Youth unemployment and welfare gains from eliminating business cycles — the case of Poland

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Abstract

During recessions, unemployment among youth rises much more dramatically than that of other cohorts. We investigate how this life-cycle heterogeneity of unemployment risk affects welfare gains from eliminating business cycles. We use an overlapping generations version of the heterogeneous agents model with aggregate risk and borrowing constraints. The parameters are calibrated to match Polish data, where youth unemployment is highly volatile. We find that the consumption of young cohorts drops due to business cycles at least a few times more than the average for the whole population and that the majority of the decline is associated with higher unemployment risk faced by young agents. We also show that stabilizing labour market fluctuations solely for young cohorts substantially reduces lifetime welfare losses related to business cycles.

Keywords: business cycle, unemployment, overlapping generations, welfare, youth
JEL classification: E24, E32, E61

1 Introduction

Many empirical studies show that recessions are particularly harmful for young people because the unemployment risk for this group rises much more dramatically than for other cohorts. In this paper, we quantify these costs in terms of welfare. More precisely, we study the welfare gains from eliminating business cycles, as well as their distribution across cohorts, while taking into account the life-cycle heterogeneity of unemployment risk.

The problem of the high relative youth unemployment rate as well as its excess sensitivity to business cycles is well documented in the literature (see for example Jimeno and Rodriguez-Palenzuela, 2002; Kawaguchi and Murao, 2012; Hoynes et al., 2012; Bruno et al., 2014). It is particularly severe in Central and Southern Europe. In Poland, for example, during the period 1997–2013, the unemployment rate for the 20–24 age group soared, on average, from 24% in booms to 33% in downturns. At the same time, the rates for the 25–60 group were 9% and 13%, respectively. Similar jumps in youth unemployment have been observed in other countries, such as Spain, Bulgaria, Slovakia and Lithuania. The recent financial crisis has been particularly harmful for young people, not only because of the rapid increase in the unemployment

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rate, which for the whole OECD area rose by 6 percentage points (Scarpetta et al., 2010; Bell and Blachflower, 2011; ILO, 2012), but also because of the high persistence of unemployment. For example, in Greece and Spain, more than 40% of young are still unemployed.

The methodology for calculating welfare gains from eliminating business cycles has been being developed since the seminal contribution of Lucas (1987). His finding that the gains for an average consumer represent less than 0.01% of lifetime consumption has been challenged from various directions. One important strand of the critique argues that the idiosyncratic consumption risk faced by individuals is much higher than the aggregate data used by Lucas suggest (Imrohoroğlu, 1989; Atkeson and Phelan, 1994; Krusell and Smith, 1999; Beaudry and Pages, 2001; Gomes et al., 2001; Storesletten et al., 2001; Mukoyama and Şahin, 2006; Krusell et al., 2009). Some of the cited papers consider not only the gain for an average agent but also study a distribution of the gains across agents. They identify some groups of people, especially the poor (Krusell and Smith, 1999; Krusell et al., 2009), the low-skilled (Mukoyama and Şahin, 2006) and the young (Storesletten et al., 2001), for whom the gains are much higher than the average for all agents.

The gain for the poor stems from their inability to insure themselves against unemployment risk, which rises periodically due to business cycles. The high gain from eliminating business cycles for low-skilled workers results mainly from the much higher unemployment risk they face during recessions compared to skilled workers. However, only Storesletten et al. (2001) analyse the life-cycle distribution of the gains. Nonetheless, they do not account for the life-cycle heterogeneity of idiosyncratic risk. In fact, in their setup, the low wealth level coupled with the precautionary motive related to high uncertainty about lifetime earnings is the logic behind the severity of business cycles for young agents.

In this paper, we develop an overlapping generations version of the heterogeneous agent model used by Krusell and Smith (1999) and particularly by Mukoyama and Şahin (2006) to account for life-cycle heterogeneity in unemployment risk. In our setup, agents differ in terms of wealth, skills, labour market status and age. They are subject to the idiosyncratic labour market risk. The transition probabilities between employment and unemployment depend not only on an agent's skill level and the aggregate shock but also on age. The model is calibrated using data from the Polish economy. In particular, the transition probabilities are set to match the age profiles of average unemployment rates and durations for workers with different skill levels in booms and recessions.

There are two reasons for using data from Poland. First, as noted already, the life-cycle heterogeneity of unemployment risk in Poland is high. However, more importantly, the mean duration of unemployment exceeds one year. This allows us to set up the OLG model at an annual frequency, which considerably facilitates the computations. There is one additional feature that makes our baseline calibration of the model unusual, namely, flat unemployment benefits, which in Poland are generally the same for all workers regardless of their previous earnings. However, we also consider a calibration with proportional unemployment benefits. Thus, apart from the unemployment duration and the benefits, our calibration does not differ much from what is used in the literature. Therefore, we think the insights gained from our study would also apply to other economies with a high volatility of youth unemployment risk.

We consider the welfare gains from two perspectives: a one-period, or momentary, utility of a single cohort and the lifetime welfare of a group of newborn agents. We use the latter perspective as a finite lifetime analogue to the standard measure introduced by Lucas (1987). In our paper, the lifetime gain is defined as a constant percentage increase in consumption of a group of newborn agents in the economy with business cycles needed to equalize the average expected lifetime utility for the agents in the economy with and without business cycles. The momentary gain is calculated in a similar manner, but now we equate the average momentary utilities for certain cohorts. Calculating the welfare gains, we explicitly take into account a transition from an economy with the aggregate risk to the world without it.

To assess the welfare gains from eliminating business cycles, we have to consider a hypothetical economy without business cycles. In particular, one should decide to what extent the idiosyncratic risk is affected by removing the aggregate risk. Due to computational difficulties, we do not apply the integration principle advocated by Krusell and Smith (1999) and Krusell et al. (2009). Instead, we follow Reiter (2012) and consider two possibilities. We assume that the transition probabilities in the economy without business cycles are either set to match the means of unemployment levels and durations for booms and recessions, as in Imrohoroğlu (1989), or simple averages of the respective transition probabilities for the two states of the economy. The latter approach is a natural consequence of the non-linearity of the unemployment rate, as recently shown by studies employing search and matching labour market models (Hairault et al., 2010; Jung and Kuester, 2011; Petrosky-Nadeau and Zhang, 2013; Iliopulos et al., 2014).

Our findings can be summarized as follows. Under mild parametrization, the gains from eliminating business cycles for young cohorts are at least two to four times higher than the average for the whole population. In other words, a decline in consumption caused by business cycles is at least a few times higher for young agents. Additionally, we identify a few other reasonable parametrizations where the differences are even more spectacular. We also show that the life-cycle heterogeneity of unemployment risk, disregarded by previous studies, is the main source of the severity of business cycles for young cohorts. In the model with homogeneous life-cycle risk, the relative gains for young people are two to three times lower. The high volatility of youth unemployment significantly increases the lifetime gains as well. We document that eliminating the business-cycle variation in the labour market risk for cohorts 20–29 exclusively can reduce the lifetime gains by as much as 70%. Due to the flat unemployment benefits in Poland, the gains are generally the highest for high-skilled workers. This, however, is no longer the case for proportional unemployment benefits. The rest of the paper is organized as follows. In section 2, we introduce the model. Then, in section 3, we present the calibration of the parameters. Section 4 discusses the issues related to the measurement of the welfare gains. Numerical results are presented in section 5. Finally, section 6 concludes the paper.

2 Model

We use an overlapping generations version of the heterogeneous agents model of Mukoyama and Şahin (2006). However, we slightly depart from their setup in a few points. First, because we are not interested in matching wealth distributions, we use the same discount factor for all agents. Second, we allow for negative wealth holdings. We hold that the no-debt requirement is an unrealistic assumption in life-cycle frameworks, which, as shown below, considerably increases the welfare gains for young cohorts. Finally, we assume that agents cannot change their skill levels.

2.1 General setup

The economy is populated by a continuum of finitely lived agents who differ in terms of age a, skill level s, employment status ε and wealth k. For simplicity, we omit the time subscripts and use primes to denote subsequent periods. Agents enter the labour market at the age of 21, work for 40 years, and then retire and live, at most, up to 100 years. The life length is stochastic.

Young agents either work ($\varepsilon = e$) or are unemployed ($\varepsilon = u$). If employed, they supply $\xi(s,a)l$ effective units of labour and obtain the net income $(1-\tau)\xi(s,a)lW$, where τ is the tax rate, W stands for the aggregate wage, l is the constant for all agents' nominal labour supply and $\xi(s,a)$ denotes the efficiency factor. The unemployed agents receive unemployment benefits. In the baseline version of the model, we assume that the benefit is proportional to the mean wage in the economy and, therefore, is equal to $\theta_u(1-\tau)\bar{\xi}lW$, where θ_u is the unemployment replacement rate and

$$\bar{\xi} = \iint (1 - \bar{u}(s,a,Z))\xi(s,a)\Gamma_\xi(s,a)dsda \tag{1}$$

denotes the mean labour efficiency across agents where $\Gamma_\xi(s,a)$ is the efficiency density, $\bar{u}(\cdot)$ represents the unemployment rate for a given cohort and Z is aggregate stochastic shock. Retirees receive pensions that are proportional to the wage of an employed agent of age 59 $\theta_r(1-\tau)\xi_{s,59}lW$, where θ_r represents the pension replacement rate. As a result, an agent's work-related income d is given by:

$$d = (1-\tau)lW\left[\xi(s,a)\cdot \mathbb{1}(a<60,\varepsilon=e) + \theta_u\bar{\xi}\cdot \mathbb{1}(a<60,\varepsilon=u) + \theta_r\xi_{s,59}\cdot \mathbb{1}(a\geq 60)\right], \tag{2}$$

where $\mathbb{1}(\cdot)$ stands for the indicator function. Moreover, the agents receive interest R on their capital stock. We allow for the negative wealth level, but debt cannot exceed some prespecified level $\underline{k}(s)$. In the baseline version of the model, we assume that newborn agents start with zero capital stock. For the robustness check, we will also consider more realistic initial capital distributions.

The production sector consists of one representative firm that hires capital and labour from agents and produces a single consumption good according to the standard Cobb-Douglas technology:

$$Y = ZK^\alpha L^{1-\alpha}, \tag{3}$$

where K and L are aggregate capital and aggregate effective labour, respectively:

$$K = \int k_j dj, \quad L = l\bar{\xi}, \tag{4}$$

and k_j represents the capital stock of the j-th agent. Because the firm operates on the competitive market, it sets the aggregate wage and interest rate equal to the marginal products of labour and capital:

$$W = (1-\alpha)ZK^\alpha L^{-\alpha}, \quad R = \alpha ZK^{\alpha-1}L^{1-\alpha}, \tag{5}$$

Every period, an agent faces the standard consumption-saving problem, which can be recursively written as:

$$V(k,a,\varepsilon,s,K,Z) = \max_{c,k'} \left\{ U(c) + \beta q_{a,a+1} \mathbb{E}\left[V(k',a+1,\varepsilon',s,K',Z') \mid \varepsilon, K, Z \right] \right\} \quad (6)$$

$$\text{s.t.} \quad k' = (1-\delta+R)k + d - c, \quad (7)$$

$$k' \geq \underline{k}(s), \quad K' = H(K,Z,Z') \quad (8)$$

where $V(\cdot)$ is an agent's value function, β is a discount coefficient, $q_{a,a+1}$ denotes the one-year surviving probability for an agent of age a and $H(\cdot)$ represents a law of motion for aggregate capital.[1] As the instantaneous utility function, we use the standard CRRA utility:

$$U(c) = \frac{c^{1-\gamma}-1}{1-\gamma}, \quad (9)$$

where γ is the risk aversion coefficient.

There is also a government in the model that imposes taxes on the income from work to finance the unemployment benefits and pensions. The tax rate is set so the government budget is always balanced. The exact derivation of the tax rate is presented in the appendix.

2.2 Stochastic structure of the economy

There are three exogenous stochastic shocks in the model. The aggregate productivity shock Z is represented by a two state Markov chain with the transition matrix P_Z. The states $Z = \{Z_b, Z_g\}$ represent recession and expansion period, respectively. The individual employment shock ε is also modelled as a two state Markov chain with the transition matrix $P_\varepsilon(s,a,Z,Z')$. Here, the transition probabilities depend on the current and future state of the economy as well as an agent's skill level and age. Finally, a lifetime in the model is stochastic. For every cohort a, there is a fraction $1-q_{a,a+1}$ of agents who die.

3 Calibration

One period in the model corresponds to one year. Although some authors also employ the yearly specification (see Storesletten et al., 2001), it is a rather rare choice regarding the cost of business cycles. However, we have at least two reasons for using this approach. First, it reduces the computational complexity of the problem. In the quarterly model, there would be 320 cohorts instead of 80 for the yearly specification. Second, it allows for a more realistic calibration of the labour market transition probabilities. This virtue is the consequence of how we construct the transition matrices. They are built in such a way that for each cohort there are only two different values of the unemployment rate — one for the recession and one for the expansion. When the aggregate state of the economy switches, the unemployment rate immediately jumps to the new level. It is much more realistic to assume that such adjustment is completed within one year than one quarter. Of course, the yearly specification also

[1] In our notation, the value function V depends on aggregate capital K. This is actually a simplification stemming from the Krusell–Smith algorithm used for approximating a solution to the decision problem. More details are given in the technical appendix.

means that the unemployment duration in the model is a multiple of a year, which is far from what we observe in the data.

We divide the parameters into two groups. In the first subsection, we describe the calibration of the structural parameters. Then, we move to the parameters governing the stochastic structure of the model.

3.1 Structural parameters

The baseline calibration of the structural parameters is presented in table 1. The capital share in the production function α is set at 0.45. This implies that the labour share in the model equals 55% — a value that lies in the middle of the average estimates for the years 1996–2013 from the European Commission's AMECO database. In the data, the mean adjusted wage share in GDP at current market prices is 50.9%, whereas the value for GDP measured at the current factor cost equals 58.3%. The capital depreciation rate $\delta = 0.055$ is calibrated to match the average ratio of investment to consumption, which is approximately 26% in Poland. We use a fairly standard value for the discount rate of $\beta = 0.98$, which results in a rather high interest rate of approximately 12% in the model. However, we do not try to lower it by increasing β, because we would end up with a number well above 1, which we find to be rather odd for the value function calculation. In our baseline calibration, we assume the coefficient of relative risk aversion $\gamma = 2$.

Table 1: Baseline calibration of the model

Description	Parameter	Value
capital share in the production function	α	0.45
capital depreciation rate	δ	0.055
discount rate	β	0.98
risk aversion	γ	2
std. dev. of aggregate shock	ΔZ	0.014
unemployment benefit replacement rate	θ_u	0.2
pension replacement rate	θ_r	0.6
relative debt level	ζ	0.75
share of low-, medium- and high-skilled workers	ω	$\{0.15; 0.7; 0.15\}$
individual labour supply	l	1

To pin down the unemployment benefit replacement rate, we use data provided by van Vliet and Caminada (2012). The average net replacement rate in the period 1996–2013 equals 0.3 for a single worker and 0.35 for a one-earner couple with two children. However, these numbers are calculated for a six-month unemployment spell. In Poland, the average unemployment duration exceeds one year, but in most cases, the unemployment benefit is paid only for six months. As a result, we set $\theta_u = 0.2$, which is approximately half the estimates mentioned above. According to the OECD database, the net pension replacement rates in Poland are close to $\theta_r = 0.6$.

The maximum debt level in the model $\underline{k}(s)$ depends on skills. We assume that it is proportional to the average net income from work for agents with a given skill level:

$$\underline{k}(s) = \zeta(1 - \bar{\tau})\xi_s l\bar{W}, \tag{10}$$

where ζ is the relative debt level, and $\bar{\tau}$ and $\bar{W}$ represent the average tax rate and wage, respectively[2]. We set $\zeta = 0.75$. As a result, approximately 4.5% of agents, primarily the youngest and oldest, in the model hold negative wealth. We do not have data on the fraction of agents with negative wealth in Poland, but compared to other countries (OECD, 2008) this is a rather low number.

Regarding the skill level, we have three groups of agents in the model: low-, medium- and high-skilled, broadly corresponding to people with a primary, secondary or tertiary education level, respectively. Their average shares in the Polish population equal 15%, 70% and 15%, respectively. The individual labour supply l is set to 1. It is a redundant parameter and has no effect on the results.

Table 2: Calibration of the individual labour efficiency factors

	Skill level				Age	
	Low	Medium	High		20–29	30–59
ξ_s	0.8	1	1.65	ξ_a	0.8	1

The individual efficiency factors $\xi(s, a)$ shown in table 2 are calibrated to match the observed differences in wages according to the Structure of Earnings Survey conducted by Eurostat in 2011. The data show that the earnings of low-skilled (primary-educated) workers are approximately 20% lower compared to medium-skilled (secondary-educated) workers. The earnings for high-skilled (tertiary-educated) workers are approximately 65% higher. Similarly, it is documented that young people (20–29 years) in Poland earn approximately 20% lower wages than the rest. The differences for other age groups are small, so we neglect them.

3.2 Parameters of the stochastic structure

To calibrate the Markov chain for the aggregate productivity, we calculate log-deviations from the HP-filtered trend of yearly GDP covering the period 1995–2013. We consider artificial GDP data that consist of consumption and investment series only. As a result, we assume symmetric business cycles where each phase lasts 3.5 years on average with productivity shock values of $Z_b = 0.986$ and $Z_g = 1.014$. The aggregate productivity transition matrix equals:

$$P_Z = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}. \tag{11}$$

The transition probabilities P_ε are pinned down to match the average level and duration of unemployment in Poland during booms and recessions for workers of different skill levels and age. We use the yearly data from the Labour Force Survey for the years 1997–2013. Because the data on the mean level of unemployment are grouped into five-year bins, the values for each year are linearly interpolated. The

[2] In fact, we do not use the exact average wage, as we have to set the debt limit $\underline{k}(s)$ at the beginning of the computations. Therefore, after some initial simulations, we make a guess on the aggregate capital stock K and use formula (5) to calculate $\bar{W}$. However, we do not have to employ the similar procedure for pinning down $\bar{\tau}$, as it solely depends on the labour supply, which is completely exogenous in the model.

data also show that the unemployment duration in Poland is virtually equal for all education levels, and they distinguish only two age groups: 20–39 and 40–59. A similar assumption on the constant unemployment duration across skill levels is used, for example, by Mukoyama and Şahin (2006).

Table 3: Labour market characteristics in Poland

Skills	Phase	Age							
		20–24	25–29	30–34	35–39	40–44	45–49	50–54	55-59
A. Unemployment rates									
Low	Rec.	46.1	37.3	29.1	26.5	23.5	22.2	19.8	13.5
	Boom	35.3	24.9	22.6	18.9	17.8	15.4	12.1	9.5
Med.	Rec.	32.4	18.5	14.4	13.2	12.8	12.5	12.0	11.1
	Boom	23.0	12.9	10.5	9.1	8.6	8.3	7.9	8.1
High	Rec.	25.9	11.3	4.3	2.8	2.4	2.7	3.4	4.2
	Boom	19.2	8.1	2.8	2.2	1.6	2.1	2.8	3.2
B. Unemployment durations (in months)									
All	Rec.	15.1	15.1	15.1	15.1	19.8	19.8	19.8	19.8
	Boom	13.0	13.0	13.0	13.0	17.4	17.4	17.4	17.4

The labour market transition probabilities are calculated in the following way. Let $\bar{u}(s,a,Z)$ denote the mean unemployment level for a cohort a with the skill level s and the aggregate state Z calculated from the data, and let $u_l(a,Z)$ be the analogous average unemployment duration. To match the unemployment duration, the probability of finding a job $p_{\varepsilon,ue}$ is a reciprocal of the unemployment duration. If there is a switch between the aggregate states of the economy, we set this probability as a reciprocal of the mean value of the unemployment duration for boom and recession:

$$p_{\varepsilon,ue}(s,a,Z,Z') = \begin{cases} u_l^{-1}(a,Z) & \text{if } Z = Z' \\ \left[0.5\left(u_l(a,Z) + u_l(a,Z')\right)\right]^{-1} & \text{if } Z \neq Z' \end{cases} \tag{12}$$

To facilitate the computations, we assume that the aggregate unemployment level can take only two values that depend on the state of the economy. To ensure that this requirement is satisfied and that the unemployment rates in the model always match the data, we set the probability of losing a job as follows:

$$p_{\varepsilon,eu}(s,a,Z,Z') = \frac{\bar{u}(s,a+1,Z') - \bar{u}(s,a,Z)p_{\varepsilon,ue}(s,a,Z,Z')}{1-\bar{u}(s,a,Z)} \tag{13}$$

Formula (13) guarantees that if the current unemployment rate equals $\bar{u}(s,a,Z)$, then, in the next period, it switches to $\bar{u}(s,a+1,Z')$.

Finally, the survival rates $q(a,a+1)$ are taken from Polish unisex lifetables from 2012.

4 Calculating the welfare gain from eliminating business cycles

To calculate the welfare gain from eliminating business cycles, we generally follow the definition proposed by Lucas (1987), with some minor modifications (see Storesletten et al., 2001). The proposal is based on a comparison of the value functions in two economies: with and without aggregate fluctuations. To approximate the value functions, we employ the standard approximate aggregation algorithm proposed by Krusell and Smith (1998). The algorithm has already been used for solving overlapping generations models by Storesletten et al. (2001) and Heer and Maussner (2009), to name but a few. Details of the computations are provided in the appendix.

4.1 Definition of the welfare gain

In his seminal paper, Lucas (1987) defined the welfare gain as a percentage increase of consumption in the economy, with business cycles needed to achieve the same utility as in an economy without business cycles. We use a similar concept with slight adjustments to the finite lifetime environment.

We define the lifetime welfare gain from eliminating business cycles as a percentage compensation in lifetime consumption of *an average newborn agent* in the economy, with aggregate fluctuations needed to achieve the same lifetime utility as the mean lifetime utility of an average newborn agent during the transition period from the economy with aggregate fluctuations to the economy without them. The transition begins in the first period, when the aggregate shock disappears, and lasts until the aggregates reach a new steady state.

Therefore, we first calculate the gains λ_t for every period $t \in [1, T]$ of the transition. In what follows, we still omit the time subscripts for most variables. We use them only in a few cases to emphasize that the gain varies during the transition. Let $\Gamma(k, a, \varepsilon, s \mid K, Z)$ denotes the conditional density of an agent's characteristics given K and Z, and let $\Gamma_{K,Z}(K, Z)$ be the unconditional density of the characteristics of the economy with aggregate fluctuations aggregate fluctuations. Moreover, $V_t(k, a, \varepsilon, s, H_t(K); \lambda_t)$ and $\Gamma_t(k, a, \varepsilon, s \mid H_t(K))$ are the value function and the density of the individual characteristics in period t of the transition, respectively, and $K_t = H_t(K)$ represents the aggregate wealth in period t given K at the beginning of the transition. Then, the gain solves the following equation:

$$\mathbb{E}V(k, 20, \varepsilon, s, K, Z) = \mathbb{E}V_t(k, 20, \varepsilon, s, H_t(K); \lambda_t), \tag{14}$$

where:

$$\mathbb{E}V(k,20,\varepsilon,s,K,Z) = \int \cdots \int V(k,20,\varepsilon,s,K,Z)\Gamma(k,20,\varepsilon,s \mid K,Z)\Gamma_{K,Z}(K,Z)dk\,d\varepsilon\,ds\,dK\,dZ, \tag{15}$$

$$\mathbb{E}V_t(k,20,\varepsilon,s,H_t(K);\lambda_t) = \int \cdots \int V_t(k,20,\varepsilon,s,H_t(K);\lambda_t)\Gamma_t(k,20,\varepsilon,s \mid H_t(K))\Gamma_{K,Z}(K,Z)dk\,d\varepsilon\,ds\,dK\,dZ, \tag{16}$$

$$V_t(k,a,\varepsilon,s,H_t(K);\lambda_t) = \max_{c,k'} \left\{U\left((1+\lambda_t)c\right) + \beta q_{a,a+1}\mathbb{E}\left[V_t(k',a+1,\varepsilon',s,H_{t+1}(K);\lambda_t) \mid \varepsilon\right]\right\} \quad \text{s.t. (7)–(8).} \tag{17}$$

Given the CRRA utility function (9), the gain can be explicitly calculated as:

$$\lambda_t = \left(\frac{\mathbb{E}V_t(k,20,\varepsilon,s,H_t(K);0)}{\mathbb{E}V(k,20,\varepsilon,s,K,Z)}\right)^{\frac{1}{1-\gamma}} - 1. \tag{18}$$

We assume that the gain is held equal across all agents of the studied cohort. The gain for the whole transition period λ, which is the main measure in our paper, is simply calculated as the average across the λ_t:

$$\lambda = \frac{1}{T}\sum_{t=1}^{T}\lambda_t. \tag{19}$$

In the next section we also report values at the beginning λ_1 and at the end λ_T of the transition to assess the general equilibrium effect for the welfare gains.

Evaluating the expectations of the value functions, we integrate over individual capital stock, labour market status, skill level and aggregate characteristics. This is also the case for the value function on the transition path, as the aggregate capital K_t depends on the initial aggregate wealth before eventually converging to some steady state value. We do not integrate over age because we compare the value functions only for newborn agents. We also consider the welfare gains for agents with different skill levels. Consequently, we do not integrate them over the skill level.

Our definition of the welfare gain from eliminating business cycles resembles the proposal of Storesletten et al. (2001). They also consider the average gain on the transition path. However, there is at least one important difference. The cited authors calculate the gains for every cohort and then average them. We report only the gain for newborn agents, as we find this measure closer to the lifetime gain usually used in infinite life frameworks. In other words, we focus on the individual's lifetime gain instead of the average economy-wide gain.

We also calculate momentary welfare gains for cohorts, where we simply compare average instantaneous utilities for every cohort. Thus, we apply formulas (18) and (19), but we replace the value functions with instantaneous utilities for particular cohorts.

4.2 Economy without business cycles

For the hypothetical stabilized economy, we have to distinguish the effects of eliminating business cycles on the aggregate and idiosyncratic risk.

For the aggregate risk, the problem is rather straightforward. Eliminating business cycles means shutting off the aggregate productivity shock. Thus, in the economy without business cycles, we set $Z = 1$.

For idiosyncratic risk, the problem is less obvious. A wide range of proposals has been discussed in the literature (see Krusell and Smith, 1999, for a more detailed discussion). In the paper, we consider two schemes: direct stabilization of unemployment rates (Imrohoroğlu, 1989; Reiter, 2012) and direct stabilization of labour market transition probabilities (Hairault et al., 2010; Jung and Kuester, 2011; Petrosky-Nadeau and Zhang, 2013; Iliopulos et al., 2014).

The stabilization of unemployment rates simply postulates that unemployment rates as well as their durations in the economy without business cycles are constant and are set as averages of the corresponding characteristics across booms and recessions. To apply this method in our paper, we calculate the mean unemployment rates and durations for different cohorts and skill-level groups and then build the two-state transition matrices using formulas (12) and (13).

According to the second method, the transition matrices in the economy without business cycles equal the weighted averages of the corresponding matrices for different phases of the business cycles. Therefore, we revert the ordering of the operations compared to the previous approach. First, we average the transition matrices. From the matrices, we can then infer the stabilized unemployment rates and durations, which are usually slightly lower than in the previous case. As a result, the welfare gain from eliminating business cycles is usually higher in this setup. The discussed method is firmly grounded in the search and matching theory of the labour market. Indeed, in a study using a standard search and matching business cycle model, Hairault et al. (2010) showed that the transition probabilities are levelled off by eliminating the aggregate risk levels.

At this point, we should also mention the third popular approach — the integration principle. Here, the aggregate shock is integrated out of the risk process faced by individuals. However, for our model, the procedure is computationally cumbersome. This is mainly because unemployment rates in the stabilized economy are not immediately constant but need some time to converge to a steady state. Therefore, one has to add unemployment rates as the new state variable, which makes the model far more computationally complex. For this reason, we do not use this approach in the paper.

5 Results

5.1 Baseline calibration

5.1.1 Lifetime gains

In table 4 we present the lifetime gains for our baseline calibration. As mentioned earlier, we always consider two stabilization schemes: the stabilized transition probabilities and the stabilized unemployment rates and durations. For each scheme, we separately report the gains for all agents as well as agents with different skill levels.

In the first row, we present λ — the average gains for all transition periods. First, one can spot the considerable differences between the stabilization schemes. Under stabilized probabilities, the gain for all agents equals 0.13% of the lifetime consumption. On the other hand, for the stabilized unemployment scheme, the gain

Table 4: Lifetime gains for the baseline calibration (in %)

	Stabilized probabilities				Stabilized unemployment			
	All	Low	Med.	High	All	Low	Med.	High
λ	0.127	0.094	0.135	0.136	0.010	-0.006	0.011	0.033
λ_1	0.127	0.094	0.135	0.136	0.022	0.007	0.024	0.045
λ_T	0.120	0.087	0.128	0.129	0.000	-0.015	0.002	0.024
λ_{un}	0.136	0.102	0.143	0.151	0.004	-0.008	0.005	0.023
λ_{em}	0.117	0.091	0.125	0.116	0.008	-0.001	0.009	0.023

λ — average gain; λ_1 — gain at the beginning of the transition; λ_T — gain at the end of the transition; λ_{un} — average gain for an unemployed agent; λ_{em} — average gain for an employed agent; Low, Med., High refer to the skill levels

is only 0.01%, which means it is approximately one order of magnitude smaller. In both cases, there are also differences in skill levels, especially between low- and high-skilled agents (0.09% to 0.14% for the stabilized probabilities policy and -0.01% to 0.03% for the stabilized unemployment scheme).

The next two rows, containing the gains at the beginning (λ_1) and the end (λ_T) of the transition period, illustrate the general equilibrium effect. The effect is relatively negligible for the stabilized probabilities case[3] but is noticeable for the stabilized unemployment scheme. Nonetheless, in both cases, the gains at the end of the transition are always lower than at the beginning. In the economy with business cycles, the aggregate capital stock is higher compared to the stabilized environment, as are wages. Because the labour-related income for an average agent exceeds the interest, the gains are partially offset by the increase in wages.

Finally, the last two rows show the gains for unemployed and employed newborn agents. Interestingly, the relative position of these two groups is different in the two stabilization schemes. Under the stabilization probability, the gains are higher for agents who enter the labour market as unemployed, whereas for stabilized unemployment, the opposite is true. To interpret these results, one has to keep in mind that in calculating the gains, we compare agents of the same type. Thus, the results show that newborn unemployed agents benefit more from stabilizing the transition probabilities compared to employed agents, whereas the unemployment stabilization policy favours agents who enter the labour market as employed.

5.1.2 Momentary gains

Figure 1 shows the age profiles of the average momentary gains for the transition period. In all cases, the gains are definitely the highest for the youngest agents. They rapidly decrease with age, reaching the minima for 35–50-year-olds. Then, for the older cohorts, the gains either remain low or grow slightly. For the youngest cohorts, we also observe that the gains are highest for high-skilled agents, which mirrors the results for the lifetime gains. For older cohorts, these differences are definitely less pronounced.

[3]Seemingly, the same results for λ and λ_1 for the stabilized probabilities scheme stem from the fact that during the transition, the gain initially grows and then decreases. As a result, the mean for the transition is incidentally close to the gain at the beginning of the transition.

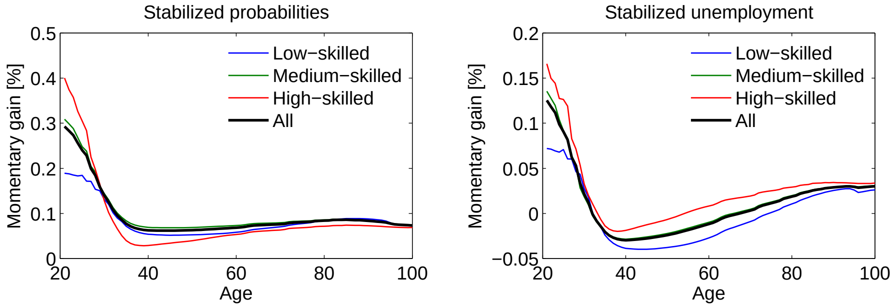

Figure 1: Momentary gains for the baseline calibration

To better capture the disparities in the momentary gains between cohorts, we calculate the average momentary gains for the key cohort bins and relate them to the average momentary gains for all generations, where in all cases, the cohorts sizes are used as weights. These results for age bins 20–24, 25–29 and 90–99 are given in table 5. The relative gains are in parentheses.

Table 5: Momentary gains for the baseline calibration

Group	Stabilized probabilities				Stabilized unemployment			
	All	Low	Med.	High	All	Low	Med.	High
$\bar{\lambda}_{20\text{-}99}$	0.100	0.084	0.105	0.091	0.006	-0.006	0.008	0.023
$\bar{\lambda}_{20\text{-}24}$	0.269	0.186	0.282	0.353	0.109	0.070	0.116	0.143
	(2.7)	(2.2)	(2.7)	(3.9)	(17.4)	–	(15.0)	(6.3)
$\bar{\lambda}_{25\text{-}29}$	0.182	0.156	0.188	0.200	0.051	0.048	0.049	0.072
	(1.8)	(1.8)	(1.8)	(2.2)	(8.0)	–	(6.3)	(3.2)
$\bar{\lambda}_{90\text{-}99}$	0.080	0.082	0.080	0.071	0.030	0.026	0.030	0.034
	(0.8)	(1.0)	(0.8)	(0.8)	(4.7)	–	(3.9)	(1.5)

Low, Med., High refer to the skill levels; In parentheses the relative gains $\bar{\lambda}_i/\bar{\lambda}_{20\text{-}99}$ are given

The first row reports the average momentary gains for all cohorts. As expected, they are generally close to the average lifetime gains λ for newborn agents. Looking at the relative measures, we can see that the gains for the young cohorts are approximately two to four times higher than the average under the stabilized probabilities scheme and three to 17 times higher for the stabilized unemployment case. The latter result is mainly due to the low average gains for all cohorts, as, in absolute terms, the gains for the young cohorts under the stabilized unemployment are still lower than under the alternative countercyclical policy. The relative gains for the oldest agents are either close to 1 (stabilized probabilities) or between 1.5 and 4 (stabilized unemployment).

5.1.3 General equilibrium effect

Finally, we also investigate the general equilibrium effect for the momentary gains, which is depicted in figure 2. The upper graphs show the age profiles of the gains at the beginning of the transition, whereas the lower graphs show the last period of the transition. Under both stabilization schemes, the general equilibrium effect lowers the gains considerably for mid-age cohorts and increases them for older agents. The former impact is especially noticeable under the stabilization unemployment scheme, where the gains for 40-year-olds drop from 0.02% at the beginning of the transition to -0.05% at the end. On the other hand, the latter effect dominates in the stabilized probabilities case, where the gains triple for agents in the last age decile.

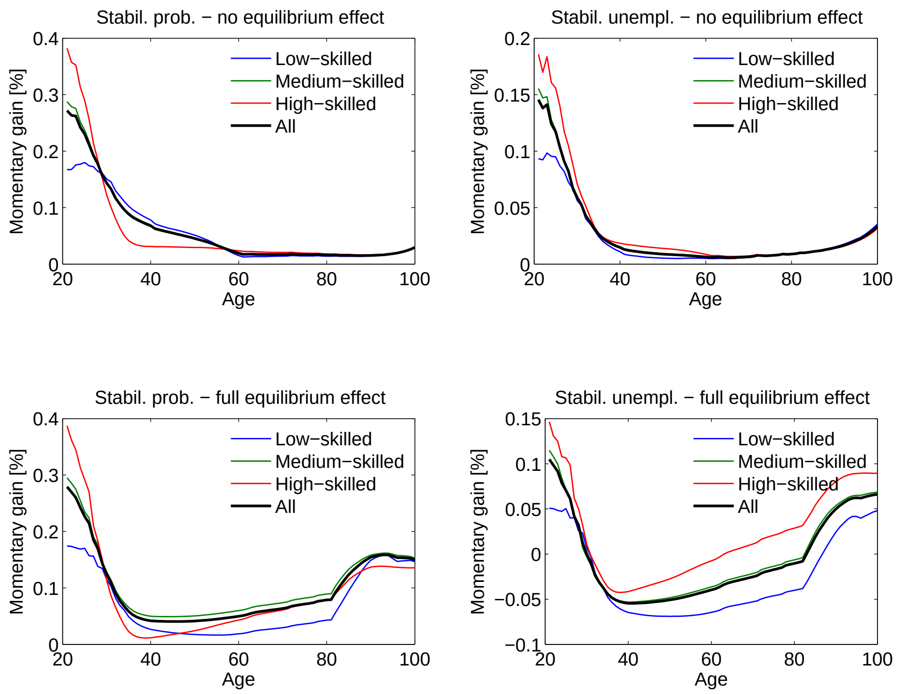

Figure 2: Momentary gains for the baseline calibration with and without the general equilibrium effect

5.2 Alternative calibrations

In this subsection, we analyse how changes in the key parameters of the model affect the results. Subsequently, we consider the models using a higher risk aversion coefficient $\gamma = 4$, no debts and newborn agents who enter the labour market with some inherited wealth. The results for these parametrizations are given in table 6.

5.2.1 Higher risk aversion

It is well known that rising risk aversion increases the lifetime welfare gains from eliminating business cycles. This is also the case for our model, where the lifetime

Table 6: Gains for the alternative calibrations

Group	Stabilized probabilities				Stabilized unemployment			
	All	Low	Med.	High	All	Low	Med.	High
	A: Higher risk aversion $\gamma = 4$							
λ	0.273	0.199	0.302	0.503	0.145	0.114	0.155	0.288
$\bar{\lambda}_{20\text{-}99}$	0.098	0.074	0.109	0.112	-0.003	-0.020	0.005	0.039
$\bar{\lambda}_{20\text{-}24}$	0.479	0.342	0.546	0.817	0.289	0.228	0.308	0.469
	(4.9)	(4.7)	(4.8)	(7.3)	–	–	(66.7)	(11.9)
$\bar{\lambda}_{25\text{-}29}$	0.378	0.293	0.396	0.613	0.208	0.186	0.213	0.356
	(3.8)	(4.0)	(3.6)	(5.5)	–	–	(46.1)	(9.0)
$\bar{\lambda}_{90\text{-}99}$	0.169	0.141	0.183	0.189	0.054	0.023	0.069	0.113
	(1.7)	(1.9)	(1.7)	(1.7)	–	–	(15.0)	(2.9)
	B: No debt allowed							
λ	0.145	0.105	0.153	0.168	0.024	0.005	0.026	0.051
$\bar{\lambda}_{20\text{-}99}$	0.100	0.074	0.108	0.103	0.011	-0.005	0.014	0.032
$\bar{\lambda}_{20\text{-}24}$	0.351	0.283	0.359	0.437	0.156	0.145	0.157	0.170
	(3.5)	(3.8)	(3.3)	(4.2)	(13.8)	–	(11.6)	(5.2)
$\bar{\lambda}_{25\text{-}29}$	0.233	0.213	0.235	0.257	0.097	0.109	0.091	0.118
	(2.3)	(2.9)	(2.2)	(2.5)	(8.6)	–	(6.7)	(3.7)
$\bar{\lambda}_{90\text{-}99}$	0.085	0.074	0.089	0.085	0.026	0.023	0.026	0.030
	(0.9)	(1.0)	(0.8)	(0.8)	(2.3)	–	(2.0)	(0.9)
	C. Newborn agents start with inherited wealth							
λ	0.108	0.094	0.113	0.099	-0.001	-0.018	0.001	0.021
$\bar{\lambda}_{20\text{-}99}$	0.084	0.088	0.085	0.068	-0.003	-0.021	0.000	0.019
$\bar{\lambda}_{20\text{-}24}$	0.188	0.155	0.196	0.203	0.047	0.028	0.050	0.064
	(2.2)	(1.8)	(2.3)	(3.0)	–	–	–	(3.4)
$\bar{\lambda}_{25\text{-}29}$	0.165	0.164	0.167	0.151	0.039	0.041	0.036	0.052
	(2.0)	(1.9)	(2.0)	(2.2)	–	–	–	(2.7)
$\bar{\lambda}_{90\text{-}99}$	0.028	0.028	0.028	0.029	0.007	-0.000	0.008	0.019
	(0.3)	(0.3)	(0.3)	(0.4)	–	–	–	(1.0)

Low, Med., High refer to the skill levels; In parentheses the relative gains $\bar{\lambda}_i/\bar{\lambda}_{20\text{-}99}$ are given

gains soar from 0.13% to 0.27% for the stabilized probabilities policy and from 0.01% to 0.15% for the alternative scheme.

We also show that disparities amongst cohorts rise as well. For the stabilized probabilities scheme, the relative momentary gains in most cases are more than doubled compared to the baseline calibration. The differences become even more striking for the unemployment stabilization policy. For example, the average gain for all agents regardless of skill level is negative, which means that, for most of their life, agents are better off in the economy with business cycles than in the stabilized environment. This, however, is not the case for the youngest and oldest generations, for which the gains in absolute terms are only approximately half the size of the gains under the stabilization probabilities scheme. In other words, the costs of business cycles are borne almost exclusively near the beginning and near the end of life.

5.2.2 No debt allowed

Not allowing agents to incur debts, which translates into setting $\zeta = 0$, has a negligible impact on the lifetime gains under the stabilized probabilities scheme but a moderate impact for the alternative policy, for which the average lifetime gains are doubled. On the other hand, the relative momentary gains remain virtually unchanged for the stabilized unemployment policy but rise considerably for the alternative scheme. Our results prove that the no-debt assumption may lead to underestimating the gains from eliminating business cycles from both lifetime and youth momentary perspectives.

5.2.3 Agents enter the labour market with inherited wealth

In this exercise, newborn agents start with exactly the same wealth as agents who die in a previous period. However, we continue to assume that leaving bequests provides no utility for testators. As a result, a vast majority of agents enter the labour market with some positive wealth, which helps protect the agents against unemployment risk during the initial periods of their job market careers. As expected, both the lifetime gains and the relative momentary gains for young and old cohorts decline, as shown in panel C. Nonetheless, the young are still two to three times worse off than the whole population due to business cycles.

5.3 Homogeneous life-cycle unemployment risk

To assess the impact of the life-cycle heterogeneity of unemployment risk, we study the model where the risk is constant across cohorts. More precisely, we calibrate the labour market transition probabilities to match the averages of the unemployment levels and the durations for all cohorts. The results of the exercise are presented in table 7.

Table 7: Gains under homogeneous life-cycle unemployment risk

Group	Stabilized probabilities				Stabilized unemployment			
	All	Low	Med.	High	All	Low	Med.	High
λ	0.129	0.124	0.133	0.103	0.008	-0.005	0.010	0.021
$\bar{\lambda}_{20\text{-}99}$	0.107	0.104	0.111	0.088	0.008	-0.003	0.009	0.020
$\bar{\lambda}_{20\text{-}24}$	0.195	0.177	0.204	0.167	0.055	0.042	0.058	0.056
	(1.8)	(1.7)	(1.8)	(1.9)	(7.0)	–	(6.2)	(2.8)
$\bar{\lambda}_{25\text{-}29}$	0.163	0.157	0.168	0.131	0.035	0.024	0.038	0.042
	(1.5)	(1.5)	(1.5)	(1.5)	(4.5)	–	(4.0)	(2.1)
$\bar{\lambda}_{90\text{-}99}$	0.071	0.070	0.071	0.065	0.016	0.013	0.017	0.020
	(0.7)	(0.7)	(0.6)	(0.7)	(2.1)	–	(1.8)	(1.0)

Low, Med., High refer to the skill levels; In parentheses the relative gains $\bar{\lambda}_i/\bar{\lambda}_{20\text{-}99}$ are given

The results for the lifetime gains do not change much (from 0.127% to 0.129% for the stabilized probabilities and from 0.01% to 0.008% for the stabilized unemployment). Nonetheless, it can be noted that high-skilled agents would no longer obtain the highest gains from the stabilized probabilities policy. However, the greatest differences are observed for the relative momentary gains. When the life-cycle heterogeneity is removed, the gains for the youngest cohorts are still higher than the average momentary gains for all generations by 50-90% under the stabilized probabilities scheme and by 110-500% for the alternative policy, excluding low-skilled agents. These are the pure effects of the low wealth level and precautionary motives regarding high uncertainty about lifetime earnings. However, when we account for the life-cycle heterogeneity, these numbers rise to 80-290% and 200-1900%, respectively. Thus, the gains double or even triple in some cases. Also the relative gains for the oldest cohorts grow with the heterogeneous life-cycle risk, particularly under the stabilized unemployment scheme.

Table 8: Gains with stabilized business cycles in labour market for young cohorts

Group	Stabilized probabilities				Stabilized unemployment			
	All	Low	Med.	High	All	Low	Med.	High
A: Business cycles on labour market stabilized for cohorts 20–24								
λ	0.100	0.093	0.104	0.081	0.005	-0.002	0.006	0.014
$\bar{\lambda}_{20\text{-}99}$	0.084	0.082	0.087	0.063	0.004	-0.004	0.005	0.013
$\bar{\lambda}_{20\text{-}24}$	0.142	0.137	0.144	0.133	0.040	0.040	0.040	0.038
	(1.7)	(1.7)	(1.7)	(2.1)	(11.4)	–	(8.7)	(2.9)
$\bar{\lambda}_{25\text{-}29}$	0.154	0.143	0.156	0.159	0.043	0.043	0.042	0.058
	(1.8)	(1.8)	(1.8)	(2.5)	(12.4)	–	(9.1)	(4.5)
B: Business cycles on labour market stabilized for cohorts 20–29								
λ	0.075	0.084	0.075	0.048	0.005	0.002	0.005	0.009
$\bar{\lambda}_{20\text{-}99}$	0.057	0.065	0.057	0.035	0.004	-0.001	0.004	0.010
$\bar{\lambda}_{20\text{-}24}$	0.067	0.083	0.066	0.038	0.017	0.020	0.017	0.014
	(1.2)	(1.3)	(1.2)	(1.1)	(4.9)	–	(4.1)	(1.4)
$\bar{\lambda}_{25\text{-}29}$	0.088	0.100	0.088	0.058	0.019	0.021	0.019	0.018
	(1.6)	(1.5)	(1.6)	(1.7)	(5.4)	–	(4.6)	(1.8)

Low, Med., High refer to the skill levels; In parentheses the relative gains $\bar{\lambda}_i/\bar{\lambda}_{20\text{-}99}$ are given

To better capture the impact of youth unemployment risk, we conduct another exercise in which we stabilize unemployment risk for young agents only and calculate the welfare gains from removing the remaining business cycle risk. The results are presented in table 8. We stabilize labour market fluctuations for the 20–24 and 20–29 cohorts in the first and second panels, respectively. We observe that the average lifetime gains drop substantially: in panel A, they decrease from 0.13% to 0.1% under the stabilized probabilities policy and from 0.01% to 0.005% under the stabilized unemployment scheme. For high-skilled agents, the effect is even more striking, as the lifetime gains are cut by 40% and 60%, respectively. Stabilizing labour market business cycles for another five-year bin reduces the gains further in quite similar proportions.

5.4 Proportional unemployment benefits

In this subsection, we study the consequences of replacing the flat unemployment benefits with the proportional benefits. Now, we assume that the benefit is related to the average labour income for a given skill level group with the same replacement rate as in the baseline calibration.

Table 9: Gains with proportional unemployment benefits

Group	Stabilized probabilities				Stabilized unemployment			
	All	Low	Med.	High	All	Low	Med.	High
	A. Baseline calibration							
λ	0.142	0.132	0.148	0.113	0.018	0.019	0.017	0.020
$\bar{\lambda}_{20\text{-}99}$	0.100	0.084	0.107	0.083	0.002	-0.011	0.004	0.017
$\bar{\lambda}_{20\text{-}24}$	0.344	0.359	0.347	0.276	0.155	0.192	0.151	0.102
	(3.4)	(4.3)	(3.3)	(3.3)	(76.8)	–	(37.1)	(6.2)
$\bar{\lambda}_{25\text{-}29}$	0.210	0.245	0.207	0.147	0.067	0.115	0.057	0.037
	(2.1)	(2.9)	(1.9)	(1.8)	(33.3)	–	(13.9)	(2.3)
$\bar{\lambda}_{90\text{-}99}$	0.095	0.080	0.100	0.095	0.032	0.014	0.035	0.050
	(1.0)	(0.9)	(0.9)	(1.1)	(15.8)	–	(8.6)	(3.0)
	B. Homogeneous life-cycle unemployment risk							
λ	0.139	0.154	0.140	0.092	0.014	0.010	0.015	0.018
$\bar{\lambda}_{20\text{-}99}$	0.111	0.118	0.113	0.082	0.010	0.004	0.011	0.018
$\bar{\lambda}_{20\text{-}24}$	0.231	0.264	0.232	0.138	0.075	0.089	0.075	0.043
	(2.1)	(2.2)	(2.1)	(1.7)	(7.3)	(23.4)	(6.7)	(2.3)
$\bar{\lambda}_{25\text{-}29}$	0.183	0.215	0.183	0.108	0.047	0.057	0.046	0.030
	(1.6)	(1.8)	(1.6)	(1.3)	(4.6)	(15.2)	(4.1)	(1.6)
$\bar{\lambda}_{90\text{-}99}$	0.080	0.075	0.082	0.075	0.016	0.012	0.017	0.020
	(0.7)	(0.6)	(0.7)	(0.9)	(1.5)	(3.2)	(1.5)	(1.1)

Low, Med., High refer to the skill levels; In parentheses the relative gains $\bar{\lambda}_i/\bar{\lambda}_{20\text{-}99}$ are given

The resulting average lifetime gains are only slightly higher than in the baseline case, but their distribution across skill level groups changes. The gains for high-skilled agents decline, whereas they grow for the two remaining groups. This reflects the changes in absolute benefits, which increase for high-skilled workers and decrease for the rest. The magnitude of the relative momentary gains remains unchanged for the stabilized probabilities scheme and even grows for the alternative scheme. Of course their distribution also changes following the pattern for the lifetime gains.

For the calibration with the proportional unemployment benefits, we also consider removing the life-cycle heterogeneity of the unemployment risk, as in the previous subsection. The results collected in panel B are similar to those for the baseline calibration. The lifetime gains move negligibly, but the relative momentary gains with the homogeneous risk are two to three times smaller.

5.5 New pension scheme

Finally, we conduct an experiment that may provide insights into the behaviour of the welfare gains in the future. We analyse the impact of the pension reforms that have recently been implemented in Poland as well as many other countries as a response to the increase of life expectancy coupled with lower fertility. As a remedy to this trend, the statutory retirement age in Poland was increased from 60 years for women and 65 years for men to 67 years for both sexes. Despite this action, the pension replacement rates are expected to drop by half.

Table 10: Gains for the new pension scheme

Group	Stabilized probabilities				Stabilized unemployment			
	All	Low	Med.	High	All	Low	Med.	High
	A. New pension scheme							
λ	0.113	0.092	0.119	0.107	0.006	-0.003	0.007	0.022
$\bar{\lambda}_{20\text{-}99}$	0.091	0.082	0.096	0.074	0.001	-0.009	0.003	0.015
$\bar{\lambda}_{20\text{-}24}$	0.290	0.238	0.297	0.343	0.124	0.116	0.125	0.128
	(3.2)	(2.9)	(3.1)	(4.6)	(87.2)	–	(44.9)	(8.4)
$\bar{\lambda}_{25\text{-}29}$	0.193	0.182	0.196	0.199	0.061	0.074	0.056	0.069
	(2.1)	(2.2)	(2.0)	(2.7)	(42.8)	–	(20.0)	(4.5)
$\bar{\lambda}_{90\text{-}99}$	0.084	0.086	0.084	0.071	0.022	0.014	0.024	0.034
	(0.9)	(1.1)	(0.9)	(1.0)	(15.8)	–	(8.4)	(2.2)
	B. New surviving probabilities only							
λ	0.121	0.087	0.129	0.133	0.003	-0.017	0.005	0.031
$\bar{\lambda}_{20\text{-}99}$	0.094	0.070	0.101	0.092	0.004	-0.012	0.006	0.025
$\bar{\lambda}_{20\text{-}24}$	0.286	0.213	0.296	0.366	0.119	0.088	0.123	0.150
	(3.1)	(3.0)	(2.9)	(4.0)	(27.6)	–	(19.3)	(6.0)
$\bar{\lambda}_{25\text{-}29}$	0.192	0.168	0.197	0.210	0.056	0.055	0.054	0.077
	(2.0)	(2.4)	(2.0)	(2.3)	(13.1)	–	(8.5)	(3.1)
$\bar{\lambda}_{90\text{-}99}$	0.096	0.085	0.099	0.092	0.032	0.028	0.033	0.038
	(1.0)	(1.2)	(1.0)	(1.0)	(7.5)	–	(5.1)	(1.5)
	C. New retirement age only							
λ	0.124	0.096	0.132	0.128	0.012	0.002	0.013	0.030
$\bar{\lambda}_{20\text{-}99}$	0.081	0.078	0.083	0.073	0.006	-0.002	0.007	0.018
$\bar{\lambda}_{20\text{-}24}$	0.255	0.176	0.268	0.336	0.110	0.076	0.0117	0.140
	(3.1)	(2.3)	(3.2)	(4.6)	(19.6)	–	(18.0)	(7.9)
$\bar{\lambda}_{25\text{-}29}$	0.187	0.170	0.191	0.200	0.051	0.054	0.049	0.068
	(2.3)	(2.2)	(2.3)	(2.8)	(9.1)	–	(7.5)	(3.9)
$\bar{\lambda}_{90\text{-}99}$	0.017	0.016	0.017	0.018	0.024	0.023	0.024	0.028
	(0.2)	(0.2)	(0.2)	(0.2)	(4.3)	–	(3.7)	(1.6)
	D. New pension replacement rate only							
λ	0.116	0.092	0.123	0.113	0.009	-0.002	0.010	0.026
$\bar{\lambda}_{20\text{-}99}$	0.094	0.080	0.100	0.080	0.007	-0.001	0.008	0.018
$\bar{\lambda}_{20\text{-}24}$	0.271	0.205	0.281	0.338	0.118	0.094	0.122	0.136
	(2.9)	(2.6)	(2.8)	(4.4)	(16.6)	–	(14.9)	(7.4)
$\bar{\lambda}_{25\text{-}29}$	0.177	0.157	0.182	0.187	0.056	0.059	0.053	0.069
	(1.9)	(2.0)	(1.8)	(2.4)	(7.9)	–	(6.5)	(3.8)
$\bar{\lambda}_{90\text{-}99}$	0.095	0.093	0.098	0.082	0.026	0.025	0.026	0.029
	(1.0)	(1.2)	(1.0)	(1.1)	(3.7)	–	(3.2)	(1.6)

Low, Med., High refer to the skill levels; In parentheses the relative gains $\bar{\lambda}_i/\bar{\lambda}_{20\text{-}99}$ are given

In our setup, the reform is represented by the alternation of three parameters. First, we proportionally reduce all death probabilities by half. As a result, life expectancy in the model rises by 7.5 years, which is close to the central demographic projection for Poland for 2050 (CSO, 2014). Second, we increase the retirement age in the model from 60 to 65 years, as we continue to assume that the actual retirement age will be lower than the statutory one. Finally, we cut the pension net replacement rate from 60% to 30% according to simulations of the Social Security Office in Poland (Kwiecińska, 2011).

The results of these reforms for the welfare gains are shown in table 10. At the beginning, we consider all changes taken together (panel A). Then, we study the effects of every action separately (panels B-D). First, we can see that under the new pension scheme, the lifetime gains decline slightly for all agents, from 0.13% to 0.11% for the stabilized probabilities and from 0.01% to 0.006% for the alternative policy. The reform is beneficial for medium- and high-skilled agents. This is mainly due to the decline in the replacement rate, resulting in lower taxes, which facilitates insurance against unemployment risk. The higher life expectancy generally decreases the gains as well because it is associated with a longer expected retired life, where the business cycle risk is minute. However, for medium- and high-skilled workers, this effect is partially neutralized by higher taxation. Exactly the opposite is true for the rise in the retirement age. However, its impact on the gains is ultimately limited.

Despite the decline in the lifetime gains, the absolute momentary gains for young cohorts are, in most cases, higher than those for the baseline pension setup. This result clearly suggests that in the new pension scheme, an even larger fraction of the business cycle costs will be borne by young generations, mainly due to the increase in the retirement age.

6 Conclusion

We study the welfare gains from eliminating business cycles in an OLG economy with the life-cycle heterogeneity of unemployment risk. We find that most of the costs associated with business cycles are borne during the early stages of a labour market career and that this result is primarily related to the much higher unemployment risk faced by young agents. We also show that a countercyclical policy that aims to stabilize business cycle fluctuations on the labour market for young agents would reduce the costs considerably, not only from the perspective of these cohorts but also from the lifetime perspective. The need for appropriate policy-actions will most likely become stronger, as the recent pension reforms triggered by the demographic changes are likely to make young people even more exposed to business cycles, despite declining lifetime gains.

Our estimates of the gains should be regarded as a downward biased because we disregarded at least a few important factors that should increase either the lifetime or the relative momentary gains for young agents. For example, we did not distinguish between normal and long-term unemployment, as in Mukoyama and Şahin (2006) and Krusell et al. (2009). Indeed, Poland has a large share of long-term unemployed, although it is unclear to what extent this results from factors unrelated to business cycle structural factors. Similarly, we did not take into account observed dependencies between the labour market conditions youth face when looking for their first job and their subsequent job market careers (Burgess et al., 2003; Kahn, 2010). The possibility that being unemployed at the beginning of a job market career reduces employment probabilities, and earnings for the next dozen or so years definitely increase the gains from eliminating business cycles for young agents. However, we leave these topics for further investigation.

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A Derivation of the tax rates

Because both, government incomes and expenditures, are related to aggregate labour supply, they depend only on the aggregate state of the economy. Let $\Gamma(k, a, \varepsilon, s \mid K, Z)$ represents agents' density. For each aggregate state Z, then, we define:

$$G_w(Z) = \tau(Z)lW(Z) \int \cdots \int \xi(s, a)\mathbb{I}(\varepsilon = e)\Gamma(k, a, \varepsilon, s \mid K, Z)dk\, da\, d\varepsilon\, ds = \tau(Z)lW(Z)L_w(Z), \quad (20)$$

$$G_u(Z) = [1 - \tau(Z)]lW(Z) \int \cdots \int \bar{\xi}\mathbb{I}(\varepsilon = u)\Gamma(k, a, \varepsilon, s \mid K, Z)dk\, da\, d\varepsilon\, ds = [1 - \tau(Z)]lW(Z)L_u(Z), \quad (21)$$

$$G_r(Z) = [1 - \tau(Z)]lW(Z) \int \cdots \int \xi(s, 59)\mathbb{I}(a \geq 60)\Gamma(k, a, \varepsilon, s \mid K, Z)dk\, da\, d\varepsilon\, ds = [1 - \tau(Z)]lW(Z)L_r(Z). \quad (22)$$

G_w represents the government income from taxes, and G_u and G_r denote expenditures on unemployment benefits and pensions, respectively. Clearly, for a budget to be balanced, we must have $G_w(Z) = G_u(Z) + G_r(Z)$, which implies that:

$$\tau(Z) = \frac{L_w(Z) + L_u(Z) + L_r(Z)}{L_u(Z) + L_r(Z)}. \quad (23)$$

B Computational details

To calculate the welfare gains, the following main steps are taken:

B.1 Solve the model without the aggregate risk to obtain a nonstochastic steady state for aggregate wealth K_{ss} and stationary density of wealth $\Gamma_{ss}(k, a, \varepsilon, s)$. These variables are then used as initial values in the subsequent steps of the procedure.

B.2 Solve the model with the aggregate risk using the Krusell–Smith approximate aggregation algorithm.

B.3 Simulate the model with aggregate risk to obtain the distribution of aggregate wealth and productivity shock $\Gamma_{K,Z}(K, Z)$. The density is approximated by a discrete density with 96 equidistant points for K and two points for Z. The simulated sample contains 100500 periods, where first 500 observations are discarded.

B.4 Solve the model without the aggregate risk *during the transition* — when K moves from the average level for the model with the aggregate risk, given by $\iint K\Gamma_{K,Z}(K, Z)dK\, dZ$, towards K_{ss}. Despite the lack of aggregate shocks, K is not constant during the transition. Thus, we have to utilize the Krusell–Smith algorithm once more. This algorithm also gives a perceived law of motion for the aggregate wealth during the transition $H_t(K)$, given K as the initial value.

B.5 Given the policy functions from steps 2 and 4 as well as the perceived law of motion $H_t(K)$, calculate the value functions $V(k, 20, \varepsilon, s, K, Z)$ and $V_t(k, a, \varepsilon, s, H_t(K); 0)$.

Because k and K are continuous, we approximate the value functions for these variables on grids. For k, we use separate polynomial grids of order 5 with 300 points for every skill level group. The polynomial grid is much denser near the borrowing constraint, where the policy function is nonlinear. For a higher k, the policy function is nearly linear, and the grid can be sparser. Because we assume different maximum debt levels for agents with different skills, we also have to use different grids for them. In most calibrations, the upper bound for individual wealth is set to 50 for high-skilled agents. It is approximately 16% lower for medium-skilled agents and 21% lower for low-skilled agents. In the case of K, we utilize an equidistant grid with 10 points over an interval $[0.9K_{ss}\ 1.1K_{ss}]$.

B.6 Integrate the value functions and calculate the welfare gains.

In what follows, we discuss the most important ingredients of the described procedure. A key computational difficulty is to solve an agent's decision problem, which is defined as:

$$V(k, a, \varepsilon, s, \Gamma, Z) = \max_{c,k'} \{U(c) + \beta q_{a,a+1}\mathbb{E}[V(k', a + 1, \varepsilon', s, \Gamma', Z') \mid \varepsilon, \Gamma, Z]\} \quad (24)$$

$$\text{s.t.} \quad k' = (1 - \delta + R)k + d - c, \quad (25)$$

$$k' \geq \underline{k}(s), \quad (26)$$

$$\Gamma' = \mathbb{H}(\Gamma, Z, Z'), \quad (27)$$

where Γ represents the distribution of agents over (k, a, ε, s). The policy functions take the forms:

$$k' = k'(k, a, \varepsilon, s, \Gamma, Z), \quad c = c(k, a, \varepsilon, s, \Gamma, Z). \tag{28}$$

Our algorithm uses three important concepts. First, the approximate aggregation property discussed by Krusell and Smith (1998) is utilized to make agents' expectations almost fully rational. Then, we use a backward iteration together with the Euler equation iteration algorithm proposed by Maliar et al. (2010) to find the explicit decision rules of agents from a given cohort. Finally, when simulating the model, we follow Heer and Maussner (2009, see p. 544–545) and iterate on the agents' density over (k, a, ε, s) instead of simulating the decisions of large group of agents, as many studies have done. Below, we discuss these concepts in more detail. In general, the whole procedure is similar to that used by Heer and Maussner (2009, algorithm 10.2.1).

B.1 Krusell–Smith algorithm for the model with aggregate risk

A fully rational agent uses the distribution Γ together with its law of motion $\mathbb{H}$ to predict an aggregate capital level that determines future interest rates and wages. Because Γ is an infinite-dimensional object, it is impossible to approximate numerically the value function and the policy functions. However, as noted by Krusell and Smith (1998), in many cases, instead of the whole distribution, it is sufficient to use only its first few moments. In fact, in our case, taking into account only the first moment K provides satisfactory accuracy. Therefore, the problem can also be written in the form presented in the main body of the paper:

$$V(k, a, \varepsilon, s, K, Z) = \max_{c,k'} \{U(c) + \beta q_{a,a+1} \mathbb{E}\left[V(k', a+1, \varepsilon', s, K', Z') \mid \varepsilon, K, Z\right]\} \tag{29}$$

$$\text{s.t.} \quad k' = (1 - \delta + R)k + d - c, \tag{30}$$

$$k' \geq \underline{k}(s), \tag{31}$$

$$K' = H(K, Z, Z'), \tag{32}$$

where we use a simple loglinear law of motion for aggregate capital:

$$\ln K' = \begin{cases} b_{0b} + b_{1b} \ln K & \text{if } Z = Z_b \\ b_{0g} + b_{1g} \ln K & \text{if } Z = Z_g \end{cases} \tag{33}$$

Krusell and Smith (1998) proposed an intuitive iterative procedure to determine the coefficients $\boldsymbol{b} = [b_{0b},\, b_{1b},\, b_{0g},\, b_{1g}]$ that makes the perceived law of motion (33) as rational as possible. The full rationality implies that the perceived law of motion coincides with the actual law of motion from the simulated model. The algorithm can be summarized as follows:

B.1.1 Set initial values of the coefficients $\boldsymbol{b}^{(0)}$.

B.1.2 For a given $\boldsymbol{b}^{(j)}$, find the decision rules $k' = k'(k, a, \varepsilon, s, K, Z)$ and $c = c(k, a, \varepsilon, s, K, Z)$ that solve the consumption–saving problem (29)–(33).

B.1.3 Given the decision rules, simulate the model for T periods and compute a time path for the mean aggregate capital.

B.1.4 Estimate the new autoregressive coefficients $\boldsymbol{b}^{(j+1)}$ using ordinary least squares.

B.1.5 If $\|\boldsymbol{b}^{(j+1)} - \boldsymbol{b}^{(j)}\|_\infty < \nu_b$, then stop; otherwise update vector $\boldsymbol{b}^{(j+1)} = \phi_b \boldsymbol{b}^{(j+1)} + (1 - \phi_b)\boldsymbol{b}^{(j)}$ and return to step 2.

In the baseline version of the model, we set $\boldsymbol{b}^{(0)} = [1,\, 0,\, 1,\, 0]$, $\nu_b = 10^{-6}$ and $\phi_b = 0.75$. The model is simulated for $T = 4200$ periods, where the first 200 periods are used as a burn-in sample. The resultant law of motion is as follows:

$$\ln K' = \begin{cases} 0.0784 + 0.9550 \ln K & \text{if } Z = Z_b \\ 0.1005 + 0.9498 \ln K & \text{if } Z = Z_g \end{cases} \tag{34}$$

The law of motion fits the simulated data well. R^2 equals 0.999947 for bad aggregate state and 0.999948 for the good aggregate state.

B.2 Solving the individual decision problem by Euler equation iteration (step B.1.2)

To approximate the solution for an agent's decision problem given $\boldsymbol{b}^{(j)}$, we use the backward iteration method, and, following Maliar et al. (2010), we employ the Euler equation iteration method for a single cohort. As shown in the cited paper, the Euler equation is given by:

$$k' = (1-\delta+R)k + d - \left[\eta + \beta q_{a,a+1}\mathbb{E}\left(\frac{1-\delta+R'}{[(1-\delta+R')k' + d' - k'']^{\gamma}}\right)\right]^{-\frac{1}{\gamma}}, \tag{35}$$

where η is a Lagrange multiplier associated with the borrowing constraint (31).

We look for the policy functions $k' = k'(k, a, \varepsilon, s, K, Z)$ and $c = c(k, a, \varepsilon, s, K, Z)$. The whole procedure can be summarized as follows:

B.1.2.1 Set the grids for individual k and aggregate wealth K.

B.1.2.2 Compute the decision rules for the last cohort:

$$k'(k, 100, \varepsilon, s, K, Z) = 0, \quad c(k, 100, \varepsilon, s, K, Z) = k + d. \tag{36}$$

We assume that the last cohort leaves neither debt nor bequest. Therefore, it simply consumes its whole wealth.

B.1.2.3 For every cohort $a = 100 - i, i = 1, 2, \ldots, 79$ and skill level group s:

(a) Set the initial policy function $k'_0(k, a, \varepsilon, s, K, Z) = k'((k, a+1, \varepsilon, s, K, Z)$, assuming that the initial policy function equals the policy for the next cohort.

(b) In (35), set $\eta = 0$. Compute the new policy function k'_{i+1} on the predefined grid from the r.h.s. of (35). The expectation term is based on the transition probabilities P_Z and P_ε. The next period interest rate R' and wage W' needed to calculate the future income d' are computed using the law of motion (33) with coefficients $\boldsymbol{b}^{(j)}$. To find a value of k'' on the grid k, we interpolate the next cohort policy $k'((k, a+1, \varepsilon, s, K, Z)$ in points obtained from the current cohort policy $k'_i((k, a, \varepsilon, s, K, Z)$. We apply a piecewise cubic Hermite interpolation.

(c) If some elements of k'_{i+1} lie outside the capital grid domain, set them to the respective boundary values.

(d) If $\|k'_{i+1} - k'_i\|_\infty < \nu_k$, then move to the next step; otherwise, update the policy function $k'_{i+1} = \phi_k k'_{i+1} + (1-\phi_k)k'_i$ and return to step (b).

(e) Compute the consumption policy from the budget constraint.

We use similar grids as in the value function approximation step. The only difference is that we now use only 150 nodes for the grids for k. Other parameters in the baseline version of the model are equal to: $\nu_k = 10^{-8}$ and $\phi_k = 0.25$.

B.3 Simulating individuals' wealth density (step B.1.3)

To simulate the dynamics of individuals' wealth density, we follow Heer and Maussner (2009, see p. 544–545) and iterate on the agents' density over (k, a, ε).

In fact, we simulate only the aggregate productivity shock. Then, for every period and skill level group, we analytically compute individual capital density functions, taking advantage of the fact that with a discretized individual capital level, their dynamics are described by a Markov chain with transition probabilities depending on the aggregate capital as well as the current and future state of the economy. Because we do not allow for skill changes, we can build separate chains for each skill group. The states of the Markov chains are then defined by a triple (k, a, ε).[4] To describe precisely how the transition matrices $P_\Gamma(s, Z, Z')$ are constructed *given some fixed aggregate capital level* K, we introduce the following probabilities:

- $p_\Gamma^{(I,J)}(s, Z, Z')$ — probability that an agent moves from state $I = (i_k, i_a, i_\varepsilon)$ to $J = (j_k, j_a, j_\varepsilon)$;

[4] Actually, the labour market status ε is redundant for retirees. In this case, we use only one value: $\varepsilon = u$.

- $p_k^{(i_k,j_k)}(a,\varepsilon,s,Z)$ — probability that an agent changes her capital stock from $k(i_k)$ to $k(j_k)$, where $i_k, j_k \in \{1,\ 2,\ ...,\ n_k\}$, and n_k represents the number of nodes in the individual capital grid:
  If $k(j_k) \leq \tilde{k}'(k(i_k),a,\varepsilon,s,Z) \leq k(j_k+1)$, then:

$$p_k^{(i_k,j_k)}(a,\varepsilon,s,Z) = \frac{\tilde{k}'(k(i_k),a,\varepsilon,s,Z) - k(j_k)}{k(j_k+1) - k(j_k)}, \tag{37}$$

$$p_k^{(i_k,j_k+1)}(a,\varepsilon,s,Z) = 1 - p_k^{(i_k,j_k)}(a,\varepsilon,s,Z); \tag{38}$$

  where $\tilde{k}'(k,a,\varepsilon,s,Z)$ denotes an interpolated policy function defined below.

- $p_Z^{(i_Z,j_Z)}$ — probability that the aggregate state of the economy switches from Z_{i_Z} to Z_{j_Z}, where $i_Z, j_Z \in \{b,\ g\}$. These probabilities are defined in the paper;

- $p_\varepsilon^{(i_\varepsilon,j_\varepsilon)}(a,s,Z,Z')$ – probability that the employment status of an agent moves from i_ε to j_ε, where $i_\varepsilon, j_\varepsilon \in \{u,\ e\}$. These probabilities are also defined in the paper.

Then:

$$p_\Gamma^{(I,J)}(s,Z,Z') = \tag{39}$$

$$= \begin{cases} (1 - q_{a(i_a),a(i_a)+1}) \cdot \bar{u}(20,s,Z(j_Z)) \cdot p_Z^{(i_Z,j_Z)} \cdot p_\varepsilon^{(i_\varepsilon,j_\varepsilon)} & \text{if } j_a = 20, j_\varepsilon = u, j_k = j_0 \quad \text{(new unempl.)} \\ (1 - q_{a(i_a),a(i_a)+1}) \cdot (1 - \bar{u}(20,s,Z(j_Z))) \cdot p_Z^{(i_Z,j_Z)} \cdot p_\varepsilon^{(i_\varepsilon,j_\varepsilon)} & \text{if } j_a = 20, j_\varepsilon = e, j_k = j_0 \quad \text{(new empl.)} \\ q_{a(i_a),a(i_a)+1} \cdot p_k^{(i_k,j_k)} \cdot p_Z^{(i_Z,j_Z)} \cdot p_\varepsilon^{(i_\varepsilon,j_\varepsilon)} & \text{if } j_a = i_a + 1, j_a < 60 \quad \text{(surv. work.)} \\ q_{a(i_a),a(i_a)+1} \cdot p_k^{(i_k,j_k)} \cdot p_Z^{(i_Z,j_Z)} & \text{if } j_a = i_a + 1, j_a \geq 60 \quad \text{(surv. retir.)} \\ 0 & \text{otherwise} \end{cases}$$

and j_0 represents a position of 0 in the grid for k whereas $\bar{u}(a,s,Z)$ is the average unemployment rate. Thus, agents who die return to the population as newborn, with no capital and either employed or unemployed. In these cases, the transition probabilities are given in the first two lines of formula (39). Then, conditional on surviving to the next periods, their further path depends on the probabilities from the last two lines, which refer to agents of working age (third line) and upon retirement (fourth line).

The whole simulation procedure consists of the following steps:

B.1.3.1 Draw a sequence of T aggregate shocks;

B.1.3.2 Discretize k and set $t = 0$ and the initial density $\Gamma_t(k,a,\varepsilon,s) = \Gamma_{ss}(k,a,\varepsilon,s)$. Here, we use a nonstochastic stationary density (stationary density from the model without aggregate shocks);

B.1.3.3 Using the density, calculate the aggregate capital K_t;

B.1.3.4 Create an interpolated policy function $\tilde{k}'(k,a,\varepsilon,s,Z)$ by interpolating the policy function $k'(k,a,\varepsilon,s,K,Z)$ at the new grid from step B.1.3.2 and aggregate the capital calculated in step B.1.3.3. It should be emphasized that, as already mentioned above, the grid for k in the wealth density simulation procedure does not necessarily coincide with the grid created for the policy iteration scheme.

B.1.3.5 For each skill level group, calculate the transition matrix $P_\Gamma(s,Z_t,Z_{t+1})$ using formulas (37)–(39);

B.1.3.6 Calculate the new density $\Gamma_{t+1}(k,a,\varepsilon,s)$. This density is a mixture of conditional densities with respect to the skill level groups, where the latter are simple products of current conditional densities and the respective transition matrices;

B.1.3.7 If $t = T$, stop; otherwise, set $t = t + 1$ and return to step B.1.3.3.

Because the procedure is invoked repeatedly by the Krusell–Smith algorithm, we always use the same sequence of aggregate shocks. Otherwise, the procedure might not converge. With 300 nodes for k, the Markov chains representing the conditional densities of individual wealth have $300 \cdot (2 \cdot 40 + 40) = 48000$ states each. As a result, the transition matrices P_Γ have approximately $2 \cdot 10^9$ entries. However, during computation,s they can easily be stored as sparse matrices, because the vast majority of their entries are zeros.

B.4 Solving the models without aggregate risk

B.4.1 Model for the transition

To solve the model without aggregate risk on the transition path, we still need to employ an algorithm similar to the Krusell–Smith procedure. This is because aggregate capital is not constant during the transition but slowly converges to the nonstochastic steady state. Therefore, one has to determine the perceived law of motion for aggregate capital that approximately coincides with rational expectations. The key deviations from the standard algorithm described in the previous subsections occur in the simulation step. First, the law of motion depends only on aggregate capital:[5]

$$K' = H_1(K) = \exp(b_0 + b_1 \ln K). \tag{40}$$

Second, the simulations always start from the same point — the average level of aggregate capital in the model with aggregate risk — and finish when the aggregate capital stabilizes, provided the minimum number of transition periods $T_{min} = 100$ is reached. Introducing the minimum duration of the transition in the Krusell–Smith procedure facilitates the convergence of the algorithm. Generally, for different values of b_0 and b_1 the duration of the transition could also differ. However, once the coefficients are eventually determined by the Krusell–Smith algorithm, we run a simulation of the transition path once more without the minimum number of periods restriction to establish the final duration of the transition. Obviously, one has to keep in mind that the policy functions during the transition no longer depend on aggregate shock.

We should stress that the transition starts from the average level of aggregate capital in the model with aggregate risk only to determine the law of motion $H_1(K)$. When we calculate the value functions, we take into account that it could start from other points as well. This is why we integrate over the density of aggregate capital. However, the duration of the transition is always kept constant, as determined by a single simulation after the Krusell–Smith algorithm.

B.4.2 Model with constant aggregate capital

When there are no aggregate shocks and the aggregate capital is kept at the nonstochastic steady state level, all other aggregates are also fixed. To solve such a model, standard procedures can be employed (Heer and Maussner, 2009). Basically, we only have to iterate on the steady state of aggregate capital, not on the law of motion, as in the Krusell–Smith method.

[5] Of course, we also have that $H_t(K) = H_{t-1}(H_1(K))$.