Fiscal Limits and Monetary Policy: Default vs. Inflation[*]

Anna Sokolova[†]

May 20, 2014

Abstract

In times of fiscal stress governments fail to adjust fiscal policy in line with the requirements for debt sustainability. Under those circumstances monetary policy impacts the probability of sovereign default alongside inflation dynamics. Uribe (2006) studies the relationship between inflation and sovereign defaults with a model in which the central bank controls the risky interest rate; he concludes that low inflation can only be maintained if the government sometimes defaults. This paper follows Uribe (2006) by examining monetary policy that controls the risky interest rate; however, it differs by the baseline assumption about the central bank's objectives. In this paper, monetary policy is not a pure inflation targeting: the central bank minimizes the probability of default while ruling out large inflation hikes. An advantage of this framework is that it avoids the issue of zero risk premium that exists in Uribe (2006) but allows a study of the relationship between the constraints on the monetary policy, default risk and the risk premium.

We show that monetary policy that controls the risky interest rate can mitigate default risks only when the central bank allows sufficient variation in inflation. When agents believe that the central bank's tolerance towards inflation hikes has increased, equilibrium risk premium goes down. It follows that information concerning changes in the central bank's preferences over inflation directly impacts default risks.

JEL classification: E61, E63, E52, F33

Keywords: inflation, default, sovereign debt, monetary policy

[†]National Research University - Higher School of Economics, 101000, Myasnitskaya Street, 20, Moscow, Russia. E-mail: asokolova@hse.ru

1 Introduction

In the aftermath of 2007-2008 crisis, some economies of the European Monetary Union (EMU) found themselves in a complex situation. On the one hand, there is a pressing need to increase budget surpluses to mitigate default risks; on the other hand, the scope of raising extra revenues through fiscal austerity is limited because such policy may lead to further recession and cause political crises. In the presence of fiscal stress, fiscal policy by itself may fail to insure the sustainability of government debt. In this environment, it is crucial to learn, what the monetary policy controlling the costs of borrowing can do to mitigate the debt crisis.

Sovereign defaults are associated with devastating consequences for the financial system. Ensuring the stability of the financial system is one of the key functions of a central bank. When government debt is nominated in national currency, the central bank is capable of resolving debt sustainability issues by causing the costs of debt servicing to be reduced.

Uribe (2006) shows that in the presence of sovereign default risks, two fundamental functions of the central bank are in conflict: ensuring debt sustainability (stability of the financial system) and maintaining low inflation. In the literature studying default risks and monetary policy, authors often presuppose that one of the two aims of the central bank is dominant; the results concerning the dynamics of inflation and the risk premium are contingent on the underlying assumption about the central bank's priorities. Specifically, in Sargent, Wallace (1981) as well as in the papers on Fiscal Theory of Price Level (FTPL)[1], the authors presuppose that the primary goal of the central bank is to avoid sovereign defaults, regardless of the costs in terms of inflation. Rational agents are aware of the central bank's preferences and thus believe that the probability of default is zero. It follows that in those models there is no risk premium on government bonds.

By contrast, Uribe (2006) and Guillard, Kempf (2012) study the case when maintaining low inflation is a primary objective of the central bank - monetary policy is conducted in a way that excludes deviations of inflation from the target. In these models defaults emerge whenever debt becomes unsustainable under the target level of inflation.

In this paper, the baseline assumption is that although the central bank is eager to minimize the probability of default arising from fiscal stress, it is constrained by formal requirements concerning inflation: there is a maximum level of inflation that the central bank may allow to avoid sovereign default. This specification of the central bank's problem can be viewed as a compromise between baseline assumptions of FTPL models and models in which the central bank does not allow any deviations of inflation from the target, such as Uribe (2006), Guillard, Kempf (2012). An advantage of this specification is that it avoids the issue of zero risk premium existing in Uribe (2006), while at the same time allowing a study of the capabilities and limitations of monetary policy

---

[1]Leeper (1991), Woodford (1995, 1998), Cochrane (2001) and others.

aimed at mitigating default risks.

We determine the threshold value of real debt that triggers sovereign default and show that this threshold is an increasing function of the upper limit on inflation. We then show that under this specification of monetary policy the equilibrium risk premium and probability of default depend on the upper limit of inflation - the higher the limit, the lower the risk premium and the probability of default. When the upper limit on inflation is high enough, monetary policy that controls the risky interest rate can ensure a zero probability of default in equilibrium. Furthermore, if agents do not possess exact information concerning inflation constraint, the central bank has incentives to create inaccurate beliefs suggesting the upper limit on inflation to be higher than the actual value in order to lower the risk premium on government bonds and reduce the probability of default. Another implication of this analysis: when the central bank is committed to mitigating default risks even if it means higher inflation, the earlier the public learns about this commitment, the lower are the costs of implementing such a policy.

Fiscal stress in the EMU

Our specification of the central bank's problem seems particularly relevant for the analysis of monetary policy within a monetary union. When the central bank of a monetary union conducts accommodative policy intended to stabilize the debt of one of the member regions, the costs in terms of inflation are spread across all member regions. Fiscally prudent governments may be unwilling to share these costs and thus may have an incentive to collectively impose an upper limit on inflation, restricting the central bank's policy choices.[2] Alternatively, the central bank may determine the upper limit on inflation by comparing the costs associated with an increase in inflation with the costs arising from a sovereign default of one of the member states.[3] Finally, the upper limit on inflation may be treated as a formal commitment of the central bank.

A study of monetary policy that controls the costs of borrowing appears to be urgent in light of the recently launched OMT program (Outright Monetary Transactions), a program presupposing that the European Central Bank would buy bonds of troubled governments to mitigate default risks given that they implement fiscal austerity.

In the EMU, the ability of the governments to flexibly adjust fiscal policy in line with the sustainability criteria is debatable. Trabandt, Uhlig (2011) show that over the past 20 years, European economies have drawn closer to the peaks

---

[2]This outcome seems reasonable if fiscal policy differs across regions. For instance, if the probability of default is rather small in the majority of regions, costs associated with an increase in inflation for these regions exceed benefits from reduction of the probability of default resulting from an increase in the upper limit of inflation.

[3]Cooper, Kempf, Peled (2010) show that in a monetary union the decision of the central bank on whether to bailout a member state or not depends on the allocation of risky bond holdings across regions. Since monetization leads to inflation growth, allocation of risky bonds might as well influence the maximum value of inflation that the central bank can tolerate to avoid defaults.

of their respective Laffer curves: the scope of raising extra tax revenues via increases in tax rates is limited since further increases in the tax rate would cause only a minor gain in a government's earnings. Cochrane (2011a) asserts that even if an economy is supposed to operate well below the Laffer curve peak, a small rise in the tax rate may cause a prominent slowdown of economic growth thereby reducing future taxable income. Bi, Leeper, Leith (2012) show that expectations of increases in fiscal surpluses may have a different impact on output growth depending on the composition of fiscal consolidation. Particularly, expectations of an increase in the labor tax rate lead to a slowdown of output growth, whereas a decrease in government expenditures promotes it. Even if tax collection capacities are to be neglected, it is plausible that a government facing a debt sustainability constraint would rather default on its debt than perform fiscal contraction even though such a move would facilitate debt service. Theoretical support for this view can be found in Eaton, Gersovitz (1981), who determine the "effective" tax rate - the highest rate it makes sense to impose before defaulting - which turns out to be lower than the rate corresponding to the Laffer curve peak.

Thus, austere tax policy has certain limitations. The scope of raising revenues through cutting transfers and government expenditures is limited as well. First, in a democratic environment it is difficult to implement such a policy without a substantial delay (see Alesina and Drazen, 1991). Second, due to adverse demographic trends on the one hand and the governments' obligations to support future retirees with appropriate benefits on the other, expenditures related to aging are expected to rise substantially in the next 50 years. According to the IMF (2009), the net present value of these promised expenses is averaging 409% of GDP across advanced G-20 countries, meaning that the transfers are not backed by tax revenues. These concerns show that fiscal stress is likely to remain a pressing issue in a long run.

Section 2 presents the model: it lays out the design of fiscal policy and the household's problem. We determine conditions insuring that government debt can be sold to households and describe the central bank's problem. In Section 3 we define equilibrium, determine conditions under which equilibrium exists, and express the default rate, the probability of default and the risk premium as functions of the risky interest rate. In Section 4 we determine conditions guaranteeing that the solution to the central bank's problem exists and characterize it, determining the risky interest rate. We explore equilibrium outcomes when households know the true value of the upper limit on inflation and when they do not know it, so the central bank can form beliefs about its value. Section 5 concludes. Appendix presents a numerical example for Greek economy.

2 The model

2.1 The government

Consider an endowment economy where the government collects lump sum taxes, pays transfers and issues one-period bonds. The economy is subject to fiscal stress: fiscal surpluses evolve exogenously and do not respond to changes in the real value of government debt − as a result, the government fails to insure debt sustainability when the inflation rate is particularly low. Using the terminology of Leeper (1991), fiscal policy is "active". We follow Uribe(2006) by assuming that fiscal surpluses (taxes minus transfers) follow an AR(1) process:

$$s_t - \bar{s} = \rho(s_{t-1} - \bar{s}) + \varepsilon_t, \tag{1}$$

where $\rho < 1$, $\varepsilon_t \sim F(0, \sigma^2)$, $\bar{s}$ is a steady state value of fiscal surplus. Government debt is risky: in period t the government defaults on a δ_t fraction of its debt. The dynamic budget constraint in period t is given by:

$$\frac{B_t}{P_t} = \frac{R_{t-1}B_{t-1}(1 - \delta_t)}{P_t} - s_t, \tag{2}$$

where B_t is the nominal debt in period t, P_t is the price level, R_{t-1} is the gross nominal interest rate. Following Bi (2012) Bi, Traum (2012), Guillard, Kempf (2012), we assume that default occurs, when the real value of debt exceeds an upper limit, $\hat{b}_t$, in which case the default rate equals δ. Thus, the default rule is given by:

$$\delta_t = \begin{cases} 0, \ if \ b_{t-1} < \hat{b}_t \\ \delta, \ if \ b_{t-1} \geq \hat{b}_t \end{cases} . \tag{3}$$

We derive $\hat{b}_t$ in section 3.1.

2.2 The household's problem

A representative household consumes c_t and purchases risky and risk-free bonds, $\tilde{B}_t$ and $\tilde{D}_t$. Risky bonds are supplied by the government; uncertainty about the return on risky bonds arises due to fiscal stress: because there is a possibility that the government might default on its debt, the value of risky bonds in period $t + 1$ is unknown in period t We further assume that whereas households cannot borrow from the government - $\tilde{B}_{t+i} \geq 0$ in each period - they may borrow from each other. We assume that private debt contracts are enforceable and private debt is risk-free; household's demand for this debt is given by $\tilde{D}_t$.[4] Let R_t^f be the gross nominal risk-free interest rate. A household also receives endowment y_t and pays the government s_t, lump-sum taxes minus transfers.

A household maximizes utility from consumption over an infinite horizon, solving:

---

[4]This assumption is necessary to characterize the behavior of the risk-free interest rate. It was also applied in Uribe (2006), Guillard, Kempf (2012) and others.

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \to max \qquad (4)$$

$$subject\ to :$$

$$dynamic\ budget\ constraint: \qquad c_t + \frac{\tilde{B}_t}{P_t} + \frac{\tilde{D}_t}{P_t} + s_t =$$

$$= y_t + \frac{(1-\delta_t)R_{t-1}\tilde{B}_{t-1}}{P_t} + \frac{R_{t-1}^f \tilde{D}_{t-1}}{P_t} \quad (5)$$

$$transversality\ condition: \qquad \lim_{j\to\infty} E_t q_{t+j} \tilde{D}_{t+j} \geq 0, \qquad (6)$$

$$where\ q_{t+j} = R_t^f R_{t+1}^f \ldots R_{t+j}^f,$$

$$\tilde{B}_{t+i} \geq 0\ for\ \forall i \qquad (7)$$

First order conditions for this problem are:

$$u'(c_t) = \beta R_t E_t \left[(1-\delta_{t+1}) \frac{P_t}{P_{t+1}} u'(c_{t+1}) \right]; \qquad (8)$$

$$u'(c_t) = \beta R_t^f E_t \left[\frac{P_t}{P_{t+1}} u'(c_{t+1}) \right]. \qquad (9)$$

In the subsequent section we specify Euler equations for the risky and risk-free interest rates corresponding to an equilibrium with a non-negative demand for each asset.

2.3   Sustainability of government debt and the relation between inflation and the default rate

For simplicity assume that endowment y_t is constant. As there is no production sector, the resource constraint is given by: $c_t = \bar{y}$.

In equilibrium risky and risk-free assets must be equally attractive for consumers. Since households are identical, equilibrium private borrowing must equal zero. The financial market equilibrium is given by:

$$B_t = \tilde{B}_t. \qquad (10)$$

We derive equilibrium conditions for the risky and risk-free interest rates from first order conditions (8) and (9):

$$1 = \beta R_t E_t (1-\delta_{t+1}) \frac{P_t}{P_{t+1}}; \qquad (11)$$

$$1 = \beta R_t^f E_t \frac{P_t}{P_{t+1}}. \qquad (12)$$

Analogously to transversality condition (5) there is a no-Ponzi game condition for government debt:

$$\lim_{j \to \infty} \beta^t E_t \left[b_{t+j} \right] \leq 0, \tag{13}$$

where $b_{t+j} = B_{t+j}/P_{t+j}$ is the real value of government debt. When condition (13) is violated, the discounted infinite sum of expected government expenditures plus debt exceeds the discounted sum of expected revenues - under these conditions rational households would not buy newly issued government bonds.

We iterate the dynamic budget constraint of the government (2) and apply Euler equation (11), obtaining:

$$\lim_{j \to \infty} E_t \beta^j \frac{B_{t+j}}{P_{t+j}} = R_{t-1} \frac{B_{t-1}}{P_t} (1 - \delta_t) - E_t \sum_{h=0}^{\infty} \beta^h s_{t+h}. \tag{14}$$

Combining (13) and (14), we derive a condition guaranteeing that the government does not engage in Ponzi schemes by choosing default rate δ_t:

$$(1 - \delta_t) b_{t-1} \leq \frac{E_t \sum_{h=0}^{\infty} \beta^h s_{t+h}}{R_{t-1}} \pi_t \equiv \frac{s_t(1 - \beta) + \bar{s}(1 - \rho)\beta}{(1 - \beta)(1 - \rho\beta) R_{t-1}} \pi_t, \tag{15}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$.

When condition (15) is violated for a set of $\{s_t; b_{t-1}; \delta_t; \pi_t; R_{t-1}\}$, the debt b_t, the debt that finances the operational deficit in period t, cannot be sold to households.[5]

Substituting $\delta_t = 0$ into (15), we obtain a condition that guarantees that the government debt is sustainable:

$$b_{t-1} \leq \frac{E_t \sum_{h=0}^{\infty} \beta^h s_{t+h}}{R_{t-1}} \pi_t \equiv \frac{s_t(1 - \beta) + \bar{s}(1 - \rho)\beta}{(1 - \beta)(1 - \rho\beta) R_{t-1}} \pi_t. \tag{16}$$

As households are rational, the discounted sum of private income must not exceed the discounted sum of private consumption. Thus, in equilibrium the discounted demand for risky assets must approach zero as t approaches infinity. Using equilibrium conditions (10) and (11), we obtain:

$$\lim_{j \to \infty} \beta^t E_t \left[b_{t+j} \right] = 0. \tag{17}$$

Equation (17) guarantees that, on the one hand, the dynamics of b_t satisfies the transversality condition and, on the other hand, consumption choices of households are rational.

When equation (17) holds, the no-Ponzi game condition (13) holds as equality. Analogously to the derivation of condition (15), using (17) we obtain an equilibrium relation between inflation and the default rate:

---

[5]See formal proof in sections 3.1.

$$\delta_t = 1 - \frac{E_t \sum\limits_{h=0}^{\infty} \beta^h s_{t+h}}{R_{t-1} b_{t-1}} \pi_t \equiv 1 - \frac{s_t(1-\beta) + \bar{s}(1-\rho)\beta}{(1-\beta)(1-\rho\beta)R_{t-1}b_{t-1}} \pi_t. \qquad (18)$$

Qualitative interpretations of equation (18) may vary, depending on how monetary policy is conducted. For example, in Uribe (2006) the central bank sets the inflation rate at $\pi_t = \pi^*$ for all t and determines the risky interest rate, R_{t-1}. Since s_t is random, from equation (18) it follows that under such policy rule, shocks to s_t result in non-zero default rates.

Such specification of a monetary policy has a disadvantage: in equilibrium, the default rate becomes negative whenever the value of fiscal shock exceeds zero. To see this, substitute the expression for δ_t from equation (18) into Euler equation for the risky interest rate (11):

$$\frac{\rho(1-\beta)s_t + (1-\rho)\bar{s}}{b_t(1-\beta\rho)(1-\beta)} = \frac{1}{\beta}. \qquad (19)$$

Equation (19) states that the bigger fiscal surpluses are, the higher real value of debt can be sustained in equilibrium. Note that when the fiscal shock is particularly small, $\varepsilon_t < -[\rho s_{t-1} + (1-\rho)\bar{s}/(1-\beta)]$, the government cannot sell debt at all: b_t is negative.

Applying (19) to (18) and assuming that $R_{t-1} = R^* = \pi^*/\beta$ as in Uribe(2006), we obtain the equilibrium default rate:

$$\delta_t = -\frac{\beta\varepsilon_t}{(1-\beta)(1-\rho\beta)}b_{t-1}. \qquad (20)$$

Thus, when $\varepsilon_t > 0$, the equilibrium default rate is negative. Moreover, it follows from (12) that the implied risk premium is always zero since $R_{t-1}^f = \pi^*/\beta \equiv R_{t-1}$.

Following Uribe (2006), this paper also focuses on monetary policy that controls the risky interest rate. However, unlike in Uribe (2006), our specification of monetary policy implies uncertainty over future inflation. Section 4 shows that the equilibrium risk premium and the probability of default are affected by agents' beliefs about the maximum value of inflation that the central bank would allow to avoid defaults: in equilibrium, the higher this value is believed to be, the lower is the risk premium.

Now assume that the default rate is fixed. By substituting a fixed default rate into (18), we uniquely determine inflation in period t because the discounted expected sum of surpluses is exogenous. This happens because under a fixed default rate and exogenous fiscal surpluses the transversality condition from the household's problem (13) holds as equality for only one value of π_t (and one value of P_t, since P_{t-1} is known in period t) - this particular level of π_t realizes as equilibrium. This result is in line with FTPL - the only difference being that in FTPL the value of δ_t is assumed to be zero. By substituting $\delta_t = 0$ we obtain the inflation rate and the price level, corresponding to the FTPL case:

$$\pi_t = \frac{R_{t-1}b_{t-1}}{E_t\sum\limits_{h=0}^{\infty}\beta^h s_{t+h}} = \frac{R_{t-1}b_{t-1}(1-\beta)(1-\rho\beta)}{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t(1-\beta)}; \qquad (21)$$

$$P_t = \frac{R_{t-1}B_{t-1}}{E_t\sum\limits_{h=0}^{\infty}\beta^h s_{t+h}} = \frac{R_{t-1}B_{t-1}(1-\beta)(1-\rho\beta)}{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t(1-\beta)}. \qquad (22)$$

The FTPL attributes this positive relation between inflation (or price level) and fiscal surpluses to the wealth effects. Suppose that in period t taxes are unexpectedly reduced (transfers are increased). With exogenous surpluses a reduction in taxes (an increase in transfers) today is not associated with a corresponding increase in taxes (reduction in transfers) in the future. Thus, if the price level in period t does not change, then households in period t become wealthier because the expected discounted sum of net taxes falls. Since households are rational, an increase in wealth leads to an increase in aggregate demand and to an escalation in the prices level (see Leeper (1991), Woodford (1995, 1998), Cochrane (2001) and among others). Similarly, when the government issues new bonds, household's wealth rises because new bonds are not backed by corresponding increases in government surpluses. Note that when the default rate differs from zero, the mechanism we have just outlined remains in place - the difference is quantitative, but not qualitative.

2.4 The central bank

We showed that in times of fiscal stress there is a negative relation between inflation and the default rate (equation (18)) - thus, when the central bank allows increases in inflation, it mitigates default risks. This trade-off between low inflation and fiscal sustainability causes a controversy between two fundamental goals of the central bank: suppressing inflation hikes and insuring stability of the financial market.

In this paper, we assume that although the central bank seeks to minimize default risks arising from fiscal stress, it has to fulfill a formal requirement regarding the maximum level of equilibrium inflation. This assumption can be viewed as a compromise between FTPL (Leeper (1991), Woodford (1995, 1998), Cochrane (2001) and others) and models where the central bank sets risk-free interest rate and does not allow any deviations of inflation from the target, such as Uribe (2006), Guillard, Kempf (2012). Formally, the central bank chooses the risky interest rate to minimize the expected default rate while insuring that inflation would not exceed an upper limit π^{max} which is set exogenously.

3 Equilibrium

We now turn to the definition of a competitive equilibrium for this economy.

Definition 1. A competitive equilibrium is a set of sequences

$\left\{ c_t, s_t, \delta_t, \pi_t, b_t \geq 0; R_t \geq 1; \hat{b}_t; P; B_t \right\}_{t=0}^{\infty}$, *if:*

1. Sequences satisfy:

- *Equilibrium condition (18):*

$$\delta_t = 1 - \frac{s_t(1-\beta) + \bar{s}(1-\rho)\beta}{(1-\beta)(1-\rho\beta)R_{t-1}b_{t-1}}\pi_t;$$

- *Default rule (3):*

$$\delta_t = \begin{cases} 0, \ if \ b_{t-1} < \hat{b}_t \\ \delta, \ if \ b_{t-1} \geq \hat{b}_t \end{cases} \quad ;$$

- *Resource constraint:*

$$c_t = \bar{y};$$

- *Euler equation for the risk-free interest rate (12):*

$$1 = \beta R_t^f E_t \frac{P_t}{P_{t+1}};$$

- *Government budget constraint (2):*

$$b_t = \frac{R_{t-1}b_{t-1}(1-\delta_t)}{\pi_t} - s_t;$$

2. The central bank chooses R_t^* to minimize the probability of default in period t+1 while maintaining $\pi_t \leq \pi^{max}$.

3. Fiscal surpluses follow (1):

$$s_t - \bar{s} = \rho(s_{t-1} - \bar{s}) + \varepsilon_t;$$

for given B_0, P_0.

In Section 3.1 we express the equilibrium value of $\hat{b}_t$ as a function of $\{s_t, R_{t-1}, \pi^{max}\}$; we find all $\{b_{t-1}; s_t; R_{t-1}; \pi^{max}\}$ such that there exists an equilibrium; assuming further that an equilibrium exists, we express δ_t as a function of $\{b_{t-1}; s_t; R_{t-1}; \pi^{max}\}$. In section 3.2 we use the equilibrium definition of $\hat{b}_t$ to derive the risk premium in period $t-1$ and the probability of default in period t and express them as functions of $\{b_{t-1}; s_{t-1}; R_{t-1}; \pi^{max}\}$. In Section 4 we solve the central bank's optimization problem and study the relation between R_{t-1} and $\{b_{t-1}; s_{t-1}; \pi^{max}\}$.

3.1 The equilibrium default rate

As shown in Section 2.3, when fiscal policy cannot adjust its surpluses and inflation is fixed, equilibrium with positive borrowing is reached through adjustment of the default rate, as in Uribe (2006). Our model differs from this setup by two assumptions: first, following Guillard, Kempf (2012), Bi, Traum (2012) and Bi (2012) we assume that when default emerges, default rate equals a known constant value $0 < \delta \le 1$.[6]; second, the central bank allows inflation to vary as long as it does not exceed π^{max}. What changes do these assumptions bring to the fiscal-monetary interaction setup? Let us consider two cases depending on the value of current fiscal surpluses in relation to the level of inherited debt.

Assume that fiscal variables are such that:

$$\frac{b_{t-1}R_{t-1}(1-\beta\rho)(1-\beta)}{s_{t-1}(1-\beta)\rho + \overline{s}(1-\rho) + \varepsilon_t(1-\beta)} \le \pi^{max}. \tag{23}$$

When condition (23) holds, in the absence of default equilibrium inflation is lower than π^{max} (from (18)). Thus, fiscal authority does not have to default on the debt in equilibrium.

Now consider the case in which restriction (23) is violated:

$$\frac{b_{t-1}R_{t-1}(1-\beta\rho)(1-\beta)}{s_{t-1}(1-\beta)\rho + \overline{s}(1-\rho) + \varepsilon_t(1-\beta)} > \pi^{max} \tag{24}$$

When for given values of b_{t-1}, R_{t-1}, s_{t-1} and ε_t relation (24) is true, equilibrium with zero default rate and inflation under π^{max} does not exist. Even if inflation equals π^{max} - the highest possible value - the government cannot cover current operational deficit by issuing new debt because such debt would not be sustainable and cannot be sold on the financial market.

Proposition 1. *If for given b_{t-1}, R_{t-1}, ε_t, s_{t-1} and $\delta_t = 0$ condition (24) holds, debt in period t cannot be sold to households.*

Proof. Suppose condition (24) holds. From the government budget constrain (2) we obtain the real value of bonds that the government must issue in period t to finance the operational deficit:

$$b_t > \frac{\beta\left[s_t(1-\beta)\rho + \overline{s}(1-\rho)\right]}{(1-\beta)(1-\rho\beta)}.$$

From the first order condition (11), households will be eager to purchase these bonds, when their expected yield satisfies:

$$\frac{1}{\beta} \le R_t E_t \frac{(1-\delta_{t+1})}{\pi_{t+1}}.$$

Applying equilibrium condition (18), we conclude that for the bonds to be sold to households, the following must hold:

---

[6]In Arellano (2008) default rate is also constant, it equals 1. In Bi (2012) the value of default rate depends on the properties of distribution of fiscal limit.

$$b_t \leq \frac{\beta \left[s_t(1-\beta)\rho + \bar{s}(1-\rho) \right]}{(1-\beta)(1-\rho\beta)}.$$

We reach a contradiction. $\square$

Thus, when (24) is true, equilibrium with both $\pi_t \leq \pi^{max}$ and $\delta_t = 0$ does not exist. It follows that under such conditions the government would be forced to default. Analogously to Proposition 1 it can be shown that an equilibrium with $\pi_t \leq \pi^{max}$ and $\delta_t = \delta$ only exists if:

$$\frac{b_{t-1}R_{t-1}(1-\beta\rho)(1-\beta)(1-\delta)}{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t(1-\beta)} \leq \pi^{max}. \tag{25}$$

We study the topic of existence further in Section 4.1.

Now we can identify the threshold value for the real debt from rule (3) using condition (24):

$$\hat{b}_t = \frac{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t(1-\beta)}{R_{t-1}(1-\beta\rho)(1-\beta)} \pi^{max}. \tag{26}$$

Debt that exceeds the threshold $\hat{b}_t$ is unsustainable when inflation is less than π^{max}. Whenever real value of debt exceeds $\hat{b}_t$, default emerges. Thus, equilibrium default rate is:

$$\delta_t = \begin{cases} 0, \; if \; b_{t-1} < \frac{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t(1-\beta)}{R_{t-1}(1-\beta\rho)(1-\beta)} \pi^{max} \\ \delta, \; if \; b_{t-1} \geq \frac{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t(1-\beta)}{R_{t-1}(1-\beta\rho)(1-\beta)} \pi^{max} \end{cases} \tag{27}$$

provided that an equilibrium exists - condition (25) holds for given $\{\pi^{max}; \delta; \varepsilon_t; s_{t-1}; b_{t-1}; R_{t-1}\}$.

3.2 The probability of default and the risk premium

In this section we study the relationship between the risky interest rate, the probability of default and the risk premium. From equation (26) using equation (19) we obtain a threshold value of fiscal shock:

$$\hat{\varepsilon}_t = \left(\frac{R_{t-1}\beta}{\pi^{max}} - 1 \right) b_{t-1} \frac{1-\beta\rho}{\beta}. \tag{28}$$

Default in period t emerges whenever the realization of fiscal shock turns out to be smaller than $\hat{\varepsilon}_t$. Note that the value of $\hat{\varepsilon}_t$ is known in period $t-1$; it goes up as R_{t-1}/π^{max} (the gross real interest rate on government bonds in case inflation reaches π^{max}) increases. It follows that the central bank can manipulate the threshold value, provided it can alter the risky interest rate or expectations over the upper limit on inflation. When the risky interest rate surpasses a certain level, namely $R_{t-1} > \beta\pi^{max}$, even positive shocks to fiscal surpluses can trigger defaults.

In the succeeding analysis we focus on cases in which fiscal disturbances are always relatively small - to highlight that even when households believe that the

range of fiscal shocks is narrow, they still demand a positive risk premium on government bonds, limiting the central bank's choices. Another motivation for this strategy: when large negative shocks occur, an equilibrium cannot form for valid values of price level and the default rate. Suppose in period t there occurs a fiscal shock such that $\varepsilon_t < -\left[\rho s_{t-1} + (1-\rho)\bar{s}/(1-\beta)\right]$. Then, from equation (18) it follows that in period t household's demand will be non-negative only under a negative price level or a default rate exceeding unity. Thus, from now on we presuppose that $\varepsilon_t \in [-\varepsilon^{max}; \varepsilon^{max}]$, and $-\varepsilon^{max} > -[\rho s_{t-1} + (1-\rho)\bar{s}/(1-\beta)]$ for all s_{t-1}. In Appendix A we provide quantitative estimates for Greek economy, showing that this assumption is realistic.

Now we can write down an estimate for the probability of default in period t, calculated in period $t-1$:

$$Pr(\varepsilon_t \leq \hat{\varepsilon}_t) = \int\limits_{-\varepsilon^{max}}^{\hat{\varepsilon}_t} f(\varepsilon)\, d\varepsilon, \tag{29}$$

where $f(\varepsilon)$ is the density of the distribution of shock to fiscal surpluses. From equation (28) it follows that the probability of default depends on the relation between the risky interest rate and the upper limit on inflation, R_{t-1}/π^{max}: the bigger the value of the gross real interest rate under maximum inflation, the higher the probability of default.

Now we can derive the risk premium on government bonds:

$$
\begin{aligned}
\frac{R_{t-1}}{R_{t-1}^f} &= R_{t-1}\beta \left[\int\limits_{-\varepsilon^{max}}^{\hat{\varepsilon}_t} \frac{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t(1-\beta)}{R_{t-1}b_{t-1}(1-\beta\rho)(1-\beta)(1-\delta)} dF(\varepsilon) + \right.\\
&\quad + \left. \int\limits_{\hat{\varepsilon}_t}^{\varepsilon^{max}} \frac{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t(1-\beta)}{R_{t-1}b_{t-1}(1-\beta\rho)(1-\beta)} dF(\varepsilon) \right] \tag{30}\\
&= \frac{1}{1-\delta} + (1 - \frac{1}{1-\delta}) \int\limits_{\hat{\varepsilon}_t}^{\varepsilon^{max}} (1 + \frac{\varepsilon\beta}{b_{t-1}(1-\rho\beta)}) dF(\varepsilon).
\end{aligned}
$$

The value of the integral on the right-hand side of the last equation is positive. When $\hat{\varepsilon}_t = -\varepsilon^{max}$, the probability of default equals zero and agents do not demand the risk premium. When $-\varepsilon^{max} < \hat{\varepsilon}_t \leq \varepsilon^{max}$, the value of the integral is smaller than unity and the risk premium is positive. An increase in R_{t-1}/π^{max} would lead to an increase in the threshold value of fiscal shock, $\hat{\varepsilon}_t$, as well as the probability of default and the risk premium.

A qualitative interpretation is as follows. Default emerges when the gap between the real value of debt and the sum of fiscal surpluses could only be eliminated through inflation that exceeds the upper limit; rational households are aware of this regularity. When the upper limit is high enough, the probability of default is relatively low. The higher the risky interest rate, the higher the

costs of debt service, the bigger the expected gap between the real value of debt and its backing - the higher the probability of default.

4 The central bank's problem: the choice of a risky interest rate

We have characterized the relation between the probability of default and the costs of debt service, $R_{t-1}b_{t-1}$. Since the probability of default depends on R_{t-1}, the risk premium is determined uniquely for a given value of the risky interest rate. In equilibrium, the central bank sets the risky interest rate, R_{t-1}, whereas the risk-free rate, R_{t-1}^f, adjusts in accordance with equation (30) which specifies the risk premium.

Before proceeding, let us verify that all variables from *Definition 1* can be determined uniquely and expressed as functions of $\{b_{t-1}; s_{t-1}; R_{t-1}; \pi^{max}\}$, provided that for a given set of $\{b_{t-1}; s_{t-1}; R_{t-1}; \pi^{max}\}$ an equilibrium exists. Knowing s_{t-1}, we determine s_t from equation (1). If for $\{b_{t-1}; s_t; R_{t-1}; \pi^{max}\}$ an equilibrium exists, $\hat{b}_t$ and δ_t can be uniquely determined from (27). Knowing δ_t, b_{t-1}, s_t, we determine π_t using (16). For given δ_t, b_{t-1}, s_t, π_t we derive b_t from the government budget constraint (2). The solution to the central bank's problem can be expressed as a function of b_t, s_t - knowing b_t, s_t, we determine the risky rate for newly issued bonds, R_t^* (see Section 4.2). For given R_t^*, s_t, b_t the risk premium and R_t^f can be uniquely determined from (30). Knowing P_{t-1}, π_t, b_t we derive B_t, P_t. Therefore, when an equilibrium exists, all variables from *Definition 1* are uniquely determined.

Now we are ready to examine how the central bank chooses the risky interest rate. In this section we study the choice of R_{t-1} and explore the relation between the upper limit on inflation, π^{max}, and the probability of default in equilibrium.

Choosing R_{t-1} in period $t-1$, the central bank minimizes the expected value of the default rate in period t, $E_{t-1}\delta_t$. Considering that in the event of default the default rate equals δ, this problem can be reduced to minimization of the probability of default in period t:

$$Pr(\varepsilon_t \le \hat{\varepsilon}_t) = \int_{-\varepsilon^{max}}^{\hat{\varepsilon}_t} f(\varepsilon)\,d\varepsilon \to \min_{R_{t-1}} \tag{31}$$

$$s.t. \quad R_{t-1} \ge 1; \tag{32}$$

$$R_{t-1} \ge \frac{1}{1-\delta} + (1 - \frac{1}{1-\delta}) \cdot \tag{33}$$

$$\cdot \int_{\hat{\varepsilon}_t}^{\varepsilon^{max}} (1 + \frac{\varepsilon\beta}{b_{t-1}(1-\rho\beta)})dF(\varepsilon) \equiv \Psi(R_{t-1});$$

where :

$$\delta_t = \begin{cases} 0, \ if \ \varepsilon_t > \hat{\varepsilon}_t \\ \delta, \ if \ \varepsilon_t \leq \hat{\varepsilon}_t \end{cases},$$

$$\hat{\varepsilon}_t = (\frac{R_{t-1}\beta}{\pi^{max}} - 1)b_{t-1}\frac{1-\beta\rho}{\beta}.$$

When the central bank chooses the value of the risky interest rate, it takes into account zero lower bound on the risk-free interest rate given by (33) - a condition obtained by substituting the risk premium from (30) into $R_{t-1}^f \geq 1$. Condition (33) ensures that for given $\{R_{t-1}; s_{t-1}; b_{t-1}\}$ the debt b_{t-1} can be sold to households in period $t-1$.

In the following sections we study, for which values of π^{max} and default rate δ the central bank's problem has solutions; we then examine the equilibrium relationship between the probability of default, the default rate δ and the upper limit on inflation π^{max}.

4.1   The existence of a solution

Let R_{t-1}^* be the solution to the central bank's problem. As shown in Section 3.1, for certain sets of $\left\{\varepsilon_t; \pi^{max}; R_{t-1}^*\right\}$ there is no equilibrium with $\pi_t \leq \pi^{max}$ (see condition (25))

Suppose in period t fiscal shock equals the largest negative value, $\varepsilon_t = -\varepsilon^{max}$. From (18) it follows that the equilibrium inflation would satisfy $\pi_t \leq \pi^{max}$ if:

$$R_{t-1}^* \leq \frac{s_{t-1}(1-\beta)\rho + \overline{s}(1-\rho) - \varepsilon^{max}(1-\beta)}{(1-\delta)(1-\beta\rho)(1-\beta)b_{t-1}}\pi^{max}. \tag{34}$$

Thus, a solution to the central bank's problem satisfying $\pi_t \leq \pi^{max}$ will exist for any $\varepsilon_t \in [-\varepsilon^{max}; \varepsilon^{max}]$, if the risky interest rate in period t-1 complies with condition (34). If it does not comply, then in the event of a large negative shock, inflation, which makes up for the gap between the real value of debt and the discounted sum of fiscal surpluses (corrected for a given δ), will exceed π^{max}.

Restriction $\pi_t \leq \pi^{max}$ is satisfied for all values of fiscal shock, when:

$$\pi^{max} \geq \frac{\beta}{1 - \varepsilon^{max}/\theta_{t-1}}\left[1 - \delta\int\limits_{e_t}^{\varepsilon^{max}}(1 + \frac{\varepsilon}{\theta_{t-1}})dF(\varepsilon)\right] \equiv \pi^c \tag{35}$$

where $e_t = \frac{\delta\theta_{t-1} - \varepsilon^{max}}{1-\delta}$, $\theta_{t-1} = \rho s_{t-1} + (1-\rho)\overline{s}/(1-\beta)$[7].

At this point we conclude that an equilibrium with inflation below π^{max} is feasible when either π^{max} is sufficiently high, or fiscal shocks are positive or relatively small.

---

[7]See Appendix B for derivation.

4.2 Solution

In this section we solve the central bank's problem. The objective function given by (31) decreases in R_{t-1} - hence, the solution to (31) is a minimum interest rate satisfying both (32) and (33). Figure 2 illustrates the two constraints; shaded area depicts the case in which both constraints are fulfilled. Note that these solutions exist for all realizations of a fiscal shock, if $\pi^{max} \geq \pi^c$.

The function $\Psi(R_{t-1})$ from the right-hand side of condition (33) is increasing in R_{t-1}:

$$\Psi'_{R_{t-1}} = \frac{\delta}{1-\delta} R_{t-1} \frac{\beta(1-\rho\beta)b_{t-1}}{(\pi^{max})^2} f(\hat{\varepsilon}_t) > 0.$$

This function is convex for all R_{t-1} such that $\hat{\varepsilon}_t \leq 0$:

$$\Psi''_{R_{t-1}} = \frac{\delta}{1-\delta} \frac{\beta(1-\rho\beta)b_{t-1}}{(\pi^{max})^2} \left[f'_{\hat{\varepsilon}} \frac{(1-\rho\beta)b_{t-1}R_{t-1}}{\pi^{max}} + f(\hat{\varepsilon}_t) \right] > 0.$$

In the following analysis we only study equilibria with $\hat{\varepsilon}_t \leq 0$ - that is to say, equilibria in which default can only be caused by negative fiscal shocks, but not positive ones. From (28) we obtain that $\hat{\varepsilon}_t \leq 0$ when $R^*_{t-1} \geq \pi^{max}/\beta$. In equilibrium, the central bank sets $R^*_{t-1} \geq \pi^{max}/\beta$ when constraint (33) is fulfilled at $R^*_{t-1} = \pi^{max}/\beta$.[8] From (33) it follows that $\pi^{max}/\beta \geq \Psi(\pi^{max}/\beta)$ if:

$$\pi^{max} \geq \frac{\beta}{1-\delta} \left[1 - \frac{\delta}{2} - \frac{\beta\delta}{b_{t-1}(1-\rho\beta)} \int\limits_0^{\varepsilon^{max}} \varepsilon dF(\varepsilon) \right] \equiv \pi^L. \tag{36}$$

Thus, we presuppose that condition (36) is fulfilled.[9]

We now determine the values of π^{max} from (36), under which the probability of default is zero. Solutions with a zero probability of default are available, if condition (33) is fulfilled for $R^*_{t-1} = 1$: since the objective function decreases in R_{t-1}, when $R^*_{t-1} = 1$ is possible, it is an equilibrium solution for which the probability of default is zero, which is the case if:

$$\pi^{max} \geq \frac{\beta [\rho s_{t-1} + (1-\rho)\bar{s}/(1-\beta)]}{[\rho s_{t-1} + (1-\rho)\bar{s}/(1-\beta)] - \varepsilon^{max}} \equiv \pi^H, \tag{37}$$

so that $\Psi(1) \leq 1$ (see Figure 2a). Thus, the probability of default is zero when the upper limit on inflation is sufficiently high. By contrast, when $\pi^{max} < \pi^H$, $\Psi(1) > 1$ (see Figure 2b). In this case an equilibrium with a zero probability of default does not exist.

---

[8] It follows from $\Psi(R_{t-1})$ being an increasing convex function under $\hat{\varepsilon}_t \leq 0$.

[9] When constraint (36) is not fulfilled, qualitative results do not change, whereas mathematical analysis becomes substantially more complicated.
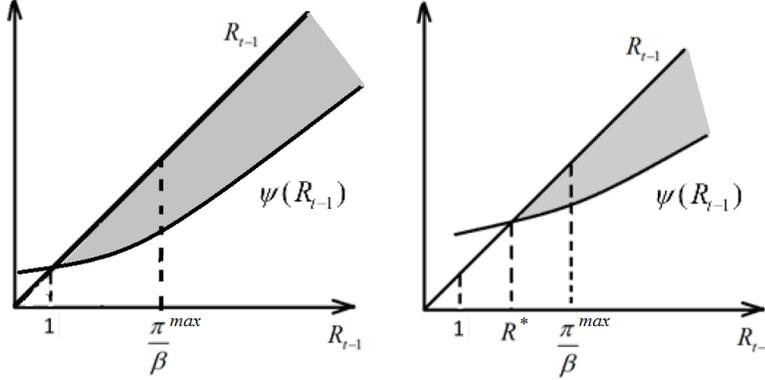
Figure 1   Zero probability of default   *Figure 2   Non-zero probability of default*

When inequality (37) holds and $R^*_{t-1} = 1$, households know that even if $\varepsilon_t = -\varepsilon^{max}$, default would not emerge because inflation that ensures debt sustainability is below π^{max}. Thus, when $R^*_{t-1} = 1$, the risk premium is zero and the zero lower bound constraint on the risk-free interest rate is fulfilled. At the same time, the central bank does not have incentives to set $R^*_{t-1} > 1$ because the corresponding probability of default is higher. Therefore, under (37), $R^*_{t-1} = 1$ is an equilibrium solution.

When the upper limit on inflation is low ($\pi^L \leq \pi^{max} < \pi^H$), an equilibrium with a zero probability of default is not feasible. Even if the central bank sets $R^*_{t-1} = 1$ in period $t-1$, there is a non-zero probability that inflation that ensures debt sustainability would exceed π^{max} in period t (and thus, default would emerge). Hence, under $R^*_{t-1} = 1$ in period $t-1$ households demand a positive risk premium on government bonds which means that the zero lower bound constraint for a risk-free interest rate is violated. Thus, solution $R^*_{t-1} = 1$ is not feasible.

To illustrate these theoretical results, Appendix A provides a numerical example for Greek economy.

4.3   Expectations over the upper limit on inflation and dynamic inconsistency

Now suppose that in period $t-1$ households do not know the true value of π^{max} and the central bank can affect household's expectations over π^{max} for period t by making a public statement in period t-1. Choosing $\tilde{\pi}^{max}_t$, the central bank solves:

$$Pr(\varepsilon_t \leq \hat{\varepsilon}_t) = \int\limits_{-\varepsilon^{max}}^{\hat{\varepsilon}_t} f(\varepsilon)\, d\varepsilon \to \min_{\tilde{\pi}^{max},R_{t-1}} \tag{38}$$

$s.t.$ $\quad R_{t-1} \geq 1;$

$$R_{t-1} \geq \frac{1}{1-\delta} + (1 - \frac{1}{1-\delta}) \cdot$$

$$\cdot \int\limits_{\hat{\varepsilon}_t^e}^{\varepsilon^{max}} (1 + \frac{\varepsilon\beta}{b_{t-1}(1-\rho\beta)}) dF(\varepsilon) \equiv \Psi(R_{t-1}),$$

where the expected threshold value of fiscal shock depends on $\tilde{\pi}_t^{max}$:

$$\hat{\varepsilon}_t^e = (\frac{R_{t-1}\beta}{\tilde{\pi}_t^{max}} - 1)b_{t-1}\frac{1-\beta\rho}{\beta}$$

and the actual threshold value of shock depends on π^{max} :

$$\hat{\varepsilon}_t = (\frac{R_{t-1}\beta}{\pi^{max}} - 1)b_{t-1}\frac{1-\beta\rho}{\beta}.$$

As noted before, the bigger the risky interest rate, R_{t-1}, the higher the expected default rate, $E_{t-1}\delta_t$. The range of R_{t-1} that satisfies the zero lower bound restriction for the risk-free rate depends on the risk premium that households demand for government bonds: the lower the risk premium, the lower the minimum value of R_{t-1} that the central bank can set. At the same time, regardless of the risk premium, the (gross) risky interest rate cannot be lower than 1. The solution $R_{t-1}^* = 1$ becomes feasible when the risk premium is zero, which would be the case under $\tilde{\pi}_t^{max} \geq \pi^H$. Thus, the solution for problem (38) is: $R_{t-1}^* = 1$; $\tilde{\pi}_t^{max} \geq \pi^H$ because for $R_{t-1}^* > 1$ the probability of default is higher, and if $\tilde{\pi}_t^{max} < \pi^H$ the solution $R_{t-1}^* = 1$ is not feasible.

Therefore, when the central bank can influence households' expectations over $\tilde{\pi}_t^{max}$, the probability of default turns out to be smaller than under $\tilde{\pi}_t^{max} = \pi^{max}$. This feature invokes the issue of dynamic inconsistency: the solution for $\tilde{\pi}_t^{max}$ does not correspond to the true value of the upper limit on inflation, if $\pi^{max} < \pi^H$.

We draw the following conclusion. When households do not know the exact boundaries of the restriction on inflation that the central bank faces, the latter has incentives to create inaccurate beliefs by suggesting that the upper limit on inflation is higher than the true value, in order to lower the risk premium and the equilibrium probability of default. On the other hand, when households believe that the upper limit on inflation is lower than its actual value, the equilibrium probability of default is higher than in case households' beliefs are accurate. Another implication of this analysis: when the central bank is committed to mitigating default risks even if it means higher inflation, it is sensible to disclose this commitment as early as possible. Such a disclosure would lead to a lower operational deficit for the government and allow it to borrow less, leading to a lower probability of default in the future.

5 Conclusion

In recent years fiscal stress has become a matter of concern for some of the developed European countries. Governments facing fiscal limits are unable to flexibly adjust their fiscal policy in line with the requirements for debt sustainability - in these countries fiscal shocks may lead to an escalation of default risks.

In these circumstances the central bank's policy affects the probability of default on government bonds alongside having an impact on inflation. In this paper we studied the capabilities and limitations of monetary policy that controls the risky interest rate in the environment where the central bank strives to minimize the probability of sovereign default while facing restrictions on the upper limit of inflation.

We have arrived at the following conclusions. The higher the upper limit on inflation, the lower the equilibrium probability of default and the risk premium on government bonds demanded by the market. When the upper limit on inflation is set relatively low, an equilibrium with inflation below the limit is only feasible when fiscal shocks are either positive or negative but small. An equilibrium with a zero probability of default is feasible when the upper limit on inflation is sufficiently high; the smaller current fiscal surpluses, the higher the value of the upper limit on inflation that ensures a zero probability of default.

Furthermore, agents' beliefs about the restrictions on inflation have a prominent effect on equilibrium outcomes. When the upper limit on inflation is believed to be higher than its actual value, the equilibrium probability of default is lower than in case when agents' beliefs reflect the true value of the upper limit on inflation. When the central bank is committed to mitigating default risks even if it means higher inflation, the earlier the public learns about this commitment, the lower the costs of implementing such a policy would be.

References

[1] Alesina, A., Drazen A. (1991). Why are Stabilizations Delayed? // American Economic Review 81, 1170-1188.

[2] Arellano, C. (2008). Default Risk and Income Fluctuations in Emerging Economies // American Economic Review, American Economic Association, vol. 98(3), pages 690-712, June.

[3] Bi, H. (2012). Sovereign default risk premia, fiscal limits, and fiscal policy // European Economic Review, Elsevier, vol. 56(3), pages 389-410

[4] Bi, H., Leeper, E., Leith, C. (2013). Uncertain Fiscal Consolidations //Economic Journal, Royal Economic Society, vol. 0, pages F31-F63, 02.

[5] Bi, H., Traum, N. (2012). Estimating Sovereign Default Risk // American Economic Review, American Economic Association, vol. 102(3), pages 161-66, May.

[6] Cochrane, J.H. (2001). Long term debt and optimal policy in the fiscal theory of the price level // Econometrica 69, 69–116

[7] Cochrane, J.H. (2011a). Understanding policy in the great recession: Some unpleasant fiscal arithmetic // European Economic Review 55 (2011) 2–30

[8] Cochrane, J.H. (2011). Determinacy and Identification with Taylor Rules // Journal of Political Economy, University of Chicago Press, vol. 119(3), pages 565 - 615.

[9] Cooper, R., Kempf, H., Peled, D. (2010). Regional debt in monetary unions: Is it inflationary? // European Economic Review, Elsevier, vol. 54(3), pages 345-358, April.

[10] Eaton, J., Gersovitz, M. (1981). Debt with Potential Repudiation: Theoretical and Empirical Analysis // Review of Economic Studies, 48(2), 289–309.

[11] Guillard, M., Kempf, H. (2012). L'insoutenable dynamique de la dette. Une analyse macroéconomique du défaut souverain // Revue d'économie politique , Dalloz, vol. 122(6), pages 921-941.

[12] International Monetary Fund (2009). Fiscal Implications of the Global Economic and Financial Crisis // IMF Staff Position Note SPN/09/13

[13] Leeper, E.(1991). Equilibria under 'Active' and 'Passive' monetary and fiscal policies // Journal of Monetary Economics 27, 129–147.

[14] Sargent, T. J., Wallace, N. (1981). Some Unpleasant Monetarist Arithmetic // Federal Reserve Bank of Minneapolis Quarterly Review, 5(Fall), 1–17.

[15] Trabandt, M., Uhlig, H. (2011). The Laffer curve revisited // Journal of Monetary Economics, Elsevier, vol. 58(4), pages 305-327

[16] Uribe, M. (2006). A fiscal theory of sovereign risk // Journal of Monetary Economics, Elsevier, vol. 53(8), pages 1857-1875, November

[17] Woodford, M. (1995). Price-Level Determinacy Without Control of a Monetary Aggregate // Carneige-Rochester Conference Series on Public Policy, 43, 1–46.

[18] Woodford, M. (1998). Control of Public Debt: A Requirement for Price Stability? // The Debt Burden and Consequences for Monetary Policy, ed. By G. Calvo, M. King, pp. 117-154. St. Martin's Press, New York

[19] Woodford, M. (2003). Interest and Prices // Princeton, NJ: Princeton Univ. Press

Appendix A

Table 1 reports annual data on the fiscal surplus as a percentage of GDP for Greece for 2000-2012. Suppose that $\rho \in [0,5; 0,8]$ and $\beta = 0,99$.[10]

According to The Stability and Growth Pact, the debt to GDP ratio in European countries must not exceed 60 percent. It follows from (19) that to support this level in the long run, $\bar{b} = 60\%$, the steady-state surplus must equal $\bar{s} = 0,6\%$.

We estimate the value of fiscal shock according to $\varepsilon_t = s_t - \bar{s} - \rho(s_{t-1} - \bar{s})$. The minimum value of fiscal shock, $-\varepsilon^{max}$, for the period 2005-2012 belongs to the interval $[-11; -2,95]$. Define $-[\rho s_{t-1} + (1-\rho)\bar{s}/(1-\beta)] = \Delta_t$. The value of Δ_t is within the range $[-28,15; -22,2]$ - thus, the assumption that ε^{max} satisfies $-\varepsilon^{max} \geq -[\rho s_{t-1} + (1-\rho)\bar{s}/(1-\beta)]$ is realistic.

Under monetary policy that controls the risky interest rate, an equilibrium with a zero probability of default is only feasible when the upper limit on inflation is believed to be very high (higher than 21% for 2004-2007 and even higher after 2007) - but this is only true if agents believe that the long-term value of surpluses equals $0,6\%$ of GDP. If the long-term surplus is believed to equal $\bar{s} = 0,7(\bar{s} = 1)$, then the upper limit on inflation for 2004-2007 must exceed $\pi^{max} = 18(\pi^{max} = 11)$ for an equilibrium with a zero probability of default to exist.

Year	s_t	Estimates for $\bar{s} = 0,6$			Estimates of π^{max}		
		Δ_t	ε_t	$-\varepsilon^{max}$	$for\ \bar{s} = 0,6$	$for\ \bar{s} = 0,7$	$for\ \bar{s} = 1$
2000	-3,7						
2001	-4,5	-28,15	-2,95	-2,95	1,10	1,09	1,05
2002	-4,8	-27,75	-2,85	-2,95	1,11	1,09	1,06
2003	-5,6	-27,6	-3,5	-3,5	1,13	1,11	1,07
2004	-7,5	-27,2	-5	-5	1,21	1,17	1,11
2005	-5,2	-26,25	-1,75	-5	1,22	1,18	1,11
2006	-5,7	-27,4	-3,4	-5	1,21	1,17	1,11
2007	-6,5	-27,15	-3,95	-5	1,21	1,17	1,11
2008	-9,8	-26,75	-6,85	-6,85	1,33	1,26	1,17
2009	-15,6	-25,1	-11	-11	1,76	1,56	1,31
2010	-10,7	-22,2	-3,2	-11	1,96	1,67	1,34
2011	-9,5	-24,65	-4,45	-11	1,78	1,57	1,32
2012	-10	-25,25	-5,55	-11	1,75	1,56	1,32

Table 1

* Data provided by *Eurostat*

**For calculation of $-\varepsilon^{max}$ we assume that it equals the minimum value of fiscal shock over the preceding period, starting from 2000, as Greece entered Eurozone. For instance, for

---

[10] Bi, Traum (2012) calibrate the model with sovereign risks for Greek economy. We follow them in assuming that $\beta = 0,99$ and rely on their results suggesting that mean values of auto-regressive coefficients for taxes, transfers and government purchases are $0,5$, $0,5$, $0,8$ respectively.

2005 with $\bar{s} = 0,6$ we obtain $-\varepsilon^{max} = min\{-2,95;\ -2,85;-3,5;-5;-1,75\} = -5$. This analysis can be generalized by adding uncertainty over ε^{max}.

Appendix B

Here we derive a condition for π^{max} that ensures existence of an equilibrium with $\pi_t \leq \pi^{max}$ for every possible realization of the fiscal shock.

Recall that the solution of problem (31) is either $R_{t-1} = \Psi(R_{t-1})$ if $\Psi(1) > 1$, where $\Psi(R_{t-1})$ is a risk premium from (33), or $R_{t-1} = 1$ if $\Psi(1) = 1$. At the same time, condition (34) gives an upper restriction on R_{t-1} that ensures the existence of an equilibrium for every possible value of ε_t:

$$R^*_{t-1} \leq \frac{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) - \varepsilon^{max}(1-\beta)}{(1-\delta)(1-\beta\rho)(1-\beta)b_{t-1}}\pi^{max} \equiv R^c_{t-1} \qquad (39)$$

Rewrite using equation (19):

$$R^c_{t-1} = \frac{\pi^{max}}{\beta(1-\delta)}\left[1 - \frac{\varepsilon^{max}}{\rho s_{t-1} + (1-\rho)\bar{s}/(1-\beta)}\right]$$

By assumption, $\varepsilon^{max} < \rho s_{t-1} + (1-\rho)\bar{s}/(1-\beta)$ - thus, R^c_{t-1} is positive. If R^*_{t-1} that solves the central bank's problem exceeds R^c_{t-1}, then for some low negative values of the fiscal shock there is no equilibrium satisfying $\pi_t \leq \pi^{max}$. This happens when $\Psi(R^c_{t-1}) > R^c_{t-1}$. Figures B1 and B2 depict possible outcomes depending on R^*_{t-1}. Figure B1 shows the case when condition (39) is violated, whereas Figure B2 illustrates the setup for which condition (39) is fulfilled.
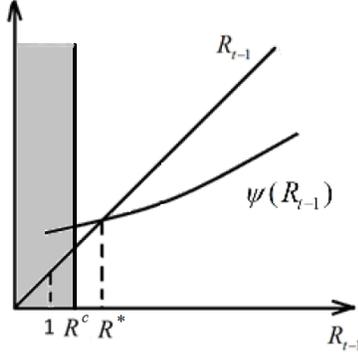

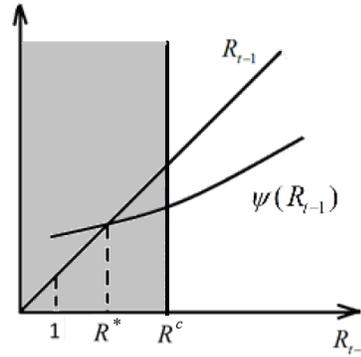Figure B1 Equilibrium exists for some values of fiscal shock

Figure B2 Equilibrium exists regardless of the value of fiscal shock

Define $\rho s_{t-1} + (1-\rho)\bar{s}/(1-\beta) \equiv \theta_{t-1}$. The condition guaranteeing existence of an equilibrium can be derived by substituting R^c_{t-1} into (30) and setting $\Psi(R^c_{t-1}) \leq R^c_{t-1}$:

$$\frac{1}{1-\delta} + (1 - \frac{1}{1-\delta}) \int\limits_{e_t}^{\varepsilon^{max}} (1 + \frac{\varepsilon\beta}{b_{t-1}(1-\rho\beta)})dF(\varepsilon) \leq \frac{\pi^{max}}{\beta(1-\delta)} \left[1 - \frac{\varepsilon^{max}}{\theta_{t-1}} \right]$$

where $e_t = \frac{\delta\theta_{t-1} - \varepsilon^{max}}{1-\delta}$. Finally, applying (19) we obtain:

$$\pi^{max} \geq \frac{\beta}{1 - \varepsilon^{max}/\theta_{t-1}} \left[1 - \delta \int\limits_{e_t}^{\varepsilon^{max}} (1 + \frac{\varepsilon}{\theta_{t-1}})dF(\varepsilon) \right].$$