# Beware of exchange rate regime choice under late payments

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#### Abstract

In countries of the eurozone periphery, payments are typically delayed by several weeks. This characteristic is introduced into a new Keynesian small open economy model by the assumption that consumption goods are paid for with a delay of one quarter. This results in a working capital requirement for firms and a backward-looking Euler equation which implies a stationary price level which is independent of the exchange rate regime. Most notably, the welfare ranking of flexible versus fixed exchange rates changes when compared with the standard New Keynesian benchmark model in that the fixed exchange rate regime is always preferred.

Keywords: Exchange rate regime, small open economy, late payments JEL classification:

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# 1 Introduction

#### TBA

**Stylized Fact:** Creditreform (2014, p.15) reports collection periods of 142 days for Greece, 105 days for Italy and 83 days for Portugal.

The rest of the paper is structured as follows. Section 2 introduces the model, section 3 shows the dynamics of the model, section 4 a welfare analysis and section 5 concludes.

## 2 The Model

The model is based on the New Keynesian small open economy model of Galí and Monacelli (2005, denoted as "GM" henceforth). A representative household maximizes his intertemporal utility and smoothes consumption by using a complete international capital market. The only difference with respect to the GM benchmark model is that he pays his consumption goods with one quarter delay. A continuum of firms produces consumption goods which are imperfect substitutes so that prices are set as a mark-up over marginal costs. Because firms receive payments for their sold consumption goods only with a one quarter delay, they need to take out a working capital loan to be able to pay their wage bill in the period in which they produced these goods. The central bank either sets the nominal interest rate in response to the domestic inflation rate and lets the nominal exchange rate float or it fixes the nominal exchange rate.

#### 2.1 Households

#### 2.1.1 Intratemporal decision

TBA (standard)

#### 2.1.2 Intertemporal decision and labor supply

The representative households maximizes the present value of his lifetime utility:

$$E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right\}$$

where [ADD VARIABLE DESCRIPTION]. He pays for his consumption only with a on-period delay so that his budget constraint is

$$P_{t-1}C_{t-1} + E_t \{Q_{t,t+1}D_{t+1}\} = W_t N_t + D_t + T_t$$

[ADD VARIABLE DESCRIPTION]. The resulting optimality conditions with respect to the labor-consumption choice is

$$\beta N_t^\phi = C_{t-1}^{-\sigma} \frac{W_t}{P_{t-1}}$$

**Interpretation:** When today's consumption is paid only with a lag of one period, today's labor pays for yesterday's consumption. Today's marginal disutility from work needs to be discounted to be equated to the marginal utility of yesterday's consumption which this effort affords. The real wage that is relevant for this optimality consideration is today's nominal wage divided by yesterday's price level.

Log-linearized and expanded by domestic prices and expressed as a deviation from the steady state we get the labor supply equation

$$ln\beta + \phi n_t = -\sigma c_{t-1} + w_t - p_{H,t} + \underbrace{p_{H,t} - p_{H,t-1}}_{=\pi_{H,t}} + \underbrace{p_{H,t-1} - p_{t-1}}_{=-\alpha s_{t-1}} \phi \widehat{n}_t = -\sigma \widehat{c}_{t-1} + \widehat{w}_t - \widehat{p}_{H,t} + \pi_{H,t} - \alpha s_{t-1}$$
(1)

The optimality condition for the intertemporal consumption decision is:

$$\frac{C_{t-1}^{-\sigma}}{\beta P_{t-1}} E_t \left\{ Q_{t,t+1} \right\} = \beta \frac{C_t^{-\sigma}}{\beta P_t}$$

which can be re-arranged to get

$$C_{t-1}^{-\sigma} = \beta R_t \frac{C_t^{-\sigma} P_{t-1}}{P_t}$$

with risk free interest rate  $R_t = \frac{1}{E_t \{Q_{t,t+1}\}}$  which we log-linearize and get

$$c_{t-1} = c_t - \frac{1}{\sigma} \left( \hat{r}_t - \pi_t \right) \tag{2}$$

with  $\hat{r}_t = r_t - \ln \beta$ . This Euler equation is backward rather than forward looking and the real rate that matters is the current period rate  $\hat{r}_t - \pi_t$ . Raising  $\hat{r}_t - \pi_t$ causes an increase in current period consumption (given past consumption). In contrast, in the standard Euler equation an increase in the (ex ante) real rate decreases current consumption (given expected consumption). This will matter below where we will see that in order to counter sunspots in inflation expectations, the central bank needs to allow the real rate to fall when inflation rises, the Taylor principle thus needs to be violated.

For the firms' price setting equations we will need the discount factor  $E_t \{Q_{t,t+k+1}\}$ . This can be shown to be

$$E_t \{ Q_{t,t+k+1} \} = \beta^{k+1} \frac{C_{t+k}^{-\sigma}}{C_{t-1}^{-\sigma}} \frac{P_{t-1}}{P_{t+k}}$$

#### 2.2Firms

Firm i maximizes its profits under the assumption that customers pay their bills only with a one period delay. This implies that they need to pay (gross) interest  $R_t^{wc}$  on a working capital loan to pay their wage bill up-front. This working capital is obtained from an international capital market. The firm's owner values future revenues with his stochastic discount factor  $E_t \{Q_{t,t+k+1}\}$ .

Under Calvo-pricing with a probability  $\theta$  of the price not being re-set between periods being re-set, the optimization problem then is

$$\max_{\overline{P}_{H,t}} \left[ \sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ Q_{t,t+k+1} \left( P_{H,t}(j) Y_{t+k}(j) \right) \right\} - \sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ Q_{t,t+k+1} \left( W_{t+k} N_{t+k}(j) \right) + \left( R_{t+k}^{wc} - 1 \right) W_{t+k} N_{t+k}(j) \right\} \right]$$
$$= \max_{\overline{P}_{H,t}} \sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ Q_{t,t+k+1} \left( P_{H,t}(j) Y_{t+k} - W_{t+k} N_{t+k}(j) R_{t+k}^{wc} \right) \right\}$$

Because the price that is relevant for the (non-discounted) revenue is identical to the benchmark model with non-delayed payments,  $\theta^k$  still prevails. However, as the revenue is paid out to the firm only one period later, the stochastic discounting is with respect to the respective following period, i.e.  $Q_{t,t+k+1}$ . The process describing the working capital interest rate will be shown below.

With linear firm specific production function

$$Y_t(j) = A_t N_t(j)$$

and goods demand  $Y_{t+k}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t+k}}\right)^{-\varepsilon} \left(C_{H,t+k} + \int_0^1 C_{H,t+k}^i di\right)$ , and  $Q_{t,t+k+1} = 0$  $\beta^{k+1} \frac{C_{t+k}^{-\sigma} P_{t-1}}{C_{t-1}^{-\sigma} P_{t+k}}$  we get the first order condition expressed in real terms<sup>1</sup>

$$\frac{c_{\pm n}}{C_{\pm -\sigma}} P_{t\pm h}$$
 V

$$\sum_{k=0}^{\infty} \theta^k \beta^k E_t \left\{ C_{t+k}^{-\sigma} Y_{t+k}(j) \frac{P_{H,t-1}}{P_{t+k}} \left( \frac{\overline{P}_{H,t}}{P_{H,t-1}} - \frac{\varepsilon}{\varepsilon - 1} \Pi_{t-1,t+k}^H M C_{t+k} \right) \right\} = 0$$

with optimal price  $\overline{P}_{H,t}$  that is identical across firms re-setting their price in period t and  $\Pi_{t-1,t+k}^{H} \equiv \frac{P_{H,t+k}}{P_{H,t-1}}$  and  $MC_t \equiv \frac{MC_t^n}{P_{H,t}}$ . (??) is exactly the same first order condition as in Galí and Monacelli (2005), apart from the different marginal cost function. Log-linearized around the zero inflation steady state with balanced trade we thus have:

$$\pi_{H,t} = \beta E_t \left\{ \pi_{H,t+1} \right\} + \lambda \widehat{mc}_t \tag{3}$$

where  $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$  and  $\widehat{mc}_t = mc_t - mc$  and  $mc_t = w_t - p_{H,t} - a_t + r_t^{wc}$ .

<sup>&</sup>lt;sup>1</sup>A detailed derivation is provided in the appendix.

Productivity follows the stationary AR(1) process

$$a_t = \rho_a \hat{a}_{t-1} + \varepsilon_t^a$$

with  $0 < \rho_a < 1$  while the interest rate on working capital  $r_t^{wc}$  equals the interest rate set by the cental bank  $r_t$  plus a risk premium  $rp_t$ :

$$\widehat{r}_t^{wc} = \widehat{r}_t + \widehat{r}p_t$$

The risk premium, in turn, follows the stationary process:

$$\hat{rp}_t = \rho_{rp}\hat{rp}_{t-1} + \varepsilon_t^{rp}$$

with  $0 < \rho_{rp} < 1$ .

#### 2.3 Monetary policy

The central bank follows the interest rate rule

$$\hat{r}_t = \phi_\pi \pi_{H,t} + \phi_e \Delta e_t + \varepsilon_t^{mp}$$

Under flexible exchange rates the central banks reacts to the domestic inflation rate  $\pi_{H,t}$  and does not target the nominal exchange rate  $e_t$  so we have  $\phi_e = 0$  and  $\phi_{\pi} > 0$  while for fixed exchange rates we have  $\phi_e \to \infty$ . The monetary policy shock  $\varepsilon_t^{mp}$  follow an i.i.d. process.

#### 2.4 General Equilibrium

The systems of equations describing the general equilibrium of the core variables of the late payments model and the GM benchmark model are shown in the appendix.

#### 2.4.1 Changed model properties in general equilibrium

In order to understand the dynamics of the model, the price level is crucial. All that is needed to understand the price level dynamics is the Euler equation and the interest rate rule of the central bank.

**Proposition 1** Under lagged payments, the price level is a) stationary under both fixed and flexible exchange rates for productivity shocksm foreign output shocks and risk premium shocks and b) non-stationary for monetary policy shocks. Under lagged payments, the price level is a) stationary under both fixed and flexible exchange rates for foreign output shocks and risk premium shocks and b) non-stationary for monetary policy shocks.

**Proof.** See appendix.  $\blacksquare$ 

## 3 Simulations

#### 3.1 Calibration

TBA

The coefficient on inflation in the interest rate rule for the late paments model needs to be below 1, i.e. violating the Taylor principle, in order to fulfil the Blanchard-Kahn conditions. We set this value to 0.9 and to 1.5 in the GM benchmark model.

#### 3.2 Productivity shocks

We first show impulse responses for a productivity shock under fixed and flexible exchange rates for both the Galí and Monacelli (2005) benchmark model and the model with late payments. Then we show the theoretical second moments.

Figure 1 shows that after an expansionary productivity shock, the model dynamics of the lagged payments model (black lines) does not differ much from the GM benchmark model (red lines) when the exchange rate is fixed. The productivity increase reduces marginal costs and domestic inflation. Consumption and output increase and the deterioration of the terms of trade (not shown) which is due to the fall in domestic prices causes net exports to increase too. However, under fixed exchange rates the central bank is unable to offset the steep drop in hours as it canot lower the nominal exchange rate as would be desirable to stabilize hours worked. Because the initial terms of trade deterioration takes several quarters to be completed due to the Calvo mechanism, exports and output have a hump-shaped form. As both in the late payments and the GM benchmark the price level is stationary, after the initial deflation prices rise again so that the price level can return back to its initial level.

We now compare these dynamics with the alternative scenario where the exchange rate is allowed to float and the central bank conducts an inflation targeting regime.

Here both models exhibit drastically different model dynamics. Inflation and hours drop dramatically under the late payments specification while inflation only falls slightly and hours even rise after the shock in the GM benchmark model. The reason for this miserable stabilization performance in the former case is the violation of the Taylor principle which is necessary to guarantee uniqueness of the adjustment paths. Although the nominal exchange rate falls significantly more in the late payments model, the real rate actually rises (not shown). This allows consumption to rise due to the backward-looking nature of the Euler equation. At the same time, the higher real interest rate causes an *appreciation* of the nominal exchange rate which significantly mitigates the terms of trade deterioration. Net exports and output thus increase only slightly. In the GM benchmark model, in contrast, the central bank allows the real interest rate to fall, boosting consumption and depreciating the nominal exchange rate which supports the terms of trade deterioration and the increase in net exports and output.



Figure 1: Impulse responses productivity shock, fixed exchange rates

The necessity for the inflation rate to bring the price level back to its original level in the late payments model causes an increase in prices after about nine quarters. Positive inflation rates in turn imply a fall in the real rate which ultimately causes the nominal exchange rate to depreciate which prevents exports from returning quickly to their initial level. Only at this later stage of the adjustment to the steady state does the nominal exchange rate "behave" as it should for stabilization policies, at least according to standard macroeconomic logic.

### 3.3 Risk premium shocks

- 3.4 Foreign output shocks
- 3.5 Monetary policy shocks

## 4 Welfare

In the next step it will be shown that the inability to stabilize hours in the late payments model under flexible exchange rates renders the fixed exchange rate regime the better option form a welfare perspective. The necessary violation of the Taylor principle under flexible exchange rates ties the hands of the central bank more than fixing the exchange rate.



Figure 2: Impulse responses productivity shock, flexible exchange rates

# 5 Conclusion

# References

- [1] Creditreform (2014): Corporate insolvencies in Europe 2012/13.
- [2] Galí, Jordi and Monacelli, Tommaso (2005): Monetary Policy and Exchange Rate Volatility in a Small Open Economy, Review of Economic Studies, vol. 72, 707–734.

# A Appendix

### A.1 Price setting

Under Calvo-pricing with a probability  $\theta$  of the price not being re-set between periods being re-set, the optimization problem is

$$\max_{\overline{P}_{H,t}} \left[ \sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ Q_{t,t+k+1} \left( \overline{P}_{H,t} Y_{t+k}(j) \right) \right\} \\ - \sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ Q_{t,t+k+1} \left( W_{t+k} N_{t+k}(j) \right) + \left( R_{t+k}^{wc} - 1 \right) W_{t+k} N_{t+k}(j) \right\} \right] \\ = \max_{\overline{P}_{H,t}} \sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ Q_{t,t+k+1} \left( \overline{P}_{H,t} Y_{t+k} - W_{t+k} N_{t+k}(j) R_{t+k}^{wc} \right) \right\}$$

Because the price that is relevant for the (non-discounted) revenue is identical to the benchmark model with non-delayed payments,  $\theta^k$  still prevails. However, as the revenue is paid out to the firm only one period later, the stochastic discounting is with respect to the respective following period, i.e.  $Q_{t,t+k+1}$ . With linear firm specific production function

$$Y_t(j) = A_t N_t(j)$$

and goods demand  $Y_{t+k}(j) = \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \left(C_{H,t+k} + \int_0^1 C^i_{H,t+k} di\right)$ , we have

$$\max_{\overline{P}_{H,t}} \sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ Q_{t,t+k+1} \left( \begin{array}{c} \overline{P}_{H,t} \left( \frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left( C_{H,t+k} + \int_{0}^{1} C_{H,t+k}^{i} di \right) \\ - \frac{W_{t+k} \left( \frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left( C_{H,t+k} + \int_{0}^{1} C_{H,t+k}^{i} di \right) \\ - \frac{A_{t+k}}{A_{t+k}} R_{t+k}^{wc} \right) \right\}$$

The first order condition is

$$\sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ Q_{t,t+k+1} \left( \begin{array}{c} \left(1-\varepsilon\right) \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \\ +\varepsilon \frac{W_{t+k}(\overline{P}_{H,t})^{-\varepsilon-1} P_{H,t+k}^{\varepsilon}}{A_{t+k}} R_{t+k}^{wc} \end{array} \right) \left( C_{H,t+k} + \int_{0}^{1} C_{H,t+k}^{i} di \right) \right\} = 0$$

$$\sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ Q_{t,t+k+1} \left( \overline{P}_{H,t} - \frac{\varepsilon}{\varepsilon-1} \frac{W_{t+k}}{A_{t+k}} R_{t+k}^{wc} \\ -MC_{t+k}^{n} \end{array} \right) \left( \frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left( C_{H,t+k} + \int_{0}^{1} C_{H,t+k}^{i} di \right) \right\} = 0$$

$$\sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ Q_{t,t+k+1} Y_{t+k}(j) \left( \overline{P}_{H,t} - \frac{\varepsilon}{\varepsilon-1} M C_{t+k}^{n} \right) \right\} = 0$$

Plugging in  $Q_{t,t+k+1} = \beta^{k+1} \frac{C_{t+k}^{-\sigma} P_{t-1}}{C_{t-1}^{-\sigma} P_{t+k}}$ :

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \beta^{k+1} \frac{C_{t+k}^{-\sigma} P_{t-1}}{C_{t-1}^{-\sigma} P_{t+k}} Y_{t+k}(j) \left( \overline{P}_{H,t} - \frac{\varepsilon}{\varepsilon - 1} M C_{t+k}^n \right) \right\} = 0$$
$$\sum_{k=0}^{\infty} \theta^k \beta^{k+1} E_t \left\{ \frac{C_{t+k}^{-\sigma}}{P_{t+k}} Y_{t+k}(j) \left( \overline{P}_{H,t} - \frac{\varepsilon}{\varepsilon - 1} M C_{t+k}^n \right) \right\} = 0$$

Re-writing in terms of real variables:

$$\sum_{k=0}^{\infty} \theta^k \beta^{k+1} E_t \left\{ C_{t+k}^{-\sigma} Y_{t+k}(j) \frac{P_{H,t-1}}{P_{t+k}} \left( \frac{\overline{P}_{H,t}}{P_{H,t-1}} - \frac{\varepsilon}{\varepsilon - 1} \underbrace{\frac{P_{H,t+k}}{P_{H,t-1}}}_{=\Pi_{t-1,t+k}^H} \underbrace{\frac{MC_{t+k}^n}{P_{H,t+k}}}_{=MC_{t+k}} \right) \right\} = 0$$

$$\sum_{k=0}^{\infty} \theta^k \beta^k E_t \left\{ C_{t+k}^{-\sigma} Y_{t+k}(j) \frac{P_{H,t-1}}{P_{t+k}} \left( \frac{\overline{P}_{H,t}}{P_{H,t-1}} - \frac{\varepsilon}{\varepsilon - 1} \Pi_{t-1,t+k}^H MC_{t+k} \right) \right\} = 0$$
(A.1)

This is exactly the same first order condition as in Galí and Monacelli (2005), apart from the different marginal cost function. We thus have in log-linearized form around the zero inflation steady state with balanced trade:

$$\pi_{H,t} = \beta E_t \left\{ \pi_{H,t+1} \right\} + \lambda \widehat{mc}_t \tag{A.2}$$

where  $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$  and  $\widehat{mc}_t = mc_t - mc$  and  $mc_t = w_t - p_{H,t} - a_t + r_t^{wc}$ .

#### A.2 Propositions

**Proposition 2** Under lagged payments, the price level is a) stationary under both fixed and flexible exchange rates for foreign output shocks and risk premium shocks and b) non-stationary for monetary policy shocks. Under lagged payments, the price level is a) stationary under both fixed and flexible exchange rates for foreign output shocks and risk premium shocks and b) non-stationary for monetary policy shocks.

**Proof.** We prove this proposition by showing that the price level returns to its initial value after the first two shocks but not the latter one. For that purpose we assume that until period t - 1 the economy was in the deterministic steady state, that in period t a shock hits the economy and that at infinity the economy has reached a steady state again. We re-arrange the Euler equation

$$\begin{split} \Delta \widehat{y}_t &= -\alpha \left( \omega - 1 \right) \Delta \widehat{y}_t^* + \frac{1}{\sigma_\alpha} \left( \widehat{r}_t - \pi_{H,t} \right) \\ \Delta \widehat{y}_t &= -\alpha \left( \omega - 1 \right) \Delta \widehat{y}_t^* + \frac{1}{\sigma_\alpha} \left( \phi_\pi \pi_{H,t} + \phi_e \Delta e_t + \varepsilon_t^{mp} - \pi_{H,t} \right) \\ \Delta \widehat{y}_t &= -\alpha \left( \omega - 1 \right) \Delta \widehat{y}_t^* + \frac{\phi_\pi - 1}{\sigma_\alpha} \pi_{H,t} + \frac{\phi_e}{\sigma_\alpha} \Delta e_t + \frac{1}{\sigma_\alpha} \varepsilon_t^{mp} \end{split}$$

 $and \ add \ future \ output \ changes$ 

$$\begin{split} E_t \left\{ \Delta \widehat{y}_{t+2} \right\} + E_t \left\{ \Delta \widehat{y}_{t+1} \right\} + \Delta \widehat{y}_t &= -\alpha \left( \omega - 1 \right) E_t \left\{ \dots + \Delta \widehat{y}_{t+2}^* + \Delta \widehat{y}_{t+1}^* + \Delta \widehat{y}_t^* \right\} \\ &+ \frac{\phi_{\pi} - 1}{\sigma_{\alpha}} E_t \left\{ \dots \pi_{H,t+2} + \pi_{H,t+1} + \pi_{H,t} \right\} \\ &+ \frac{\phi_e}{\sigma_{\alpha}} E_t \left\{ \dots \Delta e_{t+1} + \Delta e_{t+1} + \Delta e_t \right\} \\ &+ \frac{1}{\sigma_{\alpha}} E_t \left\{ \dots + \varepsilon_{t+2}^{mp} + \varepsilon_{t+1}^{mp} + \varepsilon_t^{mp} \right\} \\ \dots E_t \left\{ \widehat{y}_{t+2} \right\} - \widehat{y}_{t-1} &= -\alpha \left( \omega - 1 \right) E_t \left\{ \dots + \widehat{y}_{t+2}^* - \widehat{y}_{t-1}^* \right\} \\ &+ \frac{\phi_{\pi} - 1}{\sigma_{\alpha}} E_t \left\{ \begin{array}{c} \dots + \ln P_{H,t+2} - \ln P_{H,t+1} + \ln P_{H,t+1} \\ - \ln P_{H,t} + \ln P_{H,t-1} - \ln P_{H,t-1} \end{array} \right\} \\ &+ \frac{\phi_e}{\sigma_{\alpha}} E_t \left\{ \dots e_{t+1} - e_{t-1} \right\} + \frac{1}{\sigma_{\alpha}} \varepsilon_t^{mp} \end{split}$$

More generally

••••

$$E_t \left\{ \widehat{y}_{\infty} - \widehat{y}_{t-1} \right\} = -\alpha \left( \omega - 1 \right) E_t \left\{ \widehat{y}_{\infty}^* - \widehat{y}_{t-1}^* \right\} + \frac{\phi_{\pi} - 1}{\sigma_{\alpha}} \left( E_t \left\{ \ln P_{H,\infty} - \ln P_{H,t-1} \right\} \right) \\ + \frac{\phi_e}{\sigma_{\alpha}} E_t \left\{ e_{\infty} - e_{t-1} \right\} + \frac{1}{\sigma_{\alpha}} \varepsilon_t^{mp}$$

As the steady state output level is unique, we have an identical output level in the old and the new steady state, so that  $E_t \{ \widehat{y}_{\infty} - \widehat{y}_{t-1} \} = 0$ . The exogenous foreign output has returned to its initial value at infinity so that  $E_t \{ \widehat{y}_{\infty}^* - \widehat{y}_{t-1}^* \} = 0$ . We therefore have

$$E_t \left\{ \ln P_{H,\infty} - \ln P_{H,t-1} \right\} = \frac{-\phi_e}{\phi_\pi - 1} E_t \left\{ e_\infty - e_{t-1} \right\} + \frac{1}{\sigma_\alpha} \varepsilon_t^{mp}$$

which reduces to

$$E_t \left\{ \ln P_{H,\infty} - \ln P_{H,t-1} \right\} = \frac{1}{\sigma_\alpha} \varepsilon_t^{mp}$$

because both under fixed  $(e_{\infty} - e_{t-1} = 0)$  and flexible  $(\phi_e = 0)$  exchange rates the right hand side reduces to a term that is proportional to the monetary policy shock. This proves that the price level is stationary for both exchange rate regimes under foreign output and risk premium shocks and non-stationary for the monetary policy shock.

#### A.3 Core model equations

#### A.3.1 The late payments model

The eight endogenous variables  $\pi_{H,t}$ ,  $\widehat{mc}_t$ ,  $\widehat{y}_t$ ,  $\widehat{r}_t$ ,  $\widehat{r}_t^{wc}$ ,  $nx_t$ ,  $s_t$ ,  $e_t$  are determined by the first eight of the following equations, the driving forces are the exogenous processes  $\hat{y}_t^*$ ,  $a_t$ ,  $\hat{rp}_t$  which are shown in the last three of the following equations and the four shocks  $\varepsilon_t^{mp}$ ,  $\varepsilon_t^a$ ,  $\varepsilon_t^{rp}$ ,  $\varepsilon_t^{y^*}$ :

$$\pi_{H,t} = \beta E_t \left\{ \pi_{H,t+1} \right\} + \lambda \widehat{mc}_t \tag{A.3}$$

 $\widehat{mc}_t = \phi \widehat{y}_t + (2 - \alpha) \,\sigma_\alpha \widehat{y}_{t-1} + (\alpha \omega - 1) \,\sigma_\alpha \widehat{y}_{t-1}^* - (1 + \phi) a_t + \widehat{r}_t^{wc} - \pi_{H,t}$ (A.4)

$$0 = \Delta \hat{y}_t + \alpha \left(\omega - 1\right) \Delta \hat{y}_t^* - \frac{1}{\sigma_\alpha} \left(\hat{r}_t - \pi_{H,t}\right)$$
(A.5)

$$\Delta s_t = \sigma_\alpha \Delta \widehat{y}_t - \sigma_\alpha \Delta \widehat{y}_t^* \tag{A.6}$$

$$\Delta s_t = \Delta e_t - \pi_{H,t} \tag{A.7}$$

$$\widehat{r}_t = \phi_\pi \pi_{H,t} + \phi_e \Delta e_t + \varepsilon_t^{mp} \tag{A.8}$$

$$\hat{r}_t^{wc} = \hat{r}_t + \hat{r}\hat{p}_t \tag{A.9}$$

$$nx_t = \alpha \left(\frac{\omega}{\sigma} - 1\right) s_t \tag{A.10}$$

$$a_t = \rho \widehat{ra}_{t-1} + \varepsilon_t^a \tag{A.11}$$

$$\hat{rp}_t = \rho_{rp}\hat{rp}_{t-1} + \varepsilon_t^{rp} \tag{A.12}$$

$$\hat{y}_{t}^{*} = \rho_{y^{*}} \hat{y}_{t-1}^{*} + \varepsilon_{t}^{y^{*}}$$
(A.13)

#### A.3.2 Benchmark Gali-Monacelli model

The core model equations in the GM model (the shocks and exogenous processes are identical to the above model):

$$\begin{aligned} \pi^{GM}_{H,t} &= \beta E_t \left\{ \pi^{GM}_{H,t+1} \right\} + \lambda \widehat{mc}^{GM}_t \\ \widehat{mc}^{GM}_t &= \left( \phi + \sigma_\alpha \right) \widehat{y}^{GM}_t + \left( \sigma - \sigma_\alpha \right) \widehat{y}^*_t - (1 + \phi) a_t + \widehat{r}^{wc,GM}_t \\ 0 &= \Delta \widehat{y}^{GM}_{t+1} + \alpha \left( \omega - 1 \right) \Delta \widehat{y}^*_{t+1} - \frac{1}{\sigma_\alpha} \left( \widehat{r}^{GM}_t - \pi^{GM}_{H,t+1} \right) \\ \Delta s^{GM}_t &= \sigma_\alpha \Delta \widehat{y}^{GM}_t - \sigma_\alpha \Delta \widehat{y}^*_t \\ \Delta s^{GM}_t &= \Delta e^{GM}_t - \pi^{GM}_{H,t} \end{aligned}$$

$$\begin{split} \hat{r}_{t}^{GM} &= \phi_{\pi} \pi^{GM}_{H,t} + \phi_{e} \Delta e^{GM}_{t} \\ \hat{r}_{t}^{wc} &= \hat{r}_{t}^{GM} + \hat{rp}_{t} \\ nx_{t}^{GM} &= \alpha \left(\frac{\omega}{\sigma} - 1\right) s^{GM}_{t} \end{split}$$