# Fiscal Policy and the Term Structure of Interest Rates in a DSGE Model

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**Abstract**

We examine the role of government spending in the dynamics of the term structure of interest rates. Is the quantity of risk related government spending important for the price of risk? How does it depend on monetary policy conduct? Can fiscal policy immunize its impact on the term structure of interest rates? To answer this questions, we explore asset pricing implications of fiscal policy in what become paradigm in dynamic general equilibrium macro-finance literature. We break down the transmission of the government spending to macroeconomic attributes driving the dynamic response of the yield curve, both analytically and numerically. The novelty of our approach lies in the way we quantify the decomposition of pricing kernel. We find that rise in fiscal uncertainty amplifies the hedging property of bonds against real and nominal risks. Depending on the size of uncertainty monetary policy drives up the price of nominal risk. Spending reversals break the link between quantity and price of fiscal risk.

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## 1 Introduction

How does the term structure of interest rates respond to a rise in government purchases? Is the uncertainty about the size of government spending important for the bond prices? How does it depend on monetary policy conduct? Can fiscal policy immunize its impact on the term structure of interest rates? Neither empirical or theoretical literature provides clear answer to this questions.

It is the key piece of information for the policy makers to know the impact of their designed fiscal policies on the cost of financing in the economy. Public opinion, international affairs and political cycle often prompts policy makers to react quickly to new challenges by adjusting government spending. Higher volatility in government spending increases the uncertainty related to the fiscal policy and thus the quantity of risk in the economy. We argue that changes in the uncertainty related to government spending have consequences for the dynamic response of term structure of interest rates. Recent policy debate related to sovereign debt crises has focused on how to design fiscal and monetary policy conduct to mitigate the impact of government spending on the term structure. Greenspan's conundrum [1] pointed to the limited understanding of the link between monetary policy and term structure of interest rates. We take the debate one step further by stressing the importance of the monetary and fiscal policy mix for the bond prices. In most developed countries long-term yields has been extraordinarily low in the last decade. This could be interpreted as financial markets expecting prolonged low growth or low inflation, or both. This explanation has however little support in the data on market expectations. The balance sheet recession [2] theory has been advocated in the policy debate to

[1] In February 2005 Federal Reserve Chairman Alan Greenspan noticed that the 10-year Treasury yields failed to increase despite a 150-basis-point increase in the federal funds rate. Another is that in the early fall of 2008, as the FOMC was cutting the federal funds rate sharply, long-term interest rates actually rose, peaking in early November of that year. This could be called the .conundrum in reverse.

[2] see for example the book by Koo, Richard (2009). The Holy Grail of Macroeconomics-Lessons from Japan's Great Recession, or the discussion of Krugman and Koo at vox.org

provide clarification. Low long rates are reflective of precautionary steps taken by public and private market participants to reduce vulnerabilities to adverse outcomes. Hence, the precautionary saving motives in agents decision making may play important role in explaining the low yield decade. The fiscal policy is often put in the role of a key remedy to recover from the recession. The discussion on impacts of changes in government spending on the financial markets and precautionary saving is limited and sparse. Whether government spending raises or lowers the level and slope of the term structure is crucial for complex understanding of the impacts of government stimulus packages [3] on the economy. Stimulus packages are most often financed by issuing government debt. If the increase in government spending leads to increase of the level and slope of the term structure of interest rates it has direct adverse consequences for the current and future costs of government debt financing. Traditional cost benefit analysis of the stimulus packages may thus underestimate the true costs. This fact also points out to the gap in the vast literature on the size of multiplicator of government spending.

Our paper closely relates to De Paoli et al. (2010) in a sense that we do not attempt to solve asset pricing puzzles. Instead, we explore asset pricing implications of fiscal policy in what become paradigm in macro-finance literature. In the spirit of BansalShaliastovich2012 we evaluate how the quantity of risk coming from uncertainty about the government spending connects to the price of risk. The standard dynamic stochastic general equilibrium (DSGE) models face difficulties to match jointly macro and finance empirical moments in data e.g. Rudenbush and Swanson (2008), Hordahl et al. (2007), Jermann (1998). The inability of DSGE models to match term premium is known in literature as Backus, Gregory, and Zin (1989) puzzle. A number of models has been developed to improve the poor performance of DSGE models to price assets. Hordahl et al. (2007) shows that after modifying the standard New Keynesian DSGE model with nominal rigidities to include internal habits the model deliv-

[3] analogically the reverse applies also for austerity packages designed to downsize the public finance

ers sizable term premia. At the same time the model fits relatively well moments of consumption growth and inflation, although the results are sensitive to specific calibration. De Paoli et al. (2010) demonstrates in the similar model that implications of composition of preferences for asset prices depend on the source of the shock. Paoli and Zabczyk (2012) argue in the model with external habits that neglecting the precautionary saving motive in the model may have considerable consequences for the design of monetary policy. Rudebusch and Swanson (2012) introduce Epstein and Zin (EZ) preference structure into a basic New Keynesian (NK) DSGE model without capital. They solve the model to the third order, which allows them to match the volatility of the term premium. Ferman (2011) documents that introducing the regime switching to model with EZ preferences allow to match reasonably well the response of term premium to the different regimes of monetary policy (e.g. whether monetary policy gives a sufficient emphasis on inflation developments).

We follow different strategy than existing literature on the asset pricing within DSGE models which is mainly concerned in fitting as many empirical data features as possible. Rather than adding new modeling feature to fit additional moments in data we employ a standard DSGE model encompassing asset prices to explain in detail the interaction between government spending and the dynamics of the term structure of interest rates. To our knowledge there is not published theoretical paper asking similar question. [4]The novelty of our approach lies in modifying what is known as performance attribution analysis in asset management literature and use Brinson - Fachler methodology to link the impacts of government spending on the term structure with the macroeconomic factors. Our contribution to the literature is twofold. First, the term structure of interest rates of the risk free bonds is implicit in every DSGE model. Nevertheless, the implications for the term structure behavior in these models are not widely known. Second, neither the existing empirical nor theoretical research provides clear understanding to what is the impact of government spending on the term structure of interest rates.

[4]there is a work in progress by .... dealing with fiscal policy and term structure

We build our analysis on the variant of standard NK DSGE model (e.g. Galí (2002), De Paoli et al. (2010) or Erceg et al. (1999)) which we augment by EZ preferences as in Rudebusch and Swanson (2012), Markov switching in monetary policy rule as in Ferman (2011) and commitment to fiscal consolidation as in Giancarlo Corsetti and Müller (2009) The monetary policy rule is state dependent in the sense that there is a probability of change in the ratio of weights central bank puts on inflation output gap stabilization. We consider two fiscal scenarios. The first one is the simple fiscal setup assuming that government spending has stochastic variation and is covered by lump-sum taxes in each period (the case of balanced budget). The second fiscal arrangement allows for deficit, government debt and spending reversals a lá Giancarlo Corsetti and Müller (2009). With spending reversal the reduction in debt is aided by restraint on government purchases in the future. Giancarlo Corsetti and Müller (2009) have shown that spending reversals and, hence, higher savings of the government in the future generate crowding-in of government spending even in the present by agents anticipating lower long-term real rates that stimulate current consumption expenditures.

We show and explain why in the class of models usually used to explain asset prices: *i*) the yields jump up after rise in government spending; *ii*) uncertainty in fiscal policy amplifies the precautionary saving motive and risk aversion which leads to the drop in the level of the yield curve and rise in the term premium; *iii*) monetary policy plays crucial role in the transition mechanism of the government spending shock. *iv*) fiscal policy commitment to finance temporarily higher spending by future austerity significantly decreases the price of risk related to uncertainty about government spending

The remainder of the paper is structured as follows. In the next section we present the model. In section 3.1 we study policy implications of government spending shock for monetary policy in relation to term structure of interest rates. Section **??** utilizes the attribution analysis to decouple the transition mechanism in detail and provides robustness checks. Finally we conclude.

# 2 The model

We rely on a general equilibrium model similar to Ferman (2011) to quantitatively examine the links between government spending and dynamics of the term structure of interest rates. Our economy is populated by: *i*) households with recursive preferences who supply labor and buy public bonds, *ii*) firms operating on the final and intermediate goods market with the latter facing nominal rigidities, *iii*) government raising funding by lump-sum taxes and by issuing government bonds, *iv*) monetary policy following Taylor rule.

## 2.1 Households

The economy is inhabited by a continuum of households. The representative household chooses state-contingent paths for consumption $C$ and leisure, $L$ to maximize expected utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \tag{1}$$

where $\beta$ is the subjective discount factor of future stream of utilities, subject to an budget constraint:

$$P_t C_t + E_t Q_{t,t+1} B_{t+1} \leq B_t + D + W_t N_t + T_t \tag{2}$$

where $E_t Q_{t,t+1} B_{t+1}$ is the present value of portfolio of state-contingent bonds. $Q_{t,t+1}$ is the stochastic discount factor, $W_t N_t$ is the household labor income and $P_t$ is the aggregate price level.

The objective function in equation 1 can be written in recursive form as

$$V_t = u(C_t, L_t) + \beta E_t V_{t+1} \tag{3}$$

We follow Rudebusch and Swanson (2012) and use the following transform of Epstein and Zin (1989) preferences:

:

$$V_t = u(C_t, L_t) + \beta (E_t [V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{4}$$

when $u(C_t, L_t) > 0$.

If $u(C_t, L_t) < 0$, as in our benchmark calibration, the recursion takes the form:

$$V_t = u(C_t, L_t) - \beta(E_t[-V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{5}$$

Swanson (2012) shows the relationship of parameter $\alpha$ to the relative risk-aversion, intertemporal elasticity of substitution ($\gamma$) and coefficients.

The period utility is represented by $u(C_t, L_t) = b_t \left(\frac{C^{1-\gamma}}{1-\gamma} - \chi \frac{N^{1+\eta}}{1+\eta}\right)$ where $\xi_t$ is the time-preference shock which follows the autoregressive process:

$$b_t = \rho_b b_{t-1} + \sigma_b \epsilon_t^b \tag{6}$$

The household optimization exercise delivers Euler equation which allows us to price bond of any maturity:

$$Q_{t,t+1} = \beta \left(\frac{C_t}{C_{t+1}}\right)^{\gamma} \frac{P_t}{P_{t+1}} \left[\frac{V_{t+1}}{[E_t V_{t+1}^{1-\alpha}]^{\frac{1}{1-\alpha}}} e^{b_{t+1}-b_t}\right] \tag{7}$$

and labor supply:

$$W_t = \chi \frac{N_t^{\eta}}{C_t^{-\gamma}} \tag{8}$$

Letting $\Pi_{t+1} \equiv P_{t+1}/P_t$ denote inflation the price of a $\tau$-period nominal bond can be written as:

$$P_{\tau,t}^{bond} = E_t \left[Q_{t,t+1} P_{\tau-1,t+1}^{bond} \Pi_{t+1}^{-1}\right] \tag{9}$$

## 2.2 Firms

In the model there are final good bundlers which make use of intermediary products. Final good firms operate under perfect competition with the objective to minimize expenditures subject to the aggregate price level $P_t = \left(\int_0^1 P_t(i)^{\frac{-1}{\lambda_t}} di\right)^{-\lambda_t}$ using the technology $Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_t}} di\right)^{1+\lambda_t}$. The final good firms aggregate the continuum of intermediate goods $i$ on the interval $i \in [0, 1]$ into a single final good. Here $\lambda_t$ stands for the net-markup that is time-varying due to markup shocks (see more after the New Keynesian Phillips curve).

The cost-minimisation problem of final good firms deliver demand schedules for intermediary goods of the form:

$$Y_t(i) = \left(\frac{P_t}{P_t(i)}\right)^{\frac{1+\lambda_t}{\lambda_t}} Y_t \tag{10}$$

A continuum of intermediary firms operates in the economy. Intermediary firm $i$ uses the Cobb-Douglas technology

$$Y_t(i) = A_t \bar{K}^{\theta} N_t(i)^{1-\theta} \tag{11}$$

where $\bar{K}$ refers to the fact that firms have fixed capital[5] and $N_t(i)$ is the amount of labor employed. In equation 11 technology follows the autoregressive process:

$$A = \rho_A A_{t-1} + \sigma_A \epsilon_t^A \tag{12}$$

where $\epsilon_t^A$ is and independently and identically distributed (iid) shock with zero mean and constant variance.

Intermediaries face quadratic adjustment costs as in Rotemberg (1982): $PAC_t(i) = \frac{\zeta}{2}\left(\frac{P_t(i)}{P_{t-1}(i)}\frac{1}{\bar{\pi}} - 1\right)^2 P_t Y_t$ where $\zeta$ stands for the adjustment cost parameter. Intermediary firm chooses $P_t(i)$ so as to maximise the expected discounted sum of future profits corrected by adjustments costs:

$$E_t\{\sum_{j=0}^{\infty} Q_{t,t+j} \frac{P_t}{P_{t+j}} \left[D_{t,t+j}(i) - PAC_{t+j}(i)\right]\} \tag{13}$$

where $D_{t,t+j}(i) = P_{t+j}(i)Y_{t+j}(i) - W_{t+j}N_{t+j}(i)$ is the profit of firm $i$ between time $t$ and $t+j$ and $Q_{t,t+j}$ is the discount factor which is given by equation 7 The term $W_{t+j}N_{t+j}$ stands for the cost of labor.

The profit maximization exercise delivers the so-called New Keynesian Phillips curve

[5]Firm-specific capital can be interpreted as a model with endogenous investment that features strong adjustment costs in investment. Further, Rudebusch and Swanson (2012) emphasize the importance of firm-specific fixed factors for generating a degree of inflation persistence which can be found in the actual data.

$$MC_t = \frac{1}{1+\lambda_t} + \frac{\lambda_t}{1+\lambda_t}\zeta\left(\frac{\pi_t}{\bar{\pi}} - 1\right)\frac{\pi_t}{\bar{\pi}} - \frac{\lambda_t}{1+\lambda_t}E_t Q_{t,t+1}\zeta\left(\frac{\pi_{t+1}}{\bar{\pi}} - 1\right)\frac{\pi_{t+1}}{\bar{\pi}}\frac{Y_{t+1}}{Y_t} \tag{14}$$

where profits are discounted by $Q_{t,t+1}$ and the average real marginal cost is defined as

$$MC_t = \frac{1}{(1-\theta)\bar{K}_t^{\frac{\theta}{1-\theta}}}\left(\frac{W_{t(s)}}{A_t}\right)\left(\frac{Y_{t(s)}}{A_t}\right)^{\frac{\theta}{1-\theta}} \tag{15}$$

In equation 14 the markup (or cost-push) shock is given by:

$$log(1+\lambda_t) = (1-\rho_\lambda)log(1+\bar{\lambda}) + \rho_\lambda log(1+\lambda_{t-1}) + \sigma_\lambda \epsilon_t^\lambda \tag{16}$$

In case of flexible prices, $\zeta = 0$ and in the absence of cost-push shocks the marginal cost is constant and is equal to the inverse of the gross markup ($\frac{1}{1+\lambda}$).

## 2.3 Monetary policy

The model is closed with a monetary policy rule assuming that monetary authority sets the short-term nominal interest rate $i_t$ based on a simple Taylor rule.

$$i_t = \bar{i} + \phi_\pi \hat{\pi}_t + \phi_y \hat{\mathbf{y}}_t \tag{17}$$

where $\hat{\pi}_t$ and $\hat{\mathbf{y}}_t$ denote the percentage deviations of inflation and aggregate output from their corresponding deterministic steady states.

## 2.4 Market clearing

In equilibrium firms and household optimally choose prices with respect to their constraints and each market clears. The market clearing in the goods market requires that the aggregate demand equals to aggregate output in the economy:

$$Y_t = C_t + G_t + \bar{I} \tag{18}$$

where $G_t$ is an exogenous autoregressive process of the form:

$$G_t = \rho_G G_{t-1} + \sigma_G \epsilon_t^G \tag{19}$$

where $\epsilon_t^G$ is an iid shock with zero mean and unit variance. Parameter $\sigma_G$ scales the standard deviation of the shock. We assume in our benchmark model that government runs balanced budget financed through lump-sum taxes obtained from the household sector. The fixed nature of capital implies fixed investment that is used to replace depreciated capital: $I_t = \bar{I} = \delta \bar{K}$.

## 2.5 Extensions

We utilize the framework introduced by Giancarlo Corsetti and Müller (2009) to study the effects of fiscal consolidation on the term structure. Government consumption is financed through either lump-sum taxes, $T_t$ (taxes are in nominal terms) or the issuance of nominal debt, $D_t$, real government expenditures are denoted $G_t$.

$$T_t + Q_{t,t+1} D_{t+1} = D_t + P_t G_t \tag{20}$$

which can alternatively be expressed in real terms after dividing by the price level:

$$T_{Rt} + Q_{t,t+1} D_{Rt+1} = \frac{D_{Rt}}{\pi_t} + G_t \tag{21}$$

where $T_{Rt} = \frac{T_t}{P_t}$ are taxes in real terms and $D_{Rt} = \frac{D_t}{P_{t-1}}$ as a measure for real beginning-of-period debt.

Giancarlo Corsetti and Müller (2009) use a fiscal rule of the following form:

$$T_{Rt} = \Psi_t D_{Rt} \tag{22}$$

Spending reversals are captured by the following process for government purchases:

$$G_t = (1-\rho)G + \rho G_{t-1} - \Psi_G D_{Rt} + \eta_t \tag{23}$$

Researchers typically assume that the government spending today leads eventually to an increase in taxes. The idea of Giancarlo Corsetti and Müller

(2009) is that it is not necessary to increase taxes in response to higher government debt because government expenditures can be reduced to help settle debt. How does this work in our theoretical model? Spending reversals alter the short run effects of the government spending innovations through a financial channel that captures the combined effect of fiscal and monetary policy on long term interest rates. Households expect that the public spending will go down in the future and that monetary policy will in the forward looking Taylor rule increase short term interest rates but decreasing the long term interest rates which will boost consumption.

In another words, an increase in government spending will subsequently cause spending to fall below trend level for some time. The anticipated spending reversal does not crowd out private consumption and boost the expansionary effect of $G$ on output at the impact.

### 2.6 Calibration and solution method

To assign values to the parameters in our model we follow what become standard calibration in the literature for small closed economy models. Our calibration is similar to Rudebusch and Swanson (2012), Smets and Wouters (2007), Paoli and Zabczyk (2012) or Ferman (2011). The parameter values are summarized in table 1 and are quite standard in macro literature. We follow Ferman (2011) to set parameter $p_{11}$ which stands for the probability of staying in state one conditional on being in state one. As a result $p_{12} = 1 - p_{11}$ denotes the probability of moving to state two conditional on being in state one. The interpretations of $p_{22}$ and $p_{21}$ are analogous. Here $\zeta = \frac{\varphi\left(1-\theta+\theta\frac{1+\bar{\lambda}}{\bar{\lambda}}\right)}{(1-\varphi)(1-\varphi\beta)(1-\theta)}$ is set such that the slope of New Keynesian Phillips curve under Rotemberg price setting corresponds to the Calvo case with an average duration of price stickiness equal to $\frac{1}{1-\varphi} = 4$ quarters. The inflation coefficient of the Taylor-rule calibrated for the period of Great Moderation (state 1) is by construction higher than the one under Great Inflation (state 2) such that $\phi_{\pi(1)} > \phi_{\pi(2)}$. Important portion of the nominal term premium in the model is driven by the calibration of preference shock, elasticity of intertemporal substitution and Frisch labor elasticity. Whereas Rudebusch and Swanson (2012) picks lower than usual values of IES and Frisch elasticity we use somewhat higher persistency of preference shock and keep IES and Frisch elasticity at values standard in the literature. This is motivated by Fisher (2015) who provides structural interpretation to preference shock and identifies its increased importance since 2008.

Table 1: Calibration of the model

| Monetary Policy Rule | | | | Exogenous processes | | | |
|---|---|---|---|---|---|---|---|
| $\phi_{\pi(1)}$ | 2.19 | $\phi_{\pi(2)}$ | 0.948 | $\rho_b$ | 0.83 | $\sigma_b$ | 0.020 |
| $\phi_{y(1)}$ | 0.075 | $\phi_{y(2)}$ | 0.075 | $\rho_A$ | 0.98 | $\sigma_A$ | 0.005 |
| $p_{11}$ | 0.993 | $p_{22}$ | 0.967 | $\rho_\lambda$ | 0.18 | $\sigma_\lambda$ | 0.051 |
| | | | | $\rho_G$ | 0.94 | $\sigma_G$ | 0.008 |
| Structural Parameters | | | | The Steady-State | | | |
| $\beta$ | 0.99 | $\theta$ | 0.33 | $\bar{\Pi}$ | 1.004 | | |
| $\gamma$ | 2 | $\bar{\lambda}$ | 0.2 | $\bar{K}/(4\bar{Y})$ | 2.5 | | |
| $\eta$ | 0.40 | $\zeta$ | 233 | $\bar{G}/\bar{Y}$ | 0.2 | | |
| $\alpha$ | -108 | $\delta$ | 0.02 | | | | |

The quantity of risk under scrutiny is represented by the volatility in the innovations to government spending, $\sigma_g$, from the range of 4 bps to 6 percent. To determine this range we build on the argument forcefully put forward in Ramey (2011). She argues that defense spending are consistent with the specification of government spending in the VAR and DSGE models. The figure *xx (in appendix)* shows that defense spending accounts for most of the volatility of total government spending. The major movements in defense spending are associated with the military build up distributed around the war dates. Ramey (2011) shows that since the state and local (non-defense) spending are driven in large part by cyclical fluctuations in state revenues, aggregate VARs are not very good at capturing shocks to this type of spending. In most of the DSGE literature government spending enters exogenously the model. Thus, one has to be cautious when looking into the data for its counterpart. We check that defense spending are not correlated with the US business cycle to confirm its exogenous character, HP filter the series and use it to calculate the standard deviation of innovations.

Table 2 shows the volatility of the model consistent innovations and government spending for various sub-samples. The volatility of innovations in our data sub-samples ranges from 0.49 to 5.83 and justifies the wide range of $\sigma_g$ we use to evaluate the model. Our baseline calibration matches the long run average period between 1969 and 2009. [6]

| Period | $\sigma_g$ | std(G) |
|---|---|---|
| 1947 - 1957 | 5.83 | 17 |
| 1957 - 1967 | 1.55 | 4.53 |
| 1967 - 1977 | 1.61 | 4.71 |
| 1977 - 1987 | 0.49 | 1.43 |
| 1987 - 1997 | 0.61 | 1.79 |
| 1997 - 2007 | 0.9 | 2.63 |
| 1969 - 2009 | 0.8 | 2.43 |

Table 2: Standard deviation of defense spending and implied innovations. Results are in % deviations from the HP trend

The two-state non-linear Markov-switching model is approximated to the second-order using Dynare exploiting the fact that the model contains no state variables. The second-order approximation is necessary to break the certainty equivalence of linearized models.

[6] to calculate long run average we exclude the Korean and Vietnam war military build up as it is often done in the empirical literature - some argue that the Korean and Vietnam War were unusually large

## 2.7 Model evaluation

We report the model implied macro and finance moments along with the empirical moments for quarterly US data from 1961 to 2007. The table 3 demonstrates that the model is able to replicate the core macro-finance features reasonably well and comparably with the state of the art literature, e.i. Rudebusch and Swanson (2012), van Binsbergen et al. (2012). In the model with regime switching we report the state of the word associated with great moderation (1985 - 2007), therefore the implied term premium is higher. Similarly the volatility of inflation substantially droped to 1.39 in the period (1985 - 2007) compared to 2.9 in the period from 1969 to 1985. The version of the model with Markov switching is designed to take this regime change into account.

The resulting model set up is compromise between the complexity and clarity. We focus on matching the factors driving the nominal term premium and fiscal policy to be as closed to data as possible. Somewhat poorer match of the other variables goes on the costs of keeping the model simple and understandable. Further in the paper, we also argue that the results are robust to wide range of model specification and we analyze the sensitivity of the results to large grid of the underlying parameters values.

# 3 Transmission Mechanism: Insights from the model

In this section, we present our results and discuss the policy implications. In the second part we use the second order approximation of the pricing kernel together with the version of Brinson factor model to disentangle the fundamental drivers behind our results.

When describing the yield curve we distinguish between the immediate impact of government spending on the term structure and the long run impact on stochastic steady state. The immediate impact is the transitory contemporaneous response of the economy to a rise in government expenditure. The long

| Moments | 1961 - 2007 | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| SD($\Delta C_t$) | 2.9* | 2.51 | 3.48 | 2.58 | 2.42 | 2.51 | 2.49 | 3.22 |
| SD($C_t$) | 0.83 | 2.17 | 2.25 | 2.19 | 2.07 | 2.16 | 2.16 | 2.61 |
| SD($N_t$) | 1.71 | 2.13 | 2.61 | 2.15 | 2.22 | 2.13 | 2.08 | 2.46 |
| SD($\pi_t$) | 2.52 | 1.27 | 1.48 | 1.20 | 1.20 | 1.27 | 1.27 | 1.27 |
| SD($i_t$) | 2.71 | 2.71 | 3.16 | 2.62 | 2.61 | 2.72 | 2.72 | 2.72 |
| SD($w_t$) | 0.82 | 3.73 | 4.42 | 3.78 | 3.50 | 3.72 | 3.72 | 3.96 |
| SD($r_t$) | 2.30 | 1.96 | 2.34 | 1.93 | 1.93 | 1.96 | 1.95 | 2.08 |
| E($NTP_t$) | 1.06 | 1.43 | 0.40 | 1.21 | 1.19 | -0.01 | | |
| E($Slope_t$) | 1.43 | 1.61 | 0.18 | 1.06 | 1.19 | -0.14 | 1.62 | 1.07 |

Table 3: Moments for variants of the model. B is the baseline model with Markov Switching in the Taylor rule, C is the model with the spending reversals extension, D $\phi_y = 0$ is the benchmark model with zero weight on output gap in Taylor rule, E is the benchmark model , F benchmark model with standard CRRA preferences, G represents baseline model with $\sigma_G = 0.004$ , Hbaseline model with $\sigma_G = 0.06$

run effects represented by the change in stochastic steady state embodies the adjustment of the bond prices to rise or drop in the quantity of risk related to government spending.

### 3.1 Results

In this section we study the policy implications of changes in fiscal and monetary policy. We show that: *i*) increase in government spending raises at the impact the inflation risk and thus the whole term structure goes up *ii*) in the economy with higher uncertainty related to government spending purchase of bonds serves as a hedge against possibly large adverse effects on consumption and leisure. High demand for bonds decreases the level of the term structure. *iii*) the response of monetary policy to government spending has important consequences for the term structure. Accommodative monetary policy may help

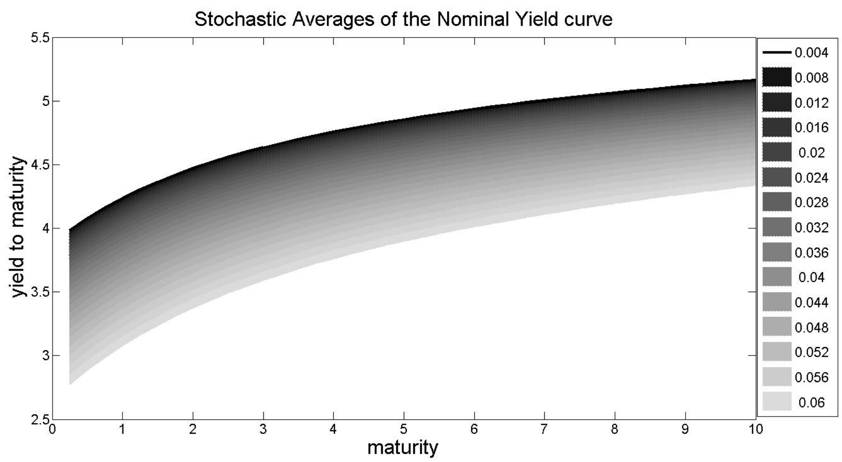


Figure 1: Term structure and varying volatility of $G$ shocks. In the legend is the volatility of the government spending innovation.

to mitigate the impact of fiscal policy uncertainty on fixed income asset prices. *iv*) fiscal authority committed to fiscal consolidation immunizes the effect of its spending on the term structure.

### 3.1.1 Fiscal Policy Uncertainty

The Figure 1 demonstrates that rise in uncertainty related to government spending decreases the level of the term structure of interest rates. The lowest volatility considered is 40 bps and the highest 6 percent. The level of term structure of interest rates decreases in turbulent times. High volatility in government spending motivates households to insure themselves against drop in their wealth. The precautionary saving motive grows with the shock volatility. High volatility of fiscal policy increases the importance of the households risk aversion for the evolution of the interest rates through the whole maturity structure. The log-linearized models abstracting from precautionary saving may thus give significantly biased policy implications. This observation is also emphasized by Paoli and Zabczyk (2012) in case of shock to productivity and preferences. In turbulent times fiscal authority in certainty equivalent world will underestimate the growth in the demand for government bonds and thus consequent drop in consumption. The increase in uncertainty will make financing of the government debt cheaper in the default free world but on the cost of causing large demand shifts away from consumption to government bonds.

The uncertainty about government spending has little impact on nominal term premium. The effect of rise in uncertainty is close to symmetric over the whole maturity profile. As we explain in the section 3.3, quantity of risk related to government spending does not contribute to the explanation of the term premium in our benchmark model. The drop in the real risk at the long tail of term structure is counterbalanced by the nominal risk. (See the section 3.3 for details )

The picture changes when monetary policy authority does not accommodate shifts in aggregate output, see figure 10. The size of fiscal policy uncertainty will matter not only for the level of the yield curve but will significantly affect also the slope and thus nominal term premium.

#### 3.1.2 Implications for Monetary Policy

The Figure 2 shows the impulse response function at the impact for the whole term structure of interest rates. Each panel represents the impact of the shock starting at 40 bps to 6 %. The red dashed line with dots is the stochastic steady state of the term structure assuming that the monetary policy authority adjust its interest rate solely in response to inflation. The red line is the term structure one period after the economy is hit by increase in government spending. The blue line is the analogy in case that the monetary policy puts non-zero weight on output stabilization. For government spending shock volatility corresponding to the calibration over last thirty years (around 1 % depending on the time period) the nominal term structure with $\rho_y = 0.075$ is roughly the same as the one with $\rho_y = 0$. In the economy with low uncertainty the level of term structure is identical for both monetary policy regimes, however slope is higher for the case when monetary policy does accommodate ($\rho_y = 0.075$) the rise in aggregate output. In the economy characterized by higher uncertainty about government spending and accommodative monetary policy regime ($\rho_y = 0.075$) the level and slope of the yield curve is lower than in the case of $\rho_y = 0$.

Figure 2 demonstrates that when government has a consistent expenditure policy keeping expenditures near its steady state and targets only inflation, this generates higher long-run averages of levels as well as slopes of the term structure when the quantity of risk in the economy is high. Monetary policy giving an emphasis on output gap triggers a lower slope and level of the yield curve.

The term structure with $\rho_y = 0.075$ is above the one with $\rho_y = 0$ when the government spending shocks are relatively large i.e. more than 2 per cent. The rest of the discussion focuses on this particular case. **?** explain that persistent and large government spending shocks are less inflationary with an output-gap coefficient of $\rho_y > 0$ because a positive output-gap coefficient raises real interest rate even more in response to positive spending shocks discouraging households from further spending in the present. Furthermore, they point out that the general equilibrium outcome of a positive government spending shock financed by lump-sum taxes is a fall in inflation and short-term nominal interest rate when $\rho_y > 0$. A rise in government purchases leads to higher future taxes (a negative wealth effect) inducing households to cut consumption expenditures and to have less leisure as long as both are normal goods. With a given time frame less leisure translates into higher hours worked (an outward shift in labour supply). The shrinkage in household spending cause firms to produce less and, therefore, demand less labour. The leftward movement of labour demand and supply curves bring about a fall in the real wage which has downward pressure on inflation through the New Keynesian Phillips curve.

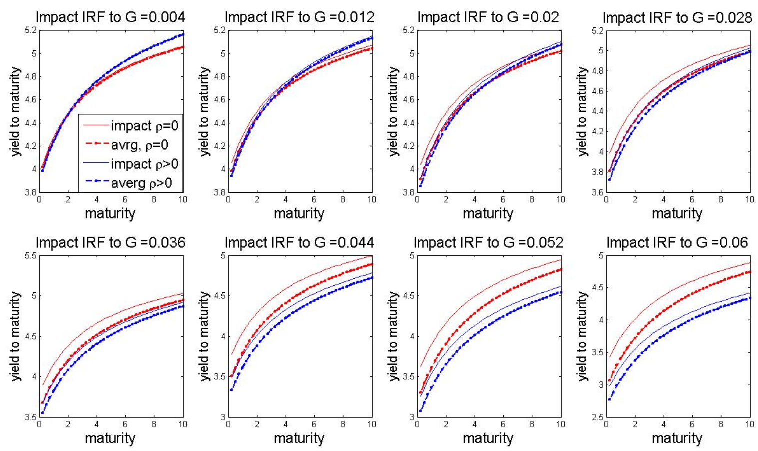


Figure 2: **Government Spending and The Term Structure of Nominal Interest Rates: The Role of Monetary Policy.** The stochastic steady state of the term structure and the impact of increase in government spending on the yield curve. The red lines are the case of zero weight on output stabilization in Taylor rule. The blue line correspond to the case of $\rho_y = 0.075$.

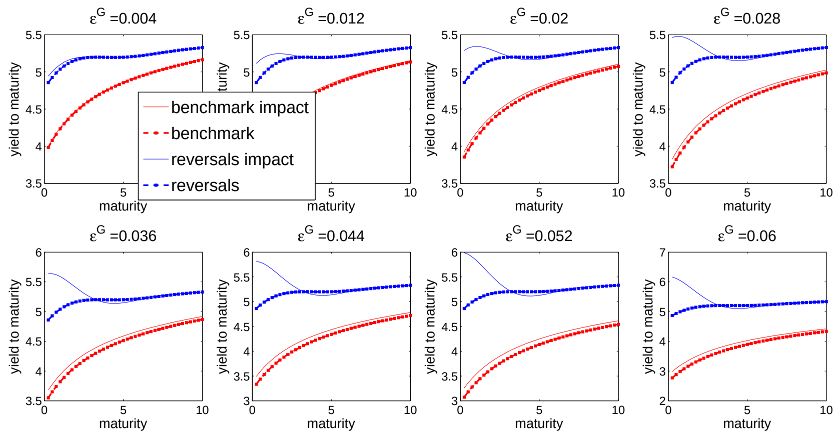


Figure 3: Impact impulse response function for different size of the shock. The stochastic steady state of the term structure and the impact of increase in government spending on the yield curve. The red lines are the benchmark case. The blue lines represents the commitment to spending consolidation.

### 3.1.3 Spending Reversals

Figure 3 depicts the impact impulse response functions of term structure of interest rates for different size of the government spending shock in case of commitment to spending consolidation and compares it with the benchmark case. The red lines represent the benchmark case and blue lines stand for the reversal in spending.

We introduce credible commitment of fiscal policy into the model such that government reduces its expenditures when government debt increases. The government spending is therefore endogenous function of the government debt and fiscal policy is characterized by spending reversals. The benchmark model augmented by these spending reversals predicts that there will be no crowding out of private investment by government. Households work more in a response to increase in aggregate demand.Higher government spending is financed through extra taxes and government debt. The price of debt is rising to encourage additional savings. Expectations about future lower than steady state government spending implies lower future taxes and debt pushing the future expected interest rates down. Higher future disposable income makes households to form expectations about future higher consumption. The intertemporaral smoothing assumption rises the current level of consumption and discourages savings.

The response of term structure of interest rates is driven solely by the intertemporal substitution effect (expectation hypothesis). The precautionary saving component of the term structure is neutralized in presence of spending reversals. [7] The transitory increase in government spending shows up at the short tail and is driven by the rise in debt and intertemporal smoothing motives from households. Fiscal policy commitment to finance temporarily higher spending by future austerity significantly decreases the price of risk related to uncertainty about government spending. Figure 11 and figure 12 demonstrates that the price of risk is negligible for both monetary policy regimes considered.

The ability to immunize the effect of government spending on the term structure has important consequences for the fiscal policy. The wealth effect crowding out private consumption in standard models plays a negligible role. Government spending does not have to be necessary financed by current or future increase in taxes. When government finances its debt by future savings households do not decrease their consumption. This has a positive impact on households welfare boosting further the aggregate demand. Therefore, introducing fiscal rule linking the government expenditure to the level of debt will accelerate the impact of fiscal policy on the whole economy. As pointed out also by Giancarlo Corsetti and Müller (2009) the increase in government spending in this case *i*) does not crowds out consumption and *ii*) enhances an expansionary effect of fiscal policy on output.

[7] We follow here the terminology of Paoli and Zabczyk (2012) and call the expectation about future short term yields as the intertemporal substitution effect and the compensation for uncertainty - intratemporal substitution as precautionary saving effect

### 3.2 The theoretical decomposition

The principal issue of the discussion by now was related to the effect of government spending on term structure of interest rates, $\frac{\partial ytm}{\partial G}$ where we denote yield curve by $ytm$ . The impact of government spending on yield curve is not direct but propagates through the macroeconomic fundamentals. We are interested in disentangling the transmission and quantitatively evaluating the importance of specific channels.

This section shows and explain the results of second order approximation. We motivate the decomposition and show how to quantitatively evaluate the channels of the transmission mechanism.

As emphasized by Kreps and Porteus (1978) and Backus et al. (2004) the recursive preferences in the form considered in this paper cannot be reduced by simply integrating out future information about the consumption process. Instead the timing of information has a direct impact on preferences and the intertemporal composition of risk matters. To illustrate how macroeconomic risk factors enter the pricing equation we analytically derive the second order approximation to the pricing kernel. In addition, our analytical results provide insight on how risk aversion and timing to solve uncertainty relates to the size and direction of risk. The unconditional mean of the price of bond with maturity $n$ can be written as [8]

$$
\begin{aligned}
E_t[\widehat{ytm}_t^n] \quad = \quad & -\frac{1}{2n}\left\{ \mathrm{Var}\sum_{j=1}^{n}\left(\hat{\zeta}_{t,t+j}\right) + \gamma^2 \mathrm{Var}\left(\Delta^n \hat{\mathbf{c}}_{t+n}\right) + \mathrm{Var}\sum_{j=1}^{n}\left(\hat{\pi}_{t,t+j}\right) + \alpha^2 \mathrm{Var} S_{t+n}\left(\cdot\right) \right\} \\
+ \quad & \frac{\gamma}{n}\mathrm{Cov}\left(\sum_{j=1}^{n}\hat{\zeta}_{t,t+j}, \Delta^n \hat{\mathbf{c}}_{t+n}\right) + \frac{1}{n}\mathrm{Cov}\left(\sum_{j=1}^{n}\hat{\zeta}_{t,t+j}, \sum_{j=1}^{n}\hat{\pi}_{t+j}\right) - \frac{\gamma}{n}\mathrm{Cov}\left(\Delta^n \hat{\mathbf{c}}_{t+n}, \sum_{j=1}^{n}\hat{\pi}_{t+j}\right) \\
+ \quad & \frac{\alpha}{n}\mathrm{Cov}\left(\sum_{j=1}^{n}\hat{\zeta}_{t,t+j}, S_{t+n}\left(\cdot\right)\right) - \frac{\gamma\alpha}{n}\mathrm{Cov}\left(\Delta^n \hat{\mathbf{c}}_{t+n}, S_{t+n}\right) - \frac{\alpha}{n}\mathrm{Cov}\left(\sum_{j=1}^{n}\hat{\pi}_{t+n}, S_{t+n}\right) \qquad (24)
\end{aligned}
$$

where $S_{t+n}\left(\sum_{j=0}^{\infty}\beta^j\left[a\hat{\zeta}_{t+j} + a\hat{\mathbf{c}}_{t+j} - b\hat{\mathbf{n}}_{t+j}\right]\right)$ is the revaluation in the expectations and can be understood as well as the news or surprise. The sign

[8] Detailed derivation can be found in appendix

of $\alpha$ determines if agents prefer early or late resolution of uncertainty. Early resolution of uncertainty means that agents wish to smooth consumption over the state of rather than over time. [9]

The variance terms represent the compensation for the real and nominal uncertainty. The covariance terms stand for the fact that adverse events may be coupled, in other words bad thing may happen at wrong time.

Variance and covariance terms associated with inflation explain the compensation for nominal risks. Cov $\left(\Delta^n \hat{\mathbf{c}}_{t+n}, \sum_{j=1}^{n} \hat{\pi}_{t+j}\right)$ says that high inflation in period of low consumption is especially hurtful for bond holders because bonds loose their value exactly when the consumption smoothing households need their savings most. This risk component was highlighted and empirically documented by Piazzesi and Schneider (2007).

Investors require extra compensation for holding bonds if the reevaluation in their expectations about future path of consumption and leisure accompanies the changes in the inflation expectations, Cov $\left(\sum_{j=1}^{n} \hat{\pi}_{t+n}, S_{t+n}\right)$. For instance, in response to negative government spending shock accommodative monetary policy regime decreases its policy rate. Consequent rise in inflation is associated with the downward revision in expectations of future consumption and leisure. Bond portfolio looses its real value exactly when bond holders expect decreasing consumption growth and leisure. Investors are willing to buy bonds only if the risk is reflected in the prices. This risk attribute plays later important role in explaining the differences of yield curve response to government spending in different monetary policy setups.

Fisher (2015) shows that preference shocks can be interpreted as shock to the demand for safe and liquid asset. Cov $\left(\sum_{j=1}^{n} \hat{\zeta}_{t,t+j}, \sum_{j=1}^{n} \hat{\pi}_{t+j}\right)$ thus represents the risk that rise in demand for safe assets will be accompanied by growth in inflation. In other words, in low inflation environment the increase in preferences for liquid assets represented by the risk free bond will push up the demand for bonds and decrease the nominal term premium. Bond holders will ask for the

[9] note that $\alpha = -108$, model is calibrated to feature preference for early resolution of uncertainty

liquidity premium if if the "flight to quality" is associated with high inflation.

The real risk premium is driven by the remaining factors. Cov $(\Delta^n \hat{\mathbf{c}}_{t+n}, S_{t+n})$, represents the risk compensation for the state of the word when the bond holder has to re-evaluate down his expectations about future path of consumption growth and leisure in the time of recession. This channel was stressed by Kaltenbrunner and Lochstoer (2010) in case of technology shocks in the model without labor-leisure choice, they argue that investors with Epstein Zin preferences demand a premium for holding assets when shocks to realized consumption growth are correlated with shocks to expected consumption growth. Positive transitory shock to government spending [10] implies that government spending are expected to revert down to its long-run trend. Thus, while the shock to realized consumption growth is negative, the shock to expected future long-run consumption growth is positive as consumption reverts to the long run trend. If agents have a preference for early resolution of uncertainty as suggested by empirical literature, and thus dislike shocks to both realized and expected consumption growth, the long run risk component acts as a hedge for shocks to realized consumption growth and the real term premium is lower. For this reason Kaltenbrunner and Lochstoer (2010) argue that investors need to form preferences for late resolution of uncertainty to match the high price of risk found in data. Previous line of logic does not necessary hold in the model with labor-leisure choice where the negative shock to realized consumption growth is followed by upward reevaluation in expected consumption growth but downward revision in expected leisure time. If the adjustment in leisure is strong enough the implication for the price of risk is reversed. Households future consumption growth is driven by even higher increase in hours worked turning the covariance term to positive. [11] Agents with a preference for early resolution of uncertainty will thus ask extra premium for holding bonds.

[10] see Kaltenbrunner and Lochstoer (2010) for the effects of transitory and permanent productivity shocks

[11] this is the case especially in RBC type of models where hours worked increases in response to positive productivity innovation. Nevertheless, as **?** argues the covariance between hours and productivity is negative or near zero in the data.

The last risk factor to be explained, Higher preferences for safe assets is positive shock to realized consumption growth and hours worked, as the consumption reverts back to long run trend the expected future consumption growth and leisure turns negative.[12] Hence, negative covariance between preference shock and long run consumption and leisure risks implies that "flight to quality" will tend to increase the real term premium.

Although the preference shock is important for the nominal term premium, it does not affect the shift in the yield curve induced by government spending shock. Transmission of change in quantity of risk coming from government spending to the price of risk is given by:

$$
\begin{aligned}
E_t\left[\Delta\widehat{ytm}_t^n\right] &= -\frac{1}{2n}\left\{\gamma^2\Delta\mathrm{Var}_t\left(\Delta^n\hat{\mathbf{c}}_{t+n}\right)+\Delta\mathrm{Var}_t\sum_{j=1}^{n}\left(\hat{\pi}_{t,t+j}\right)\right\} \\
&- \frac{1}{2n}\left\{\alpha^2\Delta\mathrm{Var}_t S_{t+n}\left(\cdot\right)\right\}-\frac{\gamma}{n}\Delta\mathrm{Cov}_t\left(\Delta^n\hat{\mathbf{c}}_{t+n},\sum_{j=1}^{n}\hat{\pi}_{t+j}\right) \\
&- \frac{\gamma\alpha}{n}\Delta\mathrm{Cov}_t\left(\Delta^n\hat{\mathbf{c}}_{t+n},S_{t+n}\left(\cdot\right)\right) \\
&- \frac{\alpha}{n}\Delta\mathrm{Cov}_t\left(\sum_{j=1}^{n}\hat{\pi}_{t+n},S_{t+n}\left(\cdot\right)\right) \qquad (25)
\end{aligned}
$$

where $E_t\left[\Delta\widehat{ytm}_t^n\right]$ is the unconditional change in nominal term structure of interest rates induced by change in government spending uncertainty.

The issue here is that in general $E_t Var_{t+j-1}x_{t+j} \neq Var_t x_{t+j}$, thus one cannot quantitatively evaluate the decomposition of the pricing kernel based on the ex-post data. We thus turn to what we call attribution analysis.

## 3.3 Attribution

In this section we propose method how to quantitatively evaluate the specific channels of the transmission mechanism as discussed in the section3.2. Agents

[12]see the impulse response functions in appendix

in the model require compensation for risk related to holding of bonds, the purpose is to uncover the relative importance of macroeconomic risk factors.

The second order approximation of the benchmark model pricing kernel points to four model endogenous risk factors driving term structure, *i*) consumption grow, *ii*) inflation, *iii*) long-run risk, *iv*) preference shock. [13] to track the propagation of exogenous shock to yields through macroeconomic factors is complicated by the fact that the effects of the factors are cross correlated. For instance, for the case of two factors, consumption growth and inflation, the term structure of interest rates can be written as a composite function $ytm(c(G,\pi(G)),\pi(G,c(G)))$, taking the derivative with respect to $G$ delivers $\frac{\partial ytm}{\partial c}\left[\frac{\partial c}{\partial G}+\frac{\partial c}{\partial \pi}\frac{\partial \pi}{\partial G}\right]+\frac{\partial ytm}{\partial \pi}\left[\frac{\partial \pi}{\partial G}+\frac{\partial \pi}{\partial c}\frac{\partial c}{\partial G}\right]$. In the following analysis we quantify the change in yields driven separately by factor stand alone effects, consumption growth $\frac{\partial ytm}{\partial c}\frac{\partial c}{\partial G}$ and inflation $\frac{\partial ytm}{\partial \pi}\frac{\partial \pi}{\partial G}$ and the interaction effect coming from the factor cross derivatives, $\frac{\partial c}{\partial \pi}\frac{\partial \pi}{\partial G}+\frac{\partial \pi}{\partial G}\frac{\partial c}{\partial G}$. For $n$ factors the derivative of composite function can be written

$$\frac{\partial ytm}{\partial G}=\sum_{i=1}^{n}\sum_{j=1}^{n-1}\frac{\partial ytm}{\partial F_i}\left[\frac{\partial F_i}{\partial G}+\frac{\partial F_i}{\partial F_j}\frac{\partial F_j}{\partial G}\right] \quad \text{for } i\neq j \tag{26}$$

where $F$ stands for the macroeconomic factor driving the yield curve dynamics. To decompose the effects of changes in government spending on the yield curve we use the idea of Brinson multi-factor model[14] (Brinson and Fachler (1985)). The figure 4 illustrates the idea behind the decomposition. Without loss of generality lets abstract from the preference shock for now and consider only the remaining three factors. We start the analysis at the deterministic steady state where the term structure is just flat line at $\frac{1}{\beta}$. Adding the stand alone risk factor increases the level of yield curve to the factor specific node, i.e. $(\Delta c_{t+1},\ \pi_{t+1},\ \left(\frac{V_{t+1}}{R_t}\right)^{-\alpha})$. In terms of equation 26 we quantify the first

[13]The long run risk may be interpreted in several ways as highlighted in Epstein and Zin (1989). The crucial point is that time to resolve uncertainty matters thus shocks to continuation value matters

[14]this version of factor model is widely use in portfolio management for return attribution analysis

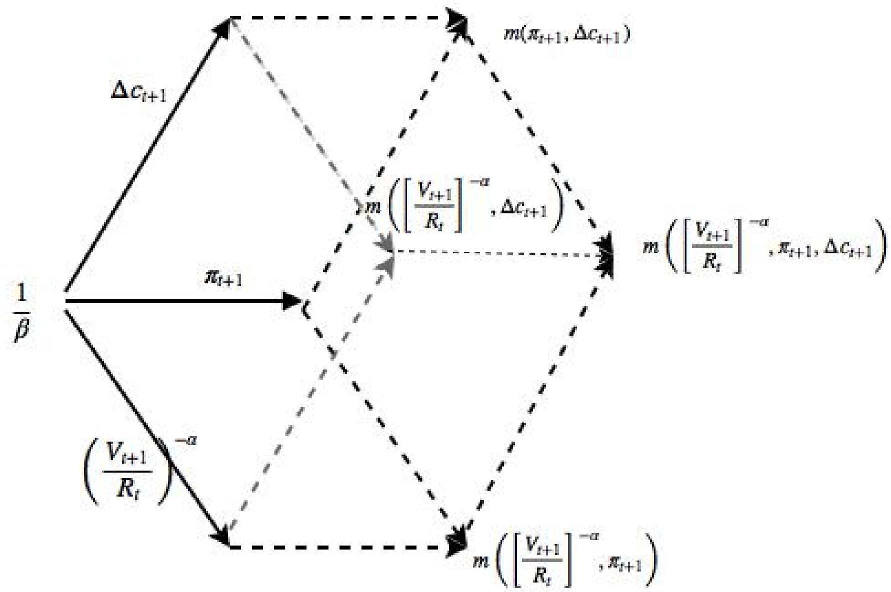


Figure 4: Diagram

term after multiplying the bracket. However, factors interactions contribute to the change in the yield curve as well. Thus, we need to calculate the factor cross derivatives as well. In the figure 4 this is represented by the nodes at the dashed lines intersection. For example, the total effect of changes in consumption growth and inflation on the yield curve is sum of the stand alone impacts, $\Delta c_{t+1}$, $\pi_{t+1}$, and their interaction, $m(\Delta c_{t+1}, \pi_{t+1})$. Considering all three factors in figure 4, the total change is sum of risks attributed to the stand alone factors, interaction of two factors and interaction of all three factors together. In general, the total effect of n-factor pricing equation can be decomposed into $n$ groups, factor risks, $1, 2 \ldots n$ level interactions.

Thus, the change in the level of the yield curve for every maturity can be written as sum the $n$ risk groups, $\frac{\partial ytm}{\partial \sigma_G} = \sum_{g=1}^{n} R_g$. The $R_g$ symbol represents the risk group.

The figure 4 demonstrates how to calculate the risk groups within our macro model. Lets again focus only on two factors, consumption growth and inflation. First, calculate the yield curve within the macro model where the pricing equation contains only consumption growth or inflation. Second, subtracting the determinist steady state. In this way we can isolate the individual contribution of inflation and consumption growth as a risk factor in pricing equation. Further, we evaluate the model with both risk factors and subtract the stand alone risks factors calculated in the previous step and subtract again the determinist steady state to find the attribution of the factors interaction. More formally,

$$R_1 = \sum_i^n (M(F_i) - M(st.st.)) \tag{27}$$

$$R_{2,i} = \sum_i^n \sum_j^n M(F_i, F_j) - R_1 - M(st.st.)) \tag{28}$$

$$R_g = \sum_g^n \sum_i^n \sum_j^n (M(F_i, F_j, \ldots F_g) - R_{n-1} - R_{n-2} \ldots R_1) \tag{29}$$

where $M(F_i)$ is the model with the risk factor $i$ and $M(st.st.)$ is the model at the steady state.

### 3.3.1 Precautionary saving effect

In the first step we decompose the level of the yield curve into the deterministic and stochastic part. Similarly as Paoli and Zabczyk (2012) we can use the equation 62 to write one period yield to 093 maturity as

$$-ytm_t = \underbrace{E_t q_{t,t+1}}_{\text{Intertemporal substitution}} + \frac{1}{2} \underbrace{\mathrm{Var}_t(q_{t,t+1})}_{\text{Precautionary savings}} \tag{30}$$

where $q_{t,t+1}$ is the log deviation of one period stochastic discount factor from its steady state. To discourage agents from savings the interest rate that clears the market is affected by the intertemporal smoothing and precautionary reasons. The unconditional mean of the intertemporal substitution part corresponds to deterministic steady state. The variance term determines how uncertainty affects interest rates through changes in precautionary savings. To analyze how the precautionary savings channel affects the transmission mechanism of shocks, we need to understand the determinants of $\text{Var}_t(\hat{\mathbf{q}}_{t,t+1})$. The table 4 summarizes the results from the factor attribution to stochastic part of the yield curve and thus quantifies the equation 24.

| Stand alone factors | Benchmark $\phi_y = 0.075$ | Benchmark only $\epsilon^G$ shock | TFP |
|---|---|---|---|
| $-\frac{\gamma^2}{2n} Var_t(\Delta C_{t+n})$ | 1.3 | 0.9 | |
| $-\frac{1}{2n} Var_t(\sum_{j=1}^{n} \pi_{t+n})$ | 85.4 | 59.7 | |
| $-\frac{\alpha^2}{2n} Var_t(S_{t+n})$ | -8.1 | -5.1 | |
| $-\frac{1}{2n} \text{Var}_t \sum_{j=1}^{n} (\zeta_{t,t+j})$ | 2.1 | 0 | |
| Factor interactions | | | |
| $-\frac{\gamma}{n} \Delta Cov_t(\Delta C_{t+n}, \sum_{j=1}^{n} \pi_{t+n})$ | -0.2 | 0 | |
| $-\frac{\gamma\alpha}{n} Cov_t(\Delta C_{t+n}, S_{t+n})$ | 29.4 | 41.8 | |
| $-\frac{\alpha}{n} Cov_t(S_{t+n}, \sum_{j=1}^{n} \pi_{t+n})$ | -43.5 | 2.6 | |
| $+\frac{\alpha}{n} Cov_t(S_{t+n}, \sum_{j=1}^{n} (\zeta_{t,t+j}))$ | 36.4 | 0 | |
| $+\frac{\gamma}{n} Cov_t(\Delta C_{t+n}, \sum_{j=1}^{n} (\zeta_{t,t+j}))$ | -1.7 | 0 | |
| $+\frac{1}{n} \Delta Cov_t(\sum_{j=1}^{n} (\zeta_{t,t+j}), \sum_{j=1}^{n} \pi_{t+n})$ | -1 | 0 | |
| Total | | | |
| $E_t[ytm_t^n]$ | -0.96 | -0.2 | |

Table 4: The table shows the factor attribution of the stochastic part of the yield curve. The reported numbers are averages over the maturity profile $n$, each column shows the percentage contribution of the attributes to the total change in the level of the yield curve. The higher order interactions are zero up to the 2nd order approximation.

In our benchmark model, the average deterministic level of the yield curve is about 4.02, and its constant over the maturity, the stochastic component is on average 0.74 and is increasing with maturity.

### 3.3.2 Variations in Uncertainty

The table 5 summarizes the decomposition of the drop [15] in the level of the yield curve in figure 1, 11 and 12. In addition we present also the factor attribution in case of TFP shock to contrast differences in the transmission.

| Stand alone factors (in %) | Benchmark $\phi_y = 0.075$ | Benchmark $\phi_y = 0$ | TFP |
|---|---|---|---|
| $-\frac{\gamma^2}{2n}\Delta Var_t(\Delta C_{t+n})$ | 0.9 | 1.3 | 1.4 |
| $-\frac{1}{2n}\Delta Var_t(\sum_{j=1}^{n}\pi_{t+n})$ | 59.7 | 78.5 | 63.3 |
| $-\frac{\alpha^2}{2n}\Delta Var_t(S_{t+n})$ | -5.1 | -8.8 | -8.8 |
| Factor interactions (in %) | | | |
| $-\frac{\gamma}{n}\Delta Cov_t(\Delta C_{t+n}, \sum_{j=1}^{n}\pi_{t+n})$ | 0.1 | -0.9 | -1 |
| $-\frac{\gamma\alpha}{n}\Delta Cov_t(\Delta C_{t+n}, S_{t+n})$ | 41.8 | 64.6 | 73.2 |
| $-\frac{\alpha}{n}\Delta Cov_t(S_{t+n}, \sum_{j=1}^{n}\pi_{t+n})$ | 2.6 | -34.7 | -28.2 |
| Total | | | |
| $\Delta E_t[ytm_t^n]$ | -0.976 | -0.55 | -0.978 |

Table 5: Quantifies the factors attribution to the total drop in the level of the term structure of interest rates when the volatility of the innovations to government spending increases from 0.4% to 6% (TFP 0.214% to 0.687%). The reported numbers are averages over the maturity profile.

The table 5 quantitatively evaluates the attributes in the transmission of the rise in fiscal uncertainty to the the level yield curve. The first column represents our benchmark model with accommodative monetary policy ($\phi_y = 0.075$). Most of the adjustment, 92 %, goes through the changes in uncertainty about inflation and nominal long run risks. The uncertainty associated with higher volatility of government spending increases the overall macroeconomic quantity of risk in the economy. This is especially hurtful in case of inflation. The increase in volatility of inflation directly translates into the volatility of real value of bond portfolio. Rise in the quantity of fiscal risk thus shows up in the asset prices.

[15] average drop over the maturity

The adjustment in bond prices in response to changes in real risk follow the argument from the section 3.2. Higher volatility of government spending rises the covariance between realized consumption growth and reevaluations in expectations about future path of consumption growth and leisure. In the specific case of our benchmark model, positive innovation in government spending lowers realized consumption growth through the wealth effect. Nevertheless, the transitory character of shocks implies positive reevaluation in expectations about future consumption growth as well as leisure. The hedging character of real long-run risk is strengthen by the update in expectations about future path of hours worked. Investors expect that they will work less in the future and at the same time their consumption will growth. The insurance property of bonds against turbulent fiscal policy can be demonstrated by considering exogenous increase in government spending, $\epsilon^G > 0$

$$\frac{\partial E_t[ytm_t^n]}{\partial \epsilon^G} = Cov\left(\frac{\partial \Delta C_{t+n}}{\partial \epsilon^G} < 0, S_{t+n}\left(\frac{\partial \Delta C_{t+n}}{\partial \epsilon^G} > 0, \frac{\partial L_{t+n}}{\partial \epsilon^G} > 0\right)\right) < 0 \tag{31}$$

Bonds provide good hedge also against negative innovations in productivity. If the economy is hit by negative $\epsilon^A < 0$ TFP shock,

$$\frac{\partial E_t[ytm_t^n]}{\partial \epsilon^A} = Cov\left(\frac{\partial \Delta C_{t+n}}{\partial \epsilon^A} < 0, S_{t+n}\left(\frac{\partial \Delta C_{t+n}}{\partial \epsilon^A} > 0, \frac{\partial L_{t+n}}{\partial \epsilon^A} > 0\right)\right) < 0 \tag{32}$$

therefore bonds can insure households against possible drop in consumption. The real part of precautionary saving effects pushes the prices of risk down due to its hedging function in response to government spending shocks. The prices of risk might increase however in case of productivity shock in the model where the substitution effect dominates at the labor market as discussed in section 3.2.

The figure 16 reports the comparison of impulse response functions for the main macroeconomic variables for the two monetary policy regimes we consider. The core difference between the regimes lies in the behavior of inflation which rises in response to positive government spending shock in the regime where monetary policy authority put zero weight on output stabilization. As we al-ready pointed out in the theoretical discussion in the section 3.2 the nominal risk drives the differences in risk price response to changes in the uncertainty related to government spending. The term $\Delta Cov_t(S_{t+n}, \sum_{j=1}^{n} \pi_{t+n}) < 0$ if $\phi_y = 0.075$ but $\Delta Cov_t(S_{t+n}, \sum_{j=1}^{n} \pi_{t+n}) > 0$ if $\phi_y = 0$. If the monetary authority lets the money supply freely adjust in response to government spending shock, $\phi_y = 0$, which is reflected in the price increase, bonds loose its real value at the time when investors receive good news about future expected consumption and leisure. The long run component serves thus as a hedge against nominal risk. Consider the case for $\phi_y = 0$

$$\frac{\partial E_t[ytm_t^n]}{\partial \epsilon^G} = Cov\left(\frac{\partial \sum_{j=1}^{n} \pi_{t+n}}{\partial \epsilon^G} > 0, S_{t+n}\left(\frac{\partial \Delta C_{t+n}}{\partial \epsilon^G} > 0, \frac{\partial L_{t+n}}{\partial \epsilon^G} > 0\right)\right) > 0 \tag{33}$$

Whereas in case of accommodative monetary policy, $\phi_y = 0.075$, bonds loose its real value at the the time when investors receive negative news about future consumption and leisure. To hold such bonds agents require extra premium to be compensated for the inflation risk

$$\frac{\partial E_t[ytm_t^n]}{\partial \epsilon^G} = Cov\left(\frac{\partial \sum_{j=1}^{n} \pi_{t+n}}{\partial \epsilon^G} > 0, S_{t+n}\left(\frac{\partial \Delta C_{t+n}}{\partial \epsilon^G} < 0, \frac{\partial L_{t+n}}{\partial \epsilon^G} < 0\right)\right) > 0 \tag{34}$$

[to be solved: model simulations provide intuitive results in line with the discussion above, but! $\alpha < 0$ thus the covariance enter the pricing equation with plus and thus suggest reverse implications. In other words, the simulated results and intuition are in line. The hedging effect (bad news compensated by different good news decreases the risk premium, if the exogenous shock implies bad news coupled with another bad news the risk premium must rise) holds throughout all simulated results. However the puzzle comes from the second order approximation of the term structure. The equation 25 implies that bad news about higher inflation comes with good news about future consumption and leisure growth. Negative covariance implicates hedging effect thus the yields should drop which is not in line with equation 25. The issue is that if I hypothetically assume there is mistake in the derivation about the sign, any change in the sign in front of the covariance of inflation with $S()$ necessarily changes sign in front of other terms as well thus it would compromise all the other results which holds perfectly! The related issue is in general about the sign of impact of $V_{t+1}$ on stochastic discount factor. Looking at the results of Uhlig, Hansen Heaton, Li and others, $V_{t+1}$ impacts stochastic discount factor negatively, in their case $(IES - CRRA)(V_{t+1} - R_{t+1})$, $CRRA < IES$, in Rudebush and Swanson case $V_{t+1}$ impacts stochastic discount factor positively, because $-\alpha(V_{t+1} - R_{t+1})$ and $\alpha < 0$ but in calibration of RS as well as Uhlig $IES < CRRA$ so agents prefer early resolution of uncertainty. I am lost! ]

Volatility of inflation is in the model with $\phi_y = 0$ lower than in the model with accommodative monetary policy. Why if the impulse response of inflation is much stronger, about 10 times, in case of $\phi_y = 0$. 307 Real risk drops somehow as well when monetary policy let the money supply to adjust. This is because consumption growth drops less in response to spending shock thus its future growth rate is slower and the insurance effect 309 much weaker. Leisure on the other hand increases more and thus there is much stronger positive news about future leisure growth.

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### 3.3.3 Transitory Shock to Government Spending

The response of therm structure of interest rates to transitory changes in government spending shocks is driven in our benchmark model by the intertemporal substitution motives. The benchmark model is approximated to 403 the second order implying that the the precautionary saving motive is constant in time and therefore does not play any role in explaining the economy dynamic response to the shocks.[16]

Here we decompose the period impact of the rise in government spending on the term structure of interest rates.

[16] it is crucial element in 405 determining the stochastic steady state. Paoli and Zabczyk (2012) quantitatively evaluate the bias from omitting second order effects

410 377 412 378 413 379 414 380 415 381 416 382 417 383 418 384 419 385 420 386 421 387 422 388 423 389 424 390 425 391 426 392 427 393 428 394 429 395 430 396 431 397 432 398 433 399 434 400 435 401 436 402 437 403 438 404 439 405 440 406 441 407 442 408 443 409 444 410 445 411 446 412 447 413 448 414 449 415 450 416 451 417 452 418

### 3.3.4 Nominal Term Premium

In the previous section we discussed the level of the yield curve, now we turn to discussion of nominal term premium which is approximately equivalent to the slope of the term structure.

Using the theoretical results from the approximation of the pricing kernel it is argued in the macro-finance literature (see Rudebusch and Swanson (2012), Paoli and Zabczyk (2012), Ferman (2011), Piazzesi and Schneider (2007)) that the covariance between inflation and consumption growth is the core element to explain nominal term premium. We check this story numerically and show that covariance of inflation 458 with intertemporal composition of risk matters as well. The table 6 shows the decomposition of the nominal term premium for our benchmark model

## 3.4 Level Premium

We follow Ferman (2011) and introduce into our analysis Markov switching in policy rule. This has important consequences for precautionary saving component of the yield curve. Precautionary savings do not impact only stochastic steady state but also the dynamic response of the model variables to the exogenous shocks. For this reason, the higher order terms in the attribution analysis have impact on the behavior of the term structure even when approximated by the second order.

$$i_t = \bar{i} + \phi_{\pi(s_t)}\hat{\pi}_t + \phi_{y(s_t)}\hat{\mathbf{y}}_t \tag{35}$$

where $\hat{\pi}_t$ and $\hat{\mathbf{y}}_t$ denote the percentage deviations of inflation and aggregate

| Factors | $NTP_40$ | B | | | | |
|---|---|---|---|---|---|---|
| $\Delta C_t)$ | 3.2* | 2.51 | 3.48 | 2.58 | 2.45 | 2.51 |
| $EZ_t$ | 0.83 | 2.17 | 2.25 | 2.19 | 2.07 | 2.16 |
| $\pi_t$ | 1.71 | 2.13 | 2.26 | 2.15 | 2.22 | 2.13 |
| $cov(\Delta C_t, \pi_t)$ | 2.52 | 1.27 | 0.37 | 0.30 | 0.30 | 0.31 |
| $cov(\Delta C_t, EZ_t)$ | 2.71 | 0.68 | 0.79 | 0.65 | 0.65 | 0.68 |
| $cov(EZ_t, \pi_t)$ | 0.82 | 3.73 | 4.42 | 3.78 | 3.50 | 3.72 |
| $cov(\Delta C_t, \pi_t)$ | 2.30 | 1.96 | 0.58 | 0.48 | 0.48 | 0.49 |
| E($NTP_t$) | 1.06 | 1.43 | 0.40 | 1.21 | 1.19 | -0.01 |
| E($Slope_t$) | 1.43 | 1.61 | 0.18 | 1.06 | 1.19 | -0.10 |

Table 6: Moments for variants of the model. B is the baseline model, C is the model with the spending reversals extension, B $\phi_y = 0$ is the benchmark model with zero weight on output gap in Taylor rule, B - MS is the benchmark model without the Markov Switching in the Taylor rule, B - EZ benchmark model with standard CRRA preferences

output from their corresponding deterministic steady states. Note that here we follow Ferman (2011) and let the reaction coefficients to inflation and output be state-dependent. The realization of state of the world $s_t \in \{1, 2\}$ translates into monetary policy regime change. The regime switching assigns the weight monetary policy puts on inflation relative to output gap stabilization. Ferman (2011) presents empirical evidence that there was a shift in US monetary policy in 1979 with respect to the strength of the reaction to inflation. In particular, he found that monetary policy during and after Volcker's chairmanship assigned stronger mandate to fighting inflation in the form of putting higher weight on inflation in the Taylor-rule. The period before 1979 is often referred to as Great Inflation and the aftermath of 1979 as Great Moderation.

Ferman (2011) convincingly argues that monetary policy switching allows to match the dependence of the slope of the term structure on the monetary policy regime found in data. We define two state of the world: $i$) active monetary policy regime where the inflation stabilization plays the main role, and *ii*) passive monetary policy regime where the inflation is less important relative to output gap stabilization.[17]. The switching process evolves through exogenous Markov Chain determined by the transition matrix $P$.

### 3.5 Robustness checks

We test robustness of our results with respect to model and parameter specification. We find that the results hold in RS (2012) as well as in Kaszab Marsal (2013).

As highlighted above, the conduct of monetary policy is important determinant of the slope and level of the term structure in response to government spending shock. For this reason, we test the robustness of our findings over the whole grid of Taylor rule estimates found in the data. Table 7 shows the estimated ranges in most influential recent studies quantifying the parameters' values in the Taylor rule.

| Study | Period | $\phi_p i$ | $\phi_y$ |
|---|---|---|---|
| Taylor (1996) | 1987 - 1997 | 1.53 | 0.77 |
| Judd and Rudebush (1998) | 1987 - 1997 | 1.54 | 0.99 |
| Clarida, Gali and Gertler (2000) | 1979 - 1996 | 2.15 | 0.93 |
| Orphanides (2003) | 1979 - 1995 | 1.89 | 0.18 |

Table 7: Taylor rule estimates for US

We take the maximal boundary values for each parameter and plot the slope, level and their changes over the whole grid of parameter combinations. The figure 5 shows [18] how the slope and level of term structure of interest rates

[17]The standard New Keynesian model exhibits indeterminacy when monetary policy reacts to a rise in inflation by less than one to one with nominal interest rate i.e. $\phi_{\pi(s_t)} < 1$. However, regime switching allows to have a regime with $\phi_{\pi(s_t)} < 1$ as long as the agents in the economy expect that the regime will change in the future back to the one where $\phi_{\pi(s_t)} > 1$

[18]We assume in this exercise that the monetary policy regimes are the same in both regimes. In other words we leave out Markov switching in Taylor rule

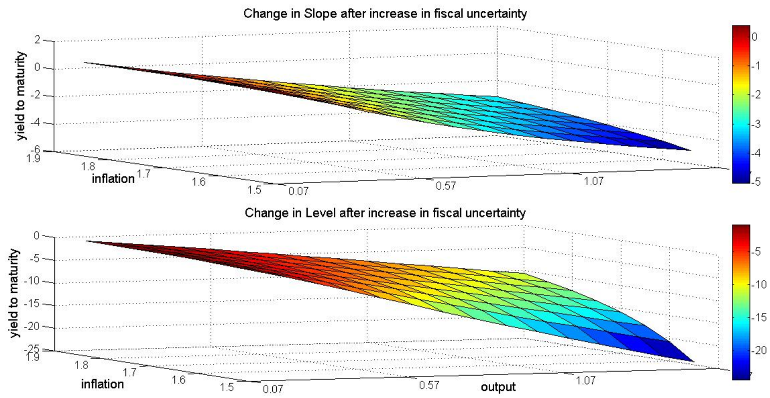


Figure 5: Changes in the level and slope of the term structure of interest rate over the grid of Taylor rule regimes after the change in volatility of government spending shock ranging from $\sigma_G = 0.004$ to $\sigma_G = 0.06$

changes with different volatility of government spending. Specifically we look what happens with the slope and level after the rise of volatility from $\sigma_G = 0.004$ to $\sigma_G = 0.06$. The bottom part of figure 5 demonstrates that the drop in the term structure of interest rates after rise in fiscal uncertainty is independent of the choice of weights in the Taylor rule. In other words, the level of the yield curve decreases after the rise in volatility of government spending for any considered combination of weights on inflation and output. The upper chart in the figure 5 [19] highlight that slope of the term structure of interest rates rises only if the weight on output gap stabilization in Taylor rule is very close to zero.

Figure 6 points out strong dependency of the model ability to match empirical level of yield curve on monetary policy conduct. For low volatility of government spending, higher the weight on output relative to inflation the higher the level of yield curve. This relation turns around in model with high volatility of government spending. Higher weight on output gap implies large negative levels of yield curve.

The figure 7 shows how the slope of the term structure of interest rates

[19]exact values used to generate this chapter figures are presented in appendix

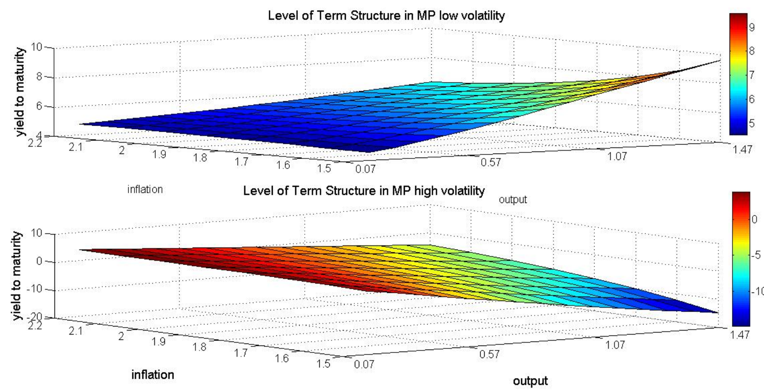


Figure 6: Level and slope of the term structure of interest rate over the grid of Taylor rule regimes for volatility (upper one) $\sigma_G = 0.004$ and volatility $\sigma_G = 0.06$ (bottom)

varies with different weights on output and inflation stabilization in Taylor rule. We show here only the case for $\sigma_G = 0.004$. It has been demonstrated in the related literature that models with standard preferences have hard time to generate positive slope of the yield curve. The picture 7 shows that one option how to increase the slope in your model would be through monetary policy.

As documented elsewhere in the literature, see for example Rudebusch and Swanson (2012), Hordahl et al. (2007) or KaszabMarsal2013, the size of the term premium is directly related to the coefficient of relative risk aversion. The micro and macro estimates of this parameters varies over the wide range of values. Horvath finds, some micro study, Binsbergen2012 finds 110. In general, standard equilibrium models used in macroeconomics require rather high risk aversion to deliver the basic asset pricing stylized facts. Here we check robustness of our results to the range of sensible values of risk aversion. Further, we reproduce the chart in Rudebusch and Swanson (2012) and KaszabMarsal2013 to directly compare relationship between the nominal term premium and coefficient of relative risk aversion.

The figure 8 shows that the level shift in stochastic average of term structure

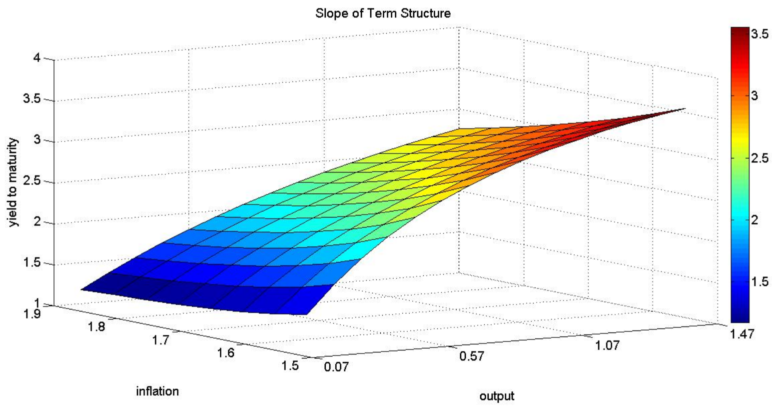


Figure 7: Slope of the term structure of interest rate over the grid of Taylor rule regimes for volatility $\sigma_G = 0.004$

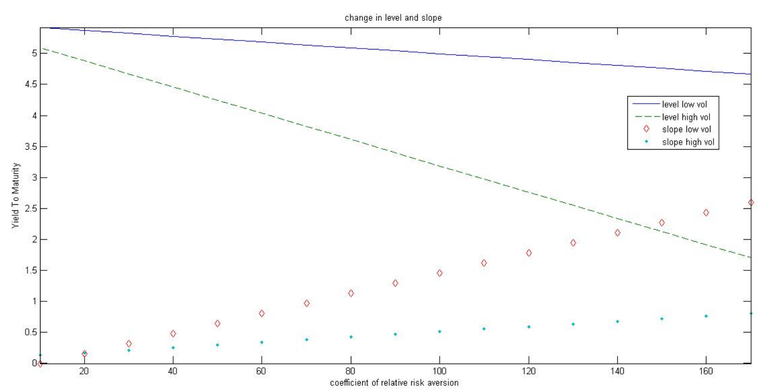


Figure 8: Sensitivity of results on risk aversion. Level and slope of yield curve for volatility of government spending 0.4 and 6 percent

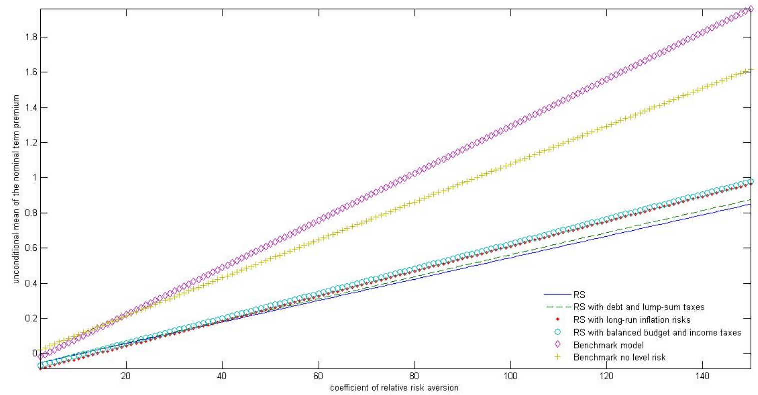


Figure 9: The relationship between the coefficient of relative risk-aversion and the mean of the nominal term premium using our benchmark model and variants of Rudebush and Swanson (2012) model

of interest rates due to increase in the volatility of government spending increases with the risk aversion coefficient. The direction of the adjustment in the yield curve is the same for the whole parameter range. The impact on slope of the term structure becomes significant only for risk aversion being higher than 20.

The figure9 illustrates the quantitative importance of the level premium (regime switching) for the slope and level of the term structure.

# 4 Conclusion

to be done

# A Note on Recursive preferences

The preferences are the crucial element driving large part of the results. Recursive preferences has been utilized increasingly in the asset pricing literature. Nevertheless, in macroeconomic literature Epstein Zin preferences belongs, yet, to group a of so called exotic preferences (see Backus (2014)). We provide detailed solution of the bond pricing equation and its second order approximation. The explicit second order solution to bond prices is useful because it helps us to better develop the intuition about the drivers of dynamics of the term structure of interest rates and relate them to macroeconomic fundamentals. The derivations are augmented by several boxes developing the intuition behind the equations.

We lay out the recursive preferences as in Weil (1990). First, we use the utility transformation as in Rudebusch and Swanson (2012) that simplifies the work with utility kernels including labor. Next, we derive and log-linearize the stochastic discount factor (SDF). To substitute out the recursive element and to get SDF just as a function of macroeconomic fundamentals we log-linearize the value function and introduce the surprise operator as in Uhlig (2010). Consequently, using the method developed by Sutherland (2002) we derive the general form of second order approximation to the bond pricing equation. Finally, we merge the results to highlight the drivers of the yield curve dynamics.

## A.1 Value function transformation

In the asset pricing literature, the recursive preferences are often formulated in the following form (see Weil (1990), Epstein and Zin (1989), Bansal and Yaron (2004), Uhlig (2010), Guvenen (2009)),

$$\tilde{V} = \left\{ c_t^{1-\rho} + \beta [E_t \tilde{V}_{t+1}^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}} \right\}^{\frac{1}{1-\rho}} \tag{36}$$

Before augmenting the model by labor it is useful to follow Rudebusch and Swanson (2012) to transform the value function. We define $\frac{1-\rho}{1-\gamma} = 1 - \alpha$

$$\tilde{V} = \left\{ c_t^{1-\rho} + \beta[E_t \tilde{V}_{t+1}^{(1-\alpha)(1-\rho)}]^{\frac{1}{1-\alpha}} \right\}^{\frac{1}{1-\rho}} \tag{37}$$

Next, we set $V_t = \tilde{V}_t^{1-\rho}$

$$V_t = u(C_t, L_t) + \beta(E_t[V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{38}$$

when $u(C_t, L_t) > 0$. If $u(C_t, L_t) < 0$, as in our benchmark calibration [20], the recursion takes the form:

$$V_t = u(C_t, L_t) - \beta(E_t[-V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{39}$$

To obtain the first order conditions, we solve for the constrain optimization problem.

## A.2 Solving for SDF

$$\frac{\partial \Lambda}{\partial c_t} = 5 \tag{40}$$

to be typed...

The optimization exercise delivers stochastic discount factor at time $t$ for stochastic payoff at time $t+1$.

$$Q_{t,t+1} = \zeta_t \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}} \left[ \frac{V_{t+1}}{R_t} \right]^{-\alpha} \tag{41}$$

where $\zeta_t$ is the preferences shock, $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the inflation between period $t$ and $t+1$, and certainty equivalent value of future consumption and leisure $R_t$ can be written

$$R_t = [E_t V_{t+1}^{1-\alpha}]^{\frac{1}{1-\alpha}} \tag{42}$$

Because of term $\left[\frac{V_{t+1}}{R_t}\right]^{-\alpha}$ [21], news at t + 1 about consumption in $c_{t+2}, c_{t+3} \dots$ and leisure in $n_{t+2}, n_{t+3} \dots$ affects marginal utility of $c_{t+1}$ and $n_{t+1}$ relative

[20] the first order conditions will be correct however in either way

[21] next periods value relative to its certainty equivalent

to marginal utility of $c_t$ and $n_t$. Good news at $t+1$ about future consumption and leisure is a positive shock to $R_{t+1}(V_{t+2})$, and therefore to $V_{t+1} = F(c_{t+1}, n_{t+1}; R_{t+1}(V_{t+2}))$. The more concave is the utility function and the more uncertain $V_{t+1}$ is, the lower is the certainty equivalent $R_t$. Note that $R_t = V_{t+1}$ if there is no uncertainty on $V_{t+1}$. [22]

There are two advantages to SDF of time-separable expected utility. First, it separates EIS from coefficient of relative risk aversion. Second, it is another source of risk premium, not just covariance with contemporaneous consumption growth, but also covariance with return to total wealth matters.

By chaining the stochastic discount factor we can price bond of any maturity:

$$Q_{t,t+n} = \beta^n \left(\frac{C_{t+n}}{C_t}\right)^{-\gamma} \prod_{j=0}^{n} \frac{\zeta_{t+j}}{\pi_{t+j+1}} \left[\frac{R_{t+j}}{V_{t+j+1}}\right]^{\alpha} \tag{43}$$

## A.3 Log-linearizing SDF

First we log linearize equation 42

LHS:

$$\bar{R}^{1-\alpha} e^{(1-\alpha)\hat{\mathbf{r}}_t} \approx \bar{R}^{1-\alpha}(1 + (1-\alpha)\hat{\mathbf{r}}_t) \tag{44}$$

RHS:

$$\bar{V}^{1-\alpha} E_t e^{(1-\alpha)\hat{\mathbf{v}}_{t+1}} \approx \bar{V}^{1-\alpha}(1 + (1-\alpha) E_t \hat{\mathbf{v}}_{t+1}) \tag{45}$$

Canceling out steady state delivers:

$$\hat{\mathbf{r}}_t = E_t \hat{\mathbf{v}}_{t+1} \tag{46}$$

Next, we log linearizing equation 41. RHS after taking Taylor expansion

$$st.st. + st.st E_t[\zeta_t - \gamma \Delta \hat{\mathbf{c}}_{t+1} - \hat{\pi}_{t+1} - \alpha(\hat{\mathbf{v}}_{t+1} - \hat{\mathbf{r}}_t) \tag{47}$$

Canceling out steady state and joining LHS with RHS we get log linearized price of one period bond:

$$q_{t,1} = \zeta_t - \gamma \Delta \hat{\mathbf{c}}_{t+1} - \hat{\pi}_{t+1} - \alpha(\hat{\mathbf{v}}_{t+1} - \hat{\mathbf{r}}_t) \tag{48}$$

[22] write box on the early resolution of uncertainty and how it depends on the calibration

Next, we substitute equation 42 into equation 41 to highlight that $\hat{\mathbf{v}}_{t+1} - \hat{\mathbf{r}}_t$ is the next periods value relative to its certainty equivalent [23]

$$q_{t,1} = E_t \left\{ \zeta_t - \gamma \Delta \hat{\mathbf{c}}_{t+1} - \hat{\pi}_{t+1} \right\} - \alpha (E_{t+1} \hat{\mathbf{v}}_{t+1} - E_t v_{t+1}) \tag{49}$$

By chaining the stochastic discount factor we derive the price of bond with any maturity $n$:

$$q_{t+n} = \sum_{j=1}^{n} E_t \zeta_{t+j} - \gamma \Delta^n E_t \hat{\mathbf{c}}_{t+n} - E_t \sum_{j=1}^{n} \hat{\pi}_{t+j} - \alpha \left[ \sum_{j=1}^{n} (\hat{\mathbf{v}}_{t+j+1} - \hat{\mathbf{r}}_{t+j}) \right] \tag{50}$$

Note that risk aversion is denoted $\rho$ and $\alpha$ is then:

$$\alpha = 1 - \frac{1-\rho}{1-\gamma} \tag{51}$$

so for EZ term to enter stochastic discount factor the risk aversion must be different of the inverse of intertemporal elasticity of substitution.

## A.4 Log-linearizing the value function

The goal is to express bond prices as a function of macroeconomic fundamentals. Therefore, we need to eliminate the recursion. Lets assume that the period utility is additively separable CRRA.

$$V_t = \left[ \frac{C^{1-\gamma}}{1-\gamma} - \chi \frac{N^{1+\eta}}{1+\eta} \right] \zeta_t + \beta (E_t [V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{52}$$

Remember that $R_t = [E_t V_{t+1}^{1-\alpha}]^{\frac{1}{1-\alpha}}$ is the certainty equivalent of next period's utility. The log-linearized equation A.4 around zero steady state is

$$\hat{\mathbf{v}}_t = \frac{\bar{\zeta} \bar{C}^{1-\gamma}}{\bar{V}} \hat{\zeta}_t + \frac{\bar{\zeta} \bar{C}^{1-\gamma}}{\bar{V}} \hat{\mathbf{c}}_t - \frac{\bar{\zeta} \bar{N}^{1-\eta}}{\bar{V}} \hat{\mathbf{n}}_t + \beta \hat{\mathbf{r}}_t \tag{53}$$

[23]note that the term $\hat{\mathbf{v}}_{t+1} - \hat{\mathbf{r}}_t$ in time $t$ expectations equals to zero. This is given by the fact that the first order approximation eliminates uncertainty from the model and thus the $E_t v_{t+1} = R_t$. Agents expectations are up to the first order identical to certainty equivalent

simplifying the notation

$$\hat{\mathbf{v}}_t = a(\hat{\zeta}_t + \hat{\mathbf{c}}_t) - b\hat{\mathbf{n}}_t + \beta\hat{\mathbf{r}}_t \tag{54}$$

where $a = \frac{\bar{\zeta}\bar{C}^{1-\gamma}}{\bar{V}}$ and $b = \frac{\bar{\zeta}\bar{N}^{1-\eta}\chi}{\bar{V}}$.

Solving the equation (53) forward we get:

$$\hat{\mathbf{v}}_t = \sum_{j=0}^{\infty} \beta^j \left( a\hat{\zeta}_{t+j} + a\hat{\mathbf{c}}_{t+j} - \sum_{j=0}^{\infty} \beta^j b\hat{\mathbf{n}}_{t+j} \right) \tag{55}$$

Next, it is convenient to follow Uhlig (2010) and introduce the "surprise" operator $S_{t+k|t}$ for any random variable $x$, given by

$$S_{t+n|t} = E_{t+k}(x) - E_t(x) \tag{56}$$

thus for the period $t+1$, $S_{t+1}$ is filtering out the surprise in conditional expectations and is defined

$$S_{t+1} = x_{t+1} - E_t[x_{t+1}] \tag{57}$$

Note that the surprise over $n$ periods is simply

$$S_{t+n} = S_{t+n} + S_{t+n-1} + \ldots + S_{t+1} \tag{58}$$

Applying the filtering, using the equation 55 in the SDF, equation 48, we can show that the bond pricing equation is determined by the period consumption growth, inflation, exogenous preference shock and the surprise or news about the future consumption and labor.

$$q_{t,1} = \zeta_{t+1} - \gamma\Delta\hat{\mathbf{c}}_{t+1} - \hat{\pi}_{t+1} - \alpha S_{t+1} \left( \sum_{j=0}^{\infty} \beta^j \left[ a\hat{\zeta}_{t+j} + a\hat{\mathbf{c}}_{t+j} - b\hat{\mathbf{n}}_{t+j} \right] \right) \tag{59}$$

Notice that that the labor enters the bond pricing equation witch is usually the case only with non-separable preferences. Nevertheless, labor affect only higher order terms. Price of bond with maturity $n$ is given by

$$q_{t+n} = \sum_{j=1}^{n} \zeta_{t+j} - \gamma \Delta^n \hat{\mathbf{c}}_{t+n} - \sum_{j=1}^{n} \hat{\pi}_{t+j} - \alpha S_{t+n} \left( \sum_{j=0}^{\infty} \beta^j \left[ a\hat{\zeta}_{t+j} + a\hat{\mathbf{c}}_{t+j} - b\hat{\mathbf{n}}_{t+j} \right] \right) \tag{60}$$

The revaluation in the expectations can be understood as well as the news or surprise. Investors require compensation for the uncertainty underlying the surprise component. Net effect of good news about $c_{t+2}, c_{t+3} \ldots$ and $n_{t+2}, n_{t+3}, \ldots$ on marginal utility of $c_{t+1}$ and $n_{t+1}$ depends on $\alpha$. If $\alpha$ is positive, news is a positive shock to SDF. Note also that news about $c_{t+1}$ directly affect the consumption growth part of SDF but it also shows up in the second part of equation 60. If there is no news about $c_{t+2}, c_{t+3}$ and $n_{t+2}, n_{t+3}$ SDF reduces to $\beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \pi^{-1}$. Each period agents make expectations about future consumption and hours worked for the remaining life of the bond and compare it with the previous period execrations. The difference between this two is the update in expectations reflected in price of bonds. The update of expectations is sum of all news (surprises) over the remaining maturity of the bond about the life time stream of consumption and leisure.

## A.5 Second order approx. to term structure

The derivations rely on Sutherland (2002) who argues that first order approximate solutions are sufficient to derive second order approximate solutions to second moments. Second order accurate solutions for second moments can be obtained by considering first-order accurate solutions to realized values because terms of order two and above in the behaviour of realized values become terms of order three and above in the squares and cross products of realized values. The first part of the derivations, which is not explicitly working with EZ preferences, is in line with Hordahl et al. (2007). The price of bond with maturity $n$ is defined $P_t^{(n)} = E_t[Q_{t,t+n}]$; in the non-stochastic steady state $\bar{P} = \bar{Q}$. Lower case letters define logarithm of their upper case counterparts.

$$
\begin{aligned}
\bar{p}(1+\hat{\mathbf{p}}_{t,n}+\frac{1}{2}\hat{\mathbf{p}}_{t,i}^2) &= E_t\left[\bar{q}(1+\hat{\mathbf{q}}_{t+n}+\frac{1}{2}\hat{\mathbf{q}}_{t+n}^2)\right] \\
&= \bar{q}_t E_t\left[1+\hat{\mathbf{q}}_{t+n}+\frac{1}{2}\hat{\mathbf{q}}_{t+n}^2\right]
\end{aligned}
$$

After canceling out steady state, we get:

$$
\hat{\mathbf{p}}_{t,n} = E_t[\hat{\mathbf{q}}_{t,t+n}+\frac{1}{2}\hat{\mathbf{q}}_{t,t+n}^2]-\frac{1}{2}\hat{\mathbf{p}}_{t,n}^2
$$

Up to the first order $\hat{\mathbf{p}}_{t,n} = E_t\{\hat{\mathbf{q}}_{t,t+n}\}$, thus we can substitute for the quadratic term $\hat{\mathbf{p}}_{t,n}^2 = (E_t\{\hat{\mathbf{q}}_{t,t+n}\})^2$. It follows that:

$$
\hat{\mathbf{p}}_{t,n} = \mathrm{E}_t\left[\hat{\mathbf{q}}_{t,t+n}+\frac{1}{2}\hat{\mathbf{q}}_{t,t+n}^2\right]-\frac{1}{2}(\mathrm{E}_t\hat{\mathbf{q}}_{t,t+n})^2
$$

From the last equation using the definition of variance [24] we can define price of one period bond.

$$
\hat{\mathbf{p}}_{t,n} = E_t[\hat{\mathbf{q}}_{t,t+n}]+\frac{1}{2}\mathrm{Var}[\hat{\mathbf{q}}_{t,t+n}] \tag{61}
$$

using the definition of yield to maturity, $\widehat{ytm_t} = -(1/n)\hat{\mathbf{q}}_{t,n}$ we can write equation **??**

$$
\widehat{ytm}_t^n = -\frac{1}{n}q_{t,t+n}-\frac{1}{2n}\mathrm{Var}(\hat{\mathbf{q}}_{t,t+n}) \tag{62}
$$

and use equation 60 and plug it into A.5

$$
\widehat{ytm}_t^n = -\frac{1}{n}\mathrm{E}_t\left\{\sum_{j=1}^{n}\left[\hat{\zeta}_{t+n}\right]-\gamma\Delta^n\hat{\mathbf{c}}_{t+n}-\sum_{j=1}^{n}\left[\hat{\pi}_{t,t+n}\right]-\alpha S_{t+n}\left(\cdot\right)\right\} \tag{63}
$$

$$
-\frac{1}{2n}\mathrm{Var}_t\left(\sum_{j=1}^{n}\left[\hat{\zeta}_{t+n}\right]-\gamma\Delta^n\hat{\mathbf{c}}_{t+n}-\sum_{j=1}^{n}\left[\hat{\pi}_{t,t+n}\right]-\alpha S_{t+n}\left(\cdot\right)\right) \tag{64}
$$

[24] $Var(x) = E[x^2]-(E[x])^2$

Unconditional mean [25] of the term structure is then

$$
\begin{aligned}
E_t[\widehat{ytm}_t^n] \quad = \quad & -\frac{1}{2n}\left\{\mathrm{Var}\sum_{j=1}^{n}\left(\hat{\zeta}_{t+j}\right)+\gamma^2\mathrm{Var}\left(\Delta^n\hat{\mathbf{c}}_{t+n}\right)+\mathrm{Var}\sum_{j=1}^{n}\left(\hat{\pi}_{t,t+j}\right)+\alpha^2\mathrm{Var}S_{t+n}\left(\cdot\right)\right\} \\
+ \quad & \frac{\gamma}{n}\mathrm{Cov}\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j},\Delta^n\hat{\mathbf{c}}_{t+n}\right)+\frac{1}{n}\mathrm{Cov}\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j},\sum_{j=1}^{n}\hat{\pi}_{t+j}\right)-\frac{\gamma}{n}\mathrm{Cov}\left(\Delta^n\hat{\mathbf{c}}_{t+n},\sum_{j=1}^{n}\hat{\pi}_{t+j}\right) \\
+ \quad & \frac{\alpha}{n}\mathrm{Cov}\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j},S_{t+n}\left(\cdot\right)\right)-\frac{\gamma\alpha}{n}\mathrm{Cov}\left(\Delta^n\hat{\mathbf{c}}_{t+n},S_{t+n}\right)-\frac{\alpha}{n}\mathrm{Cov}\left(\sum_{j=1}^{n}\hat{\pi}_{t+n},S_{t+n}\right) \qquad (65)
\end{aligned}
$$

$S_{t+n}$ embodies how much surprises I get from consumption, leisure and preference shocks over the maturity horizon.

The covariance terms can be further rewritten using correlations. Thus, we can separate the hedging property from the quantity of macroeconomic risk represented by the standard deviations. Note, that to save the notation we omit subscripts at the standard deviations $\sigma$.

$$
\begin{aligned}
E_t[\widehat{ytm}_t^n] \quad = \quad & -\frac{1}{2n}\left\{\mathrm{Var}\sum_{j=1}^{n}\left(\hat{\zeta}_{t+j}\right)+\gamma^2\mathrm{Var}\left(\Delta^n\hat{\mathbf{c}}_{t+n}\right)+\mathrm{Var}\sum_{j=1}^{n}\left(\hat{\pi}_{t,t+j}\right)+\alpha^2\mathrm{Var}S_{t+n}\left(\cdot\right)\right\} \\
+ \quad & \frac{\gamma}{n}\sigma_{\hat{\zeta}}\sigma_{\Delta\hat{\mathbf{c}}}\mathrm{Corr}\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j},\Delta^n\hat{\mathbf{c}}_{t+n}\right)+\frac{1}{n}\sigma_{\hat{\zeta}}\sigma_{\hat{\pi}}\mathrm{Corr}\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j},\sum_{j=1}^{n}\hat{\pi}_{t+j}\right) \\
- \quad & \frac{\gamma}{n}\sigma_{\Delta\hat{\mathbf{c}}}\sigma_{\hat{\pi}}\mathrm{Corr}\left(\Delta^n\hat{\mathbf{c}}_{t+n},\sum_{j=1}^{n}\hat{\pi}_{t+j}\right)+\frac{\alpha}{n}\sigma_{\hat{\zeta}}\sigma_S\mathrm{Corr}\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j},S_{t+n}\left(\cdot\right)\right) \\
- \quad & \frac{\gamma\alpha}{n}\sigma_{\hat{\mathbf{c}}}\sigma S\mathrm{Corr}\left(\Delta^n\hat{\mathbf{c}}_{t+n},S_{t+n}\right)-\frac{\alpha}{n}\sigma_{\hat{\pi}}\sigma_S\mathrm{Corr}\left(\sum_{j=1}^{n}\hat{\pi}_{t+n},S_{t+n}\right) \qquad (66)
\end{aligned}
$$

We study in the paper how the change in the volatility of government spending affects the unconditional mean of the term structure, thus

[25] For a random variable $ytm_t$, the unconditional mean is simply the expected value, $E(ytm_t)$. In contrast, the conditional mean of $ytm_t$ is the expected value of $ytm_t$ given a conditioning set of variables, $\Omega_t$. The term under the expectations in the equation A.5 is on average zero. The term thus corresponds to the determinist steady state. The variance components represent the deviation between stochastic and deterministic steady state

$$
\begin{aligned}
E_t\left[\widehat{ytm}^n_{t|\sigma_{g,L}} - \widehat{ytm}^n_{t|\sigma_{g,H}}\right] &= -\frac{1}{2n}\left\{\gamma^2 \mathrm{Var}\left(\Delta^n \hat{\mathbf{c}}_{t+n}\right) + \mathrm{Var}\sum_{j=1}^{n}\left(\hat{\pi}_{t,t+j}\right) + \alpha^2 \mathrm{Var} S_{t+n}\left(\cdot\right)\right\} \\
&- \frac{\gamma}{n}\mathrm{Cov}\left(\Delta^n \hat{\mathbf{c}}_{t+n}, \sum_{j=1}^{n}\hat{\pi}_{t+j}\right) - \frac{\gamma\alpha}{n}\mathrm{Cov}\left(\Delta^n \hat{\mathbf{c}}_{t+n}, S_{t+n}\right) \\
&- \frac{\alpha}{n}\mathrm{Cov}\left(\sum_{j=1}^{n}\hat{\pi}_{t+n}, S_{t+n}\right)
\end{aligned} \tag{67}
$$

We assume zero correlation between preference and government spending shock, thus $\hat{\zeta}_{t,t+j}$ becomes a constant in the expectations.

We can again go one step further and use the correlations.

$$
\begin{aligned}
E_t\left[\widehat{ytm}^n_{t|\sigma_{g,L}} - \widehat{ytm}^n_{t|\sigma_{g,H}}\right] &= -\frac{1}{2n}\left\{\gamma^2 \mathrm{Var}\left(\Delta^n \hat{\mathbf{c}}_{t+n}\right) + \mathrm{Var}\sum_{j=1}^{n}\left(\hat{\pi}_{t,t+j}\right) + \alpha^2 \mathrm{Var} S_{t+n}\left(\cdot\right)\right\} \\
&- \frac{\gamma}{n}\sigma_{\hat{\mathbf{c}}}\sigma_{\hat{\pi}}\mathrm{Corr}\left(\Delta^n \hat{\mathbf{c}}_{t+n}, \sum_{j=1}^{n}\hat{\pi}_{t+j}\right) - \frac{\gamma\alpha}{n}\sigma_{\hat{\mathbf{c}}}\sigma_S\mathrm{Corr}\left(\Delta^n \hat{\mathbf{c}}_{t+n}, S_{t+n}\right) \\
&- \frac{\alpha}{n}\sigma_{\hat{\pi}}\sigma_S\mathrm{Corr}\left(\sum_{j=1}^{n}\hat{\pi}_{t+n}, S_{t+n}\right)
\end{aligned} \tag{68}
$$

The rest will be ready soon.

U

$$
\widehat{ytm_t}^{(n)} = \frac{1}{n}\left\{\begin{array}{l} -E_t[\Delta^{(n)}\hat{\lambda}_{t+j}] + \sum_{j=1}^{n} E_t[\hat{\pi}_{t+j}] - \frac{1}{2}Var_t\left[\Delta^{(n)}\hat{\lambda}_{t+j}\right] \\ -\frac{1}{2}Var_t\left[\sum_{j=1}^{n}\hat{\pi}_{t+j}\right] + Cov_t\left[\sum_{j=1}^{n}\hat{\pi}_{t+j}, \Delta^{(n)}\hat{\lambda}_{t+j}\right] \end{array}\right\} \tag{69}
$$

where $\hat{\lambda}_t$

Further, I define slope of the term structure as a difference between yield to maturity of $n$ period bond and one period bond and take unconditional expectations to get rid off the level terms (expectation hypothesis terms which are on average zero)

$$
\begin{aligned}
E[\widehat{ytm_t}^{(n)} - \hat{\mathbf{i}}_t] &= -\frac{1}{2n}\left(E[Var_t(\Delta^{(n)}\hat{\lambda}_{t+j})]\right) - E[Var_t(\Delta\hat{\lambda}_{t+1})] \\
&- \frac{1}{2}\left(\frac{1}{n}E[Var_t(\sum_{j=1}^{n}\hat{\pi}_{t+j})] - E[Var_t(\hat{\pi}_{t+1})]\right) \\
&+ \frac{1}{n}E[Cov_t(\sum_{j=1}^{n}\hat{\pi}_{t+j}, \Delta^{(n)}\hat{\lambda}_{t+j})] - E[Cov_t(\hat{\pi}_{t+1}, \Delta\hat{\lambda}_{t+1})] \qquad (70)
\end{aligned}
$$

The intuition and interpretation of equation 70 is discussed in detail in Hordahl et al. (2007). [26]

Using equation 60 in the equation **??**:

$$
\begin{aligned}
s40_t &= -\frac{1}{2n}E\left\{Var_t\left(\sum_{j=1}^{n}\zeta_{t+j} - \gamma\Delta^n E_t\hat{\mathbf{c}}_{t+j} - \alpha\left[\sum_{j=1}^{n}(\hat{\mathbf{r}}_{t+j} - \hat{\mathbf{v}}_{t+j+1})\right]\right)\right\} \\
&+ \frac{1}{2}E[Var_t(\zeta_t - \gamma\Delta E_t\hat{\mathbf{c}}_{t+1} - \alpha(\hat{\mathbf{r}}_t - \hat{\mathbf{v}}_{t+1})] \\
&- \frac{1}{2n}\left(E[Var_t(\sum_{j=1}^{n}\hat{\pi}_{t+j})] - E[Var_t(\hat{\pi}_{t+1})]\right) \\
&+ \frac{1}{n}E\left[Cov_t\left(\sum_{j=1}^{n}\hat{\pi}_{t+j}, \sum_{j=1}^{n}\zeta_{t+j}\right)\right] - E\left[Cov_t\left(\hat{\pi}_{t+1}, \zeta_t\right)\right] \\
&- \frac{\gamma}{n}E\left[Cov_t\left(\sum_{j=1}^{n}\hat{\pi}_{t+j}, \Delta^n E_t\hat{\mathbf{c}}_{t+j}\right)\right] + E\left[Cov_t\left(\hat{\pi}_{t+1}, \Delta E_t\hat{\mathbf{c}}_{t+1}\right)\right] \\
&- \frac{\alpha}{n}E\left[Cov_t\left(\sum_{j=1}^{n}\hat{\pi}_{t+j}, \sum_{j=1}^{n}(\hat{\mathbf{v}}_{t+j+1} - \hat{\mathbf{r}}_{t+j})\right)\right] + \alpha E[Cov_t\left(\hat{\pi}_{t+1}, (v_{t+1} - \hat{\mathbf{r}}_t)\right))]
\end{aligned} \qquad (71)
$$

Next, I substitute out the recursive element from the unconditional slope of the term structure.

[26] create box discussing the intuition

$$
\begin{aligned}
s40_t &= -\frac{1}{2n}E\left\{Var_t\left(\sum_{j=1}^{n}\zeta_{t+j} - \gamma\Delta^n E_t\hat{\mathbf{c}}_{t+j} - \alpha SS_{t+n}\right)\right\} \\
&+ \frac{1}{2}E[Var_t(\zeta_t - \gamma\Delta E_t\hat{\mathbf{c}}_{t+1} - \alpha SS_{t+1}] \\
&- \frac{1}{2n}\left(E[Var_t(\sum_{j=1}^{n}\hat{\pi}_{t+j})] - E[Var_t(\hat{\pi}_{t+1})]\right) \\
&+ \frac{1}{n}E\left[Cov_t\left(\sum_{j=1}^{n}\hat{\pi}_{t+j}, \sum_{j=1}^{n}\zeta_{t+j}\right)\right] - E\left[Cov_t\left(\hat{\pi}_{t+1}, \zeta_t\right)\right] \\
&- \frac{\gamma}{n}E\left[Cov_t\left(\sum_{j=1}^{n}\hat{\pi}_{t+j}, \Delta^n E_t\hat{\mathbf{c}}_{t+j}\right)\right] + E\left[Cov_t\left(\hat{\pi}_{t+1}, \Delta E_t\hat{\mathbf{c}}_{t+1}\right)\right] \\
&- \frac{\alpha}{n}E\left[Cov_t\left(\sum_{j=1}^{n}\hat{\pi}_{t+j}, SS_{t+n}\right)\right] + \alpha E[Cov_t\left(\hat{\pi}_{t+1}, SS_{t+1}\right)]
\end{aligned} \tag{72}
$$

where $SS_{t+n} = S_{t+n}\left(\sum_{j=0}^{\infty}\beta^j a\hat{\mathbf{c}}_{t+j} - \sum_{j=0}^{\infty}\beta^j b\hat{\mathbf{n}}_{t+j}\right)$

In the next step I rewrite the variance of linearized stochastic discount factor:

$$
\begin{aligned}
s40_t &= -\frac{1}{2n}E\left[Var_t\left(\sum_{j=1}^{n}\zeta_{t+j}\right)\right] + \frac{1}{2}E[Var_t(\zeta_{t+1}] \\
&- \frac{\gamma^2}{2n}Var_t(\Delta^n E_t\hat{\mathbf{c}}_{t+j}) + \frac{\gamma}{2}Var_t(\Delta E_t\hat{\mathbf{c}}_{t+1}) \\
&- \frac{\alpha^2}{2n}Var_t(SS_{t+n}) + \frac{\alpha^2}{2}Var_t SS_{t+1} \\
&+ \frac{1}{n}Cov_t\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j}, \Delta^n E_t\hat{\mathbf{c}}_{t+j}\right) - \frac{1}{2}Cov_t\left(\hat{\zeta}_{t+1}, \Delta E_t\hat{\mathbf{c}}_{t+1}\right) \\
&- \frac{\gamma\alpha}{n}Cov_t\left(\sum_{j=1}^{n}SS_{t+n}, \Delta^n E_t\hat{\mathbf{c}}_{t+j}\right) + \gamma\alpha Cov_t\left(SS_{t+1}, \Delta E_t\hat{\mathbf{c}}_{t+1}\right) \\
&+ \frac{\alpha}{n}Cov_t\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j}, SS_{t+j}\right) - \alpha Cov_t\left(\hat{\zeta}_{t+1}, SS_{t+1}\right) \\
&- \frac{1}{2n}\left(E[Var_t(\sum_{j=1}^{n}\hat{\pi}_{t+j})] - E[Var_t(\hat{\pi}_{t+1})]\right) \\
&+ \frac{1}{n}E\left[Cov_t\left(\sum_{j=1}^{n}\hat{\pi}_{t+j}, \sum_{j=1}^{n}\zeta_{t+j}\right)\right] - E\left[Cov_t\left(\hat{\pi}_{t+1}, \zeta_t\right)\right] \\
&- \frac{\gamma}{n}E\left[Cov_t\left(\sum_{j=1}^{n}\hat{\pi}_{t+j}, \Delta^n E_t\hat{\mathbf{c}}_{t+j}\right)\right] + \gamma E\left[Cov_t\left(\hat{\pi}_{t+1}, \Delta E_t\hat{\mathbf{c}}_{t+1}\right)\right] \\
&- \frac{\alpha}{n}E\left[Cov_t\left(\sum_{j=1}^{n}\hat{\pi}_{t+j}, SS_{t+n}\right)\right] + \alpha E[Cov_t\left(\hat{\pi}_{t+1}, SS_{t+1}\right)]
\end{aligned} \tag{73}
$$

The slope of the term structure decomposition reveals that if over long horizon there is:[27]

- higher variance of preference shock, consumption growth, and bad news decreases slope due to precautionary saving motive
- bad news come with low consumption growth the slope decreases
- bad news come with negative preference shock slope increases

[27] keep in mind that more negative $\alpha$ means higher risk aversion

- bad news comes with high inflation slope increases

To sum up, equation (60) determines the level of the yield curve. Equation (73) sheds light on the slope determinants.

# B Deriving model steady state

Labor supply in steady state

$$\frac{W}{C^\gamma} = \chi N^\eta \tag{74}$$

$\chi$ is calibrated in such way that steady state hours worked are $N = 1$.

$$N = \left[\frac{W}{C^\gamma \chi}\right]^{\frac{1}{\eta}} = 1 \tag{75}$$

Next, from the Philips curve I get

$$MC = \frac{1}{1+\lambda} \tag{76}$$

so then from the definition of aggregate marginal costs

$$\frac{1}{1+\lambda} = W \frac{1}{1-\theta} \left(\frac{Y}{K}\right)^{\frac{\theta}{1-\theta}} \tag{77}$$

so I can express $W$. Capital and government spending as s fraction of output are calibrated thus known.

$$W = \frac{1-\theta}{1+\lambda} \left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}} \tag{78}$$

Thus, we know steady state wage just as a function of parameters. Now I can get $\chi$ as a function of parameters.

Plugging equation 78 into 74 and using the market clearing condition $Y = C + G + \delta K$ and consequently $C = (1 - \frac{G}{Y} - \delta \frac{K}{Y})Y$

$$\left\{ \frac{\frac{1-\theta}{1+\lambda} \left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}}}{\left[(1 - \frac{G}{Y} - \delta \frac{K}{Y})Y\right]^\gamma \chi} \right\}^{\frac{1}{\eta}} = N \tag{79}$$

So we search for $\chi$ making $N = 1$:

$$\left\{\frac{\frac{1-\theta}{1+\lambda}\left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}}}{\left[(1-\frac{G}{Y}-\delta\frac{K}{Y})Y\right]^{\gamma}}\right\} = 1^{\eta}\chi \tag{80}$$

From production function:

$$N = \left(\frac{K^{\theta}}{Y}\right)^{\frac{-1}{1-\theta}} = \left(\frac{K}{Y}\right)^{-\frac{\theta}{1-\theta}} Y \tag{81}$$

So it should follow that if $N = 1$ then:

$$Y = \left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}} \tag{82}$$

and also,

$$K^{\theta} = Y \tag{83}$$

From equation 80,

$$\left\{\frac{\frac{1-\theta}{1+\lambda}\left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}}}{\left[(1-\frac{G}{Y}-\delta\frac{K}{Y})\right]^{\gamma}}\right\} = 1^{\eta}\chi Y^{\gamma} \tag{84}$$

Now using equation 82

$$\left\{\frac{\frac{1-\theta}{1+\lambda}\left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}}}{\left[(1-\frac{G}{Y}-\delta\frac{K}{Y})\right]^{\gamma}}\right\} = 1^{\eta}\chi\left(\left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}}\right)^{\gamma} \tag{85}$$

simplifying we derive value for $\chi$ making the $N = 1$.

$$\chi = \frac{\frac{1-\theta}{1+\lambda}\left(\frac{K}{Y}\right)^{\frac{\theta(1-\gamma)}{1-\theta}}}{\left[(1-\frac{G}{Y}-\delta\frac{K}{Y})\right]^{\gamma}} \tag{86}$$

So using equation 79 and plugging in from production function I get:

$$\left\{\frac{\frac{1-\theta}{1+\lambda}\left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}}}{\left[(1-\frac{G}{Y}-\delta\frac{K}{Y})Y\right]^{\gamma}\chi}\right\}^{\frac{1}{\eta}} = \left(\frac{K}{Y}\right)^{-\frac{\theta}{1-\theta}} Y \tag{87}$$

getting out output in steady state:

$$Y = \left\{\frac{\frac{1-\theta}{1+\lambda}\left(\frac{K}{Y}\right)^{\frac{\theta(1+\eta)}{1-\theta}}}{\left[(1-\frac{G}{Y}-\delta\frac{K}{Y})\right]^{\gamma}\chi}\right\}^{\frac{1}{\eta+\gamma}} \tag{88}$$

Knowing steady state output I can back up steady state hours worked from equation 81. From the market clearing condition I can get steady state consumption $C = (1 - \frac{G}{Y} - \delta\frac{K}{Y})Y$.

Consequently, $\frac{K}{Y} = 10$ so $K = 10Y$. Capital is at its steady state value so the investment must exactly offset capital depreciation. Therefore, $I = \delta K$. Similarly, $G = 0.2Y$.

The last steady state value is the value function. Note that you can rewrite the value function as infinite geometric sum by iteration. In steady state

$$V = u(C,N) + \beta[u(C,N) + \beta\{u(C,N) + \beta u(C,N)\ldots\}] \tag{89}$$

taking the utility out of the bracket

$$V = u(C,N)[1 + \beta + \beta^2 + \beta^3 \ldots] \tag{90}$$

so the steady state is

$$V = u(C,N)\frac{1}{1-\beta} \tag{91}$$

# C Charts

# References

David Backus, Allan W. Gregory, and Stanley E. Zin. Risk premiums in the term structure. *Journal of Monetary Economics*, 1989.

David Backus, Bryan Routledge, and Stanley Zin. Exotic Preferences for Macroeconomists. NBER Working Papers 10597, National Bureau of Economic Research, Inc, June 2004. URL `http://ideas.repec.org/p/nbr/nberwo/10597.html`.

Ravi Bansal and Amir Yaron. Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles. *Journal of Finance*, 59(4):1481–1509, 08 2004. URL `http://ideas.repec.org/a/bla/jfinan/v59y2004i4p1481-1509.html`.

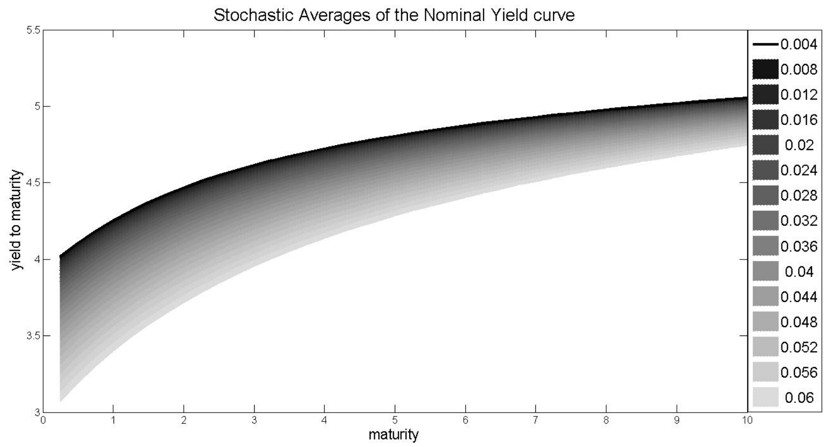


Figure 10: Term structure and varying volatility of $G$ shocks in the benchmark model when central bank puts zero weight on output gap stabilization. In the legend is the volatility of the government spending innovation.

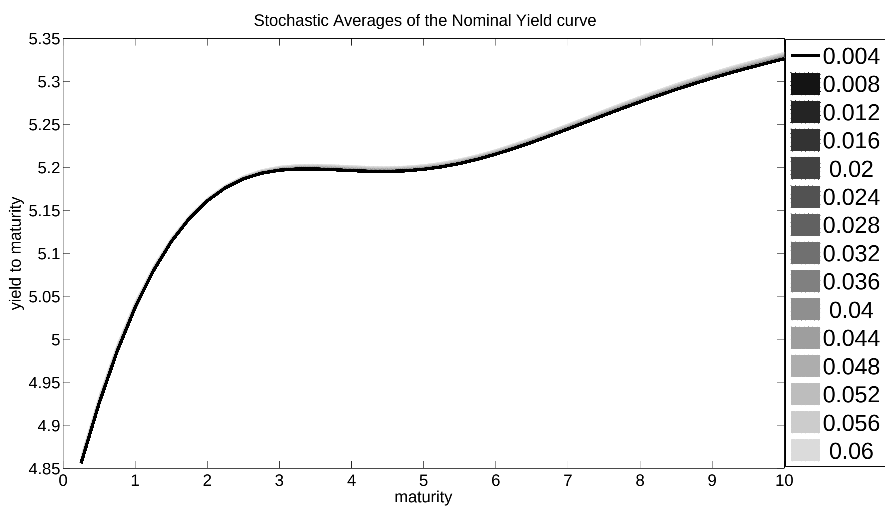


Figure 11: Term structure and varying volatility of $G$ shocks in the model with spending reversals when central bank puts $\phi_y = 0.075$ on output gap stabilization. In the legend is the volatility of the government spending innovation.

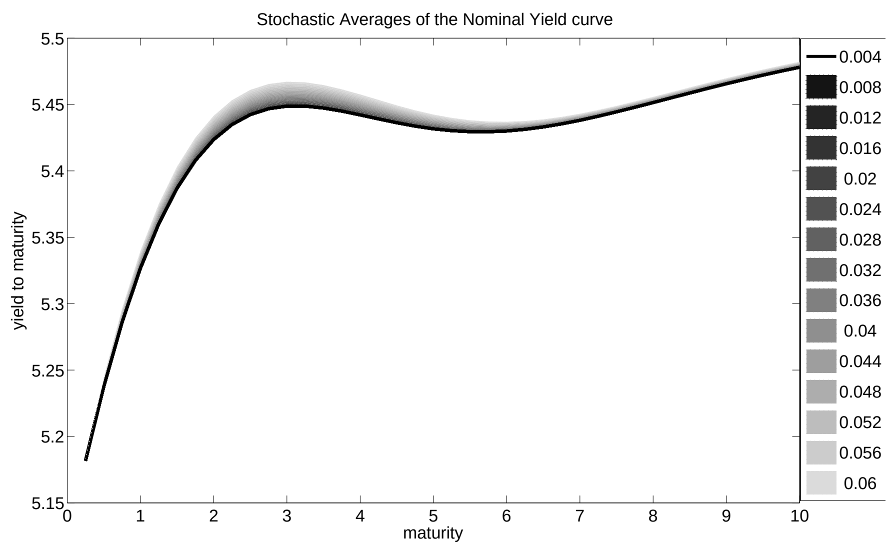


Figure 12: Term structure and varying volatility of $G$ shocks in the model with spending reversals when central bank puts $\phi_y = 0$ on output gap stabilization. In the legend is the volatility of the government spending innovation.

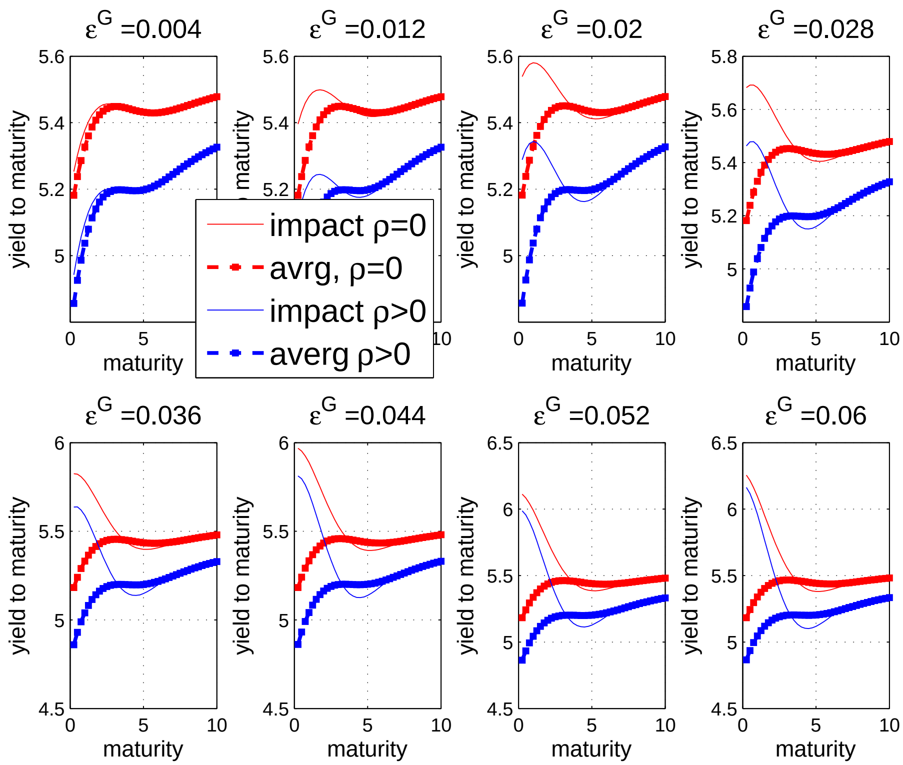


Figure 13: Term structure and varying volatility of $G$ shocks in the model with spending reversals when central bank puts $\phi_y = 0$ on output gap stabilization. In the legend is the volatility of the government spending innovation.

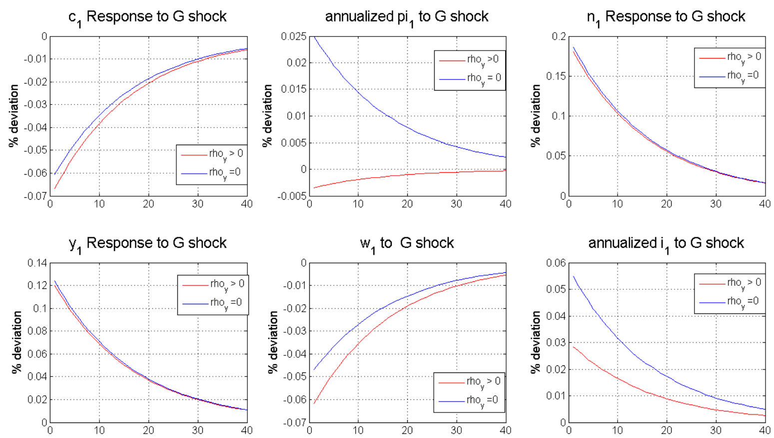


Figure 14: IR functions to 0.8% shock in $G$

innovations. Compares the benchmark model case, $\phi_y = 0.075$ to $\phi_y = 0$

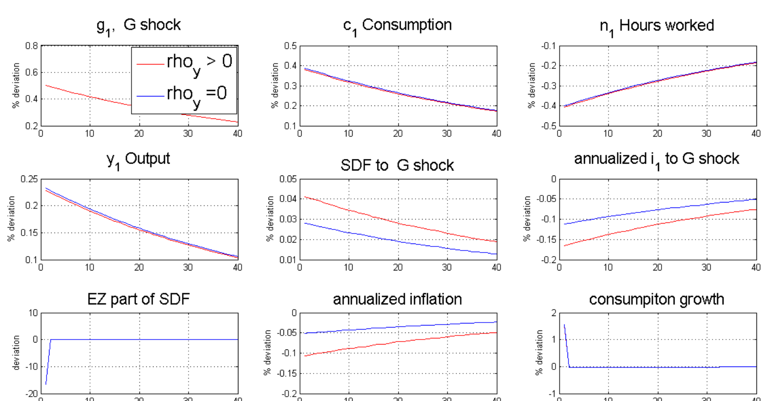


Figure 15: IR functions to 0.8% shock in $A$

innovations. Compares the benchmark model case, $\phi_y = 0.075$ to $\phi_y = 0$

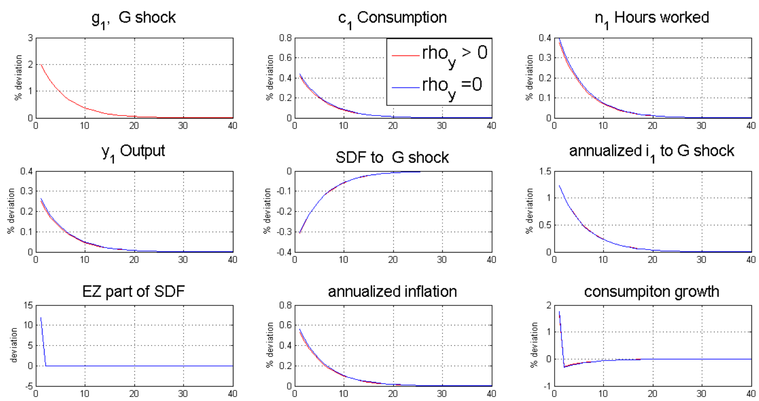


Figure 16: IR functions to 0.8% shock in $Pref$ innovations. Compares the benchmark model case, $\phi_y = 0.075$ to $\phi_y = 0$

Gary P. Brinson and Nimrod Fachler. Measuring non-us equity portfolio performance. *Journal of Portfolio Management*, 11(3):73–76, 1985.

Bianca De Paoli, Alasdair Scott, and Olaf Weeken. Asset pricing implications of a New Keynesian model. *Journal of Economic Dynamics and Control*, 34(10): 2056–2073, October 2010. URL http://ideas.repec.org/a/eee/dyncon/v34y2010i10p2056-2073.html.

Larry G Epstein and Stanley E Zin. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, 57(4):937–69, July 1989. URL http://ideas.repec.org/a/ecm/emetrp/v57y1989i4p937-69.html.

Christopher J. Erceg, Dale W. Henderson, and Andrew T. Levin. Optimal monetary policy with staggered wage and price contracts. International Finance Discussion Papers 640, Board of Governors of the Federal Reserve System (U.S.), March 1999. URL http://ideas.repec.org/p/fip/fedgif/640.html.

Marcelo Ferman. Switching monetary policy regimes and the nominal term structure. Dynare Working Papers 5, CEPREMAP, May 2011. URL http://ideas.repec.org/p/cpm/dynare/005.html.

Jonas D.M. Fisher. On the structural interpretation of the smetswouters risk premium shock. *Journal of Money, Credit and Banking*, 47(2-3):511–516, 2015. ISSN 1538-4616. doi: 10.1111/jmcb.12184. URL http://dx.doi.org/10.1111/jmcb.12184.

Jordi Galí. Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *NBER Working Paper*, 2002.

Andre Meier Giancarlo Corsetti and Gernot Müller. Fiscal stimulus with spending reversals. IMF Working Papers 09/106, International Monetary Fund, May 2009. URL http://ideas.repec.org/p/imf/imfwpa/09-106.html.

Peter Hordahl, Oreste Tristani, and Dvaid Vestin. The Yield Curve and Macroeconomic Dynamics. *Working paper series 832*, 2007. URL http://www.ecb.europa.eu.

Urban J. Jermann. Asset pricing in production economies. *Journal of Monetary Economics*, 41(2):257–275, April 1998. URL http://ideas.repec.org/a/eee/moneco/v41y1998i2p257-275.html.

Georg Kaltenbrunner and Lars A. Lochstoer. Long-Run Risk through Consumption Smoothing. *Review of Financial Studies*, 23(8):3190–3224, August 2010. URL https://ideas.repec.org/a/oup/rfinst/v23y2010i8p3190-3224.html.

David M Kreps and Evan L Porteus. Temporal resolution of uncertainty and dynamic choice theory. *Econometrica*, 46(1):185–200, January 1978. URL http://ideas.repec.org/a/ecm/emetrp/v46y1978i1p185-200.html.

Bianca De Paoli and Pawel Zabczyk. Cyclical risk aversion, precautionary saving and monetary policy. CEP Discussion Papers dp1132, Centre for Economic Performance, LSE, March 2012. URL http://ideas.repec.org/p/cep/cepdps/dp1132.html.

Monika Piazzesi and Martin Schneider. Equilibrium Yield Curves. In *NBER Macroeconomics Annual 2006, Volume 21*, NBER Chapters, pages 389–472. National Bureau of Economic Research, Inc, October 2007. URL http://ideas.repec.org/h/nbr/nberch/11182.html.

Valerie A. Ramey. Identifying Government Spending Shocks: It's all in the Timing. *The Quarterly Journal of Economics*, 126(1):1–50, 2011. URL http://ideas.repec.org/a/oup/qjecon/v126y2011i1p1-50.html.

Glenn D. Rudebusch and Eric T. Swanson. The bond premium in a dsge model with long-run real and nominal risks. *American Economic Journal: Macroeconomics*, 4(1):105–43, January 2012. URL http://ideas.repec.org/a/aea/aejmac/v4y2012i1p105-43.html.

Glenn Rudenbush and Eric T. Swanson. Examining The Bond Premium Puzzle with a DSGE Model. *Journal of Monetary Economics*, 2008.

Frank Smets and Rafael Wouters. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97 (3):586–606, June 2007. URL http://ideas.repec.org/a/aea/aecrev/v97y2007i3p586-606.html.

Alan Sutherland. A Simple Second-Order Solution Method for Dynamic General Equilibrium Models. Discussion Paper Series, Department of Economics 200211, Department of Economics, University of St. Andrews, December 2002. URL http://ideas.repec.org/p/san/wpecon/0211.html.

Harald Uhlig. Easy ez. unpablished manuscript, 2010.

Jules H. van Binsbergen, Jess Fernndez-Villaverde, Ralph S.J. Koijen, and Juan Rubio-Ramrez. The term structure of interest rates in a

DSGE model with recursive preferences. *Journal of Monetary Economics*, 59(7):634–648, 2012. URL http://ideas.repec.org/a/eee/moneco/v59y2012i7p634-648.html.

Philippe Weil. Nonexpected utility in macroeconomics. *The Quarterly Journal of Economics*, 105(1):29–42, 1990. doi: 10.2307/2937817. URL http://qje.oxfordjournals.org/content/105/1/29.abstract.