The relationship of simple sum and Divisia monetary aggregates with real GDP and inflation: a wavelet analysis for the U.S.

Preliminary version, do not quote

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Abstract

We apply wavelet analysis to compare the relationship between simple sum and Divisa monetary aggregates with real GDP and CPI inflation for the U.S. using data from 1967 to 2013. Wavelet analysis allows to account for variations in the relationships both across the frequency spectrum and across time. While we find evidence for a weaker comovement of Divisia compared to simple sum monetary aggregates with real GDP the relationship between money growth and inflation is estimated to be much tighter between Divisia monetary aggregates and CPI inflation than for simple sum aggregates, in particular at lower frequencies. Furthermore, for the Divisia indices for broader monetary aggregates (M2, M2M, MZM) we estimate a stable lead before CPI inflation of about four to five years.

Keywords: money growth, Divisia aggregates, inflation, wavelet analysis

1 Introduction

A central proposition of monetary theory is that, over the long-term, money growth is strongly correlated with inflation. According to the quantity theory this correlation should be one-to-one. Furthermore, given a stable money demand function, i.e. a stable velocity of circulation, there should also exist a strong correlation between real money and real output as more money is required for an increasing volume of economic transactions. A vast empirical literature

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has studied the relationship between money, output and inflation using different empirical methodologies and different measures.

In the U.S. there was a strong relationship between the narrow monetary aggregate M1 and nominal GDP (Lucas, 1980) which became unstable in the mid 1980s and 1990s, probably due to regulatory changes (e.g. lifting of Regulation Q) and technological innovations, such as ATMs (Teles and Zhou, 2005). Beginning in the 1970s, conventional money demand functions, linking nominal money holdings to the price level, output and opportunity cost measures also showed signs of instability (e.g. Judd and Scadding, 1992). One strategy aiming to recover a stable relationship between money, output and prices was to adjust the definition of monetary aggregates. For example, based on the concept of liquidity as a basis for defining money Motley (1988) and Poole (1991) proposed using the monetary aggregate MZM (money zero maturity). Combining M1 with MZM Teles and Zhou (2005) found evidence for a stable money demand function and a stable relationship between money and output. Another empirical strategy was to allow for time variation in the macroeconomic relationships and to focus at specific frequency ranges in the time series, especially on the low frequency, i.e. trend components. For example, Valle e Azevedo and Pereira (2010) show that there is a strong correlation between the low-frequency components in M2 and MZM money growth and U.S. inflation, Benati (2005) analyzes the relationship between inflation and the growth rates M1 and M2 for US for the sample period of 1870 to 2004. He finds marked differences between frequency ranges. For fluctuations between 8 and 30 years, and particularly for even lower frequency components he finds stable correlations. Benati (2009) studies the relationship for many countries, including observations for U.S. from 1820-2008. His analysis incorporates time-variation by using rolling windows. For the U.S. the cross-spectral coherency at frequency zero, i.e. for permanent innovations is consistently close to one while the cross-spectral gain displays strong time variation and is significantly below one for long time periods. This implies, that while innovations in money growth account for almost all of the long-run variance of inflation, there is no one-to-one relationship between these two variables. His interpretation of these results is that, in times of low inflation, the money growth-inflation relationship is obscured by velocity shocks and that the relationship is only uncovered in these rare episodes in which surges in inflation and money growth occur. Sargent and Surico (2011) provide an analysis of the correlation of M2 growth and inflation for quarterly U.S. data over the sample period 1875-2007. They decompose their estimate of a time-varying VAR with stochastic volatility with respect to its frequency content and present evidence that there is quite some variation in the comovement between these series which might be due to different monetary regimes. In particular, they show that the cross-spectral gain gain at frequency zero has moved far below one after the early 1980s. Haug and Dewald (2012) also find evidence for time variation of the cyclical components at business cycle periodicities, but a stable correlation between money growth and inflation at lower frequencies.

However, the monetary aggregates under consideration usually were all of the class of so-called simple sum aggregates, i.e. monetary aggregates constructed

as the sum of financial assets considered as money. Barnett (1980) critized this kind of aggregation as inconsistent with economic theory as well as aggregation and index number theory and proposed Divisa aggregates as an alternative. Instead of assigning equal weights to all components of a monetary aggregate the Divisia approach weights each component according to its user cost which is measured relative to a benchmark yield, i.e. the return on an asset which is not considered as money. Accordingly, the different monetary components are aggregated according to their degree of "moneyness" which is measured by their opportunity costs. Divisia aggregates are theoretically appealing because they are micro-founded based on utility maximizing behavior and firmly grounded in index number theory. For an overview, see Barnett (2012), Barnett and Binner (2004), Barnett and Chauvet (2011b), and Barnett and Serletis (2000). The difference between Divisa and simple sum monetary aggregates emerges, when monetary components are paid interest rates above zero. Changes in the remuneration of the components of the money supply, both relative to each other and relative to the benchmark yield, result in Divisia aggregates exhibiting different growth rates from simple sum ones. Since Divisia monetary aggregates are more closely related to the liquidity services provided by money their relationship to money and output might be closer and more stable than for simple sum monetary aggregates. In a theoretical model, Belongia and Ireland (2012) show that simple sum aggregates can move inconsistently with the model-implied "true" monetary aggregate, whereas Divisia measures provide the correct signals. Kelly et al. (2011) show that the liquidity puzzle, commonly encounterd in VAR models, that shows an increase in money growth following a restrictive monetary policy shock disappears if one uses Divisia instead of simple sum monetary aggregates. Hendrickson (2013) tests the stability of money demand for simple sum and Divisia aggregates for various monetary aggregates and finds stable money demand relationships only for Divisia measures. Granger causality of money growth for nominal GDP growth is not rejected for all monetary aggregates before 1979, but only for Divisia aggregates afterwards. Serletis and Gogas (2014) estimate money demand functions for the U.S. using Divisia aggregates and are able to restrict the income elasticity to one, i.e. to impose a long-run one-to-one relationship between real money and output.

This paper analyzes the money-inflation relationship as well as the relationship between money and real economic activity over time and frequency for the U.S. by applying the methodology of wavelet analysis which, by design, is especially suited for this kind of questions. Wavelet analysis allows for variability in the money growth - inflation/output growth relationship both across time and across the frequency spectrum. Thus, focusing on a given frequency range, we can study whether comovements between the variables have become stronger or weaker through time. Alternatively, for a given point in time, we can analyze whether the relationship between money and inflation or output growth changes for different frequencies. Furthermore, wavelet analysis provides information on lead-lag patterns, again allowing for change through time and across frequencies.

We study, whether simple sum or Divisia aggregates are more useful when looking at the link between monetary aggregates, output and inflation in the

U.S. We focus on the U.S. because of the availability of a range of Divisia aggregates provided by the Federal Reserve Bank of St. Louis and by the Center for Financial Stability. The relatively long time series for the U.S. enable us to consider the relationship between money growth and the other variables at lower frequencies. Our results show no evidence for a tighter link between fluctuations in the growth rates of Divisia monetary aggregates with those in real GDP growth than for simple sum monetary aggregates. For inflation, however, our estimates indicate much stronger comovements between Divisia money growth of M2 and related aggregates and inflation than for simple sum aggregates, in particular at lower frequencies with fluctuations of periods of 12 years and longer. This result becomes even stronger after adjusting money growth for real GDP growth and accounting for changes in velocity. Furthermore, Divisia M2 and related aggregates have a stable lead before CPI inflation while the lead-lag pattern of the simple sum aggregates is much less stable.

Wavelet analysis so far has only sparsely been applied to the analysis of the money growth - output growth/inflation relationship. Mostly, researchers have used the wavelet approach to extract frequency components from the time series and then performed regression analysis using the extracted components. For example, Andersson (2011) uses band spectrum regression methodology to analyse the effects of changes in money growth on consumer price inflation and asset price inflation in the U.S. The frequency components are extracted using a discrete wavelet transform. He finds that money growth affects mainly asset prices in the short-run, whereas it takes around ten years to affect consumer price inflation. Whitcher (2001) uses the discrete wavelet transform to extract specific frequency bands from Mexican money growth and inflation time series and then tests for Granger causality between money growth and inflation. A similar approach is taken by Greiber and Neumann (2007) who apply a discrete wavelet transform to the money growth series only, which might generate "spurious" Granger causality. More closely related to to this paper are the analyses in Rua (2012) and in Mandler and Scharnagl (2013). There, wavelet analysis is applied to the study of the money growth-inflation relationship in the Euro area using the M3 simple sum monetary aggregate and consumer price inflation using wavelet coherency, phase difference and, in the case of Mandler and Scharnagl (2013) cross-spectral gains. While Rua (2012), in part due to using self-constructed inflation data, finds a significant and stable relationship between money growth and inflation throughout his sample period (1970-2007), Mandler and Scharnagl (2013) using the official inflation series do not estimate significant comovements between the low frequency components in the growth rates of the monetary aggregate M3 and consumer price inflation over 1970-2012.

2 Wavelet analysis

Wavelet analysis is an extension of spectral analysis.[1] Spectral analysis measures the contribution of periodic cycles of specific frequencies to the variance of a time

[1] For an introduction to wavelet analysis, see Aguiar-Conraria and Soares (2013).

series or to identify frequencies with dominating coherencies between multiple time series. However, it is restricted to stationary time series as the cycles have an infinite support. If an AR(1) process exhibits a structural break in its persistence, both dominant frequencies will show up in the spectrum. It is not possible to assign the frequencies to subsamples, i.e. spectral analysis has no resolution over time.

Wavelet analysis removes this restriction.[2] It provides the possibility of uncovering transient relations (Aguiar-Conraria et al., 2008). As wavelets have only finite support ("small wave" compared with sine function which can be interpreted as a "big wave"), they are ideally suited to locally approximating variables in time or space (Crowley et al., 2006). The multiresolution decomposition (MRD) allows for a decomposition of a time series into trend, cyclical component and noise. This kind of analysis asks different questions compared to the time series type of analysis. It looks for correlations at specific frequencies. It does not analyze the propagation mechanism by means of impulse responses and it also does not try to identify shocks to the system.

The starting point is a so called mother wavelet ψ. By scaling and translation a variety of wavelets can be generated.

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right), \tag{1}$$

where s is a scaling or dilation factor. It controls the width of the wavelet. $1/\sqrt{|s|}$ guarantees the preservation of energy, i.e. $\left|\left|\psi_{\tau,s}\right|\right| = |\psi|$. τ is a translation parameter controlling the location of the wavelet.

The function ψ has to fullfil some requirements to have properties of wavelets (Percival and Walden, 2002): The integral of $\psi(u)$ is zero:

$$\int_{-\infty}^{\infty} \psi(u)\, du = 0$$

Over time ψ has to be below and above zero. An admissibility condition is a sufficient decay:

$$\Psi(0) = \int_{-\infty}^{\infty} \psi(t)\, dt = 0$$

These properties are necessary for an "effective localization in both time and frequency" (Aguiar-Conraria and Soares (2011)).[3]

The continuous wavelet transform (CWT), $W_x(\tau, s)$, is obtained by projecting $x(t)$ onto the family $\{\psi_{\tau,s}\}$

$$W_x(\tau, s) = \int_{-\infty}^{\infty} x(t)\, \frac{1}{\sqrt{|s|}}\psi^*\left(\frac{t-\tau}{s}\right) dt, \tag{2}$$

[2] It was initially proposed by Ramsey and Lampart (1998) and Ramsey (2002) for applications in economics and finance. See also Crowley (2005).

[3] The CWT is an exploratory data analysis tool. As it is two dimensional but depending on a one dimensional signal, it contains a lot of redundancy. When moving into larger scales there is little difference in CWT between adjacent scales and there are slow variations across time at any fixed large scale.

where * denotes the complex conjugate.

There is a variety of different wavelet functions. In the empirical section the so-called Morlet wavelet is used.

$$\psi_{\omega_0}(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{-\frac{t^2}{2}}, \tag{3}$$

where $\omega_0 = 6$. This specific choice for the parameter ω yields a simple relation between scales and frequencies: $f \approx \frac{1}{s}$.

The wavelet power spectrum is defined as

$$WPS_x(\tau, s) = |W_x(\tau, s)|^2.$$

In the case of a complex-valued Morlet wavelet, the corresponding wavelet transform is also complex–valued, allowing for an analysis of phase differences, i.e. lead-lag structures. It can de decomposed into a real part, the amplitude, $|W_x(\tau, s)|$, and its imaginary part, the phase, $|W_x(\tau, s)| \; e^{i\phi_x(\tau,s)}$. The phase-angle $\phi_x(\tau, s)$ can be obtained by

$$\phi_x(\tau, s) = \arctan\left(\frac{\Im\{W_x(\tau, s)\}}{\Re\{W_x(\tau, s)\}}\right) \tag{4}$$

where $\Im$ denotes the imaginary part and $\Re$ the real part.

Aguiar-Conraria and Soares (2011) use an analytical wavelet. This type is defined as one whose Fourier transform is supported on the positive real-axis only. The corresponding wavelet transform is the analytic wavelet transform.

The cross wavelet transform is defined as

$$W_{xy} = W_x W_y^* \tag{5}$$

where * denotes the complex conjugate. The cross wavelet power spectrum is defined as $|W_{xy}|$. Wavelet coherency between two time series x and y can be interpreted as local correlation and is defined as

$$R_{xy}(s) = \frac{\left|S\left(s^{-1} W_{xy}(s)\right)\right|}{\sqrt{S\left(s^{-1}|W_x|\right)}\sqrt{S\left(s^{-1}|W_y|\right)}} \tag{6}$$

where S is a smoothing operator with respect to time and scale. The wavelet phase difference can be computed via (4)

If $\phi_{x,y}(s, \tau) = 0$, the series x and y move together at the specified frequencies. If $\phi_{x,y}(s, \tau) \in \left(0, \frac{\pi}{2}\right)$, series x leads y. The time lag between both series can be calculated as

$$\Delta T(s, \tau) = \frac{\phi_{x,y}(s, \tau)}{2\pi f(\tau)}. \tag{7}$$

The cross spectral gain is defined as

$$G_{xy}(s) = \frac{\left|S\left(s^{-1} W_{xy}(s)\right)\right|}{S\left(s^{-1}|W_y|^2\right)}. \tag{8}$$

3 Empirical application

3.1 Data

The data set consists of U.S. quarterly time series for four monetary aggregates M1, M2, M2M, MZM, for real GDP and the CPI.[4] M2M is equal to M2 minus small-denomination time deposits. MZM (money zero maturity) is equal to M2 minus small-denomination time deposits plus institutional money funds.[5] For each monetary aggregate we use time series for the simple sum aggregate and for two Divisia aggregates, first, the Divisia index from the Center for Financial Stability (CFS), and second, the Monetary Services Index (MSI) from the Federal Reserve Bank of St.Louis. Except for the CFS Divisia index all series are available from the Federal Reserve Bank of St. Louis' FRED data base.[6] The CFS Divisia index can be downloaded from the CFS website.[7] All series are seasonally adjusted. The sample starts in 1967Q1, which is the earliest availability of the Divisia-type series and ends in 2013Q2. The monetary aggregates are normalized to 100 at the start of the sample period. The CFS version of Divisia differs from the MSI because of a different construction of the benchmark rates and differences in the treatment of sweep accounts, for details see Barnett et al. (2012), Jones et al. (2005).

In our analysis, we use annual log-differences (annual growth rates) [8] Figure 1 shows the annual log-differences for the various simple sum and Divisia aggregates, Figure 2 those for real GDP, nominal GDP and the CPI. For M1 the growth rates of simple sum and Divisia aggregates start to diverge in the middle 1970s and that for the simple sum version stays above those for the both Divisia aggregates until the end of the 1980s. In contrast, from 1995 right to the beginning of the financial crisis M1 Divisia grows faster than the simple sum aggregate, markedly so from the mid 1990s to the early 2000s. Afterwards, the growth rate of the simple sum aggregate again exceeds that of the Divisia aggregate. For M2 (second row from top) the growth rates of the Divisia aggregates are below those of the simple sum aggregate for M2 growth rates almost throughout the 1970s until the mid1980s, again in the late 1980s, mid-1990s, around 2000 and from 2005 to the beginning of the financial crisis. Faster growth in Divisia M2 relative to simple sum M2 can be observed only in

[4] In addition, the GDP Deflator, nominal GDP and the three-month Treasury Bill rate are used for various adjustments to the money growth series.

[5] M1 comprises currency in circulation, traveler's checks of nonbank issuers, demand deposits and other checkable deposits. M2 consists of M1 plus savings deposits (including money market deposit accounts), small-denomination time deposits and balances held in retail money market mutual funds.

[6] http://research.stlouisfed.org/fred2/

[7] http://www.centerforfinancialstability.org/amfm_data.php.

The CFS provides Divisia aggregates for broader definitions of money (M3, M4) and a MSI version of M3 is also available from the Federal Reserve Bank of St. Louis. Since, however, the corresponding simple sum aggregate has either been discontinued as in case of M3 or has never been compiled and, hence, a comparison of simple sum and Divisia aggregates is not possible, we do not include them in this study.

[8] The results are robust to using quarterly growth rates instead.

the mid 1980s, from about 1990 to 1995 and in the early 2000s. For the M2M and MZM monetary aggregates Divisia and simple sum versions follow a similar relative pattern as for M2, although the differences between both types of aggregates are less pronounced. Generally for M2M and MZM, money growth rates are very close to each other when they are falling, while a gap tends to open when they are on the rise - a phenomenon which is better visible in MZM than in M2M. The observable jumps in the growth rates of M2 and, particularly M2M and MZM in 1982 result from a redefinition of M2 in 1982 which removed the volatile wholesale component of money market mutual funds from M2 and included retail repurchase agreements within M2 (Whitesell and Collins, 1997). The convergence of simple sum and Divisia growth rates for M2 and its variants during the financial crisis can be explained by the compression of the yield curve in the low interest-rate environment which leads to a convergence in the user cost and hence in the weights of the different components in the Divisia aggregates which, thus, become close to simple sum monetary aggregates.

Barnett and Chauvet (2011a) argue that growth rates of Divisia aggregates are much lower before recessions or at the beginning of high interest rate periods. Looking at the growth rates of M2 Divisia and real GDP this seems to be the case for the 1980 and 1981/82 recessions but for the other recessions it is not that obvious. Turning to the correlation with CPI inflation at least for M2 the more pronounced fall in the Divisia growth rates at the end of the 1970s seem to be stronger correlated with the decline in inflation rates than the roughly constant evolution of the growth rates of the simple sum aggregate, and indicates tighter monetary conditions compared to the simple sum version of M2.

3.2 Econometric analysis

We estimate the wavelet coherency between the annual growth rates of the monetary aggregates under consideration, the annual growth rate of real GDP, and annual CPI inflation.[9] Figure 3 presents the estimated wavelet coherencies for each monetary aggregate with real GDP growth across time (horizontal axis) and frequencies/periods (vertical axis, in years). Numerical values of the estimated coherencies are represented by different colours and are increasing from blue to red. Black lines indicate coherency different from zero at the 5%-level. However, only estimates between the curved red bands (cone of influence) should be interpreted, since the estimates in the outer regions, i.e. outside of the cone of influence, are based on relatively few observations.[10]

[9] All estimations were performed using the AST-toolbox for MATLAB by Aguiar-Conraria and Soares. https://sites.google.com/site/aguiarconraria/joanasoares-wavelets/

[10] If there is only an insufficient number of past or future observations available to estimate the wavelet power spectra and the wavelet cross spectrum at a given point in time the algorithm extends the sample backwards or forward by "reflection", i.e. double using some observations close to the edges. In the region outside of the red lines the results are affected by this procedure.

The cone of influence becomes smaller for lower frequencies because the flexible window length that enters the wavelet transform increases and more observations are required for extracting lower frequency components. This effect limits the range of frequencies that can

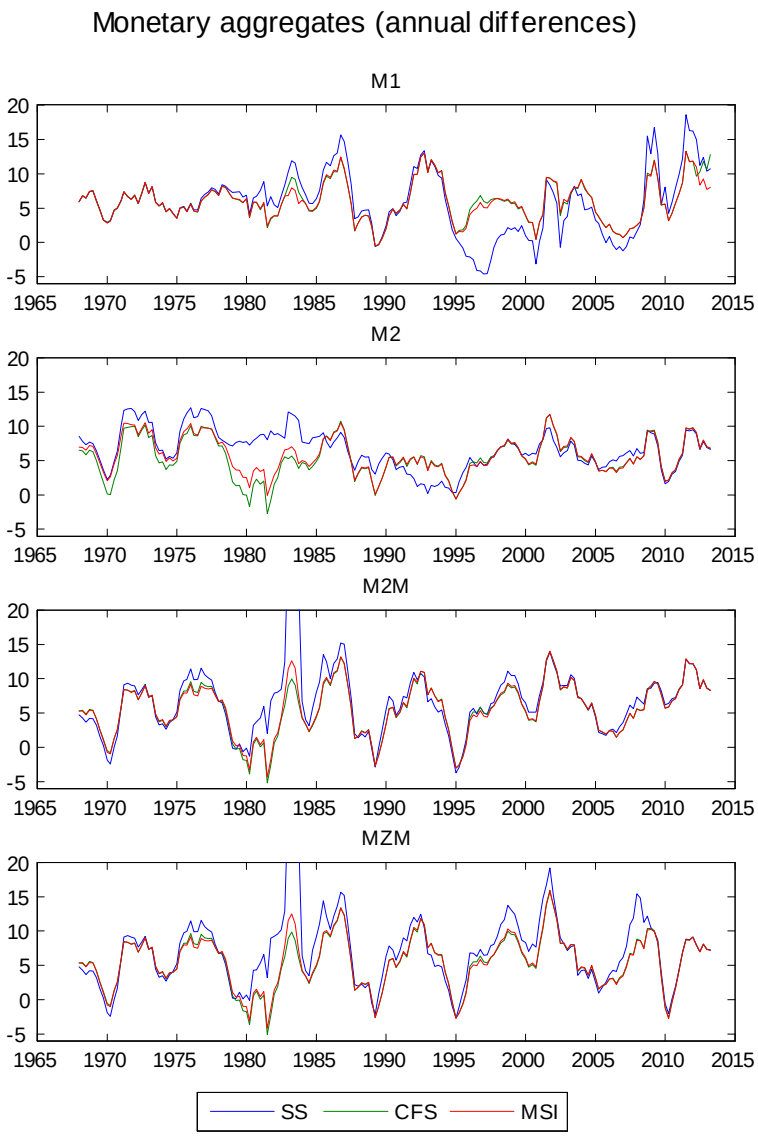

Figure 1: Annual growth rates of monetary aggregates.

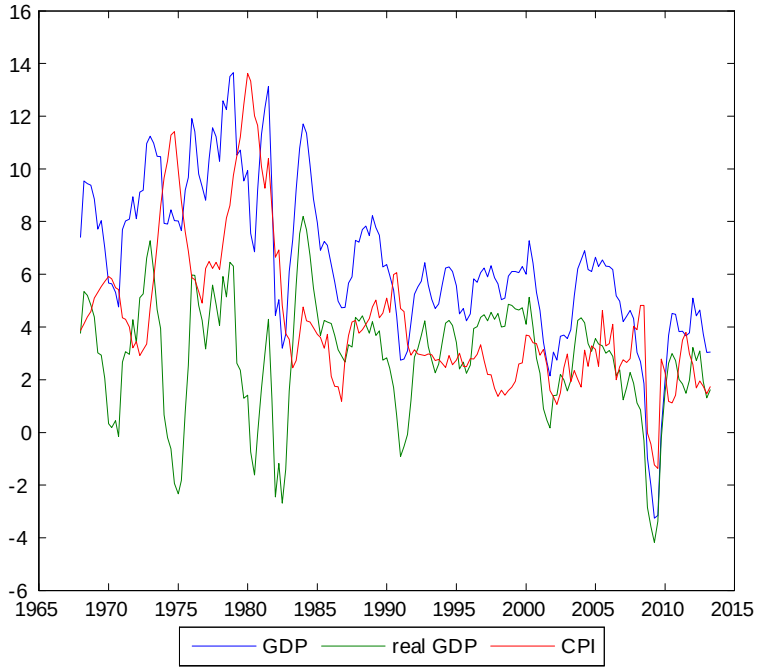

Figure 2: Annual growth rates of real GDP, nominal GDP and CPI.

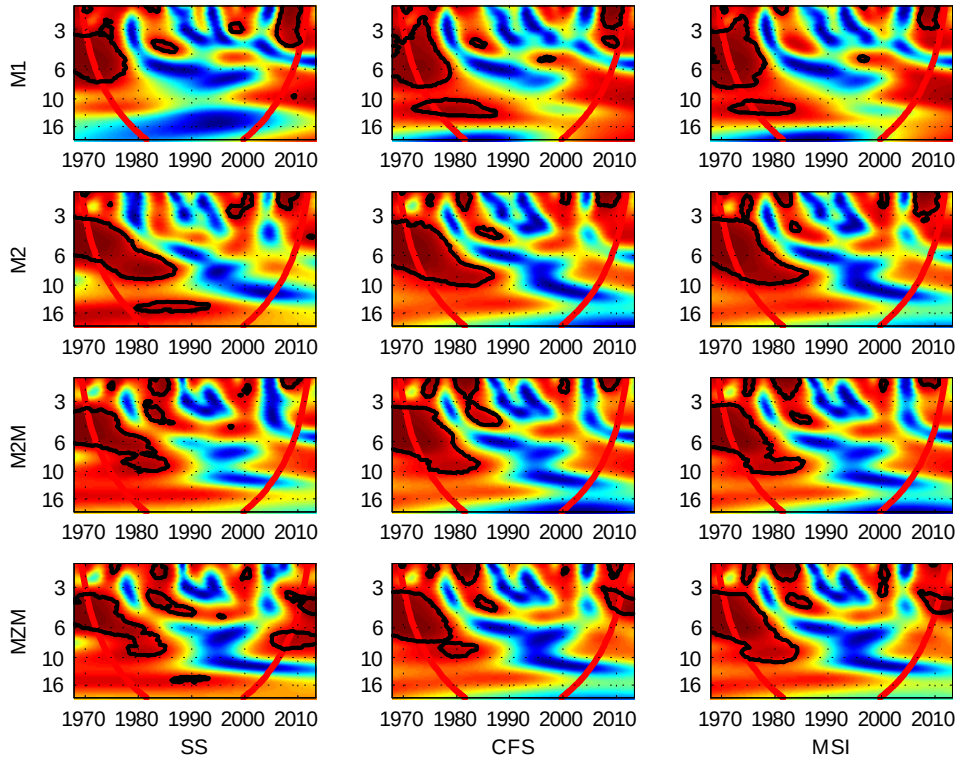

Figure 3: Wavelet coherency of annual growth rate of real monetary aggregates an annual growth rate of real GDP.

The left column in Figure 3 presents the results for the simple sum monetary aggregates while the middle and right columns show those for the Divisia aggregates (CFS in the middle column, MSI in the right one). A stable long-run function relating demand for real balances to real output should reflect in high and stable coherencies at low frequencies. In most cases (eight out of twelve) we estimate marked coherencies between money growth and real GDP growth at low frequencies which appear to be relatively stable over time in contrast to the more fluctuating coherencies at middle-to-high frequencies, indicating a more stable relationship between the longer-term trend components in the series than in the faster moving components. The top row shows that switching for M1 from the simple sum aggregate to the Divisia aggregates leads to a strong increase in the wavelet coherency across the low frequency spectrum. (fluctuations of 12 years and longer). In particular, when using the CFS Divisia aggregate we estimate a stable wavelet coherency with real GDP growth at low frequencies that is close to one and that is statistically significant at the 5%-level. For the MSI Divisia aggregate the results are somewhat weaker but the relationship between real money growth and output growth at low frequencies is still much stronger than for the simple sum aggregate and is stable throughout the sample period. Turning to the broader monetary aggregates (M2, M2M, MZM) the results are mostly the reverse. While we estimate high - and for the late 1980s up to the mid 1990s significant - coherencies with GDP growth at low frequencies for the simple sum aggregates, the estimates for the Divisia aggregates at these frequencies are lower. Furthermore, at low frequencies the relationship between the growth rates of the divisa aggregates and real GDP growth tends to weaken over time with the exception of M2-MSI. At middle-to-higher frequencies (i.e. fluctuations of less than ten years) use of Divisia aggregates strengthens the cohereny with real GDP in the late 1970s and early 1980s for M2 but weakens them for M2M. For MZM the results for the Divisia aggregates are broadly similar to those for the simple sum aggregates. Overall, these results indicate that Divisia aggregates improve the real money growth - output growth relationship for the narrow monetary aggregate M1 while there is no such improvement for M2 and its variants.

The results for the relationship between money growth and CPI inflation are shown in Figure 4. For M1 (top row) the Divisia aggregates lead to no improvement compared to the simple sum aggregate in the coherencies at low frequencies. However, comovements between money growth and inflation turn out to be somewhat more pronounced for the Divisia aggregates in the middle-frequency band with fluctuations between four to eight years where the coherencies become higher from the mid 1990s onwards.For M2, M2M and MZM we estimate higher coherencies between the low frequency components (fluctuations of ten to twelfe years) of the Divisia monetary aggregates and inflation than for the simple sum aggregates and these coherencies turn out more stable through time. The improvement is particularly strong for M2 where the estimates using Divisia become statistically significant until the early-to-mid 1990s and where

be studied using a data set of given length.

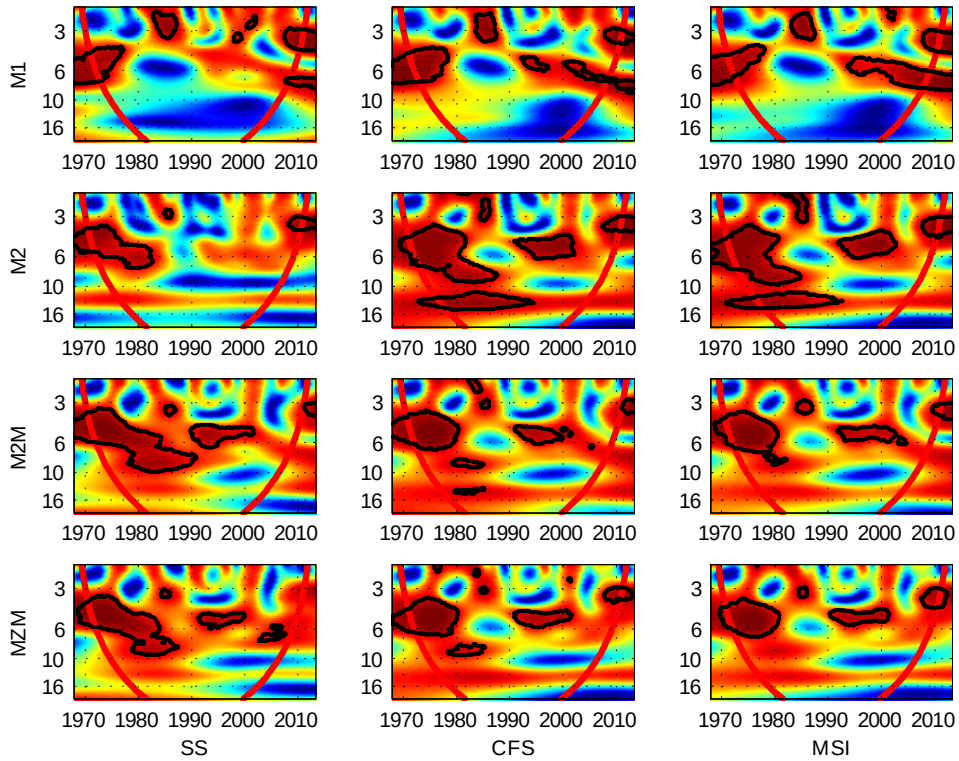

Figure 4: Wavelet coherency of annual growth rate of monetary aggregates and annual CPI inflation.

we observe also a strong improvement in the estimated relationship at middle-to-higher frequencies from the late 1970s to the late 1980s. Nevertheless, the coherency estimated using the Divisia aggregates becomes slightly weaker when moving throught the 1990s and 2000s with the MSI aggregates apparently more affected than the CFS aggregates.

Since the quantity equation suggests that money growth driven by the expansion in the transaction volume, i.e. by growth in the real economy, should not be inflationary, we follow Teles and Uhlig (2010) and adjust the monetary aggregates for real GDP growth by subtracting the annual growth rate in real GDP from the annual growth rates of the monetary aggregates.[11] The estimated

[11] In fact, the coefficient on real GDP in a regression of the simple sum aggregates real M2, real M2M and real MZM on real GDP and an opportunity cost measure is close to one. Hence, the results presented in the main text should be considered conservative estimates, as they are based on a correction of monetary aggregates for real GDP growth that reflects the empirical relationship for the simple sum aggregates more closely than that for the Divisia aggregates

wavelet coherencies of these adjusted money growth series with CPI inflation are displayed in Figure 5. Comparing Figures 5 and 4, the adjustment leaves the results for M1 mostly unchanged. While for both simple sum and Divisia aggregates estimated coherencies increase somewhat for low frequencies they decline for middle frequencies in the early sample period. Turning to M2, M2M and MZM, the adjustment leads to a strong improvement in estimated coherencies for the simple sum aggregates along a middle frequency range of about three to ten years where we estimate high and mostly significant coherencies throughout the sample period. Across this frequency spectrum coherencies for the Divisia aggregates improve less and thus the coherencies with inflation turn out less pronounced than those using the simple sum aggregates. In fact, using the Divisia series there appears a "gap" with relatively low coherencies in the 1980s which does not emerge for the simple sum aggregates. The Divisia aggregates, nevertheless, remain superior to the simple sum ones for the long-term fluctuations. Using the adjusted series leads to high and stable coherencies between the long-term growth components of Divisia aggregates and CPI inflation across a broad frequency band of fluctuations of 12 years and longer. In contrast, for the simple sum aggregates the adjustment for real GDP growth does not lead to a strengthening of the empirical relationship with inflation at low frequencies.

The quantity theory also suggests that in order to uncover the relationship between money growth and inflation, money growth should not only be adjusted for changes in real GDP but also for changes in velocity. Hence, as in Teles and Uhlig (2010) we adjust the growth rates of all monetary aggregates not only by subtracting real GDP growth but also by the estimated change in velocity. Using the quantity equation in growth rates ($\Delta m_t + \Delta v_t = \Delta p_t + \Delta y_t$), with m: nominal money, v: velocity, p: price level, and y: real GDP (all in logarithms), adjusted money growth is given by (using annual differences)

$$\Delta m_t^* = \Delta m_t + \Delta v_t - \Delta y_t. \tag{9}$$

Following Teles and Uhlig (2010) we model the change in velocity as

$$\Delta v_t = \beta_1 \cdot oc_t, \tag{10}$$

with oc as the opportunity cost of holding money. Teles and Uhlig obtain β_1 from the OLS regression

$$\log V_t = \log\left(\frac{M_t}{P_t Y_t}\right) = \beta_0 + \beta_1 oc_t, \tag{11}$$

with $P_t Y_t$ representing nominal GDP. Adjusted money growth is then defined as

$$\Delta m_t^* = \Delta m_t - \Delta y_t + \widehat{\beta}_1 \Delta oc_t, \tag{12}$$

for which the coefficient is markedly lower.

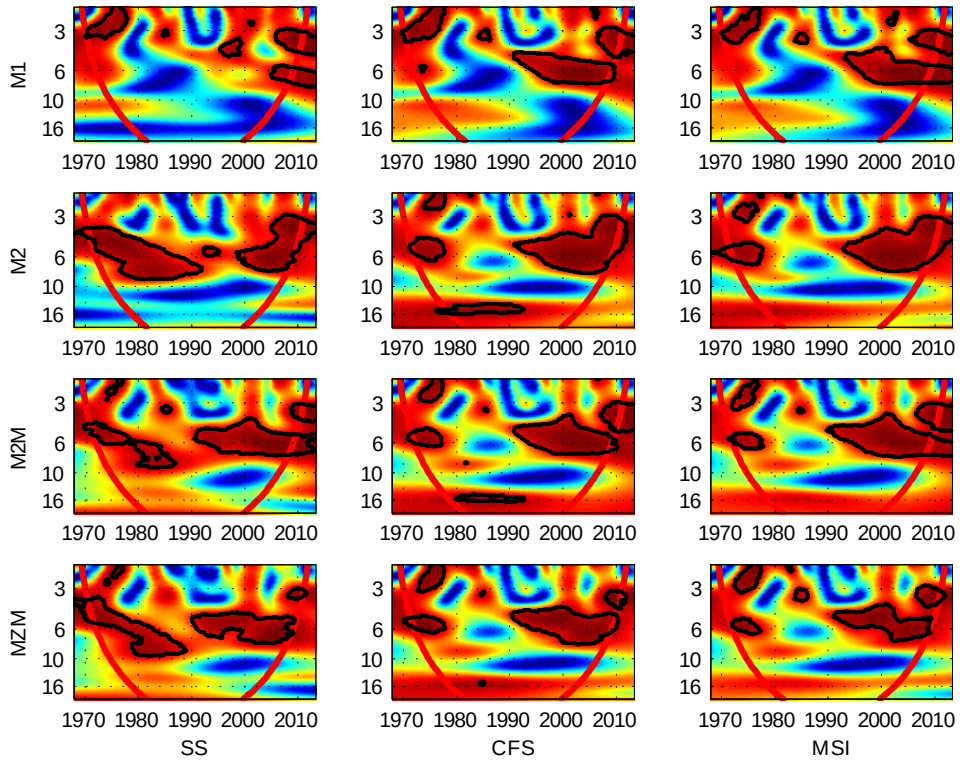

Figure 5: Wavelet coherency of annual growth rate of monetary aggregates(adjusted for real GDP growth) and annual CPI inflation.

with $\widehat{\beta}_1$as the OLS estimate of the interest rate semi-elasticity of velocity.

For estimation, the opportunity cost of holding money is represented by the three-month Treasury Bill rate. The interest-rate semi-elasticities vary widely across the monetary aggregates. The results shown below are based on a value of -4 which is close to the average of the estimates for the simple sum monetary aggregates. Figure 6 presents the estimated wavelet coherencies of adjusted money growth and annual CPI inflation. The changes are most striking for the Divisia aggregates M2, M2M and MZM which now exhibit strong and significant wavelet coherencies with inflation at frequencies with periods of four years and longer extending almost all the way through the interpretable region. At frequencies between four and eight years there are almost continuous significant coherencies throughout the sample period and also at the lower frequency spectrum there is evidence for a strong and stable relationship between Divisia money growth and inflation although there is some weakening in the 2000s. This could reflect effects from the financial crisis, since, at these frequencies the variable windows of the wavelet decomposition already start in the early 2000s to include observations from this period. For MZM the strength of the relationship is more concentrated at the lower frequencies and not so much in the middle band. The improvements for M1 compared to Figure 5 are not as impressive as for the broader aggregates but the estimated coherencies, nevertheless, increase strongly in the middle and lower frequency bands. Turning to the simple sum aggregates (left column) the adjustment of the monetary aggregates for changes in velocity leads only to a modest increase in the wavelet coherencies at low frequencies and, for M2M and MZM tends to weaken the estimated relationship over the middle frequency range compared to Figure 5. The results for simple sum M1 are very similar to those for the Divisa aggregates. In summary, our estimates indicate strong comovements between the adjusted Divisia money growth series and CPI inflation which are much more pronounced than for simple sum monetary aggregates. Comparison of the results for the CFS and MSI Divisia aggregates shows both to lead to very similar results.

The estimated time differences (see (7)) indicate whether money growth is preceding or following inflation in time. From the perspective of monetary policy a useful indicator property of money growth for future inflation requires a sufficient lead in money growth before price level developments. Figure 7 displays the estimated time differences for the unadjusted money growth series with CPI inflation. Positive (negative) values indicate a lag (lead) of money growth before inflation. For fluctuations with periods between 12 and 16 years. . We focus on these lower freuquencies as in our experience with many applications the time differences tend to become very unstable for medium to higher frequencies. Our results show that the estimated time differences for the simple sum aggregates exhibit instabilities at various points in time, first increasing (M2, M2M, MZM) in the late 1970s and the falling again in the middle 1990s. More importantly, the time differences for simple sum M2M and MZM switch from money growth lagging inflation until the mid 1990s to a lead of money growth afterwards. Only for M2 the estimates indicate a consistent lead before inflation although with a changing time difference. In contrast, the Divisia

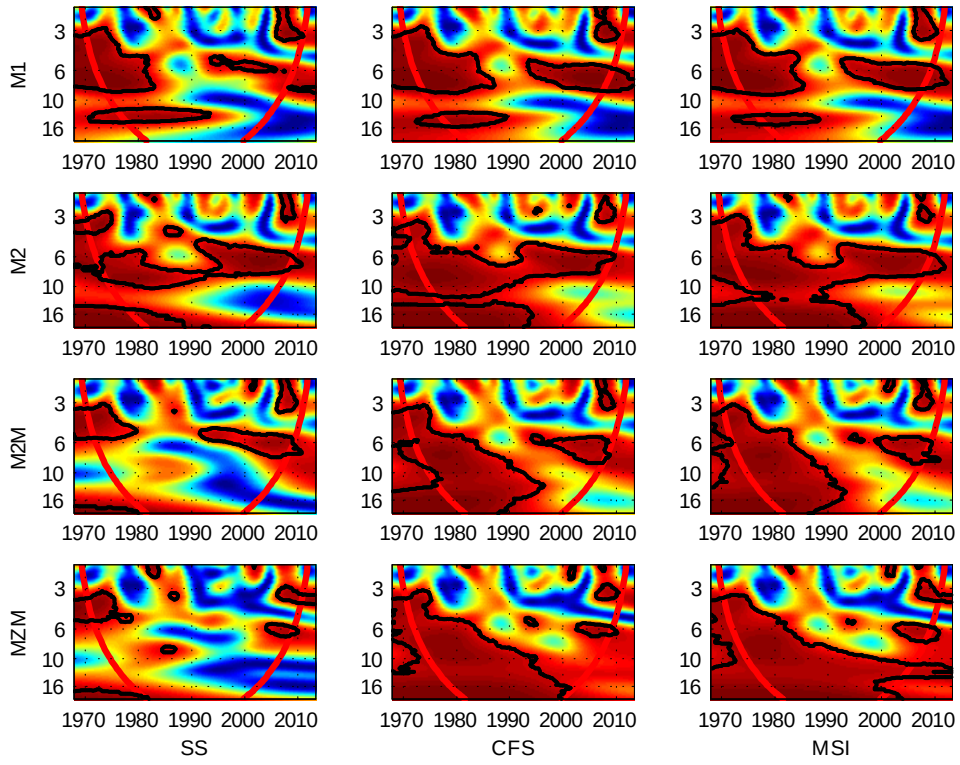

Figure 6: Wavelet coherency of annual growth rate of monetary aggregates (adjusted for real GDP growth and changes in velocity) and annual CPI inflation.

aggregates for M2, M2M and MZM are estimated to have a very stable lead before inflation of about four to five years. Only for M1 the growth in the Divisia aggregate exhibits a lag for most of our sample period and then switches to a lead in the early 2000s, possible due to the effects of the financial crisis on the estimates.[12] These results are confirmed if we compute the time differences for money growth adjusted for real GDP growth (Figure 8) and if we use the money growth rates adjusted for real GDP growth and for changes in velocity (Figure 9). In the latter case the lead of adjusted Divisia money growth before inflation turns out somewhat shorter with about 1.5 to two years but remains stable through time. For M1, the adjustment for velocity changes results in an almost contemporaneous relationship between M1 growth and inflation for all the aggregates for most of the sample period.

While the cross-spectral coherency measures the extent of covariability in the fluctuations in both time series at a given frequency range, comparable to the R^2 in a regression analysis,. the cross-spectral gain (8) is a measure of the relative size of the comovements in both series at a given frequency range. It can be interpreted as a regression coefficient in a regression of specific frequency components of inflation on the corresponding frequency components in money growth. At frequency zero it captures the long-run (permanent) relationship between money growth and inflation, as in the Lucas regressions (Lucas, 1980). An estimated gain of one at frequency zero would imply that the quantity theory holds in the long run, i.e. that inflation and money growth move one to one. In wavelet analysis the length of the sample period limits the frequency spectrum that can be analysed, as estimation for lower frequencies requires more and more data. As a consequence, we are unable to present evidence for the relationship at frequency zero and approximate the long run be looking at fluctuations with periods between twelve and sixteen years.[13]

Figures 10 to 12 show the estimated cross-spectral gains for fluctuations with periods between 12 and 16 years of CPI inflation with respect to unadjusted

[12] Since the time differences are based on phase differences between the cycles of the two time series in question, the methodology effectively distinguishes a lead from a lag by proximity. A phase difference between 0 and π, which equals a time difference of up to half of the period of fluctations, is interpreted as a lag of money growth behind inflation, while a phase difference between $-\pi$ and 0 is interpreted as the equivalent lead of money before inflation. Hence, the separation of leads from lags is somewhat arbitrary. For example, for fluctations with periods of 16 years a phase difference interpreted as a lag of money growth after inflation of seven years could also be interpreted as a lead of money growth of nine years.

[13] Estimating the gain at frequency zero would, in fact, require an infinitely long sample period. Parametric methods can be used to estimate cross spectra at low frequency even on finite data sets as they allow to "extrapolate" the relationships between the variables to frequencies not actually contained in the data. See, for example, Sargent and Surico (2011) who estimate a time-varying VAR and derive the cross-spectral gain at frequency zero from the estimated VAR coefficients.

Given our sample size, the maximum length of period we can get estimates for is about 23 years. This however, would us only allow to get one estimate at the middle of the sample and, hence, would leave no scope for studying time-variation in the relationship. Against this background, looking at periods with length between 12 and 16 years is a reasonable compromise.

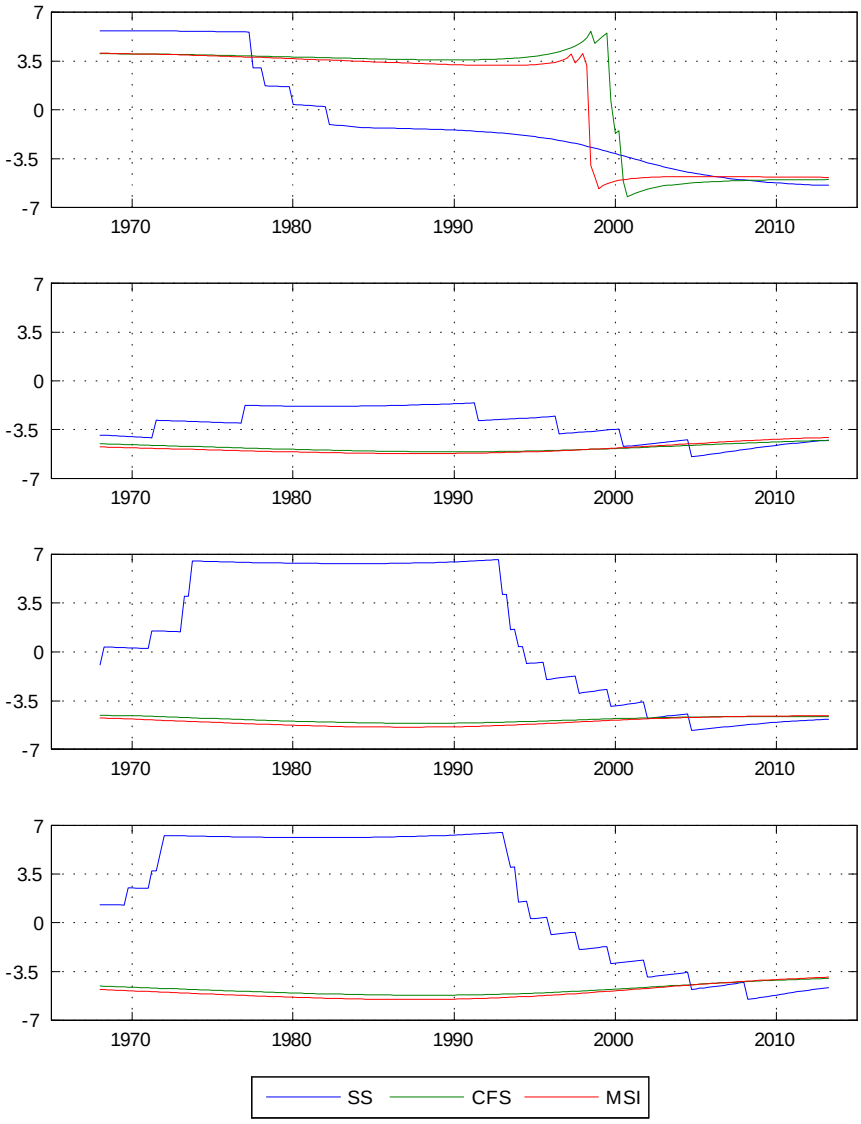

Figure 7: Time difference between annual money growth and annual CPI inflation. (Positive numbers indicate lag of money growth, negative numbers indicate lead. Frequencies with fluctuations of 12-16 years.)

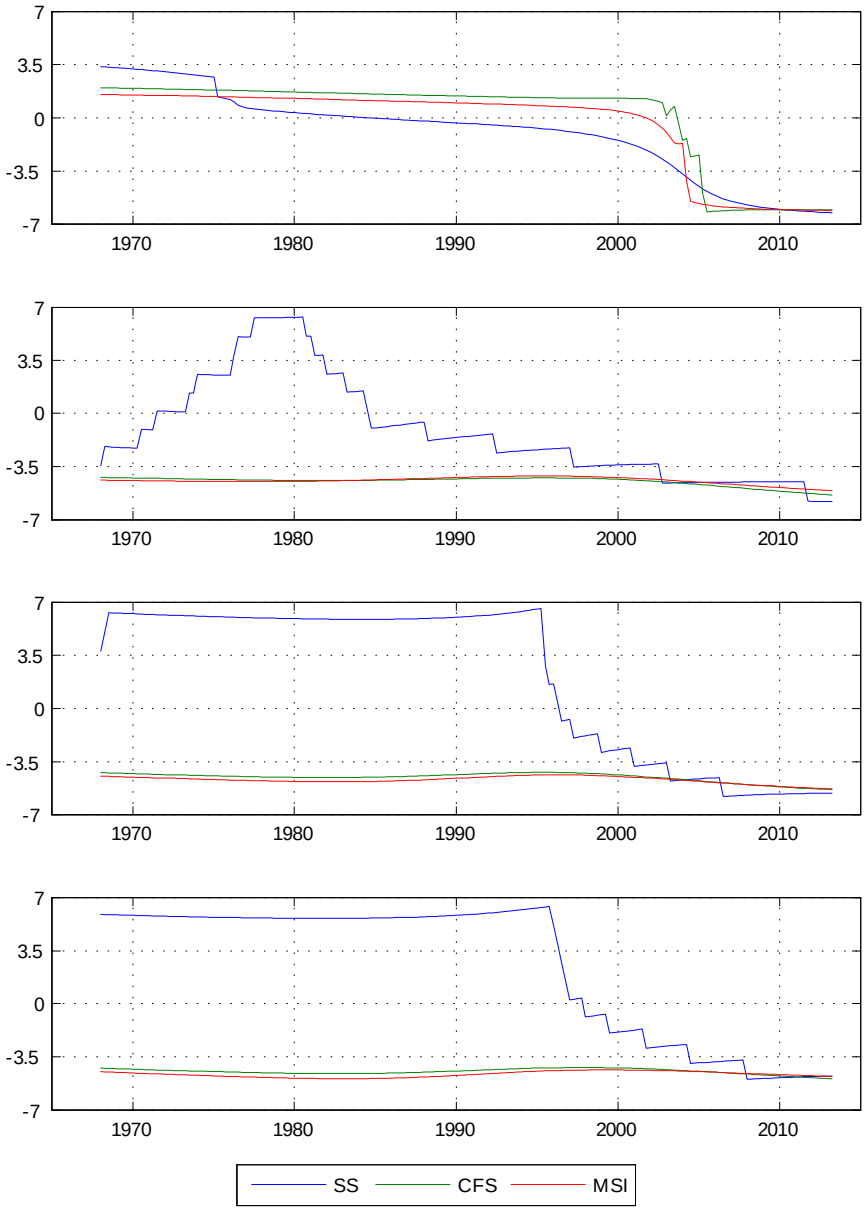

Figure 8: Time difference between annual money growth (adjusted for real GDP growth) and annual CPI inflation. (Positive numbers indicate lag of money growth, negative numbers indicate lead. Frequencies with fluctuations of 12-16 years)

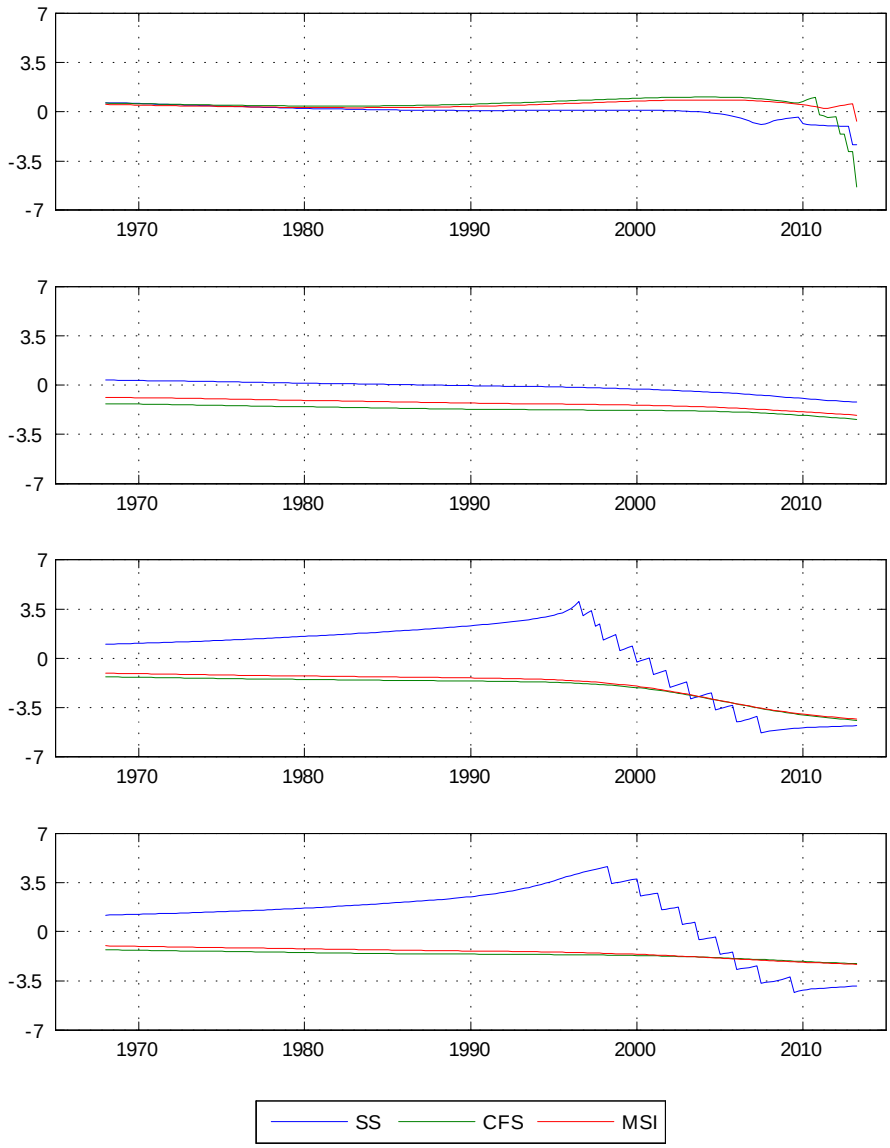

Figure 9: Time difference between annual money growth (adjusted for real GDP growth and changes in velocity) and annual CPI inflation. (Positive numbers indicate lag of money growth, negative numbers indicate lead. Frequencies with fluctuations of 12-16 years.)

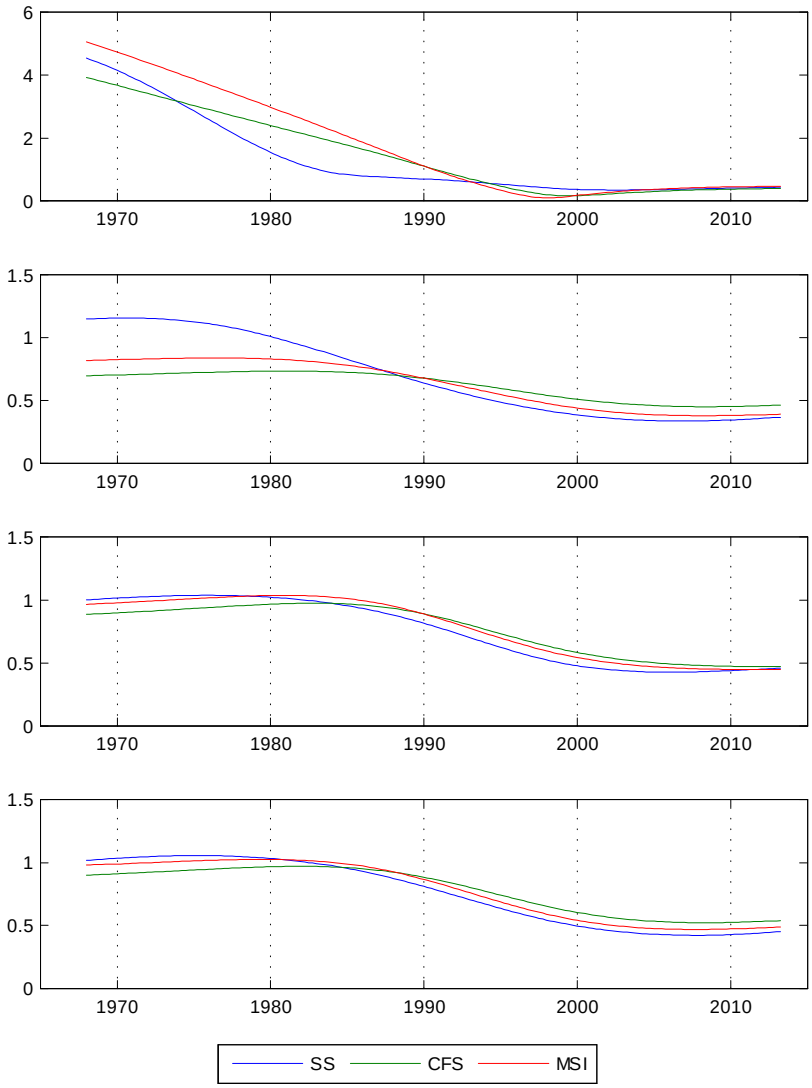

Figure 10: Cross spectral gain between CPI inflation and money growth (Rows: M1, M2, M2M, MZM, fluctuations with periods between 12 and 16 years).

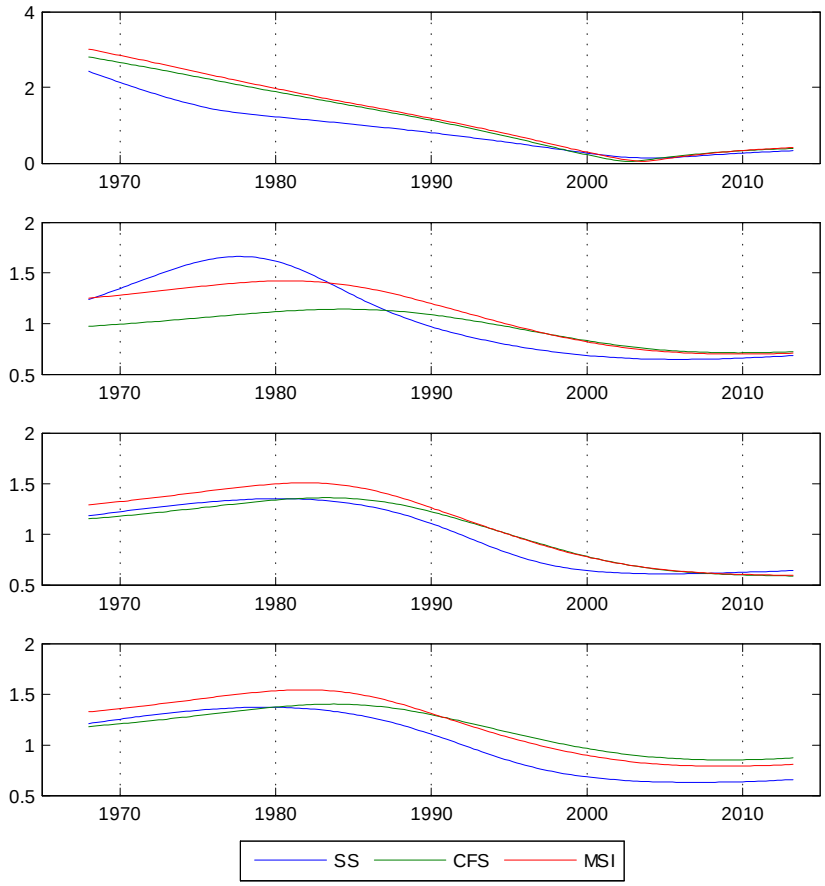

Figure 11: Cross spectral gain between CPI inflation and money growth (adjusted for real GDP growth) (Rows: M1, M2, M2M, MZM, fluctuations with periods between 12 and 16 years).

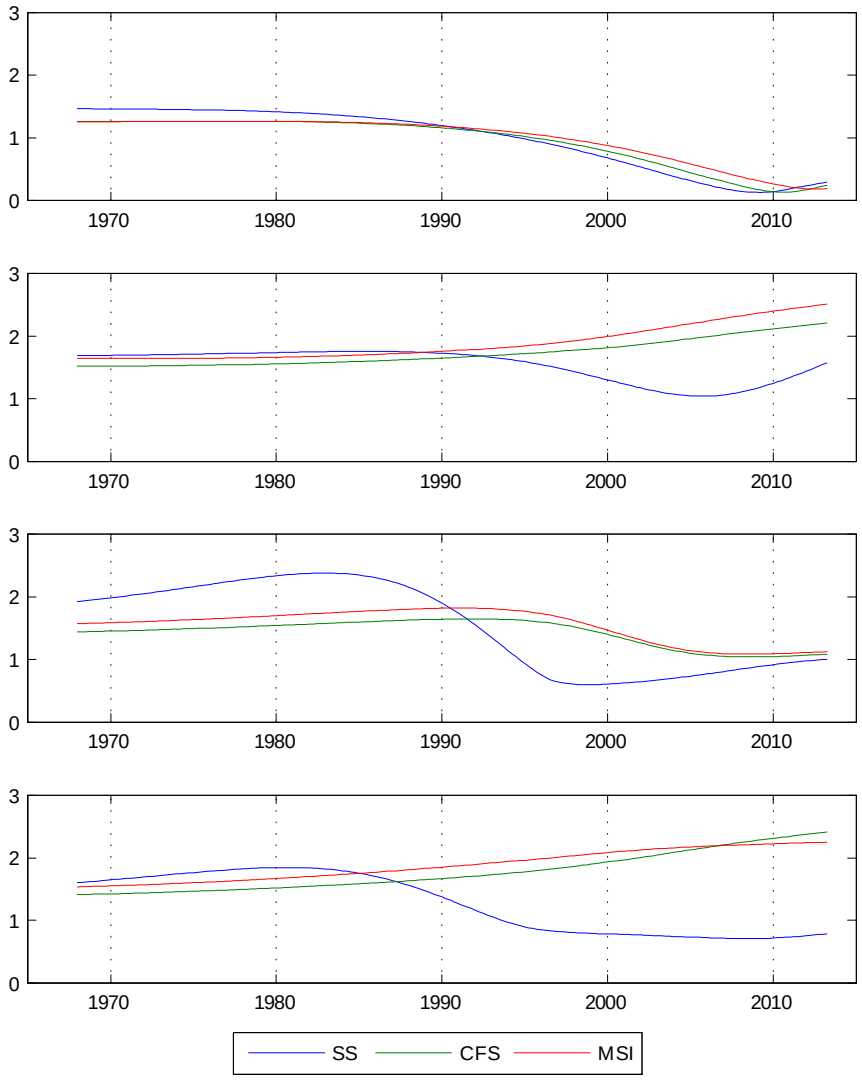

Figure 12: Cross spectral gain between CPI inflation and money growth (adjusted for real GDP growth and changes in velocity) (Rows: M1, M2, M2M, MZM, fluctuations with periods between 12 and 16 years).

money growth, money growth adjusted for GDP growth and money growth adjusted for GDP growth and changes in velocity.[14] For the unadjusted M1 series the gain starts out very high and then drops towards one with similar results for simple sum and Divisia aggregates. For M2 the gain using the simple sum aggregate starts out above one with the one estimated using the Divisia aggregates slightly below one, then the gain declines for all aggregates converging to a value of about 0.5 or slightly below, in case of the simple sum aggregate. Using a time-varying VAR for the U.S. Sargent and Surico (2011) estimate cross spectral gains between money growth (M2) and inflation (measured by the growth in the GDP deflator). Their results indicate that the cross-spectral gain at frequency zero is not significantly different from one in the 1970s but declines significantly below one after 1980 with point estimates around 0.25. Our results for simple sum M2 are broadly consistent with this evidence, for the Divisia aggregates the change is, however, less pronounced. Sargent and Surico attribute the strong decline in the cross spectral gain at frequency zero to a more aggressive monetary policy reaction function which implies a shift in the cross spectral gain towards zero. Based on a DSGE model they show that a cross spectral gain at low frequencies around one results from the central bank allowing persistent innovations in money by reacting to weakly to inflationary pressures. In contrast, a monetary policy reaction function in which the central bank responds aggressively to inflationary pressures leads to a gain close to zero.

For M2M and MZM the results for the growth rates based on unadjusted simple sum and Divisia aggregates are very close to each other - they start around one and ten slowly decline towards 0.5. Adjusting for real GDP growth (Figure 11) leads to lower estimated gains for M1 with a similar qualitative pattern as before. For the other monetary aggregates the gains turn out somewhat higher than in Figure 10 at the beginning of the sample. They start out somewhat above one and then decline to somewhat below with not much difference between simple sum and Divisia for M2M and MZM. For M2, however, the gain estimated for the simple sum aggregate is slightly less stable than for the Divisia monetary aggregates. Finally, using the money growth rates adjusted for both real GDP growth and changes in velocity the gain estimated using the Divisia aggregates for M2, M2M and MZM is initially above one and, beginning in the mid 1990s increases even further for M2 and MZM while declining towards one for M2M (Figure 12) For the simple sum aggregates the gain behaves less systematically and tends to be lower in the late sample than at the beginning. The gain for M1 evolves similarly for all types of monetary aggregates and from the early 1990s onward falls from about one to zero. Overall the gains computed for the Divisia aggregates tend to behave more stable than those for the simple

[14] The figures do not indicate rejection regions around zero or one to test for statistical significance. Simulated probability bands for these null hypotheses turned out to be extremely wide. For example, the 90-percent band for the null hypothesis of a cross-spectral gain of zero always also covers one. This however, is not specific to gains estimated using wavelet analysis. For example, the estimated gains for money growth and inflation in Sargent and Surico (2011) which are computed from the coefficients of an estimated VAR model exhibit similar problems and the probability bands around their estimates often cover zero and one simultaneously.

sum aggregates.

4 Summary and conclusions

Comparing the results from our wavelet analyses for simple sum and Divisia monetary aggregates we find no evidence for a tighter link between fluctuations in the growth rates of Divisia monetary aggregates with those in real GDP growth than for simple sum monetary aggregates. In fact, except for M1 the relationship between real money growth and real GDP growth is estimated to be weaker, especially at low frequencies when using the Divisia aggregates.

Turning to the relationship between money growth and inflation, our results show much stronger comovements between CPI inflation and growth in Divisia monetary aggregates than between inflation and growth rates in simple sum aggregates, especially at low frequencies. Adjusting money growth for real GDP growth and accounting for changes in velocity strengthens these results even more. The relationship between Divisia money and inflation remains strong throughout our sample period. Only for M1 the results for simple sum and Divisia aggregates turn out to be very similar. Furthermore, while the lead-lag patterns between money growth and inflation exhibits some instabilities for simple sum monetary aggregates, growth rates in broader Divisia monetary aggregates are estimated to have a stable lead before CPI of about four to five years. Adjusting money growth for real GDP growth and changes in opportunity costs shortens this lead to about 1.5 to two years. The results for the cross-spectral gains at low frequencies, i.e. for the level relationship between changes in money growth and changes in inflation is less conclusive but the estimated gains tend to be more stable for the Divisia than for the simple sum aggregates. Comparison of the results for CFS (Center for Financial Stability) and MSI (Federal Reserve Bank of St.Louis) types of Divisia aggregates does not suggest marked differences between them. Overall, our results provide evidence that the indicator properties of long-term changes in money growth for future long-term changes in inflation in the U.S. are more pronounced when using Divisia monetary aggregates instead of simple sum aggregates. This suggests, that the Federal Reserve should emphasize the role of Divisia monetary aggregates over that of conventional monetary aggregates.

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