The effects of public expenditures shocks on the labor market: focus on households' preferences

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Preliminary version

Abstract

This paper deals with the effects of public spending shocks on the labor market. The core of the paper is a medium-scale DSGE model with a detailed fiscal sector including public consumption and public investment. The financing of the spending can be tax-based (taxes on consumption and on labor income) or debt-based. The focus in on the labor supply side and the main findings are: 1) I find strong negative fiscal multipliers with rapport to the literature. A 1% of GDP shock of the public consumption decreases the unemployment rate by 0.6% and a similar public investment shock decreases unemployment by about 1%. These results are in the case of a debt-based expenditures expansion. 2) The results are robust to changes in the parameters values concerning the households preferences. I can not find with this model positive unemployment fiscal multipliers even when setting polar values for the households' preferences parameters. These results are more sensitive to deep parameters like nominal rigidities and to parameters driving the labor demand 3) The tax on consumption has a positive effect on the labor supply and a tax on labor income has the opposite effect. In both cases, the introduction of a tax funding the half of the expenditures does not produce positive unemployment fiscal multipliers: the results are robust to the introduction of both taxes. A general conclusion is that under the different scenarios the unemployment fiscal mutlipliers are always negative even if the size of the multipliers varies.

Keywords: Fiscal multipliers, labor market, DSGE models, preferences, unemployment
JEL classification: E32, F77

1 Introduction

The effects of the fiscal policy is a very old question in macroeconomics. However, there is no real consensus about issues like the size of the fiscal multiplier at the short term, the transmission channels at the macroeconomic level or the cost of the fiscal policy on growth at the long term.

Especially, with the current struggling economic situation for most of the developed countries, in terms of growth, unemployment and levels of the public

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debt, issues concerning the abilities of the fiscal policy as a cyclical stabilizer particularly matter and receive new attention from the academics.

A very large literature grown up recently and deals with the impact of the fiscal policy at the short term. Firstly, in the new-Keynesian paradigm, interesting questions have been investigated with the help of the well known DSGE models such as the size of the fiscal multiplier, the response of the private consumption to a spending shock (Gali, Lopez-Salido and Valles, 2007, Coenen and Straub, 2005), or the effectiveness of the fiscal policy when the interest-rate lower bound binds (Hall,2010, among others). Secondly, numerous empirical studies try to measure the effects of the fiscal policy, with a large debate on the best way to identify fiscal shocks. Unlikely, no real consensus arise due to methodological discrepancies (See Ramey (2010) for an extensive survey), notably ib the response of the private consumption to public spending shocks. The narrative approach primarily developed in Ramey and Shapiro (1998) concludes generally for a large decrease of the private consumption. Conversely, in the SVAR approach initiated in Blanchard and Perotti (2002), public spending shocks cause an increase of the private consumption.

Euro-Area countries face currently very high rates of unemployment (12.1% for the Euro Area in July 2013, sources Eurostat), especially for countries in where strong austerity plans have been implemented (26.26% for Spain at the second quarter of 2013, 26.9% for Greece). It is complicated to exactly know in what extent the current fiscal contractions contribute to this sharp degradation of the unemployment rate. Investigating this issue requires a precise knowledge as for the effects of the fiscal policy on the labor market.

Interestingly, the effects of the fiscal policy on the labor market have been studied only recently. The reason is mainly methodological. First generation of RBC/DSGE models does not allow for a good interpretation of the short-term dynamic of the labor market. These models include a Walrasian labor market which is not able to reproduce a performing description of the real behavior of the labor market. More recently, many papers reconcile the two largest paradigms in the modern theoretical macroeconomics : A DSGE structure with a labor market à la Mortensen and Pissarides (2000). Applied to fiscal matters, Mayer, Moyen and Stähler (2010) or Monacelli, Perotti and Trigari (2011) use this class of models for analyzing the effects of public spending shocks on the labor market.

Gali, Smets and Wouters (2011) propose an other way to describe a non-Walrasian labor market, by linking the unemployment rate and the mark-up on wages. Following the authors, this model respects the original insight behind the Phillips curve and allows to split between labor supply shocks and shocks on the wage markup.

The model used in this article is very close to the one developed in Gali, Smets and Wouters (2011), excepted for the fiscal side of the model. The latter is well detailed and allows to various policy simulations. In this paper, the aim is to contribute to the literature dealing with the effects of public spending shocks on the labor market. This paper can be related to two recent papers. Firstly,

this paper is in the spirit of Monacelli and Perotti (2008) in the sense that this paper focus on the behavior of the optimizing households to explain the size of the fiscal multiplier on output. Monacelli and Perotti (2008) use different kinds of preferences for the households and focus on the presence (or not) of a wealth effect of the private consumption on the labor supply. In my paper, I also focus on preferences' modeling but investigate the effects of public spendings shocks on the unemployment rate. Secondly, this paper is close to Mayer, Moyen and Stähler (2010). The authors develop an highly developed DSGE model with a labor market à la Mortensen et Pissarides (2000) and investigate what parameters of the model drive the response of the unemployment rate to a public spending shock. The main conclusions of this paper are that the drivers are: the degree of price stickiness, the degree of wage stickiness, the introduction of non-Ricardian households and the financing of the public spending (debt or taxes). The authors conclude in a positive effect of the fiscal expenditures shocks on the unemployment rate. My paper differs from Mayer, Moyen and Stähler (2010) on at least two aspects. Firstly, my model is significantly different, especially for the labor market. As already mentioned, I model the labor market following Gali, Smets and Wouters (2011). Secondly, I focus on other elements for explaining the channel transmissions of public spending shocks on the labor market. I investigate the way the preferences are modeled and attempt to give new conclusions about the reaction of the labor supply following public expenditures shocks. In several papers, authors find a response of the labor supply larger than the response of the labor demand. I show in this paper that I can not reproduce this fact in my model even if changing dramatically the values of the parameters included in the households preferences. For instance, introducing a very large elasticity of the labor supply obviously hikes the response of the labor supply but it remains significantly lower than the positive response of the labor demand. The main contribution is that, if changes in the parameters values for the households can alter the unemployment fiscal multiplier, the amplitude of these changes remain low. In order to give some robustness to these results, I lead a sensitivity analysis using the Global Sensitivity Analysis Toolbox working with the Dynare program, which has not been yet used for analyzing this precise issue. I show that parameters included in the production side of the model influence more the final results as well as deep parameters and notably the degree of price stickiness and the degree of nominal wage of rigidity.

Two different public spending are introduced in the model: public consumption and public investment. I find strong negative multipliers regarding to the literature: a 1% GDP increase of the public consumption decreases the unemployment rate by 0.4% to 0.6% according to the different simulated scenarios. Multipliers for public investments are higher due to an additional productive effect on the supply side of the model, allowing for an higher labor demand addressed by firms. In this case, the multiplier reaches -1%. In the present model, under various calibrations, I do not observe in any case an increase of the unemployment rate following a positive public spending shock unlike few recent papers like Poppa and Bruckner (2010). In fact, a consensus is that both the labor demand and the labor supply increase following a government spending shock: the total effect on the unemployment rate thus depends on the relative strength of these two effects. In this paper, the increase of the labor demand is largely higher than the rise on labor supply. One important feature in this

model is the introduction of the households' preferences like in Jaimovich and Rebelo (2009). Such preferences allow to introduce a short term wealth effect of the consumption on the labor supply ans also allow to test different sizes for this wealth effect. The introduction of a very strong wealth effect of consumption on the labor supply clearly has a weak impact on the fiscal multiplier on unemployment, all things being equals in the model. This wealth effect means that when private consumption decreases, the marginal utility of labor increases for the households thus the labor supply increases. The sign of this impact on unemployment depends on the response of the private consumption. In DSGE models, the private consumption generally decreases following a public spending shock. This is due to the taxes which dampen the disposable income in the case of tax-financed spending but also to a rise on the nominal interest rate even in the case of debt-financed fiscal shocks. In many empirical studies, private consumption would increase following a spending shock (Blanchard and Perotti,2002, Perotti, 2005), this is why I illustrate the analysis with a case where public consumption increases, by introducing a positive complementarity between private consumption and public consumption like in Bouakez and Rebei (2007). There is no clear conclusion about the complementarity between private and public consumption despite empirical investigations in the literature like Aschauer (1985), Ni (1995) and McGrattan (1994). More recently, Bouakez and Rebei (2007) estimate a DSGE model for the US with a complementarity between private and public consumption. The posterior indicates a strong Edgeworth complementarity which leads to a positive response of the private consumption following a government consumption shock. In this case, and following my DSGE model, a rise of the consumption has a negative effect on the labor supply and thus hikes the fiscal multiplier on the unemployment rate. The response of the private consumption influences the effects of fiscal expenditures shocks on the labor market.

The section 2 presents the log-linearized version of the model and the different alternatives investigated. The section 3 presents the results and discuss what elements drive the response of the unemployment rate . A fourth section concludes this paper.

2 The log-linearized model

The model is a medium-scale DSGE model with a detailed fiscal sector. The core of the model is the Gali, Smets and Wouters' model (2011), namely the Smets and Wouters' model (2007) with a non-Walrasian labor market where the unemployment rate is observable. For the fiscal sector, public consumption of goods and services and public investment constitute the public expenditures. The expansion of the expenditures are in part funded by debt and by a tax on consumption and a tax on labor income levied by the government.

This paper focus on the response of the households to public expenditures expansions, thus two alternative specifications for the preferences of the households are proposed. First, preferences introduce a short term wealth effect of

consumption on the labor supply, à la Jeimovich et Rebelo (2009). Like Gali, Smets and Wouters (2011), I differ from Jeimovich and Rebelo (2009) since I add habit formations for consumption. Three key parameters are introduced in these preferences and allow for different simulations: the elasticity of substitution of the labor supply, the size of the wealth effect and the degree of habit formation. Secondly, I will introduce a complementarity between the private consumption and the public consumption, like in Bouakez and Rebei (2007). I will use this case to observe what (strong) multiplier I can find with a positive response of the private consumption produces by such a complementarity. Whether the degree of complementarity between private and public consumption is uncertain, I use the value found in Bouakez and RebeÃ¯ (2007) in order to find a plausible value of the unemployment fiscal multiplier in the case where the private consumption increases with the shock.

2.1 Households preferences and labor supply: Jaimovich and Rebelo (2009) with habit formation

Both preferences mentioned above are presented and derived in this subsection. Also, I detail the introduction of non-Ricardian households.

There is a continuum of Ricardian households on $[0, 1]$ maximizing their preferences given the following lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t(C_t, L_t(i)) = E_0 \sum_{t=0}^{\infty} \beta^t (log\tilde{C}_t - X_t \Delta_t L_t^{1+\phi}/1+\phi) \tag{1}$$

The representative household earns utility from consumption $\tilde{C}_t$ and disutility from labor supply L_t. $\tilde{C}_t$ contains habit formations for consumption such as: $\tilde{C}_t = C_t - hC_{t-1}$. β^t is a discount factor and ϕ is the labor elasticity of substitution. X_t is a preference shock, increasing current disutility for work.

Δ_t introduces the wealth effect on the labor supply. Thus,Δ_t is function of consumption, such as:

$$\Delta_t = Z_t/\tilde{C}_t \tag{2}$$

with $Z_t = Z_{t-1}^{1-\nu}(C_t - hC_{t-1})^{\nu}$. The households face the following budget constrait:

$$(1+\tau_t^c)P_tC_t + I_tP_t + \frac{E_tB_{t+1}}{1+R_t} \leq (1-\tau_t^w)W_tL_t + B_t + R_t^k \nu_t K_{t-1} - f(\nu_t)\nu_t K_{t-1} + Div_t \tag{3}$$

P_t is the level of prices, R_t the quarterly nominal interest rate , W_t is the nominal wage for employment and B_t is the government bonds held by the households. As it is shown in the Appendix describing the whole model, the households invest in capital, I_t representing the level of investment and K_t the accumulated capital. They loan this capital to the firms at le rate R_t^k. ν_t is the degree of

capital utilization and $f(\nu_t)$ is a function characterizing the cost for the households following a change on the degree of capital accumulation. Finally, Div_t is the profit from the firms redistributed to the households.

τ_t^c and τ_t^w are respectively the tax on consumption and the tax on labor income levied by the government.

Maximization of (1) with respect to L_t subject to (3) yields the marginal rate of substitution between consumption and labor, such as:

$$(1 - \tau_t^w)W_t/P_t = X_t\tilde{C}_t\Delta_t L_t^{\phi} \tag{4}$$

$$(1 - \tau_t^w)W_t/P_t = X_t Z_t L_t^{\phi} \tag{5}$$

Finally, taking in logs I obtain:

$$(1 - \tau_t^w)(w_t - p_t) = x_t + z_t + \phi l_t \tag{6}$$

The equation (6) allows to obtain the labor supplied by the households, namely the labor force participation. This specification for the labor supply allows to analyze different changes on parameters values. Firstly, the parameter ϕ captures the sensitivity of the labor supply to both the real wage and the smoothed consumption. Moreover, changes on the parameter ν uniquely describe the sensitivity of the labor supply to the consumption wealth effect, including the tax on consumption levied by the government.

In the polar case where $\nu = 1$, (6) represents standard additively separable preferences (King, Plosser and Rebelo, 1988) such as:

$$w_t - p_t = x_t + (1 + \tau_c)(c_t - hc_{t-1}) + \phi l_t \tag{7}$$

(7) contains a greater wealth effect from consumption on the labor supply. The other polar case is the GHH preferences, avoiding any wealth effect from consumption, such as:

$$(1 - \tau_t^w)w_t - p_t = x_t + \phi l_t \tag{8}$$

2.2 The fiscal sector

As previously said, the fiscal spending are composed by public purchases of goods and services and public investment, respectively defined by C_t^g and I_t^g. The financing of these spending is assumed to be partly tax-based and partly debt-based. Furthermore, following Furceri and Mourougane (2010), I explicit the spread between the interest rate and the government bond interest rate, such as:

$$r_t^g - r_t = E_t d_{t+1} \tag{9}$$

with r_t^g the government bond interest rate. Equation (9) defines the premium as a function of the expected deficit at the period $t + 1$.

The total spending for the government g_t is defined by:

$$g_t = c_t^g + i_t^g \tag{10}$$

Each spending is introduced as an AR(1) shock, such as in logs:

$$c_t^g = \rho^{c,g} c_{t-1}^g + \epsilon_t^{c,g} \tag{11}$$

$$i_t^g = \rho^{i,g} i_{t-1}^g + \epsilon_t^{i,g} \tag{12}$$

Two taxes are levied by the government : a tax on consumption and a tax on labor income. Introducing such taxes is interesting because both will change the optimal choice of the households for consumption and labor supply. Introducing a lump-sum tax is not very informative since a lump-sum tax will only change the size of the multiplier but not the decisions for consumption and labor supply which are the focus of this work.

The total revenue for the government is given by:

$$R_t^g = \tau_t^c c_t + \tau_t^w (\frac{w_t}{p_t} l_t) \tag{13}$$

Thus, for each period the deficit is expressed as:

$$d_t = g_t - R_t^g \tag{14}$$

and the debt is predetermined such as:

$$b_t = (1 + r_t^g) b_{t-1} + d_t \tag{15}$$

Many empirical studies tend to conclude for the consideration of the levels of debt and deficit when the government chooses its fiscal standing: the government tries to sustain a given level of debt. A deficit-sustainability objective for the government is introduced in the working of the fiscal sector. The adjustment variables are the two taxes and each one responds to the level of deficit, such as:

$$\tau_t^c = \rho^{\tau,c} \tau_{t-1}^c + \alpha^{\tau,c} d_t \tag{16}$$

$$\tau_t^w = \rho^{\tau,w} \tau_{t-1}^w + \alpha^{\tau,w} d_t \tag{17}$$

with $\alpha^{\tau,c}, \alpha^{\tau,w} \in [0; 1]$. These parameters represent the degree of reaction of the taxes to the level of deficit. The introduction of such rules is relevant theoretically to mimic the real behavior of a government. Moreover, such a rule has a practical aspect: it allows to simulate different financing scenarios. If $\alpha^{\tau,c} = \alpha^{\tau,w} = 0$, the spending is totally debt-financed. In an other polar case where $\alpha^{\tau,c} = 1$, the spending is funded totally by the tax on consumption and so on. It is also relevant to introduce a degree of inertia in the tax-rules, defined by the parameters $\rho^{\tau,c}$ and $\rho^{\tau,w}$ since tax rates can not change dramatically in few quarters.

For the public investment, I have to describe the law of motion of the public capital accumulation, such as:

$$k_t^g = (1 - \delta) k_{t-1}^g + i_t^g \tag{18}$$

The introduction of the public investment is shown hereinafter when presenting the production process of the firms.

2.3 The remaining log-linearized model

I present in this subsection the log-linearized model, see the Appendix for a total derivation of the model.

2.3.1 Optimal decisions for the households

Maximization of (1) subject to (3) with rapport to $\tilde{C}_t$ yields the consumption Euler equation such as:

$$c_t = \alpha_{c,1} c_{t-1} + (1 - \alpha_{c,1}) c_{t+1} - \alpha_{c,2}(r_t - E_t \pi_{t+1} + \epsilon_t^b) \tag{19}$$

with $\alpha_{c,1} = h/(1+h)$ and $\alpha_{c,2} = (1-h)/(1+h)$. π_{t+1} defines the inflation rate such as: $\pi_{t+1} = p_{t+1} - p_t$. Thus, consumption depends positively on the expected future consumption and on the past consumption. The trade off between consumption and saving is defined by the negative relationship between the real interest rate $(r_t - E_t \pi_{t+1})$ and the current consumption.

Optimal decisions for the real investment I_t are characterized by the following investment Euler equation:

$$i_t = \alpha_{i,1} i_{t-1} + (1 - \alpha_{i,2}) i_{t+1} + \alpha_{i,2} q_t + \epsilon_t^q \tag{20}$$

with $\alpha_{i,1} = 1/(1+\beta)$ and $\alpha_{i,2} = \alpha_{i,1}/\psi$ where ψ defines an adaptation cost to changes on investment. q_t is the value of the installed capital stock, defined by:

$$q_t = q_1 E_t r_{t-1}^k + (1 - q_1) E_t q_{t+1} - (r_t - E_t \pi_{t+1} + \epsilon_t^b) \tag{21}$$

Investment decision determines the evolution of the stock of capital:

$$\tilde{k}_t = \alpha_{k,1} \tilde{k}_{t-1} + (1 - \alpha_{k,1}) i_t + \alpha_{k,2} \epsilon_t^q \tag{22}$$

with $\alpha_{k,1} = 1 - (I/\tilde{K})$ where I and $\tilde{K}$ respectively are the steady-state values of investment and capital accumulation. $\alpha_{k,2} = (I/\tilde{K})(1+\beta)\psi$. Finally, fixed capital are used only one period after their formation and effectively used capital depends on the rate of capital utilization ν_t, such as:

$$k_t = \nu_t + \tilde{k}_{t-1} \tag{23}$$

and

$$\nu_t = ((1 - \psi)/\psi) r_t^k \tag{24}$$

2.3.2 Wage setting

Workers maximize their nominal wage constrained by a nominal rigidity introduced à la Calvo (1983). At each period, only a fraction of workers can reset their wage, defined by the parameter Θ_w. When the workers cannot reset their wage, an automatic and partial indexation on the past inflation. The degree of indexation is defined by the parameter γ_w. The log-linearized dynamic for the nominal wages is given by:

$$\pi_t^w - \gamma_w \pi_{t-1}^p = \beta(E_t \pi_{t+1}^w - \gamma_w \pi_t^p) - \lambda_w(\mu_{w,t} - \mu_{w,t}^n) \tag{25}$$

where π_t^w defines the wage inflation, γ_w the degree of wage indexation and $\lambda_w = (1-\beta\Theta_w)(1-\Theta_w)/[\Theta_w(1+\xi_w\phi)]$. $\mu_{w,t}$ defines the mark-up on wage for the workers, expressed as:

$$\mu_{w,t} = w_t - p_t - (z_t + x_t + \phi n_t) \tag{26}$$

Rearranging the expression using the equation (6):

$$\mu_{w,t} = \phi(l_t - n_t) \tag{27}$$

$$\mu_{w,t} = \phi u_t \tag{28}$$

where $u_t = l_t - n_t$ defines the unemployment rate.

Finally, the natural mark-up on wages is assumed to be constant over time in the absence of a specific shock, such as:

$$\mu_{w,t}^n = 100 * \epsilon_t^w \tag{29}$$

where ϵ_t^w is an AR(1) shock on the wage mark-up. As said previously, such a specification allows to differentiate between a shock on the labor supply and a shock on the wage mark-up. As shown the equation (25), the wage inflation depends on the expected future wage inflation, past inflation and on the wage mark-up. Equations (25) and (26) show clearly that this mark-up is linked to the unemployment rate u_t. As argues in Gali, Smets and Wouters (2011), this framework allows to base theoretically the initial description of the Phillips' curve, linking negatively the wages and the unemployment rate.

2.3.3 Firms

In this model, a continuum of intermediate firms produce differentiated goods and services using private capital, labor and public capital (public investment). Introducing public capital, the production function is defined by:

$$y_t = M_p(\alpha k_t + (1-\alpha)n_t + \alpha_g k_{t-1}^g + \epsilon_t^a) \tag{30}$$

I introduce in the production function the lagged public capital. The reason behind this statement is that when the government decides to invest in productive capital, the latter becomes effective later (length of building-up). Public investment could even take more than a quarter to become usable since large public works can take more than one year to be completely built. However, I introduce just one lag in the production function for convenience as it is done commonly in the literature. At this stage, I can formulate a remark concerning public investment. The latter has the same effects than government consumption (hiking the demand for goods and services and causing a negative wealth effect on the households) but has an additional effect: a productive effect on the supply side. In fact, as described in the equation (30), public investment is similar to the technological shock ϵ_t^a, excepted the demand effect observable in the equation (38).

The marginal cost for the firms can be expresses as:

$$mc_t = (1-\alpha)(w_t - p_t) + \alpha r_t^k - \epsilon_t^a - \alpha^g k_{t-1}^g \tag{31}$$

Minimizing the cost of production, firms choose their demand for labor and capital such as this optimal condition:

$$k_t = (wt - p_t) - r_t^k + n_t \tag{32}$$

with n_t defines the demand for labor. The demand for each input is negatively function of the cost of the considered input and positively function of the cost of the other input (substitution effect).

2.3.4 Price setting

The price setting mechanism is similar to the wage setting: firms can reset their price at each period with a probability Θ_p. Also, a indexation mechanism is assumed, where current inflation is function of past inflation with a degree of indexation γ_p. The dynamic of the price inflation is defined by:

$$\pi_t^p - \gamma_p \pi_{t-1}^p = \beta(E_t \pi_{t+1}^p - \gamma_p \pi_t^p) - \Gamma(\mu_{p,t} - \mu_{p,t}^n) \tag{33}$$

with $\Gamma = (1 - \beta\Theta_p)(1 - \Theta_p)/[\Theta_p(1 + (M_p - 1)\kappa]$. $\mu_{p,t}$ represents the mark-up on price, defined by the inverse of the marginal cost such as:

$$\mu_{p,t} = -(1 - \alpha)(w_t - p_t) - \alpha r_t^k + \epsilon_t^a + \alpha^g k_{t-1}^g \tag{34}$$

with ϵ_t^a an AR(1) technological shock which diminishes the overall cost of production. The natural mark-up is assumed to be constant until the occurrence of an exogenous shock, such as:

$$\mu_{p,t}^n = 100 * \epsilon_t^p \tag{35}$$

with ϵ_t^p an AR(1) exogenous process.

2.3.5 Unemployment

The unemployment rate is simply defined as the difference between the labor supply and the labor demand:

$$u_t = l_t - n_t \tag{36}$$

2.3.6 Monetary policy

The monetary policy is introduced in the usual manner, namely a Taylor rule. The nominal interest reacts to the variations of output and to the price inflation.

$$r_t = \rho^r r_{t-1} + (1 - \rho_r)(\Phi^y \Delta y_t + \Phi^\pi \pi_t^p) \tag{37}$$

where ρ^r is a degree of inertia of the nominal interest rate and $\Delta y_t = y_t - y_{t-1}$. Φ^y and Φ^π define respectively the weight given in the Taylor rule for the stabilization of the output and of the inflation rate.

2.3.7 Market clearing

In order to clear the model, the total demand is defined by:

$$y_t = c_y c_t + i_y i_t + g_y c_t^g + v_y v_t + ig_y i_t^g \tag{38}$$

with $c_y = C/Y$, $i_y = I/Y$,$g_y = C^g/Y$, $v_y = V/Y$ and $ig_y = I^g/Y$, which correspond to the steady state share of each component in the total demand function.

3 Calibration, simulations and analysis

3.1 Some comments about the calibration and the sensitivity analysis exercise

The values for the parameters are chosen following the estimation of the model with US data in Gali, Smets and Wouters (2011). Furthermore, Smets, Warne and Wouters (2013) estimate the same model but with data for the Euro-Area. Great differences exist for some parameters, notably the Euro-Area has a larger price and wage rigidity than the US (respectively θ_p and θ_w), an higher elasticity of substitution of the labor supply ϕ and an higher share of capital α in the production function.

Parameter	Value
h	0.8
ϕ	2
ψ	6
δ	0.025
r^k	$\beta^{-1} = 1.005$
β	0.995
c_y	0.5
i_y	0.2
g_y	0.2
v_y	0.1
ig_y	0.1
α	0.18
M_p	1.75
Θ_p	0.5
γ_p	0.5
κ	10
γ_w	0.16
Θ_w	0.5
ξ_w	1.5
Θ^y	0.5
Θ^π	1.5
ρ^r	0.9
α^τ	0.5
Ξ	0.5
α^g	0.05

Figure 1: Initial calibration of the model

Concerning the degree of habit formation for consumption, two values are tested. These habits can be very strong (around 0.8) but the degree varies according to the estimations: for instance, Smets, Warne and Wouters find for the Euro-Area $h = 0.65$. In this paper, I initially set $h = 0.8$ but simulate also with a low/medium value, $h = 0.4$. The inverse of the elasticity of the labor supply ϕ is set to 2. This parameter mainly drives two transmission channels: firstly the

wealth effect of the consumption on the labor supply (but also the sensitivity of the labor supply to the real wages). Secondly, this parameter enters in the wage equation. Equation (22) indicates that higher is the elasticity of the labor supply, higher is the effect of the unemployment rate on the wage setting. In order to assess the importance of this parameter, the model is simulated also with $\phi = 0.5$. This is a lower level with rapport to estimates in the literature but a plausible value characterizing a strong elasticity of the labor supply. As previously said in the third section, the parameter ν allows to implement different degrees of wealth effects of the private consumption on the labor supply. In Gali, Smets and Wouters (2011), the value of the parameter depends drastically on the fact that the model is estimated with the unemployment rate as observable or not: I set initially $\nu = 0.4$. In order to test a high level of wealth effect, I simulate also the model with $\nu = 1$.

3.2 Case of a government consumption shock funded by debt

Parameter	Initial value	Tested value
ν	0.4	1
ϕ	2	0.5
h	0.8	0.4

Figure 2: Changes in parameter values for the different simulations

A government consumption shock tends to hike both the labor demand and the labor supply and that is the case in this model. The shock rises the total demand addressed to the firms which also increase their demand for input, that is capital and labor. Following this increase of the labor demand, the unemployment rate decreases thus the real wages tends to increase. Facing this additional demand, firms set higher prices according to the degree of price rigidity leading to an higher inflation. The central bank reacts by rising its interest rate thus households' consumption decreases. With the introduction of the preferences à la Jaimovich and Rebelo (2009) allowing for a wealth effect on the labor supply, a lower consumption causes an higher labor supply already boosted by the rise of the real wages.

With the initial calibration, the rise of the labor demand exceeds the rise of the labor demand, thus the unemployment rate decreases. In this case where the public spending is debt-financed, no additional effects due by taxes changes alters the decisions taken by the agents. As illustrated in the figure 2, the unemployment rate falls by 0.56% and the peak of this effect is at the first period. This result is in line with other studies. The GDP fiscal multiplier is around 1 when the shock occurs, which is a reasonable value since the shock is debt-financed. The both effects on GDP and unemployment are short-lasting. With $\rho^{c,g} = 0.6$ that is a medium (but usual) value for the length of the fiscal chock, the effects on unemployment and GDP are different from zero only for the ten

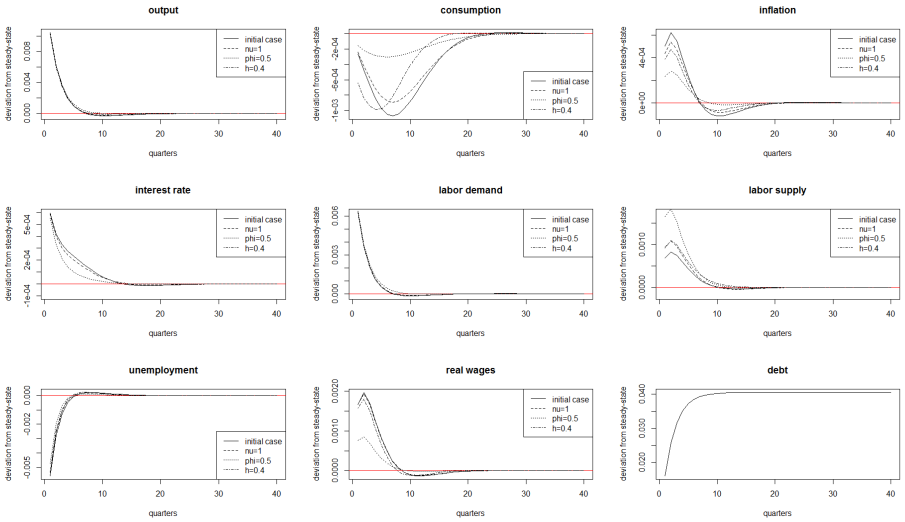


Figure 3: Effects of an increase of public consumption (corresponding to 1% of GDP)

	Initial case	$\nu = 1$	$\phi = 0.5$	$h = 0.4$
1st period	1.04%	1.04%	1.04%	1.02%
5 periods	2.31%	2.32%	2.36%	2.25%
10 periods	2.29%	2.33%	2.48%	2.29%

Figure 4: Cumulative GDP multipliers for a 1% of GDP increase of public consumption

first periods. Regarding to the literature, I find strong (negative) multipliers for the public consumption shock, in the line of Monacelli, Perotti and Trigari (2010) or Ravn and Simonelli (2007). A common result in this literature is that the effects on the unemployment rate are short-lasting. This is shown in the Figure (2) and this is due to short-lasting effects on both the labor demand and the labor supply in the case of a public consumption shock. For the labor supply, two effects drive the response: the rise of the real wage and the fall of the private consumption. Figure (3) indicates that the response of the real wage is very short-lasting leading to a fast return of the unemployment rate to its steady-state, even if the response of the private consumption is more

	Initial case	$\nu = 1$	$\phi = 0.5$	$h = 0.4$
1st period	-0.56%	-0.54%	-0.47%	-0.53%
5 periods	-1.02%	-0.94%	-0.75%	-0.89%
10 periods	-0.94%	-0.87%	-0.71%	-0.80%

Figure 5: Cumulative unemployment multipliers for a 1% of GDP increase of public consumption

long-lasting. For the labor demand, the strongly temporary response is due to the response of the real-wage and to the effect of the supplementary exogenous demand deeply dampened by the strong U-shaped decrease of the private consumption. The U-shape of the private consumption is obtained by introducing a degree of habit formation for the real consumption. In the case where the real consumption is fully flexible (corresponding to $h = 0$), the consumption has a monotonic reaction, decreasing strongly on impact and gradually returning to its steady-state value.

This is the labor demand which drives the unemployment fiscal multiplier. The labor demand increases by 0.65%. The labor supply has a weaker reaction, having a peak of 0.17% in the situation where the labor supply is strongly elastic ($\phi = 0.5$). In such a configuration, this model argues for a co-movement of the labor demand and of the labor supply, but the labor supply has no chance to exceed the labor demand thus obtaining a positive response of the unemployment rate. In Bruckner and Pappa (2010), the authors obtain a positive response of the unemployment rate for several OECD countries. They explain this fact by the strong increase of the labor supply following a public spending expansion, higher than the hike of the labor demand.

In order to analyze the robustness of these results, let take a look at the different scenarios tested and summarized in the Figure 2. The aim is to test the importance of the model parameters introduced in the households preferences. The four scenarios are: a simulation with the initial calibration. Then, I introduce an higher wealth effect on the labor supply by setting $\nu = 1$ instead of $\nu = 0.4$ initially. Thirdly, I set $\phi = 0.5$ instead of $\phi = 2$ which amounts to introduce a stronger elasticity of substitution. Finally, I set initially a large habit formation in consumption with $h = 0.8$, I investigate the case of a lower real rigidity on consumption by setting $h = 0.4$. The Figures 4 and 5 summarize the values of the multiplier for the 4 scenarios.

The cumulative unemployment fiscal multipliers (Figure 4) indicate that the introduction of theses large changes in the value of ν, ϕ and h does not change dramatically the effects at the medium run of the public consumption on the unemployment rate. However, the effects are different according to what parameter is considered. The most influencing parameter is the inverse of the elasticity of substitution ϕ. A rise of this elasticity dampens the unemployment fiscal multiplier by increasing the reaction of the labor supply (by about 100%) to an higher real wage and a weaker consumption. The parameter ϕ has an additional effect through the nominal wage setting. In the equation (25), ϕ influences positively the response of the nominal wage to a change in the wage mark-up. Thus, lower is the elasticity of substitution, lower is the reaction of the nominal wage to a change in the wage-mark-up, thus to the unemployment rate. As a consequence, the real wage hikes less strongly than in the initial case, constraining the rise of the labor supply. The way ϕ is included in the wage setting dampens the initial positive effect of an higher elasticity of substitution of labor on the unemployment rate. If the volatility of the labor supply doubles when introducing a strong elasticity of substitution, the final effect on the unemployment rate is significant but low since the labor supply has a weak initial dynamic.

Concerning the parameters h and ν, the change in their values has some significant but low effects on the total effect of the consumption shock on the unemployment rate. Introducing a lower real rigidity for consumption ($h = 0.4$), the consumption reacts more quickly to the shock, increasing more quickly the labor supply through the wealth effect. Introducing a strong wealth effect of the private consumption on the labor supply ($\nu = 1$), the labor supply increases more and absorbs partially the negative effect of the shock on the consumption. If introducing these calibration changes affects the effects of the public consumption shock on the unemployment rate, the changes are large and the consequences are weak . Indeed, I attempt to argue here that the results concerning the unemployment fiscal multiplier are quite robust to changes on the values of the parameters introduce in the households preferences. Even if I can introduce different dynamics for the labor supply, the amplitude of these changes are sufficiently low to observe large and negative effects on the unemployment rate in all cases. In the next subsection, I investigate more deeply what parameters drive the response of the unemployment rate following the shock by achieving a sensitivity analysis.

3.2.1 What parameters drive the unemployment fiscal multiplier? A sensitivity analysis

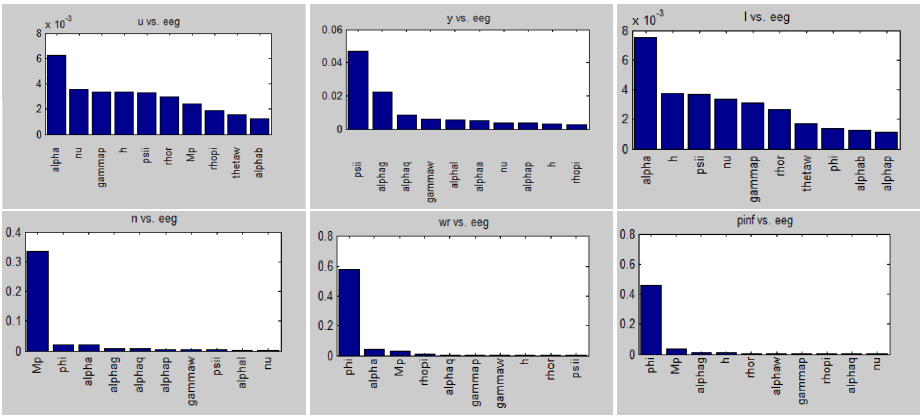


Figure 6: Sensitivity analysis results to the government consumption shock.

I compute the following sensitivity analysis using the Dynare program and more precisely the "Global Sensitivity Analysis Toolbox". Dynare runs a Monte Carlo process from the structural model generating 3000 data (and I offset the 1000 first draws). Then, with the prior for the parameters as given (first and second moments, the distribution shape), Dynare analyses the importance of each parameter for each variable and for each shock.

The results are summarized in the Figure 6. The parameters included in the households' preferences affect the response of the unemployment rate, the labor demand and the labor supply but are not predominant: other parameters are of

interest. Especially, deep parameters like the degree of price stickiness strongly drive the unemployment fiscal multiplier. Also, two structural parameters are important concerning the response of the labor demand: the degree of return to scale in the production function Mp ans the share of labor $(1-\alpha)$. For the degree of return to scale, I set initially $Mp = 1.75$: this is the value estimated in Gali, Smets and Wouters (2012). This is a plausible but high value. It is most likely that this parameter take a lower value than a higher value. This paramater drives the response of the firms to a supplementary demand. Higher is the value of this parameter, lower is the increase of the demand for capital and labor by the firms following a positive demand shock. If I decrease the value of Mp, the labor demand tends to be higher following the public expenditures shock than in the initial case. Since I already use a high value for this parameter, there is no chance than this parameter could explain and produce positive response of the unemployment rate in response to positive fiscal shocks.

For the value of the parameter α which defines the share of capital (and thus of labor) in the production function, I set $\alpha = 0.18$. This value can vary among countries but not in a large range. It is unlikely that this parameter can take extreme values allowing for a positive response of the unemployment rate.

The sensitivity analysis indicates that this model tends to always predict a negative effect of the fiscal policy on the unemployment rate.

3.3 The case of a public investment shock

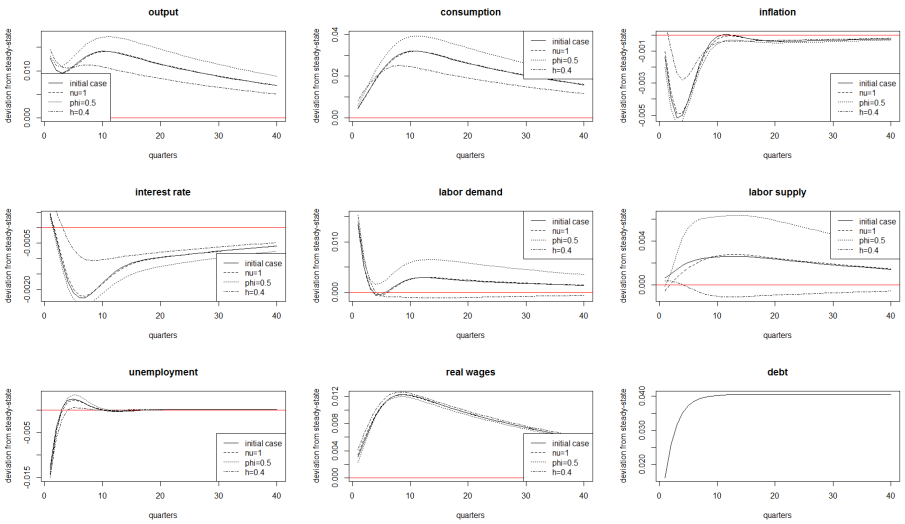


Figure 7: Effects of an increase of public investment (corresponding to a 1% of GDP)

The effects of a rise of the public investment are clearly different than government consumption albeit both tools share partly some transmission channels.

	Initial case	$\nu = 1$	$\phi = 0.5$	$h = 0.4$
1st period	1.27%	1.27%	1.32%	1.45%
5 periods	5.29%	5.23%	5.95%	5.81%
10 periods	11.72%	11.59%	13.68%	11.16%

Figure 8: Cumulative GDP multipliers for a 1% of GDP increase of public investment

	Initial case	$\nu = 1$	$\phi = 0.5$	$h = 0.4$
1st period	-1.28%	-1.40%	-1.42%	-1.49%
5 periods	-1.18%	-1.50%	-1.27%	-2.29%
10 periods	-0.76%	-1.04%	-0.51%	-2.23%

Figure 9: Cumulative unemployment multipliers for a 1% of GDP increase of public investment

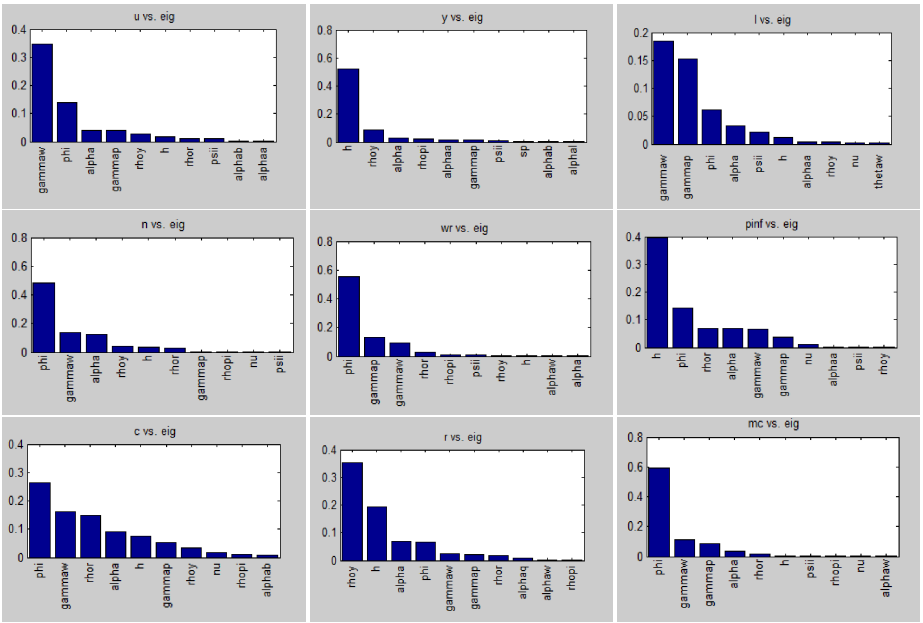


Figure 10: Sensitivity analysis results for the government investment shock.

Public investment is introduced in the total demand function like government consumption (see equation 38). A rise of government investment with respect to equation (38) will have the same effect than a rise of government consumption. Public investment has also an specific feature in the sense that public capital has a productive effect in the supply side of the model. As shown in the equation (30), the public capital enters as an input in the production function. Moreover, and as a consequence, the public capital influences negatively the marginal cost of the firms. Thus, when the government invests in capital, the marginal cost of the firms decreases (like in the case of a technological shock). The markup on price hikes thus the firms diminish prices. The interest rate decreases and private consumption hikes. Thus, the GDP fiscal multiplier for public investment is higher than for public consumption and the effects on the GDP are much more long lasting than in the case of the government consumption shock, as it's shown in the figure 7. The multipliers for the four scenarios are summarized in the figures 8 and 9.

The rise of the consumption has an other effect. Since I introduce a wealth effect in the labor supply's decision, captured by the parameter ν, a rise of consumption tends to dampen the response of the labor supply to higher real wages. As observed in the IRFs, labor supply fluctuates less strongly than in the case of a public investment shock. Combined with a larger response of the labor demand than in the case of the public consumption shock, the effects on the unemployment rate are stronger. At the first period, unemployment falls by 1.28% with the initial calibration.

It's interestingly that the response of the unemployment becomes positive after few periods causing lower cumulative multipliers over time (Figure 9). An important fact is that the effects of the shock on the labor supply is long-lasting, contrary to the case of the government consumption shock, while the effects on the labor demand remain short-lasting. This is why I obtain positive effects on the unemployment rate after few periods. In this model, the long lasting effect on the labor supply are due to the nominal wages. Just after the shock, the unemployment rate decreases very strongly. Since the nominal wage is set according to the evolution of the unemployment rate, the nominal wage increases dramatically. With the introduction of a nominal rigidity on the wage, the strong fall of the unemployment rate causes a long-lasting increase of the nominal wage. In this case, the real wage reacts more strongly than in the public consumption case and in a more long-lasting manner. Finally, these mechanisms cause a more long-lasting reaction of the labor supply. The total effect is in this case more ambiguous. In the case of a public investment shock, the multiplier at its peak is higher than the multiplier for a public consumption shock.

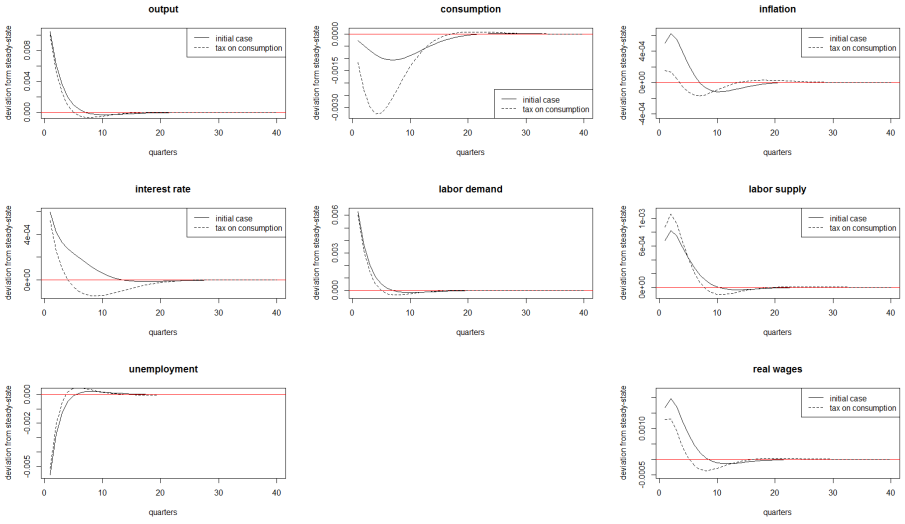
3.4 Introduction of taxes: different ways to fund the public expenditures

Until now, I assumed that the spending expansion was debt-based. I introduce now two taxes, namely a tax on consumption and a tax on the labor income.As said previously, taxes are introduced as simple rules and react to the variations

of the deficit. According to the values given to $\alpha^{\tau,c}$ and $\alpha^{\tau,w}$, different scenarios of financing are testable. In this section, I attempt to investigate whether the introduction of the taxes in order to fund the spending expansion can produce positive unemployment fiscal mutlipliers. The tax on consumption can be seen as a relevant choice to investigate the case of a tax decreasing the demand and having a direct negative effect on the GDP. Moreover, since this paper focuses on the households' preferences and the tax on consumption is closely related to the households. Since I introduce a wealth effect of the consumption on the labor supply, the model allows to precisely analyze the impact of the rise of the tax on consumption on the labor market, both at the demand and the supply side. The tax on labor income is also directly related to the household's behavior and on the labor market. Moreover, a tax on the labor income has a direct in effect on the private demand since there is a link between consumption and labor supply in the case of preferences á la Jaimovich and Rebelo (2009).

For both taxes, the methdology is the same. I set the parameters $\alpha^{\tau,c}$ and $\alpha^{\tau,w}$ in order to obtain a deficit two times lower than in the case of the totally debt-based expenditures. I then analyse whether the cumulative unemployment fiscal multipliers are still negative in these cases. For the next IRFs, I reproduce the cases with and without the tax in order to make the comparison easier.

3.4.1 The tax on consumption



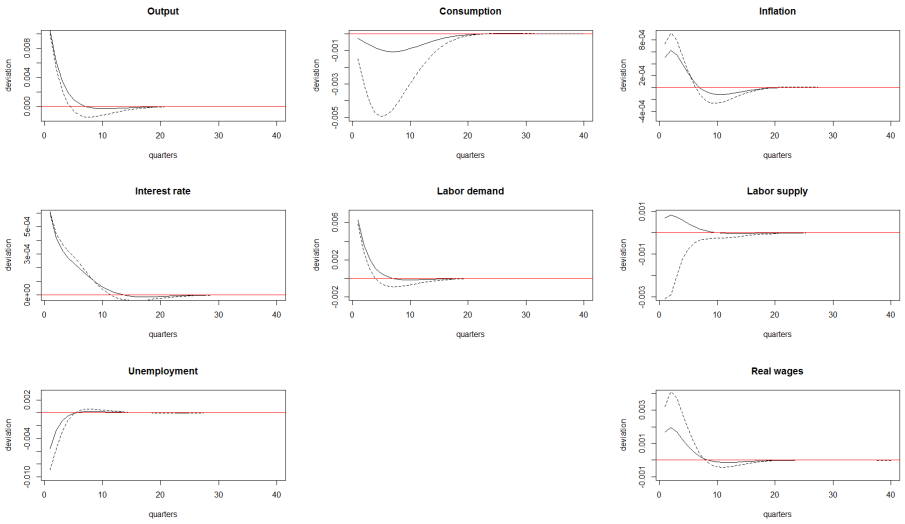
Lecture: The black line represents the initial case where the spending is funded by debt. The dashed line is the case with the tax on consumption.

Figure 11: Introduction of the tax on consumption

With a tax on consumption, the public expenditures crowd out much more the private consumption. It tends to reduce the positive effect of the public consumption shock on the GDP. A consequence is a lower hike of the labor de-

mand. Moreover, since the private consumption drops sharply, the labor supply increases more on impact via the wealth effect. The tax rule is set in order to absorb the half of the deficit which can be seen as a strong will by the government to sustain the level of deficit whithin the context of a countercyclical fiscal policy. Even in the case of this assumption, the unemployment fiscal multiplier remains strongly negative.

3.4.2 The tax on labor income



Lecture: The black line represents the initial case where the spending is funded by debt. The dashed line is the case with the tax on labor income.

Figure 12: Introduction of the tax on labor income.

In this case, the tax on labor income produces an higher unemployment fiscal multiplier. With the tax, the marginal utility from working is lower thus the households adress a lower labor supply. However, the drop of the disposable income reduces private consumption causing a lower GDP fiscal multiplier and a lower labor demand than in the inital (debt-based) case. The consequence on the labor supply is larger in absolute value than the consequence on the labor demand pruducing higher negative effects on the unemployment rate in the case of a shock partly funded by a tax on labor income.

4 Conclusion

I argue in this paper that I obtain with this model strong negative multipliers in the lines of Monacelli, Perotti and Trigari (2010) or Ravn ans Simonelli (2007). In this model, it is unlikely that the increase of the labor supply exceeds the rise of the labor demand. This is the case under various calibrations for the households preferences and especially even in the case of a large elasticity of substitution of the labor supply and of a strong wealth effect of consumption on the labor supply. A sensitivity analysis exercise indicates that the unemployment rate is more sensitive to changes on parameters included in the production side, namely the degree of return to scale and the capital/labor ration in the production function. However, as it is discussed in the result section, it is unlikely that these two parameters explain a positive response of the unemployment rate. Finally, the deep parameters and especially those determining the degree of nominal rigidities and the wage setting are the parameters influencing more the dynamic of the unemployment rate, as it is highlighted already in the literature.

This model contains two transmission channels of the fiscal policy on the labor supply: the rise of the real wage and the wealth effect via the introduction of the consumption in the labor supply equation. A third transmission channel is not present in this model: a "call effect". It means that when the labor demand increases, the probability of finding a job is higher: some inactive people can choose to return to the labor market thus rising the labor supply. This transmission channel can be highlighted in a Mortensen and Pissarides model for the labor market but in any case in this present model. A call effect could reduce the size of the multiplier but regarding to the multipliers found in this paper, the call effect would have to be very large in order to obtain a positive response of the unemployment rate after a rise of the public spending.

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Appendix

In this appendix, I derive entirely the model used in the paper. Especially, I derive the FOCs for the households and for the firms. Within the paper, the variables in lower case are expressed in log-deviation around the steady-state. In what follows, all the previous variables are expressed in level in upper case letters.

A The households

The representative household consumes the final goods and services supplied by the firms and supplies labor. In this framework, the continuum of households are monopolist for labor thus each worker supplies a diffiretiated kind of labor to the firms. The assumption of monopolist competition on the supply side of the labor market allows to introduce a wage-setting process led by the workers (or, identically, by unions representing the workers).

Defining the aggregate labor supply as $l_t = \int_0^1 l_t(i)\,\mathrm{d}i$ allows to rewrite the optimization program for the representative household as:

$$E_0 \sum_{t=0}^{\infty} \beta^t (log\tilde{C}_t - \frac{X_t \Delta_t L_t^{1+\phi}}{1+\phi}) \tag{39}$$

Originally, $\tilde{C}_t = C_t - h\bar{C}_{t-1}$ with $\bar{C}_{t-1}$ the aggregate past consumption representing a consumption index over the continuum of differentiated households. Expressing the utility function for a representative houshold and assuming that there is a perfect risk sharing within the households concerning the level of consumption in the spirit of Merz (1995), I simplify such as: $C_t = \bar{C}_t$.

The labor supply decision is already highlighted in the section 1 of the paper thus I still have to descibe the FOCs for consumption, investment and the degree of capital utilization since the households are supposed to be the final owners of the intermediate firms.

The maximization program can be expressed as:

$$max E_0 \sum_{t=0}^{\infty} \beta^t (log\tilde{C}_t - \frac{X_t \Delta_t L_t^{1+\phi}}{1+\phi}) \tag{40}$$

In order to transform the maximisation program, I use the indirect utility function defined by:

$$V(\Lambda_t) = U(C_t^\star, L_t^\star) \tag{41}$$

where $V(\Lambda_t)$ is the maximal utility given the level of the exogeneous and predetermined variables included in the column vector Λ_t. $C_t^\star$ and $L_t^\star$ represent the optimal values for consumption and labor allowing for the maximal utility.

I can define the indirect utility at the period 0 as:

$$V(\Lambda_0) = U(C_0^\star, L_0^\star) + E_0\beta U(C_1^\star, L_1^\star) + E_0E_1\beta^2 U(C_2^\star, L_2^\star) + ... \tag{42}$$

And using the fact that $E_tE_{t-1}x_{t+2} = E_tx_{t+2}$, It yealds:

$$V(\Lambda_0) = U(C_0^\star, L_0^\star) + E_0\beta U(C_1^\star, L_1^\star) + E_0\beta^2 U(C_2^\star, L_2^\star) + ... \tag{43}$$

In order to obtain an expression in terms of variables at the periods 0 and 1:

$$V(\Lambda_0) = U(C_0^\star, L_0^\star) + \beta V(\Lambda_1) \tag{44}$$

For a general expression, I can rewrite this equation in terms of the periods t and $t+1$ such as:

$$V(\Lambda_t) = U(C_t^\star, L_t^\star) + \beta V(\Lambda_{t+1}) \tag{45}$$

The budget contraint and the capital accumulation equation are introduced in the Lagrangian program:

$$(1+\tau_t^c)P_tC_t + I_tP_t + \frac{E_tB_{t+1}}{1+R_t} \leq (1-\tau_t^w)W_tL_t + B_t + R_t^k\nu_tK_{t-1} - f(\nu_t)\nu_tK_{t-1} + Div_t \tag{46}$$

and

$$K_t = (1-\delta)K_{t-1} + \epsilon_t^i[1 - S(\frac{I_t}{I_{t-1}})]I_t \tag{47}$$

The FOCs respectively for consumption, labor, debt, investment, capital accumulation and capital utilization are:

$$\lambda_t = \frac{U'_{C,t}}{P_t(1+\tau_t^c)} \tag{48}$$

$$\lambda_t = \frac{(1-\tau_t^w)U'_{l,t}}{w_t} \tag{49}$$

$$\lambda_t = \lambda_{t-1}(1+R_t) \tag{50}$$

$$\lambda_tP_t = \Omega_t\epsilon_t^i(1 - S(\frac{I_t}{I_{t-1}}) - S'(\frac{I_t}{I_{t-1}})(\frac{I_t}{I_{t-1}})) + E_t\Omega_{t+1}\epsilon_{t+1}^i(1 - S'(\frac{I_{t+1}}{I_t})(\frac{I_{t+1}}{I_t})^2) \tag{51}$$

$$\Omega_t = \beta E_t[\lambda_{t+1}(R_{t+1}^kV_{t+1} - P_{t+1}f'(v_{t+1})) + \Omega_{t+1}(1-\delta)] \tag{52}$$

$$\frac{R_t^k}{P_t} = f'(v_t) \tag{53}$$

where λ_t and Ω_t are respectively the Lagragian multipliers corresponding to the budget constraint and to the capital accumulation equation.

Including (10) in (12) allows us to obtain the consumption Euler equation:

$$\frac{U'_{c,t}}{U'_{c,t+1}} = \frac{1+R_t}{\Pi_{t+1}}\frac{1+\tau_t^c}{1+\tau_{t+1}^c} \tag{54}$$

and including (10) in (12) yields the marginal rate of substition between consumption and leisure, such as:

$$\frac{(1-\tau_t^w)U'_l}{(1+\tau_t^c)U'_C} = \frac{W_t}{P_t} \tag{55}$$

B Firms

In this two-sector model for the firms, a continuum of differentiated intermediate firms over [0,1] produce goods in a monopolistic competition and so these firms can set their price constrained by a Calvo sticky-wage process. Their production technology is a standard Cobb-Douglas function in which is added the public accumulated capital. The final firm then purchases a basket of the intermediate goods and retails a package of the goods at the consumers.

B.1 Intermediate firms

The technologic process of the productive firms is defined by the following Cobb-Douglas function:

$$Y_t(i) = \epsilon_t^a K_t^{\alpha}(i) N_t^{1-\alpha}(i)(K_{t-1}^g(i))^{\alpha_g} \tag{56}$$

As said previously in the paper, the public capital enters in the production function assuming that this governement investment is productivity-ehnancing for the private sector.

The profit of the firm is expressed as:

$$\Pi_t(i) = P_t(i)Y_t(i) - W_t N_t(i) - R_t^k K_t(i) \tag{57}$$

Maximization of (19) subject to (18) gives the following FOCs for capital and labor, such as:

$$\frac{\partial \Pi_t(i)}{\partial N_t(i)} = 0 \Leftrightarrow (1-\alpha)\epsilon_t^a K_t^{\alpha} N_t^{-\alpha}(K_{t-1}^g)^{\alpha_g} = W_t \nabla_t \tag{58}$$

$$\frac{\partial \Pi_t(i)}{\partial K_t(i)} = 0 \Leftrightarrow \alpha\epsilon_t^a K_t^{\alpha-1} N_t^{1-\alpha}(K_{t-1}^g)^{\alpha_g} = R_t^k \nabla_t \tag{59}$$

where ∇_t is the Lagrangian multiplier associated with the technologic constraint (18). By rearranging equations (20) and (21) I find the demand function for each input, such as:

$$K_t = \frac{W_t}{R_t^k} N_t \tag{60}$$

Finally, the marginal cost for the firms can be expressed as:

$$mc_t = \frac{(R_t^k)^{\alpha} W^{1-\alpha}}{\epsilon_t^a + (K_t^g)^{\alpha_g}} \tag{61}$$

B.2 Final goods firms

(Appendix to be finished)