Switching from Patents to an Intertemporal Bounty in a Non-Scale Growth Model: Transitional Dynamics and Welfare Evaluation

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Abstract

Using a non-scale R&D-based growth model, an intertemporal bounty regime is shown to be a Pareto-optimal alternative to the patent regime. Under this alternative, any innovative capital good is *freely* licensed and the innovator is rewarded with a *perpetual bounty claim* to an infinite stream of future bounties, payable on the marginal-cost sales of the bountiable capital good at a rate determined by the government. With bounties paid by lump-sum taxes, an active asset market for bounty claims determines the equilibrium asset price and delivers a signal to R&D. Calibrating the model to the US, this paper solves numerically a four-dimensional nonlinear dynamic system with two state variables (capital & knowledge stocks). The Pareto optimal bounty rate is computed and the regime switch from patents to bounties generates a welfare gain equivalent to an increase of about 10 to 20 percent in per capita consumption after taking transitional dynamics into account.

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1 Introduction

Profit-oriented R&D (research & development) is a critical driver of technological innovation and long-run economic growth. Innovative technologies are often non-rival and have public goods features. Without a systemic arrangement to prevent free riding on innovative effort, private agents would not engage in R&D.[1] Patents are designed to promote innovation by creating transient monopoly power for the innovator to profit under the markup pricing of patented goods. A patent system must seek to balance short-run static (monopoly) distortion against long-run dynamic innovation (Nordhaus 1969).

Optimizing the patent system has been a focus in the long literature on the economics of patents. Many studies seek to improve efficiency by fine-tuning a patent's *length*, or *scope*, or both; see, for instance, Judd (1985), Gilbert Shapino (1990), Klemperer (1990), Matutes et al. (1996), and O'Donoghue and Zweimüller (2004).[2] Increasing the *length* of patents extends the *duration* of monopoly power over patented goods, whereas broadening the *scope* of patents raises the *degree* of the monopoly power. True, how to fix a patent's length and scope is an important policy question. But patents designed to function by introducing monopolies are doomed to be suboptimal, no matter how one fine-tunes the length and scope. In addition to the well-known static deadweight losses, the patent system provides insufficient incentives for innovative effort. This is because patented firms merely reap a fraction of the entire social surplus that would otherwise result under marginal cost pricing.[3]

Nevertheless, the patent system is inflexible in practice to cope with externalities such as the knowledge spillovers or research congestion. This involves statutory changes in the length or scope of patents. Even if these statutory changes can be made precisely to adjust innovating incentives to internalize these externalities, the resulting monopoly distortion may get worse, albeit not always. For instance, to achieve the goal of better exploiting a knowledge spillovers externality, it is desirable to raise innovative incentives by extending the patent length. Yet, this dynamic efficiency gain must go against a static efficiency loss

[1]Seminal papers on R&D-based endogenous growth models all presume a patent system issuing infinitely lived patents to incentivize innovative activities; see Romer (1990), Segerstrom et al. (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992).

[2]A patent is typically in three dimensions: length, breadth, and height. A patent's scope refers to *breadth* and *height*. The breadth (or " lagging breadth") defines the range of a patent's claims against imitation, whereas the height (or "leading breadth") confers protection against future improvements too close to the patented good; see van Dijk (1992) and O'Donoghue et al (1998).

[3]See Kremer (1998) and Grinols and Lin (2011) for discussion of problems with patents.

from the longer duration of monopolies. Optimizing the "length-scope" mix does not lead to Pareto efficiency whatsoever.

In this paper, I analyze the *intertemporal bounty* (IB) system as an alternative to the patent system that prevails in a dynamic macroeconomy. The IB system, proposed by Grinols and Henderson (2007), was recently examined by Grinols and Lin (2011) using a Pharmaceutical sectoral equilibrium model. The present paper instead precisely formulates an IB system in a macro-dynamic general equilibrium context. In this context, the IB system requires three core institutional arrangements: (i) any bountiable technical innovation (design) must be *freely* licensed so as to force markets to be perfectively competitive (marginal cost pricing); (ii) For any bountiable innovation, the innovator is rewarded with a *perpetual bounty claim* to an infinite stream of future bounties, payable on the marginal-cost sales of the bountiable capital good at a rate determined by the government; and (iii) the dynamic bounty flow are paid by lump-sum taxes.[4] Under the IB system, bounty claims are valuable and tradeable. The price of bounty claims reflects the discounted present value of the ongoing bounty flow. If the price is above (below) the cost of innovating a new good, more (less) resources flow to R&D. In equilibrium, they both are equalized. The asset market for bounty claims therefore delivers a signal to R&D.

The IB system is actually an implementation of the idea of the "Hotelling proposal" discussed more than seven decades ago (Hotelling 1939):[5]

> "... in a specified sense maximum welfare requires that the quantity of each good consumed or produced by an individual shall be that corresponding to all sales being at marginal cost. This proposition has revolutionary implications, for example in electric-power and railway economics, in showing that society would do well to cut rates drastically and replace the revenue thus lost by subsidies derived largely from income and inheritance taxes and the site value of land."

From the Hotelling proposal, *non-convex* firms (e.g., electric-power or railway businesses),

[4]The innovator does not have to keep the bounty claim forever, for there is an active market for bounty claims. Whoever holds the bounty claim receives the bounty over time (annually), and wherever competitive producers are, they adopts the same freely-licensed design to contribute to the overall market sales of the *bountiable good*.

[5]To the best of my knowledge, the present paper is the first to formulate a decentralized market mechanism to implement the Hotelling proposal using a dynamic macroeconomic growth model. Pioneered by Guesnerie (1975), there has been an active "marginal cost pricing equilibrium" literature on Hotelling's proposal examined in the static Arrow-Debreu microeconomic framework. See Quinzii (1992) for a survey of this literature.

which operate under increasing returns, should be regulated to charge marginal-cost rates while society should pay taxes to fully subsidize these firms' fixed costs, in order to maximize social welfare. Similarly but not equally, the IB system is designed to force the otherwise *non-convex* firms to transform into constant-return *convex* firms by making innovative technologies freely available in the public domain, while collecting lump-sum taxes to pay bounties in support of an active bounty-claim market whereby to direct resources into R&D activities. The IB system can thus remove monopoly distortions while keeping technological innovation viable over time.[6] This is evidenced in the paper by simulating a R&D-based growth model calibrated to advanced economies such as the United States. Besides, the government can adjust the bounty rate to internalize the externalities such as knowledge spillovers or research congestion. In so doing, a decentralized market mechanism can duplicate the model's social planner optimum in the long run. That is, an IB system with the right bounty rate can be Pareto optimal in the long run, whereas the patent system is always suboptimal.

For the analysis, a patent system with a finite patent length is firstly embedded in a non-scale endogenous growth model with population growing exogenously at a constant rate and private agents devoting foregone consumption to R&D and physical capital accumulation.[7] Patents incentivize R&D to innovate new varieties of patentable capital (or durable) goods. Initially, the modeled economy is in the steady state along a balanced-growth path, where the stock of "knowledge" – the variety of capital goods – keeps growing at a constant innovation rate. At a point in time, the patent regime is switched to an IB alternative according to the aforementioned institutional arrangements. The regime switch forces the economy to transition over time to a bounty-regime balanced growth path. The dynamics in transition is governed by a nonlinear system of four first-order ordinary differential equations (ODE) in two state variables (capital stock, knowledge stock) and two control-like variables (consumption, bounty-claim price). This presents a nonlinear boundary value problem (BVP) and is solved numerically with a Python package that wraps a Fortran BVP_Solver.

Under a set of plausible benchmark parameters, the computed transition paths indicate

[6]The paper presumes that lump-sum taxes are available. If such taxes are unavailable, the IB system must turn to taxes on factor incomes and may create a distortion.

[7]The model of the paper is a modification of Romer (1990) in three dimensions. First, patents are finitely lived. Second, as in Rivera-Batiz and (1991), research input is from forgone consumption rather than from labor (human capital). Third, as in Jones (1995), the R&D (innovation) function displays diminishing marginal returns to the research input, thereby removing the empirically implausible scale effects.

that switching to the Pareto optimal IB regime enables the calibrated U.S. economy to transition to one much more intensive in both capital and knowledge than under the initial patent regime (20 years patent length). For instance, on the bounty-regime balanced growth path, the stock of knowledge is *seventeen times* as much as on the initial patent-regime balanced growth path, the stock of capital is *three times* as much, and per capita consumption is *two times* as much. Although households experience temporary consumption sacrifices to accommodate faster capital and knowledge accumulation in transition, the entire cumulative consumption gain after taking into account the short-run losses remains about 10 to 20 percent more than under the patent regime. The market mechanism responsible for these results and the robustness checks are analyzed in the paper. The central message is that an intertemporal bounty regime can be a Pareto-optimal alternative to the world's long-standing patent system.

Relating to the patent reform literature, the intertemporal bounty represents a tax-financed public reward system designed to sustain both technological innovation and perfectly competitive provision of innovative goods. Detailed discussions of other reward designs can be seen in Kremer (1998), Hopenhayn el al (2006), and Grinols and Henderson (2007). Of the alternative reward designs, the "patent buyout" system is close to the intertemporal bounty and can date back to Polanvyi (1944). The basic idea of this system is for the government to purchase patents at a price to reward the innovator once and for all, while placing these purchased patents in the public domain. In practice, the price of patents is difficult to estimate, though Kremer proposes an auction mechanism to determine the patent buyout price. Under the IB system, however, the government has no need to buy out bounty claims and is flexible to adjust the bounty rate to determine the bounty flow payable on the ex post market sales of bountiable goods under marginal cost pricing.

The rest of the paper is organized as follows. Section 2 lays out a patent-regime R&D-based growth model featuring multiple distortions from patents, knowledge spillovers, and research congestion. Section 3 evaluates the patent regime against the model's social planner optimum. Section 4 introduces an intertemporal bounty regime, its dynamic system, and the Pareto-optimal bounty rate. Section 5 calibrates the model to the initial patent-regime steady state that largely mimics an innovating economy like the United States and solves the nonlinear boundary value problem for a four-dimensional ODE system. Transition paths, welfare gains, and the robustness checks are computed and analyzed. Section 6 concludes.

2 The Model for Patent Regime

In this paper I formulated a patent regime for a closed economy using a Romer-style variety-expansion model of endogenous growth. This economy is endowed with labor L that grows exogenously. Labor and physical capital are employed to produce final output Y, which can be used for consumption or for investment in either durable (capital) goods X or R&D (research and development). Investing in durable goods X contributes to the accumulation of physical capital K, while investing in R&D is to innovate technical designs for new types of durable goods. Technical designs are indexed by i in a closed interval $[0,V] \subset \mathbb{R}^+$. For each design, the innovator receives a patent for a finite term denoted by $T \in (0,\infty)$. Patents are tradable and perfectly enforced. Within the finite patent term, every patented firm has the exclusive power to produce and sell a specific type of durable good $X[i]$. Yet, once a patent expires, the granted monopoly power vanishes and the out-of-patent firm reduces to a perfectly competitive firm immediately. All durables are horizontally differentiated. Variable V represents not only the number of patents that have been issued, but also a measure of the variety of available durable goods. Like K measuring the stock of capital, V is a proxy of the knowledge stock in the model. The growth of V allows for a finer division of capital and leads to an increase in the level of TFP (total factor productivity) for the economy.

The model described below is for an economy that implements a patent system. It has four sectors referred to as household, final good, durable (capital) good, and R&D.

2.1 Household

There is a continuum of identical households with measure of one in the economy. Each household has L identical workers, which also represent the economy's population of labor. The population of labor is normalized to one at time $t=0$ and is assumed to grow at an exogenous rate, n. So, the size of a typical household at time t is given by $L[t]=e^{nt}$. Define $C[t]$ as the rate of the economy's aggregate consumption of final goods and $c[t] \equiv \frac{C[t]}{L[t]}$ as the rate of per capita (or per worker) consumption at a point in time. For each household, its family lifetime utility $U[t]$ at time t is given by

$$U[t] = \int_t^\infty \left(\frac{c[s-t]^{1-\gamma}-1}{1-\gamma}\right) e^{-(\rho-n)(s-t)} ds, \ \rho > 0, \ \gamma > 0 \tag{1}$$

where ρ and γ are two preference parameters with ρ measuring the rate of time preference and $\frac{1}{\gamma}$ the elasticity of intertemporal substitution. All labor services are used for final goods production. Each household owns assets including physical capital and patents. Its per capita asset stock is $a[t]$ at a point in time and earns the net capital income of $r[t]a[t]$ at the market rate of interest. From each household, every worker earns wage income $w[t]$ at a point in time from a perfectly competitive labor market. Hence, a typical household's flow budget constraint is given by

$$\dot{a}[t] \equiv \frac{da[t]}{dt} = (r[t]-n)a[t] + w[t] - c[t] \tag{2}$$

Households take as given all market prices such as $w[t]$ and $r[t]$. From a household's intertemporal optimization, the familiar Euler condition is

$$\hat{c}[t] \equiv \frac{\dot{c}[t]}{c[t]} = \frac{r[t]-\rho}{\gamma}. \tag{3}$$

From (1) and (3), for the family lifetime utility to be bounded in the steady state, we invoke the parameter constraint that $\rho > n + (1-\gamma)\hat{c}_0$, where $\hat{c}_0$ represents a constant steady-state growth rate of per capita consumption under a patent regime. Throughout the paper, an overdot defines a time derivative of the associated variable and the symbol ^ indicates its instantaneous growth rate. Henceforth, the time variable t will be suppressed in most equations unless otherwise necessary.

2.2 Final goods

We chose final good Y as the numeraire. In the final goods sector there are many identical (competitive) producers and the aggregate production function is given by

$$Y = L^{1-\alpha}\left(\int_0^V X[i]^{\sigma\alpha}di\right)^{1/\sigma}, \ 0 < \sigma < \frac{1}{\alpha}, \tag{4}$$

where Y is the economy's aggregate final output or real GDP produced with labor L supplied from households and V types of capital goods $X[i]$ for $i \in [0, V]$ that have been invented. As the functional form implies, production of Y exhibits constant returns in labor and capital goods and these capital goods are imperfectly substitutable. Each capital good, if patented, is priced at a markup equal to $\eta \equiv \frac{1}{\sigma\alpha} > 1$.[8] Profit maximization yields the following first-order conditions:

$$w = (1-\alpha)Y/L \tag{5a}$$

$$p[i] = \alpha L^{1-\alpha}\left(\int_0^V X[i]^{\sigma\alpha}di\right)^{(1-\sigma)/\sigma} X[i]^{\sigma\alpha-1} \tag{5b}$$

where $p[i]$ is the price of capital good i, the fraction of $1-\alpha$ in (5a) represents the labor share, and (5b) is an inverse demand function for any capital good. This demand function implies that the price elasticity of demand for each durable is equal to $\varepsilon = \frac{1}{1-\sigma\alpha} > 1$. The markup for any patented durable is therefore given by $\eta = \frac{\varepsilon}{\varepsilon-1} = \frac{1}{\sigma\alpha} > 1$. As for out-of-patent (or unpatented) durables, they are priced at marginal cost.

Final output Y is allocated for consumption C, capital investment I_K, and R&D investment I_V. As Y represents the economy's real GDP, the national income identity requires

$$Y = C + S = C + I_K + I_V \tag{6}$$

where S is the flow of aggregate saving (or forgone consumption) and must be equal to $I_K + I_V$ in equilibrium. Define $s \equiv \frac{S}{Y} = (Y-C)/Y$ as the saving rate, $s_K \equiv \frac{I_K}{Y}$ as the the rate of capital investment, and $s_V \equiv \frac{I_V}{Y}$ as the rate of R&D investment (or called the economy's R&D intensity). Then the flow equilibrium in saving and investment is given by

$$s = 1 - \frac{C}{Y} = s_K + s_V \tag{7}$$

Investment in capital goods (I_K) accumulates the stock of capital K, while investment in R&D (I_V) creates new varieties of capital goods and raises the stock of knowledge V. The two stocks of the economy evolve over time according to

$$\dot{K} \equiv \frac{dK}{dt} = I_K - \delta K = s_K Y - \delta K \tag{8a}$$

$$\dot{V} \equiv \frac{dV}{dt} = \xi I_V = \xi s_V Y \tag{8b}$$

[8] If parameter σ were set equal to one, production function (4) would reduce to Romer's (1990). As such, durables would become neither substitutable nor complementary and the markup $\eta \equiv \frac{1}{\sigma\alpha}$ would be $\frac{1}{\alpha}$. In this paper, σ is less than $1/\alpha$ so that the markup η is allowed to deviate from the inverse of the capital share (α), as in Jones and Williams (2000) and Eicher and Turnovsky (2001).

where δ is the rate of capital depreciation and ξ is an endogenous measure of research productivity. In contrast to physical capital, the stock of knowledge does not depreciate over time, as in most endogenous growth models, whereas Eicher and Turnovsky (2001) introduce knowledge depreciation. The measure of research productivity in (8b) is endogenously determined in terms of $\xi = \mu V^{\phi} I_V^{\lambda-1}$, with $\mu > 0$, $0 < \phi < \bar{\phi} < 1$, and $0 < \lambda < 1$. The parameter of μ is an exogenous technology term. The parameter of ϕ is positive to capture the phenomenon of knowledge spillovers, but ϕ must be less than an upper bound $\bar{\phi} < 1$ to ensure a bounded long-run innovation rate, denoted by $\lim_{t\to\infty} \hat{V}[t] < \infty$, where $\hat{V} \equiv \frac{\dot{V}}{V}$.[9] The parameter of λ is positive but less than one, meaning that research input I_V displays social diminishing marginal return on research productivity. This captures the effect of research congestion, which may result from too many research firms engaged in similar projects in patent races.

For simplicity, each unit of foregone consumption can presumably produce a unit of each type of capital goods, so that the stock of capital is given by $K = \int_0^V X[i]di$. Foregone consumption is partially invested in R&D, thereby enabling the economy to introduce new types of durables over time. These new capital goods are patentable as soon as they are invented. Active R&D therefore keeps changing the composition of patented and unpatented durable goods under a patent system, as will be analyzed later. What follows describes individual firms' pricing behavior in the durable goods sector.

2.3 Durable (capital) goods

Firms of capital goods may engage in monopolistic competition or perfect competition, depending on whether or not they are under patent protection. As each unit of any type of durable goods requires one unit of forgone consumption, $r + \delta$ measures the user cost of capital (interest rate plus capital depreciation). Given $p[i]$ (the rental price of durable i), $p[i] - (r + \delta)$ measures a firm's unit profit. The profit function is then given by

$$\pi[i] = (p[i] - r - \delta)X[i], i \in [0, V]. \tag{9}$$

At any point in time, there are V firms (or V varieties of available durables) in the capital goods sector. Of these available durable goods, there are V_p patented durables supplied by

[9] From (8b), the instantaneous innovation rate is given by $\hat{V} = \mu V^{\phi-1}(s_V Y)^{\lambda}$. On a balanced-growth path, it can be shown that the long-run steady-state innovation rate is determined by $\lim_{t\to\infty} \hat{V}[t] = \frac{\lambda}{1-\phi-\lambda/(\varepsilon\sigma(1-\alpha))}$, which is bounded and positive if and only if $\phi < \bar{\phi} \equiv 1 - \lambda/[\varepsilon\sigma(1-\alpha)]$.

monopolistic firms and V_{np} unpatented (out-of-patent) durables by competitive firms. That is, $V = V_p + V_{np}$. Patented durables are priced at the markup of $\eta \equiv \frac{1}{\sigma\alpha} > 1$, while while unpatented ones obey marginal cost pricing. These pricing conditions are given by

$$p[i] = \begin{cases} p_{np} = r + \delta & \text{for } i \in [0, V_{np}] \\ p_p = \eta(r + \delta) & \text{for } i \in (V_{np}, V] \end{cases} \tag{10}$$

where p_{np} (= marginal cost) represents the competitive price of unpatented durables and p_p the monopolistic price of patented durables. From (10) and (5b), demands for patented and unpatented durables are related in terms of:

$$X_{np} = \eta^{\varepsilon} X_p \tag{11}$$

where $X_{np} = X[i]$ for $i \in [0, V_{np}]$, $X_p = X[i]$ for $i \in (V_{np}, V]$, and $X_{np} > X_p$ due to $\eta^{\varepsilon} > 1$. That is, on a per-durable basis, the economy uses more of an unpatented capital good than a patented one, since each patented durable costs more ($p_p > p_{np}$). Define $\zeta \equiv \frac{V_p}{V}$ as the fraction of patented durables and $1 - \zeta \equiv \frac{V_{np}}{V}$ as the fraction for unpatented durables. Then one can use (11) to derive the stock of capital,

$$K = \int_0^V X[i]di = VX_{np}(\eta^{-\varepsilon}\zeta + 1 - \zeta) \tag{12}$$

Note that equation (12) actually represents a market clearing condition for capital goods, because K is the stock of forgone consumption borrowed to purchase both patented and unpatented durables.

2.4 Innovation under a patent system

This subsection discusses the innovation function (8b) and explains how the R&D-driven innovation rate interacts with the dynamic evolution from patented to unpatented durables over time, given that patents are finitely lived.

2.4.1 Innovation, patent value, and R&D intensity

In the research sector there is a stream of identical firms with measure of one. The innovation function (8b) thus applies to the entire economy and any individual research firm as well. Under perfect competition research firms borrow forgone consumption to develop new designs for capital goods. At the firm level, research input I_V and the flow $\dot{V}$ of capital-good designs are linearly related, since the research productivity measure of ξ in (8b) is taken as given in an individual research firm's decision problem. Under a patent system, the flow of new designs $\dot{V}dt$ during an instant means the flow of newly issued patents and each of these patents has a finite patent term denoted by $T > 0$. Each newly issued patent gives birth to a new firm producing a new type of capital goods. Patents are tradable. The market for patents is perfectly competitive. The price υ of a fresh patent issued at time t must reflect its future profitability during an entire patent life. Hence,

$$\upsilon[t] = \int_t^{t+T} e^{-R[\tau]} \pi[\tau] d\tau \tag{13}$$

where $R[\tau] \equiv \int_t^\tau r[\omega] d\omega$ is the cumulative sum of interest rates from t to τ and π measures the profit flow to any firm holding a legally live patent (see (9)). The vintage of a live patent has no effects on the profit flow. Under non-arbitrage conditions, the equilibrium price of a fresh patent must match the cost of developing a new design. That is,

$$\upsilon[t] = 1/\xi[t] \equiv 1/\left(\mu V[t]^\phi (I_V[t])^{\lambda-1}\right) \tag{14}$$

where the inverse of $\xi[t]$ measures the development cost according to (8b).[10] If $\upsilon > 1/\xi$ there would be unbounded R&D demand for foregone consumption. If $\upsilon < 1/\xi$, there would be unbounded demand for patents. With well functioning markets, equilibrium R&D investment I_V must be such that the price of fresh patents equals the cost of capital-good innovation and determines the instantaneous rate of innovation,

$$\hat{V} \equiv \frac{\dot{V}}{V} = \mu V^{\phi-1} I_V^\lambda, \tag{15}$$

using (8b). With the identity of $V = V_p + V_{np}$, the innovation rate $\hat{V}$ is a weighted average of the growth rate $\hat{V}_p$ of patented durables and the growth rate $\hat{V}_{np}$ of unpatented durables. That is, $\hat{V} = \zeta \hat{V}_p + (1-\zeta)\hat{V}_{np}$ with $\hat{V}_p \equiv \frac{\dot{V}_p}{V_p}$ and $\hat{V}_{np} \equiv \frac{\dot{V}_{np}}{V_{np}}$, where the fraction of patented durables $\zeta \equiv V_P/V$ and the fraction of unpatented durables $1-\zeta = V_{np}/V$ evolve over time until the economy is in a steady state. The patent length T plays a role in the motions of ζ, V_p, and V_{np}. We describe their relationships in what follows.

[10]The research input of I_V generates $\dot{V}$ patents at time t. So, the present-time cost of developing a patent (or a new design) is measured by $I_V/\dot{V} = 1/\xi$ in terms of (8b).

2.4.2 Patent length and the dynamic evolution from patented to unpatented durables

First, with an active R&D sector, there are $\dot{V}[t]$ patents issued at a point in time and these newly issued patents will expire at a future date, $t+T$. That is, it holds that $\dot{V}[t] = \dot{V}_{np}[t+T]$ and this equality relation implies $\dot{V}_{np}[t] = \dot{V}[t-T]$. As such, $\dot{V}_{np}[t]$ is pre-determined by previous innovations at a time of $t-T$. With a finite patent length, the growth rates of unpatented and patented durables are therefore given by[11]

$$\hat{V}_{np}[t] = \left(\frac{1}{1-\zeta[t]}\right)\hat{V}[t-T]e^{-\int_{t-T}^{t^-}\hat{V}[\tau]d\tau} \tag{16a}$$

$$\hat{V}_p[t] = \left(\frac{1}{\zeta[t]}\right)(\hat{V}[t] - \hat{V}[t-T]e^{-\int_{t-T}^{t^-}\hat{V}[\tau]d\tau}) \tag{16b}$$

where t^- is asymptotically close to t and $\int_{t-T}^{t^-}\hat{V}[\tau]d\tau$ represents the cumulative sum of innovation rates between $t-T$ and t. Differentiating $\zeta \equiv \frac{V_p}{V}$ with respect to time t yields

$$\dot{\zeta}[t] = (\hat{V}_p[t] - \hat{V}[t])\zeta[t] \tag{17}$$

As indicated, $\dot{\zeta} > 0$ if the growth rate of patented durables, $\hat{V}_p$, exceeds the innovation rate, $\hat{V}$. If it is the case, there is an *increase* in the fraction of patented durables and a *decrease* in the fraction of unpatented durables. From (17), ζ is driven by the two motions of $\hat{V}$ and $\hat{V}_p$ in terms of (17). These motions underlie the patent-created innovating mechanism in a dynamic general-equilibrium context. This mechanism contributes to total factor productivity (TFP) by introducing new types of capital goods, but it can also induce a technical distortion to offset the TFP enhancement to some extent. The following analysis will unfold this problem.

2.5 Innovation and monopoly distortion

In the model patents incentivize innovation of new capital goods by creating monopoly distortions. In this environment, the relative prices of patented and unpatented capital goods are distorted and prevent society from adopting the technically optimal mix of durables, although individual final goods producers always choose their cost-minimizing techniques

[11] (16a) can be obtained using: (i) $\dot{V}_{np}[t] = \dot{V}[t-T]$, (ii) $\hat{V}_{np}[t] \equiv \frac{\dot{V}_{np}[t]}{V_{np}[t]} = \frac{\dot{V}[t-T]}{V[t-T]} \times \frac{V[t-T]}{V[t]} \times \frac{V[t]}{V_{np}[t]}$, (iii) $\frac{V[t]}{V_{np}} = \frac{1}{1-\zeta[t]}$, and (iv) $V[t] = V[t-T]e^{\int_{t-T}^{t^-}\hat{V}[\tau]d\tau}$, where the upper bound t^- is asymptotically close to t. Then with (16a) and the identity $\hat{V} = \zeta\hat{V}_p + (1-\zeta)\hat{V}_{np}$, (16b) holds evidently.

for whatever relative prices. This problem is missing in almost all endogenous growth models where patents are presumed to be infinitely lived.

2.5.1 Monopoly-distorted TFP

Consider symmetries of capital goods permitting to use X_{np} for $X[i]$, $i \in [0, V_{np}]$ and X_p for $X[i]$, $i \in (V_{np}, V]$. From (11) and (12), the final goods production function can reduce to :

$$Y \quad = F[L, K; A, z, \alpha,] = \quad z(AL)^{1-\alpha} K^{\alpha} \tag{18}$$

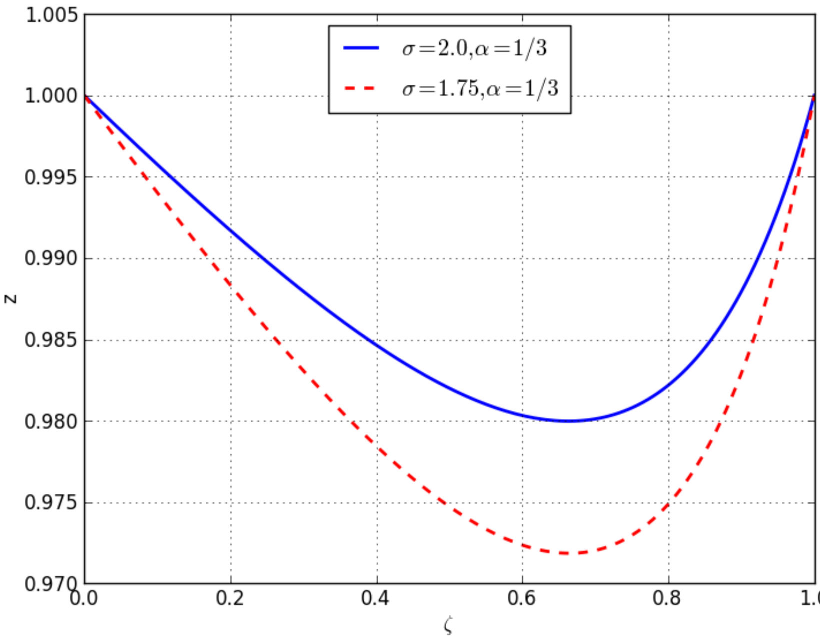
where $A \equiv V^{\frac{1}{\varepsilon\sigma(1-\alpha)}}$ is an endogenous technology term, $zA^{1-\alpha}$ ($= zV^{\frac{1}{\varepsilon\sigma}}$) is a measure of TFP, and z is the monopoly-induced technical distortion term defined as

$$z = z[\zeta] = \frac{\left(\eta^{1-\varepsilon}\zeta + 1 - \zeta\right)^{1/\sigma}}{\left(\eta^{-\varepsilon}\zeta + 1 - \zeta\right)^{\alpha}} \tag{19}$$

It can be verified that z bears a convex relationship with $\zeta \in [0,1]$, has a local minimum at $\zeta = \varepsilon([1-(\frac{1}{\eta})^{\varepsilon}]^{-1} - (\frac{1}{\eta})[1-(\frac{1}{\eta})^{\varepsilon-1}]^{-1}) > 0$, and rises to one if ζ moves to either end of its domain. This relationship is demonstrated in Figure 1, which presumes $\alpha = 1/3$ and $\sigma = 1.75$ or 2.00.[12]. Note that the socially optimal technique for final goods production ought to involve *symmetric* use of all available capital goods in accordance with (4). But under the patent regime the relative prices of patented and unpatented durables are distorted, causing final goods producers' use of capital goods biased toward those that are marginal cost priced. This bias gives rise to a socially inefficient technique in production of final goods and translates into an economy-wide drag on the level of TFP (due to $zA^{1-\alpha} < A^{1-\alpha}$). Thus, TFP fails to attain its potential level ($= A^{1-\alpha}$), unless patents are either infinitely lived ($T \to \infty$) or non-existent ($T = 0$). In the latter case, the R&D sector must vanish, however. R&D-based growth models often presume perpetual patents ($T \to \infty$, $\zeta = 1$) and therefore ignore a finitely-lived patent's negative effects on the level of TFP. For instance, these models include Romer (1990), Segerstrom (1990), Grossman Helpman (1991), Aghion and Howitt (1992), Jones (1995), Jones and Williams (2000), and Eicher and Turnovsky (2001).

[12]Recall that $0 < \sigma < \frac{1}{\alpha}$, $\varepsilon = \frac{1}{1-\sigma\alpha} > 1$, $\eta = \frac{1}{\sigma\alpha} > 1$, and $0 < \zeta < 1$. Given these parameters and (19), if $\zeta = 1$, $z = \eta^{(1-\varepsilon)/\sigma + \varepsilon\alpha} = 1$ due to $(1-\varepsilon)/\sigma + \varepsilon\alpha = 0$. As well, if $\zeta = 0$, the result of $z = 1$ is self-evident. Certainly, both z and ζ are determined simultaneously in equilibrium.

Figure 1: The convex relationship between z and $\zeta = V_p/V$



2.5.2 Monopoly-distorted factor income shares

Next, let us look at how factor income shares are influenced by monopoly distortions. Consider the flow of gross capital income $\int_0^V p[i]X[i]di$ that can be decomposed into aggregate (gross) rental income $(r+\delta)K$ and aggregate profit $\Pi = V_p\pi = \zeta V\pi$, where ζV is the number of patented firms. The labor share is $\frac{wL}{Y} = 1-\alpha$, so parameter α represents the gross capital income share and must satisfy the identity relationship that $\alpha = \frac{\Pi}{Y} + \frac{(r+\delta)K}{Y}$, where the shares of profit and gross rental income are given by[13]

$$\frac{\Pi}{Y} = m\alpha, \text{ with } m = \frac{\zeta(\eta-1)}{(1-\zeta)\eta^{\varepsilon}+\zeta\eta} \in (0,1) \tag{20a}$$

$$\frac{(r+\delta)K}{Y} = (1-m)\alpha \tag{20b}$$

Note that the fraction term m determines how much of the the gross capital income share, α, goes to monopolistic profits. Certainly, $(1-m)\alpha$ is the remaining capital income share

[13] Aggregate profit is $\Pi = \zeta V\pi = (\eta-1)(r+\delta)\zeta VX_p$ based on (9) and (10) and aggregate rental income is $(r+\delta)K = (r+\delta)VX_{np}(\eta^{-\varepsilon}\zeta+1-\zeta)$ according to (12). Hence, using (11), the ratio of $\Pi/[(r+\delta)K]$ is determined by $\frac{\zeta(\eta-1)}{\zeta+(1-\zeta)\eta^{\varepsilon}}$. Using this ratio, we can solve the identity $\alpha = \frac{\Pi}{Y} + \frac{(r+\delta)K}{Y}$ for (20a) and (20b).

going for rents and capital depreciation, subject to the relative magnitude of r and δ. Since $\frac{\partial m}{\partial \zeta} = \eta^{\varepsilon} > 0$, the profit share, Π/Y, rises if patented firms account for a larger fraction in the entire capital goods sector. If this fraction is one ($\zeta = 1$), the profit share reaches its maximum at $\left(\frac{\eta-1}{\eta}\right)\alpha$ and the gross rental share drops to its minimum at $\frac{\alpha}{\eta}$. To the contrary, $\zeta = 0$ (no monopoly) dictates that the entire capital share, α, goes for gross rents.

2.6 Patent regime's steady-state dynamic system

The patent-regime economy is in the steady state by assumption. Let us close the patent regime model by presenting its balanced-growth dynamic system and its core variables' steady state equilibrium in this subsection. Later the economy will be perturbed at time $t = 0$ by replacing the patent regime with an intertemporal-bounty (IB) regime. To differentiate one regime from the other other, let us label the initial patent steady state with subscript 0 and a new IB steady state with subscript 1. From (8a), (8b), (3), and (13), the patent-regime dynamic system on a balanced-growth path is represented by four ordinary differential equations:[14]

$$\dot{K}_0[t] = (1 - s_{V0})Y_0[t] - C_0[t] - \delta K_0[t] \tag{21a}$$

$$\dot{V}_0[t] = \mu V_0[t]^{\phi}(s_{V0}Y_0[t])^{\lambda} \tag{21b}$$

$$\dot{C}_0[t] = C_0\left(\frac{r-\rho}{\gamma} + n\right) \tag{21c}$$

$$\dot{\upsilon}_0[t] = r_0\upsilon_0[t] - \pi_0[t](1 - e^{-(r_0-\hat{\pi}_0)T}) \tag{21d}$$

where $Y_0[t] = z_0(A_0[t]L[t])^{1-\alpha}K_0^{\alpha}$ and $\pi_0[t] = \alpha m_0 Y_0[t]/\zeta_0 V_0[t]$ according to (18) and (20a). On the balanced-growth path, the *non-stationary* variables including $K_0[t]$, $V_0[t]$, $Y_0[t]$, $C_0[t]$, $\upsilon_0[t]$, and $\pi_0[t]$ continue to grow over time at their constant growth rates. In contrast, the *stationary* variables such as interest rate r_0, R&D investment rate s_{V0}, capital investment rate s_{K0}, and patented durables share ζ_0 all stay unchanged over time. For those non-stationary variables, their constant steady-state growth rates are given as follows (see Appendix A for derivations):

$$\hat{K}_0 = \hat{Y}_0 = \hat{C}_0 = \theta_K n \tag{22a}$$

$$\hat{V}_0 = \theta_V n; \quad \hat{A}_0 = \theta_A n \tag{22b}$$

$$\hat{\upsilon}_0 = \hat{\pi}_0 = \theta_{\upsilon} n \equiv (\theta_K - \theta_V)n \tag{22c}$$

[14]By Leibniz's rule, differentiating (13) on a balanced growth path yields the differential equation of (21d).

where each of terms θ_V, θ_A, θ_K and θ_υ is a structural composite of growth-relevant parameters for a specific variable or for a set of specific variables, as given below,

$$\theta_V = \frac{\lambda}{1-\phi-\lambda/[\varepsilon\sigma(1-\alpha)]};\ \theta_A = \frac{\lambda}{(1-\phi)[\varepsilon\sigma(1-\alpha)]-\lambda};\ \theta_K = \theta_A + 1 = (\frac{1-\phi}{\lambda})\theta_V. \tag{23}$$

To facilitate subsequent discussions, let us call these structural composites "*growth kernels*," which readily translate the population growth rate n into a relevant variable's long-run growth rate. For instance, $\theta_V n$ is the long-run innovation rate of V, $\theta_A n$ the long-run growth rate of A, and $\theta_K n$ the long-run growth rate of K, Y, and C.[15] Certainly, on per capita terms, $(\theta_K - 1)n$ $(= \theta_A n)$ determines the long-run growth rate of k, y, and c due to $\hat{k} = \hat{K} - n$, $\hat{y} = \hat{Y} - n$ and $\hat{c} = \hat{C} - n$. As for a fresh patent's price υ and a patented firm's profit flow π, they share the same growth kernel represented by θ_υ (or θ_K minus θ_V), and thereby their long-run growth rate is given by $\theta_\upsilon n$ or $(\theta_K - \theta_V)n$, as indicated by (22c). This actually makes intuitive sense, because final output growth ($\theta_K n$) expands the market for capital goods, whereas the arrival of newer capital goods ($\theta_V n$) dilutes the market.

Note that the structural parameters (α, σ, ε λ, ϕ) that form the growth kernels are from the production functions of final goods and technical designs of capital goods. The population growth rate n is a parameter, too. Therefore, the long-run innovation rate $\hat{V}_0$ and all other long-run growth rates mentioned above are exogenous, independent of the size of the economy and the patent length. Also, as noted earlier (see Footnote 9), a bounded, positive long-run innovation rate, $\hat{V}_0$, requires that ϕ be less than the upper bound $\bar{\phi}$ $(= 1-\lambda/[\varepsilon\sigma(1-\alpha)] < 1)$. Once $\hat{V}_0$ is bounded, all other non-stationary variables are bounded as well.

In the model ongoing growth makes non-stationary variables unbounded and grow exponentially at different paces in the long run. These features call for multiple normalization factors to transform non-stationary into stationary variables. The "growth-kernel powered labor forces" defined by L^{θ_K}, L^{θ_V}, L^{θ_A} and $L^{\theta_K-\theta_V}$ provide a systemic way to build such normalization factors. For instance, K, V, A and υ can be normalized to $\widetilde{K} \equiv K/L^{\theta_K}$, $\widetilde{V} \equiv V/L^{\theta_V}$, $\widetilde{A} \equiv A/L^{\theta_A}$, and $\widetilde{\upsilon} \equiv \upsilon/L^{\theta_\upsilon}$, respectively. In so doing, each variable with tilde is a bounded, measurable scale-adjusted quantity, because $\hat{\tilde{x}}[t] = \hat{x}[t] - \theta_x n = 0$, $x \in \{K, V, A, \upsilon\}$ as t goes to infinity.[16] We close the patent regime model by deriving the long-run steady-state equilibria of a set of scale-adjusted variables

[15] The economy's long-run TFP growth rate is determined by $(1-\alpha)\theta_A n$ in terms of (18).

[16] Similar normalization factors are used in Eicher and Turnovsky (2001).

for further analysis (see Appendix A for detailed derivations):

$$\widetilde{K}_0 = \widetilde{A}_0 \left(\frac{z_0 s_{K0}}{\delta + \theta_K n} \right)^{1/(1-\alpha)} \tag{24a}$$

$$\widetilde{V}_0 = \left[s_{V0} \left(\frac{z_0 s_{K0}}{\delta + \theta_K n} \right)^{\alpha/(1-\alpha)} \left(\frac{\mu}{\theta_V n} \right)^{1/\lambda} \right]^{\theta_V} \tag{24b}$$

$$\widetilde{C}_0 = (1 - s_0) \widetilde{Y}_0 \tag{24c}$$

$$\widetilde{\upsilon}_0 = \frac{s_{V0} \widetilde{Y}_0}{\hat{V}_0 \widetilde{V}_0} = \frac{\alpha m_0 \widetilde{Y}_0}{\zeta_0 \widetilde{V}_0} \left(\frac{1 - e^{-(r_0 - \hat{Y}_0 + \hat{V}_0)T}}{r_0 - \hat{Y}_0 + \hat{V}_0)} \right) \tag{24d}$$

where

$$s_{K0} = \alpha (1 - m_0) \left(\frac{\delta + n + \hat{A}_0}{r_0 + \delta} \right), \tag{25a}$$

$$s_{V0} \quad = \hat{V}_0 \quad \left(\frac{\alpha m_0}{\zeta_0} \right) \left(\frac{1 - e^{-(r_0 - \hat{Y}_0 + \hat{V}_0)T}}{r_0 - \hat{Y}_0 + \hat{V}_0)} \right), \tag{25b}$$

$$r_0 = \rho + \gamma \hat{A}_0, \tag{25c}$$

$$\zeta_0 = 1 - e^{-\hat{V}_0 T}, \tag{25d}$$

and our earlier definitions suffice to recognize that $\widetilde{A}_0 = \widetilde{V}_0^{1/(\varepsilon\sigma(1-\alpha))}$ is a scale-adjusted technology term, $\widetilde{Y}_0 = z_0 \widetilde{A}_0^{1-\alpha} \widetilde{K}_0^{\alpha}$ is a scale-adjusted GDP, z_0 is a monopoly-induced technical distortion on TFP (see (19), $\alpha(1 - m_0)$ is the gross rental income share, αm_0 is the profit share (see (20a) and (20b)), and $s_0 \equiv s_{K0} + s_{V0}$ is the steady-state saving rate.[17] Equations (24a) - (25b) imply the roles of patent length (T) and patent-created monopoly distortions (z_0, m_0). For instance, given the long-run innovation rate $\hat{V}_0 = \theta_V n$, equation (25d) says that the patent length determines the size of the monopolistic sector: the greater the patent length, the greater is the share ζ_0 of monopolistic (patented) firms in the capital goods sector. The patent length and the resulting monopoly distortions then exert effects on investment rates (s_{K0}, s_{V0}), which in turn have influences on the stocks of capital and knowledge ($\widetilde{K}_0$, $\widetilde{V}_0$). The paper is not to fine-tune the patent length for an existing patent regime, however.

[17] $A = V^{\frac{1}{\varepsilon\sigma(1-\alpha)}}$ implies $\widetilde{A} = \widetilde{V}^{\frac{1}{\varepsilon\sigma(1-\alpha)}}$ using $A = \widetilde{A} L^{\theta_A}$, $V = \widetilde{V} L^{\theta_V}$, and $\frac{\theta_V}{\varepsilon\sigma(1-\alpha)} - \theta_A = 0$; see (23).

The paper is instead aimed at examining whether or not it is socially desirable to replace the entire patent regime with an intertemporal bounty regime that will be laid out later in Section 4. To this end, it is conductive to evaluate the patent regime and the proposed alternative, respectively, against the economy's Pareto optimality. The next section evaluates the patent regime.

3 Evaluating Patent Regime against Pareto Optimality

How efficient is a decentralized patent-regime economy in allocating resources? This question relates to private agents' saving and investment decisions, but let us firstly look at the economy's Pareto optimal allocation as a benchmark.

3.1 Pareto optimality

The Pareto optimal allocation can be obtained by maximizing household's lifetime utility, subject to the economy's initial resource endowments (K_0, V_0, $L_0 = 1$) and technological constraints. Hence, the social planner optimization problem is given by[18]

$$\max_{c, I_K, I_V} \int_0^\infty \frac{c^{1-\gamma}-1}{1-\gamma} e^{-(\rho-n)t} dt, \qquad \text{s.t.:} \tag{26a}$$

$$Y = (AL)^{1-\alpha} K^\alpha = C + I_K + I_V, \;\; C = cL \tag{26b}$$

$$\dot{K} = I_K - \delta K, \;\; K[0] = K_0; \quad \dot{V} = \mu V^\phi I_V^\lambda, \;\; V[0] = V_0 \tag{26c}$$

where $L[t] = e^{nt}$. With no market imperfections in such a centralized economy, we need to set ζ at $\zeta^* = 0$ and z at $z^* = 1$.[19] This makes final goods production function (18) reduce to (26b). We can solve the social planner problem for c, I_K, and I_V, which can then translate into s, s_K, and s_V, respectively.[20] Using these Pareto-optimal investment rates,

[18]Romer (1990) examines the social planner problem in an endogenous R&D-based growth model with the scale effects, where the long-run socially optimal innovation rate is derived. The present paper uses a non-scale growth model with a long-run innovation rate parametrized by $\theta_V n$ (see (22b)), so the focus is on the socially optimal allocation of resources, as in Jones and Williams (2000), and Eicher and Turnovsky (2001).

[19]Hereafter, a variable with an asterisk indicates a centralized economy's steady-state equilibrium.

[20]As a standard procedure, one can obtain the socially optimal solution by maximizing the Hamiltonian,

$$\mathscr{H} \equiv \left(\frac{c^{1-\gamma}-1}{1-\gamma}\right) e^{-(\rho-n)t} + \psi_K \cdot \left(V^{\frac{1}{\varepsilon\sigma}} L^{1-\alpha} K^\alpha - cL - I_V - \delta K\right) + \psi_V \cdot \mu V^\phi I_V^\lambda$$

we can derive the Pareto-optimal scale-adjusted stocks of capital and knowledge in the same way as we did earlier for the patent regime. Also, as we have analyzed, the long-run growth rates of non-stationary variables are parametrized by their associated growth kernels and the population growth rate (see (22a) - (22c)). Certainly, those long-run growth rates hold for the centralized economy; that is, $\hat{K}^* = \hat{Y}^* = \hat{C}^* = \theta_K n$, $\hat{V}^* = \theta_V n$, $\hat{A}^* = \theta_A n$, and $\hat{\upsilon}^* = \hat{\pi}^* = (\theta_K - \theta_V)n$. However, except for the real interest rate r, the centralized economy has a different set of stationary equilibria given below:

$$\widetilde{K}^* \equiv \frac{K}{L^{\theta_K}} = \widetilde{A}^* \left(\frac{s_K^*}{\delta + n + \hat{A}^*} \right)^{1/(1-\alpha)} \tag{27a}$$

$$\widetilde{V}^* \equiv \frac{V}{L^{\theta_V}} = \left[s_V^* \left(\frac{s_K^*}{\delta + \theta_K n} \right)^{\alpha/(1-\alpha)} \left(\frac{\mu}{\theta_V n} \right)^{1/\lambda} \right]^{\theta_V} \tag{27b}$$

$$s_K^* = \alpha \cdot \left(\frac{\delta + n + \hat{A}^*}{r^* + \delta} \right) \tag{27c}$$

$$s_V^* \quad = \quad \hat{V}_0 \left(\frac{1}{\varepsilon\sigma} \right) \left(\frac{\lambda}{r^* - (\hat{Y}^* - \hat{V}^*) - \phi \hat{V}^*} \right) \tag{27d}$$

$$r^* = r_0, \quad \zeta^* = 0 \tag{27e}$$

where $s_K^* + s_V^* = s^*$ is the long-run saving rate. By comparing equations (27a) - (27e) to equations (24a), (24b), (25a) and (25b), one can verify that a decentralized patent-regime economy can never attain Pareto optimality in the long run, no matter how one fine-tunes the patent regime. The analysis is presented below.

3.2 Patent regime is suboptimal

To show that a patent-regime economy is always suboptimal, we derive the ratios of (24a) to (27a), (24b) to (27b), (25a) to (27c), and (25b) to (27d):

$$\frac{\widetilde{K}_0}{\widetilde{K}^*} = \left(\frac{\widetilde{V}_0}{\widetilde{V}^*} \right)^{1/[\varepsilon\sigma(1-\alpha)]} \left(\frac{z_0 s_{K0}}{s_K^*} \right)^{1/(1-\alpha)} \tag{28a}$$

where ψ_K and ψ_V are Lagrangian multipliers.

$$\frac{\widetilde{V}_0}{\widetilde{V}^*} = \left(\frac{s_{V0}}{s_V^*}\right)^{\theta_V} \left(\frac{z_0 s_{K0}}{s_K^*}\right)^{\alpha\theta_V/(1-\alpha)} \tag{28b}$$

$$\frac{s_{K0}}{s_K^*} = 1 - m_0 \tag{28c}$$

$$\frac{s_{V0}}{s_V^*} \quad = \quad \left(\frac{\alpha m_0/\zeta_0}{1/(\varepsilon\sigma)}\right)\left(\frac{1-e^{-(r_0-\hat{Y}_0+\hat{V}_0)T}}{1}\right)\left(\frac{1/(r_0-\hat{Y}_0+\hat{V}_0)}{1/(r_0-\hat{Y}_0+\hat{V}_0-\phi\hat{V}_0)}\right)\left(\frac{1}{\lambda}\right) \tag{28d}$$

It is clear from (28a) that as long as there is a non-zero patent length ($T > 0$), patent-regime agents must underinvest in the capital stock (because $\frac{s_{K0}}{s_K^*} < 1$ due to $0 < m_0 < 1$): the greater the share m_0 of monopoly profits in the economy's gross capital income, the greater is the extent of underinvestment. The market mechanism is straightforward: patent-created monopolies raise the prices of capital goods above marginal costs and decrease the market incentive to build the stock of capital. Further, as we had mentioned earlier, there is a monopoly-induced technical distortion, denoted by z_0, that can decrease the economy's total factor productivity (see (19)). As a result, through the $s_{K0} - \text{cum} - z_0$ channel, a patent regime's stock of capital must fall below the Pareto optimal level. This is exactly what (28a) indicates. As well, through the same $s_{K0} - \text{cum} - z_0$ channel, a patent-regime's stock of knowledge must be short of the Pareto optimal level according to (28b). The monopoly-restrained knowledge stock, in turn, puts a drag on the stock of capital, because the knowledge-stock ratio $\widetilde{V}_0/\widetilde{V}^*$ is also a determinant of the capital-stock ratio $\widetilde{K}_0/\widetilde{K}^*$ in equation (28a).

Aside from the aforementioned $s_{K0} - \text{cum} - z_0$ channel, there is the s_{V0} channel through which the patent-regime steady-state capital and knowledge stocks can be affected. However, this channel is complicated. For purposes of exposition, we break the determinants of the R&D investment ratio s_{V0}/s_V^* in (28d) into four sources:

(i) From the first parenthesized term, the private valuation of a differentiated capital good is always less than its social valuation at any point in time (i.e. $\alpha m_0 \zeta_0 < 1/(\varepsilon\sigma)$).[21] This is a *static valuation distortion*, making patent-regime agents **underinvest** in R&D. (ii)

[21] Recall that $\varepsilon = 1/(1-\sigma\alpha) > 1$ is the price elasticity of demand for a capital good. The social valuation of the variety of capital goods is reflected in term $1/(\varepsilon\sigma)$, which is the elasticity of final output Y with respect to the measure of variety V. That is, $\frac{V}{Y}\frac{\partial Y}{\partial V} = 1/(\varepsilon\sigma)$ in terms of (18). The private valuation of a patented capital good is $\alpha m_0/\zeta_0 = \alpha\left(\frac{\eta-1}{(1-\zeta)\eta^{\varepsilon}+\zeta\eta}\right) = \left(\frac{\zeta\eta}{(1-\zeta)\eta^{\varepsilon}+\zeta\eta}\right)\cdot\alpha/\varepsilon$ according to (20a) and $\eta = \varepsilon/(\varepsilon-1) > 1$. Since $\frac{\zeta\eta}{(1-\zeta)\eta^{\varepsilon}+\zeta\eta} < 1$ and $\alpha < \frac{1}{\sigma}$ from (4), the private valuation must be unambiguously less than the social valuation.

From the second term, patent-regime agents **underinvest** in R&D again, because a finite patent length is always too short to reflect an innovative durable's permanent contribution to household's welfare. This is a *dynamic valuation distortion* inherent in any patent system that rewards an innovation only for a finite duration. The static and dynamic valuation distortions combine to create the "surplus-appropriability" problem. (iii) The third term represents a positive *technical externality*, making patent-regime agents **underinvest** in R&D, because they do not internalize knowledge spillovers ($0 < \phi < 1$) in their private R&D decisions (see (8b)). (iv) Lastly, from the fourth term, $1/\lambda$ is greater than one, due to the *research congestion* externality ($\lambda < 1$) (see (8b)). For this negative *technical externality*, patent-regime agents **overinvest** in R&D. In contrast to patent-regime agents, the social planner internalizes both positive and negative externalities.

The above analysis indicates more distortions tending to cause underinvestment in R&D in a decentralized patent-regime economy, whereas only the research-congestion distortion tends to the opposite. Empirical estimates seem to suggest that the research-congestion distortion alone should not dominate.[22] Thus, compared to the Pareto optimum, the patent regime should tend to have a smaller R&D intensity (s_V) and should tend to build a smaller stock of capital and knowledge, respectively, on account of (28a) and (28b). Simulation of the model later will lend support to these tendencies. Regardless, a patent system is apparently suboptimal, with no hope to duplicate the social planner's optimal solution.[23]

4 Switching to the Intertemporal Bounty Regime

Can a decentralized economy attain the Pareto optimum characterized by (27a) - (27e) if the patent regime is replaced with an alternative that rewards technological innovation with intertemporal bounties payable on the marginal-cost sales of bountiable goods? This is the central question of the paper. To facilitate the analysis, I firstly describe institutional arrangements for the intertemporal bounty regime to be a patent replacement.

[22]For instance, Jones and Williams (1998) find that an innovating economy's socially optimal R&D investment is at least four time greater than actual spending.

[23]For instance, if the patent length goes to infinity ($T \to \infty$), $z_0 \to 1$, $\zeta_0 \to 1$, $m_0 \to (\eta - 1)/\eta = 1/\varepsilon$, and $\frac{\alpha m_0/\zeta_0}{1/(\varepsilon\sigma)} \to \sigma\alpha < 1$. These changes only mitigate to some extent the monopoly-created distortions. Adjusting the patent breadth (equivalent to changing the markup rate, η) cannot fix the suboptimality problem, either.

4.1 Institutional arrangements

A feasible intertemporal bounty regime requires the following institutional arrangements:[24]

First, any bountiable innovation of a differentiated capital good must be *freely* licensed. This creates a free entry condition to establish a perfectly competitive market for the bountiable durable that must sell at marginal cost. That is, $p = r + \delta$ with $\eta = 1$ (see (10)).

Second, the innovator is rewarded with a *perpetual bounty claim* to an infinite stream of future bounties, payable on the ex post marginal-cost sales of a specific bountiable capital good at a rate determined by the government. Denote by β the bounty rate. Then given the ex post market sales pX of the bountiable good, the bounty flow, denoted by b, paid to the holder of a bounty claim at a point in time is determined by $b = \beta pX$. Note that the number of bountiable goods is the same as the number of bounty claims.

Third, the government collects taxes from households to make bounty payments. For simplicity, the paper presumes a non-distortionary lump-sum tax available to fund such intertemporal bounties. The tax-financed bounty payment must be creditable so as to form an active asset market for tradeable bounty claims.[25] The equilibrium price of a typical bounty claim must therefore reflect the discounted present value of an infinite stream of future bounties, delivering a signal to R&D investment. Households are the owners of bounty claims. So, as a government transfer payment, the collected lump-sum tax must return to the household sector.

Fourth, the intertemporal-bounty regime grandfathers all those previously innovated durables whose patents are still legally live at the moment of the regime switch, which occurs at $t = 0$ with live patents amounting to $V_p[0] = \zeta_0 V[0]$. Thus, at $t = 0$, the government needs to issue $\zeta_0 V[0]$ bounty claims immediately, and variable ζ henceforth needs to be redefined as the fraction of *bountied* (not *patented*) durables. Eliminating the grandfather clause would allow bounties and patents to co-exist within the remaining patent life. This would complicate the model and delay the gain from removing monopolies.

Under the above arrangements, all capital goods, either bountied or non-bountied, sell at the same marginal cost. This simplifies calculation of bounties. The government's aggregate bounty payments, denoted by B, at a point in time are equal to $B = \beta\alpha Y\zeta = \beta(r+\delta)K\zeta$, where αY or $(r+\delta)K$ is aggregate capital spending. That is, the government

[24] These arrangements bear similarities to Grinols and Henderson (2007) and Grinols and Lin (2011) except for discussions of an asset market for bounty claims and tax-financed bounties in a dynamic general equilibrium framework.

[25] In practice, a bounty claim can split into shares to make it more liquid on the market.

needs to collect from households a lump-sum tax equal to B, which results from all final goods producers' spending decisions on capital goods. Certainly, $b = B/\zeta V = \beta\alpha Y/V$, since b is the bounty flow to an individual bounty claim and ζV is the number of bounty claims. We now can make the patent-regime model switch to a bounty-regime model by changing some parameters, as will be shown below.

4.2 Dynamic system under the intertemporal bounty regime

Now, we replace $\eta > 1$ with $\eta = 1$, monopoly profit π with bounty b, and $T < \infty$ with $T \to \infty$ in the patent-regime model.[26] In so doing, we parameterized a regime switch from patents to intertemporal bounties. This switch pushes the economy off the initial patent-regime balanced-growth path defined by (21a) - (21d) and gives rise to a bounty-regime dynamic system that evolves for $t \in [0, \infty)$ as given below:

$$\dot{K}[t] = (1 - s_V[t])Y[t] - C[t] - \delta K[t] \tag{29a}$$

$$\dot{V}[t] = \mu V[t]^{\phi}(s_V[t]Y[t])^{\lambda} \tag{29b}$$

$$\dot{C}[t] = C[t](\frac{r[t] - \rho}{\gamma} + n) \tag{29c}$$

$$\dot{\upsilon}[t] = r[t]\upsilon[t] - b[t] \tag{29d}$$

where $Y[t]$, $s_V[t]$, $r[t]$ and $b[t]$ are determined by

$$Y[t] = (A[t]L[t])^{1-\alpha}K[t]^{\alpha} \tag{30a}$$

$$s_V[t] = \upsilon[t]^{1/(1-\lambda)}(\mu V[t]^{\phi})^{1/(1-\lambda)}/Y[t] \tag{30b}$$

$$r[t] = \frac{\alpha Y[t]}{K[t]} - \delta \tag{30c}$$

$$b[t] = \beta\alpha Y[t]/V[t] \tag{30d}$$

$$\zeta[t] = 1 - (1 - \zeta_0)e^{-\int_0^t \hat{V}[\tau]d\tau}, \;\; \zeta[0] = \zeta_0, \;\; \zeta[\infty] = 1 \tag{30e}$$

[26]The bounty regime requires $T \to \infty$ for bounty claims are perpetual and some other adjustments. For the flow budget constraint (2), per-capita lump-sum tax needs to included so that $\dot{a} = (r - n)a + w - c - \frac{B}{L}$, where asset stock a includes physical capital and bounty claims that earn net capital income consisting of the earned bounty flow. Individual households treat the lump sum tax as given in their intertemporal decisions. Also, we need to re-interpret p_p (p_{np}) as the price of bountied (unbountied) durables, and V_p (V_{np}) as the number of bountied (unbountied) durables.

This is a nonlinear dynamic system of four first-order ordinary differential equations. This system start at $t = 0$, driving the economy to transition to a new balanced-growth path in the long run. It contains new elements that warrant attention in some respects:

First, in contrast to (21d), (29d) is a no-arbitrage condition applying to infinitely lived bounty claims, where $\upsilon[t]$ is reinterpreted as the market price of a bounty claim (not a patent) and $b[t]$ is the bounty flow to the holder of a typical bounty claim. Second, the bounty flow $b[t]$ is based on the government-determined bounty rate β and the market size of a bountied durable, $\alpha Y[t]/V[t]$, according to (30d). Third, the TFP-distortion term z jumps to one as η is set to one at the moment of the regime switch, thereby raising TFP to the distortion-free level, as implied by (30a). Fourth, using (14) and $s_V Y = I_V$, one can readily derive (30b) to determine the R&D investment rate $s_V[t]$. Fifth, setting $m = 0$ (due to $\eta = 1$) in (20b) yields equation (30c) for the interest rate $r[t]$ at any moment. Lastly, (30e) determines the fraction $\zeta[t]$ of bountied durables. This is a state variable evolving to one in the long run, but it does not enter the dynamic system, since bountied and unbountied durables are equally priced.[27]

To examine how the regime switch is to impact the economy, we must solve the dynamic system of (29a) - (29d) for a nonlinear stable manifold in the two-dimensional state space of capital and knowledge stocks. This is tantamount to computing the transition paths of $K[t]$, $V[t]$, $C[t]$ and $\upsilon[t]$, $t \in [0, \infty)$. But this is infeasible in that these variables are unbounded in the long run. Thus as under the patent regime, the bounty-regime dynamic system must be transformed into a stationary one.

4.3 Normalization of bounty-regime dynamic system

Normalizing the dynamic system of (29a) - (29d) requires four normalization factors, as we did for the patent regime. More details are given in what follows.

4.3.1 Growth kernel-powered labor forces as normalization factors

Note that long-run growth rates are regime-independent in the model. They are parametrized by their associated growth kernels (θ_V, θ_A, θ_K, θ_υ) and the population growth rate (n) in accordance with (22a) - (22c) and (23). As under the patent regime, these growth kernel-powered labor forces (L^{θ_K}, L^{θ_V}, L^{θ_A}, L^{θ_υ}) serve as normalization factors such that

[27] Perpetual bounty claims implies $\dot{V}_p = \dot{V}$ and $\hat{V}_p = \frac{V}{V_p}\frac{\dot{V}}{V} = \hat{V}/\zeta$. Therefore, (17) needs to be changed to $\dot{\zeta} = (1-\zeta)\hat{V}$, which solves for 30e.

$\widetilde{K}=K/L^{\theta_K}$, $\widetilde{V}=V/L^{\theta_V}$, $\widetilde{A}=A/L^{\theta_A}$, $\widetilde{C}=C/L^{\theta_K}$, $\widetilde{Y}=Y/L^{\theta_K}$, $\widetilde{\upsilon}=\upsilon/L^{\theta_\upsilon}$, and $\widetilde{b}=b/L^{\theta_\upsilon}$.[28] These scale-adjusted quantities ($\widetilde{K}$, $\widetilde{V}$, $\widetilde{A}$, $\widetilde{C}$, $\widetilde{Y}$, $\widetilde{\upsilon}$, $\widetilde{b}$) are stationary and measurable as t goes to infinity. Using this normalization approach and noting how those growth kernels are interrelated in terms of (23), we can transform (29a) - (29d) into a stationary dynamic system as given below:

$$\dot{\widetilde{K}}[t]=f_1[\vec{\omega}[t]]\equiv s_K[t]\widetilde{Y}[t]-(\delta+\theta_K n)\widetilde{K}[t] \tag{31a}$$

$$\dot{\widetilde{V}}[t]=f_2[\vec{\omega}[t]]\equiv \mu\widetilde{V}[t]^{\phi}(s_V[t]\widetilde{Y}[t])^{\lambda}-\theta_V n\widetilde{V}[t] \tag{31b}$$

$$\dot{\widetilde{C}}[t]=f_3[\vec{\omega}[t]]\equiv \widetilde{C}[t]\left(\frac{r[t]-\rho}{\gamma}-\theta_A n\right) \tag{31c}$$

$$\dot{\widetilde{\upsilon}}[t]=f_4[\vec{\omega}[t]]\equiv r[t]\widetilde{\upsilon}[t]-\widetilde{b}[t]-(\theta_K-\theta_V)n\widetilde{\upsilon}[t] \tag{31d}$$

subject to

$$\widetilde{K}[0]=\widetilde{K}_0,\ \widetilde{V}[0]=\widetilde{V}_0;\ \ \widetilde{K}[\infty]=\widetilde{K}_1,\ \widetilde{V}[\infty]=\widetilde{V}_1,\ \widetilde{C}[\infty]=\widetilde{C}_1,\ \widetilde{\upsilon}[\infty]=\widetilde{\upsilon}_1 \tag{31e}$$

where $\vec{\omega}[t]\equiv[\widetilde{K}[t],\widetilde{V}[t],\widetilde{C}[t],\ \widetilde{\upsilon}[t]]$ is a vector of four unknown scale-adjusted quantities and the other variables that depend on $\vec{\omega}[t]$ are scale-adjusted final output $\widetilde{Y}$, scale-adjusted bounty flow $\widetilde{b}$, capital investment rate s_K, R&D investment rate s_V, interest rate r, subject to the following static equilibrium conditions:

$$\widetilde{Y}[t]=\widetilde{A}[t]^{1-\alpha}\widetilde{K}[t]^{\alpha},\ \text{with}\ \widetilde{A}[t]=\widetilde{V}^{1/(\varepsilon\sigma(1-\alpha))} \tag{32a}$$

$$\widetilde{b}[t]=\beta\alpha\widetilde{Y}[t]/\widetilde{V}[t] \tag{32b}$$

$$s_K[t]=(1-s_V[t])-\frac{\widetilde{C}[t]}{\widetilde{Y}[t]} \tag{32c}$$

$$s_V[t]=(\widetilde{\upsilon}[t]\cdot\mu\widetilde{V}[t]^{\phi})^{1/(1-\lambda)}/\widetilde{Y}[t] \tag{32d}$$

$$r[t]=\frac{\alpha\widetilde{Y}[t]}{\widetilde{K}[t]}-\delta \tag{32e}$$

[28] From (30d), the instantaneous growth rate of the bounty flow b is $\hat{b}[t]=\hat{Y}[t]-\hat{V}[t]$ and $\hat{b}[\infty]=(\theta_K-\theta_V)n\equiv\theta_\upsilon n$ in steady state. Therefore, L^{θ_υ} serves as a normalization factor for b, as in the case of υ.

$$\zeta[t] = 1 - (1 - \zeta_0)e^{-\int_0^t (\hat{\widetilde{V}}[\tau] + \theta_V n)d\tau}, \quad \zeta[0] = \zeta_0, \quad \zeta[\infty] = 1 \tag{32f}$$

Equation (32f) results from (30e) by replacing the innovation rate $\hat{V}$ with $\hat{\widetilde{V}}[\tau] + \theta_V n$ due to $V = \widetilde{V}L$. The normalized system features two state variables ($\widetilde{K}$, $\widetilde{V}$) and jump variables ($\widetilde{C}$, $\widetilde{\upsilon}$). The two state variables – capital and knowledge stocks – are predetermined at any point in time and must evolve smoothly over time, whereas the two jump variables – aggregate consumption and price of bounty claim – are free to make a discrete change in response to shocks. The normalized system is subject to two initial-value boundary conditions represented by $\widetilde{K}[0] = \widetilde{K}_0$ and $\widetilde{V}[0] = \widetilde{V}_0$ at $t = 0$ as well as four other boundary conditions represented by $\widetilde{K}[\infty] = \widetilde{K}_1$, $\widetilde{V}[\infty] = \widetilde{V}_1$, $\widetilde{C}[\infty] = \widetilde{C}_1$, and $\widetilde{\upsilon}[\infty] = \widetilde{\upsilon}_1$ at $t \to \infty$.[29] Hence, this is a nonlinear two-point boundary value problem and can only be solved numerically. Yet, before we proceed to solve the boundary value problem in Section 5, the long-run Pareto optimal bounty rate must be determined for the bounty regime. This is because the bounty rate has effects on the boundary conditions at $t \to \infty$.

4.3.2 Optimal bounty rate and the new balanced-growth path

Setting $\dot{\widetilde{K}} = \dot{\widetilde{V}} = \dot{\widetilde{C}} = \dot{\widetilde{\upsilon}} = 0$ in (31a) - (31d) allows us to solve for the steady-state boundary conditions at $t \to \infty$ (see Appendix B for derivations):

$$\widetilde{K}_1 = \left(\frac{s_{K1}}{\delta + \theta_K n}\right)^{1/(1-\alpha)} \widetilde{A}_1 \tag{33a}$$

$$\widetilde{V}_1 = \left[s_{V1} \left(\frac{s_{K1}}{\delta + \theta_K n}\right)^{\alpha/(1-\alpha)} \left(\frac{\mu}{\theta_V n}\right)^{1/\lambda} \right]^{\theta_V} \tag{33b}$$

$$\widetilde{C}_1 = (1 - s_1)\widetilde{Y}_1 \tag{33c}$$

$$\widetilde{\upsilon}_1 = \frac{s_{V1}\widetilde{Y}_1}{\hat{V}_1 \widetilde{V}_1} = \frac{\beta\alpha\hat{Y}_1}{\widetilde{V}_1}\left(\frac{1}{r_1 + \hat{V}_1 - \hat{Y}_1}\right) \tag{33d}$$

where

$$s_{K1} = s_K^*, \quad s_{V1} = \frac{\beta\alpha\hat{V}_1}{r_1 + \hat{V}_1 - \hat{Y}_1}, \quad \zeta_1^{Bountied} = 1 \tag{33e}$$

Note that $\widetilde{A}_1 = \widetilde{V}_1^{1/(\varepsilon\sigma(1-\alpha))}$ and $\widetilde{Y}_1 = \widetilde{A}_1^{1-\alpha}\widetilde{K}_1^{\alpha}$ due to $z = 1$, while $\zeta_1^{Bountied} = \zeta[\infty] = 1$ in terms of (32f). On the bounty-regime balanced-growth path, we have $\hat{K}_1 = \hat{C}_1 = \hat{Y}_1 =$

[29] Recall that we label a variable with subscript 0 (1) to indicate the variable associated with the balanced-growth patent (bounty) regime.

$\theta_K n$, $\hat{V}_1 = \theta_V n$, $\hat{A}_1 = \theta_A n$, and $\hat{\upsilon}_1 = \hat{b}_1 = (\theta_K - \theta_V)n$, for these long-run growth rates are regime-independent. The long-run interest rate is parametrized too, with $r_1 = r_0 = r^*$. As (33e) indicates, on the balanced-growth path, the decentralized bounty regime always has the same physical capital investment rate as the social planner optimum (i.e., $s_{K1} = s_K^*$), irrespective of the magnitude of the bounty rate of β. However, the R&D investment rate for the bounty regime differs from the Pareto optimal level in general (i.e., $s_{V1} \neq s_V^*$).[30] That is, unless the bounty rate, β, is set rightly, the bounty regime will not see a R&D investment rate that matches the Pareto optimal level. This problem can be fixed, however. Solving the equation, $s_{V1} = s_V^*$, based on (27d) and (33e), actually yields a bounty rate, β^*, that is Pareto optimal:

$$\beta^* = \frac{\lambda}{\varepsilon - 1} \cdot \left(\frac{r_0 + \hat{V}_0 - \hat{Y}_0}{r_0 + (1-\phi)\hat{V}_0 - \hat{Y}_0} \right) \tag{34}$$

Replacing β with β^* in (33e) therefore ensures that the resulting bounty regime has the same R&D investment rate as the socially planner optimum. As such, with $s_{K1} = s_K^*$ and $s_{V1} = s_V^*$, it must hold that $\widetilde{K}_1 = \widetilde{K}^*$ and $\widetilde{V}_1$ and $\widetilde{V}^*$. Such a bounty regime fully removes monopoly distortions and builds a distortion-free capital stock, while enabling an active bounty claims market that delivers the right signal to allocate the Pareto optimal level of social resources (labor) to accumulate knowledge in the new steady state.

5 Numerical Analysis: Transition Paths & Welfare

The regime switch cannot make the economy jump all the way to the new steady state, however. The capital (K) and knowledge (V) stocks can only evolve smoothly through time. It is therefore critical to observe how the economy behaves in transitional periods, before one can evaluate the cumulative welfare impact of replacing patents with bounties. To this end, this section presents numerical analysis.

5.1 Calibration of the model and steady-state simulations

At the outset we use a set of benchmark parameters to calibrate the patent-regime model to an innovating economy like the United States. These parameters, given in Table 1, are

[30] There is a short cut to deriving either s_{K1} or s_{V1}: setting $m_0 = 0$ in (25a) yields s_{K1}, while replacing $\alpha m_0 \hat{V}_0$ with $\beta \alpha \hat{V}_1$ and letting $T \to \infty$ in (25b) yields s_{V1}. In Appendix B, s_{K1} and s_{V1} are derived from steady-state equilibrium conditions.

identical to or close to those used in previous calibration exercises.[31]

Table 1: Benchmark parameters for model calibration

Production:	$\alpha = 0.36,$	$\sigma = 1.80,$	$\mu = 0.21$	$\phi = 0.50,$	$\lambda = 0.65,$
Preference:	$\rho = 0.04,$	$\gamma = 1.0,$			
Depre. & Pop.:	$\delta = 0.05,$	$n = 0.015,$			
Patent Length:	$T = 20.$				

In calibration of the final goods production function, the parameter of α is set equal to 0.36, meaning that the gross capital income share in GDP is 36% and the labor share is 64%. The parameter of σ is set equal to 1.80, which reflects the substitutability of capital goods and must be less than $1/\alpha$, as noted earlier. The two parameters lead to a price elasticity of demand for a typical capital good equal to 2.84 ($= \varepsilon = 1/(1-\sigma\alpha)$) and the markup rate of about 1.54 ($= \eta = \frac{1}{\sigma\alpha}$) for patented capital goods.[32] For the production function of durable-good designs, the knowledge-spillover parameter, ϕ, is set equal to 0.50, which is less than the upper bound of $\bar{\phi} = 1 - \lambda/(\varepsilon\sigma(1-\alpha)) \approx 0.80$ (see Footnote 9); the research-congestion parameter, λ, is set equal to 0.65; but the research productivity parameter, μ, requires more elaboration and is to be calibrated later. For the preference parameters, the rate of time preference, ρ, is constant at 0.04 and the elasticity of intertemporal substitution is assumed to be $1/\gamma = 1$ (that is, $\gamma = 1$). Another three parameters are the rate of physical capital depreciation set at $\delta = 0.05$, the growth rate of labor supply (population) at $n = 0.015$, and the patent length at $T = 20$. The parameter of $T = 20$ means that any newly granted patent expires in a finite period of twenty years, which is consistent with US and WTO (World Trade Organization) patent policy.

Now let us return to calibration of the research productivity parameter, μ. First, the above-calibrated production parameters (α, σ, ϕ, λ) are sufficient to compute the three growth kernels in terms of (23), which are $\theta_V = 2.1567$, $\theta_A = 0.6590$, and $\theta_K = 1.6590$. The labor supply growth rate of $n = 0.015$ multiplied by these computed growth kernels then deliver the following long-run growth rates of important macroeconomic variables for the benchmark patent regime: $\hat{V}_0 = 0.0324$, $\hat{A}_0 = 0.0099$, $T\hat{F}P_0 = (1-\alpha)\hat{A}_0 = 0.0063$, and

[31] See, for instance, Jones and Williams (2000) and Eicher and Turnovsky (2001).

[32] In contrast, the markup rate for unpatented durables equals one. Given the share of patented durables less than 50% (as will be shown in the benchmark case, the share is 47.6%), the industry-wide average markup rate is about 1.26, which falls onto the empirical evidence range (see Norrbin 1993 and Basu 1996).

$\hat{K} = \hat{Y} = \hat{C} = 0.0249$. Second, since labor supply L is exogenous, we chose $L[0] = L_0 = 1$. Also, since there is only one remaining parameter, μ, to be calibrated, we have a degree of freedom to choose either K_0 or V_0 at $t = 0$, and we chose $V_0 = 1$, which implies $\widetilde{V}_0 = V_0/L_0^{\theta_V} = 1$. Third, with $\widetilde{V}_0 = 1$ and all the already-calibrated parameters, we can solve the eight equations of (24a) - (24d) and (25a) - (25d) for parameter μ and the other seven variables ($\widetilde{K}_0$, $\widetilde{A}_0$, $\widetilde{\zeta}_0$, $\widetilde{C}_0$, r_0, s_{K0}, s_{V0}). Using this procedure, parameter μ is calibrated at 0.21, as indicated in Table 1.

From the above calibration of the μ parameter, we have generated the initial patent-regime steady state. Now with a complete set of benchmark parameters (1), we can also solve (33a) - (33e) and (34) for the Pareto optimal steady state under an intertemporal bounty regime that implements the Pareto-optimal bounty rate. The steady state results for the two regimes are summarized in Table 2 and analyzed as follows:

First, for the decentralized bounty-regime economy to mimic the Pareto optimum, the required optimal bounty rate is set at $\beta^* = 0.49$. That is, for each \$1.00 sales in a year of a typical bountied durable under marginal cost pricing, the government pays a bounty flow of about 49 cents annually to the bounty-claim holder. The resulting annual aggregate bounty flows (B) account for 18% of GDP ($= Y$), which is funded by a lump-sum tax. In contrast, under the patent regime with a markup rate (η) of 1.54, the patent holder receives a profit margin of 54 cents from each \$1.54 sales of a patented durable. Such patent-created monopoly profits (Π) account for about 4% of GDP.

Second, the steady-state of the benchmark patent regime pretty much matches the United States macroeconomic features including the R&D intensity (s_{V0}) of about 3%, the physical capital investment rate (s_{K0}) of 24%, the consumption/GDP ratio ($\widetilde{C}_0/\widetilde{Y}_0$) of 73% ($= 1 - s_{K0} - s_{V0}$), the capital/output ratio ($\widetilde{K}_0/\widetilde{Y}_0$) of 3.24, in addition to the real interest rate r and long-run growth rates of GDP and TFP (see Remarks of Table 2). The knowledge/output ratio ($\widetilde{V}_0/\widetilde{Y}_0$) is 0.53. Empirically, this ratio is not precisely available. Under the patent regime, 48% of durable goods firms are patented ($\zeta_0 \approx 0.48$) and the resulting monopoly-induced technical distortion makes TFP about 2% below its potential level, as implied by $z_0 \approx 0.98$.

Third, the regime switch removes monopoly distortions and optimizes the allocation of resources. This allows the economy to attain the Pareto optimum in the long-run. As the figures indicate, the economy becomes much more relatively intensive in both capital and knowledge under the bounty regime. The stock of scale-adjusted capital ($\widetilde{K}$) rises from 6.07 to17.05. Even more significant is a remarkable increase in the stock of scale-adjusted knowledge ($\widetilde{V}$) from 1.00 to 16.66. As a result, the regime switch raises the capital/output ratio from 3.24 to 3.60 and the knowledge/output ratio sharply from 0.53 to 3.43. This explains why the regime switch generates a long-run consumption gain of about 124% by comparing $\widetilde{C}_1 \approx 3.06$ to $\widetilde{C}_0 \approx 1.36$. A drop of the consumption/GDP ratio from 0.73 to 0.63 is, in fact, indispensable, since the bounty regime requires more foregone consumption to establish a more knowledge-intensive, more capital-intensive economy.

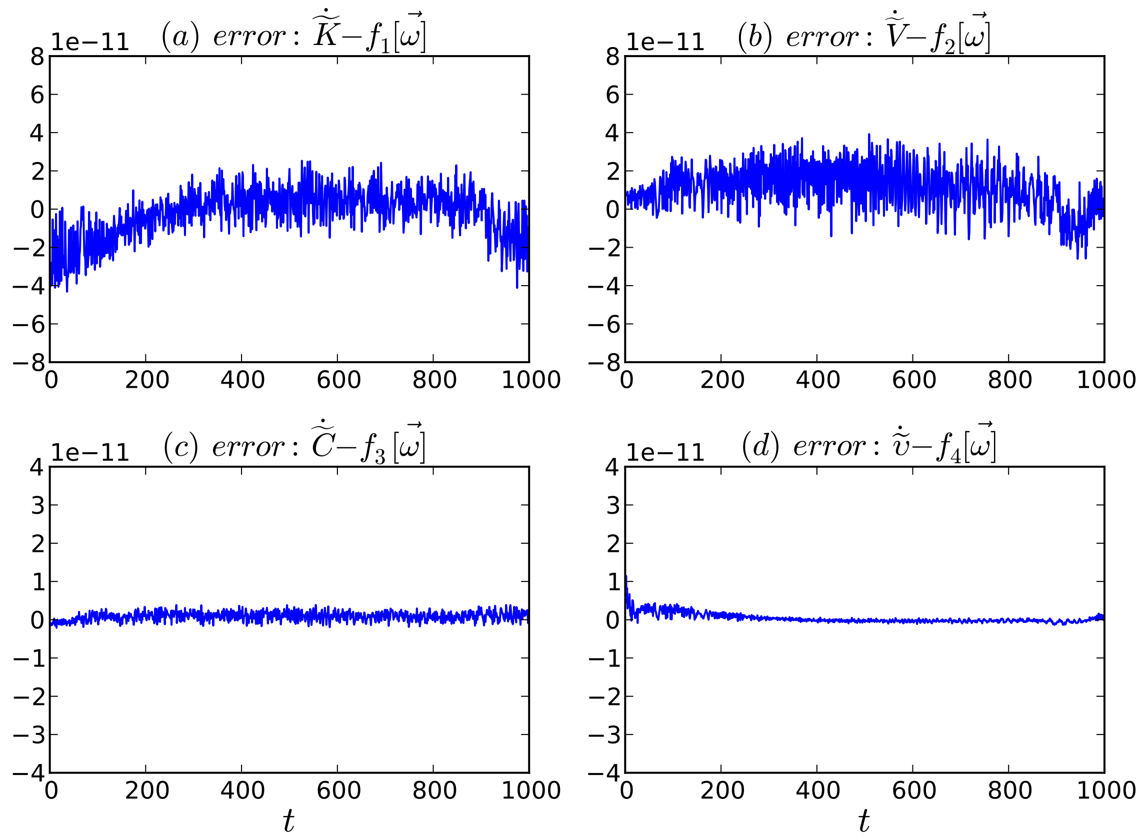
Table 2: Steady states of patent and Pareto-optimal bounty regimes

Patent Regime Steady State (benchmark)		Bounty Regime Steady State	
$\widetilde{K}_0$	6.07	$\widetilde{K}_1$	17.51
$\widetilde{V}_0$	1.00	$\widetilde{V}_1$	16.66
$\widetilde{Y}_0$	1.87	$\widetilde{Y}_1$	4.86
$\widetilde{C}_0$	1.36	$\widetilde{C}_1$	3.06
$\widetilde{\upsilon}_0$	1.72	$\widetilde{\upsilon}_1$	0.90
s_{K0}	0.24	s_{K1}	0.27
s_{V0}	0.0297	s_{V1}	0.0999
$\widetilde{C}_0/\widetilde{Y}_0 = C_0/Y_0$	0.73	$\widetilde{C}_1/\widetilde{Y}_1 = C_1/Y_1$	0.63
$\widetilde{K}_0/\widetilde{Y}_0 = K_0/Y_0$	3.24	$\widetilde{K}_1/\widetilde{Y}_1 = K_1/Y_1$	3.60
$\widetilde{V}_0/\widetilde{Y}_0$	0.53	$\widetilde{V}_1/\widetilde{Y}_1$	3.43
z_0	0.98	z_1	1.00
$\zeta_0^{Patented}$	0.48	$\zeta_1^{Bountied}$	1.00
$\eta - 1$	0.54	$\beta = \beta^*$	0.49
$\Pi/Y = \alpha m_0$	0.04	$B/Y = \alpha\beta$	0.18

Remarks: Independent of the regime change are the real interest of $r_0 = r_1 \approx 0.0499$ and the long-run growth rates: $\hat{V}_0 = \hat{V}_1 \approx 0.0324$; $\hat{K}_0 = \hat{Y}_0 = \hat{C}_0 = \hat{K}_1 = \hat{Y}_1 = \hat{C}_1 \approx 0.025$, $T\hat{F}P_0 = T\hat{F}P_1 \approx 0.063$. Computing Π/Y requires $m_0 = 0.1022$. Figures are computed using the benchmark parameters in Table 1.

In sum, the steady-state welfare improvement from the regime switch is enormous. However, to present a complete welfare analysis, we must take transitional dynamics into account. The next subsection is to trap how the dynamic system evolves in transition.

Figure 2: Absolute errors of solving (31a) -(31d) with Python's **scikits.bvp_solver**



Remarks: $\vec{\omega} \equiv [\widetilde{K}, \widetilde{V}, \widetilde{C}, \widetilde{\upsilon}]$ for $t \in [0, 1000]$ and $-1 \times 10^{-10} < error < 1 \times 10^{-10}$.

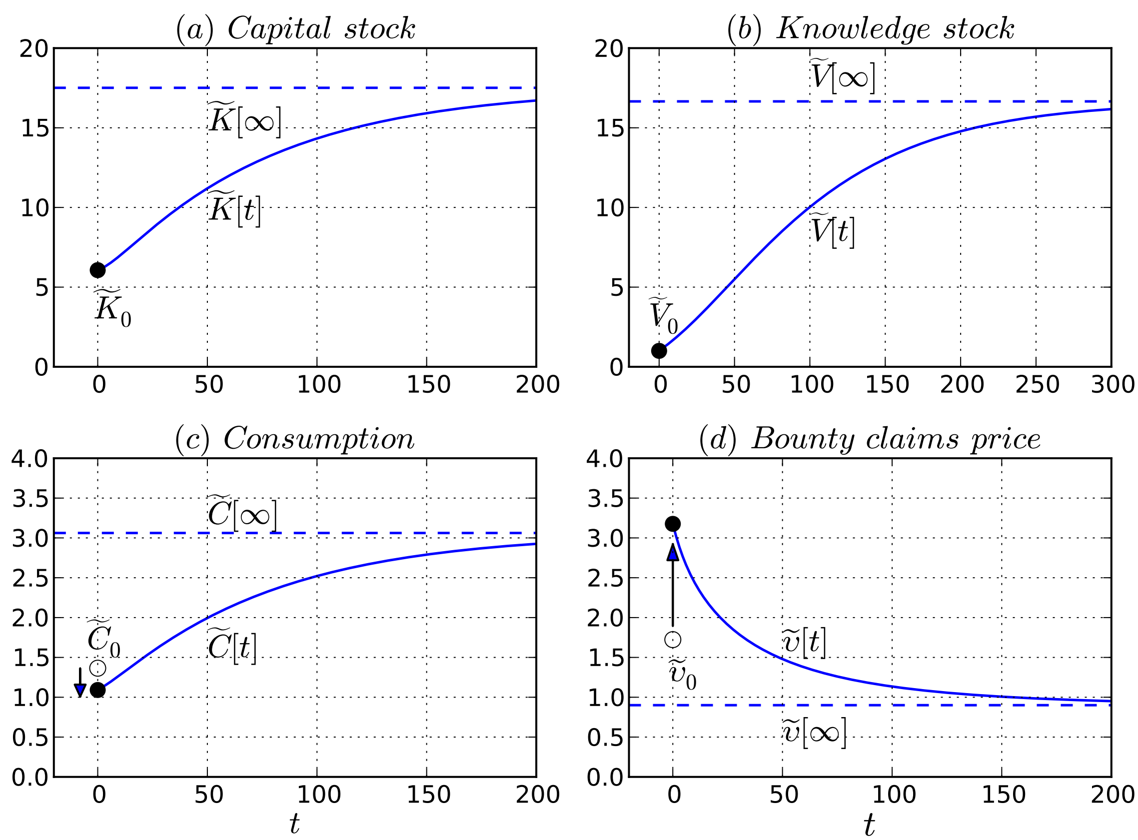
5.2 Dynamics in transition

Numerical solution The dynamic system of (31a) - (31d) calibrated by the benchmark parameter set (Table 1) is found to display saddle path stability around its long-run stationary equilibrium, which refers to the bounty-regime steady state, $[\widetilde{K}_1, \widetilde{V}_1, \widetilde{C}_1, \widetilde{\upsilon}_1]$, shown in Table 2. The regime switch from patents to the intertemporal bounty means a structural change taking place at $t = 0$. Computing the transitional dynamics should proceed by trapping the calibrated model's nonlinear stable manifold rather than its linearized model's local saddle path around the bounty-regime steady state. I used a Python boundary value problem solver (**scikits.bvp_solver 1.1**) to solve the calibrated system of (31a) - (31d) for the approximate solution $\vec{\omega} = [\widetilde{K}, \widetilde{V}, \widetilde{C}, \widetilde{\upsilon}]$.[33] The calibrated system is defined on $[0, \infty)$,

[33] **scikits.bvp_solver** (http://pypi.python.org/pypi/scikits.bvp_solver) is a Python package that wraps a Fortran BVP_Solver based on Mono Implicit Runge Kutta methods; see Shampine et al. (2006). All compu-

a semi-infinite time interval. To apply the Python bvp solver, I truncated it into a finite time interval, $[0, t_{max}]$, where $t_{max} = 1000$, and it turns out that one thousand years is long enough for economy to work out all transitional effects.[34] Indeed, the Python bvp solver proved fast in just a few seconds and generated precise enough results. As Figure 2 indicates, the approximate solution is precise to the extent that its absolute errors (or residuals) are all less than 10^{-10} in absolute value.

Figure 3: Numerical solution of dynamic system (31a) - (31d)



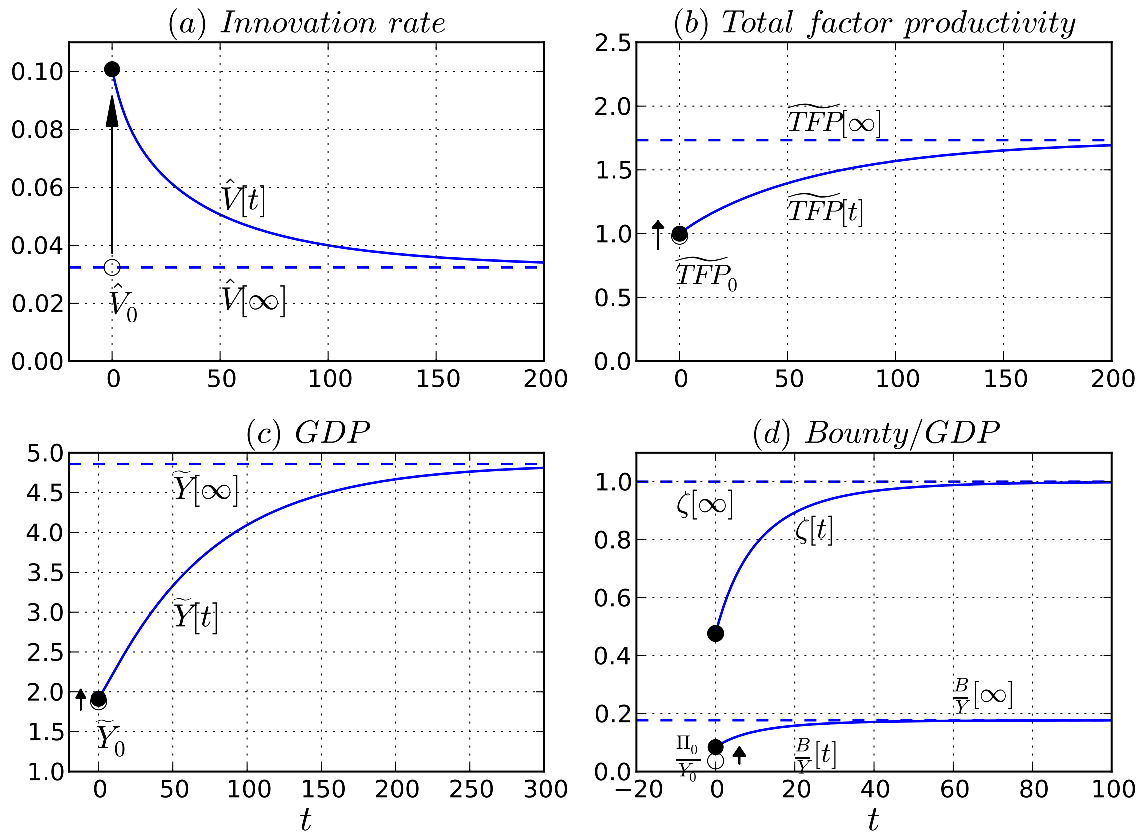
The approximate solution is shown in Figure 3 including the transition paths of the dynamic system's two state variables $(\widetilde{K}, \widetilde{V})$ and two jump variables $(\widetilde{C}, \widetilde{v})$. The stocks of capital $\widetilde{K}[t]$ and knowledge $\widetilde{V}[t]$ have continued to rise in transition until they both reached their long-run steady state equilibrium, respectively. At the moment of the regime switch,

tations for the present paper are implemented using Python 2.7.3 on Mac OS 10.6.8. The Python code is available from the author upon request. A Matlab code using **bvp4c** is also available.

[34]The boundary conditions at $t \to \infty$ need to be approximately satisfied at t_{max}. If not, we must increase the value of t_{max} until the errors are negligible.

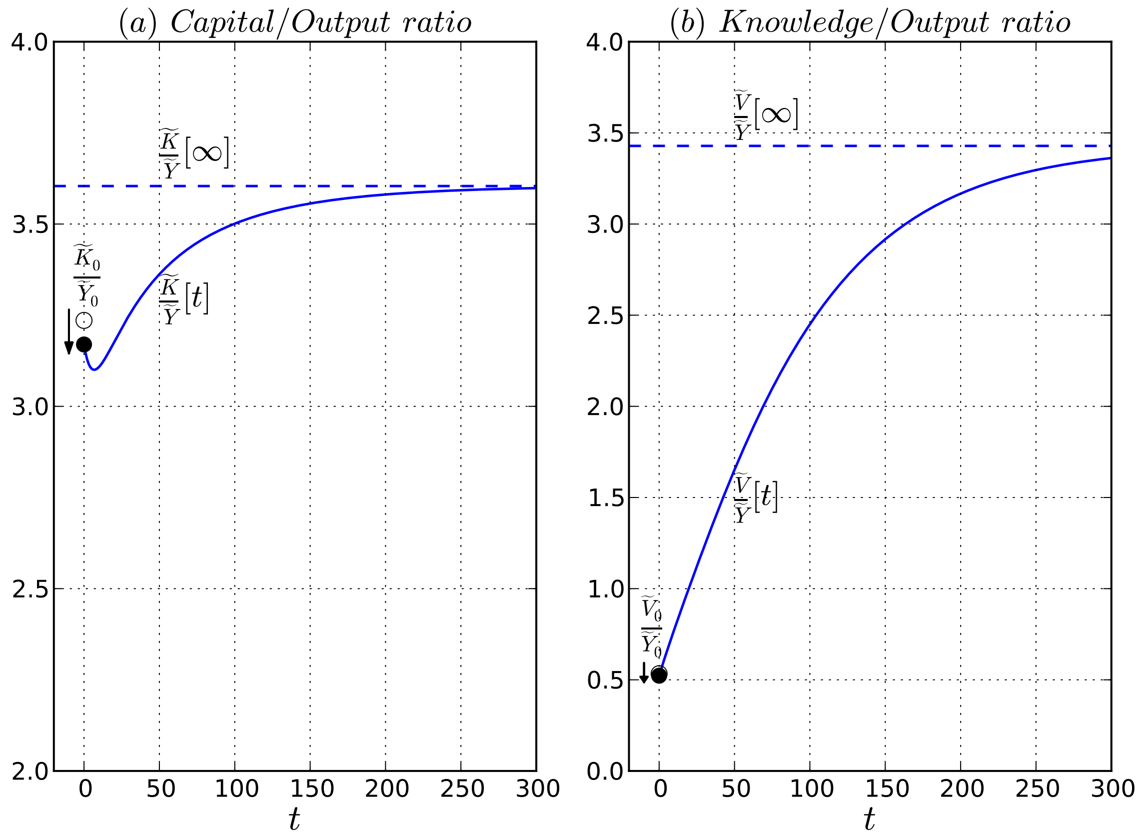
the consumption flow $\widetilde{C}$ jumps down from $\widetilde{C}_0$ to $\widetilde{C}[0]$ and then starts to grow over time toward a higher steady-state level. To the contrary, the price of the bounty claim $\widetilde{\upsilon}$ jumps up from $\widetilde{\upsilon}_0$ to $\widetilde{\upsilon}[0]$ and then starts to fall over time toward a lower steady-state level.

Figure 4: Transition paths: $\hat{V}$, $\widetilde{TFP}$, $\widetilde{Y}$, ζ, and B/Y



The above-described solution allows to compute other variables' trajectories. Figure 4 indicates that: (i) at the moment of the regime switch, the innovation rate $\hat{V} \equiv \dot{V}/V$ jumps up remarkably from $\hat{V}_0$ to $\hat{V}[0]$ and then drifts down smoothly in transition to the same steady state level ($\hat{V}_0 = \hat{V}_1 = \theta_V n$) (panel (a)); (ii) to a lesser degree, both total factor productivity $\widetilde{TFP}$ ($= \widetilde{A}^{1-\alpha} = \widetilde{V}^{1/(\varepsilon\sigma)}$) and final output $\widetilde{Y}$ jump up at $t = 0$, but unlike the innovation rate, they both continue to rise in transition to their long-run steady state levels (panels (b) & (c)); and (iii) driven by persistent innovation ($\hat{V}[t] > 0$), the bountied fraction ζ of capital goods keeps rising all the way toward one (its long-run steady state level), while the bounty/output ratio B/Y also keeps increasing in transition to a higher steady state ratio equal to 0.18 – this ratio is greater than the patent-regime steady state profit/output ratio

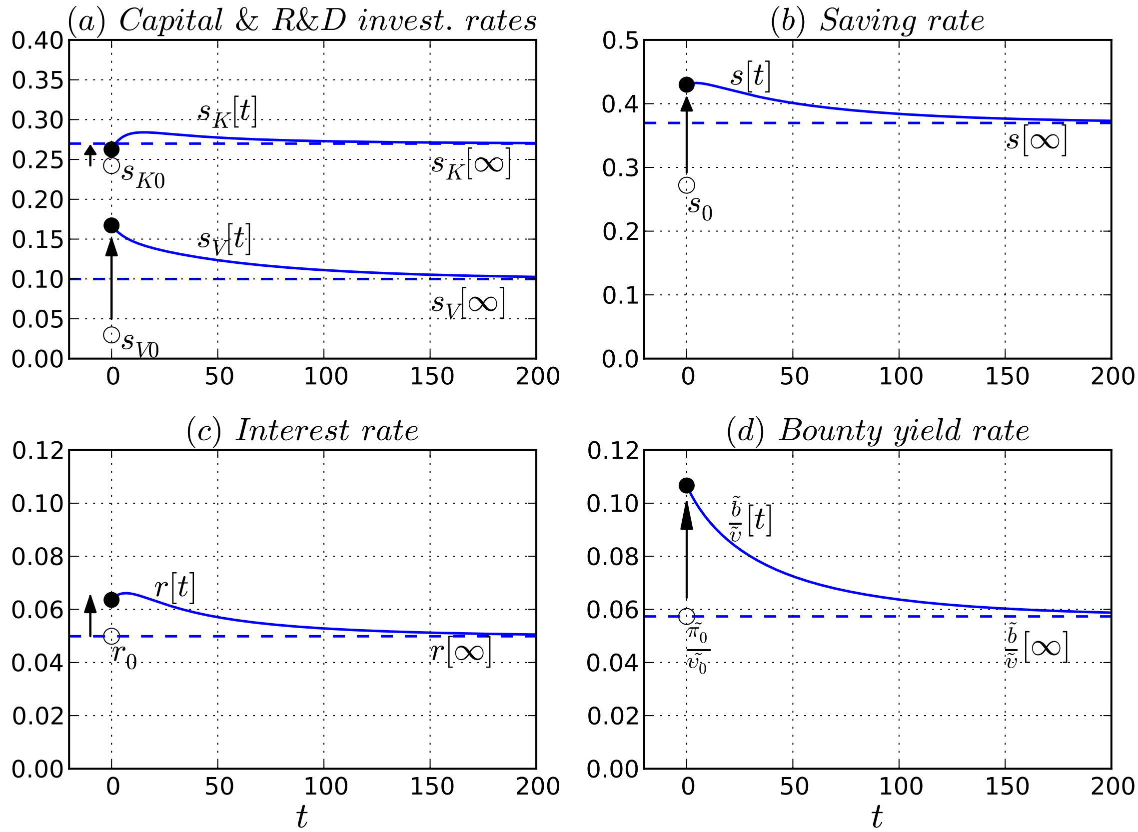
Figure 5: Transition paths: $\frac{\widetilde{K}}{\widetilde{Y}}$, $\frac{\widetilde{V}}{\widetilde{Y}}$



$\Pi_0/Y_0 \approx 0.04$ (Table 2 or pane d)

Notably, the regime switch makes the economy transition to one much more intensive in both capital and knowledge. Figure 5 indicates this feature: the capital/output and knowledge/output ratios ($\frac{\widetilde{K}}{\widetilde{Y}}$, $\frac{\widetilde{V}}{\widetilde{Y}}$) have trended upward over time remarkably to their new steady state equilibrium, although these ratios experience a small drop at the moment of the regime switch when final output Y jumps up slightly. All these transition paths described above make intuitive sense: a more capital-intensive, more knowledge-intensive economy must require the dual occurrences of short-run consumption sacrifices and bounty-claim appreciation (compared to the initial patent value $\widetilde{\upsilon}_0$) so as to draw more resources into capital accumulation and R&D investment. The simulated bounty regime that implements the Pareto bounty rate β^* can therefore raise the long-run productive capacity to sustain an eventually higher level of consumption than does the initial patent regime.

Figure 6: Transition paths: s_K, s_V, s, r, $\frac{\tilde{b}}{\tilde{v}}$



Market mechanism How does the decentralized market mechanism work to drive a patent-regime economy initially in its steady state to transition to the Pareto optimal bounty-regime balanced growth path, as characterized above by Figures 3 - 5? Ostensibly, this involves a removal of all-patent created distortions and the initiating of the Pareto-optimal bounty rate that fixes another two distortions stemming from the externalities of knowledge spillovers and research congestion while also presenting research rewards strong enough to strengthen an innovative sector. As such, at the moment of the regime switch, the market for capital goods becomes perfectly competitive immediately and all durables sell at marginal cost. This boosts market demand especially for those durables whose patents have not yet expired and removes the patent-induced technical distortion on total factor productivity (i.e., z_0 jumps to $z[0] = 1$). This market mechanism explains why both total factor productivity and GDP ($\widetilde{TFP}$, $\widetilde{Y}$) jump up at $t = 0$ (Figure 4 (b) & (c)), accompanied by the capital /output ratio ($\frac{\widetilde{K}}{\widetilde{Y}}$) and the knowledge/output ratio ($\frac{\widetilde{V}}{\widetilde{Y}}$) jumping down at the same time (Figure 5), given that $\widetilde{K}$ and $\widetilde{V}$ are predetermined at any point in time.

Further, at the moment of the regime switch, the Pareto-optimal bounty rate β^* is strong enough to reward technological innovation, thereby not only creating bounty claims more valuable than patents ($\widetilde{\upsilon}$ jumps up at $t=0$; Figure 3 (d)) in the asset market, but also generating a bounty flow relative to the value of a bounty claim, $\frac{\widetilde{b}}{\widetilde{\upsilon}}$, greater than the monopoly profit flow relative to the value of a fresh patent, $\frac{\widetilde{\pi}}{\widetilde{\upsilon}}$ (Figure 6 (d)). Thus, switching to the bounty regime presents strong incentives to boost investment in both physical capital and R&D, causing both s_K and s_V to jump up at $t=0$ (Figure 6 (a)). The resulting strong demands for foregone consumption therefore make the real interest rate r (Figure 6 (c)) and the saving rate s (Figure 6 (b)) jump up, respectively, at the moment of the regime switch. A more capital-intensive, more knowledge-intensive economy is then in the making in transition.

5.3 Welfare change in transition to the bounty-regime steady state

Using the numerical trajectory of consumption shown earlier in Figure 3 (c), one can compute the welfare change for the regime switch from patents to an intertemporal bounty that allows a decentralized economy to attain Pareto optimality in the long run. Following Lucas (1987), the welfare change is the measure of $\Omega[t]$ that obeys [35]

$$U[\{(1+\Omega[t])c[\tau]^{Patent}\}_{\tau=0}^{t}] = U[\{c[\tau]^{Bounty}\}_{\tau=0}^{t}], \;\; 0 \le \tau \le t < \infty \tag{35}$$

where $\{c[\tau]^{Patent}\}_{\tau=0}^{t}$ is a stream of per capita consumption for the time interval of $[0,t]$ under the steady-state patent regime and $\{c[\tau]^{Bounty}\}_{\tau=0}^{t}$ is the stream of per capita consumption for the same time interval under the bounty regime. As such, starting from the moment of the regime switch, $\Omega[t]$ measures a consumption gain for a time interval with the length of duration equal to t, whereas $\Omega[\infty]$ is the consumption gain for the entire semi-infinite time interval, $[0,\infty)$. For instance, if $\Omega[t]$ turns out to be positive, it means that in time interval $[0,t]$, the regime switch generates a welfare gain equivalent to a percent $(=\Omega[t]\times 100\%)$ increase from the patent-regime steady state.

Formulae for Ω Note that per capita consumption $c[t] \equiv C[t]/L[t]$ can be written as $c[t] = \widetilde{C}[t]L[t]^{\theta_K - 1} = \widetilde{C}[t]L[t]^{\theta_A} = \widetilde{C}[t]e^{\theta_A n}$ in terms of (23) and $L[t] = e^{nt}$. Now, given

[35] The same approach was used in King and Rebelo (1990), Ireland (1994), and Lin and Russo (1999).

the initial steady state consumption of $\widetilde{C}_0$ and the bounty-regime transition path of $\widetilde{C}[t]$, one can compute the welfare measure of $\Omega[t]$, $t \in [0, \infty)$, based on the following formulae:

$$\Omega[t] = \begin{cases} \psi_a[t]^{1/(1-\gamma)} - 1, \ \text{with} \ \psi_a[t] = \frac{\int_0^t \widetilde{C}[\tau]^{1-\gamma} e^{-(\rho-n-(1-\gamma)\theta_A n)\tau} d\tau}{\int_0^t \widetilde{C}_0^{1-\gamma} e^{-(\rho-n-(1-\gamma)\theta_A n)\tau} d\tau}, & \gamma \neq 1 \\ \\ e^{\psi_b[t]} - 1, \ \text{with} \ \psi_b[t] = \frac{\int_0^t \log[\widetilde{C}[\tau]/\widetilde{C}_0] e^{-(\rho-n)\tau} d\tau}{\int_0^t e^{-(\rho-n)\tau} d\tau}, & \gamma = 1 \end{cases} \tag{36}$$

If one overlooked the transitional impacts by presuming that the economy jumps from the initial patent-regime balanced-growth path directly to the new bounty-regime balanced-growth path, then the bounty-regime scale-adjusted consumption $\widetilde{C}$ would jump to $\widetilde{C}_1$ from $\widetilde{C}_0$ at $t = 0$ and formulae (36) would reduce to

$$\Omega[t] = \frac{\widetilde{C}_1}{\widetilde{C}_0} - 1 \equiv \overline{\Omega}, \ t \in [0, \infty), \tag{37}$$

irrespective of whether $\gamma \neq 1$ or $\gamma = 1$.

Magnitudes of $\Omega[t]$, $\Omega[\infty]$ and $\overline{\Omega}$ From the forgoing transitional analysis, the time profile of the consumption flow (Figure 3 (c)) must imply that that $\Omega[t] \leq \Omega[\infty] < \overline{\Omega}$ for $t \in [0, \infty)$. Indeed, under the benchmark parameter set, $\overline{\Omega} \approx 1.24$ implies that the welfare improvement resulting from the regime switch is equivalent to a remarkable increase of 124% in steady-state consumption. If one takes into account short-run consumption sacrifices in transition, then the measure of $\Omega[\infty] \approx 0.2180$ implies a consumption gain of 21.80%, which remains impressive enough, though. Figure 7 demonstrates the transition path of $\Omega[t]$ as the calibrated economy evolves toward a new balanced-growth path. As indicated, the initial welfare loss from the regime switch appears significant: for the first year ($t = 1$), this loss is about a 20% decrease in consumption, but it keeps diminishing over time. By the time of $t \approx 35$, the economy has fully recouped the losses of the earlier periods and will eventually secure a net gain up to 21.80% in the long run ($\Omega[t]$ converges to 21.80 as $t \to \infty$).

Robustness checks The dynamic system of (31a) - (31d) exhibits saddle-path stability. This feature appears robust to parameter changes according to numerous experiments. The welfare measure of $\Omega[\infty]$, however, is sort of sensitive to the change in γ (intertemporal substitution) and rather sensitive to the change in λ (research congestion). As shown in

Figure 7: Changing welfare in transition based benchmark parameters including $\gamma = 1$

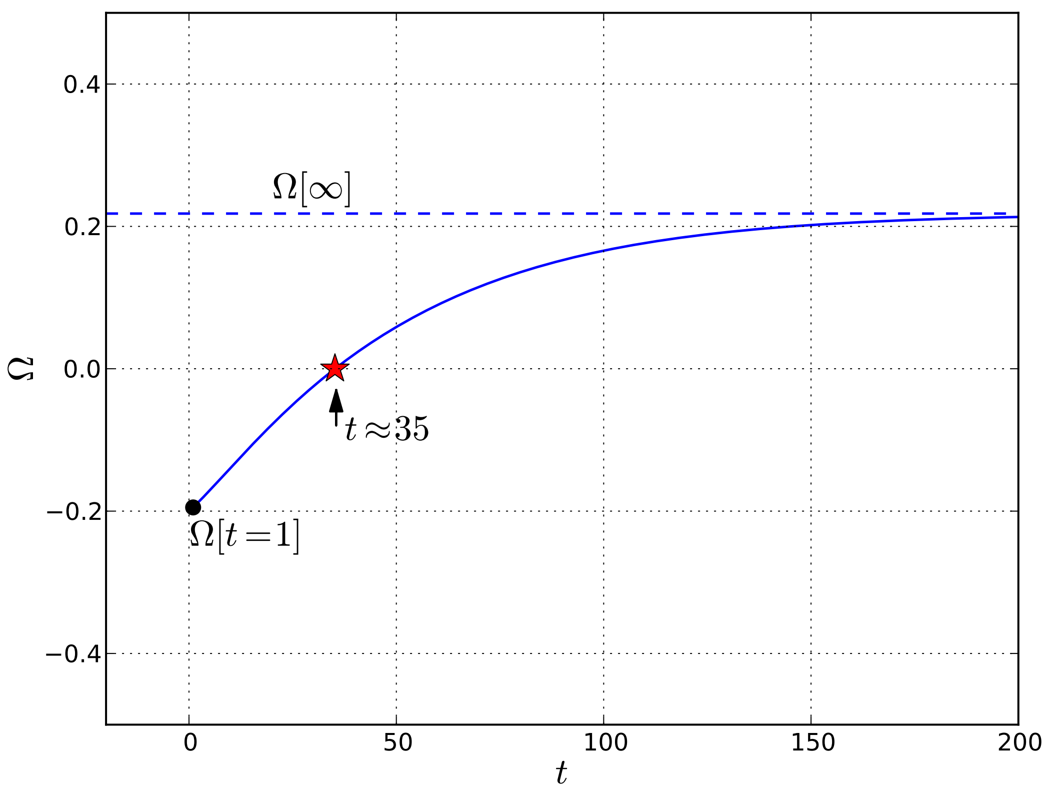


Table 3, for instance, an increase in the value of γ from 1.00 (benchmark) to 2.00 causes $\Omega[\infty]$ to drop from 0.2180 to 0.1273 under the benchmark parameter of $\lambda = 0.65$. That is, if the elasticity of intertemporal substitution in consumption $(1/\gamma)$ is decreased from 1.00 to $0.5(= 1/2.00)$, the percent gain in consumption drops significantly from 21.80% to 12.73% for the entire time horizon, although the steady-state consumption gain, measured by $\overline{\Omega}$, appears quite insensitive to this parameter change. This is consistent with our economic intuition: if future consumption is less substitutable for present consumption, the measure of $\Omega[\infty]$ must get smaller.

Even more sensitive is the measure of $\Omega[\infty]$ with respect to parameter λ, which inversely reflects the extent of research congestion. For instance, given the benchmark parameter of $\gamma = 1.00$, a decrease in λ from 0.65 to 0.50 causes the measure of $\Omega[\infty]$ to drop sharply from 0.2180 to 0.0966 and the measure of $\overline{\Omega}$ to fall from 1.24 to 0.64. Namely, if the extent of research congestion is so significant that λ decreases to 0.50, the welfare improvement resulting from the regime switch is reduced to a gain of 9.66% in consumption, compared to the 21.80% gain under the benchmark parameter set. Regardless, the central message of these numerical experiments maintains that an intertemporal bounty regime can be a welfare-improving Pareto optimal alternative to the world's patent system.

Table 3: Sensitivity analysis of parameters γ and λ on welfare change

γ	$\lambda = 0.65^*$		$\lambda = 0.50$	
	$\overline{\Omega}$	$\Omega[\infty]$	$\overline{\Omega}$	$\Omega[\infty]$
2.00	1.12	0.1273	0.58	0.0668
1.50	1.18	0.1624	0.61	0.0792
1.00*	1.24	0.2180	0.64	0.0966
0.75	1.28	0.2582	0.65	0.1082
0.50	1.32	0.3109	0.67	0.1233

Remark: $*$ indicates a benchmark parameter.

6 Concluding Remarks

First-generation endogenous growth models feature the empirically implausible scale effects, implying a faster steady-state growth rate for a larger economy. The progression to non-scale growth models removes the unwanted scale effects, but at the expense of raising the dimensionality. The present paper is no exception. It has to deal with a four dimensional dynamic system that governs the transitional dynamics when the initial patent regime is switched to an intertemporal bounty regime that implements the Pareto optimal bounty rate to reward technological innovation. This system consists of two state variables – capital & knowledge stocks – evolving at different paces and must be normalized into a stationary one using multiple normalization factors. Python's boundary value problem solver (**scikits.bvp_solver**) proved fast and precise in solving the normalized dynamic system.

The paper has demonstrated that a decentralized market mechanism via the proposed intertemporal bounty system can duplicate Pareto optimality, provided that the rate of bounty is set rightly to internalize externalities. The paper can extend by allowing bounty claims to be finitely lived. In general, infinitely-lived bounty claims and the associated Pareto optimal bounty rate should not be the only institutional arrangement that can attain Pareto optimality. Instead, there should be a stream of bounty terms (length) and rate combinations that are Pareto efficient in the long run. So, what is the optimal mix for a bounty's length and rate? This question is important and will become complicated if the intertemporal bounty must be financed by factor income taxes rather than the lump-sum tax presumed for the present study. The socially optimal mix of bounty length and rate should be the one that can minimize the tax-induced efficiency loss and short-run consumption sacrifices needed to enable the economy to evolve toward Pareto optimality in the long run.

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Appendix

A Patent regime's balanced growth paths

Along a balanced-growth path, every variable grows at a constant rate, including zero growth. Such steady-state growth rates ($\hat{V}_0 \equiv \frac{\dot{V}_0}{V_0}$, $\hat{A}_0 \equiv \frac{\dot{A}_0}{A_0}$, $\hat{K}_0 \equiv \frac{\dot{K}_0}{K_0}$ and $\hat{Y}_0 \equiv \frac{\dot{Y}_0}{Y_0}$) can be obtained by solving the four equations system:

$$\hat{Y}_0 - \hat{K}_0 = 0 \tag{A.1a}$$

$$(\phi - 1)\hat{V}_0 + \lambda \hat{Y}_0 = 0 \tag{A.1b}$$

$$\hat{Y}_0 - (1-\alpha)\hat{A}_0 - \alpha \hat{K}_0 = (1-\alpha)n \tag{A.1c}$$

$$\hat{A}_0 - \frac{1}{\varepsilon\sigma(1-\alpha)}\hat{V}_0 = 0 \tag{A.1d}$$

These equations result from (21a), (21b), (18), and the definition of A. For instance, (21a) implies $\hat{Y}_0 = \hat{K}_0 = \hat{C}_0$. Solving (A.1a) - (A.1d) yields (22a) - (22c). From (14) and (8b), the patent price equation is given by $\upsilon = s_V Y/\dot{V} = s_V Y/(\hat{V}V)$. Along a balanced-growth path, both s_V and $\hat{V}$ are stationary. Thus, differentiating the patent price equation yields

$$\hat{\upsilon}_0 \equiv \frac{\dot{\upsilon}_0}{\upsilon_0} = \hat{Y}_0 - \hat{V}_0 = (\theta_K - \theta_V)n \tag{A.2}$$

Derivations of the remaining stationary variables are in order. First, from the Euler condition (3) and using the growth kernel θ_A, the steady-state interest rate is given by

$$r_0 = \rho + \gamma\theta_A n \tag{A.3}$$

Next, from (20b), the steady-state capital/output ratio is given below

$$\frac{K_0}{Y_0} = \frac{\alpha(1-m_0)}{r_0+\delta} \tag{A.4}$$

This result and (8a) combine to imply the steady-state capital investment rate,

$$s_{K0} = \frac{\alpha(1-m_0)}{r_0+\delta}(\delta+\theta_K n) \tag{A.5}$$

Recall that Π denotes aggregate profit flows and $\pi = \Pi/(\zeta V)$ measures a patented firm's profit flow. Using (20a) and (13) yields the steady-state price of a newly issued patent,

$$\upsilon_0[t] = \frac{\alpha m_0 Y_0[t]}{\zeta_0 V_0[t]}\left(\frac{1-e^{-(r_0-\hat{Y}_0+\hat{V}_0)T}}{r_0-\hat{Y}_0+\hat{V}_0)}\right), \text{ or } \widetilde{\upsilon}_0 = \frac{\alpha m_0 \widetilde{Y}_0}{\zeta_0 \widetilde{V}_0}\left(\frac{1-e^{-(r_0-\hat{Y}_0+\hat{V}_0)T}}{r_0-\hat{Y}_0+\hat{V}_0)}\right) \tag{A.6}$$

where $\widetilde{\upsilon}_0 = \upsilon_0[t]/L[t]^{\theta_K-\theta_V}$, $\widetilde{Y}_0 = Y_0[t]/L[t]^{\theta_K}$, and $\widetilde{V}_0 = V_0[t]/L[t]^{\theta_V}$ are constant on a balanced-growth path. Also, with $\dot{\zeta}_0 = 0$ and $\hat{V} = \hat{V}_p = \hat{V}_{np} = \theta_V n$ in the steady state, either (16a) or (16b) implies the equilibrium patented goods fraction,

$$\zeta_0 = 1 - e^{-\hat{V}_0 T} \tag{A.7}$$

The R&D equilibrium condition (14) requires $\xi[t]\upsilon[t] = 1$ at any instant, which implies $\upsilon[t]\dot{V}[t] = s_V[t]Y[t]$ in terms of (8b). Along a balanced growth, we therefore have $s_{V0} =$

$\upsilon_0 \hat{V}_0 V_0 / Y_0$, which, together with (A.6), implies

$$s_{V0} = \hat{V}_0 \left(\frac{\alpha m_0}{\zeta_0} \right) \left(\frac{1 - e^{-(r_0 - \hat{Y}_0 + \hat{V}_0)T}}{r_0 - \hat{Y}_0 + \hat{V}_0} \right) \tag{A.8}$$

Lastly, the two stock variables (K, V) are non-stationary. Using these scale-adjusted quantities, $\widetilde{K} \equiv K/L^{\theta_K}$, $\widetilde{Y} \equiv Y/L^{\theta_K}$, and $\widetilde{A} = A/L^{\theta_A}$, we can transform the final goods production function (18) into its intensive form, $\widetilde{Y} = z\widetilde{A}^{1-\alpha}\widetilde{K}^{\alpha}$, and the motion of capital accumulation (12) into $\dot{\widetilde{K}} \equiv \frac{d\widetilde{K}}{dt} = s_K \widetilde{Y} - (\delta + \theta_K n)\widetilde{K}$. Setting $\dot{\widetilde{K}} = 0$ then yields the steady state scale-adjusted stock of capital,

$$\widetilde{K}_0 = \widetilde{A}_0 \left(\frac{z_0 s_{K0}}{\delta + \theta_K n} \right)^{1/(1-\alpha)}, \tag{A.9}$$

On a balanced-growth path, the stock of knowledge capital V keeps growing at a rate equal to $\theta_V n$. But we can normalize V to $\widetilde{V} \equiv V/L^{\theta_V}$. Then dividing each side of the innovation equation (21b) by V_0 while replacing V_0 and Y_0 with $\widetilde{V}_0 L^{\theta_V}$ and $\widetilde{Y}_0 L^{\theta_K}$ in this equation, we obtain

$$\frac{\dot{V}_0}{V_0} = \theta_V n = \mu \widetilde{V}_0^{\phi-1} (s_{V0}\widetilde{Y}_0)^{\lambda} L^{(\phi-1)\theta_V + \lambda\theta_K} \tag{A.10}$$

where term $L^{(\phi-1)\theta_V+\lambda\theta_K}$ can drop out due to $(\phi-1)\theta_V + \lambda\theta_K = 0$ (see (23)). Now using $\widetilde{Y}_0 = z_0 \widetilde{A}_0^{1-\alpha} \widetilde{K}_0^{\alpha}$, $\widetilde{A}_0 = \widetilde{V}_0^{1/(\varepsilon\sigma(1-\alpha))}$, and result (A.9), we can easily solve (A.10) for the steady state scale-adjusted stock of knowledge, which is given in (24b).

B Steady state of the normalized dynamic system

First, setting $\dot{\widetilde{K}} = 0$ in (29a) and using (32a) yield the stationary equilibrium of $\widetilde{K}$,

$$\widetilde{K}_1 = \left(\frac{s_K}{\delta + \theta_K n} \right)^{1/(1-\alpha)} \widetilde{A}_1 \tag{B.1}$$

Second, setting $\dot{\widetilde{K}} = 0$ in (29a) and using (32e) yield the equilibrium capital investment rate,

$$s_{K1} = (\delta + \theta_K n) \frac{\widetilde{K}_1}{\widetilde{Y}_1} = (\delta + \theta_K n) \left(\frac{\alpha}{r_1 + \delta} \right) \tag{B.2}$$

Third, setting $\dot{\widetilde{V}} = 0$ in (31b) yields $(s_{V1}\widetilde{Y}_1)^{\lambda} = \frac{\theta_V n}{\mu \widetilde{V}_1^{\phi-1}}$ and (32d) implies $(s_{V1}\widetilde{Y}_1)^{1-\lambda} =$

$\mu\widetilde{\upsilon}_1\widetilde{V}_1^{\phi}$. Combining these two equations yields the equilibrium level of R&D,

$$s_{V1}\widetilde{Y}_1 = \widetilde{\upsilon}_1\widetilde{V}_1\theta_V n \tag{B.3}$$

Fourth, replacing $\widetilde{b}$ with (32b) and setting $\dot{\widetilde{\upsilon}} = 0$ in (31d), we obtain

$$r_1 - \beta\alpha\frac{\widetilde{Y}_1}{\widetilde{\upsilon}_1\widetilde{V}_1} = (\theta_K - \theta_V)n \tag{B.4}$$

Then replacing $\widetilde{\upsilon}_1\widetilde{V}_1$ in (B.4) with $s_{V1}\widetilde{Y}_1/\theta_V n$ based on the result of (B.3), we obtain the equilibrium R&D investment rate,

$$s_{V1} = \frac{\beta\alpha\cdot\theta_V n}{r_1 - (\theta_K - \theta_V)n} \tag{B.5}$$

where $r_1 = r_0 = \rho + \gamma\theta_A n$. Substituting s_{V1}into (B.3) yields (33d) in the text.

Fifth, using $\widetilde{Y} = \widetilde{A}^{1-\alpha}\widetilde{K}^{\alpha}$ and (B.1), we can rewrite $(s_{V1}\widetilde{Y}_1)^{\lambda} = \frac{\theta_V n}{\mu\widetilde{V}_1^{\phi-1}}$ (from step 3) as

$$s_{V1}^{\lambda}\left(\frac{s_{K1}}{\delta+\theta_K n}\right)^{\lambda\alpha/(1-\alpha)}\widetilde{A}_1^{\lambda} = \frac{\theta_V n}{\mu\widetilde{V}_1^{\phi-1}} \tag{B.6}$$

Then replacing $\widetilde{A}_1$ with $\widetilde{V}_1^{1/(\varepsilon\sigma(1-\alpha))}$ and collect the $\widetilde{V}_1$ terms in (B.6), we obtain (33b) in the text.

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