Combining time-variation and mixed-frequencies: an analysis of government spending multipliers in Italy[*]

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December 26, 2013

Abstract

We propose a time-varying VAR model, estimated on mixed-frequency data, to study the macroeconomic effects of government spending shocks. Time-variation in the model allows to capture regime shifts and structural changes in the transmission of fiscal shocks, a feature that can be particularly important in the context of the current crisis. The model is estimated exploiting information from both quarterly and annual fiscal series. This is very useful for fiscal policy analysis given the absence of long time series of quarterly fiscal data for most advanced economies, but at the same time the availability of relatively long time series of annual data. Based on this model, we investigate how the transmission of government spending shocks has changed over time in Italy, over the period 1988Q4-2012Q2. We find that, for this country, government spending shocks tend to have positive effects on output. The fiscal multiplier appears to follow a U-shape over the sample considered: it peaks at around 1.5 at the beginning of the sample to stabilizes between 0.9 and 1 during the run-up phase to the EMU, before rising again to above unity in the context of the crisis.

JEL Classification: C32, E62, H30, H50.
Keywords: Time variation, Mixed-frequency data, government spending multiplier.

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1 Introduction

During the recent economic and financial crisis, fiscal policy has regained prominence in the academic and policy debate. In the first phase of the crisis - during 2008 and 2009 - governments around the world enacted unprecedented fiscal stimulus packages in response to rapidly deteriorating macroeconomic conditions. This was reflected in the adoption of the European Economic Recovery Plan (EERP) and the American Recovery and Reinvestment Act (ARRA), respectively, in the European Union and the United States.

As a result of such fiscal stimulus packages, of the operation of automatic stabilizers in a contracting economy, and of other factors (e.g., recapitalization of distressed banks), government deficit and debt ratios skyrocketed in many industrialized countries leading to a sovereign debt crisis since mid-2010. Therefore, in response to heightened concerns regarding the long-term sustainability of public finances and to market pressure, since 2010 many governments adopted sizeable fiscal consolidation measures with a view of bringing down deficit and debt levels to sustainable levels.

More recently, some commentators claimed that fiscal austerity may have gone too far, and that too severe fiscal consolidation policies may lead to 'self-defeating' effects, i.e. the debt-to-GDP ratio may increase despite the adoption of fiscal adjustment measures (see, e.g., Corsetti (2012)). In this context, it has been sometimes argued that - to exit from the current recessionary period, still experienced by many countries, especially in Europe - fiscal stimulus policies should be revamped (see, notably, Krugman (2013)).

Overall, despite this renewed interest for the active use of fiscal policy - both as a tool for macroeconomic stimulus, or to restore the long-term sustainability of public finances - there is still uncertainty regarding the macroeconomic impact of discretionary fiscal policies, and notably of government expenditure measures. Indeed, the empirical literature on the effects of government spending shocks is limited and overall quite inconclusive so far, especially as concerns euro area

countries.

One of the main factors explaining the scarcity of empirical research on the effects of fiscal policy for euro area countries is data (un)availability: while, for most European countries, annual time series for fiscal variables are available starting from the 1980s; accrual quarterly time series have been produced by European statistical agencies only since the inception of the European Monetary Union. The absence of sufficiently long data set of quarterly data limits the use of time series models, such as vector autoregression (VAR) models. In fact, three or four decades of quarterly data are typically needed for the estimation of such models and the identification of fiscal shocks.

Another limitation that prevented a rapid development of the empirical literature on fiscal policy is that - until recently - econometric approaches that allow to capture structural changes, as for example the ones likely to have occurred during the recent crisis, have not been available to practitioners. While this limitation has been recently overcome through the development of time-varying parameters models (see, for example, Primiceri (2005)), to the best of our knowledge so far no authors have adapted time-varying parameter VAR models to a mixed-frequency framework.

Against this background, this paper contributes to the macroeconomic literature, and in particular to the empirical research on fiscal policy, by proposing a time-varying parameters model estimated on both annual and quarterly data in a unique framework.

We apply such methodology to study the effects of government spending shocks in Italy, over the period 1988Q4-2012Q2. Italy represents an interesting case study: it is the third euro zone economy, it has experienced a severe economic downturn since 2009, it has a fragile fiscal position with the government debt ratio expected to reach around 130% of GDP in 2013, and it has been at the centre of sovereign market tensions for long periods during the crisis. Therefore, the analysis of the macroeconomic effects of fiscal policy actions for this country is particularly relevant in our view. Italy is a relevant case for the application of a mixed-frequency approach because - similarly

to other European countries - the national statistical agency (ISTAT) started to produce quarterly time series for the main fiscal variables only as of 1999, while annual time series are available as of the beginning of the 1980s for most fiscal variables.

Our results indicate that, for Italy, the fiscal multiplier tends to follow a U-shape over the 1988Q4-2012Q2 sample: it peaks at around 1.5 in the late 1980s and in the beginning of the 1990s, it then stabilizes between 0.9 and 1 during the run-up phase to the EMU, before rising again to above unity in the context of the recent global crisis. This tends to be in line with recent research highlighting stronger effects of fiscal policies during economic contractions (see, e.g., Callegari et al. (2012), Baum et al. (2012)), although evidence on Italy has been so far missing.

The reminder of the paper is organized as follows. Section 2 discusses related literature, the description of the model is outlined in section 4, section 3 presents the Italian data used in the analysis and section 5 illustrates our results. Finally, Section 6 concludes.

2   Related literature

Since the seminal work by Blanchard and Perotti (2002), the empirical research on the effects of fiscal policy shocks has developed relatively rapidly. However, studies on European countries are scarce, mainly due to unavailability of long time series of quarterly fiscal data complied on accrual basis. In fact, with the exception of a few countries (e.g., Germany), before 1999 most European national statistical offices did not produce comprehensive quarterly time series of fiscal data in accordance with internationally agreed accounting standards (e.g. ESA95 accounting rules).

One of the first paper to estimate the effects of fiscal shocks for European countries is Marcellino (2006). Based on semi-annual data from the OECD (whose production has been discontinued), this paper estimates the effects of fiscal shocks in Germany (and other three EU countries) over the period 1981-2001. It is shown that spending shocks have weak effects on output in Germany,

whereas the effects of tax shocks are shown to be more sizeable and of the expected negative sign.

As regards Italy, there are very few studies on the effects of fiscal shocks. One exception is Giordano et al. (2007). The authors construct a quarterly cash data set for selected fiscal variables for the period 1982-2004, mainly relying on the information contained in the Italian Treasury Quarterly Reports. The paper suggests that a one percent government spending shock increases private real GDP by 0.6 per cent after 3 quarters. Such response fades away after two years. More recently, Caprioli and Momigliano (2011) propose new estimates of expenditure and revenue shocks for Italy: they find that fiscal shocks tend to have significant effects on economic activity. These effects appear to be stronger, as well as more precisely estimated and robust, for expenditure shocks.

As regards the analysis of time-variation in the transmission of fiscal policy shocks, one of the earlier study is Cimadomo and Bénassy-Quéré (2012). Based on rolling-window estimates of fiscal VAR models for Germany (in addition to the UK and US), it is found that the net tax multiplier follows a humped-shaped curve, peaking in the middle of the 1990s, declining thereafter, before rising again during the recent crisis. Government spending shocks are found to be more powerful to stimulate output after the German reunification.

A Bayesian time-varying model for the analysis of fiscal shocks has been proposed by Kirchner et al. (2010) (see Cogley and Sargent (2005) and Primiceri (2005) for earlier studies on monetary policy). The paper focuses on the aggregate euro area, based on the fiscal data set complied by Paredes et al. (2009). The results show that, for the aggregate euro zone, the short-run effectiveness of government spending in stimulating real GDP and private consumption has increased until the end-1980s but it has decreased thereafter. Another paper using a time-varying Bayesian VAR model for the analysis of fiscal shocks is Pereira and Lopes (2010). Based on a dataset for the United States and covering the period 1965-2009, the paper suggests that fiscal policy lost some capacity to stimulate output over time, but this trend is more pronounced for taxes net of transfers

than for government expenditure, whose effectiveness declines only slightly.

Finally, while the development of models for variables sampled at different frequencies, i.e. of mixed-frequency models, has accelerated substantially in the recent period (see Foroni and Marcellino (2013) for a survey), so far little has been done in the field of fiscal policy analysis. Most importantly, to the best of our knowledge there are no papers proposing such models in combination to the analysis of time variation. In this paper, we also aim at filling this gap in the literature.

3 Data

Our benchmark VAR includes three variables: government spending, i.e. government consumption plus investment expenditure, GDP and the short-term nominal interest rate. For the latter, we use the average interest rate on Italian government T-bills, i.e. government securities with maturity of less than one year.[1] In line with the reference literature (see, e.g. Perotti (2007)), we transform government spending and GDP in real per capita terms: first, we divide the nominal series by the GDP deflator. Then, we take the ratio of the real series to total population. The interest rate is not transformed.

The data set covers the period 1981Q4-2012Q2 and includes quarterly observations for GDP and the interest rate, given that quarterly times series for these variables are available for these two variable over this period. For government spending, quarterly time series are available only as of 1999Q1. For the previous period, we use therefore annual data. This represents the mixed-frequency feature of our data set.

Incorporating GDP allows to analyse how the government spending multiplier, i.e., the percentage change of GDP following a 1% of GDP shock to government spending, has evolved over time in Italy, including large part of the recent global crisis. At the same time, using an interest rate on

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[1]The use of long-term interest rates does not lead to significantly different results in our analysis. Additional results are reported in the robustness section.

sovereign securities allows to capture the interdependencies between fiscal policies and the sovereign market, and to address the following questions: how does sovereign interest rate react to an expansionary (or contractionary) fiscal shock? and, viceversa, how does government spending respond to a shock in the interest rate on a government securities? These issues are particularly relevant for Italy. In fact, Italy has witnessed a remarkable decline in sovereign yields in the run-up phase to the EMU, coupled with a tightening of government expenditure (see Figure 1). Interest rates on sovereign securities stabilized during the first phase of the EMU, while government spending rose again until the recent crisis. As of 2008, Italy has experienced a new spending contraction - triggered by the consolidation policies adopted in the context of the crisis - which has been accompanied by a sharply declining GDP and interest rates on government securities rising again in some periods. Against this background, analysing the joint interrelations between government spending, GDP and interest rates is of great interest in our view.

The model is estimated on both annual and quarterly data for the three variables in the benchmark VAR. Annual and quarterly data for government consumption, investment and GDP are taken from Eurostat, which validates - according to ESA95 accounting standards - the national account statistics produced by the Italian statistical agency, i.e., ISTAT. The data on interest rates on government securities are retrieved from the IMF's International Financial Statistics dataset.

Our resulting dataset is unbalanced given that - while the last observation for all variables is 2012 (for annual data) or 2012Q2 (for quarterly data) - the same time series generally have different starting points. For annual data, the first available observation is 1979 for government spending. For GDP and the interest rate, data are available before that date, but we need to truncate the time series of these variables to 1979 for consistency with the available data on spending. For quarterly data, the first available observation is 1999Q1 for government spending. Quarterly real GDP is available as of 1981Q1, whereas the quarterly time series for the short-term interest rate is available

as of 1979Q1.

The methodology presented in section 4 allows to back-cast the quarterly profile of the variables with annual frequency (government spendig). The model produces such back-cast by exploiting essentially the cross-sectional covariation across variables.

4 Model

In this section we describe the basic characerestics of the model, which allows to study the impact of a government spending shock in a mixed frequency environment, in addition we also allow parameters and volatility to change over-time. The model can be summarized as follows.

Let us assume that a vector of endogenous variables, eventually sampled at different frequencies, $Y_t = [y'_{1,t}, y'_{2,t}, ..., y'_{N,t}]'$ can be written as:

$$Y_t = C(L)\widetilde{Y}_t + v_t \tag{1}$$

where $C_t(L) = [I_N + 0_N L + ... + 0_N L^l] \ \forall t$, $\widetilde{Y}_t$ is a vector of states[2] and $v_t \sim N(0, R_t)$ with R_t a $(N \times N)$ diagonal matrix. We denote as $r_{i,t}$ the ith element on the diagonal; r_i^{th} can only take two values, 0 if $\widetilde{y}_{i,t}$ is avaivable, ∞ otherwise.

We also assume that $\widetilde{Y}_t$ can be written as:

$$\widetilde{Y}_t = A_{0,t} + A(L)_t \widetilde{Y}_{t-1} + \varepsilon_t \tag{2}$$

where, $A_{0,t}$ is the vector of time-varying intercepts, $A(L)$ is a matrix polynomial in the lag operator L of time-varying coefficients and ε_t is the vector of is a vector of innovations.

Let $A_t = [A_{0,t}, A_{1,t}, ...A_{l,t}]$ and $\theta_t = vec(A'_t)$, where $vec(\cdot)$ is the column stacking operator. The

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[2]In a mixed frequency environment states are known if data are available or unknown if they are not.

law of motion for θ_t is assumed to be such that

$$\theta_t = \theta_{t-1} + \omega_t,$$

where ω_t is a Gaussian white noise with zero mean and covariance Ω.

The innovations in equation (2) are assumed to Gaussian white noises with zero mean and time-varying covariance Σ_t that is factorized as

$$\Sigma_t = F_t D_t F_t',$$

where F_t is lower triangular, with ones on the main diagonal, and D_t a diagonal matrix. Let σ_t be the vector of the diagonal elements of $D_t^{1/2}$ and the off-diagonal element of the matrix F_t^{-1}. We assume that the standard deviations, σ_t, evolve as geometric random walks, belonging to the class of models known as stochastic volatility. The contemporaneous relationships ϕ_{it} in each equation of the VAR are assumed to evolve as an independent random walk, leading to the following specifications

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\begin{aligned}
\log \sigma_t &= \log \sigma_{t-1} + \zeta_t \\
\phi_{it} &= \phi_{it-1} + \varphi_{it}
\end{aligned}
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where ζ_t and φ_{it} are Gaussian white noise with zero mean and covariance Ξ and Ψ_i, respectively. We assume that $\varepsilon_t, \omega_t, \zeta_t$, and φ_{it} are mutually uncorrelated at all leads and lags and that φ_{it} is independent of φ_{jt} for $i \neq j$.

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4.1 Priors Specification and initial values

The model is estimated using Bayesian methods. While the details of the posterior simulation are accurately described in the Appendix, in this section we briefly discuss the specification of our priors and initial values. The unknow state vector $\widetilde{Y}_t$ is initialized by linear interpolation. That is, in our framework with quarterly and annual observations, missing quarterly observations are computed as linear interpolation of annual data points.[3]

Following Primiceri (2005), we make the following assumptions for the priors densities. First, the coefficients of the covariances of the log volatilities and the hyperparameters are assumed to be independent of each other. The priors for the initial states θ_0, ϕ_0 and $\log \sigma_0$ are assumed to be normally distributed. The priors for the hyperparameters, Ω, Ξ and Ψ are assumed to be distributed as independent inverse-Wishart. More precisely, we have the following priors:

- Time varying coefficients: $P(\theta_0) = N(\hat{\theta}, \hat{V}_\theta)$ and $P(\Omega) = IW(\Omega_0^{-1}, \rho_1)$;

- Diagonal elements: $P(\log \sigma_0) = N(\log \hat{\sigma}, I_n)$ and $P(\Psi_i) = IW(\Psi_{0i}^{-1}, \rho_{3i})$;

- Off-diagonal elements: $P(\phi_{i0}) = N(\hat{\phi}_i, \hat{V}_{\phi_i})$ and $P(\Xi) = IW(\Xi_0^{-1}, \rho_2)$;

where the scale matrices are parametrized as follows $\Omega_0^{-1} = \lambda_1 \rho_1 \hat{V}_\theta$, $\Psi_{0i} = \lambda_{3i} \rho_{3i} \hat{V}_{\phi_i}$ and $\Xi_0 = \lambda_2 \rho_2 I_n$. The hyper-parameters are calibrated using a time invariant recursive VAR estimated using a sub-sample consisting of the first $T_0 = 28$ observations. For the initial states θ_0 and the contemporaneous relations ϕ_{i0}, we set the means, $\hat{\theta}$ and $\hat{\phi}_i$, and the variances, $\hat{V}_\theta$ and $\hat{V}_{\phi_i}$, to be the maximum likelihood point estimates and four times its variance. For the initial states of the log volatilities, $\log \sigma_0$, the mean of the distribution is chosen to be the logarithm of the point estimates of the standard errors of the residuals of the estimated time invariant VAR. The degrees of freedom for the covariance matrix of the drifting coefficient's innovations are set to be equal to

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[3]The expectation and the covariance of the initial states, $E(\widetilde{Y}_{0|0})$ and $\widetilde{P}_{0|0}$, are initialized respectively with the unconditional mean and the identity matrix.

T_0, the size of the initial-sample. The degrees of freedom for the priors on the covariance of the stochastic volatilities' innovations, are set to be equal to the minimum necessary to insure that the prior is proper. In particular, ρ_1 and ρ_2 are equal to the number of rows Ξ_0^{-1} and Ψ_{0i}^{-1} plus one respectively. The parameter λ_1 is fixed to 0.004, while λ_2 and λ_3 to 0.0001. Estimation is performed by discarding the explosive draws.

Figure 2 shows the quarterly government spending data for available sample (1999Q1-2012Q1), together with the quarterly series produced by the model described above for the period 1989Q1-1998Q4. Blue bars represent 68% confidence bands around the generated data.

Figure 3 illustrate the fit of the model, using 68% confidence bands around the true data for the period 1999Q1-2012Q1. As apparent from this Figure, the model performs very well in-sample.

5   Results

Figure 4, 5 and 6 show the main results of the paper. Figure 4 plots the impulse response of government spending (first figure), GDP (second figure) and the interest rate (third figure) to a government spending shock equal to 1% of GDP over the 1988Q4-2012Q2 sample. A horizon of 10 years (40 quarters) is considered.

The impulse response of government spending to its own shock is very stable over the entire sample, for all quarters in the sample. Instead, the response of GDP to the spending shock appears to be very unstable. In particular, GDP reacts strongly in the first part of the sample, from 1988 until the beginning of the 1990s, and the multiplier peaks at around 1.5 at short horizons, i.e. up to 1 years after the shock. Then, the GDP reaction declines below unity in the run-up phase to the EMU and until the recent crisis. During the crisis, we observe a further increase in the short-term GDP multiplier, which reaches values above one in the last part of the crisis period, i.e. in 2011 and 2012. This seems to corroborate the hypothesis that spending shocks have stronger effects on output

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during slowdowns, due for example to the presence of a higher number of credit-constrained agents in these phases of the business cycle. As regards the interest rate response to the spending shock, a declining patterns emerges from the third chart of Figure 4: the interest rate response appears to be much stronger in the first part of the sample, coinciding with the early 1990s and run-up phase to the single currency, compared to the EMU period. This might be due to higher concerns for the sustainability of public finances in the eraly 1990s, which is mirrored in a more volatile sovereign debt market and in a higher risk premium requested by investors in Italian securities following expansive fiscal policies. During the EMU period, the interest rate response appears quite muted. The crisis period is characterized by a somewhat more unstable reaction of the interest rate to the spending shock, which might be due to a somewhat erratic behaviour of investors in sovereign securities during this period.

Figure 5 shows the impulse responses of GDP (first column) and the interest rate (second column) to the government spending shock in three selected quarters: 1989Q1, 1999Q1, 2012Q1, together with 68% confidence bands. The GDP response is statistically significant in the short terms, up to the 6-8 quarters. This is particularly true for the first quarter considered, i.e. 1989Q1. For longer horizons, the GDP impulse responses are generally not-significant, although for the last sample considered, the lower band is very close to the zero line thus signalling less uncertainty in the crisis period. The impulse response of the interest rate to the spending shock is also significant at short horizons, but only for the first quarter considered and, marginally, for the second.

Finally, Figure 6 illustrates the cross impulse responses for shocks to spending, GDP and the interest rate to the three variables in the VAR. Here, the first column represents again the responses of government spending, GDP and the interest rate to a government spending shock equal to 1% of GDP, as in Figure 4. The second column shows the impulse response of government spending, GDP and the interest rate to GDP shock of 1%. Interestingly, a GDP shock exerted a pro-cyclical

reaction of spending, which increases over the whole sample. The interest rate reacts positively at short horizons, consistently with the idea that monetary policy tends to tighten during follwing expansive shocks which may lead to inflation. Figures in the the third column are impulse response of government spending, GDP and the interest rate to an interest rate shock of 1 percentage point. GDP reacts negatively to a rise in the interest rate, and government spending contracts. This may be due to the reaction of fiscal policy to tighter financing conditions for the government, which may led to a fiscal adjustment reflected here in a spending contraction.

6 Conclusion

The recent global economic crisis has revived interest for issues related to fiscal policy, and in particular for the active use of fiscal policies in stimulating economic growth. Yet, there is a lot of uncertainty on the effects of discretionary fiscal actions. This is particularly true for euro area countries. In fact, for most of these countries, there is lack of sufficiently long time series of fiscal data. This limits significantly the possibility of estimating commonly used econometric models - and to draw robust conclusions regarding the effects of fiscal shocks - based on these models.

In this paper, we propose a VAR model which allows to overcome this limitation, and which is therefore particularly suitable for the analysis of fiscal policies for countries characterized by a limited availability of quarterly data. The model is estimated by exploiting information from both quarterly and annual fiscal series. In addition, we study possible regime shifts and instability in the transmission of government spending shocks by allowing time-variation in the VAR parameters and in the covariance matrix of VAR residuals. These features are particularly important in the context of the current crisis, given the structural changes that many advanced economies are likely to have experienced during this period.

Based on this model, we investigate how the transmission of government spending shocks has

changed over time in Italy, over a period of three decades. Our results indicate that, for Italy, the fiscal multiplier tends to follow a U-shape over the 1988Q4-2012Q2 sample: it peaks at around 1.5 at the beginning of the 1990s, it then stabilizes at around 0.9 during the run-up phase to the EMU, before rising again to above unity in the context of the recent global crisis. We also show that the reaction of interest rate on Italian short-term securities to the spending shock was stronger in the 1990s than in the EMU period and in the recent crisis, possibly due to higher concerns of the sustainability of Italian public finances in the first part of the sample considered.

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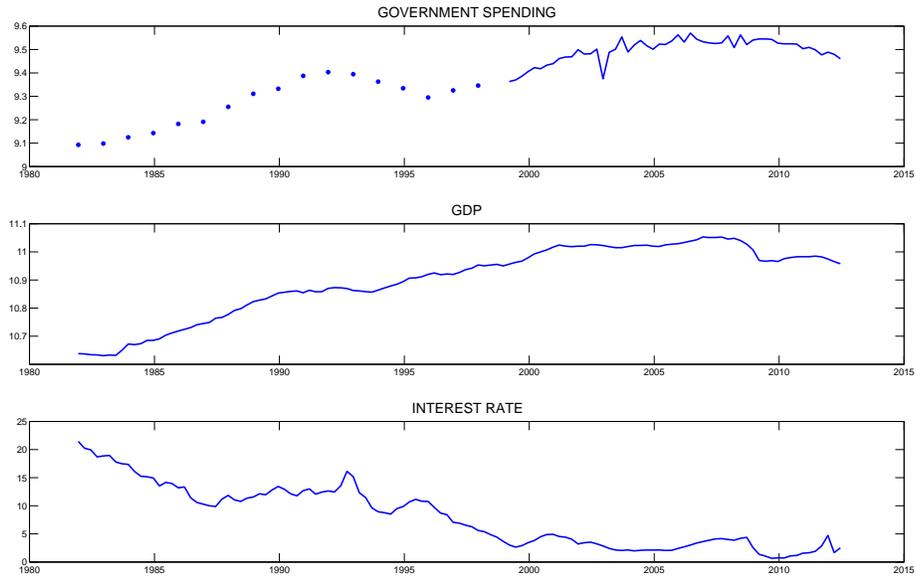
Figure 1: Variables in the baseline VAR: government spending, i.e. government consumption plus investment expenditure, GDP and the short-term nominal interest rate. The latter is the average interest rate on Italian government T-bills, i.e. government securities with maturity of less than one year. Government spending and GDP are in real per capita terms, and are transformed taking logs. The interested rate is in levels. Sample: 1980Q1-2012Q1.
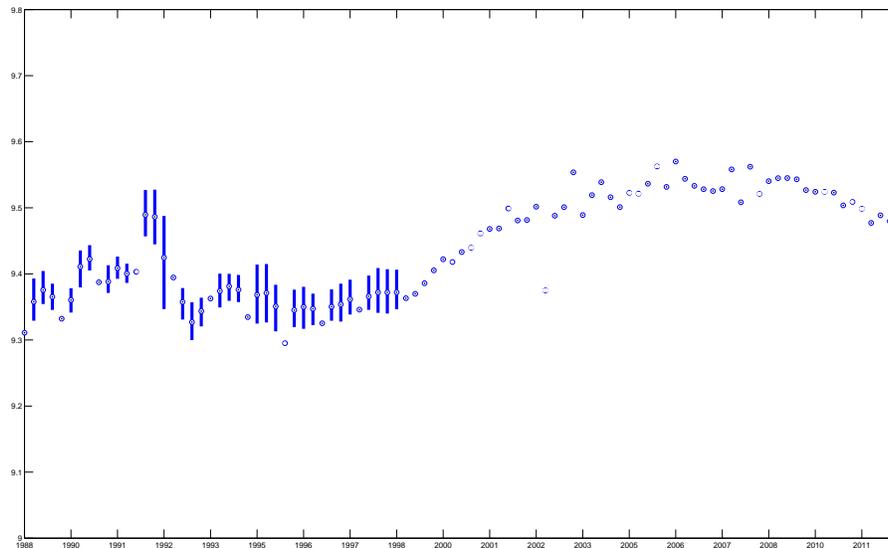

Figure 2: Quarterly government spending included in the baseline VAR. 1999Q1-2012Q1: available data from EUROSTAT; 1989Q1-1998Q1: data generated by the model described in Section 4. Bars represent [68%] confidence bands around the generated data.
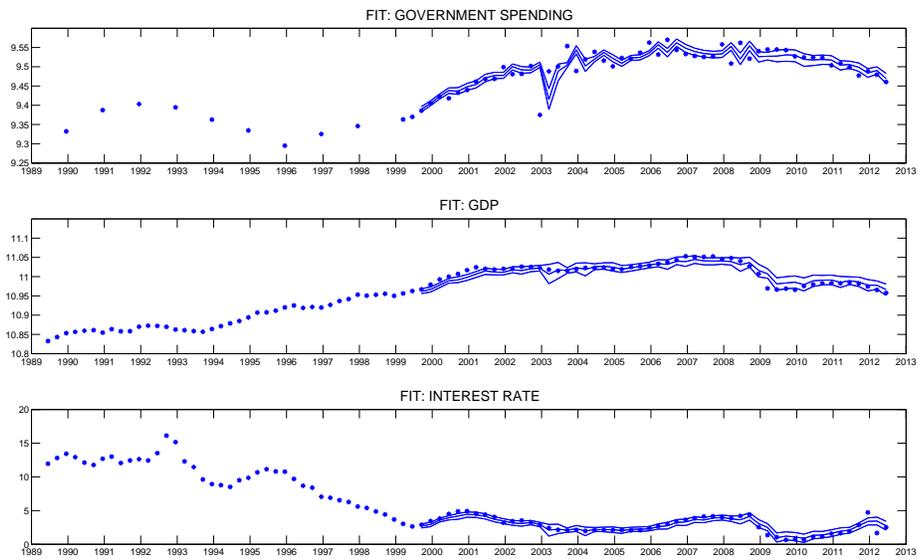
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Figure 3: Model fit over the sample 1999Q1-2012Q1. Stars are true data, solid lines the [68%] confidence bands around the data generated by the model.
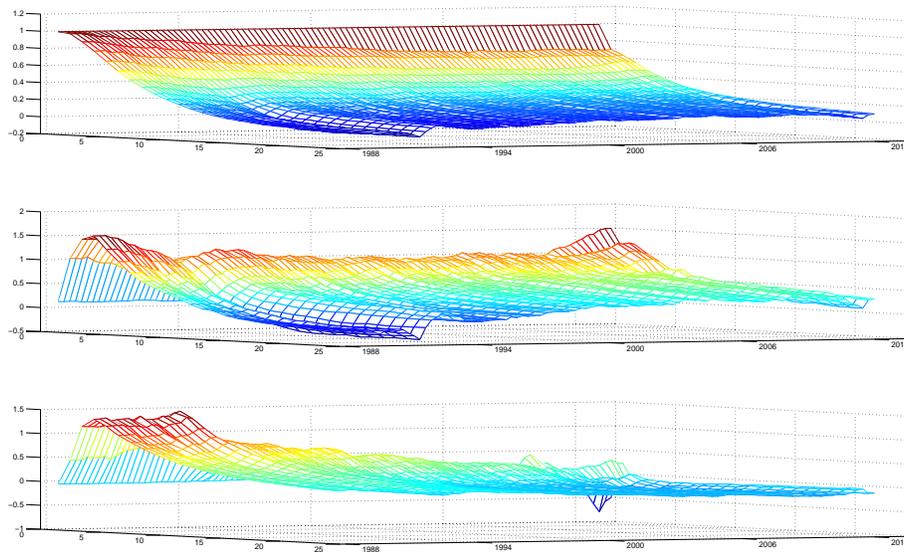

Figure 4: Impulse response of government spending (first figure), GDP (second figure) and the interest rate (third figure) to a government spending shock equal to 1% of GDP. Sample: 1989Q1-2012Q1.
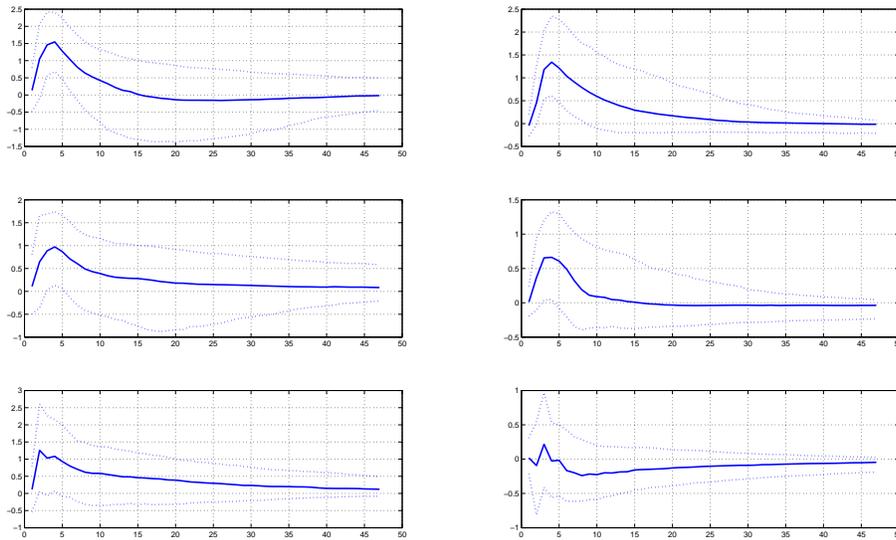
Figure 5: Impulse responses of GDP (first column) and the interest rate (second column) to a government spending shock equal to 1% of GDP in three selected quarters [1989Q1, 1999Q1, 2012Q1], together with 68% confidence bands.
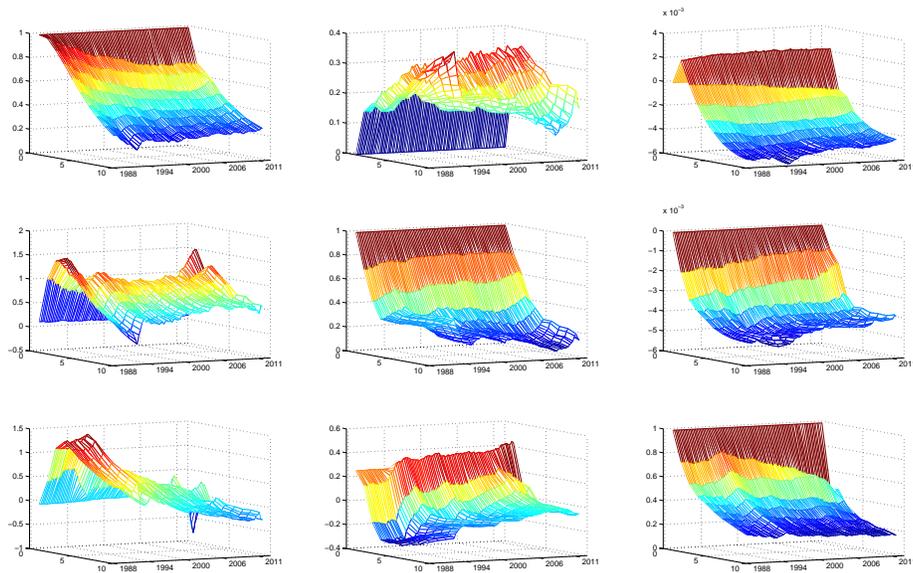

Figure 6: First column: impulse response of government spending, GDP and the interest rate to a government spending shock equal to 1% of GDP. Second column: impulse response of government spending, GDP and the interest rate to GDP shock of 1%. Third column: impulse response of government spending, GDP and the interest rate to an interest rate shock of 1 percentage point. Sample: 1989Q1-2012Q1.

Appendix: the bayesian algorithm

Estimation is done using Bayesian methods. To draw from the joint posterior distribution of model parameters we use a Gibbs sampling algorithm. The basic idea of the algorithm is to draw sets of coefficients from known conditional posterior distributions. The algorithm is initialized at some values and, under some regularity conditions, the draws converge to a draw from the joint posterior after a burn in period. Let z be $(q \times 1)$ vector, we denote z^T the sequence $[z_1', ..., z_T']'$. Each repetition is composed of the following steps:

1. $p(\widetilde{y}^T | y^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi, s^T)$

2. $p(s^T | y^T, \widetilde{y}^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi)$[4]

3. $p(\sigma^T | y^T, \widetilde{y}^T, \theta^T, \phi^T, \Omega, \Xi, \Psi, s^T)$

4. $p(\phi^T | y^T, \widetilde{y}^T, \theta^T, \sigma^T, \Omega, \Xi, \Psi, s^T)$

5. $p(\theta^T | y^T, \widetilde{y}^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi, s^T)$

6. $p(\Omega | y^T, \widetilde{y}^T, \theta^T, \sigma^T, \phi^T, \Xi, \Psi, s^T)$

7. $p(\Xi | y^T, \widetilde{y}^T, \theta^T, \sigma^T, \phi^T, \Omega, \Psi, s^T)$

8. $p(\Psi | y^T, \widetilde{y}^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, s^T)$

Gibbs sampling algorithm

- Step 1: sample from $p(\widetilde{y}^T |, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi, s^T)$

  Draws for $\widetilde{y}_t$ can be obtained from a $N(\widetilde{y}_{t|t+1}, \widetilde{P}_{t|t+1})$, where $\widetilde{y}_{t|t+1} = E(\widetilde{y}_t | \widetilde{y}_{t+1}, y^T, \theta^T, \sigma^T, \Omega, \Xi, \Psi, s^T)$ and $\widetilde{P}_{t|t+1} = Var(\widetilde{y}_t | \widetilde{y}_{t+1}, y^T, \theta^T, \sigma^T, \Omega, \Xi, \Psi, s^T)$ are obtained with the algorithm of Carter and R.Kohn (1994).

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[4]See below the definition of s^T.

•Step 2: sample from $p(s^T|y^T, \widetilde{y}^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi)$

Conditional on $y_{i,t}^{**}$ and r^T, we independently sample each $s_{i,t}$ from the discrete density defined

by $Pr(s_{i,t} = j|y_{i,t}^{**}, r_{i,t}) \propto f_N(y_{i,t}^{**}|2r_{i,t} + m_j - 1.2704, v_j^2)$, where $f_N(y|\mu, \sigma^2)$ denotes a normal density

with mean μ and variance σ^2.

• Step 3: sample from $p(\sigma^T|y^T, \widetilde{y}^T, \theta^T, \phi^T, \Omega, \Xi, \Psi, s^T)$

To draw σ^T we use the algorithm of Kim, Shephard and Chibb (KSC) (1998). Consider the

system of equations $y_t^* \equiv F_t^{-1}(y_t - X_t'\theta_t) = D_t^{1/2}u_t$, where $u_t \sim N(0, I)$, $X_t = (I_n \otimes x_t')$, and

$x_t = [1_n, y_{t-1}...y_{t-p}]$. Conditional on y^T, θ^T, and ϕ^T, y_t^* is observable. Squaring and taking the

logarithm, we obtain

$$y_t^{**} = 2r_t + v_t \tag{3}$$

$$r_t = r_{t-1} + \xi_t \tag{4}$$

where $y_{i,t}^{**} = \log((y_{i,t}^*)^2 + 0.001)$ - the constant (0.001) is added to make estimation more robust -

$v_{i,t} = \log(u_{i,t}^2)$ and $r_t = \log \sigma_{i,t}$. Since, the innovation in (3) is distributed as $\log \chi^2(1)$, we use,

following KSC, a mixture of 7 normal densities with component probabilities q_j, means $m_j - 1.2704$,

and variances v_j^2 (j=1,...,7) to transform the system in a Gaussian one, where $\{q_j, m_j, v_j^2\}$ are chosen

to match the moments of the $\log \chi^2(1)$ distribution. The values are:

Table A1: Parameters Specification

j	q_j	m_j	v_j^2
1.0000	0.0073	-10.1300	5.7960
2.0000	0.1056	-3.9728	2.6137
3.0000	0.0000	-8.5669	5.1795
4.0000	0.0440	2.7779	0.1674
5.0000	0.3400	0.6194	0.6401
6.0000	0.2457	1.7952	0.3402
7.0000	0.2575	-1.0882	1.2626

Let $s^T = [s_1, ..., s_T]'$ be a matrix of indicators selecting the member of the mixture to be used for each element of v_t at each point in time. Conditional on s^T, $(v_{i,t}|s_{i,t} = j) \sim N(m_j - 1.2704, v_j^2)$. Therefore we can use the algorithm of Carter and R.Kohn (1994) to draw r_t (t=1,...,T) from $N(r_{t|t+1}, R_{t|t+1})$, where $r_{t|t+1} = E(r_t|r_{t+1}, y^t, \theta^T, \phi^T, \Omega, \Xi, \Psi, s^T,)$ and $R_{t|t+1} = Var(r_t|r_{t+1}, y^t, \theta^T, \phi^T, \Omega, \Xi, \Psi, s^T)$.

- Step 4: sample from $p(\phi^T|y^T, \widetilde{y}^T, \theta^T, \sigma^T, \Omega, \Xi, \Psi, s^T)$

Consider again the system of equations $F_t^{-1}(y_t - X_t'\theta_t) = F_t^{-1}\hat{y}_t = D_t^{1/2}u_t$. Conditional on θ^T, $\hat{y}_t$ is observable. Since F_t^{-1} is lower triangular with ones in the main diagonal, each equation in the above system can be written as

$$\hat{y}_{1,t} = \sigma_{1,t}u_{1,t} \tag{5}$$

$$\hat{y}_{i,t} = -\hat{y}_{[1,i-1],t}\phi_{i,t} + \sigma_{i,t}u_{i,t} \quad i = 2, ..., n \tag{6}$$

where $\sigma_{i,t}$ and $u_{i,t}$ are the ith elements of σ_t and u_t respectively, $\hat{y}_{[1,i-1],t} = [\hat{y}_{1,t}, ..., \hat{y}_{i-1,t}]$. Under the

block diagonality of Ψ, the algorithm of Carter and R.Kohn (1994) can be applied equation by equation, obtaining draws for $\phi_{i,t}$ from a $N(\phi_{i,t|t+1}, \Phi_{i,t|t+1})$, where $\phi_{i,t|t+1} = E(\phi_{i,t}|\phi_{i,t+1}, y^t, \theta^T, \sigma^T, \Omega, \Xi, \Psi)$ and $\Phi_{i,t|t+1} = Var(\phi_{i,t}|\phi_{i,t+1}, y^t, \theta^T, \sigma^T, \Omega, \Xi, \Psi)$.

- Step 5: sample from $p(\theta^T|y^T, \widetilde{y}^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi, s^T)$

Conditional on all other parameters and the observables we have

$$\widetilde{y}_t = X_t'\theta_t + \varepsilon_t \tag{7}$$

$$\theta_t = \theta_{t-1} + \omega_t \tag{8}$$

Draws for θ_t can be obtained from a $N(\theta_{t|t+1}, P_{t|t+1})$, where $\theta_{t|t+1} = E(\theta_t|\theta_{t+1}, y^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi)$ and $P_{t|t+1} = Var(\theta_t|\theta_{t+1}, y^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi)$ are obtained with the algorithm of Carter and R.Kohn (1994).

- Step 6: sample from $p(\Omega|y^T, \widetilde{y}^T, \theta^T, \sigma^T, \phi^T, \Xi, \Psi, s^T)$

Conditional on the other coefficients and the data, Ω has an Inverse-Wishart posterior density with scale matrix $\Omega_1^{-1} = (\Omega_0 + \sum_{t=1}^{T} \Delta\theta_t(\Delta\theta_t)')^{-1}$ and degrees of freedom $df_{\Omega_1} = df_{\Omega_0} + T$, where Ω_0^{-1} is the prior scale matrix, df_{Ω_0} are the prior degrees of freedom and T is length of the sample use for estimation. To draw a realization for Ω make df_{Ω_1} independent draws z_i ($i=1,...,df_{\Omega_1}$) from $N(0, \Omega_1^{-1})$ and compute $\Omega = (\sum_{i=1}^{df_{\Omega_1}} z_i z_i')^{-1}$ (see Gelman et. al., 1995).

- Step 7: sample from $p(\Xi|y^T, \widetilde{y}^T, \theta^T, \sigma^T, \phi^T, \Omega, \Psi, s^T)$

Conditional the other coefficients and the data, Ξ has an Inverse-Wishart posterior density with scale matrix $\Xi_1^{-1} = (\Xi_0 + \sum_{t=1}^{T} \Delta\log\sigma_t(\Delta\log\sigma_t)')^{-1}$ and degrees of freedom $df_{\Xi_1} = df_{\Xi_0} + T$ where Ξ_0^{-1} is the prior scale matrix and df_{Ξ_0} the prior degrees of freedom. Draws are obtained as in step 5.

- Step 8: sample from $p(\Psi|y^T, \widetilde{y}^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, s^T)$.

Conditional on the other coefficients and the data, Ψ_i has an Inverse-Wishart posterior density with scale matrix $\Psi_{i,1}^{-1} = (\Psi_{i,0} + \sum_{t=1}^{T} \Delta\phi_{i,t}(\Delta\phi_{i,t})')^{-1}$ and degrees of freedom $df_{\Psi_{i,1}} = df_{\Psi_{i,0}} + T$ where $\Psi_{i,0}^{-1}$ is the prior scale matrix and $df_{\Psi_{i,0}}$ the prior degrees of freedom. Draws are obtained as in step 5 for all i.

The estimations are performed with 12000 repetitions discarding the first 10000 and collecting one out of five draws.