Fiscal Policy Implications of Social Discounting

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Abstract

At what rate should policy-makers discount the future? One long-debated and influential view holds that the social rate of discount should be lower than the private rate. This idea has risen to renewed academic and policy prominence in ongoing debates on the economics of climate change. This paper explores the broader fiscal implications of social discounting by formalizing and quantifying the portfolio of optimal tax policies in a dynamic general equilibrium model of fiscal and climate policy. The main finding is that adopting a different social than private discount rate dramatically alters optimal tax policy compared to the classic prescriptions from the literature. First, I theoretically show that, if the government discounts the future less than households, decentralizing the optimal allocation requires (i) capital income subsidies, (ii) labor income taxes that are decreasing over time, and/or (iii) consumption taxes that are increasing over time. These policies stand in stark contrast to the common prescriptions of zero capital income taxes, constant labor income taxes, and constant (uniform) consumption taxes. Second, I show that the magnitude of these policy changes is very large for the social discount factors advocated in the climate change literature. Decentralizing the optimal allocation may require capital income subsidies ranging from $30 - 65\%$, and a temporary labor income tax increase to 53% followed by a continual decline to 36% by the end of the 21st Century. Regardless of whether policy-makers are motivated to adopt differential social discounting for ethical reasons related to climate change in particular, or intergenerational equity in general, they should thus be aware that meeting their policy objectives may require fundamental fiscal policy reform.

1   Introduction

At what rate should policy-makers discount the future? Economists have long debated the appropriate social rate of discount. One particularly influential view holds that the social rate

of discount should be *lower* than the private rate for reasons such as inter-generational equity. Since the 1960s, a rich set of theoretical studies has debated the validity and potential micro-foundations of this idea (see, e.g., Baumol, 1968; Sen, 1982; Lind, 1982; Caplin and Leahy, 2004, etc.). However, the broader macroeconomic policy implications of such differential discounting remain an open question. Recent work by Fahri and Werning (2005, 2007, 2010) demonstrates that progressive estate taxation can decentralize the optimal allocation in an economy where the social planner values future generations' welfare differently than current private agents. While their framework focuses on a stochastic endowment economy with overlapping generations (OLG), it thus seems clear that differential discounting could also change fiscal policy prescriptions in a production economy with factor taxation - the focus of this paper.

Importantly, the issue of differential discounting has once again risen to the forefront of both academic and policy debates in the context of the economics of climate change (see, e.g., Arrow et al., 1995). As the impacts of greenhouse gas emissions occur over extremely long time horizons, optimal climate policy and emissions taxes depend critically on the chosen rate of discount (Nordhaus, 2007; Interagency Working Group, 2010). A number of economists have once again argued that it is immoral for policy makers to discount future generations' welfare at a rate consistent with market interest rates, a view often associated with Nicholas Stern (2006) in this context.[1] Stern thus imposes a "prescriptive" pure rate of social time preference (0.1% per year) in his climate-economy modeling work, rather than a "descriptive" value consistent with macroeconomic data (e.g., 1.5% per year). Perhaps the most common critique of this approach is that it generates savings rates that are too high compared to actual market data (Nordhaus, 2007; see also, e.g., Gerlagh and Liski, 2014). Consequently, several recent studies have incorporated differential discount factors for households (β_h) and the government (β_g) into climate-economy models (see, e.g., Kaplow, Moyer, and Weisbach, 2010; Goulder and Williams, 2012; Gerlagh and Liski, 2014; von Below, 2012). Again, however, the broader policy implications of this normative choice have received limited attention thus far. In particular, while several studies have noted that there would likely be broader implications such as for capital allocations over time (e.g., Goulder and Williams, 2012; Manne, 1995), there have been very few formalizations and quantifications of these implications. Von Below (2012) demonstrates the necessity of capital income subsidies (alongside carbon taxes) to decentralize the optimal allocation if the government is more patient than the household. Building on this insight, Barrage (2016) studies differential discounting in general equilibrium, finding that (i) capital income subsidies may be a higher policy priority for a 'Sternian' social planner than high carbon taxes, depending on the intertemporal elasticity of substitution, and (ii) that the nature of climate damages - e.g., whether the climate affects the marginal return to

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[1]Earlier studies and authors - such as Cline (1992) - have made similar arguments as well.

capital investments - becomes critically important in determining constrained-optimal policy with differential discounting. Both these studies nonetheless remain narrow in focusing on first-best environmental and tax policies. Whether these results are robust to a more realistic fiscal policy environment - such as where revenues for capital income subsidies have to be raised through distortionary tax instruments - remains an open question.

This paper formalizes and quantifies the implications of differential discounting for the portfolio of optimal distortionary fiscal policies in the Ramsey tradition. More specifically, I theoretically characterize and empirically quantify optimal tax policies in a dynamic, general equilibrium growth model with differential planner-household discounting. The fiscal side of the model incorporates the need for government expenditure through distortionary taxes in the Ramsey tradition (see, e.g., Chari and Kehoe, 1999), and builds on previous computational work on optimal Ramsey policies (e.g., Jones, Manuelli, Rossi, 1993). The environmental side of the model is based on (i) the seminal climate-economy modeling framework of the DICE/RICE model family (Nordhaus, 2008, 2010; Nordhaus and Boyer, 2000, etc.), (ii) the growing macroeconomic literature on optimal dynamic environmental policy in general equilibrium (e.g., Golosov, Hassler, Krusell, and Tsyvinski, 2014; Gerlagh and Liski, 2014; Acemoglu, Aghion, Bursztyn, and Hemous, 2012; Leach, 2009, etc.), and (iii) the rich literature on interactions between distortionary taxes and environmental policy (e.g., Goulder, 1995; Bovenberg and Goulder, 1996, 2002, etc.), most closely on the COMET fiscal climate-economy model of Barrage (2015). The central contribution of the paper is thus to study the implications of differential social discounting - a topic of interest in both the public finance and climate change literatures - within a single, coherent theoretical and quantitative general equilibrium model of both optimal taxation and climate change.

The main finding is that adopting a different social than private discount rate dramatically alters optimal fiscal policy compared to the classic prescriptions from the literature. First, I theoretically show that, if the government discounts the future less than households, decentralizing the optimal allocation requires (i) capital income subsidies, (ii) labor income taxes that are decreasing over time, and/or (iii) consumption taxes that are increasing over time. These policies stand in stark contrast to the standard setting optimal policy prescriptions of zero capital income taxation, constant labor income taxes, and constant (uniform) consumption taxes. Second, I show that the magnitude of the required fiscal policy changes is very large for the social discount factors advocated in the climate economics literature. In particular, adopting the social pure rate of time preference advocated by Stern (2006) increases optimal climate policy stringency significantly, but also has the broader policy implication that a transition to capital income subsidies is needed, beginning with a 30% subsidy within two model periods, and increasing to a 65% subsidy by the end of the Century. This policy stands in sharp contrast

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to both current policy of positive capital income taxes ($\sim 40\%$ average effective tax rate), and to the optimal policy prescription of zero capital income taxes in the standard setting where the government adopts the same discount factor as households. Similarly, decentralizing the optimal allocation with differential "Stern" discounting changes the optimal labor tax prescription from a constant optimal rate of 41% to a high initial rate of 53%, followed by a continual decline to 36% by the end of the 21st Century. Regardless of whether policy-makers are motivated to adopt differential social discounting for ethical reasons related to climate change in particular or intergenerational equity in general, they should thus be aware that meeting their policy objectives may then require fundamental fiscal policy reform.

The remainder of this paper proceeds as follows. Section 2 sets up the model and provides the main theoretical results. Section 3 describes the empirical calibration. Section 4 provides the quantitative results and concluding thoughts.

2 Benchmark Theoretical Model

Households

An infinitely-lived representative household has well-behaved preferences over consumption C_t and labor supply L_t, with lifetime utility:

$$U_{0,h} \equiv \sum_{t=0}^{\infty} \beta_h^t U(C_t, L_t)$$

In the decentralized economy, the representative household faces the following flow budget constraint in every period:

$$(1 + \tau_{ct})C_t + \rho_t B_{t+1} + K_{t+1} \leq w_t(1 - \tau_{lt})L_t + \{1 + (r_t - \delta)(1 - \tau_{kt})\} K_t + B_t \qquad (1)$$

Here, the consumption-investment good is the numeraire, τ_{ct} denotes the consumption tax in period t, B_{t+1} is purchases of one-period government bonds at price ρ_t, K_{t+1} denotes the household's capital holdings in period $t+1$, w_t the gross wage, τ_{lt} the linear labor income tax, τ_{kt} the linear net-of-depreciation capital income tax, r_t the return on capital, δ the depreciation rate, and B_t repayments of government bond holdings. An alternative specification, which facilitates a closed-form solution for capital subsidies, specifies capital income taxes as falling on the total return on capital holdings $\widetilde{\tau_{kt+1}}$, yielding budget constraint:

$$(1 + \tau_{ct})C_t + \rho_t B_{t+1} + K_{t+1} \leq w_t(1 - \tau_{lt})L_t + \{(1 - \delta + r_{t+1})(1 - \widetilde{\tau_{kt}})\} K_t + B_t \qquad (2)$$

Letting U_{it} denote the derivative of the household's felicity function with respect to argument i at time t, the household's first-order conditions can be used to derive the standard optimal labor-consumption and savings decisions, respectively:

$$\frac{-U_{lt}}{U_{ct}} = \frac{w_t(1 - \tau_{lt})}{(1 + \tau_{ct})} \tag{3}$$

$$\frac{U_{ct}}{U_{ct+1}} = \beta_h \left\{1 + (r_{t+1} - \delta)(1 - \tau_{kt+1})\right\} \frac{(1 + \tau_{ct})}{(1 + \tau_{ct+1})} \tag{4}$$

or:

$$\frac{U_{ct}}{U_{ct+1}} = \beta_h \left\{1 - \delta + r_{t+1}\right\} (1 - \widetilde{\tau_{kt+1}}) \frac{(1 + \tau_{ct})}{(1 + \tau_{ct+1})} \tag{5}$$

Production

In the benchmark version of the model, the aggregate consumption-investment good is produced by a competitive representative firm with constant returns to scale production technology $F(A_t, K_t, L_t)$ that satisfies the Inada conditions. Here, A_t denotes the level of total factor productivity in period t. Profit-maximization requires the firm to equate factor prices with their marginal products, denoted by F_{it} for factor i at time t :

$$
\begin{aligned}
F_{lt} &= w_t \\
F_{kt} &= r_t
\end{aligned}
\tag{6}
$$

Government

The social planner seeks to maximize the household's lifetime utility, but discounts the future at a potentially different rate β_g than the household:

$$U_{0,g} = \sum_{t=0}^{\infty} \beta_g^t U(C_t, L_t) \tag{7}$$

The government must finance an exogenously given sequence of revenue requirements $\{G_t\}_{t=0}^{\infty}$. As is common in the literature, I first model G_t as wasteful government consumption (e.g., Atkeson, Chari, and Kehoe, 1999; Chari and Kehoe, 1999, etc.). The expanded quantitative version of the model further adds government transfers to households G_t^T to government consumption requirements in order to match IMF Government Finance Statistics.

The government can raise revenues by issuing bonds and levying taxes on factor income and

consumption, with corresponding flow budget constraint:

$$G_t + B_t^G = \tau_{lt}w_t L_t + \tau_{ct}C_t + \tau_{kt}(r_t - \delta)K_t + \rho_t B_{t+1}^G \tag{8}$$

or:

$$G_t + B_t^G = \tau_{lt}w_t L_t + \tau_{ct}C_t + \widetilde{\tau_{kt}}[1 - \delta + r_t]K_t + \rho_t B_{t+1}^G \tag{9}$$

It should be noted that, among τ_k, τ_l, and τ_c, only two out of the three are needed to form a "complete" tax system (Chari and Kehoe, 1999). That is, the same allocation can be decentralized by many different tax systems that correspond to the same overall wedge between the relevant marginal rates of substitution (MRS), as can be readily seen for the use of either τ_{lt} or τ_{ct} to create a wedge in the consumption-leisure tradeoff (3). I thus consider both a version of the model where the untaxed numeraire is the consumption good or capital investments, respectively.

The bond market clearing condition is given by:

$$B_{t+1}^G = B_{t+1} \tag{10}$$

Competitive Equilibrium

The definition of competitive equilibrium in this economy thus far is standard as it requires no modification for differential discounting:

Definition 1 *A competitive equilibrium consists of an allocation* $\{C_t, L_t, K_{t+1}\}$, *a set of prices* $\{p_t.r_t, w_t, \rho_t\}$ *and a set of policies* $\{\tau_{kt}, \tau_{lt}, \tau_{ct}, B_{t+1}^G\}$ *such that*
    (i) the allocations solve the consumer's and the firm's problems given prices and policies,
    (ii) the government budget constraint is satisfied in every period, and
    (iii) markets clear.

The government's objective is to implement the competitive equilibrium that yields the highest household lifetime utility (7) for a given set of initial conditions (K_0, B_0, $\overline{\tau_{k0}}$, $\overline{\tau_{c0}}$). As is standard in the Ramsey optimal taxation literature, the initial capital income tax $\overline{\tau_{k0}}$ and consumption tax $\overline{\tau_{c0}}$ are assumed to be exogenously given as they can otherwise be used as effective lump-sum taxes. I also assume that the government can commit to a sequence of capital income taxes, in line with much (see discussion in Chari and Kehoe, 1999) but not all (e.g., Klein and Rios-Rull, 2003; Benhabib and Rustichini, 1997) of the Ramsey taxation literature.

The set of allocations that can be decentralized as a competitive equilibrium with a set of taxes and prices can be characterized by two constraints: feasibility and an "implementability"

constraint that captures the optimizing behavior of households and firms. The following proposition formalizes this point. Note that the key difference compared to the standard version of, e.g., Chari and Kehoe (1999) is that the present setting requires careful differentiation between the household's and the planner's discount factors in the intertemporal aggregation.

Proposition 2 *The allocations* $\{C_t, L_t, K_{t+1}\}$, *along with initial bond holdings* B_0, *initial capital* K_0, *initial capital tax* $\overline{\tau_{k0}}$ *and consumption tax* $\overline{\tau_{c0}}$ *in a competitive equilibrium satisfy:*

$$F(A_t, L_t, K_t)_t + (1 - \delta)K_t \leq C_t + G_t + K_{t+1} \tag{RC}$$

and

$$\sum_{t=0}^{\infty} \beta_h^t \left[U_{ct} C_t + U_{lt} L_t \right] = \frac{U_{c0}}{1 + \overline{\tau_{c0}}} \left[K_0 \left\{ 1 + (F_{k0} - \delta)(1 - \overline{\tau_{k0}}) \right\} + B_0 \right] \tag{IMP}$$

In addition, given an allocation that satisfies (RC)-(IMP), one can construct prices, debt holdings, and policies such that those allocations constitute a competitive equilibrium.

The Proof follows the standard procedure as outlined in Chari and Kehoe (1999), see also Barrage (2015). The only difference in the present framework is that one must be careful to employ the household's discount factor β_h rather than the planner's β_g in the derivation of the implementability constraint (IMP), which captures the optimizing behavior of households. This is the key benefit of *differential* discounting models such as this one: they allow the planner to select a social discount factor for ethical reasons but take as given the behavior of agents in the economy (see discussions in, e.g., Kaplow, Moyer, and Weisbach, 2010; Goulder and Williams, 2012; von Below, 2012).

2.1   Optimal Allocations and Tax Wedges

The social planner's problem is to maximize (7) subject to (RC) and (IMP):

$$\max \sum_{t=0}^{\infty} \beta_g^t [U(C_t, L_t,)] \tag{SWF}$$

$$+ \sum_{t=0}^{\infty} \lambda_t \left[\{ F(A_t, L_t, K_t) \} + (1 - \delta)K_t - C_t - G_t - K_{t+1} \right] \tag{RC}$$

$$+ \phi \left[\sum_{t=0}^{\infty} \beta_h^t \underbrace{(U_{ct} C_t + U_{lt} L_t)}_{\equiv W_t} - \left\{ \frac{U_{c0}}{1 + \overline{\tau_{c0}}} \left[K_0 \left\{ 1 + (F_{k0} - \delta)(1 - \overline{\tau_{k0}}) \right\} \right] \right\} \right] \tag{IMP}$$

The first-order conditions (FOCs) for the planner's problem can be combined to derive expressions for the optimal wedges (between marginal rates of substitution and transformation) in the economy. Comparison with the optimality conditions governing households' and firms' behavior, in turn, yields expressions for the corresponding taxes that can decentralize the optimal allocation.

2.1.1 Capital Income Taxes

The planner's FOCs imply that the net return on investment optimally evolves according to:

$$\beta_h \left[(1 - \delta) + F_{kt+1} \right] = \frac{\left[\left(\frac{\beta_g^t}{\beta_h^t} \right) U_{ct} + \phi W_{ct} \right]}{\left[\left(\frac{\beta_g^{t+1}}{\beta_h^{t+1}} \right) U_{ct+1} + \phi W_{ct+1} \right]} \tag{11}$$

where W_{ct} denotes the partial derivative of expression W_t in the implementability constraint (IMP) with respect to consumption at time t. In the decentralized economy, the representative household's optimality condition governing savings for a given capital income tax τ_{kt+1} (when consumption is the untaxed numeraire) is conversely given by:

$$\beta_h \left[1 + (F_{kt+1} - \delta)(1 - \tau_{kt+1}) \right] = \frac{U_{ct}}{U_{ct+1}} \tag{12}$$

or:

$$\beta_h \left[(1 - \delta) + F_{kt+1} \right] (1 - \widetilde{\tau_{kt+1}}) = \frac{U_{ct}}{U_{ct+1}} \tag{13}$$

In the standard setting where $\beta_g = \beta_h$, as is well known, for any set of preferences such that that the right-hand side of (11) reduces to U_{ct}/U_{ct+1}, optimal effective capital income taxes are zero (for $t > 1$) (Atkeson, Chari, and Kehoe, 1999; Judd,1999; Chari and Kehoe, 1999, etc.). However, as can be seen in (11), the introduction of $\beta_G \neq \beta_H$ introduces a wedge that will remain even if the other terms would otherwise reduce to U_{ct}/U_{ct+1}. As the optimal tax implied by this wedge depends on the utility function, I next consider two commonly used constant elasticity of substitution preferences that satisfy consistency with balanced growth (as per King, Plosser, and Rebelo, 2001):

$$U(C_t, L_t) = \log C_t + v(L_t) \tag{A}$$
$$U(C_t, L_t) = \frac{(C_t L_t^{-\gamma})^{1-\sigma}}{1 - \sigma} \tag{B}$$

where $v(L_t)$ is some function that is increasing and concave in leisure $(1 - L_t)$.

With preferences of the form (A), it is straightforward to show that $W_{ct} = 0$. Consequently,

the planner's optimality condition for investment (11) becomes:

$$\beta_h \left[(1 - \delta) + F_{kt+1}\right] = \frac{U_{ct}}{U_{ct+1}} \left(\frac{\beta_h}{\beta_g}\right) \tag{14}$$

Comparing (12) and (14), it immediately follows that the capital income tax that decentralizes the optimal allocation for $t > 0$ is given by:

$$\tau^*_{kt+1} = \left(\frac{\beta_h - \beta_g}{\beta_h}\right) \frac{(F_{kt+1} - \delta + 1)}{(F_{kt+1} - \delta)}. \tag{15}$$

or:

$$\widetilde{\tau_{kt+1}}^* = 1 - \frac{\beta_g}{\beta_h} \tag{16}$$

With standard discounting, $\beta_g = \beta_h$, and that the optimal capital income tax for $t > 1$ is thus equal to zero for both specifications. However, differential discounting $\beta_g > \beta_h$ clearly implies the optimality of a capital income subsidy (as both (15) and (16) are then negative).

Proposition 3 *If the social planner's discount factor β_g exceeds the household's discount factor β_h, and if preferences are of the form (A), then the optimal tax policy requires a capital income subsidy for all periods $t > 1$.*

For non-separable preferences of the form (B), one can easily show that:

$$W_{ct} = U_{ct}[1 - \sigma - \gamma(1 - \sigma)] \tag{17}$$
$$W_{lt} = U_{lt}[1 - \sigma - \gamma(1 - \sigma)] \tag{18}$$

Substituting (17) into the planner's optimality condition for savings (11) yields:

$$\beta_h \left[(1 - \delta) + F_{kt+1}\right] = \frac{U_{ct}}{U_{ct+1}} \frac{\left[\left(\frac{\beta_g}{\beta_h}\right)^t + \phi[1 - \sigma - \gamma(1 - \sigma)]\right]}{\left[\left(\frac{\beta_g}{\beta_h}\right)^{t+1} + \phi[1 - \sigma - \gamma(1 - \sigma)]\right]} \tag{19}$$

Let the overall wedge term in (19) be denoted ω_t:

$$\omega_t \equiv \frac{\left[\left(\frac{\beta_g}{\beta_h}\right)^t + \phi[1 - \sigma - \gamma(1 - \sigma)]\right]}{\left[\left(\frac{\beta_g}{\beta_h}\right)^{t+1} + \phi[1 - \sigma - \gamma(1 - \sigma)]\right]} \tag{20}$$

One can then easily show (see Appendix) that the capital income tax that decentralizes the

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optimal allocation defined by (19) is defined by:

$$\tau^*_{kt+1} = \left(\frac{\omega_t - 1}{\omega_t}\right) \frac{(F_{kt+1} - \delta + 1)}{(F_{kt+1} - \delta)} \tag{21}$$

It is thus immediately obvious that, if $\beta_g = \beta_h$, the wedge term (20) reduces to unity, and the optimal capital income tax (21) equals zero. However, if $\beta_g > \beta_h$, we have the following result:

Proposition 4 *If the social planner's discount factor β_g exceeds the households's discount factor β_h, and if preferences are of the form (B) and consistent with balanced growth, then it is optimal to subsidize capital income for all periods $t + 1 > 1$.*

Proof. First, note that, for utility parameters consistent with balanced growth, one can easily show (see Appendix) that:

$$\phi[1 - \sigma - \gamma(1 - \sigma)] \geq 0 \tag{22}$$

Second, noting that $\beta_g > \beta_h$ implies that $\left(\frac{\beta_g}{\beta_h}\right)^{t+1} > \left(\frac{\beta_g}{\beta_h}\right)^t$, it is clear that the numerator of the wedge term (20) is smaller than the denominator. Consequently, $0 < \omega_t < 1$. As can readily be seen from equation (21), the optimal capital income tax thus consists of a subsidy $(\tau^*_{kt+1} < 0)$. Finally, the restriction of this result to $t + 1 > 1$ stems from the fact that the planner's first-order conditions are non-stationary, implying a different optimal capital income tax rule for $t = 0$ versus all subsequent periods. However, the optimality condition (19) is valid for all $t > 0$, giving the desired result. ∎

2.1.2 Labor Income Taxes

Combining the planner's FOCs yields the following condition characterizing the optimal labor-consumption wedge:

$$\frac{(-U_{lt}/U_{ct})}{F_{lt}} = 1 + \phi \left[\frac{W_{ct}}{U_{ct}} + \frac{W_{lt}}{U_{ct}F_{lt}}\right] \left(\frac{\beta_g}{\beta_h}\right)^{-t} \tag{23}$$

In the decentralized economy, the household's optimal choices for a given labor tax satisfy:

$$\frac{-U_{lt}/U_{ct}}{F_{lt}} = (1 - \tau_{lt}) \tag{24}$$

Comparing (23) and (24) reveals that the optimal allocation can be decentralized by a labor income tax implicitly defined by:

$$\tau_{lt} = (-\phi) \left[\frac{W_{ct}}{U_{ct}} + \frac{W_{lt}}{U_{ct}F_{lt}}\right] \left(\frac{\beta_g}{\beta_h}\right)^{-t} \tag{25}$$

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For a given intratemporal allocation, differential discounting with $\beta_g > \beta_h$ would thus imply lower and decreasing labor income taxes compared to the standard case where $\beta_g = \beta_h$. However, as the optimal allocation is endogenous, one must again consider specific utility functions in order to assess the effect of differential discounting on optimal tax policy. In particular, consider the following functional form for $v(L_t)$ in (A):

$$U(C_t, L_t) = \log C_t - v_1 L_t^{v_2} \tag{A'}$$

It is easy to show that (A') satisfies the key properties of being increasing and concave in leisure as long as $v_1 > 0$ and $v_2 > 1$. If preferences are of the form (A'), it follows that:

$$W_{lt} = U_{lt} v_2 \tag{26}$$

Substituting (26) into the planner's optimality condition (23), rearranging terms, and comparing with the household's labor supply condition (24) demonstrates that the net-of-tax rate on labor income that decentralizes the optimal allocation for $t > 0$ is implicitly defined by:

$$(1 - \tau_{lt}) = \frac{1}{\left[1 + \phi v_2 \left(\frac{\beta_h}{\beta_g}\right)^t\right]} \tag{27}$$

With standard or descriptive discounting ($\beta_g = \beta_h$), it is clear from (27) that the optimal labor income tax would be constant over time. However, with differential or prescriptive discounting ($\beta_g > \beta_h$), this standard optimal tax smoothing result is overturned. Unfortunately, one cannot draw unambiguous conclusions on the effect of $\beta_g > \beta_h$ on the optimal labor tax *level* since the Lagrange multiplier on the implementability constraint, ϕ, is endogenous. However, the *rate of change* in the optimal labor tax over time can be determined:

Proposition 5 *If the social planner's discount factor β_g exceeds the household's discount factor β_h, and if preferences are of the form (A'), then the optimal tax policy requires labor income taxes to be decreasing over time for all t after $t = 1$.*

Proof. Consider the optimal labor tax wedge for any period $t > 0$ (27). The optimal tax at $t + 1$ is defined by:

$$(1 - \tau_{lt+1}) = \frac{1}{\left[1 + \phi v_2 \left(\frac{\beta_h}{\beta_g}\right)^{t+1}\right]}$$

If $\beta_g > \beta_h$, then $\left(\frac{\beta_h}{\beta_g}\right)^{t+1} < \left(\frac{\beta_h}{\beta_g}\right)^t$. Noting that $\phi \geq 0$ and $v_2 > 0$, this, in turn, implies that $(1 - \tau_{lt+1}) > (1 - \tau_{lt})$. ∎

Next, with non-separable preferences of the form (B), optimal labor income tax wedges for $t > 0$ are unaffected by the assumption that $\beta_g > \beta_h$. In fact, the optimal wedge is zero for $t > 0$ with this type of preferences, implying that all revenue should be raised through initial labor income taxes and period $t = 1$ capital income taxes.[2] Although similar results have been found, e.g., by Jones, Manuelli, and Rossi (1997) in a Ramsey taxation model with human capital with regards to optimal long-run tax rates, in the current setting this result is due do the specific choice of utility function (B). With the more common variant of non-separable preferences in this class of utility functions,

$$U(C_t, L_t) = \frac{(C_t(1 - L_t)^v)^{1-\sigma}}{1 - \sigma} \tag{B'}$$

the optimal net-of-labor tax rate in the current setting is given by (see Appendix for derivation):

$$(1 - \tau_{lt}^*) = \frac{1 + \phi[1 - \sigma + A(L_t)] \left(\frac{\beta_h}{\beta_g}\right)^t}{1 + \phi[1 - \sigma + A(L_t) + B(L_t)] \left(\frac{\beta_h}{\beta_g}\right)^t} \tag{28}$$

where $A(L_t) = (-v)(1-\sigma)(1-L_t)^{-1}L_t$ and $B(L_t) = (1-L_t)^{-1}$. As in the case with separable preferences (A) considered above, the endogeneity of terms such as ϕ (the Lagrange multiplier on the implementability constraint in the planner's problem) preclude one from drawing definitive conclusions about the effect of $\beta_g > \beta_h$ on the optimal labor tax wedge level. Unfortunately, in contrast to the case with separable preferences, here even the *rate of change* in the optimal labor tax wedge over time is endogenous due to the labor supply term L_t in (28). Section 3 thus solves for optimal tax rates numerically in an integrated assessment climate-economy model with fiscal policy optimization.

2.1.3 Consumption Taxes

While the discussion thus far has treated the consumption good as the untaxed numeraire, the optimal allocation can also be decentralized by a fiscal system of consumption- and labor (or other combinations of) taxes. In particular, consider a decentralized economy where capital investments are the untaxed numeraire. The household's optimal savings condition (4) then becomes:

$$\frac{U_{ct}}{U_{ct+1}} = \beta_h \left[(1 - \delta) + F_{kt+1}\right] \frac{(1 + \tau_{ct})}{(1 + \tau_{ct+1})} \tag{29}$$

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[2]This can be shown straightforwardly by substituting the terms (17)-(18) into the optimal labor wedge expression (23) and cancelling terms.

For separable preferences of the form (A), comparison of (29) with the planner's optimality condition for savings (14) shows that, on order to decentralize the optimal allocation, consumption taxes for $t > 0$ must satisfy:

$$\frac{(1 + \tau_{ct+1}^*)}{(1 + \tau_{ct}^*)} = \frac{\beta_h}{\beta_g} \tag{30}$$

Condition (30) immediately implies that optimized consumption taxes must be decreasing over time (for $t > 0$) if the planner discounts the future less than households $(\beta_g > \beta_h)$. The intuition for this result is as follows. Capital income taxes are equivalent to ever-increasing consumption taxes, as shown and discussed in detail by Judd (1999). Consequently, the desirability of capital income subsidies - of incentives to increase savings and delay consumption - can be met through ever-decreasing consumption taxes. For example, in the debate surrounding differential discounting in the climate change literature, it is common to assume $\beta_h = 0.985$ and $\beta_g = 0.999$. Condition (30) implies that, in order to decentralize the optimal allocation in this setting, the after-tax price of consumption should decrease by 1.4% per year.

Finally, for non-separable preferences (B), comparison of the household's Euler Equation (29) with the planner's optimality condition (19) shows that the optimal consumption tax sequence must satisfy:

$$\frac{(1 + \tau_{ct+1}^*)}{(1 + \tau_{ct}^*)} = \frac{\left[\left(\frac{\beta_g}{\beta_h}\right)^t + \phi[1 - \sigma - \gamma(1 - \sigma)]\right]}{\left[\left(\frac{\beta_g}{\beta_h}\right)^{t+1} + \phi[1 - \sigma - \gamma(1 - \sigma)]\right]} \tag{31}$$

As discussed above, if the government discounts the future less than households $(\beta_g > \beta_h)$, then the denominator in the right-hand side term of (31) is larger than the numerator. Consequently, the optimal consumption tax sequence must again be decreasing over time.

Proposition 6 *If the social planner's discount factor β_g exceeds the household's discount factor β_h, and if preferences are of the form (A) or (B), then the optimal consumption tax rate is decreasing over time for all t after $t = 1$.*

In contrast, with standard discounting $(\beta_g = \beta_h)$, it is clear from (30) and (31) that the optimal consumption tax (for $t > 0$) is constant over time. Intuitively, this finding can be understood both through the lens of the classic uniform commodity taxation result applied to consumption over time (see, e.g., Chari and Kehoe, 1999), and in terms of the equally classic 'Chamley-Judd' result on the desirability of no intertemporal distortions in a wide range of settings (see, e.g., Chamley, 1985; Judd, 1986, 1999; Atkeson, Chari, and Kehoe, 1999; Acemoglu, Golosov, and Tsyvinski, 2011).

2.2 Theory Results Summary

The central theoretical finding is that the adoption of different social than private discount rates drastically alters classic optimal tax policy prescriptions. In the benchmark setting with descriptive discounting - where the planner adopts the household's discount rate $\beta_g = \beta_h$ - the optimal allocation can be decentralized by zero capital income taxes, constant labor taxes, and/or constant consumption taxes. In contrast, if the planner decides to discount the future at a lower rate than households ($\beta_g > \beta_h$), I find that the optimal tax system features capital income subsidies, labor income taxes that are decreasing over time, and/or consumption taxes that are decreasing over time. To the best of my knowledge, these far-reaching implications of differential discounting have not been previously formalized as such, nor incorporated in the ongoing policy and academic debate on discounting in the economics of climate change.

3   Quantitative Analysis

In order to assess the quantitative importance of the theoretical results in a policy-relevant setting, I build on the global fiscal climate-economy model of the COMET (Barrage, 2015) to solve the planner's problem numerically. The COMET is a dynamic general equilibrium growth model of the global economy calibrated to incorporate fiscal and climate policy. The climate-economy modeling side of the COMET builds on the seminal DICE framework of Nordhaus (see, e.g., 2008, 2010, etc.). DICE is one of three models currently used by the U.S. Government to estimate the social cost of carbon emissions (Interagency Working Group, 2010). The COMET expands upon DICE in several ways to include distortionary fiscal policy. The representative household has non-separable CES preferences similar to (B):

$$U(C_t, L_t, T_t) = \left\{ \frac{[C_t \cdot (1 - \phi L_t)^\gamma]^{1-\sigma}}{1 - \sigma} \right\} + \alpha_0(T_t)^2 \tag{32}$$

The additional parameter ϕ is introduced in order to ensure that the calibration can simultaneously match (i) a desired intertemporal elasticity of substitution ($\sigma = 1.5$), (ii) a Frisch elasticity of labor supply of 0.78 based on a survey by Chetty, Guren, Manoli, and Weber (2011), and (iii) and to rationalize base year (2005) labor supply as estimated from OECD data ($l_{2005} = 0.227$, see Barrage (2015) for details). The specification maintains consistency with balanced growth. Next, T_t denotes mean atmospheric surface temperature change over pre-industrial levels, and captures the severity of global climate change. This specification of preferences permits a representation of the direct effects of climate change on utility (through, e.g., lost existence value for species suffering extinction). The additively separable formulation,

however, ensures that optimal tax wedges are not affected by climate change (see Barrage, 2015), and that the theoretical results derived above correspond closely to the quantitative implementation.

Government revenue requirements and expenditure patterns are calibrated based on IMF Government Finance Statistics. In the model base year 2005, the PPP-adjusted GDP-weighted average share of government expenditure is 33.75% of GDP. I further break down observed expenditures into government consumption ($G_t^C \sim 57\%$) and social transfers (unemployment insurance, disability insurance, etc.) ($G_t^T \sim 43\%$). On the macroeconomic side, the model adopts the productivity and population growth rate projections for the 21st and 22nd Centuries of the DICE/RICE model family (Nordhaus, 2008, 2010). For example, TFP growth is assumed to decline from 1.37% per year in the first model decade (2010-2020) to 0.76% by the end of the century. In line with other fiscal computable general equilibrium and optimization models (e.g., Jones, Manuelli, and Rossi, 1993; Goulder, 1995), I then assume that the level of total government expenditure G_t grows at the rates of labor productivity and population growth, with G_t^C and G_t^T evolving at constant shares proportional to G_t (e.g., $G_t^C = G_t (.57)$).

The initial year tax rate estimates are based on effective tax rate estimates across countries by Carey and Rabesona (2002). The PPP-adjusted GDP-weighted average rates for 1995-2000 based on the set of OECD countries in their study are given by:

$$\text{Labor \& Consumption:} \quad 35.19\% \tag{33}$$
$$\text{Capital:} \quad 43.27\%$$

There are two production sectors: the final consumption-investment good is produced using capital, labor, and energy inputs, and energy is produced from capital and labor. Both sectors assume a Cobb-Douglas production technology, with factor shares based on the literature and U.S. Bureau of Economic Analysis Data (see Barrage, 2015, for details). There are two types of energy: carbon-based (e.g., oil, coal, gas) and clean. Production of the former leads to carbon emissions which accumulate in the atmosphere and change the climate. The climate system is modeled as in DICE, with three carbon reservoirs (lower ocean, upper ocean/biosphere, atmosphere) and an exogenous path of projected land-based emissions. Climate change affects welfare through two channels: direct utility damages as per (32), and output losses in the final consumption-investment good production sector. Total damages estimates are taken from the DICE model (Nordhaus, 2008; Nordhaus and Boyer, 2000), and the split into output and utility damages is based on a calculation presented in Barrage (2015). The majority ($\sim 75\%$) of global damages are assumed to affect production processes, with the remainder ($\sim 25\%$) projected to occur outside the production sector.

Given the non-stationary nature of the optimal Ramsey taxation problem, I numerically solve the model using a similar but different procedure as in Jones, Manuelli, and Rossi (1993). The model optimizes directly over all allocations for T periods as well as over the continuation gross savings rate for after period T (with $T = 25$, representing 250 years). Then, in order to enable the global climate to reach a new steady-state before imposing a balanced growth path, I simulate the model for another 100 years with the continuation savings rate and other variables (e.g., the clean energy share) locked in at their time T values. Finally, I assume a balanced growth path after the year 2365 (35 periods), and compute continuation values for the infinite time horizon. The computation uses Matlab's optimization package.

4   Quantitative Results and Conclusions

The central quantitative result is that the values of β_g currently debated and employed in the climate change economics literature would require massive changes in fiscal policy in order to decentralize the optimal allocation. The figures below display optimal tax rates over time for different values of β_g. Households' pure rate of social time preference is held constant at 1.5% per year ($\beta_h = 0.985$), consistent with macroeconomic data at the assumed intertemporal elasticity of substitution parameter $\sigma = 1.5$ (Nordhaus, 2008). In contrast, for the planner's discount factor, a range from $\beta_g = 0.985$ (standard or descriptive discounting) to the value of $\beta_g = 0.999$ adopted by the influential Stern Report (Stern, 2006). Figure 1 displays the results for optimal capital income taxes over time:

Optimal Capital Income Tax Across Social Discount Rates
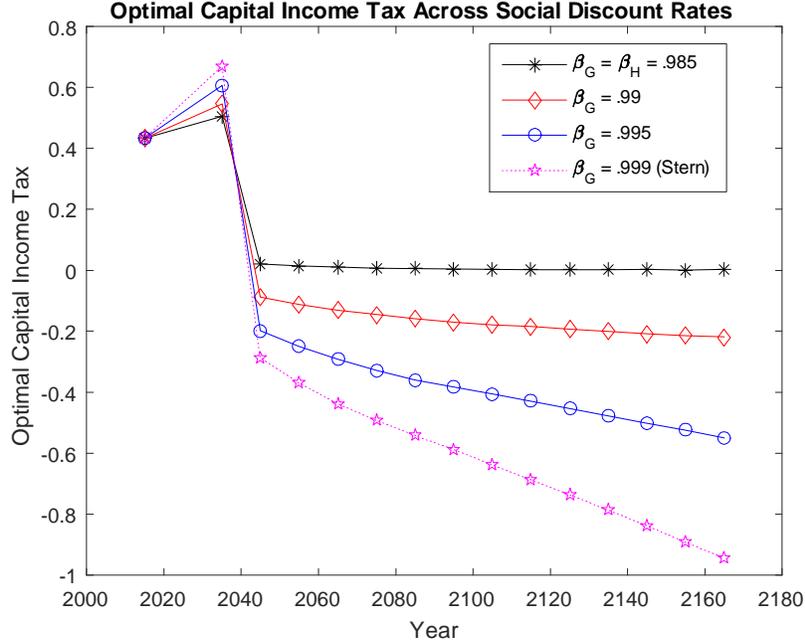
Optimal Capital Income Tax

In line with the theoretical results, the optimal capital income tax with standard discounting $(\beta_g = \beta_h)$ equals zero after the first optimization period. In contrast, with differential discounting, large capital income subsidies become necessary. For the "Stern" pure rate of social time preference $(\beta_g = 0.999)$, a 29% subsidy (or -29% capital income tax) becomes necessary in the third model period, increasing to a 64% subsidy ($\tau_k = -64\%$) by the end of the century. These values undoubtedly represent great departures both from standard recommendations and from current policy practice, thus potentially suggesting some degree of caution in adopting the prescriptive or differential discounting approach.

Figure 2 displays the labor income taxes that can decentralize the optimal allocation for different social vs. private discount factors. Again in line with the theoretical results and the standard prescription from the literature, the optimal labor income tax is constant at $\sim 41\%$ after the first period with equal discounting $(\beta_g = \beta_h)$. In contrast, if the planner decides to discount the future at a lower rate than households $(\beta_g > \beta_h)$, labor income taxes must be decreasing after period $t = 1$. For the social pure rate of social time preference advocated by Stern (2006), this would initially require the imposition of very high labor taxes $\sim 53\%$ (in order to meet the government revenue requirement), declining to $\sim 36\%$ by the end of the 21st Century.

Optimal Labor Income Tax
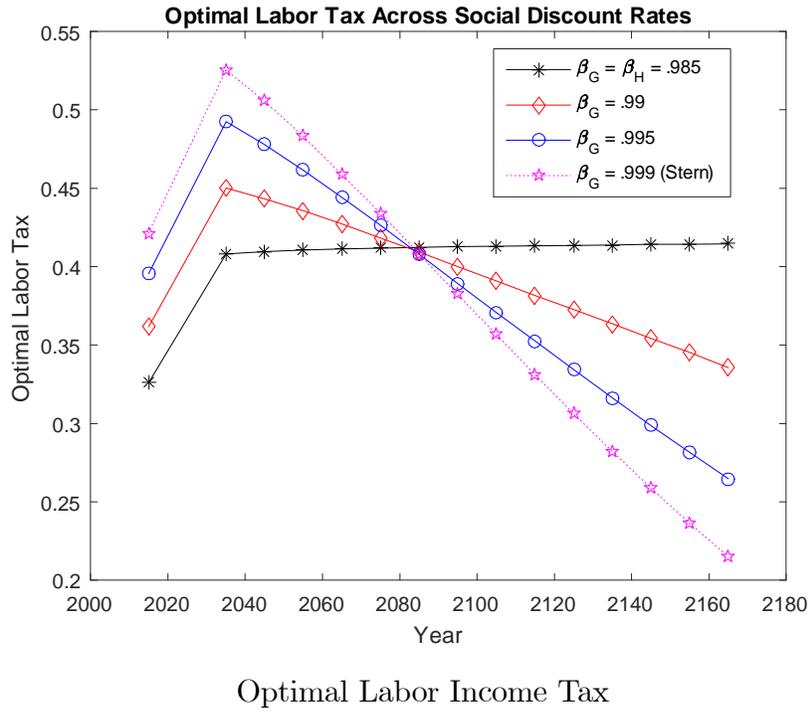
Finally, in order to place these results in the appropriate policy context, Figures 3 and 4 display optimal climate change and carbon pollution taxes across social discount rates.
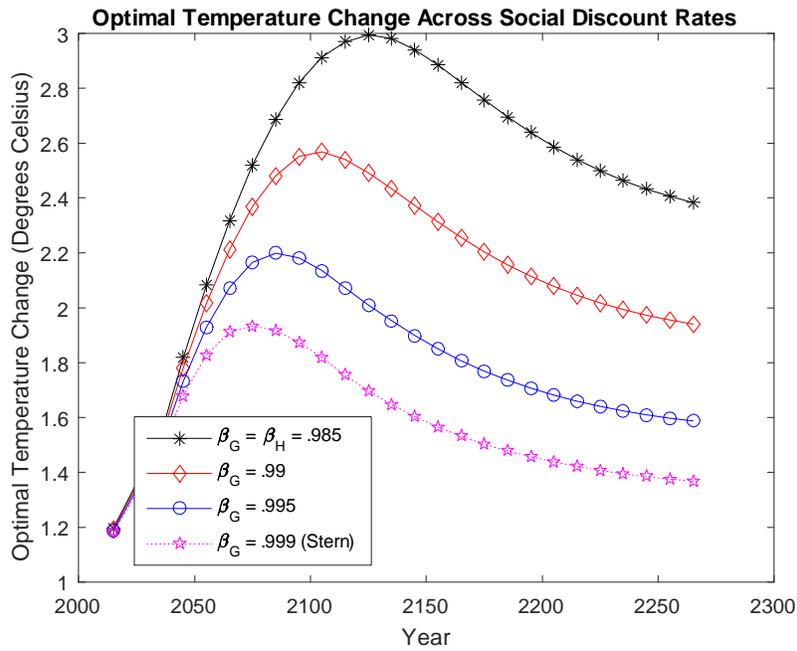
Optimal Temperature Change
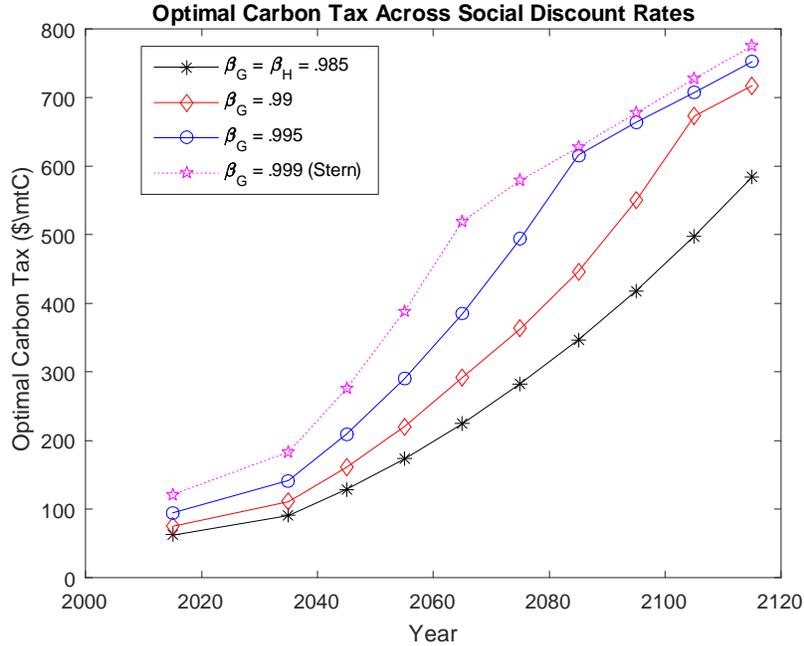
Optimal Carbon Taxes

As has been the focus of the literature, a planner adopting the low social discount rates advocated by authors such as Stern (2006) and Cline (1992) would seek to impose stringent

environmental policies in order to limit global temperature change to below $2°C$ (specifically $1.93C°$). In contrast, if governments adopt a discount factor consistent with household behavior ($\beta_g = \beta_h = 0.985$), the optimal policy limits climate change to around $3°C$, in line with the seminal results of the DICE/RICE model family (Nordhaus, 2008, 2010, etc.). The corresponding optimal carbon tax levels required to decentralize these environmental outcomes range from $62 per metric ton carbon (mtC) in 2015 ($2005) for standard discounting to $121/mtC with "Stern" discounting ($\beta_g = 0.999$). While these climate policy results for differential discounting are thus well-known, the key contribution and finding of this model is thus to demonstrate that, in a richer macroeconomic context, the axiomatic choice to impose a social discount rate based on ethical considerations has dramatic implications for optimal fiscal policy. Both qualitatively and quantitatively, decentralization of the optimal allocation with differential social discounting requires changing income tax policy in a way that runs contrary to both the standard prescriptions from the literature, and to current policy practice (specifically a switch to subsidizing capital income at an ever-increasing rate, and a sharp increase and subsequent decline in labor income taxes over time).

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5 Appendix

5.1 Derivations for Non-Separable Preferences (B)

5.1.1 Optimal Capital Income Taxes

This section derives the optimal capital income tax expression as a function of the wedge ω_t :

$$
\omega_t \equiv \frac{\left[\left(\frac{\beta_G}{\beta_H}\right)^t + \phi[1 - \sigma - \gamma(1 - \sigma)]\right]}{\left[\left(\frac{\beta_G}{\beta_H}\right)^{t+1} + \phi[1 - \sigma - \gamma(1 - \sigma)]\right]}
$$

First, substitute this term into the planner's first order condition for capital (19) to obtain:

$$
\beta_H \left[(1 - \delta) + F_{kt+1}\right] = \frac{U_{ct}}{U_{ct+1}}\omega_t \tag{34}
$$

Next, consider the household's Euler equation for a given capital income tax:

$$
\beta_H \left[1 + (F_{kt+1} - \delta)(1 - \tau_{kt+1})\right] = \frac{U_{ct}}{U_{ct+1}}
$$

Rearranging terms allows one to express the capital income tax that decentralizes a given allocation as:

$$\tau_{kt+1} = 1 - \frac{\frac{U_{ct}}{\beta_H U_{ct+1}}}{(F_{kt+1} - \delta)} + \frac{1}{(F_{kt+1} - \delta)} \tag{35}$$

Finally, rearrange terms in the planner's optimality condition (34) as follows:

$$[1 + F_{kt+1} - \delta]\frac{1}{\omega_t} = \frac{U_{ct}}{\beta_H U_{ct+1}}$$

$$1 - \frac{[1 + F_{kt+1} - \delta]\frac{1}{\omega_t}}{(F_{kt+1} - \delta)} + \frac{1}{(F_{kt+1} - \delta)} = 1 - \frac{(U_{ct}/\beta_H U_{ct+1})}{(F_{kt+1} - \delta)} + \frac{1}{(F_{kt+1} - \delta)} \tag{36}$$

Comparing (35) and (36), we thus see that the capital income tax that decentralizes the optimal allocation at $t+1$ for $t > 0$ is defined by:

$$\tau^*_{kt+1} = 1 - \frac{[1 + F_{kt+1} - \delta]\frac{1}{\omega_t}}{(F_{kt+1} - \delta)} + \frac{1}{(F_{kt+1} - \delta)}$$

$$= \left(\frac{\omega_t - 1}{\omega_t}\right)\frac{(F_{kt+1} - \delta + 1)}{(F_{kt+1} - \delta)}$$

5.1.2   Consistency with Balanced Growth

This section derives the parametric restrictions required to ensure consistency with balanced growth for preferences (B):

$$U(C_t, L_t) = \frac{(C_t L_t^{-\gamma})^{1-\sigma}}{1 - \sigma}$$

Let leisure be denoted $\mathcal{L}_t = 1 - L_t$, and re-write the utility function in terms of leisure as:

$$U(C_t, \mathcal{L}_t) = \frac{(C_t(1 - \mathcal{L}_t)^{-\gamma})^{1-\sigma}}{1 - \sigma} \tag{37}$$

The King-Plosser-Rebelo conditions for (37) to be consistent with a balanced growth path are as follows. Focusing on the case where $\sigma > 1$, the sub-function $v(\mathcal{L}) = (1 - \mathcal{L}_t)^{-\gamma(1-\sigma)}$ must be decreasing and convex.

$$v(\mathcal{L}) = (1 - \mathcal{L}_t)^{-\gamma(1-\sigma)} \tag{38}$$
$$v'(\mathcal{L}) = \gamma(1 - \sigma)(1 - \mathcal{L}_t)^{-\gamma(1-\sigma)-1} \tag{39}$$
$$v''(\mathcal{L}) = = \gamma(1 - \sigma)(\gamma(1 - \sigma) + 1)(1 - \mathcal{L}_t)^{-\gamma(1-\sigma)-2} \tag{40}$$

1) Decreasing:

In order for $v(\mathcal{L}_t)$ to be decreasing, as per (39), what is needed is that:

$$\gamma(1 - \sigma) < 0$$

For $\sigma > 1$, this condition is satisfied as long as $\gamma > 0$.

2) Convex:

Convexity of $v(\mathcal{L}_t)$ requires, based on (40), that:

$$\gamma(1 - \sigma)(\gamma(1 - \sigma) + 1) > 0$$

As we already have that $\gamma(1 - \sigma) < 0$, all that is further required is that:

$$\gamma > \frac{-1}{(1 - \sigma)}$$

3) Overall concavity:

Finally, in order to ensure the overall concavity of the utility function, the leisure preference function must satisfy:

$$(-\sigma)\frac{v''(\mathcal{L})}{v'(\mathcal{L})} > (1 - \sigma)\frac{v'(\mathcal{L})}{v(\mathcal{L})}$$

$$\Rightarrow (-\sigma)v''(\mathcal{L})v(\mathcal{L}) < (1 - \sigma)[v'(\mathcal{L})]^2 \tag{41}$$

Substituting in from (38)-(40), the requirement (41) reduces to:

$$\frac{-\sigma}{(1 - \sigma)} < \gamma \tag{42}$$

It should be noted that this final condition (42) implies that the other conditions for consistency with balanced growth are satisfied as well.

Finally, consider the claim in Proposition 3 (22) that:

$$\phi[1 - \sigma - \gamma(1 - \sigma)] \geq 0$$

Combining the implications of (42) with the observation that the Lagrange multiplier on (IMP), ϕ, is necessarily weakly positive, confirms this claim.

5.2 Derivations for Non-Separable Preferences (B')

5.2.1 Optimal Labor Income Taxes

This section derives an expression for optimal labor income taxes when preferences are of the non-separable form (B'):

$$U(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma}(1-L_t)^{v(1-\sigma)} \tag{B'}$$

First, note that (B') implies that:

$$W_{ct} = U_{ct}\left[1-\sigma+(-v)(1-\sigma)(1-L_t)^{-1}L_t\right] \tag{43}$$

$$W_{lt} = U_{lt}\left[1-\sigma+1+(-1)(v(1-\sigma)-1)(1-L_t)^{-1}L_t\right] \tag{44a}$$

Substituting (43)-(44a) into the planner's leisure-consumption optimality condition,

$$1+\phi\left[\frac{W_{ct}}{U_{ct}}+\frac{W_{lt}}{U_{ct}F_{lt}}\right]\left(\frac{\beta_G^t}{\beta_H^t}\right)^{-1} = \frac{(-U_{lt}/U_{ct})}{F_{lt}}$$

and rearranging terms leads to:

$$\frac{1+\phi\left[1-\sigma+(-v)(1-\sigma)(1-L_t)^{-1}L_t\right]\left(\frac{\beta_G^t}{\beta_H^t}\right)^{-1}}{\left[1+\phi[1-\sigma+(-v)(1-\sigma)(1-L_t)^{-1}L_t+(1-L_t)^{-1}]\left(\frac{\beta_G^t}{\beta_H^t}\right)^{-1}\right]} = \frac{(-U_{lt}/U_{ct})}{F_{lt}}$$

Defining $A(L_t) = (-v)(1-\sigma)(1-L_t)^{-1}L_t$ and $B(L_t) = (1-L_t)^{-1}$ thus yields:

$$\frac{1+\phi[1-\sigma+A(L_t)]\left(\frac{\beta_H^t}{\beta_G^t}\right)}{1+\phi[1-\sigma+A(L_t)+B(L_t)]\left(\frac{\beta_H^t}{\beta_G^t}\right)} = \frac{(-U_{lt}/U_{ct})}{F_{lt}} \tag{45}$$

Finally, note that the household's optimality condition for the consumption-leisure tradeoff

$$(1-\tau_{lt}) = \frac{-U_{lt}/U_{ct}}{F_{lt}}$$

thus implies the desired result that the left-hand-side of (45) defines the net-of-labor tax rate that decentralizes the optimal allocation.