Improve volatility forecasting with realized semivariance

- Evidences from intra-day large data sets in Chinese

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Abstract Barndorff-Nielsen, Kinnebrock and Shephard (2008) introduce a new measure of variation called realized semivariance. This estimator is reported more informative than realized variance. This paper employs a new modeling approach for the realized semivariance inspired by Chou (2005) in order to better capture the asymmetry of volatility in financial markets. With high frequency data from Shanghai stock market in Chinese, the empirical results using four types of volatility proxies including squared daily returns, daily high-low price ranges, realized variance, and realized range consistently indicate that this model sharpens the forecast power of existing volatility models in terms of GARCH type models. Four loss functions are employed for the assessments in out of the sample forecasting.

Keywords: realized volatility; semi-variance; MEM model; loss function

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1. Introduction

Volatility has been a traditional measure of risk. It plays a key role in the areas of asset pricing, portfolio allocation, and risk management. As transaction data is becoming more widely available, great interest has been drawn into the use of high frequency data for measuring and forecasting volatility. This is called the realized volatility approach. One advantage of the new emerging nonparametric volatility approach is that it can fully exploits intraday information and deliver a observable proxy for the volatility and therefore make the direct modeling volatility possible and avoid complicated estimation procedures needed for the unobservable volatility approach – using the GARCH type and stochastic volatility models.

Barndorff-Nielsen, Kinnebrock and Shephard (2008) introduce a new measure for the variation of asset prices based on high frequency data. It is called realized semivariance (RS) and is reported more informative than the simple realized variance. Inspired by Chou (2005), we adopted the same methodology in that paper for the realized semivariance to better capture the asymmetry in financial markets. Intuitively this modeling approach combined with realized semivariance can sharpen the forecast power of existing volatility models. We also confirm this in our empirical study through a comparison of four GARCH-type models for non-negative series, proposed by Engle (2002) and known as Multiplicative Error Model (MEM). We employ Shanghai composite index data of one minute's frequency to obtain our daily and realized volatility estimators. According to Engle (2005), different volatility proxies contain different information about volatility. Therefore, we use six different volatility proxies of both daily frequency and high frequency as the measure volatility: squared daily returns, absolute daily returns, daily high-low price ranges, realized variance, realized range, and realized bipower variation. They consistently indicate that our modeling approach sharpens the forecast power of non-negative series GARCH type models. We use four loss functions in Hansen and Lunde (2005) as criterions for assessing the forecasting ability of the models. All in sample and out of sample prediction consistently confirmed our intuition that this modeling approach combined with realized semivariance is able to sharpen the forecast power of non-negative series GARCH type models. The rest of this paper has the following structure. In section 2 we will discuss the theory of realized volatility and semivariances. Section 3 introduces our empirical funds. Section 4 is the model comparisons. Section 5 concludes.

2. Realized volatility, realized semivariance and the model

Realized variance estimates the ex-post variance of asset prices over a fixed time interval. Since we are going to carry out our empirical analysis based in trading time, we define realized variance as:

$$RV_t = \sum_{i=1}^{I} (P_{t,j+1} - P_{t,j})^2$$

RV_t is then the sum of squared intraday returns. Though the data arrives into our database at irregular points in time, according to Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) these irregularly spaced observations can be thought of as being equally spaced observations on a new time-changed process in the same stochastic class. Thus there is no intellectual loss in initially considering equally spaced returns. In arbitrage free markets, P is often considered to follow a semimartingale process. Then as we have more and more data in one day's time interval RV_t must converge into:

$$RV_t = \sum_{i=1}^{I} (P_{t,j+1} - P_{t,j})^2 \xrightarrow{p} \int_0^1 \sigma_s^2 ds$$

Where $P_t = \int_0^t \mu_s ds + \int_0^t \sigma_s ds$. μ is a locally bounded predictable drift process and σ is a cadlag volatility process, which adapted to some common filtration F_t. Barndorff-Nielsen, Kinnebrock and Shephard (2008) introduce a new measure of variation called realized semivariance. This kind of estimator is solely determined by the single side (upward and downward) moves in high frequency asset prices defined as:

$$RS_t^- = \sum_{i=1}^{I}(P_{t,j+1} - P_{t,j})^2 1_{P_{t,j+1}-P_{t,j}\le 0}$$

$$RS_t^+ = \sum_{i=1}^{I}(P_{t,j+1} - P_{t,j})^2 1_{P_{t,j+1}-P_{t,j}\ge 0}$$

Where 1_P is the indicator function taking the value 1 of the argument is true and 0 otherwise. If P is a semi martingale without jumps as $P_t = \int_0^t \mu_s ds + \int_0^t \sigma_s^2 ds$, then there would be no difference between RS_t^- and RS_t^+. They both converge into:

$$RS_t^- \xrightarrow{p} \frac{1}{2}\int_0^1 \sigma_s^2 ds \xleftarrow{p} RS_t^+$$

Under in-fill asymptotics. But if there are jumps in the process of P:

$$P_t = \int_0^t \mu_s ds + \int_0^t \sigma_s ds + J_t$$

Then the realized variance of P converges into:

$$RV_t = \sum_{i=1}^{I}(P_{t,j+1} - P_{t,j})^2 \xrightarrow{p} \int_0^1 \sigma_s^2 ds + \sum_{s\le t} J_t^2$$

And the downward realized semivariance and upward realized semivariance will converge into different limits under in-fill asymptotic:

$$RS_t^- \xrightarrow{p} \frac{1}{2}\int_0^1 \sigma_s^2 ds + \sum_{s\le t} J_t^2 1_{P_{t,j+1}-P_{t,j}\le 0}$$

$$RS_t^+ \xrightarrow{p} \frac{1}{2}\int_0^1 \sigma_s^2 ds + \sum_{s\le t} J_t^2 1_{P_{t,j+1}-P_{t,j}\ge 0}$$

From above, we can easily see that: $RV_t \overset{p}{=} RS_t^- + RS_t^+$. But since the two components of RV_t can be distinguished, it must be more informative than mixed together. For the purpose of volatility measuring, we also introduce two another realized measures here. The first one is called realized range, proposed by Christensen and Podolskij (2005) and Martens and van Dijk (2007). This estimator is inspired by the idea of Parkinson (1980) that range-based variance estimator is much more efficient than return-based estimator. And this one is indeed reported more efficient and less contaminated by micro noises in empirical study. It is defined as follows:

$$RR_t = \frac{1}{4\log 2}\sum_{i=1}^{I}(H_{t,j+1} - L_{t,j})^2 \xrightarrow{p} \int_0^1 \sigma_s^2 ds + \sum_{s\le t} J_t^2$$

In a driftless martingale process, this estimator also converges to quadratic variation. Usually for the estimation of one day's volatility, driftless martingale process assumption is not a bad one. The second one is called realized bipower variation. This estimator is proposed by Barndorff-Nielsen and Shephard (2002). It is defined as:

$$RB_t = \frac{1}{\mu_1^2}\sum_{i=1}^{I} | P_{t,j+1} - P_{t,j} \| P_{t,j} - P_{t,j-1} | \xrightarrow{p} \int_0^1 \sigma_s^2 ds$$

Where μ_1 is a normalization factor. And in a semimartingale process with finite jumps, realized bipower variation converges to integrated variation but not quadratic variation.

Inspired by Chou (2005), we can see that his model can be naturally extended to model the upward (downward) realized semivariances with a little modification:

$$RS_t^{+(-)} = \lambda_t^{+(-)}\varepsilon_t^{+(-)}, \quad \varepsilon_t^{+(-)} \sim iid.f^{+(-)}(\cdot)$$

$$\lambda_t^{+} = \omega^{+} + \sum_{i=1}^{p}\alpha_i^{+} RS_{t-i}^{+} + \sum_{j=1}^{q}\beta_j^{+}\lambda_{t-j}^{+}, \lambda_t^{-} = \omega^{-} + \sum_{i=1}^{p}\alpha_i^{-} RS_{t-i}^{-} + \sum_{j=1}^{q}\beta_j^{-}\lambda_{t-j}^{-}$$

$$E[RV_{t+1} | I_t] = E[RS_{t+1}^{+} + RS_{t+1}^{-} | I_t] = E[RS_{t+1}^{+} | I_t] + E[RS_{t+1}^{-} | I_t] = \lambda_t^{+} + \lambda_t^{-}$$

We call this model as Asymmetric Multiplicative Error Model (AMEM), see Engle and Gallo(2006), for Realized Semivariance (AMEM-RS). In the following empirical study, we compare volatility forecasting power in context of out-of-sample forecast of four different models: MEM-RV, MEM-RV with lagged return, AMEM-RS and AMEM-RS with lagged return.

3. Empirical results

To calibrate our models, we employ high frequency Shanghai composite index data in this paper. The data contain observations from January 1, 2007 to January 4, 2013. After deleting the days of unavailable and insufficient information, we have 1570 days' observations of 1 minute's frequency data. The data is from the Windin database. Table 1 gives out the descriptive statistics of raw data and daily estimators obtained from raw data in everyday.

Table 1: The descriptive statistics of raw and daily data

	Raw prices	Raw returns	Daily returns	Squared returns	Absolute returns	Ranges
Mean	2957	-9.5E-06	-3.3E-04	1.512	0.921	1.709
Median	2774	0.000	0.0337	0.489	0.699	1.481
Maximum	6092	2.951	6.940	48.167	6.940	7.731
Minimum	1707	-5.653	-5.801	0.000	0.000	0.398

Std. Dev.	882.5	0.064	1.230	3.087	0.815	0.921
Skewness	1.524	-1.600	0.012	6.220	1.897	1.945
Kurtosis	4.845	252.7	5.164	62.965	8.932	9.113
Jarque-Bera	36726	1.6E+09	306	245344	3244	3434
Probability	0.000	0.000	0.000	0.000	0.000	0.000
Sum	7.8E+08	-5.970	-0.515	2374	1446	2684
Observations	628934	628933	1570	1570	1570	1570

*Raw returns, daily returns and range are all multiplied by 100; squared returns and absolute returns are respectively the squared value and absolute value of daily returns;

In order to compare models in terms of their prediction accuracy, we need to use proper proxies for underlying unobservable true volatility. According to Engle and Gallo (2006), there is still no consensus about a “true” or "best" measure of volatility. And “many ways exist to measure and model financial asset volatility”. Here we employ six measures of asset volatility for our model comparison. Three of them are three ordinary daily measures: absolute daily returns, daily Parkinson high-low range estimator and the most usual squared daily returns. We give their statistics description in Table 1. The other three of them are realized volatility measures: realized variance, realized range and realized bipower variation with the most used 5 minutes’ frequency. Table 2 gives their statistics description together with RS+ and RS-.

Table 2: The descriptive statistics of realized estimators

	RV	RR	RB	RS+	RS-
Mean	1.642	1.110	1.636	0.811	0.831
Median	1.057	0.758	1.112	0.501	0.493
Maximum	35.663	23.719	21.293	19.202	33.875
Minimum	0.132	0.117	0.157	0.074	0.046
Std. Dev.	2.114	1.393	1.866	1.146	1.375
Skewness	6.303	6.837	4.896	6.321	11.779
Kurtosis	69.475	79.844	38.567	67.219	238.107
Jarque-Bera	299464	398511	89026	280241	3652223
Probability	0.000	0.000	0.000	0.000	0.000
Sum	2577	1743	2569	1273	1304
Observations	1570	1570	1570	1570	1570

Figure 2 presents the time series of RS+ against RS-. These two parts of realized variance do look very different from each other, and therefore two separately models for each of them is necessary and maybe fruitful.

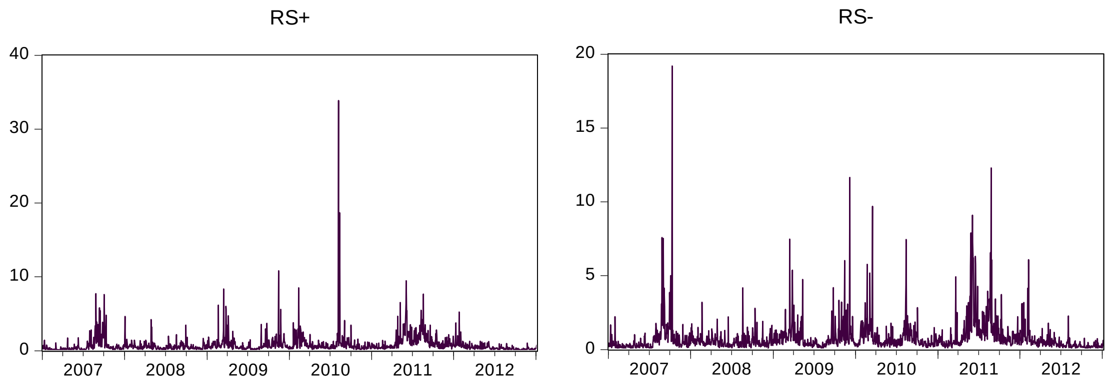

Fig 1: Upside and downside realized semivariance

In order to better incorporate the leverage effects of lagged returns, we estimate four models in this section: MEM-RV, MEM-RV with lagged returns, AMEM-RS and AMEM-RS with lagged returns. We employ the simplest form GARCH model for all of the four models – GARCH (1, 1), which is already adequacy in most applications according to Bollerslev, Chou, and Kroner (1992). Table 3 presents the estimated parameters of the four models.

Table 3: MEM type models for realized volatility and semivariance

	MEM-RV		MEM-RS			
			RS+	RS-	RS+	RS-
Constant	0.037 (0.050)	0.035 (0.032)	0.016 (0.017)	0.017 (0.016)	0.015 (0.011)	0.026 (0.011)**
ARCH	0.367 (0.097)***	0.219 (0.087)**	0.257 (0.078)***	0.282 (0.058)***	0.158 (0.052)***	0.152 (0.054)***
GARCH	0.623 (0.090)***	0.760 (0.086)***	0.731 (0.071)***	0.710 (0.065)***	0.823 (0.051)***	0.814 (0.057)***
Return(-1)		-0.097 (0.049)**				-0.065 (0.015)***
Return(-2)					-0.046 (0.022)*	
Log-L	-2446.0	-2438.9	-1910.8	-1912.5	-1902.5	-1896.1

Model selection is based on AIC and BIC and numbers in parenthesis are the standard deviations, and stars refer to significance level of 10% (*), 5% (**) and 1% (***).

4. Models comparison

According to Hansen and Lunde (2005) we continue to use the four loss functions employed by them as criterions for model:

$$MSE = n^{-1}\sum_{i=1}^{n}(MV_t - FV_t)^2$$

$$MAE = n^{-1}\sum_{i=1}^{n}| MV_t - FV_t |$$

$$QLIKE = n^{-1}\sum_{i=1}^{n}(\ln FV_t - MV_t / FV_t)^2$$

$$R^2LOG = n^{-1}\sum_{i=1}^{n}\ln^2(MV_t / FV_t)$$

The first two loss functions are regular ones. *QLIKE* is proposed by Bollerslev (1994), and is also called Gaussian quasi-maximum likelihood function, which can easily see that it is originated from the likelihood function of GARCH model from its formulation. R^2LOG is proposed by Pagan and Schwert (1990), it aims to give some penalty to the asymmetry of the volatility forecasting. Different from the quadratic loss function, it was a proportional loss function. We focus on the out of sample comparisons for finding useful models in prediction of real world. In table four, r^2, |r|, range, realized volatility, realized range and realized bipower variance are used as measurement volatility (MV) to judge the out of sample forecasting of the four models. It is clear that with most loss functions the lagged realized semivariance (RS-Lag) performs better than other forecasted volatilities (FV).

Table 4: out of sample forecasting comparisons with different loss functions

	r^2	|r|	Range	RV	RR	RB
			Loss function: MSE			
RV	1311.01	316.10	173.71	161.84	166.19	147.48
RV-Lag	1291.31	264.08	130.04	159.16	128.32	154.23
RS	1314.82	299.29	156.18	159.38	156.75	151.35
RS-Lag	1275.95	227.06	104.69	159.07	110.93	155.88
			Loss function: MAE			
RV	177.15	110.01	79.57	70.02	75.87	61.68
RV-Lag	175.46	104.57	74.08	69.30	71.58	62.83
RS	178.71	109.48	76.87	70.22	76.95	62.96
RS-Lag	173.93	100.14	69.41	69.15	69.96	62.58
			Loss function: QLIKE			
RV	296.25	137.68	267.57	119.68	79.97	109.97
RV-Lag	280.83	130.42	257.72	120.26	74.55	110.61
RS	274.01	120.77	232.00	104.60	68.97	95.95
RS-Lag	256.58	117.03	233.38	110.12	65.46	101.07
			Loss function: R2LOG			
RV	581.08	184.60	30.35	23.71	45.23	18.45
RV-Lag	575.44	180.40	28.11	23.20	41.84	18.39
RS	586.71	186.37	27.38	24.15	46.95	19.09
RS-Lag	576.70	179.22	25.32	23.73	41.91	18.90

For RS models we use upside RS and downside RS forecasting to synthesize RV forecasting and the model with minimum forecasting errors under four types of loss functions and six types of "true volatility" measurements are highlighted.

In the Appendix, we also presented the in-sample forecasting performance under the same criterions.

5. Conclusion

Volatility is one of the core problems in many financial practices but the asymmetry of volatility is often confused in arbitrage and risk management because of downside volatility is definitely not equal to upside volatility in these fields. Separately modeling the two sides of volatility would be more informative than just mixing them together.

In this paper, we use a new modeling approach to model the realized semi variance with high frequency data in Chinese financial markets. Then the empirical study shows that when measured by six different volatility proxies, the realized semi variance (RS) performs better than the traditional realized volatility estimator (RV).

These findings mean when measuring volatility or fluctuations of financial assets, the usage of our new estimator will increase the performance of many financial practices like pricing or risk management. With the development of the information technology, high frequency data are more and more available. Developing an accurate and robust estimation of assets volatility is more and more important. One feasible way to extend this paper is to incorporate the correlation effects between upside realized semivariance and downside ones.

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Appendix: In sample comparison

Table 5: in sample forecasting comparisons with different loss functions

	r^2	|r|	Range	RV	RR	RB
			Loss function: MSE			
RV	917.34	281.96	175.48	329.61	239.66	229.96
RV-Lag	868.62	223.06	126.37	304.65	194.04	206.50
RS	900.61	258.05	152.80	318.47	223.53	220.72
RS-Lag	857.14	195.18	105.52	299.71	178.72	202.70
			Loss function: MAE			
RV	161.01	103.54	80.22	81.53	84.83	72.35
RV-Lag	155.57	97.17	72.51	77.34	78.68	68.61
RS	160.62	102.74	77.13	81.24	85.25	72.52
RS-Lag	154.68	94.45	68.36	76.89	77.82	68.17
			Loss function: QLIKE			
RV	318.62	137.03	265.86	151.53	97.56	136.82
RV-Lag	296.85	129.43	253.80	146.52	88.02	131.02
RS	291.97	120.12	232.62	138.01	85.96	122.61
RS-Lag	280.73	114.25	224.65	133.55	77.94	118.88
			Loss function: R2LOG			
RV	567.85	178.04	32.61	32.08	55.92	25.60
RV-Lag	552.63	170.76	28.97	29.77	50.91	23.26
RS	572.36	179.32	29.69	32.67	57.67	26.03
RS-Lag	556.11	170.91	26.11	30.29	52.32	23.71