Long memory in log-range series: Do structural breaks matter?[☆]

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Abstract

This paper examines whether the observed long memory behavior of log-range series is to some extent spurious and whether it can be explained by the presence of structural breaks. Utilizing stock market data we show that the characterization of log-range series as long memory processes is a strong assumption. Moreover, we find that all examined series experience a large number of significant breaks whereas not all breaks are sudden. Once the breaks are accounted for, the volatility persistence is eliminated. Overall, the findings suggest that volatility can be adequately represented through a multiple breaks process and a short run component.

Key words: Structural breaks, Long memory, Log-range volatility proxy, Stock market
JEL classification codes: C22, C58, G10

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1. Introduction

The modelling of financial time series volatility has been a flourishing field of research. A number of theoretical and empirical studies focus on the apparent persistence in volatility manifested by slowly decaying autocorrelation functions which induces the frequent characterization of volatility as a long memory process (see Ding et al., 1993).

At the same time, many studies point out that structural breaks or regime switches may induce spurious long memory effects in time series (see for example Liu, 2000; Diebold and Inoue, 2001; Granger and Hyung, 2004; Starica and Granger, 2005 and Davidson and Sibbertsen, 2005). They provide examples in which long memory can be easily confused with structural breaks, concluding that it is very difficult to distinguish between true and spurious long memory processes (see for instance Berkes et al., 2006 and Zhang et al., 2007). A growing strand of literature has tried to address the issue by developing tests that distinguish between true and spurious long memory. For example we refer the Berkes et al. (2006), Ohanissian et al. (2008), Perron and Qu (2010), Qu (2011) and Shao (2011) tests. For reviews on structural breaks and long memory, we refer to Sibbersten (2004), Banerjee and Urga (2005) and Perron (2006).

From an empirical point of view, although the existing literature that examines long memory or structural changes is prominent, studies that focus on their interaction are limited but steadily growing. The distinction between long memory and structural breaks has not produced, yet, a clear answer as to which feature characterizes volatility time series or which feature is dominant. However the correct classification of volatility as either long memory process or a process subjected to structural breaks or both, can lead in measurable forecasting gains. Choi et al. (2010) examine the existence of structural breaks and long memory in daily exchange rate realized volatility series, establishing that part of long memory is due to structural breaks. McMillan and Ruiz (2009) find that the long memory property largely disappears when volatility time-variation is taken into account in absolute stock returns. Bisaglia and Gerolimetto (2009) examine the existence of long memory and occasional breaks in daily log absolute returns, concluding that the series is characterized by structural breaks and not by long memory. Morana and Beltratti (2004) find that while long memory is evident in the daily exchange rate realized volatility, this feature is partially explained once changes are accounted for.

This paper provides empirical evidence as to whether long memory in daily log-range series could be explained by the presence of structural breaks. Along these lines, our analysis contributes to relevant literature in three important ways. First, we use the daily log-range as a volatility proxy. This is originated from the fact that the most commonly used proxies such as the absolute or square daily returns are less informative compared to range based volatility proxies (see for instance Garman and Klass, 1980; Parkinson, 1980 and Alizadeh et al., 2002). Particularly, the increased noise levels present in the absolute or square returns might mask the presence and/or the number of breaks. Second, a sequential break search procedure is adopted that does not fix the upper bound of allowed breaks given prior empirical evidence that volatility regimes might be extremely short lived (see Liu, 2000). Restricting the upper bound of allowed breaks to values typically encountered in applied macroeconomic series, e.g. 3 to 5 breaks, might leave a large number of breaks undetected. Third, we consider a smooth transition trend model that allows abrupt shifts, smooth shifts or a combination. The existing tests for structural breaks impose only abrupt changes, but changes could also be characterized as smooth, a feature that abrupt break tests ignore. Smooth changes may be more realistic because volatility usually evolves over time in a continuous manner (McMillan and Ruiz, 2009).

Overall, the aim of this paper is first to examine empirically whether the long memory behavior observed in daily log-range series could be spurious and second, drawing on these findings, to investigate empirically if the long memory behavior can be explained by the presence of structural breaks. Using data from the US stock market, we find strong evidence of long memory in log-range series, while the results from stochastic volatility models reveal that a high persistent component in volatility is a too strong assumption. The break analysis indicates that all series under scrutiny experienced a large number of structural breaks. However a number of identified breaks are better described as smooth transitions. After controlling for the changes, the long memory feature is no longer supported.

The rest of the paper is organized as follows. Section 2 presents the log-range volatility proxy and the data. Section 3 presents the long memory and stochastic volatility approaches to modelling the long run component of volatility and includes the discussion of our results. Section 4 provides the structural break analysis and the subsequent discussion while Section 5 concludes.

2. Volatility proxy and Data

In this study, volatility is approximated by the log-range. Range based volatility proxies are more informative compare to the classical log return-based volatility estimators, as mentioned by Garman and Klass (1980) and Parkinson (1980) among others. Following Alizadeh et al. (2002), we formulate the log-range volatility proxy as the difference between the highest and lowest log prices

$$R_t = \ln(\ln(H_t) - \ln(L_t)) = \ln(\ln(H_t/L_t))$$

where H_t and L_t denote the highest and the lowest price of the t day. The superior efficiency of the log-range is demonstrated by Alizadeh et al. (2002) who find that under benchmark assumptions on the data generating process, the log-range standard deviation is about one quarter of the standard deviation of the log absolute returns. As such, the log-range volatility proxy outperforms the usual volatility proxies of log absolute or squared returns since its adoption curtails the impact of noise present in the absolute or squared log-return measures of volatility. In addition, as shown by Alizadeh et al. (2002), range based volatility estimation can be powerful and convenient due its apparent near log-normality. The log-range is nearly normally distributed[1] with mean $0.43 + h_t$ and variance 0.29^2, with h_t the daily log-volatility ($h_t = \ln \sigma_t$)

$$R_t \approx N\left(0.43 + h_t, 0.29^2\right)$$

while it is robust toward microstructure effects, particularly in liquid markets.

We study the S&P 500 and Dow Jones Industrial Average (DJIA) indices along with the thirty stocks that were components of the Dow Jow Industrial Average index as of 20/06/2011, namely AA, AXP, BA, BAC, CAT, CSCO, CVX, DD, DIS, GE, HD, HPQ, IBM, INTC, JNJ, JPM, KFT, KO, MCD, MMM, MRK, MSFT, PFE, PG, T, TRV, UTX, VZ, WMT and XOM. The data sample runs from January 2nd 2002 to June 20th 2011, covering the period after the dotcom bubble and the recent financial crisis, resulting in a total of 2384 daily observations.

Table 1 presents descriptive statistics for all log range series. All series have low kurtosis and display the typical long memory pattern of slowly

[1] See also, Brandt and Jones (2006).

decaying autocorrelations[2]. Figure 1 shows the AA log-range series and its autocorrelation function as an example[3]. The top panel presents the log range series, while the bottom the autocorrelation function up to $T-1$ lag. The autocorrelation function decreases to zero approximately at lag 400, reaches a minimum value at lag 1000 and goes back to near zero values at distant lags. Perron and Qu (2010) demonstrate that this shape of the autocorrelation function could characterize a short memory process with level shifts. Though, if we restrict our attention to autocorrelations up to lag 400, the function decays in a hyperbolic pattern akin to a long-memory process. Approximate normality is rejected mostly on the basis of positive skewness ranging from 0.269 to 0.798 across series. Alizadeh et al. (2002) demonstrate that the theoretical skewness of log-range series is 0.17. However, we emphasize that if we consider subsamples up to the end of 2006, excluding the period during the financial crisis, the feature of skewness is no longer present in all series.

[Table 1 here]

[Figure 1 here]

3. Long memory and stochastic volatility models

3.1. Long memory

Baillie (1996) provides a detailed survey of econometric work on long memory and its application in economics and finance. One definition of long memory for a stationary discrete time series process, y_t, is that

$$\lim_{n\to\infty} \sum_{j=-n}^{n} \left|\rho_j\right| = \infty$$

[2]Albeit, in all series, the empirical autocorrelation magnitudes are significantly lower than those produced by the corresponding absolute return series.

[3]In order to save space, we use the AA DJIA component as a representative series for the figures.

with ρ_j the autocorrelation function of y_t at lag j. Furthermore, the spectral density of the series is unbounded at low frequencies. A more general definition of long memory can be given in terms of decay rates of autocorrelations. The autocorrelation function of a long memory process is given by

$$\rho_k \sim k^{2d-1} \text{ as } k \to \infty$$

where $d > 0$ means that the autocorrelations of this process decay in a hyperbolic fashion, in contrast to the exponential rate of a stationary short-memory series.

The most widely adopted class of long memory models employed in econometrics is the class of fractionally integrated models. This class provides an attractive alternative to the $I(1)/I(0)$ dichotomy in econometrics with long memory nonstationarity versus short memory stationarity replaced by processes with a continuum of memory or persistence characteristics defined through a single parameter $d \in \left(-\frac{1}{2}, 1\right]$, the order of integration. That class of models is based on the representation

$$(1 - L)^d \, y_t = u_t$$

where u_t is a stationary, weakly dependent, zero mean process. The $ARFIMA(p, d, q)$ model developed by Granger (1980), Granger and Joyeux (1980) and Hosking (1981) belongs to this class of models. For $-0.5 < d < 0$, y_t is characterized as a stationary short-memory series and addressed as antipersistent. For $d = 0$ we obtain an $I(0)$ series. For $0 < d < 0.5$ the series is regarded as persistent (long memory) but stationary and mean-reverting and finally, for $0.5 < d < 1$ the series is nonstationary long memory but mean reverting. For $d = 1$ we obtain an $I(1)$ series. There are several parametric and semi-parametric methods in order to estimate the fractional differencing parameter d (see Banerjee and Urga, 2005). Although the latter are preferable due to their robustness to model mispecification, the semi-parametric estimators are less efficient.

We implement two semi-parametric procedures, the most commonly used in empirical studies, the log-periodogram estimator of Geweke and Porter-Hudak (1983), GPH, and the Gaussian semi-parametric estimator of Robinson (1995), GSP. Moreover, we estimate parametric $ARFIMA(p, d, q)$ models based on the maximum likelihood procedures explained in Doornik and Ooms (2004).

Recent studies show that the long memory characteristic might be spurious due to unaccounted structural changes or regime switches, e.g. Liu (2000) and Diebold and Inoue (2001). In order to examine the null hypothesis of long memory against the alternative of a short memory process contaminated by level shifts, we employ five test statistics proposed by Perron and Qu (2010) and Qu (2011). The first group of tests are the $t_d, \sup -t_d$ and $mean - t_d$ of Perron and Qu (2010). The t_d test statistic is given by

$$t_d\left(a, c_1; b, c_2\right) = \sqrt{24c_1\left[T^a\right]/\pi^2}\left(\hat{d}_{a,c_1} - \hat{d}_{b,c2}\right)$$

where $\hat{d}_{a,c}$ denote the log-periodogram estimate of d when $m = c\left[T^a\right]$ frequencies are used. The $\sup -t_d$ is the supremum of the $t_d\left(1/3, c_1; 1/2, 1\right)$,

$$\sup -t_d = \sup_{c_1 \in [1,2]} t_d\left(1/3, c_1; 1/2, 1\right)$$

and the $mean - t_d$ is defined as the average of the $t_d\left(1/3, c_1; 1/2, 1\right)$,

$$mean - t_d = mean_{c_1 \in [1,2]} t_d\left(1/3, c_1; 1/2, 1\right)$$

(see Perron and Qu (2010) for an extensive analysis). The t_d statistic is asymptotically normal distributed. Since the limit distribution of the $\sup -t_d$ and $mean - t_d$ is not available, we use a parametric bootstrap procedure to compute the relevant critical values, following Perron and Qu (2010).

The remaining two test statistics, W and W "prewhitening", are due to Qu (2011). The former test is based on the profiled likelihood function of the local Whittle estimator, while the latter is an extension of the W so as to control the test size in the presence of short memory (see Qu (2011) for details). Both tests include a trimming parameter, ε, which ensures a reliable asymptotic approximation even in small samples. Qu (2011) suggests to choose $\varepsilon = 0.02$ for large samples, while the asymptotic critical values for the tests are 1.118, 1.252 and 1.517 for 10%, 5% and 1% significant level, correspondingly.

3.2. Long memory estimation and test results

We estimate the long memory parameter using the GPH and GSP semiparametric approaches for different truncation values, $m = T^{0.5}, T^{0.6}$ and $T^{0.7}$ and parametric $ARFIMA\left(p, d, q\right)$ models, specifically an $ARFIMA\left(0, d, 0\right)$ and $ARFIMA\left(1, d, 0\right)$. Table 2 provides a brief overview of the estimated d values for each log-range series. All semiparametric estimates are statistically different from zero and close to or greater than 0.5 suggesting near nonstationary or nonstationary but mean reverting behavior of the log-range volatility series. Notice that benchmark unit root tests reject the null of a unit root.

[Table 2 here]

The estimated values of d by the GPH and GSP fall within both the stationary and nonstationary regions depending on the number of periodogram ordinates used. For $m = T^{0.5}$ the GPH estimates of the long memory parameter spread over the range 0.462 to 0.741, for $m = T^{0.6}$ lie between 0.469 and 0.679 and when $m = T^{0.7}$ is employed, they reach the lowest values ranging from 0.399 to 0.595. The GSP estimates of d are close to that of GPH estimates. Under the hypothesis of no level shifts, the results imply that indices and DJIA stock components have volatility that exhibits either stationary or nonstationary long memory characteristics depending on the different truncation values m.

Perron and Qu (2010) showed that the estimates of d by the log periodogram regression for a short memory series with breaks will vary with m. As m increases the short memory component becomes more important relative to the level shift component and the estimate of d falls. On the contrary, for a truly long memory process, d is independent of m. Our results for all series reveal that the value of d indeed declines as the truncation value increases advocating in favor of simultaneous presence of a level shift component and short memory dynamics. Following Perron and Qu (2010) we compute the log periodogram estimates of d (GPH) for truncation values m ranging from 10 to $T^{3/4}$ for all series. Figure 2 presents the long memory parameter estimates for the AA log range series[4]. The memory parameter estimates are nonstationary for small values of m and as m increases the estimate of d gradually decreases. The results support the theoretical findings of Perron and Qu (2010) as d declines with m.

[Figure 2 here]

A further aspect worth mentioning from Table 2, is the memory estimates from maximum likelihood methods. It appears that maximum likelihood estimation produces results that are not in accordance with semiparametric

[4]The behavior of the long memory parameter is similar across all examined series.

estimates, at least with respect to the presence of nonstationarity in the underlying series (right panel of Table 2). Estimation of $ARFIMA\,(0,d,0)$ models produces estimates of d with mean 0.34 across series whereas estimates based on an $ARFIMA\,(1,d,0)$ model are centred around $\hat{d}=0.42$ with the autoregressive parameter being suspiciously negative around a mean value of -0.17. The negative value of the autoregressive parameter is suggestive of overdifferencing, also reflected in the inflated $\hat{d}$ estimate. We consider $ARFIMA\,(p,d,0)$ models with $p>1$, as well. The addition of autoregressive parameters inflates further the value of the estimated long memory parameter whereas AR parameters remain consistently low and negative. Such model estimates become meaningless in economic terms and resemble near root cancellation in ARMA models with redundant parameters.

So far, descriptive measures along with semiparametric and parametric estimation of long memory models indicate that the log-range series might display spurious long memory characteristics. Motivated by the results, we apply the test statistics proposed by Perron and Qu (2010) and Qu (2011). Table 3 summarizes the test statistic results along with the simulated critical values for the $\sup -t_d$ and $mean-t_d$ tests. The rejection of the long memory null hypothesis is almost uniform. The null is rejected at least by two tests in each series. These findings suggest that the evidence of long memory is not as strong as considered.

[**Table 3 here**]

3.3. Log-range based stochastic volatility factor models

If the log-range series can be decomposed to the sum of two processes, a persistent long term process plus a short memory process which can be even noise, then the findings of the GPH, GSP and maximum likelihood procedures regarding d could be justified. Superposition of independent short memory processes can mimic empirically observed slow decaying autocorrelation functions or power laws (see LeBaron, 2001; Barndorff-Nielsen, 2001 and Barndorff-Nielsen and Shephard, 2001 among others). In the context of GARCH models Granger and Ding (1996) suggested a two-component GARCH model, one component describing short-run dynamics whereas the persistent component specified as an IGARCH process determines the long run behavior of volatility.

In light of Alizadeh et al. (2002) analysis, we proceed with estimation of factor stochastic volatility models. A single factor stochastic volatility model for the log-range, R_t, expressed as a linear state space system of equations (see Harvey et al., 1994) can take the form

$$\begin{aligned} R_t &= 0.43 + h_t + \varepsilon_t \\ h_t &= \bar{h} + \phi\left(h_{t-1} - \bar{h}\right) + \eta_t \end{aligned} \tag{1}$$

where h_t determines the latent log-volatility, $\bar{h}$ is the mean of h_t, ϕ is the autoregressive parameter, $\eta_t \sim nid\left(0, \sigma_\eta^2\right)$ and $\varepsilon_t \sim nid\left(0, 0.29^2\right)$. The first equation in (1) is the signal equation that relates the log volatility proxy to the underlying latent log volatility, while the second is the state or transition equation and represents the dynamics of the latent volatility.

In the case of (1) being inadequate to fit the data, Alizadeh et al. (2002) proposed a two factor model. In the two factor model the state equation is enhanced by including a second latent component so that

$$\begin{aligned} R_t &= 0.43 + \bar{h} + h_{1,t} + h_{2,t} + \varepsilon_t \\ h_{1,t} &= \phi_1 h_{1,t-1} + \eta_{1,t} \\ h_{2,t} &= \phi_2 h_{2,t-1} + \eta_{2,t} \end{aligned} \tag{2}$$

with latent volatility given by the sum $h_t = \bar{h} + h_{1,t} + h_{2,t}$ and ϕ_1, ϕ_2 are the autoregressive parameters for each volatility component. The error terms $\eta_{1,t}$ and $\eta_{2,t}$ are assumed contemporaneously and serially independent $N\left(0, \sigma_{i,\eta}^2\right)$ random variables.

Estimation of (1) and (2) is based on Kalman filter implementation and Gaussian quasi-maximum likelihood methods. The approximate Gaussianity of the log-range produces highly efficient parameter estimates and extractions of latent volatility.

3.4. Estimation of stochastic volatility models

The left panel of Table 4 reports the estimates of the one factor stochastic volatility model (1). The estimated autoregressive volatility parameter ϕ for all DJIA components and indices is high stretching from 0.853 to 0.992 and points towards a rather persistent or even unit root process being present. This is in contrast to the Alizadeh et al. (2002) results for future exchange rate series where a short memory dependence structure was revealed. However, when we employed recursive and rolling windows estimation of the autoregressive parameter ϕ, we came across considerable time varying behavior.

In particular, during "calm" sub-sample intervals characterized by relatively low volatility and an adequate number of observations, the estimated values of ϕ were around a mean value of 0.6 as in Alizadeh et al. (2002).

[**Table 4 here**]

The residuals from the signal equation are correlated, however substantial negative residual serial correlation is uniformly present in all DJIA components and S&P 500 index, whereas DJIA index residuals appear to have low positive correlation, especially at lags 1 and 2. The negative residual correlation is similar to the ARFIMA results mentioned earlier and is suggestive of some type of overdifferencing or mispecification. It is also in contrast to the Alizadeh et al. (2002) residual inspection results where positive leftover correlation was detected. In order to get over the deficiencies of the one factor stochastic volatility model, and in light of the remaining residual positive serial correlation, Alizadeh et al. (2002) proceed in the estimation of a two factor stochastic volatility model. Although in our case this is not suggested by the residual descriptives, we also proceed with estimating model (2) for comparison reasons[5].

The estimated autoregressive parameters from the two factor stochastic volatility model are presented in the right panel of Table 4. Parameter ϕ_1 estimates point to high persistence, possibly to a unit root component, since they vary from 0.980 to 0.997. The second factor appears to be transient with its estimated parameter ϕ_2 varying from a low of 0.111 to a high of 0.674. The estimate of parameter ϕ_2 for the S&P 500 is negative and equals to -0.235. All estimates are highly statistically significant suggesting the existence of at least two latent volatility components, a persistent long run and a transient short run component. However, the autocorrelation diagnostics for residuals from the two factor stochastic volatility model still show the same uniform negative serial correlation pattern at least at lag 1 that suggests overdifferencing or that the near unit root persistent component is estimated with a positive bias. The introduction of more factors into the model becomes quickly non-intuitive in economic terms and does not solve the "overdifferencing effect".

[5] The log range series of the DJIA index is adequately modeled by the one factor model.

4. Structural break analysis

4.1. Multiple abrupt mean break model

Granger and Hyung (2004) recommend that a way to explain overdifference is neglected nonlinearity, such as level shifts, smooth transitions or nonlinear trends. We test the null hypothesis of constant unconditional mean against the alternative of multiple instantaneous breaks in the unconditional mean of the daily log-range series R_t. Under the alternative, a model with m breaks, that is $m+1$ mean regimes, is considered

$$R_t = \mu_j + u_t \ , \ t = T_{j-1}+1, ..., T_j \ , \ j = 1, ..., m+1 \tag{3}$$

where T denotes sample size with $T_0 = 1$ and $T_{m+1} = T$ and u_t a linear process of martingale differences.

Given the large sample size at hand and the fact that we do not want to overly restrict the minimal subsample length and subsequently the number of multiple breaks "allowed", we adopt the Bai (1997b) sequential procedure for detecting multiple structural breaks in the mean. The first (if $m > 1$) break point is identified using the test statistic

$$\max_{k \in \{[\pi T], [\pi T]+1, ..., [(1-\pi)T]\}} F_T(k) = \max \frac{S_T - S_T(k)}{\hat{\sigma}^2}$$

where S_T represents the restricted residual sum of squares under the null of no break from (3), $S_T(k)$ is the unrestricted residual sum of squares under the alternative of a single break at date k and $\hat{\sigma}^2$ is a consistent estimator[6] of the long run variance of R_t. We set $\pi = 0.15$ so that both pre- and post-break periods contain at least 15% of the available observations and [.] denotes the integer part. We use the method of Hansen (1997) to obtain approximate asymptotic p-values and employ a 5% significance level throughout. The estimated break date is given by

$$\hat{k} = \arg \max_{k \in \{[\pi T], [\pi T]+1, ..., [(1-\pi)T]\}} F_T(k)$$

and corresponds to the date where $F_T(k)$ is maximized[7].

[6] Three different estimators are employed, the Newey and West (1987, 1994) and the Andrews (1991) quadratic spectral kernel estimator.

[7] Confidence intervals for $\hat{k}$ can be computed using the methods developed by Bai (1997a).

After identifying the first break date, the sample is divided into two subsamples where hypothesis testing of parameter constancy for each subsample is performed. If the constancy test fails then the corresponding subsample is further divided into subsamples at the newly estimated break point and the parameter constancy test is applied for the hierarchically obtained subsamples. The minimum subsample length is set to 10 working days or two weeks in order to avoid the application of autocorrelation robust estimates on very small samples. This procedure is repeated until the parameter constancy test is not rejected for all subsamples. The number of break points is equal to the number of subsamples minus 1. The limit distributions of the break date estimates $\hat{\tau}_1 = \left[\frac{\hat{T}_1}{T}\right], ..., \hat{\tau}_m = \left[\frac{\hat{T}_m}{T}\right]$ depend on the parameters in all segments of the sample, particularly on the relative break positions and magnitudes. To remedy this problem, Bai (1997b) suggested a "repartition" procedure that amounts to re-estimating each break date conditional on the adjacent break dates. For example, let the initial estimates of the m break dates be denoted by $\left(\hat{T}_1^{(0)}, ..., \hat{T}_m^{(0)}\right)$. The second round estimate (first repartition run) for the i^{th} break date is obtained by fitting a one break model to the segment starting at $\hat{T}_{i-1}^{(0)}$ and ending at date $\hat{T}_{i+1}^{(0)}$ (with $\hat{T}_0^{(.)} = 0$ and $\hat{T}_{m+1}^{(.)} = T$). The number of rounds continues until there is no change in the number of breaks and the maximum difference of a newly found break in the last round from the corresponding date in the previous round does not exceed 10 working days. The estimates obtained from this repartition procedure have the same limit distributions as those obtained by a simultaneous, but more computationally demanding, procedure[8].

A central feature of the testing procedure based on model (3) is that structural change occurs instantaneously. The mean volatility level "jumps" at $T_{j-1} + 1$ to a new mean level μ_j so that the transition from μ_{j-1} to μ_j is completed within a day interval. Such instant changes are intuitive given that the arrival of major news is rapidly incorporated by market participants and triggers volatility jumps or switches in volatility.

Empirical models with abrupt level shifts similar to (3) have been consider by Liu (2000), Gourieroux and Jasiak (2001), Granger and Hyung (2004), Davidson and Sibbertsen (2005), Lu and Perron (2010), Perron and Qu (2010) and references therein as generating long-memory characteristics.

[8]See Bai and Perron (1998, 2003a,b).

4.2. Multiple abrupt structural break results

Table 5 presents the estimated number of breaks for the multiple break model based on the Bai's methodology. Also, it reports descriptive statistics on the residuals $\hat{u}_t$, that is the break-adjusted log-range series. In particular, the estimated autoregressive lag order, the sum of the autoregressive parameters and the estimates of the long memory parameter d from competing $ARFIMA\,(0,d,0)$ and $ARFIMA\,(1,d,0)$ specifications on $\hat{u}_t$ are tabulated. The estimated structural break dates per month are summarized in Figure 3.

[Table 5 here]
[Figure 3 here]

A number of immediate findings can be derived from inspecting Table 5. First, there are numerous volatility mean level shifts in all series under scrutiny. The total number of level shifts is 763. The number of estimated breaks ranges from 17, in the BA and MRK stocks, to 41 level shifts in the JPM. The median number of detected shifts across stocks equals 25. The estimated number of level shifts for the S&P500 index lie between these extremes, while the DJIA index appears to be more volatile as the estimated number of shifts is 44.

The number of identified shifts are high compared to other studies that examine stock market series. For instance, Granger and Hyung (2004) use the same methodology with this study to detect level shifts in the log absolute returns of 12 subperiods of S&P 500 index, covering the period from 1928 to 2002. The number of identified shifts ranges from 4 to 13, while each period contains 1705 observations (only the last period includes 1113 observations). McMillan and Ruiz (2009) examine for breaks in log absolute index returns from ten countries over the period 1990 - 2005. They implement the methodology by Bai and Perron (2003a), which allows for a maximum number of breaks, usually set at five breaks. The maximum number of breaks is 4 for the Germany index. Bisaglia and Gerolimetto (2009) identified 13 shifts in S&P 500 log absolute returns for the time period from January 2nd 1988 to June 15th 2005, following the methodology of Bai and Perron (1998, 2003a). Lu & Perron (2010) proceed in the estimation of a random level shift model in order to detect level shifts in the absolute log returns of the S&P 500, AMEX, Dow Jones and NASDAQ indices. Although the long examined time period, they identify a few shifts; 15 for the S&P 500, 28 for the AMEX, 12 for the Dow Jones and 7 for the NASDAQ.

The difference in the number of the identified breaks in this study is due to the volatility proxy and the methodology. The log-range approach reduces the noise encompassed in the volatility proxy resulting in a more "clear" proxy. Moreover, the Bai (1997)'s approximation used in this study does not impose any bounds on the number of identified breaks.

In order to verify - ex post - the statistical significance of shifts and guard against the possibility of overestimating the number of breaks, we examine the equality of mean log-range estimates across subsamples by employing Wald tests of the null hypothesis of pairwise coefficient equality $\mu_j - \mu_{j+1} = 0$ in the following model,

$$R_t = \sum_{j=1}^{m+1} \mu_j D_{j,t} + u_t$$

where m denotes the number of detected level shifts and $D_{j,t}$ denotes a dummy variable taking the value of 1 within the j^{th} subsample interval. The null hypothesis of equal mean estimates across the subsamples defined by the level shifts is strongly rejected for all cases and further strengthens the case of level shifts.

Based on the break dates as illustrated in Figure 3 we are able to separate the full sample into three distinct sub-sample periods. Two of the periods are marked by episodes of increased instability, while one of the sub-samples marks a period of tranquility. The first period runs from January 2002 to November 2002, the second covers the time period between December 2002 and May 2007 and the third extends from June 2007 to June 2011. The first time period can be associated with the general stock market downturn during 2002 in the aftermath of the dotcom bubble. Stock markets faced dramatic declines in this period, especially during July and September. In this period the number of identified level shifts is relatively high, as we estimate a total of 106 breaks in the stocks and 10 in indices. June, July, August and November can be characterized as the most volatile months, as we detect 18, 13, 16 and 22 breaks in stocks, respectively and a shift per month in each index.

The second period is relatively calm. Despite the high number of identified breaks (194 shifts in DJIA components and 17 in indices), it covers a long lasting period of four and a half years.

The final period is the most unstable (volatile) and is related to the recent financial crisis. The majority of the identified shifts is located in this period. We detect 463 firm oriented level shifts and 49 market and industry oriented. Most of the identified shifts are related to major economic events. We focus on the months that experienced the highest number of shifts in this period, namely, September, October and December 2008. The financial institution crisis hit its peak during these months. Several major institutions, such as Lehman Brothers, Merrill Lynch, Fannie Mae, Freddie Mac, Washington Mutual, Wachovia, Citigroup and AIG, either failed or were subject to government takeover.

In September, we detect level shifts in 26 out of 30 DJIA stocks, while in three of them, namely CAT, IBM and JPM, two shifts are identified (29 total). Moreover, two shifts are identified in the DJIA index and one in the S&P 500. Despite that we detect shifts in 18 stocks during October, ten of them (AA, AXP, BA, DD, KO, MMM, MRK, T, VZ and XOM) are the most affected, as they faced with two level shifts (28 total). Also, two level shifts are identified in each index. Finally, in December 28 stocks faced with level shifts, with the exception of KO stock which faced two shifts (29 total). In this month, we identify a level shift in each index, as well.

[Figure 4 here]

Another interesting feature of the detected shifts is related to their duration. Figure 4 shows the distribution of the duration of the identified volatility regimes based of the number and location of the detected breaks. Notice that the breaks cannot be termed "infrequent" as the median regime duration is 64 trading days, that corresponds to three trading months, for the DJIA stocks and 41 days for the indices. There are many short-lasting regimes and a few with long duration which generate a heavy tail duration distribution. This pattern of duration could be responsible for the long memory behavior of series according to Liu (2000) who points out that if the duration of regimes has a heavy-tail distribution, then the underlying series can display long memory characteristics. The large number of short regimes is cancelled out by the short duration of these regimes and as a result the long-lasting regimes induce persistence.

The third and fourth columns of Table 5 present diagnostic checks of remaining correlation on the break-adjusted residual series $\hat{u}_t = R_t - \sum_{j=1}^{m+1} \hat{\mu}_j D_{j,t}$, as a tool of assessing the impact of breaks on the series dynamics. It appears that the break adjusted series $\hat{u}_t$ exhibit positive but small correlation. A low order AR model with short memory can be identified for each residual series. If we use the sum of the autoregressive parameters as an indicator of persistence, we obtain sums that stretch from 0.145 to 0.394 suggesting short leftover memory. The positive autoregressive coefficient signs for all series corroborate against overfitting by the structural breaks procedure. The autoregressive lag order is selected using the AIC criterion with maximum lag set at 12. An alternative way to capture the remaining "persistence" is through the long memory parameter from a fractional white noise model, $ARFIMA(0, d, 0)$. Once the level shifts are accounted for the picture is completely different, the long memory parameter estimates are severely downsized, as can be shown from the fifth column of Table 5. The values of $\hat{d}$ from the break-adjusted series have been substantially reduced, being below 0.18 (expect for the BAC stock) and the leftover memory is of no economic and practical importance given the small $\hat{d}$ values.

Finally, notice that when we estimate $ARFIMA(1, d, 0)$ models, we find that the $AR(1)$ coefficient estimates (sixth column of Table 5) are highly inflated, with a median value of 0.95, while the estimates of the long memory parameter (seventh column of Table 5) are highly negative inflated which is a sign of mispecification for the $ARFIMA(p, d, 0)$ model with $p > 0$.

4.3. Smooth transitions

The analysis so far reveals that volatility dynamics can be represented through a model with multiple abrupt level shifts plus a short run component that exhibits low positive correlation.

Despite the fact that volatility breaks need not happen instantly or within a trading day, almost all existing tests are constructed under the assumption of abrupt breaks. In periods of increased economic uncertainty, for example when uncertainty is originated by macroeconomic instability, volatility may increase slowly enough as to produce a smooth transition pattern, at least for a short period of time. Such patterns might also arise when market participants do not react simultaneously to changes in uncertainty levels, for example due to arbitrage limits. Although we do not argue in favor of long-term trend movements in volatility, there are no a priori restrictions as to the instantaneous character of a volatility level shift. Generally, as Hansen (2001) points out "while it may seem unlikely that a structural break could be immediate and might seem more reasonable to allow a structural change to take a period of time to take effect, we most often focus on the simple case of an immediate structural break for simplicity and parsimony".

In order to encompass the possibility of a smooth transition change in mean volatility levels, we consider an alternative model specification around each candidate break date. In particular, we employ the logistic smooth transition specification which, for the limited case of a single break, admits the form

$$\begin{aligned} R_t &= \bar{h}_1 + \bar{h}_2 S_t\left(\frac{t}{T};\gamma,\tau\right) + u_t \\ S_t\left(\frac{t}{T};\gamma,\tau\right) &= \left[1+\exp\left\{-\gamma\left(\frac{t}{T}-\tau\right)\right\}\right]^{-1} \ , \gamma > 0 \end{aligned} \tag{4}$$

Parameter τ determines transition midpoint since $S_{-\infty} = 0$, $S_{\tau T} = \frac{1}{2}$ and $S_{+\infty} = 1$ while parameter γ determines transition speed. For relatively small γ values, function S_t slowly traverses the interval $(0,1)$ implying a log-range mean level that smoothly moves from $\bar{h}_1$ to $\bar{h}_1 + \bar{h}_2$. As γ increases, the transition of S_t from 0 to 1 becomes so rapid that resembles the instantaneous break case.

González and Teräsvirta (2008) provide a procedure (QuickShift) for a sequence of specification tests that can estimate the number of smooth transition break points when we generalize (4) to the multiple break case. However, it becomes numerically difficult to obtain exact maximum likelihood or nonlinear least square estimates from model

$$\begin{aligned} R_t &= \bar{h}_1 + \sum_{j=2}^{m} \bar{h}_j S_t\left(\frac{t}{T};\gamma_j,\tau_j\right) + u_t \\ \gamma_j &> 0 \ , 0 \le \tau_j \le 1 \ , \tau_j < \tau_{j+1} \end{aligned} \tag{5}$$

as the number of breaks m increases. Given that the instantaneous breaks procedure supports the presence of numerous breaks in volatility levels, we will attempt only subsample local estimation of smooth transition models around the break points suggested by the instantaneous breaks procedure. The final fitted values $\hat{R}_t$ proxying the latent log-volatility will be a combination of instant and smooth transition changes.

4.4. Smooth transition results

Table 6 reports the results of the aforementioned experiment that locally employs logistic smooth transition functions as a candidate transition mechanism. The percentage of the identified level shifts that admits a smooth transition representation for each series is shown at the second column. The third and fourth columns display the estimated lag order and the sum of the autoregressive parameters from the break adjusted residual series where the adjustment permits the combination of abrupt and smooth breaks. Figure 5 represents six of the identified smooth level shifts for the AA DJIA component as a visual aid.

A large number of level shifts previously identified as abrupt are now replaced by smooth transitions in all DJIA component and indices. For some of the series almost the half abrupt level shifts are replaced by smooth transitions, while the indices, S&P 500 and DJIA, exhibit a higher number of smooth transitions, 15 and 22 correspondingly, compared to stocks. The median value of replaced abrupt changes by smooth transitions in stocks equals 10. Moreover, it is interesting to note that smooth transitions, in both stocks and indices, last from a few days up to a maximum of one business month (near 22 days) thus the daily data do not support the existence of any long-term trend in volatility. Finally, residual correlation after incorporating smooth transitions is marginally lower, while the autoregressive lag order is reduced in some residual series. Thus, structural changes, abrupt and smooth, account for nearly all of the long memory part present in log-range series and only a short run component in volatility is evident.

[Table 6 here]
[Figure 5 here]

Our conclusion in favor of a short memory process with multiple level shifts, instead of a long memory process, is in line with the results of Varneskov and Perron (2011) despite their different approach. In particular, they combine the level shifts and the long memory by employing a random level shift ARFIMA model in logarithm daily absolute returns.

5. Conclusions

This paper adds on a previous stream of research that questions empirically two data characterizations of volatility, namely long memory and structural breaks. We employ the log-range as a volatility proxy in order to minimize noise effects on a latent volatility component that potentially undergoes level shifts. Our first aim was to evaluate whether the evidence for long memory may be considered to some extent spurious. We find that the latent volatility process can mimic unit root or near unit root behavior as well as behavior that resembles mean reverting long memory processes when level shifts are unaccounted. We then conduct a multiple mean break analysis that produced evidence for multiple significant structural breaks in all examined series that cannot be characterized as occasional. A large number of breaks is not instantaneous, however slowly evolving trends do not exceed one business month in length. When accounting for the level shifts, the evidence in favor of long-memory in the log-range series disappears. The most appealing volatility representation comes from a multiple level shift component plus a short run component that is adequately modelled as a low order $AR(p)$ process with positive correlation.

It will be of interest in future work to attempt univariate or multivariate model specifications where breaks are frequent but considered endogenous. That case, presents potential gains from forecasting the timing and size of breaks and needs to be explored further.

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Results are generated using Ox version 5.10 (see Doornik, 2007), the Arfima package version 1.04 (see Doornik and Ooms, 2003), the R software and EViews 7.

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Table 1. Descriptive Statistics

	mean	median	max	min	stdev	skew	kurt	ACF(1)	ACF(2)
					Indices				
S&P500	-4.421	-4.458	-2.216	-6.036	0.613	0.379	3.249	0.610	0.640
DJIA	-3.951	-4.007	-1.929	-5.757	0.433	0.549	4.853	0.745	0.745
					Dow Jones Industrial Average components				
AA	-3.572	-3.622	-1.271	-4.859	0.531	0.553	3.493	0.587	0.563
AXP	-3.819	-3.858	-1.316	-5.988	0.696	0.380	2.922	0.755	0.733
BA	-3.823	-3.862	-1.949	-5.326	0.495	0.323	3.200	0.542	0.505
BAC	-3.889	-3.990	-0.733	-5.510	0.763	0.798	3.598	0.819	0.778
CAT	-3.762	-3.815	-1.553	-5.090	0.500	0.529	3.381	0.571	0.525
CSCO	-3.721	-3.749	-1.939	-5.786	0.520	0.269	3.180	0.595	0.552
CVX	-3.991	-4.030	-1.768	-5.361	0.483	0.692	4.413	0.576	0.533
DD	-3.896	-3.937	-1.902	-5.571	0.509	0.492	3.446	0.578	0.556
DIS	-3.868	-3.918	-1.973	-5.256	0.540	0.473	3.166	0.632	0.574
GE	-3.947	-4.001	-1.555	-5.729	0.629	0.571	3.355	0.699	0.685
HD	-3.799	-3.848	-1.500	-5.203	0.529	0.465	3.234	0.610	0.578
HPQ	-3.771	-3.814	-1.538	-5.457	0.533	0.348	3.174	0.571	0.519
IBM	-4.106	-4.163	-2.184	-5.834	0.528	0.510	3.352	0.619	0.603
INTC	-3.709	-3.734	-2.142	-5.165	0.497	0.305	2.952	0.586	0.556
JNJ	-4.324	-4.367	-2.078	-5.804	0.537	0.434	3.368	0.618	0.577
JPM	-3.756	-3.818	-1.336	-5.456	0.674	0.487	3.115	0.771	0.733
KFT	-4.114	-4.140	-2.158	-5.635	0.494	0.409	3.399	0.474	0.375
KO	-4.234	-4.278	-2.192	-5.957	0.507	0.501	3.580	0.572	0.544
MCD	-4.020	-4.033	-1.459	-5.770	0.512	0.339	3.394	0.547	0.506
MMM	-4.097	-4.137	-1.302	-5.436	0.476	0.660	4.129	0.512	0.474
MRK	-3.907	-3.951	-1.711	-5.338	0.519	0.535	3.685	0.536	0.492
MSFT	-3.965	-4.005	-2.065	-5.363	0.531	0.389	3.027	0.595	0.556
PFE	-3.956	-3.991	-1.784	-5.353	0.488	0.512	3.650	0.529	0.476
PG	-4.286	-4.331	-0.766	-5.635	0.495	0.795	5.036	0.529	0.481
T	-3.927	-3.990	-1.761	-5.501	0.560	0.463	3.078	0.655	0.611
TRV	-3.911	-3.979	-1.381	-5.447	0.596	0.702	3.687	0.647	0.624
UTX	-4.014	-4.034	-1.852	-5.460	0.514	0.412	3.440	0.565	0.542
VZ	-3.974	-4.017	-1.625	-5.389	0.539	0.450	3.141	0.634	0.590
WMT	-4.107	-4.145	-2.005	-5.688	0.489	0.434	3.541	0.554	0.537
XOM	-4.032	-4.063	-1.811	-5.393	0.492	0.581	4.109	0.595	0.554

Table 2. Semi-parametric and maximum likelihood estimation of the long memory parameter

	GPH			GSP			$ARFIMA\,(0,d,0)$	$ARFIMA\,(1,d,0)$	
Levels	$T^{0.5}$	$T^{0.6}$	$T^{0.7}$	$T^{0.5}$	$T^{0.6}$	$T^{0.7}$	$\hat{d}$	$\hat{d}$	$AR\,(1)$
					Indices				
S&P500	0.620	0.638	0.531	0.628	0.645	0.574	0.333	0.465	-0.311
DJIA	0.642	0.658	0.595	0.643	0.683	0.601	0.401	0.494	-0.243
				Dow Jones Industrial Average components					
AA	0.718	0.469	0.416	0.649	0.518	0.457	0.330	0.411	-0.173
AXP	0.688	0.605	0.454	0.706	0.653	0.530	0.388	0.468	-0.189
BA	0.706	0.582	0.444	0.669	0.559	0.445	0.315	0.386	-0.155
BAC	0.741	0.610	0.544	0.704	0.641	0.545	0.431	0.475	-0.101
CAT	0.616	0.599	0.498	0.596	0.566	0.484	0.330	0.397	-0.147
CSCO	0.692	0.630	0.484	0.629	0.607	0.484	0.326	0.394	-0.152
CVX	0.498	0.596	0.522	0.530	0.571	0.528	0.346	0.423	-0.160
DD	0.669	0.567	0.481	0.664	0.572	0.480	0.324	0.407	-0.190
DIS	0.700	0.642	0.504	0.623	0.643	0.487	0.355	0.404	-0.108
GE	0.709	0.627	0.495	0.673	0.623	0.533	0.365	0.459	-0.219
HD	0.592	0.610	0.463	0.660	0.633	0.493	0.339	0.415	-0.175
HPQ	0.599	0.599	0.443	0.599	0.603	0.476	0.337	0.397	-0.127
IBM	0.651	0.582	0.588	0.624	0.584	0.559	0.347	0.444	-0.217
INTC	0.714	0.608	0.488	0.682	0.633	0.493	0.327	0.402	-0.178
JNJ	0.658	0.589	0.508	0.612	0.572	0.498	0.355	0.427	-0.155
JPM	0.740	0.592	0.501	0.708	0.648	0.544	0.406	0.466	-0.138
KFT	0.462	0.555	0.437	0.467	0.530	0.428	0.304	0.336	-0.067
KO	0.619	0.639	0.486	0.644	0.618	0.492	0.327	0.403	-0.167
MCD	0.617	0.513	0.435	0.724	0.582	0.475	0.318	0.385	-0.151
MMM	0.624	0.547	0.514	0.580	0.517	0.452	0.311	0.382	-0.150
MRK	0.580	0.515	0.413	0.554	0.557	0.453	0.323	0.391	-0.143
MSFT	0.641	0.679	0.483	0.600	0.613	0.497	0.330	0.401	-0.168
PFE	0.627	0.550	0.399	0.618	0.541	0.446	0.321	0.381	-0.123
PG	0.593	0.490	0.452	0.580	0.511	0.481	0.322	0.390	-0.143
T	0.697	0.622	0.492	0.654	0.632	0.530	0.356	0.421	-0.149
TRV	0.691	0.605	0.546	0.636	0.613	0.536	0.360	0.448	-0.193
UTX	0.660	0.646	0.507	0.625	0.608	0.501	0.327	0.413	-0.190
VZ	0.667	0.611	0.462	0.653	0.624	0.504	0.352	0.417	-0.147
WMT	0.570	0.602	0.511	0.614	0.613	0.513	0.322	0.407	-0.187
XOM	0.559	0.591	0.535	0.563	0.584	0.503	0.359	0.436	-0.156

Table 3. Tests of spurious long memory

	Tests					Critical Values	
	$W\,(\varepsilon=0.02)$	W "prewhitening"	$t_d\,(1/2,1;4/5,1)$	$\sup -t_d$	$mean-t_d$	$\sup -t_d$	$mean-t_d$
Indices							
S&P500	1.23*	1.27***	2.03***	2.60**	2.04**	2.17,2.61,3.36	1.22,1.54,2.21
DJIA	1.26**	3.43***	2.05***	2.97***	2.33***	0.80,1.19,1.84	-0.06,0.23,0.85
Dow Jones Industrial Average Components							
AA	2.10***	2.13***	3.86***	3.05**	2.30***	2.17,2.59,3.38	1.16,1.54,2.16
AXP	2.55***	2.40***	3.32***	3.23**	2.40**	2.62,3.11,3.66	1.73,2.02,2.66
BA	2.21***	3.40***	3.75***	1.74	1.04	2.17,2.56,3.37	1.25,1.64,2.33
BAC	2.34***	2.95***	3.45***	3.12	2.59**	3.14,3.58,4.60	2.18,2.53,3.22
CAT	1.67***	4.13***	2.52***	2.35**	1.88***	1.24,1.66,2.46	0.44,0.74,1.43
CSCO	2.03***	2.08***	3.44***	2.36**	1.49*	2.17,2.58,3.39	1.18,1.54,2.20
CVX	1.05	3.67***	1.02	1.30**	0.96**	0.82,1.34,2.29	-0.08,0.29,1.09
DD	1.72***	1.86***	3.28***	2.31**	1.54*	2.17,2.65,3.28	1.17,1.56,2.11
DIS	2.06***	4.44***	3.50***	1.67**	1.23**	1.32,1.77,2.54	0.51,0.78,1.48
GE	1.96***	2.04***	3.47***	3.64***	2.88***	2.17,2.64,3.35	1.21,1.57,2.18
HD	2.31***	3.99***	2.32***	2.36**	1.84***	1.57,2.11,2.80	0.75,1.09,1.76
HPQ	1.69***	1.75***	2.77***	1.33	0.63	2.15,2.63,3.33	1.19,1.53,2.22
IBM	1.05	1.24*	2.38***	1.65	0.98	2.17,2.61,3.32	1.22,1.57,2.18
INTC	2.68***	3.96***	3.67***	2.49***	1.76***	1.21,1.55,2.34	0.34,0.64,1.30
JNJ	1.54***	3.58***	2.85***	1.21**	0.57*	1.04,1.58,2.25	0.24,0.64,1.26
JPM	2.28***	3.12***	3.47***	3.53**	2.44**	2.90,3.30,4.15	2.07,2.34,3.00
KFT	1.05	4.32***	1.49	2.20***	1.47***	1.03,1.45,2.13	0.19,0.50,1.14
KO	1.83***	3.75***	2.75***	2.47**	1.39**	1.63,1.96,2.67	0.79,1.12,1.58
MCD	1.92***	3.55***	3.25***	0.26	-0.41	1.84,2.29,3.14	0.94,1.34,1.99
MMM	1.51**	1.63***	2.92***	2.04	1.24*	2.15,2.63,3.38	1.19,1.54,2.22
MRK	1.61***	3.84***	2.50***	2.81**	2.22***	1.84,2.23,2.94	0.96,1.31,1.82
MSFT	1.78***	4.10***	2.99***	2.54***	1.81***	1.20,1.58,2.40	0.37,0.67,1.39
PFE	1.79***	2.23***	3.17***	2.50**	1.66**	2.08,2.44,3.16	1.12,1.58,2.13
PG	1.35**	3.69***	2.46***	3.09***	1.88***	1.37,1.80,2.44	0.54,0.89,1.45
T	1.94***	2.94***	3.68***	1.82	1.23*	2.02,2.44,3.39	1.15,1.53,2.18
TRV	1.57***	3.92***	3.22***	1.43*	0.88**	1.22,1.59,2.56	0.36,0.72,1.39
UTX	1.61***	3.33***	3.01***	1.59*	0.96**	1.21,1.63,2.33	0.34,0.68,1.49
VZ	1.96***	3.64***	3.24***	1.68*	0.70*	1.33,1.78,2.58	0.57,0.91,1.56
WMT	1.70***	3.40***	2.45***	1.74**	1.07**	1.34,1.71,2.62	0.54,0.79,1.45
XOM	1.09	4.08**	1.63	1.45**	0.78**	0.91,1.24,2.05	0.02,0.41,0.99

Note: *, ** and *** denote significance at 10%, 5% and 1% level. The simulated critical values correspond to 10%, 5% and 1% significance levels and are based on 1000 replications.

Table 4. Quasi-maximum Likelihood Estimates of One and Two Factor stochastic volatility models and residuals diagnostics

	One-Factor model					Two-Factor model						
	levels			residuals		levels					residuals	
	$\hat{\phi}$	$\bar{h}$	$var(\eta)$	$ACF(1)$	$ACF(2)$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\bar{h}$	$var(\eta_1)$	$var(\eta_2)$	$ACF(1)$	$ACF(2)$
Indices												
S&P500	0.970	-4.205	0.015	-0.292	-0.072	0.986	-0.235	-4.852	0.007	0.045	-0.027	-0.102
DJIA	0.992	-4.383	0.002	0.010	0.042							
Dow Jones Industrial Average components												
AA	0.953	-4.002	0.017	-0.198	-0.109	0.994	0.527	-4.017	0.002	0.040	-0.235	-0.037
AXP	0.976	-4.251	0.018	-0.190	-0.133	0.995	0.467	-4.247	0.003	0.043	-0.224	-0.061
BA	0.956	-4.256	0.012	-0.155	-0.129	0.993	0.462	-4.262	0.002	0.036	-0.191	-0.061
BAC	0.978	-4.318	0.021	-0.132	-0.193	0.997	0.674	-4.317	0.002	0.038	-0.192	-0.156
CAT	0.965	-4.194	0.010	-0.127	-0.147	0.988	0.406	-4.192	0.003	0.031	-0.159	-0.087
CSCO	0.950	-4.162	0.017	-0.184	-0.138	0.991	0.350	-4.141	0.003	0.045	-0.174	-0.044
CVX	0.968	-4.418	0.009	-0.113	-0.134	0.980	0.374	-4.426	0.005	0.018	-0.137	-0.096
DD	0.972	-4.327	0.009	-0.144	-0.117	0.991	0.421	-4.324	0.002	0.029	-0.183	-0.058
DIS	0.961	-4.304	0.015	-0.121	-0.157	0.993	0.490	-4.299	0.002	0.037	-0.163	-0.089
GE	0.974	-4.384	0.015	-0.202	-0.122	0.991	0.310	-4.372	0.005	0.039	-0.192	-0.044
HD	0.975	-4.231	0.009	-0.122	-0.128	0.991	0.361	-4.240	0.003	0.031	-0.156	-0.067
HPQ	0.927	-4.208	0.026	-0.217	-0.148	0.989	0.391	-4.197	0.003	0.049	-0.189	-0.056
IBM	0.975	-4.545	0.009	-0.160	-0.119	0.983	0.111	-4.544	0.006	0.021	-0.123	-0.075
INTC	0.981	-4.155	0.005	-0.086	-0.107	0.992	0.402	-4.135	0.002	0.023	-0.141	-0.066
JNJ	0.960	-4.760	0.015	-0.161	-0.147	0.986	0.424	-4.751	0.004	0.033	-0.182	-0.084
JPM	0.978	-4.192	0.015	-0.124	-0.163	0.996	0.641	-4.182	0.002	0.032	-0.193	-0.129
KFT	0.853	-4.547	0.040	-0.254	-0.213	0.980	0.292	-4.545	0.004	0.058	-0.143	-0.119
KO	0.964	-4.673	0.011	-0.136	-0.099	0.992	0.440	-4.664	0.002	0.034	-0.182	-0.032
MCD	0.953	-4.454	0.015	-0.171	-0.143	0.991	0.327	-4.466	0.002	0.044	-0.161	-0.05
MMM	0.951	-4.528	0.012	-0.155	-0.128	0.988	0.429	-4.528	0.002	0.034	-0.179	-0.055
MRK	0.918	-4.339	0.027	-0.244	-0.148	0.986	0.356	-4.349	0.003	0.051	-0.194	-0.053
MSFT	0.971	-4.402	0.010	-0.143	-0.162	0.989	0.240	-4.393	0.003	0.035	-0.131	-0.088
PFE	0.933	-4.388	0.018	-0.178	-0.142	0.990	0.491	-4.395	0.002	0.039	-0.194	-0.069
PG	0.942	-4.719	0.016	-0.180	-0.152	0.984	0.352	-4.719	0.004	0.039	-0.17	-0.068
T	0.972	-4.367	0.012	-0.126	-0.154	0.993	0.469	-4.368	0.003	0.033	-0.175	-0.094
TRV	0.958	-4.346	0.021	-0.224	-0.126	0.988	0.278	-4.358	0.005	0.048	-0.183	-0.036
UTX	0.965	-4.450	0.011	-0.164	-0.118	0.988	0.271	-4.444	0.003	0.036	-0.154	-0.04
VZ	0.970	-4.410	0.011	-0.122	-0.148	0.992	0.482	-4.412	0.003	0.031	-0.172	-0.094
WMT	0.972	-4.542	0.008	-0.117	-0.079	0.990	0.386	-4.542	0.002	0.028	-0.163	-0.021
XOM	0.962	-4.462	0.011	-0.122	-0.117	0.983	0.558	-4.466	0.004	0.020	-0.167	-0.087

Table 5. Results of abrupt level shifts detection.

	# of breaks	p	$sum\ AR\,(p)$	$ARFIMA(0,d,0)$ $\hat{d}$	$ARFIMA(1,d,0)$ $\hat{d}$	$ARFIMA(1,d,0)$ $AR(1)$
				Indices		
S&P500	32	4	0.278	0.081	-0.896	0.977
DJIA	44	2	0.219	0.108	-0.838	0.947
			Dow Jones Industrial Average components			
AA	21	2	0.256	0.147	-0.799	0.952
AXP	34	3	0.278	0.141	-0.795	0.940
BA	17	3	0.284	0.146	-0.814	0.964
BAC	30	3	0.360	0.219	-0.712	0.945
CAT	24	3	0.257	0.136	-0.812	0.954
CSCO	24	2	0.174	0.101	-0.836	0.950
CVX	34	3	0.204	0.103	-0.843	0.954
DD	28	4	0.264	0.111	-0.855	0.968
DIS	19	3	0.316	0.169	0.091	0.111
GE	23	3	0.304	0.139	-0.809	0.951
HD	21	3	0.226	0.116	-0.844	0.969
HPQ	23	2	0.255	0.160	-0.797	0.965
IBM	25	4	0.348	0.144	-0.813	0.959
INTC	26	2	0.145	0.088	0.000	0.000
JNJ	32	3	0.272	0.144	-0.798	0.947
JPM	41	3	0.258	0.151	-0.777	0.935
KFT	26	2	0.215	0.116	0.000	0.170
KO	23	2	0.216	0.124	0.104	0.000
MCD	26	2	0.145	0.085	0.000	0.107
MMM	19	7	0.357	0.151	0.152	0.000
MRK	17	3	0.313	0.167	-0.794	0.969
MSFT	22	4	0.327	0.146	-0.813	0.962
PFE	23	2	0.222	0.134	-0.792	0.934
PG	30	2	0.158	0.094	-0.824	0.927
T	31	2	0.176	0.108	-0.813	0.937
TRV	20	5	0.394	0.176	-0.800	0.979
UTX	23	4	0.285	0.126	-0.835	0.968
VZ	21	4	0.324	0.162	-0.801	0.969
WMT	28	2	0.186	0.100	0.087	0.000
XOM	32	3	0.278	0.150	-0.806	0.960

Column "# of breaks" reports estimated number of breaks, Column "p" reports estimated residual lag order using the AIC criterion with maximum lag set at 12. Column "sum AR(p)" reports the sum of statistically significant autoregressive coefficients in the corresponding residual AR(p) model. Column "ARFIMA(0,d,0)" reports statistically significant maximum likelihood estimates of d for the residual series. Columns "ARFIMA(1,d,0)" report statistically significant maximum likelihood estimates of d and of the AR(1) coefficient for the residual series.

Table 6. Results of smooth transitions

	% smooth transitions	p	$sum\ AR\,(p)$
	Indices		
S&P500	46.88	3	0.241
DJIA	50.00	2	0.189
	Dow Jones Industrial components		
AA	47.62	2	0.250
AXP	41.18	3	0.274
BA	41.18	3	0.280
BAC	26.67	3	0.355
CAT	50.00	3	0.253
CSCO	45.83	2	0.197
CVX	35.29	3	0.193
DD	35.71	4	0.220
DIS	26.32	3	0.304
GE	26.09	3	0.298
HD	38.10	3	0.215
HPQ	47.83	2	0.245
IBM	52.00	4	0.345
INTC	34.62	2	0.141
JNJ	46.88	3	0.265
JPM	26.83	2	0.221
KFT	38.46	2	0.218
KO	39.13	2	0.215
MCD	42.31	2	0.146
MMM	36.84	6	0.292
MRK	35.29	3	0.311
MSFT	36.36	4	0.314
PFE	26.09	2	0.218
PG	43.33	1	0.121
T	38.71	2	0.179
TRV	30.00	5	0.388
UTX	39.13	4	0.271
VZ	23.81	4	0.320
WMT	39.29	2	0.180
XOM	37.50	2	0.230

Note: Column "% smooth transitions" expresses the proportion of smooth transition shifts relative to the total number of level shifts. Column "p" reports estimated residual lag order. Column "sum AR(p)" reports the sum of statistically significant autoregressive coefficients in the corresponding residual AR(p) model.

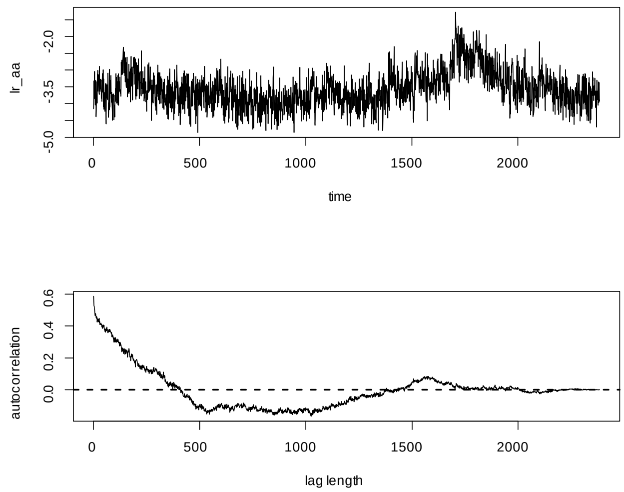

Figure 1: Log-range series (top panel) and autocorrelation function (bottom panel) of AA stock.

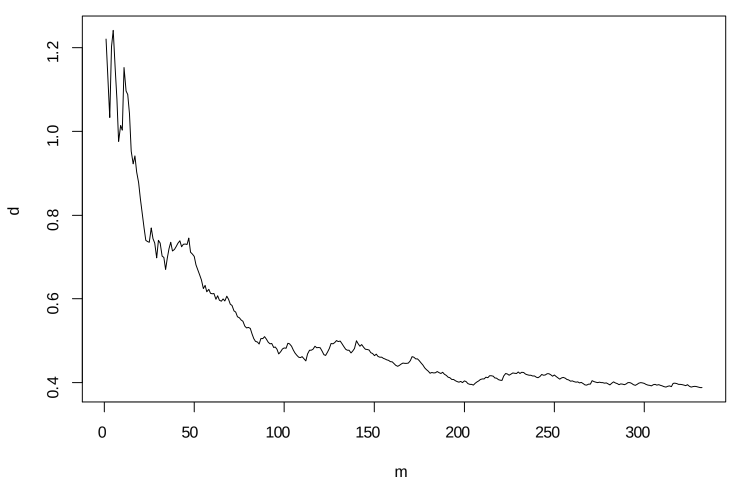

Figure 2: GPH estimates across different number of frequency ordinates (m) for AA log-range series.

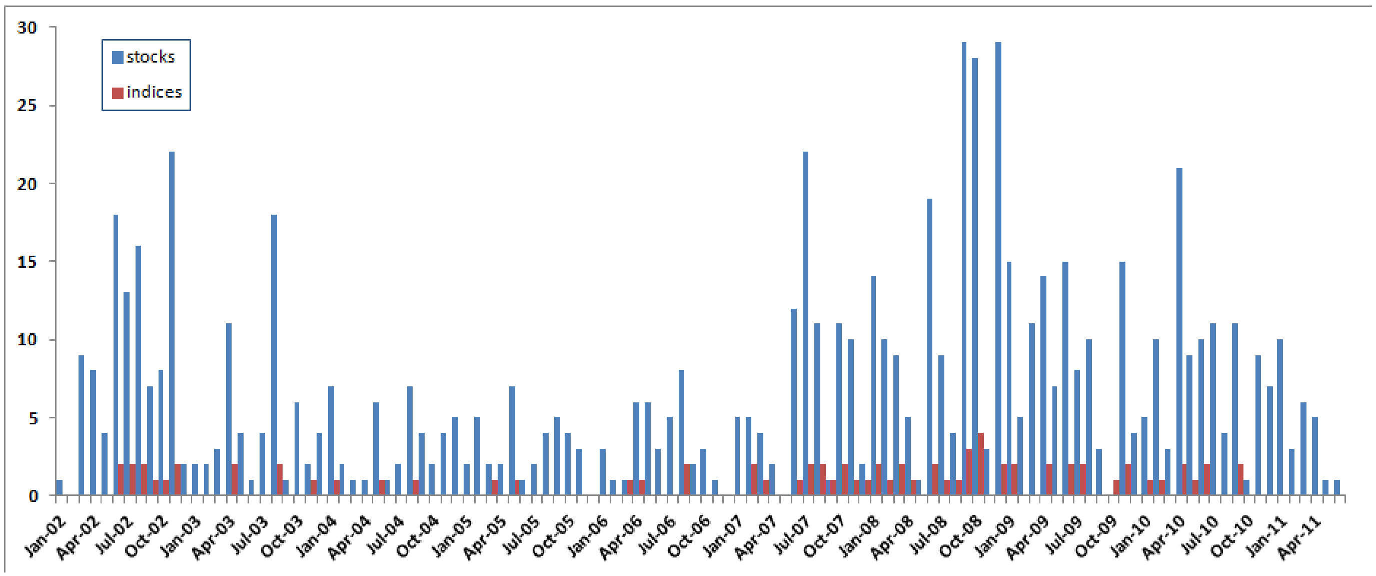

Figure 3: Identified structural breaks per month.

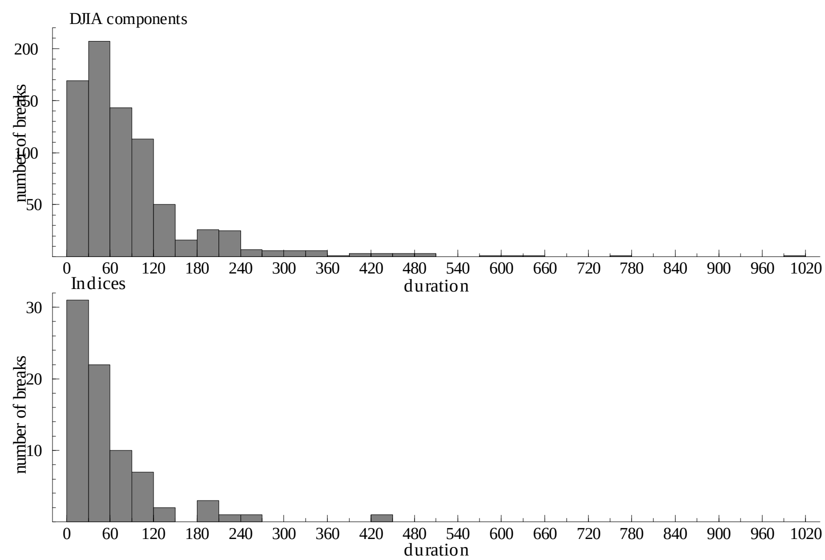

Figure 4: Distribution of breaks duration

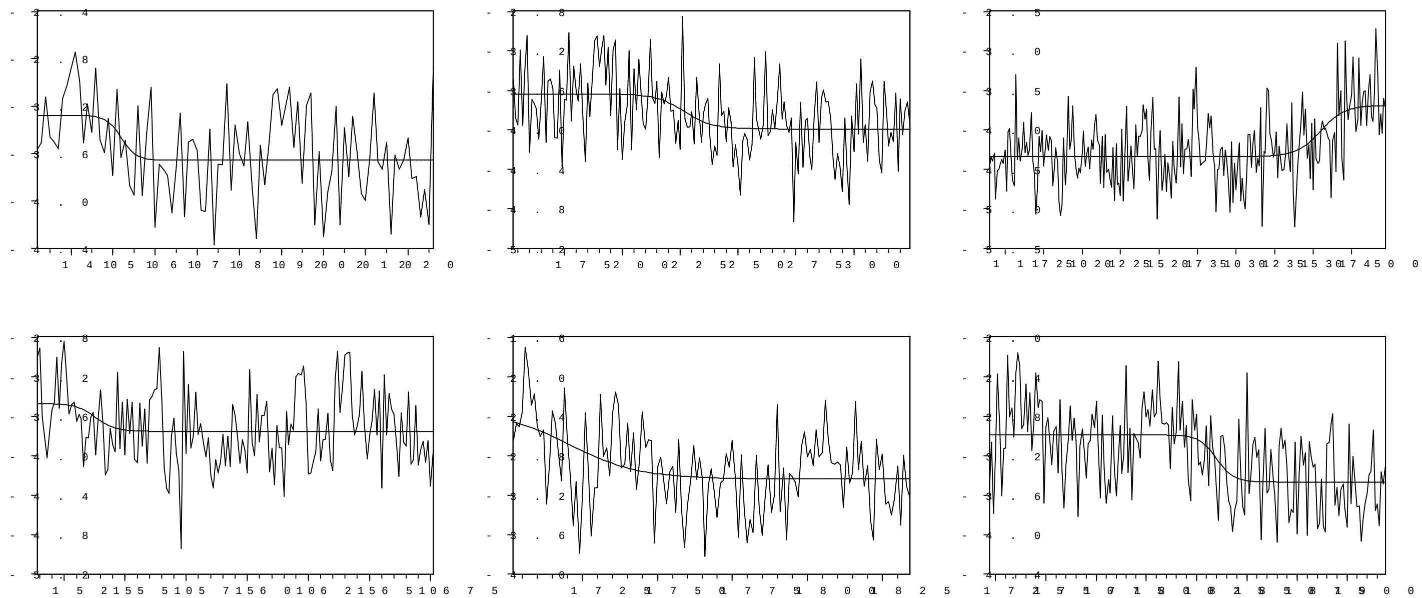

Figure 5: Smooth level shifts for AA log-range series.