\frac{\kappa_p}{\sigma\left(1-\beta\right)}+\frac{\kappa_p+\kappa_w}{\left(\sigma+\phi\right)\kappa_w}$$

and

$$\chi<\frac{\sigma\left(\kappa_p+\kappa_w\right)\left[1+3\beta\right]+\left(\sigma+\phi\right)\kappa_p\kappa_w+4\sigma\left(1+\beta\right)^2}{\sigma\left(\sigma+\phi\right)\kappa_w\left[1+3\beta\right]}.$$

Under this conditions, since the first two coefficients $(\tilde{a}_4, \tilde{a}_3)$ are negative and the last is positive, then it must be that $\tilde{a}_2 > 0$ and $\tilde{a}_1 < 0$ to have three signs inversions.

Condition for $\tilde{a}_2 > 0 \Rightarrow N_{\tilde{a}_2} > 0 \Rightarrow$

$\chi > \frac{\kappa_p}{\sigma(1+\beta)} - \frac{2(1-\beta)}{\kappa_w(\sigma+\phi)} + \frac{\kappa_p+\kappa_w}{\kappa_w(\sigma+\phi)}$.

Condition for $\tilde{a}_1 < 0 \Rightarrow N_{\tilde{a}_1} < 0 \Rightarrow$

$\phi_\pi > \bar{\phi}_\pi^{b,CR} - \frac{4\sigma\left(1-\beta^2\right)}{\kappa_p\kappa_w(\phi+\sigma)}$.

So determinacy can occur iff all the following conditions are jointly satisfied:

$$\begin{cases} \phi_\pi > -1 - \frac{2\sigma(1+\beta)[2(1+\beta)+(\kappa_p+\kappa_w)-(\sigma+\phi)\chi\kappa_w]}{(\sigma+\phi)\kappa_p\kappa_w} = \bar{\phi}_\pi^{a,CR} \\ \phi_\pi < \frac{\sigma(1-\beta)[(\sigma+\phi)\chi\kappa_w-(\kappa_p+\kappa_w)]}{(\sigma+\phi)\kappa_p\kappa_w} = \bar{\phi}_\pi^{b,CR} \\ \phi_\pi > 1 \\ \phi_\pi > \bar{\phi}_\pi^{b,CR} - \frac{4\sigma\left(1-\beta^2\right)}{\kappa_p\kappa_w(\phi+\sigma)} \\ \chi > \frac{\kappa_p}{\sigma(1+\beta)} - \frac{2(1-\beta)}{\kappa_w(\sigma+\phi)} + \frac{\kappa_p+\kappa_w}{\kappa_w(\sigma+\phi)} \\ \chi > \frac{\kappa_p}{\sigma(1-\beta)} + \frac{\kappa_p+\kappa_w}{(\sigma+\phi)\kappa_w} \\ \chi < \frac{\sigma(\kappa_p+\kappa_w)[1+3\beta]+(\sigma+\phi)\kappa_p\kappa_w+4\sigma(1+\beta)^2}{\sigma(\sigma+\phi)\kappa_w[1+3\beta]} \end{cases}$$

It is easy to show that this case is extremely unlikely. First, since it should be $\bar{\phi}_\pi^{b,CR} - \frac{4\sigma\left(1-\beta^2\right)}{\kappa_p\kappa_w(\phi+\sigma)} < \phi_\pi < \bar{\phi}_\pi^{b,CR}$, then if $\beta = 1$ this case does not admit determinacy. Moreover, for $\beta \to 1$, also the set that define the conditions on χ becomes empty. Second, for our benchmark calibration for the conditions above that define the admissible values of χ imply: $\chi > \frac{\kappa_p}{\sigma(1+\beta)} - \frac{2(1-\beta)}{\kappa_w(\sigma+\phi)} + \frac{\kappa_p+\kappa_w}{\kappa_w(\sigma+\phi)} = 0.42246$; $\chi > \frac{\kappa_p}{\sigma(1-\beta)} + \frac{\kappa_p+\kappa_w}{(\sigma+\phi)\kappa_w} = 9.3275$; $\chi < \frac{\sigma(\kappa_p+\kappa_w)[1+3\beta]+(\sigma+\phi)\kappa_p\kappa_w+4\sigma(1+\beta)^2}{\sigma(\sigma+\phi)\kappa_w[1+3\beta]} = 4.8919$, that can not be jointly satisfied.

Finally, $\bar{\phi}_\pi^{a,CR}$ and $\bar{\phi}_\pi^{b,CR}$ are equal for a value of χ that implies $\bar{\phi}_\pi^{a,CR} = \bar{\phi}_\pi^{b,CR} = \frac{\sigma(1-\beta)\left[(\sigma+\phi)\kappa_p\kappa_w+4\sigma(1+\beta)^2\right]}{(\sigma+\phi)\kappa_p\kappa_w\sigma[1+3\beta]}$. It is sufficient to assume that $\frac{\sigma(1-\beta)\left[(\sigma+\phi)\kappa_p\kappa_w+4\sigma(1+\beta)^2\right]}{(\sigma+\phi)\kappa_p\kappa_w\sigma[1+3\beta]}$ is less than one to get rid of this case. So in what follows we will assume this mild condition, that is very likely to be satisfied.

Case $\phi_\pi < 1$.

$\tilde{a}_4$) In this case $N_{\tilde{a}_4} > 0$, hence D must be negative. Thus:

$$\phi_\pi < -1 - \frac{2\sigma\left(1+\beta\right)\left[2\left(1+\beta\right)+\left(\kappa_p+\kappa_w\right)-\left(\sigma+\phi\right)\chi\kappa_w\right]}{\left(\sigma+\phi\right)\kappa_p\kappa_w} = \bar{\phi}_\pi^{a,CR} \tag{66}$$

$\tilde{a}_3$) Then, since $D < 0$, there are two cases:

i) $N_{\tilde{a}_3} < 0 \Rightarrow \tilde{a}_3 > 0$, that happens for

$$\phi_\pi < \frac{\sigma\left(1-\beta\right)\left[\left(\sigma+\phi\right)\chi\kappa_w-\left(\kappa_p+\kappa_w\right)\right]}{\left(\sigma+\phi\right)\kappa_p\kappa_w} = \bar{\phi}_\pi^{b,CR} \tag{67}$$

Note that in this case $\tilde{a}_1 < 0$, since $\tilde{a}_1 = -\tilde{a}_3 - 8(1-\beta^2)$. It follows that, whatever the sign of $\tilde{a}_2$, $P_H(s)$ exhibits three sign changes, while $P_H(-s)$ only one. So there will be 3 positive roots and 1 negative root. This proves that if is $\phi_\pi > \min\left\{1; \bar{\phi}_\pi^{a,CR}; \bar{\phi}_\pi^{b,CR}\right\}$, the REE is determinate.

ii) $N_{\tilde{a}_3} > 0 \Rightarrow \tilde{a}_3 < 0$, that happens for: $\phi_\pi > \frac{\sigma(1-\beta)[(\sigma+\phi)\chi\kappa_w-(\kappa_p+\kappa_w)]}{(\sigma+\phi)\kappa_p\kappa_w} = \bar{\phi}_\pi^{b,CR}$. Since anyway, it should be $\phi_\pi < 1$ and $\phi_\pi < \bar{\phi}_\pi^{a,CR}$, the condition $\phi_\pi > \bar{\phi}_\pi^{b,CR}$ then requires $\bar{\phi}_\pi^{b,CR} < 1$ and $\bar{\phi}_\pi^{b,CR} < \bar{\phi}_\pi^{a,CR}$ to have an interval where the 3 conditions are all jointly satisfied. This implies the following conditions:

$$\chi > \frac{\kappa_p}{\sigma(1-\beta)} + \frac{\kappa_p + \kappa_w}{(\sigma+\phi)\kappa_w}$$

and

$$\chi < \frac{\sigma(\kappa_p+\kappa_w)[1+3\beta] + (\sigma+\phi)\kappa_p\kappa_w + 4\sigma(1+\beta)^2}{\sigma(\sigma+\phi)\kappa_w[1+3\beta]}.$$

Under this conditions, since the first two coefficients $(\tilde{a}_4, \tilde{a}_3)$ are negative and the last is positive, then it must be that $\tilde{a}_2 > 0$ and $\tilde{a}_1 < 0$ to have three signs inversions.

Condition for $\tilde{a}_2 > 0 \Rightarrow N_{\tilde{a}_2} < 0 \Rightarrow$

$\chi < \frac{\kappa_p}{\sigma(1+\beta)} - \frac{2(1-\beta)}{\kappa_w(\sigma+\phi)} + \frac{\kappa_p+\kappa_w}{\kappa_w(\sigma+\phi)}$.

Condition for $\tilde{a}_1 < 0 \Rightarrow N_{\tilde{a}_1} > 0 \Rightarrow$

$\phi_\pi < \bar{\phi}_\pi^{b,CR} - \frac{4\sigma(1-\beta^2)}{\kappa_p\kappa_w(\phi+\sigma)}$.

This latter condition, however, contradicts the condition above that yields $\tilde{a}_3 < 0$, that is: $\phi_\pi > \bar{\phi}_\pi^{b,CR}$. Hence this case does not admit determinacy of REE.

The two conditions that are necessary and sufficient for the determinacy of the equilibrium are therefore: $\phi_\pi > \max\left\{1; \bar{\phi}_\pi^{a,CR}; \bar{\phi}_\pi^{b,CR}\right\}$ or $\phi_\pi < \min\left\{1; \bar{\phi}_\pi^{a,CR}; \bar{\phi}_\pi^{b,CR}\right\}$. QED

A.3.6 Proof of Proposition 5: Case $\phi_{\pi^w} \neq 0$

Let's now consider the general case, where the Hurwitz polynomial is (63).

A first important result is that $1 = \phi_\pi + \phi_{\pi^w}$ identifies a zero root. This analytically suggests that the numerical result in Galí (2008) regarding the model of Erceg et al. (2000) still survives in a model with LAMP.

Here we are just looking for a necessary condition that involves $\phi_\pi + \phi_{\pi^w} \lesseqgtr 1$. We know that a necessary condition is $\tilde{a}_4 < 0$. This is satisfied iff:

1) either $N_{\tilde{a}_4} > 0, D < 0$

2) or $N_{\tilde{a}_4} < 0, D > 0$.

That implies:

1) either $(\phi_\pi + \phi_{\pi^w}) < \min\left\{1, \bar{\phi}_{\pi,\pi^w}\right\}$

2) or $(\phi_\pi + \phi_{\pi^w}) > \max\left\{1, \bar{\phi}_{\pi,\pi^w}\right\}$.

QED

A.4 Proposition 6: derivation of the Welfare-based Loss Function

Remember that the steady state of our economy is efficient, therefore:

$$\frac{v_{L,H}}{u_{C,H}} = \frac{v_{L,S}}{u_{C,S}} = \frac{W}{P} = \frac{Y}{L} = 1 \tag{68}$$

where $L_H = L_S = L = Y$ and $C_H = C_S = C = Y$. The last equality in (68) holds since the economy production function is: $Y_t = L_t A_t$, where $A = 1$ in steady state.

As in Bilbiie (2008) we assume that the Central Bank maximizes a convex combination of the utilities of two types of households, weighted by the mass of agents of each type, i.e.:

$$W_t = \lambda \left[u \left(C_{H,t} \right) - v(L_{H,t}) \right] + (1 - \lambda) \left[u \left(C_{S,t} \right) - v(L_{S,t}) \right] \tag{69}$$

we know that in our model, because of the presence of the union, $L_{H,t} = L_{S,t} = L_t$ for each t, this means that (69) can be rewritten as

$$W_t = \lambda u \left(C_{H,t} \right) + (1 - \lambda) u \left(C_{S,t} \right) - v \left(L_t \right) \tag{70}$$

A second order approximation of $\lambda u \left(C_{H,t} \right)$ and $(1 - \lambda) u \left(C_{S,t} \right)$ delivers

$$\lambda u \left(C_{H,t} \right) - \lambda u \left(C_H \right) \simeq \lambda u_{C_H} C_H \left(c_{h,t} + \frac{1}{2} (1 - \sigma) c_{h,t}^2 + c_{h,t} \psi_t \right) + tip \tag{71}$$

$$(1 - \lambda) u \left(C_{s,t} \right) - \lambda u \left(C_s \right) \simeq (1 - \lambda) u_{C_s} C_s \left(c_{s,t} + \frac{1}{2} (1 - \sigma) c_{s,t}^2 + c_{s,t} \psi_t \right) + tip \tag{72}$$

Also a second order approximation to $v \left(L_t \right)$ yields:

$$v \left(L_t \right) - v \left(L \right) \simeq v_L L \left(l_t + \frac{1 + \phi}{2} l_t^2 \right) \tag{73}$$

Summing all the terms and considering steady state consumption levels of the two households are identical

$$\begin{aligned} W_t - W = & \lambda u_C C \left(c_{h,t} + \frac{1}{2} (1 - \sigma) c_{h,t}^2 \right) + u_C C c_t \psi_t \\ & + (1 - \lambda) u_C C \left(c_{s,t} + \frac{1}{2} (1 - \sigma) c_{s,t}^2 \right) - v_L L \left(l_t + \frac{1 + \phi}{2} l_t^2 \right) + tip \end{aligned} \tag{74}$$

>From the economy production function we know that

$$l_t = y_t + d_{w,t} + d_{p,t} - a_t \tag{75}$$

where $d_{w,t} = \log \int_0^1 \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} dj$ is the log of the wage dispersion and $d_{p,t} = \log \int_0^1 \left(\frac{P_t^i}{P_t} \right)^{-\theta_p} di$ is the log of the price dispersion. Both terms are of second order and therefore they cannot be neglected in a second order approximation. Notice that

$$l_t^2 = \left(\hat{y}_t + d_{w,t} + d_{p,t} - a_t \right)^2 = y_t^2 + a_t^2 - 2 y_t a_t \tag{76}$$

using (75), the efficient steady state condition $u_C C = v_L L$, the equilibrium condition $c_t = y_t$ we get:

$$\begin{aligned} \frac{W_t - W}{u_C C} = & y_t + \frac{(1 - \sigma)}{2} \left[\lambda c_{h,t}^2 + (1 - \lambda) c_{s,t}^2 \right] + c_t \psi_t + \\ & - \left(y_t + d_{w,t} + d_{p,t} - a_t + \frac{1 + \phi}{2} y_t^2 - (1 + \phi) y_t a_t \right) + tip \end{aligned} \tag{77}$$

Next notice that $c_{H,t} = w_t + l_t$,then

$$\begin{aligned} c_{H,t}^2 &= w_t^2 + l_t^2 + 2w_t l_t \\ &= w_t^2 + y_t^2 + a_t^2 - 2y_t a_t + 2w_t y_t - 2w_t a_t \\ &= (y_t - a_t)^2 + w_t^2 + 2w_t y_t - 2w_t a_t \end{aligned}$$

and $c_{S,t} = \frac{1}{1-\lambda} c_t - \frac{\lambda}{1-\lambda} c_{H,t}$,thus

$$\begin{aligned} c_{S,t}^2 &= \frac{1}{(1-\lambda)^2} c_t^2 + \left(\frac{\lambda}{1-\lambda}\right)^2 c_{H,t}^2 - 2\left(\frac{1}{1-\lambda}\right)\left(\frac{\lambda}{1-\lambda}\right) c_t c_{H,t} \\ &= \frac{1}{(1-\lambda)^2} c_t^2 + \left(\frac{\lambda}{1-\lambda}\right)^2 \left(\hat{w}_t^2 + l_t^2 + 2\hat{w}_t l_t\right) - \frac{2\lambda}{(1-\lambda)^2} c_t \left(\hat{w}_t + l_t\right) \\ &= \frac{1}{(1-\lambda)^2} \hat{y}_t^2 + \left(\frac{\lambda}{1-\lambda}\right)^2 \left(\hat{w}_t^2 + \hat{y}_t^2 + a_t^2 - 2\hat{y}_t a_t + 2\hat{w}_t \hat{y}_t - 2\hat{w}_t a_t\right) \\ &\quad - \frac{2\lambda}{(1-\lambda)^2} \left(\hat{y}_t \hat{w}_t + \hat{y}_t^2 - y_t a_t\right) \end{aligned}$$

then

$$\begin{aligned} &\left(\lambda c_{H,t}^2 + (1-\lambda) c_{S,t}^2\right) \\ = \;& \lambda \left(y_t^2 + a_t^2 - 2y_t a_t + \hat{w}_t^2 + 2\hat{w}_t \hat{y}_t - 2w_t a_t\right) + \\ & \frac{1}{(1-\lambda)} \hat{y}_t^2 + \frac{\lambda^2}{(1-\lambda)} \left(\hat{w}_t^2 + \hat{y}_t^2 + a_t^2 - 2\hat{y}_t a_t + 2\hat{w}_t \hat{y}_t - 2\hat{w}_t a_t\right) - \frac{2\lambda}{(1-\lambda)} \left(\hat{y}_t \hat{w}_t + \hat{y}_t^2 - y_t a_t\right) \end{aligned}$$

collecting terms and simplifying

$$\left(\lambda c_{H,t}^2 + (1-\lambda) c_{S,t}^2\right) = \left(\frac{\lambda}{(1-\lambda)}\right) w_t^2 + y_t^2 + \frac{\lambda}{(1-\lambda)} a_t^2 - 2\left(\frac{\lambda}{(1-\lambda)}\right) w_t a_t$$

Using this results and considering that a_t is independent of policy the welfare function can be rewritten as

$$\begin{aligned} \frac{W_t - W}{u_C C} &= \frac{1}{2}\left[\frac{(1-\sigma)\lambda}{(1-\lambda)} w_t^2 - (\sigma + \phi) y_t^2 - 2\frac{(1-\sigma)\lambda}{(1-\lambda)} w_t a_t + 2y_t \psi_t + 2(1+\phi) y_t a_t\right] \\ &\quad - (d_{w,t} + d_{p,t}) + tip \end{aligned}$$

Next we have to rewrite some terms. Recall that $(\sigma + \phi) y_t^{Eff} = (1+\phi) a_t + \psi_t$, thus

$$(\sigma + \phi) y_t y_t^{Eff} = (1+\phi) y_t a_t + y_t \psi_t$$

and

$$\begin{aligned} (\sigma + \phi)\left(y_t - y_t^{eff}\right)^2 &= (\sigma + \phi)\left(y_t^2 + \left(y_t^{eff}\right)^2 - 2y_t y_t^{eff}\right) \\ &= (\sigma + \phi)\left(y_t^2 + \left(y_t^{eff}\right)^2\right) - 2(\sigma + \phi) y_t y_t^{eff} \end{aligned}$$

substituting for the previous result

$$(\sigma + \phi)\left(y_t - y_t^{eff}\right)^2 = (\sigma + \phi)\left(y_t^2 + \left(y_t^{eff}\right)^2\right) - 2(1+\phi) y_t a_t - 2y_t \psi_t$$

In this case

$$\frac{W_t - W}{u_C C} = \frac{1}{2}\left[\frac{(1-\sigma)\lambda}{(1-\lambda)}\left(w_t^2 - 2w_t a_t\right) - (\sigma+\phi)\, x_t^2\right] - (d_{w,t} + d_{p,t}) + tip$$

where $x_t = \left(y_t - y_t^{Eff}\right)$ and given that y_t^{Eff} is independent of policy. Also notice that $w_t^{eff} = a_t$, which is a term independent of policy. Multiplying w_t^{Eff} by w_t we get: $w_t w_t^{eff} = w_t a_t$, and therefore

$$\left(w_t - w_t^{eff}\right)^2 = w_t^2 + \left(w_t^{eff}\right)^2 - 2w_t w_t^{eff} = w_t^2 - 2w_t a_t + \left(w_t^{eff}\right)^2$$

which implies

$$w_t^2 - 2w_t a_t = \left(w_t - w_t^{eff}\right)^2 - \left(w_t^{eff}\right)^2 = \tilde{\omega}_t^2 - \left(w_t^{eff}\right)^2$$

Substituting the latter into the welfare loss function and considering that w_t^{eff} is a term independent of policy, we get

$$\frac{W_t - W}{u_C C} = \frac{1}{2}\left[\frac{(1-\sigma)\lambda}{(1-\lambda)}\tilde{\omega}_t^2 - (\sigma+\phi)\, x_t^2\right] - (d_{w,t} + d_{p,t}) + tip$$

Using Woodford Lemma 1 and Lemma 2, we can finally write the present discounted value of the Central Bank loss function as

$$L = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\frac{(\sigma-1)\lambda}{(1-\lambda)}\tilde{\omega}_t^2 + (\sigma+\phi)\, x_t^2 + \frac{\theta_w}{\kappa_w}(\pi_t^w)^2 + \frac{\theta_p}{\kappa_p}\pi_t^2\right) + tip$$

Notice that if $\sigma < 1$ deviation of the real wage from its efficient level leads to a lower society's loss.

A.4.1 Derivation of the welfare function under flexible wages

Remember that in the case in which wages are fully flexible, the labor supply is:

$$\omega_t = \sigma c_t + \phi l_t - \psi_t = (\sigma+\phi)\, y_t - \phi a_t - \psi_t - \phi d_{p,t} \tag{78}$$

hence, subtracting the efficient equilibrium to the LHS and the RHS of the previous equation

$$\tilde{\omega}_t = (\sigma+\phi)\, x_t - \phi d_{p,t} \tag{79}$$

where we use the fact that $d_{p,t} - d_{p,t}^{Eff} = d_{p,t}$ (given that $d_{p,t}^{Eff} = 0$). Moreover, we know $a_t = a_t^{Eff}$ and that $\psi_t = \psi_t^{Eff}$ and terms multiplied by $-\phi d_{p,t}$ are terms higher than second order. Then

$$\tilde{\omega}_t^2 = (\sigma+\phi)^2\, x_t^2$$

this means that the welfare-loss can be re-written as follows:

$$L = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left((\sigma+\phi)\left[1 + (\sigma-1)(\sigma+\phi)\frac{\lambda}{1-\lambda}\right]x_t^2 + \frac{\theta_p}{\kappa_p}\pi_t^2\right)$$

Notwithstanding wage flexibility there is and additional term with respect to a fully Ricardian framework, given by $\frac{(\sigma+\phi)(\sigma-1)\lambda}{1-\lambda}x_t^2$. Two conditions are necessary for the presence of this additional term. Once again this is due to the presence of rot agents and similarly it disappears when $\sigma = 1$. Also, when $\sigma < 1$, the identified additional term leads to a reduction in society's welfare loss.

A.4.2 Proofs of Proposition 8

Given $(M1)$ it follows immediately that strict price inflation targeting and strict wage gap targeting are equivalent. Indeed, $\pi_t = 0, \forall t \Leftrightarrow \tilde{\omega}_t = 0, \forall t$.In this case the model reduces to

$$(M2) \qquad \pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_w(\sigma + \phi)x_t$$

$$(M3) \qquad \pi_t^w = \Delta\omega_t^{Eff}$$

$$(M4) \qquad x_t = E_t x_{t+1} - \tfrac{1}{\sigma} E_t \left(i_t - r_t^{Eff}\right)$$

from which we can determine the path $\{\pi_t, \pi_t^w, x_t\}_{t=0}^{\infty}$ independently of λ. The loss function also does not depend on λ. From $(M3)$ and Given $a_t = \rho_a a_{t-1} + \varepsilon_t^a$, then $\pi_t^w = \Delta\omega_t^{Eff} = \Delta a_t = (\rho_a - 1)a_{t-1} + \varepsilon_t^a$. For $\rho_a < 1$

$$Var(\pi^w) = Var(\Delta a_t) = (\rho_a - 1)^2 Var(a_t) + \sigma_a^2 = \frac{2}{1+\rho_a}\sigma_a^2$$

Then substituting $(M3)$ into $(M2)$:

$$\begin{aligned} \pi_t^w &= \beta E_t \pi_{t+1}^w + \kappa_w(\sigma + \phi)x_t \\ \Delta\omega_{tt}^{Eff} &= \beta E_t \Delta\omega_{t+1}^{Eff} + \kappa_w(\sigma + \phi)x_t \end{aligned}$$

then assuming that a_t is known at t it follows that

$$x_t = \frac{1}{\kappa_w(\sigma+\phi)}\left[\Delta a_t - \beta E_t \Delta a_{t+1}\right] = \frac{1}{\kappa_w(\sigma+\phi)}\left[(\rho_a - 1)\left(1 - \beta\rho_a\right)a_{t-1} + \left(1 - \beta(\rho_a - 1)\right)\varepsilon_t^a\right]$$

One can found a value for the variance of the output gap as

$$Var\left(x_t\right) = \left(\frac{\left(\rho_a - 1\right)\left(1 - \beta\rho_a\right)}{\kappa_w(\sigma+\phi)}\right)^2 \frac{\sigma_a^2}{1-\rho_a^2} + \left(\frac{1 - \beta(\rho_a - 1)}{\kappa_w(\sigma+\phi)}\right)^2 \sigma_a^2$$

Substitute the unconditional variances in unconditional expectation of the loss function to get unconditional society's loss.

B Figures

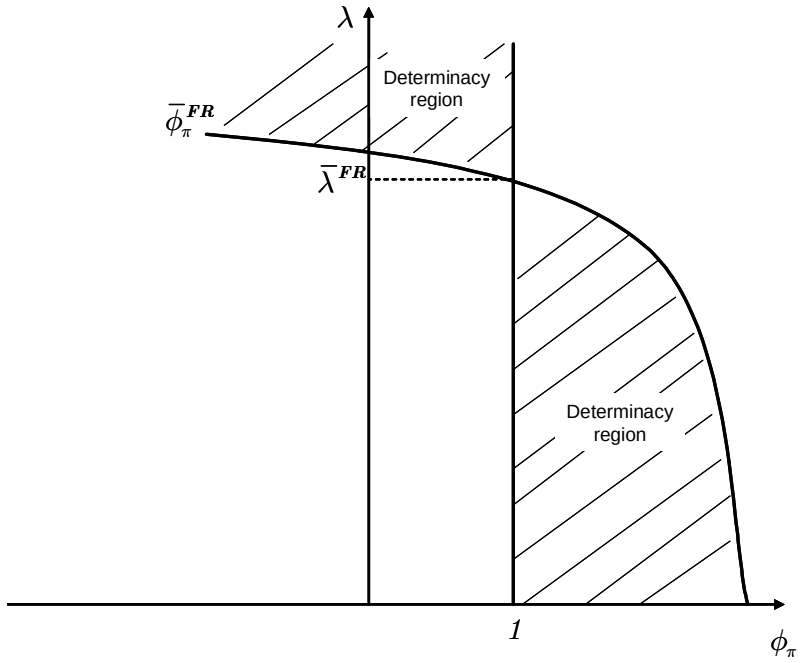

Figure 1. Determinacy and Indeterminacy regions when $i_t = \phi_\pi \pi_{t+1}$

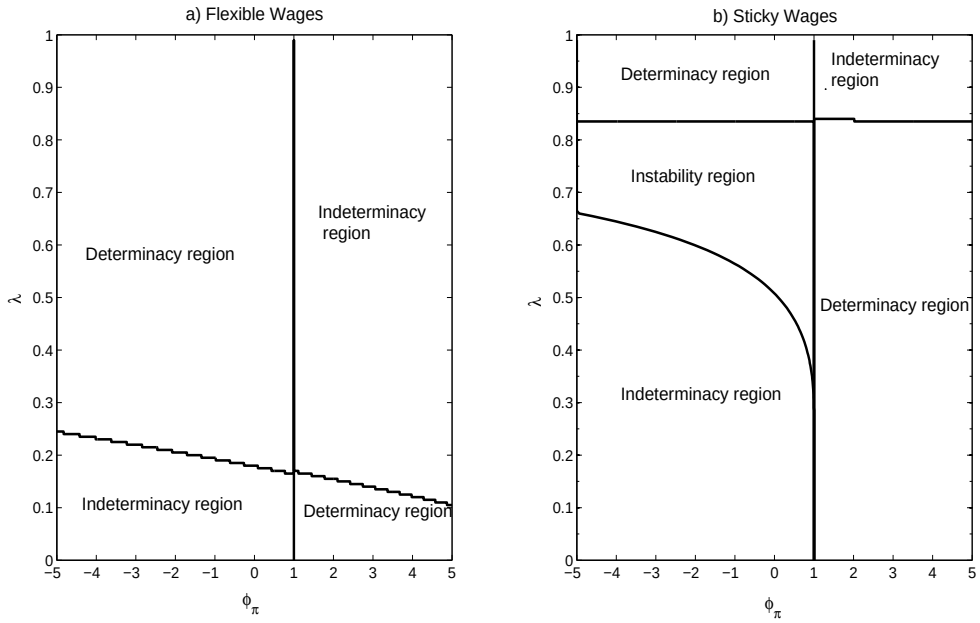

Figure 2. Determinacy and Indeterminacy regions under the rule: $i_t = \phi_\pi \pi_{t+1}$. Panel a): flexible wages, Panel b): sticky wages

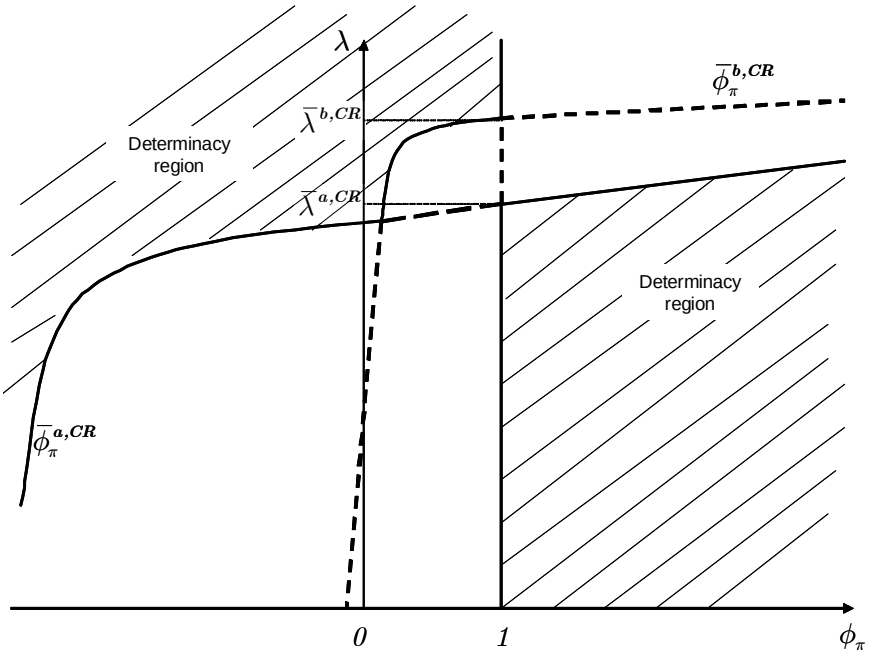

Figure 3. Determinacy and Indeterminacy regions when $i_t = \phi_\pi \pi_t$.

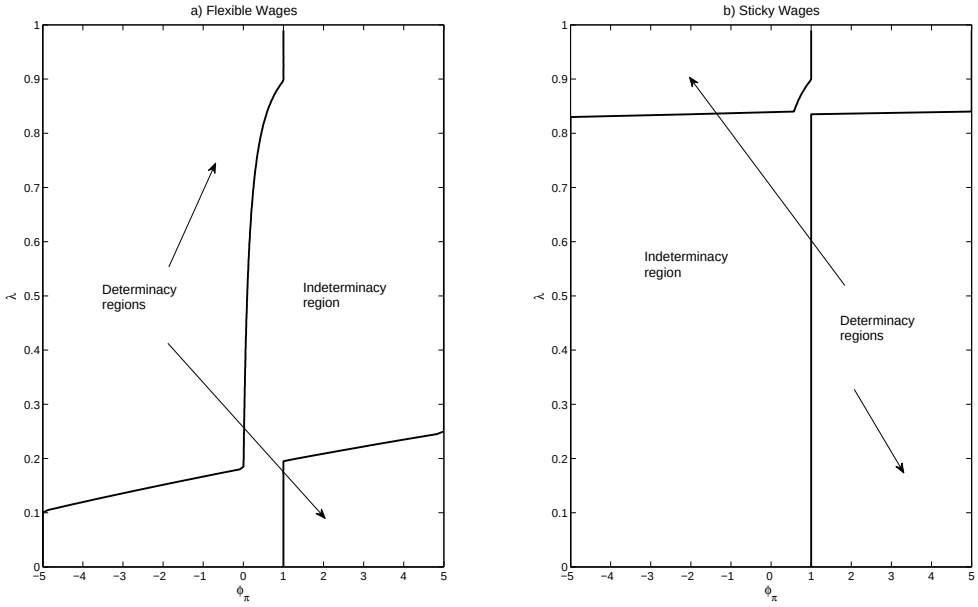

Figure 4. Determinacy and Indeterminacy regions under the rule: $i_t = \phi_\pi \pi_t$. Panel a): flexible wages, Panel b): sticky wages.

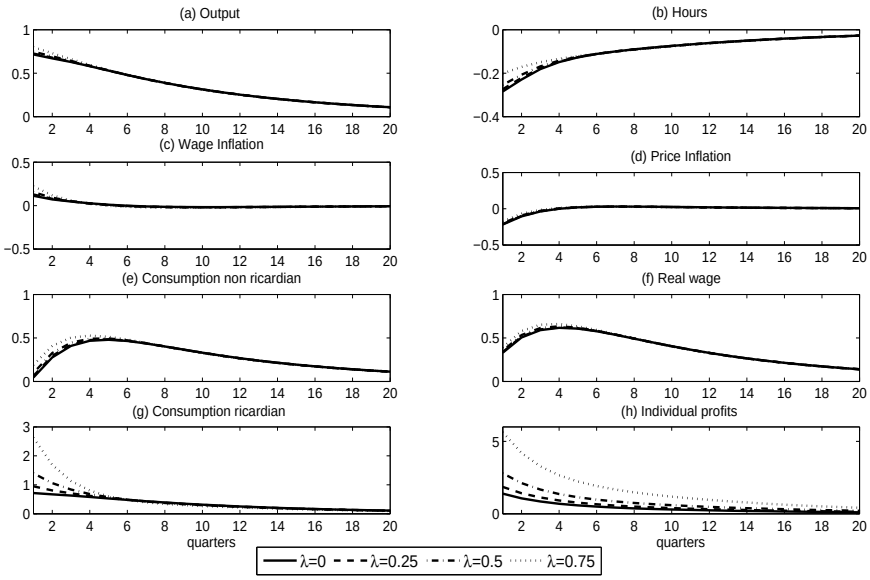

Figure 5. Impulse response function to a technology shock under full commitment for alternative values of the share of non-Ricardian agents (λ)

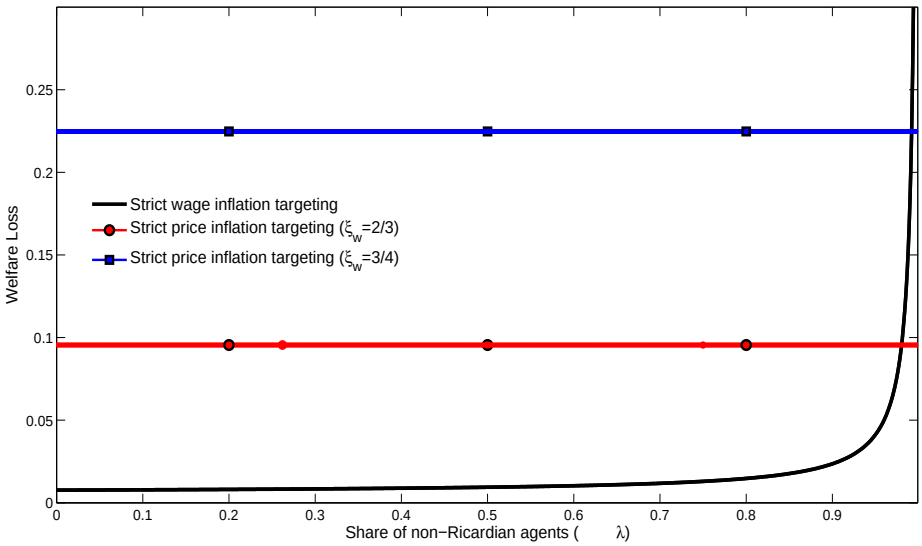

Figure 6. Unconditional welfare loss under strict wage inflation targeting and strict price inflation targeting. The latter is reported for two alternative average durations of wage contracts: 3 quarters ($\xi_w = 2/3$) and 4 quarters ($\xi_w = 3/4$).

C Tables

Average duration of wage contracts $(1-\xi_w)^{-1}$	$\lambda = 0$	$\lambda = 0.25$	$\lambda = 0.5$	$\lambda = 0.75$
Full Commitment				
1	0	0	0	0
2	0.0046	0.0054	0.007	0.0108
3	0.0059	0.0066	0.008	0.0125
4	0.0066	0.0071	0.0084	0.0125
5	0.0069	0.0075	0.0086	0.0124

Table 1: Unconditional welfare loss under full commitment. We consider alternative parameterizations for the share of non-Ricardian consumers and alternative average duration of wage contracts. The welfare loss is expressed as a percentage of the efficient steady state level of consumption, while the average duration of wage contracts is expressed in quarters.

Average duration of wage contracts $(1-\xi_w)^{-1}$	$\lambda = 0$	$\lambda = 0.25$	$\lambda = 0.5$	$\lambda = 0.75$
A) $i_t = \phi_\pi \pi_t$				
1	10,0.03	-10,0.02	-10,0.02	-10,0.02
2	5,1.3	5.1,1.1	6.1,0.9	-10,4.8
3	4.3,2.1	4.5,1.9	5.4,1.6	10,1.4
4	4.3,2.8	4.6,2.5	5.4,2.1	9.6,1.9
5	4.4,3.5	4.8,3.2	5.6,2.8	9.4,2.4
B) $i_t = \phi_\pi E_t \pi_{t+1}$				
1	10,0.06	-5.2,0.1	-10,0.06	-10,0.06
2	7.8,1	8.4,0.9	10,0.8	-10,0.6
3	6.5,1.6	7.1,1.4	9.4,1.3	10,2.2
4	6.4,2.2	7.1,2	9,1.8	10,2.7
5	6.7,2.8	7.3,2.6	9.1,2.3	10,3.4

Table 2: Panel A: Optimal contemporaneuos inflation response coefficient (left), welfare loss (right). Panel B: Optimal expected inflation response coefficient (left), welfare loss (right). The welfare loss is expressed as a fraction of the efficeint steady state consumption multiplied by one hundred. The average duration of wage contracts is expressed in quarters

Average duration of wage contracts $(1-\xi_w)^{-1}$	$\lambda = 0$	$\lambda = 0.25$	$\lambda = 0.5$	$\lambda = 0.75$
A) $i_t = \phi_\pi \pi_t + \phi_y y_t$				
1	10,0.05,0.04	-10,0.04,0.03	-10,0.05,0.03	-10,0.05,0.03
2	5.5,0.2,1.2	5.6,0.16,1.1	6.45,0.08,0.9	-10,-1.3,3.4
3	4.42,0.15,1.9	4.7,0.12,1.8	5.59,0.07,1.5	10,-0.07,1.4
4	4.8,0.2,2	5.04,0.17,1.8	5.8,0.1,1.5	9.5,-0.04,1.8
5	5.05,0.2,3.2	5.37,0.18,3	6.5,0.13,2.7	9.4,-0.01,2.4
B) $i_t = \phi_\pi \pi_t + \phi_\pi \pi_t^w$				
1	10,-0.006,0.04	-10,-0.13,0.02	-10,-0.11,0.03	-9.2,0.5,0.04
2	10,7.24,0.4	10,6.13,0.5	10,4.18,0.6	-10,-10,0.7
3	6.75,10,0.6	7.9,10,0.7	10,10,0.8	10,7.8,1.2
4	4.33,10,0.7	5.2,10,0.8	7.2,10,0.9	10,10,1.2
5	3.3,10,0.8	4.12,10,0.8	5.82,10,0.9	10,10,1.3

Table 3: Panel A: Optimal inflation response coefficient (left), optimal output response coefficient (center), welfare loss (right). Panel B: Optimal inflation response coefficient (left), optimal wage inflation response coefficient (center), welfare loss (right). The welfare loss is expressed as a fraction of the efficeint steady state consumption multiplied by one hundred. The average duration of wage contracts is expressed in quarters