# Does it Pay to Invest in Art? A Selection-corrected Returns Perspective 

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#### Abstract

This paper shows the importance of correcting for sample selection when investing in illiquid assets with endogenous trading. Using a large sample of 20,538 paintings that were sold repeatedly at auction between 1972 and 2010, we find that paintings with higher price appreciation are more likely to trade. This strongly biases estimates of returns. The selectioncorrected average annual index return is 6.5 percent, down from 10 percent for traditional uncorrected repeat sales regressions, and Sharpe Ratios drop from 0.24 to 0.04 . From a pure financial perspective, passive index investing in paintings is not a viable investment strategy once selection bias is accounted for. Our results have important implications for other illiquid asset classes that trade endogenously.


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Over the last two decades investors are allocating increasingly larger shares of their portfolios to alternative assets. Many of the alternative asset classes, such as private equity and real estate, and even certain traditional assets such as corporate bonds, are highly illiquid. This complicates return evaluation, especially when trades are endogenously related to the performance of the asset, giving rise to a sample selection problem. This paper demonstrates the empirical first-order importance of correcting for sample selection when evaluating performance and constructing optimal portfolios that include alternative assets.

Among the alternative assets, paintings (and other collectibles) are often considered a comparatively safe investment in times of financial turmoil (Dimson and Spaenjers, 2011). Finding low or negative correlations between art ${ }^{1}$ and public equity markets, several recent academic studies argue that art investments should be included in optimal asset allocations (see, for example, Mei and Moses, 2002; Taylor and Coleman, 2011). Indeed, investors allocate about 6 percent of their total wealth to so-called passion investments ${ }^{2}$, and several art funds have been created to allow for diversified investments in art.

The returns and risks of art investments are, however, not beyond debate, and indeed not well known. Constructing an art index and computing the return to art investing is a non-trivial exercise, as prices are not observed at fixed intervals, but only when the artwork trades. Goetzmann $(1993,1996)$ argues that these trades are endogenous, and he conjectures that paintings that have appreciated in value are more likely to come to market, resulting in high observed returns for paintings that sell, relative to the population. As a result, the observed price appreciation is not representative of the entire market for paintings. In fact, in periods with few

[^0]trades, it is possible to observe high and positive returns even though overall values of paintings are declining.

We present an econometric model of art indices, based on the framework developed by Korteweg and Sorensen (2010, 2013), that generalizes the standard repeat sales regression (RSR; see Bailey et al., 1963, and Case and Shiller, 1987) to correct for selection bias in the sample of observed sales. This model explicitly specifies the entire path of unobserved valuation and returns between sales, as well as the probability of observing a trade at each point in time, and estimates the selection-corrected price for each individual artwork at each point in time, even when it is not traded. We estimate the model using a unique proprietary auction database from which we construct one of the largest samples of repeat sales of paintings in the literature to date, with 20,538 paintings being sold a total of 42,548 times between 1972 and 2010.

We find that selection bias is of first-order importance. Paintings are indeed more likely to sell when they have risen in value, consistent with Goetzmann's hypothesis. The difference between our selection-corrected index and the standard (non-corrected) RSR index is economically and statistically large, and robust across specifications. Normalizing indices at 100 in 1972, the RSR index is 4,278 in 2010, the end of our sample period, whereas the selectioncorrected index ends around 1,000 to 1,300 , depending on the specification of the selection model.

The selection correction has important implications for asset allocation decisions. The annual return to the standard RSR art index over the period 1972 to 2010 is on average 10 percent, with volatility of 17 percent, a Sharpe Ratio of 0.24 , and a correlation with world equity returns of 0.3 . Given these statistics, a mean-variance investor would allocate roughly equal fractions of her portfolio to art and to public equities, and she would appear to earn a portfolio

Sharpe Ratio of 0.31 , which is $17 \%$ higher than the Sharpe Ratio of 0.26 on a portfolio of equities only.

However, these returns are not what a passive art investor actually earns, unless she is able to pick a portfolio of "winners" that rise in value in similar fashion to the paintings that come to auction (such that the RSR index is representative for her portfolio). Instead, it is the selectioncorrected returns that are more representative for the experience of an investor who has invested in a well-diversified, passive portfolio of paintings. ${ }^{3}$ After correcting for selection bias we find an average annual art index return of 6.5 percent, which is 3.5 percentage points lower than the non-corrected index, and a Sharpe Ratio of 0.04 , down from 0.24 without the selection correction. This reduces the attractiveness of art as an investment to the point that the passive index investor optimally assigns zero weight to art in her portfolio. If the investor chooses to ignore the sample selection problem and assigns equal weight to art and stocks instead, she ends up reaping a portfolio Sharpe Ratio that is $18 \%$ lower than what she would earn by investing in stocks alone. This main portfolio allocation result is robust to broadening the investment opportunity set to other asset classes, to the effects of illiquidity and transaction costs on portfolio allocations, and to considering higher moments of art returns.

Our model also allows us to separately assess various styles of paintings. We distinguish between five styles: Post-war and Contemporary, Impressionist and Modern, Old Masters, American, and $19^{\text {th }}$ Century European. In addition, we consider returns to top selling artists. Although we observe interesting differences between the selection-corrected returns per style,

[^1]and positive portfolio allocations to either Post-war and Contemporary paintings or paintings from top artists, the portfolio weights and improvements in Sharpe ratios are small at best.

To our knowledge, this is the first paper to show the statistical and economic importance of selection effects for price indices of collectibles, and the first to show the importance of selection bias for performance evaluation and portfolio allocation for any asset class. A number of papers in the literature estimate art returns and assess their usefulness for optimal portfolio construction without accounting for selection bias (see Ashenfelter and Graddy, 2003, for an excellent overview of this literature). The real estate literature on the other hand has recognized the sample selection problem for index construction (for example, Case et al., 1997, and Korteweg and Sorensen, 2013, show that appreciating properties trade faster), but the impact on performance and portfolio allocations has not been examined.

In summary, our results suggest that sample selection can be of first-order importance for performance evaluation and portfolio allocation in illiquid markets with endogenous trading. Exploring the severity of this effect in other asset classes is an important avenue for future research.

The remainder of the paper is organized as follows. In Section 1, we describe our model for prices and trades of artworks. Section 2 describes our data. We discuss the art indices in Section 3, both with and without correcting for sample selection, and perform additional tests on the existence and strength of the selection problem in Section 4. In Section 5 we analyze optimal asset allocations, and Section 6 concludes.

## 1. A selection model of prices and sales of artworks

### 1.1. Selection model

We decompose the log return of an artwork, $i$, from time $t$ - 1 to $t$, into two components,

$$
\begin{equation*}
r_{i}(t)=\delta(t)+\varepsilon_{i}(t) \tag{1}
\end{equation*}
$$

The first return component, $\delta(t)$, is the log price change of the aggregate art market from time $t-1$ to $t$. We show below how to use the time series of $\delta$ to construct a price index for the market. The second return component, $\varepsilon_{i}(t)$, is an idiosyncratic return that is particular to the individual artwork. We assume that $\varepsilon$ has a normal distribution with mean zero and variance $\sigma^{2}$, and is independent over time and across artworks.

If we observe sales of artworks at both time $t-1$ and $t$, then the returns are observed, and estimation is straightforward. However, art is sold very infrequently. Typically, years pass between consecutive sales. With a sale at time $t-h$ and at time $t$, the observed $h$-period log return is derived from the single-period returns in equation (1) by summation:

$$
\begin{equation*}
r_{i}^{h}(t)=\sum_{\tau=t-h+1}^{t} \delta(\tau)+\varepsilon_{i}^{h}(t) \tag{2}
\end{equation*}
$$

The error term $\varepsilon_{i}^{h}(t)$ is normally distributed with mean zero and variance $h \sigma^{2}$. By defining indicator variables for the periods between sales, the $\delta$ 's can be estimated by standard generalized least squares (GLS) regression techniques, scaling return observations by $1 / \sqrt{ } h$ to correct for heteroskedasticity. This is the repeat sales regression (RSR) technique that is standard in the literature (Bailey et al., 1963, and Case and Shiller, 1987).

The $\delta$ estimates are consistent as long as the indicator variables in the RSR regression are uncorrelated with the error term, i.e., if the probability of a sale is unrelated to the idiosyncratic return component. However, in their survey of the literature, Ashenfelter and Graddy (2003)
highlight the concern that art prices may be exacerbated during booms as "better" paintings may come up for sale. Similarly, Goetzmann $(1993,1996)$ argues that selection biases are important in art data because the decision by an owner to sell a work of art may be conditional on whether or not the value of the artwork has increased, either for rational reasons or due to the disposition effect. ${ }^{4}$

To correct the repeat sales model for selection bias of this nature, we specify the sales behavior of art following the model of Korteweg and Sorensen $(2010,2013)$ that was developed for venture capital and real estate. Suppose a sale of artwork $i$ at time $t$ occurs whenever the latent variable $w_{i}(t)$ is greater than zero, and remains untraded otherwise, i.e.,

$$
\begin{equation*}
w_{i}(t)=W_{i}(t)^{\prime} \alpha_{0}+r_{i}^{0}(t) \alpha_{r}+\eta_{i}(t) \tag{3}
\end{equation*}
$$

where $r_{i}^{0}(t)$ is the return since the last sale of the artwork, and is mostly unobserved, except when $w_{i}(t)>0$. The vector $W_{i}(t)$ contains observed covariates. The error term, $\eta_{i}(t)$, is i.i.d. normal with mean zero and variance normalized to one, and independent of $\varepsilon_{i}(t) .{ }^{5}$

The selection model, consisting of the observation equation, (1), and the selection equation, (3), nests the classic RSR model: If the selection coefficient, $\alpha_{r}$, equals zero, then trades occur for reasons unrelated to price, there is no selection bias, and we recover the standard RSR model. Conversely, if Goetzmann's conjecture is correct and artworks that have risen in value are more likely to sell, then we should find a positive selection coefficient. ${ }^{6}$ By estimating and testing the selection coefficient, we allow the data to speak to the direction and importance of selection bias.

[^2]From an econometric perspective, the model is a dynamic extension of Heckman’s (1979, 1990) selection model. As in Heckman's model, our model adjusts not only for selection on observable variables, such as the size or style of a painting, but also controls for selection on unobservable variables. However, Heckman's model assumes that observations are independent, implying that observations for which price data are missing, are only informative for estimating the selection model, (3), in the first stage, but do not carry any further information for the price index in equation (1) in the second stage. Since prices are path-dependent, this independence assumption fails to hold. Each observation carries information about not only the current price, but also about past and future prices of a painting, even at times when the artwork does not trade. Unlike the standard selection model, our model does not impose the independence assumption, and uses all information to make inference about the price path of individual artworks, and the parameters of interest, $\alpha, \delta$, and $\sigma^{2}$.

The downside of allowing for the dependencies between observed and missing data is that it makes estimation more difficult relative to the standard selection model. We use Markov chain Monte Carlo (MCMC), a Bayesian estimation technique, to estimate the model parameters (see Korteweg and Sorensen, 2013, for details).

Our model extends Korteweg and Sorensen (2013) by allowing for separate indices for different styles of artwork, or whether the artist belongs to the top 100 artists. For these specifications, we replace equation (1) with

$$
\begin{equation*}
r_{i}(t)=X_{i}^{\prime} \cdot \delta(t)+\varepsilon_{i}(t) \tag{4}
\end{equation*}
$$

The vector $X_{i}$ is a set of dummy variables that indicate to which category the painting belongs. The categories need not be mutually exclusive. For example, a painting can belong both to the "Old Masters" style and be in the "Top 100 Artists" category (to be defined below).

### 1.2. Price index construction

The art literature defines the price index, $\Pi(t)$, across $N$ artworks relative to base year 0 as

$$
\begin{equation*}
\Pi(t) \equiv \sum_{i=1}^{N} P_{i}(t) / \sum_{i=1}^{N} P_{i}(0) \tag{5}
\end{equation*}
$$

where $P_{i}(t)$ is the price of painting $i$ at time $t$. Goetzmann (1992) and Goetzmann and Peng (2002) show that the index can be constructed from the model estimates in a sequential manner starting from the base year index normalized at 100,

$$
\begin{equation*}
\Pi(t)=\Pi(t-1) \cdot \exp \left(\delta(t)+\frac{1}{2} \sigma^{2}\right) \tag{6}
\end{equation*}
$$

The $\frac{1}{2} \sigma^{2}$ adjusts for the Jensen inequality term due to the log operator in (1). ${ }^{7}$

## 2. Art data

We construct a sample of repeat sales from the Blouin Art Sales Index (BASI), an online database that provides data on artworks that are sold at auction at over 350 auction houses worldwide. ${ }^{8}$ The BASI database is presently the largest known database of artworks, containing roughly 4.6 million works of art by more than 225,000 individual artists over the period 1922 to 2010. We solely focus on paintings, which represent 2.3 million artworks in the database.

For each auction record, the database contains information on the artist, the artwork, and the sale. We observe the artist's name, nationality, year of birth, and year of death (if applicable). For the artwork, we know its title, year of creation, medium, size, and style, and whether it is

[^3]signed or stamped. For the sale, we have data on the auction house, date of the auction, lot number, hammer price (the price for which the artwork was sold, excluding any premiums paid from the buyer to the auction house, and converted to U.S. dollars at the prevailing spot price), and whether the artwork has been "bought in" or was withdrawn. ${ }^{9}$

We identify repeat sales by matching auction sales records using artists' names, artwork names, painting size, medium, and signature (similar to the matching procedures in Taylor and Coleman, 2011, and Renneboog and Spaenjers, 2013). We start our search in 1972, due to the limited coverage of the database before that time. We drop buy-ins to avoid Goetzmann's (1993) concern that particular auction records are wrongly classified as sales when the painting fails to meet the seller's reservation price. To eliminate false matches, we remove paintings from the same artist with the generic titles "untitled" and "landscape." We further check whether the remaining potential repeat sales are true repeat sales by manually searching for the artwork's provenance, which shows the chronology of the artwork's earlier sales. The provenance is typically found on the websites of the auction houses. For instance, Christie's and Sotheby's provide online provenance information on all auction sales since 1998. When we are in doubt about whether we are dealing with a true repeat sale, we delete it from our sample. Our final sample includes 42,548 sales of 20,538 unique paintings.

Panel A of Figure 1 shows the number of sales in the repeat sales sample, broken down by the number of first, second, and third or more sales for the artworks in our repeat sales sample. ${ }^{10}$ For comparison, Panel B shows the number for the full BASI dataset since 1972. The full sample shows substantial growth in sales over time, peaking in 2006, whereas the repeat sales sample has the highest number of sales in 1989. This difference is due to the drop in the number of first

[^4]sales, as paintings that sell for the first time in the later part of our sample period have a smaller probability of being sold for a second time by the end of the sample period and thus have a smaller chance of being included in the repeat sales sample.

## [ Please insert Figure 1 here ]

Panel A of Table 1 shows descriptive statistics of the paintings in our repeat sales sample. The average hammer price in the full sample is $\$ 61,939$, with a long right tail of extremely expensive paintings. The average surface of the paintings is about $547,000 \mathrm{~mm}^{2}$, or $0.55 \mathrm{~m}^{2}$. Around 22 percent of sales take place at Christie's auction house, and 25 percent at Sotheby's. For 20 percent of sales, the auction house is located in London, and another 20 percent are sold in New York. Using the same style classifications as Christie's and Sotheby's, the BASI database distinguishes between six broad styles. The Impressionist and Modern style accounts for one third of sales, followed by European $19^{\text {th }}$ Century paintings with one fourth of sales. About 16 percent of sales are of the Post-war and Contemporary style, 12 percent are American paintings, and 5 percent are Old Masters. The residual "Other style" category makes up the remaining 9 percent of sales. Nearly 10 percent of sold paintings are by artists with total dollar sales in the top 100 of the BASI database over the sample period. ${ }^{11}$ Finally, more than two percent of sales occur within two years after the artist has deceased.

## [ Please insert Table 1 here ]

Panel A also shows the descriptive statistics for the full BASI sample over the same period. Compared to the full sample, more expensive paintings of higher quality are more likely to be sold repeatedly, underscoring the importance of correcting for sample selection. It should be noted that even if the repeat sales sample were statistically indistinguishable from the full sample

[^5]of sales, the sample selection issue that we address in this paper may still be present, as even the full sample of sales may not be representative of the underlying population of paintings.

Panel B of Table 1 provides information about the sale-to-sale returns in the repeat sales sample. The arithmetic price increase between two consecutive sales of the same painting is 123.5 percent on average. The median return is 42.4 percent, and the standard deviation is 368.5 percent. With an average time between sales of 7.6 years, this translates to an average (median) annualized return of 16.5 percent ( 7.5 percent), with a standard deviation of 32.7 percent. Log returns are lower, on average 43.9 percent ( 6.9 percent annualized), with a median of 35.3 percent (5.7 percent annualized) and a standard deviation of 78.1 percent (16.7 percent annualized).

## [ Please insert Figure 2 here ]

Figure 2 shows the distribution of the annualized sale-to-sale returns. Although the average return is positive, the distribution shows negative returns occur regularly. Annualized returns below - 30 percent or above 70 percent are rare.

## 3. Art indices

In this section we first present the results of estimating art indices without taking selection into account. Then we turn to our selection-corrected indices.

### 3.1. Art indices without selection correction

Figure 3 plots two estimated art price indices that ignore selection bias. The first index is constructed from the standard repeat sales regression estimated by GLS, weighing each observation by the inverse of the square root of the time between trades, to correct for potential
heteroskedasticity (the "GLS index"). The second is a MCMC specification that ignores selection by forcing $\alpha_{r}$ in equation (3) to equal zero (the "MCMC index"). We assign an index value of 100 to the year 1972 and construct annual end-of-year arithmetic indices as shown in equations (5) and (6).

## [ Please insert Figure 3 here ]

The GLS and MCMC indices practically coincide, mitigating concerns about distributional assumptions of the MCMC estimator. Over the early part of the sample period, the indices rise until they peak at 1,300 in 1990. After bottoming out at 900 in 1993, following the Japanese real estate collapse in the early 1990s, the indices climb again until peaking at around 4,300 in 2007, showing particularly high growth after 2001. In 2010, the end of our sample period, the price indices have largely recovered from the dip in the global financial crisis of 2008-2009, and are nearing the 4,300 level again.

### 3.2. Selection models

The selection models require us to take a stance on what drives the sale of an artwork. We estimate three specifications of the selection equation. All models include the log return since last sale, to capture the direction and strength of the sample selection effect. We also include the time since the last sale, both linearly and squared, in all specifications. Time since last sale functions as an instrument to identify the model from more than distributional assumptions alone: it changes the probability of a sale in the next period without affecting the return on the artwork going forward, based on the common-place assumption that prices incorporate all available public information, which includes the date that the painting last sold. Other variables that may be important for the probability of sale are the size of the painting, an indicator whether the artist
deceased in the past two years, and the growth in worldwide GDP. The size of the painting may be related to the probability of sale, because smaller paintings are easier to hang and transport. The death of the artist might be relevant as there is a popular belief that artworks are more likely to be sold when the artist has recently deceased. We include worldwide GDP growth since Goetzmann (1993), Hiraki et al. (2009), Goetzmann et al. (2011), and Pownall et al. (2013) establish an important relation between art and wealth.
[ Please insert Table 2 here ]
Table 2 shows the estimated coefficients of the selection models. Model A only includes the log return and the time variables. Most importantly, paintings with higher returns since the prior sale are more likely to sell, and hence appear more frequently in the sales data, confirming that selection is important in evaluating art returns. As a result, standard indices exaggerate the price appreciation of the overall market.

The magnitude of the selection coefficient is not only statistically significant, it is also economically meaningful. For example, a painting that was last sold one year ago and has not changed in value since, has a 3.6 percent probability of being sold in the next year. Had the painting increased in value by one standard deviation of the annualized return, then the probability of sale in the next year would be 4.1 percent. For a two standard deviation increase in price, the sale probability in the next year is 4.7 percent.

Controlling for price appreciation, the time since the prior sale has a non-linear relation to the probability of a sale. An artwork is less likely to sell again in the first year after a trade, but as the time since last sale increases, the probability of a sale rises as the coefficient on the squared time since sale starts to dominate. All time effects are significant at the one percent
significance level, indicating that the time variables improve the statistical identification of the model.

The estimates for Model B show that our main finding that returns and the probability of sale are positively related, is robust to adding additional variables to the selection model. The coefficients on these variables provide further insight into what drives sales behavior. A painting is more likely to sell within two years after the artist deceases, and when world GDP is declining. The latter result is not straightforward to interpret given that we also control for price appreciation separately, but it is in line with situations in which owners are forced to sell in bad times (Campbell, 2008). We do not find a significant effect of a painting's size.

Model C in Table 2 allows for different selection coefficients and intercepts by decade, where we group the 1970s with the 1980s (due to data sparseness), and we split the 2000s in the pre and post financial crisis years. Thus, our periods are 1972 to 1989,1990 to 1999, 2000 to 2006, and 2007 to 2010. We find evidence for selection bias in every sub-period. The effect of returns on the probability of trade is especially large in the two periods after 2000. This could indicate that the selection bias has become more important since the turn of century. However, it may also mean that, although sample selection is important even in the earliest years of the sample, the selection problem is worse in more recent periods, as many paintings with slow price appreciation have not yet been sold. This is related to the well-known problem of revision bias in repeat sales indices (Clapham et al., 2006), which the selection correction model helps to ameliorate (as shown in Korteweg and Sorensen, 2013).

We use Models A, B, and C to construct three selection-corrected indices, which we denote Indices A, B, and C, respectively. Panel A of Figure 4 shows the selection-corrected price indices over time. The selection correction is quite robust across models, as the differences
among the three selection-corrected indices are not very large. Panel A also includes the MCMC model without selection correction. The differences between the non-corrected and the selectioncorrected indices are striking. First, the selection-corrected indices are considerably lower than the non-corrected indices, conform the intuition of the effect of a positive selection coefficient. The peak in 1990, which occurred at a 1,300 index level in the non-selection-corrected model, occurs at around 850 in the corrected indices. The 2007 peak is around 1,300 (or 1,000 for model C), rather than the 4,300 index level of the non-corrected model. Second, the selection-corrected indices show an additional peak around the year 2003, which does not occur in the non-corrected indices. Third, the selection-corrected indices do not recover after the global financial crisis of 2008, unlike the non-corrected indices.

## [ Please insert Figure 4 here ]

In Panel B, we plot the difference between the natural logarithm of the non-selectioncorrected index and each of the selection-corrected indices. As foreshadowed by the timevarying selection coefficients in model C , the graph shows that the deviation between the nonselection and the selection-corrected indices already starts in the first years of our sample period, increases steadily over time, and accelerates in the new millennium.

Next, we consider potential differences between art styles. Buelens and Ginsburgh (1993) find differential performance among styles, and possibly, different styles are in favor in different periods. Figure 5 plots the estimated selection-corrected indices.

## [ Please insert Figure 5 here ]

We observe that the price index of Post-war and Contemporary paintings peaks around 1990 and 2007. Impressionist and Modern paintings show large increases in the early period but are hit heavily in 1990, which is in line with the popular interpretation of art observers that the Japanese
real estate bubble (which burst in 1990) and the corresponding strong yen in the 1980s had strong effects on the prices of Impressionist paintings (see for example Wood, 1992). Old Masters do not increase as much in value as the other styles over the sample period.

Figure 5 also shows the selection-corrected price index for paintings of the top 100 artists in terms of the total dollar value of sales in our sample period. Top artists outperform all of the styles, and the index peaks both around the years 1990 and 2007. This result relates to the "masterpiece effect", the general belief among art dealers and critics that highly priced paintings are the best buy (e.g., Adam, 2008). Several prior academic studies examine masterpieces, but generally find that masterpieces underperform (Pesando, 1993; Mei and Moses, 2002; 2005). ${ }^{12}$ Our results suggest that selection bias is an important determinant of this discrepancy, as not controlling for selection biases artificially drives up the returns for artworks with which masterpieces are compared. Paintings from top artists do experience relatively volatile returns, so it is not clear whether they should get higher weights in optimal portfolios. We discuss portfolio allocations after we present further evidence in support of the sample selection problem in the next section.

## 4. Further supporting evidence for sample selection

The results from the econometric model show that paintings are more likely to trade when they have experienced a high return since prior sale, and that correcting for the sample selection problem is economically important for index construction. In this section we show further corroborating evidence of the existence and strength of the selection problem in the raw data, without relying on the econometric machinery of the selection model.

[^6]First, we consider the relationship between the time between sales and the annualized sale-to-sale return. For each painting in the repeat sales sample, we compute the annualized log return between two adjacent sales, and the number of years between the two sales. Both these quantities are observed in the raw data. In Figure 6, we graph the average annualized sale-to-sale return against the return horizon. For example, for all sale-to-sale returns that occurred over a span of one year or less, the average annualized return is 13 percent, compared to 8 percent for sales that took place between one and two years apart. For longer return horizons the average annualized return is even lower. We also show the median annualized return against the return horizon, which follows a nearly identical pattern.

## [ Please insert Figure 6 here ]

If there were no sample selection problem (i.e., if the selection coefficient, $\alpha_{r}$, equals zero in the econometric model), then there should be no systematic relation between annualized returns and the time between sales, and Figure 6 should show a flat, horizontal line. Instead, the line is downward-sloping, which is consistent with a sample selection problem in which paintings with high returns are more likely to trade. To take an oversimplified example, suppose that paintings trade as soon as the return since last sale hits a fixed threshold, say, 10 percent (not annualized). Then the paintings that happen (by chance) to have a high return soon after the prior sale, will trade quickly and show a high annualized return. The paintings that are slower to hit the threshold will trade at a later date and exhibit lower annualized returns.

Figure 6 also shows that the selection problem is plausibly large. The mean annual index return from the RSR regressions is essentially an average of the annualized observed returns over all horizons. In a selected sample where the paintings with higher returns are more likely to sell, the selection-corrected average annualized return must be lower than the observed returns, and
thus lower than even the long-horizon returns. Based on Figure 6, a difference of a few percentage points in the mean annual return between the RSR and the selection-corrected indices is not surprising.

The second piece of evidence suggesting sample selection is the relation between annualized returns and trading intensity. In Figure 7, we plot the time series of annualized log returns, where we take the average sale-to-sale returns for all second sales of a sales pair in a given year. We only show the time series starting in 1980, because for many sales in the 1970s the first sale takes place prior to the start of our sample in 1972, and we see only the shorthorizon returns in the 1970s. In the same graph, we show the time series of trading intensity, defined as the percentage of paintings that sold in the calendar year, calculated from the full BASI dataset of 2.3 million observations (the results are very similar if we use the repeat sales sample instead).

## [ Please insert Figure 7 here ]

Without selection, i.e. when paintings trade for reasons unrelated to returns, there should not be a systematic relation between trading intensity and annualized returns. This is clearly not what we observe in Figure 7, which shows a strong positive correlation between trading intensity and the annualized returns. This relation is consistent with the sample selection problem identified in this paper, where a positive shock to the value of paintings results in more paintings that are likely to trade, and a higher average (annualized) realized return, and vice versa for negative shocks.

Our third exercise further exploits the variation in the data, by considering the relation between trading intensity and annualized returns for individual styles of paintings. Consistent with the aggregate results, Table 3 Panel A shows that both the average and median annualized return of a specific style of painting are strongly positively correlated with the market share of
that style, where market share is defined as the number of paintings of the style that sold over the year relative to the total number of paintings that traded across all styles. Panel B shows that this correlation is highly statistically significant in a pooled regression analysis that controls for style fixed effects.

## [ Please insert Table 3 here ]

To summarize, the evidence in this section is supportive of the result from the econometric model that the probability that a painting trades is positively related to its return since the prior sale.

## 5. Optimal portfolio allocation

In this section, we show that our estimated indices provide important insights regarding the role of paintings for diversification and optimal portfolio allocation. More broadly, the results underscore the importance of adjusting for sample selection for performance evaluation and portfolio optimization in the presence of illiquid assets when trading is endogenous.

### 5.1 Descriptive Statistics

We start our analysis by describing the returns to the art indices. We follow the artinvestments literature (e.g., Campbell, 2008, and Renneboog and Spaenjers, 2013) and use the technique pioneered in real estate by Geltner (1991) to unsmooth the index to deal with spurious first-order autocorrelation caused by time-aggregation of sales (Working, 1960, and Schwert, 1990). The unsmoothing procedure causes us to lose the first observation, so all returns in this section start in 1973. ${ }^{13}$ Table 4 reports means, standard deviations and Sharpe Ratios of the

[^7]annual arithmetic returns on the art indices. For clarity, it should be noted that these are the returns to the indices, and thus are different from the sale-to-sale returns reported in Table 1 and Figures 2, 6, and 7.

## [ Please insert Table 4 here ]

Panel A shows that the standard repeat sales GLS index that does not correct for selection has an average annual return of 10.1 percent with a standard deviation of 16.7 percent, and an annual Sharpe ratio of 0.24 . The non-selection-corrected MCMC index returns are nearly identical to the GLS index. In contrast, the selection-corrected indices have considerably lower average returns, standard deviations and Sharpe Ratios. The average return ranges from 6.0 percent to 6.6 percent depending on the selection model specification. The standard deviations are in the range of 12.8 percent to 13.0 percent, and Sharpe Ratios range from -0.01 to 0.04 .

It is important to point out that, as a measure of performance, the standard non-selectioncorrected indices implicitly assume that an investor can either pick "winners" that rise in value and are thus more likely to sell, or assume that there is no selection problem and that all other holdings of the investor follow the same price path as the paintings that are auctioned off. The selection-corrected indices do not make such assumptions but rather measure the rise in value of the overall portfolio of paintings, both those that sold and those that did not. The selectioncorrected returns are therefore more representative of the experience of an investor who has invested in a well-diversified, passive portfolio of paintings.

Panel A of Table 4 also presents the descriptive statistics for sample period returns of a broad portfolio of global equities (the MSCI world total return index, which includes distributions), corporate bonds (the Merrill Lynch US Corporate Master Bond index), commodities (the GSCI Commodity total return index for commodity futures), and real estate
(the U.S. residential real estate index from Shiller, 2009). For the risk-free asset we use the oneyear U.S. Treasury bill rate at the beginning of the year. Being our risk-free asset, we do not report the Sharpe Ratio on Treasuries in Table 4.

Despite the low Sharpe Ratios on the selection-corrected art indices relative to the Sharpe Ratios on stocks, corporate bonds, and commodities of $0.26,0.33$, and 0.18 , respectively ${ }^{14}$, investing in paintings may still be useful for constructing optimal portfolios if the correlations between art and the other asset classes are low. Panel B of Table 4 shows that the art indices have a correlation of 0.32 with world stock returns. The correlations of art with commodities and real estate are of similar magnitude, and the correlation with corporate bonds is negative, although not statistically significant.

### 5.2 Optimal Portfolio Allocations

To examine the portfolio allocation decision formally, we construct optimal portfolios based on the following base case assumptions that are common in the literature. First, investors have mean-variance utility, and allocate their portfolio among the risk-free asset, a welldiversified stock index, and a well-diversified, passive art index (we will consider other assets below). Second, borrowing and short sales are not allowed. Third, there are no transaction costs to constructing the indices. Fourth, there is no illiquidity return premium on paintings. Fifth, investing in the art index does not provide the investor with access to the artworks underlying the index, and we therefore do not consider any consumption utility of owning art.
[ Please insert Table 5 here ]

[^8]Panel A of Table 5 shows the portfolio weights for the tangency portfolio of stocks and art (i.e., the portfolio with the maximum Sharpe Ratio in the presence of a risk-free asset). An investor who does not correct for selection bias in art returns would want to assign considerable weight to art. Based on the GLS returns, art receives a portfolio weight of 47 percent, with the remainder assigned to stocks. The perceived portfolio Sharpe ratio of 0.31 is $17 \%$ higher than the Sharpe Ratio of 0.26 that is achieved with stocks alone, an economically significant improvement. The MCMC index that ignores selection gives weights that are nearly identical to the GLS index, and for brevity we omit them from the portfolio weights tables. The non-selection-corrected indices thus suggest that paintings should play an important role in asset allocation.

In contrast, an investor who corrects for sample selection optimally assigns zero weight to paintings across all selection model specifications, as the diversification benefit from investing in art as an asset class does not outweigh its low Sharpe Ratio. This stark result underscores the importance of correcting for sample selection when making optimal portfolio decisions, as the results can be dramatically different. Had the investor allocated 47\% of her portfolio to art, her realized portfolio Sharpe Ratio would be only 0.21 , based on the selection-corrected art returns, rather than the 0.26 Sharpe Ratio that she would have earned on an all-stock portfolio, which corresponds to a loss of $18 \%$ (this result is not tabulated).

Panels B, C, and D of Table 5 show the portfolio allocations to paintings, stocks, and the risk-free asset for a mean-variance utility investor with a risk aversion coefficient equal to two, five, and ten, respectively. The results are similar to the tangency portfolio results and underscore our main result: an investor who does not correct for sample selection would allocate roughly the same fraction of her portfolio to paintings as to public equity. With the selection
correction, the same investor would put all her weight on stocks and Treasuries, and zero weight on paintings. The Sharpe Ratios are naturally the same across all panels of Table 5.

We explore two extensions of the base case scenario. In the first extension we broaden the investment opportunity set of risky assets to include corporate bonds, commodities, and real estate in addition to stocks and art.

## [ Please insert Table 6 here ]

Table 6 shows that the results are similar to the base case. A benchmark portfolio that includes all assets except art earns a Sharpe ratio of 0.41 . Adding art to the investment opportunity set would appear to raise the portfolio Sharpe ratio by 0.05 to 0.46 , a $13 \%$ increase, if selection is ignored. However, with the selection correction the optimal weight on art is again zero, and the optimal portfolio is identical to the benchmark portfolio.

In the second extension we consider whether investing in particular styles of paintings, or in top-selling artists, would be beneficial for forming portfolios. Panel A of Table 7 reports the results when restricting the set of risky assets to stocks and art only. Panel B extends the opportunity set to incorporate corporate bonds, commodities, and real estate. All results in this table are based on the selection-corrected returns.

## [ Please insert Table 7 here ]

Across the two panels, the only style that receives a non-zero portfolio allocation is the Post-war and Contemporary style. Investing in this style and stocks yields a portfolio Sharpe Ratio of 0.28 if we only consider stocks and art (compared to 0.26 for stocks only), and 0.43 if we consider the broader opportunity set (compared to 0.41 for the benchmark portfolio that excludes art). Including the index of the Top 100 artists as a separate asset drives out the allocation to Post-war and Contemporary paintings, while raising the Sharpe Ratio slightly to
0.29 and 0.45 in panels A and B, respectively. Thus, there appears to be a small improvement in performance when considering narrower categories of paintings. Still, we should be careful in drawing strong conclusions from this exercise, as the styles do not show consistent performance relative to each other (as seen in Figure 5), and the Top 100 index is defined rather endogenously: it might not be surprising that the top-selling artists over the sample period outperformed, but it is not obvious that these artists could be identified ex ante.

### 5.3 Robustness

In this section we show that the zero portfolio allocation to art after selection correction is robust to various measurement and other issues. First, Ang et al. (2013) show that allocation to an illiquid asset is lower when the illiquid nature of the asset is taken into account. For example, using their model and our non-selection-corrected returns, a power utility investor with risk aversion of two allocates $10 \%$ of the tangency portfolio to art and $90 \%$ to stocks, down from roughly equal allocations. Not surprisingly, the allocation to art after correcting for selection remains at zero for all levels of risk aversion. ${ }^{15}$

Second, if investors have, for example, power utility rather than mean-variance utility, they may care about higher moments of returns such as skewness (e.g. Ball et al., 1995, and Harvey and Siddique, 2000) and kurtosis. However, art returns are negatively skewed and exhibit excess kurtosis (results not tabulated), which only serves to make art less attractive relative to stocks.

Third, there are sizeable transaction costs in the art market: the typical buyer's premium in art is up to 17.5 percent of the hammer price, and on top of that there are storage and insurance

[^9]fees. These costs make paintings less appealing for optimal portfolio allocation and hence reinforce our main result.

In other robustness tests, we confirm robustness to the choices we made regarding assets and methodology. As an alternative to unsmoothing the art returns, we correct for nonsynchronous trading using Dimson's (1979) method with one year leads and lags of stock returns. Our main result holds: correcting for sample selection, an investor would optimally not invest in art. We also reran the portfolio allocations without unsmoothing the returns (i.e., using the original index returns). Without unsmoothing, the volatility of art returns is lower and the correlation of art with stocks is closer to zero. This makes art more attractive as an investment, but after controlling for sample selection we find that there is still virtually no gain in the Sharpe Ratio when including art in the investor's opportunity set.

Next, we take a U.S.-centric approach and use the value-weighted or the equally-weighted CRSP index (both including distributions) or the S\&P composite index instead of the MSCI world total return index. The correlation between art and U.S. stock returns is around 0.09 , considerably lower than the 0.32 correlation with global stock returns, making art more attractive for diversification to an investor with a U.S. focus. For comparison, Renneboog and Spaenjers (2013) find a correlation coefficient of -0.03 between art and the S\&P 500 index, and Taylor and Coleman (2011) find a negative correlation (of about -0.30) between Australian stocks and aboriginal art. Our finding underscores that art is truly a global asset and its return is more closely linked to the global economy than to the U.S. economy. Still, despite the lower correlation with U.S. stocks, our main result carries through, and investors correcting for sample selection optimally forego investing in art.

Finally, our portfolio allocation result is also robust to using the longer 1926 to 2010 period to estimate the average market return and its standard deviation (computing the covariance with art from the correlation over the 1972 to 2010 period but using the standard deviation from the longer time series) ${ }^{16}$, to using the return to the Citigroup World Government Bond Index instead of the one-year T-bill rate as the risk-free asset, and to using logarithmic rather than arithmetic returns.

## 6. Conclusion

We estimate an empirical model that adjusts for selection bias in illiquid asset markets with endogenous trading, using a large dataset of auction sales of paintings. We find a large selection effect of the kind hypothesized by Goetzmann (1993, 1996), namely that paintings that have increased in value are more likely to sell. This has a first-order impact on art indices, lowering the average annual price increase from 10 percent for a standard repeat sales index to 6.5 percent for selection-corrected indices, and resulting in a drop in annual Sharpe Ratios from 0.24 to 0.04 .

If passive index investors ignore the sample selection problem, they would allocate roughly equal fractions of their portfolios to art and stocks, and they would appear to reap large portfolio Sharpe Ratios on the order of 0.31 annually, or $17 \%$ more than the 0.26 Sharpe Ratio on a portfolio of stocks only. However, this strategy turns out to hurt them as they in fact reap Sharpe Ratios that are $18 \%$ lower than what they would earn on a portfolio of only stocks, once sample selection is accounted for. In summary, the diversification benefits of art do not outweigh its lower Sharpe Ratio. Our results show that investors should optimally forego investing in paintings, even without considering transaction and insurance costs, and the risks of forgeries,

[^10]thefts, and physical damage, unless they are able to pick winners or there is substantial nonmonetary utility from owning and enjoying art. This result is robust to considering a broader investment opportunity set that includes corporate bonds, commodities, and real estate, as well as to considering the effects of illiquidity and higher moments of art returns on optimal portfolio allocations.

To our knowledge, this is the first paper to show the importance of the endogenous trading sample selection problem for performance evaluation and optimal portfolio allocation. It stands to reason that other illiquid asset classes exhibit a similar selection problem, and evaluating the returns to these other asset classes is an important task for future work.

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Figure 1: Number of sales
This figure shows the number of auction sales of paintings in the repeat sales sample (panel A) and the full sample (panel B) by calendar year. In the repeat sales sample in panel A, we distinguish between the first, second, and third or more sales of an artwork.

Panel A: Repeat sales sample


Panel B: Full sample


Figure 2: Annualized sale-to-sale return distribution
This figure shows histograms of the annualized sale-to-sale returns of paintings in the repeat sales sample. Panel A shows the annualized arithmetic sale-to-sale returns, and Panel B shows the natural logarithm of the annualized sale-to-sale returns.

Panel A: Arithmetic returns




## Figure 3: Non-selection-corrected price indices

This figure shows repeat sales arithmetic price indices that do not correct for sample selection. The indices are normalized to an index value of 100 in 1972. The GLS index is the standard repeat sales regression indices as estimated by generalized least squares, with weights that are inverse proportional to the square root of the time between sales. The MCMC index is the index estimated by the Markov chain Monte Carlo algorithm when the sample selection problem is forcibly ignored, i.e., $\alpha_{r}$ in equation (3) is set to zero.


## Figure 4: Selection-corrected price indices

Panel A shows price indices corrected for sample selection, and normalized to an index value of 100 in 1972. Models A through C correspond to the specifications of the selection equation as shown in Table 2. For comparison, the figure also shows the MCMC non-selection-corrected index over this same period (denoted No selection. This is the same index as the MCMC index in Figure 3). Panel B graphs the difference between the natural logarithm of the No selection index and the selection-corrected indices.

Panel A: Price indices



## Figure 5: Selection-corrected price indices per style

This figure shows selection-corrected price indices for each style classification, normalized to an index value of 100 in 1972. Top 100 refers to the index of paintings by top 100 artists based on the total value of sales (in U.S. dollars) of all paintings by the artist over the sample period.


Figure 6: Annualized sale-to-sale returns by return horizon
This figure shows the relation between the annualized log sale-to-sale returns (on the left-hand vertical axis) and the time between sales (in years, on the horizontal axis) in the repeat sales data. The solid and striped lines represent the average and the median annualized log return between observed sales, respectively. The vertical bars are the number of observations in each bin, measured on the right-hand vertical axis.


Figure 7: Time series of annualized returns and trading intensity
This figure graphs the time series of the average annualized log sale-to-sale returns (represented on the left-hand vertical axis) and trading intensity (on the right-hand axis). The average log sale-to-sale return is computed over consecutive sales of paintings for which the second sale falls in the given year. Trading intensity is calculated as the number of sales in a calendar year as a percentage of all sales over the 1980 to 2010 period.


## Table 1: Summary statistics

This table reports summary statistics for the sample of paintings in the Blouin Art Sales Index (BASI) dataset from 1972 to 2010. Panel A presents descriptive statistics for the repeat sales sample that contains paintings that sold at least twice during the sample period (left columns), and the full BASI dataset (right columns). The unit of observation is a sale of a painting at auction. Hammer price is the auction price in thousands of U.S. dollars. Surface is the surface of the painting in thousands of squared millimetres. Deceased $<2 y r s$ is a dummy variable equal to one when the sale occurs within two years after the artist deceases, and zero otherwise. Christie's and Sotheby's are dummy variables that equal one if the painting is auctioned at Christie's or Sotheby's, respectively, and London and New York are dummy variables that equal one if the painting is auctioned in London or New York, respectively. Top 100 Artists is a dummy variable equal to one when the artist is in the top 100 in terms of total value of sales (in U.S. dollars) over 1972 to 2010, and zero otherwise. The remaining variables in Panel A represent style classifications. The last column shows $t$-statistics for difference in means tests (for Hammer price and Surface) and $z$-statistics for difference in proportions tests (for the other variables) between the full and the repeat sales samples. The sale-to-sale returns in Panel B are for the repeat sales sample only, and calculated as the natural logarithm of the ratio of the current and prior hammer price of a painting. ${ }^{* * *},{ }^{* *}$ and ${ }^{*}$ indicate statistical significance at the 1,5 and 10 percent level, respectively.

Panel A. Descriptive statistics for the full and repeat sales sample

|  | Repeat sales sample <br> (42,548 sales) |  |  | Full sample$(2,302,738$ sales $)$ |  |  | Difference statistic |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Median | St. Dev. | Mean | Median | St. Dev. |  |
| Hammer price (\$000s) | 61.9 | 6.2 | 444.2 | 28.6 | 3.0 | 395.5 | -15.39*** |
| Surface | 546.9 | 331.0 | 792.1 | 491.8 | 306.5 | 811.8 | -14.19*** |
| Deceased < 2yrs | 2.12\% |  |  | 1.59\% |  |  | -8.71*** |
| Christie's | 21.54\% |  |  | 15.57\% |  |  | -33.59*** |
| Sotheby's | 25.46\% |  |  | 15.82\% |  |  | -53.75*** |
| London | 19.47\% |  |  | 14.15\% |  |  | -31.07*** |
| New York | 21.12\% |  |  | 9.75\% |  |  | -77.66*** |
| Top 100 Artists | 9.62\% |  |  | 2.77\% |  |  | -83.52*** |
| Post-war and Contemporary | 15.81\% |  |  | 11.75\% |  |  | -25.69*** |
| Impressionist and Modern | 34.03\% |  |  | 22.32\% |  |  | -57.29*** |
| Old Masters | 5.32\% |  |  | 9.87\% |  |  | 31.26*** |
| American | 11.57\% |  |  | 7.29\% |  |  | -33.47*** |
| European 19 ${ }^{\text {th }}$ Century | 23.83\% |  |  | 31.28\% |  |  | 32.89*** |
| Other Style | 9.44\% |  |  | 17.49\% |  |  | 43.46*** |

Panel B. Sale-to-sale returns for the repeat sales sample (22,010 returns for 20,538 paintings)

|  | Mean | Median | St. Dev. |
| :--- | ---: | ---: | ---: |
| Sale-to-sale return |  |  |  |
| Arithmetic return | $123.50 \%$ | $42.37 \%$ | $368.54 \%$ |
| Log return | 7.61 | $35.33 \%$ | $78.08 \%$ |
| Years between sales |  |  | 6.55 |
| Annualized sale-to-sale return | $16.50 \%$ | $7.53 \%$ | $32.68 \%$ |
| Arithmetic return | $6.90 \%$ | $5.65 \%$ | $16.69 \%$ |
| Log return |  |  |  |

Table 2: Selection equation coefficients
This table presents the parameter estimates of three specifications of the selection equation (equation (3) in the text). Log return is the natural logarithm of the return since the prior sale of a painting. Time is the time in years since the prior sale. Log surface is the natural logarithm of the painting's surface in thousands of $\mathrm{mm}^{2}$. World GDP growth is the yearly increase in worldwide GDP, obtained from Historical Statistics of the World Economy. The other variables are as defined in Table 1. Sigma is the standard deviation of the error term in equation (1). Standard errors are in parentheses. ${ }^{* * *},{ }^{* *}$ and * indicate statistical significance at the 1,5 and $10 \%$ level.

|  | A | B | C |
| :---: | :---: | :---: | :---: |
| Log return | $\begin{aligned} & 0.375 * * * \\ & (0.008) \end{aligned}$ | $\begin{aligned} & 0.372 \text { *** } \\ & (0.008) \end{aligned}$ |  |
| 1972-1989 |  |  | $\begin{aligned} & 0.300^{* * *} \\ & (0.014) \end{aligned}$ |
| 1990-1999 |  |  | $\begin{aligned} & 0.278 \text { *** } \\ & (0.009) \end{aligned}$ |
| 2000-2006 |  |  | $\begin{aligned} & 0.581^{* * *} \\ & (0.013) \end{aligned}$ |
| 2007-2010 |  |  | $\begin{aligned} & 0.772 \text { *** } \\ & (0.027) \end{aligned}$ |
| Time (yrs) | $\begin{aligned} & -0.401 * * \\ & (0.019) \end{aligned}$ | $\begin{aligned} & -0.033^{* * *} \\ & (0.002) \end{aligned}$ | $\begin{aligned} & -0.022 \text { *** } \\ & (0.002) \end{aligned}$ |
| Time squared | $\begin{aligned} & 0.090^{* * *} \\ & (0.009) \end{aligned}$ | $\begin{aligned} & 0.001^{* * *} \\ & (0.000) \end{aligned}$ | $\begin{aligned} & 0.000^{* * *} \\ & (0.000) \end{aligned}$ |
| Log surface |  | $\begin{array}{r} 0.005 \\ (0.007) \end{array}$ | $\begin{array}{r} 0.002 \\ (0.008) \end{array}$ |
| Deceased < 2yrs |  | $\begin{aligned} & 0.104^{* * *} \\ & (0.031) \end{aligned}$ | $\begin{aligned} & 0.094^{* * *} \\ & (0.031) \end{aligned}$ |
| World GDP growth |  | $\begin{aligned} & -1.008^{* * *} \\ & (0.253) \end{aligned}$ | $\begin{aligned} & -1.273^{* * *} \\ & (0.309) \end{aligned}$ |
| Intercept | $\begin{aligned} & -1.517 * * * \\ & (0.007) \end{aligned}$ | $\begin{aligned} & -1.512 * * * \\ & (0.042) \end{aligned}$ |  |
| 1972-1989 |  |  | $\begin{aligned} & -1.487 * * * \\ & (0.044) \end{aligned}$ |
| 1990-1999 |  |  | $\begin{aligned} & -1.490 \text { *** } \\ & (0.044) \end{aligned}$ |
| 2000-2006 |  |  | $\begin{aligned} & -1.518 * * * \\ & (0.045) \end{aligned}$ |
| 2007-2010 |  |  | $\begin{aligned} & -1.660 \text { *** } \\ & (0.046) \\ & \hline \end{aligned}$ |
| Sigma | $\begin{aligned} & 0.282 * * * \\ & (0.001) \\ & \hline \end{aligned}$ | $\begin{gathered} 0.282 * * * \\ (0.001) \\ \hline \end{gathered}$ | $\begin{gathered} 0.288^{* * *} \\ (0.001) \\ \hline \end{gathered}$ |

Table 3: Relation between annualized returns and market shares by style
Panel A shows the correlation between the yearly market share of each style and the mean (left column) and median (right column) annualized log sale-to-sale return for the style, computed over the returns for which the second sale falls in the given year. The market share of a specific style is the total sales of this style in a given year relative to all sales in that year, calculated from the full BASI dataset. Panel B shows the coefficients of a regression of the yearly style market shares on the annualized sale-to-sale returns by style, and dummy variables representing the styles (Other Style is the omitted variable). Standard errors are in parentheses. ***, ** and * indicate statistical significance at the 1,5 and $10 \%$ level, respectively.

Panel A. Correlation coefficients between market shares and returns per style

|  | Mean annualized sale-to- <br> sale return |  | Median annualized <br> sale-to-sale return |
| :--- | :---: | :---: | :---: |
| Post-war and Contemporary | 0.235 | 0.257 |  |
| Impressionist and Modern | 0.411 | 0.426 |  |
| Old Masters | 0.083 | -0.053 |  |
| American | 0.391 | 0.467 |  |
| European 19 ${ }^{\text {th }}$ Century | 0.224 | 0.295 |  |
| Other Style | -0.080 | -0.178 |  |
| Top 100 Artists | 0.373 | 0.296 |  |

Panel B. Regression analysis (Dependent variable = Yearly market share by style)

|  | I | II |
| :---: | :---: | :---: |
| Mean annualized sale-to-sale return | $\begin{aligned} & 0.116^{* * *} \\ & (0.035) \end{aligned}$ |  |
| Median annualized sale-to-sale return |  | $\begin{aligned} & 0.130^{* * *} \\ & (0.034) \end{aligned}$ |
| Post-war and Contemporary dummy | $\begin{aligned} & 0.069 \text { *** } \\ & (0.009) \end{aligned}$ | $\begin{aligned} & 0.069 \text { *** } \\ & (0.009) \end{aligned}$ |
| Impressionist and Modern dummy | $\begin{aligned} & 0.240^{* * *} \\ & (0.009) \end{aligned}$ | $\begin{aligned} & 0.240^{* * *} \\ & (0.009) \end{aligned}$ |
| Old Masters dummy | $\begin{aligned} & -0.033^{* * *} \\ & (0.009) \end{aligned}$ | $\begin{aligned} & -0.033^{* * *} \\ & (0.009) \end{aligned}$ |
| American dummy | $\begin{aligned} & 0.019 * * \\ & (0.009) \end{aligned}$ | $\begin{aligned} & 0.019 * * \\ & (0.009) \end{aligned}$ |
| European $19^{\text {th }}$ Century dummy | $\begin{aligned} & 0.145 \text { *** } \\ & (0.009) \end{aligned}$ | $\begin{aligned} & 0.144^{* * *} \\ & (0.009) \end{aligned}$ |
| Top 100 Artists dummy | $\begin{aligned} & -0.001 \\ & (0.009) \end{aligned}$ | $\begin{array}{r} -0.001 \\ (0.009) \end{array}$ |
| Intercept | $\begin{aligned} & 0.086^{* * *} \\ & (0.007) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.087 * * * \\ & (0.007) \\ & \hline \end{aligned}$ |
| Adjusted R ${ }^{2}$ | 83.7\% | 83.7\% |
| Number of observations | 264 | 264 |

## Table 4: Descriptive statistics of annual index returns

Panel A reports descriptive statistics of the annual arithmetic returns to indices of paintings and other assets over the period 1973 to 2010. GLS is the standard repeat sales index of paintings as estimated by generalized least squares. The MCMC index is the non-selection-corrected index from our Markov chain Monte Carlo estimator. The selection-corrected art indices $A$ through $C$ are as described in Table 2. All art returns are unsmoothed using the Geltner (1991) method. Stocks is the MSCI world total return index including distributions. Corporate bonds is the Merrill Lynch US Corporate Master Bond index. Commodities is the GSCI Commodity total return index for commodity futures. Real estate is the U.S. residential real estate index from Shiller (2009). Treasuries are one-year U.S. Treasury bills. The Sharpe Ratio is annualized. Panel B reports correlation coefficients between excess returns on the various art indices and the other assets. ${ }^{* * *},{ }^{* *}$ and ${ }^{*}$ indicate statistical significance at the 1,5 and $10 \%$ level, respectively.

Panel A. Descriptive statistics of annual returns

|  | Mean | St. dev. | Sharpe <br> Ratio |
| :--- | :---: | :---: | :---: |
| Returns on non-selection-corrected art | indices: |  |  |
| GLS | $10.08 \%$ | $16.70 \%$ | 0.235 |
| MCMC | $10.07 \%$ | $16.85 \%$ | 0.232 |

Returns on selection-corrected art indices:

| A | $6.53 \%$ | $12.83 \%$ | 0.035 |
| :--- | :--- | :--- | :--- |
| B | $6.57 \%$ | $12.83 \%$ | 0.038 |
| C | $5.95 \%$ | $13.00 \%$ | -0.012 |

Returns on other assets:

| Stocks | $10.95 \%$ | $18.49 \%$ | 0.261 |
| :--- | ---: | ---: | :---: |
| Corporate bonds | $8.94 \%$ | $8.75 \%$ | 0.332 |
| Commodities | $10.21 \%$ | $23.44 \%$ | 0.175 |
| Real estate | $5.03 \%$ | $6.66 \%$ | -0.150 |
| Treasuries | $6.09 \%$ | $3.38 \%$ | - |

Panel B. Correlation coefficient between annual excess returns on art and other assets

| Art | Stocks | Corporate bonds | Commodities | Real estate |
| :--- | :--- | :---: | :---: | :---: |
| GLS | $0.327^{* *}$ | -0.189 | $0.367^{* *}$ | $0.397^{* *}$ |
| MCMC | $0.341^{* *}$ | -0.185 | $0.364^{* *}$ | $0.403^{* *}$ |
| A | $0.324^{*}$ | -0.250 | $0.354^{* *}$ | $0.357^{* *}$ |
| B | $0.326^{* *}$ | -0.246 | $0.358^{* *}$ | $0.360^{* *}$ |
| C | $0.317^{*}$ | $-0.269^{*}$ | $0.275^{*}$ | $0.300^{*}$ |

## Table 5: Optimal asset allocation between art and stocks

Panel A shows the mean-variance tangency portfolio weights on paintings (using the non-selection-corrected GLS index, and the selection-corrected indices A, B, and C from Table 2) and stocks (the MSCI world total return index including distributions). Panels B, C and D show the optimal weights for a one-period mean-variance utility investor with risk aversion ( $\gamma$ ) equal to two, five, and ten, respectively, where one-year Treasuries are the risk-free asset. Short sales and borrowing are not allowed. Returns are measured over 1973 to 2010. Sharpe Ratios are annualized.

|  | GLS | Selection-corrected |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  |  | A | B | C |
| Panel A. Tangency portfolio weights |  |  |  |  |
| Paintings | 0.470 | 0 | 0 | 0 |
| Stocks | 0.530 | 1 | 1 | 1 |
| Sharpe Ratio | 0.305 | 0.261 | 0.261 | 0.261 |
|  |  |  |  |  |
| Panel B. Mean-variance utility, risk aversion $\gamma=2$ |  |  |  |  |
| Paintings | 0.470 | 0 | 0 | 0 |
| Stocks | 0.530 | 0.706 | 0.706 | 0.706 |
| Treasuries | 0 | 0.294 | 0.294 | 0.294 |
| Sharpe Ratio | 0.305 | 0.261 | 0.261 | 0.261 |
|  |  |  |  |  |
| Panel C. Mean-variance utility, risk aversion $\gamma=5$ |  |  |  |  |
| Paintings | 0.208 | 0 | 0 | 0 |
| Stocks | 0.225 | 0.284 | 0.284 | 0.284 |
| Treasuries | 0.566 | 0.716 | 0.716 | 0.716 |
| Sharpe Ratio | 0.305 | 0.261 | 0.261 | 0.261 |
|  |  |  |  |  |
| Panel D. Mean-variance utility, risk aversion $\gamma=10$ |  |  |  |  |
| Paintings | 0.104 | 0 | 0 | 0 |
| Stocks | 0.113 | 0.142 | 0.142 | 0.142 |
| Treasuries | 0.783 | 0.858 | 0.858 | 0.858 |
| Sharpe Ratio | 0.305 | 0.261 | 0.261 | 0.261 |

Table 6: Optimal asset allocation between art and other assets
Panel A shows the mean-variance tangency portfolio weights on art (using the non-selectioncorrected GLS index, and the selection-corrected indices A, B, and C from Table 2) and other assets (as defined in Table 4). The Benchmark portfolio leaves art out of the opportunity set. Panels B, C and D show the optimal weights for a one-period mean-variance utility investor with risk aversion ( $\gamma$ ) equal to two, five, and ten, respectively. Short sales and borrowing are not allowed. Returns are measured over 1973 to 2010. Sharpe Ratios are annualized.

|  | Bench mark | GLS | Selection-corrected |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | A | B | C |
| Panel A. Tangency portfolio weights |  |  |  |  |  |
| Paintings | - | 0.231 | 0 | 0 | 0 |
| Corporate bonds | 0.692 | 0.677 | 0.692 | 0.692 | 0.692 |
| Commodities | 0.164 | 0.073 | 0.164 | 0.164 | 0.164 |
| Real estate | 0 | 0 | 0 | 0 | 0 |
| Stocks | 0.144 | 0.018 | 0.144 | 0.144 | 0.144 |
| Sharpe Ratio | 0.409 | 0.461 | 0.409 | 0.409 | 0.409 |
| Panel B. Mean-variance utility, risk aversion $\gamma=2$ |  |  |  |  |  |
| Paintings | - | 0.222 | 0 | 0 | 0 |
| Corporate bonds | 0.441 | 0.392 | 0.441 | 0.441 | 0.441 |
| Commodities | 0.210 | 0.127 | 0.210 | 0.210 | 0.210 |
| Real estate | 0 | 0 | 0 | 0 | 0 |
| Stocks | 0.350 | 0.259 | 0.350 | 0.350 | 0.350 |
| Treasuries | 0 | 0 | 0 | 0 | 0 |
| Sharpe Ratio | 0.380 | 0.409 | 0.380 | 0.380 | 0.380 |

Panel C. Mean-variance utility, risk aversion $\gamma=5$

| Paintings | - | 0.243 | 0 | 0 | 0 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Corporate bonds | 0.674 | 0.620 | 0.674 | 0.674 | 0.674 |
| Commodities | 0.169 | 0.083 | 0.169 | 0.169 | 0.169 |
| Real estate | 0 | 0 | 0 | 0 | 0 |
| Stocks | 0.155 | 0.055 | 0.155 | 0.155 | 0.155 |
| Treasuries | 0.001 | 0 | 0.001 | 0.001 | 0.001 |
| Sharpe Ratio | 0.409 | 0.458 | 0.409 | 0.409 | 0.409 |

Panel D. Mean-variance utility, risk aversion $\gamma=10$

| Paintings | - | 0.163 | 0 | 0 | 0 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Corporate bonds | 0.333 | 0.440 | 0.333 | 0.333 | 0.333 |
| Commodities | 0.084 | 0.050 | 0.084 | 0.084 | 0.084 |
| Real estate | 0 | 0 | 0 | 0 | 0 |
| Stocks | 0.078 | 0.014 | 0.078 | 0.078 | 0.078 |
| Treasuries | 0.506 | 0.333 | 0.506 | 0.506 | 0.506 |
| Sharpe Ratio | 0.409 | 0.461 | 0.409 | 0.409 | 0.409 |

Table 7: Optimal asset allocation to painting styles
This table shows mean-variance tangency portfolio weights (in the column Tang ptf), and optimal weights on paintings of different styles, for a one-period mean-variance utility investor with risk aversion $\gamma$. Painting styles are as described in the text, and the assets are described in Table 4. Top 100 Artists is based on total value of sales over the sample period. Short sales and borrowing are not allowed. Returns are over 1973 to 2010. Sharpe Ratios are annualized.

Panel A. Painting styles and stocks only

|  | Tang ptf | Risk aversion ( $\gamma$ ) |  |  | Tang ptf | Risk aversion ( $\gamma$ ) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 5 | 10 |  | 2 | 5 | 10 |
| Post-war and Contemporary | 0.306 | 0.289 | 0.117 | 0.058 | 0 | 0 | 0 | 0 |
| Impressionist and Modern | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Old Masters | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| American | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| European 19th Century | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Other Styles | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Top 100 Artists | - | - | - | - | 0.276 | 0.240 | 0.094 | 0.048 |
| Stocks | 0.694 | 0.656 | 0.618 | 0.133 | 0.725 | 0.651 | 0.260 | 0.130 |
| Treasuries | - | 0.052 | 0.278 | 0.810 | - | 0.110 | 0.646 | 0.822 |
| Sharpe Ratio | 0.278 | 0.278 | 0.278 | 0.278 | 0.292 | 0.292 | 0.292 | 0.292 |

Panel B. Painting styles and all assets

|  | Tang ptf | Risk aversion ( $\gamma$ ) |  |  | Tang ptf | Risk aversion ( $\gamma$ ) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 5 | 10 |  | 2 | 5 | 10 |
| Post-war and Contemporary | 0.033 | 0 | 0.067 | 0.030 | 0 | 0 | 0 | 0 |
| Impressionist and Modern | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Old Masters | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| American | 0.075 | 0 | 0 | 0.012 | 0.052 | 0 | 0 | 0 |
| European 19th Century | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Other Styles | 0.107 | 0 | 0.063 | 0.070 | 0.064 | 0 | 0.012 | 0.043 |
| Top 100 Artists | - | - | - | - | 0.082 | 0.134 | 0.108 | 0.053 |
| Corporate bonds | 0.644 | 0.441 | 0.623 | 0.399 | 0.670 | 0.418 | 0.653 | 0.400 |
| Commodities | 0.098 | 0.210 | 0.133 | 0.075 | 0.090 | 0.151 | 0.119 | 0.069 |
| Real estate | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Stocks | 0.042 | 0.350 | 0.113 | 0.043 | 0.044 | 0.297 | 0.108 | 0.043 |
| Treasuries | - | 0 | 0 | 0.371 | - | 0 | 0 | 0.392 |
| Sharpe Ratio | 0.432 | 0.380 | 0.424 | 0.432 | 0.448 | 0.402 | 0.443 | 0.448 |


[^0]:    ${ }^{1}$ Following the literature, we use the terms "art" and "paintings" interchangeably throughout the paper.
    ${ }^{2}$ See the article "Follow your heart" in the Wall Street Journal on September 20, 2010. Passion investments include art, wine, and jewelry.

[^1]:    ${ }^{3}$ Index investing is a feasible strategy for investors who cannot afford a diversified exposure to the art market through the purchase of individual, often expensive paintings. With this view, we ignore any aesthetic return on art, i.e., we assume no consumption utility of owning art, since the investor does not have access to the artworks underlying the index.

[^2]:    ${ }^{4}$ The disposition effect states that investors are more likely to sell assets that have risen in price since purchase while holding on to those that have dropped in value, and has been documented in real estate (e.g., Genesove and Mayer, 2001) and public equities (e.g., Odean, 1998, though Ben-David and Hirshleifer, 2012, offer a different and more nuanced view of the empirical evidence for stocks).
    ${ }^{5}$ The normalization is necessary, but without loss of generality, because the parameters in equation (3) are only identified up to scale, as in a standard binary probit model.
    ${ }^{6}$ We follow Goetzmann's conjecture and model the selection process as a function of the return since prior sale. Similar selection effects occur if selection is on price rather than return, as suggested by Pénasse et al. (2013).

[^3]:    ${ }^{7}$ Shiller (1991) develops a repeat sales index estimator that corrects for the Jensen inequality term in the estimation, but in our model it is computationally more convenient to do the adjustment post-estimation as it preserves the linearity of the state space in the filtering problem.
    ${ }^{8}$ Art is not only sold in auction but also privately, for example through dealers. Renneboog and Spaenjers (2013) note that it is generally accepted that auction prices set a benchmark that is also used in the private market.

[^4]:    ${ }^{9}$ An artwork is "bought in" when the bidding does not reach the reserve price, and the artwork goes unsold.
    ${ }^{10}$ Observing more than three sales is extremely rare in our sample period: we only observe 26 paintings with four sales, and one painting with five sales.

[^5]:    ${ }^{11}$ We keep the artists in the top 100 category fixed throughout the sample period.

[^6]:    ${ }^{12}$ A notable exception is Renneboog and Spaenjers (2013), who find no evidence that masterpieces underperform.

[^7]:    ${ }^{13}$ The portfolio allocation results are qualitatively the same if we do not unsmooth the returns and use the original index returns instead. We discuss this in more detail in the robustness section below.

[^8]:    ${ }^{14}$ Real estate had a Sharpe Ratio of -0.15 over the period, though this ignores any dividends from consuming housing.

[^9]:    ${ }^{15}$ We thank Andrew Ang, Dimitris Papanikolaou, and Mark Westerfield for generously sharing their code for the portfolio optimization problem in their paper.

[^10]:    ${ }^{16}$ For the 1926 to 2010 period we use the CRSP stock return index (including distributions) and the one-month Tbill rate to compute excess market returns, as the MSCI world index is only available starting 1969, and the one-year T-bill rate is only available starting 1959.

