Spillovers in a New Keynesian Continuous Time Framework with Financial Markets [*]

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Abstract

One lesson to be learned from the recent financial and real economic crisis is that macroeconomists need to more sufficiently account for the influence of financial markets. This paper explores the consequences of treating the interaction between different financial markets, monetary policy, and the real economy seriously by developing a fully dynamic theoretical modelling framework. Using a standard New Keynesian framework as a starting point, we extend the model with stochastic differential equations so as to analyse spillover effects and steady-state properties. We solve the model for theoretically derived parameters, distinguishing between (almost) closed, equally-sized, and differently-sized economies. In an empirical application, we estimate model parameters for the case of Canada and the United States. Using Lyapunov techniques, we find evidence for instabilities in the US and Canadian financial systems.

Keywords: New Keynesian model, Philipps Curve, Taylor Rule, Stochastic Differential Equations.

JEL Classification Numbers: C02, C63, E44, E47, E52 F41

1 Introduction

The recent sovereign debt crisis emphasises how important financial markets are for macroeconomic stability. This should not be a surprise, as financial markets played important roles in previous major crises, too, such as the Great Depression and the Asian financial crisis. Arguably, these crises were triggered by financial market turmoil and each required a different stabilisation policy to avoid further (financial) contagion. The Great Depression was initiated by a stock market crash, the Asian crisis was caused by speculative attacks on the Thai Baht, and the European sovereign debt crisis was triggered by speculative attacks on sovereign bonds due to an increase in public debt, at least partly caused by government intervention to support the banking sector. It is the latter crisis which particularly underlines the fact that globalisation has resulted in much greater international economic interconnectedness than was the case in the past. Thus, policymakers need to be aware not only of financial market developments in their own economy but also of those occurring abroad. In addition, they have to keep in mind differences in the size and structure of financial markets so as to be able to design appropriate policy. In a world still largely dominated by national policymaking, this is a considerable hurdle to the development of optimal policy.

In light of this situation, we believe a better understanding of the role different financial markets play in a macroeconomy is crucial. Furthermore, observing not only financial market actions but also reactions to different policies is helpful in designing appropriate (monetary) policy. Policymakers' responses to shocks could be improved by the availability of a thorough theoretical framework. However, common macroeconomic approaches used by central banks, both analytically as well as empirically, tend to downplay the role of financial markets in the conduct of monetary policy, and rarely, if ever, take into account different markets.

We argue that academic research can help guide central banks' efforts by placing a greater emphasis on financial markets and moving beyond common

macroeconomic models. Therefore, the purpose of this paper is to explore the consequences of treating the interaction between financial markets, monetary policy, and the real economy in a globalised world seriously by developing a fully dynamic theoretical modelling framework. We are particularly interested in shedding light on the relationship between financial markets and monetary policy, which can be characterised by a substantial degree of simultaneity.

In mainstream macroeconomic research, the New Keynesian (NK) model (Blinder 1997; Clarida, Gali, and Gertler 1999; Romer 2000; Woodford 1999) has become a frequently chosen starting point for the analysis of monetary policy. We follow this strand of literature and adopt the three-equation NK model as our baseline model. Two equations, aggregate demand and aggregate supply, describe the structure of the economy. The third equation models monetary policy reactions in the form of a Taylor rule and reflects policymakers' predominant targets. A general overview of the properties of the Taylor rule is provided by (Asso, Kahn, and Leeson 2007). The NK model is a mix of traditional Keynesian and New Classical approaches. It is largely consistent with important features of real business cycle models, but also includes monopolistic competition and nominal rigidities. Arguably, including the Taylor rule as the monetary policy reaction function helps link theory and practice (Gali 2008). Empirical tests of the conventional NK model show some support for this class of models, but the evidence is not overwhelmingly strong (Christiano and Eichenbaum 2005; Dennis 2004; Gali, Smets, and Wouters 2011; Ireland 2001; Lubik and Smets 2005; Negro et al. 2007; Peersman and Straub 2006; Rotemberg and Woodford 1998; Schorfheide 2011; Smets and Wouters 2003).

As our main interest lies in the analysis of monetary policy and its interaction with financial markets, we need to extend the NK model further. Our starting point is that monetary policy reacts to financial markets and financial markets react to monetary policy and that this relationship is characterised by a notable degree of simultaneity. This is not a new idea, as the literature contains studies taking such simultaneity into account. However, most of these are empirical in nature, e.g., Bjornland and Leitemo (2009); Rigobon (2003); Rigobon and Sack (2003). Rigobon (2003); Rigobon and Sack (2003) observe

the effects of bonds and stocks on monetary policy and vice versa by applying a novel approach (identification through heteroscedasticity) to circumvent simultaneity issues. Bjornland and Leitemo (2009) adopt a VAR approach for studying the question of how financial markets affect monetary policy. Technically, they deal with the simultaneity issue by imposing a priori short-run and long-run restrictions. Both studies find strong evidence of monetary policy reaction to financial market developments.

In a less formal manner, Hildebrand (2006) argues that financial markets are the link between monetary policy and the real economy and are thus an important part of the transmission mechanism for monetary policy. Moreover, he argues that financial markets reflect expectations about future inflation and output and hence are also affected by monetary policy. Christiano et al. (2008) put forward a more formal empirical approach. Considering the problem of boom-bust cycles in the economy, they find evidence that it might be more expedient for monetary policymakers to target credit growth instead of inflation. Faia and Monacelli (2007) find evidence that monetary policy should respond to asset price hikes. Reflecting these considerations, Belke and Klose (2010) estimate Taylor rules for the European Central Bank (ECB) and the Federal Reserve (Fed) and include asset prices as additional monetary policy targets. They find statistically significant evidence that short-term interest rates respond to financial market developments, but the quantitative economic effects appear to be small.

The studies discussed so far focus on stock or bond markets. However, another important financial market is the one for foreign exchange. Clarida, Gali, and Gertler (2002) incorporate the exchange rate in a NK two-country model, where domestic and foreign households share the same preferences. They find, under quite restrictive assumptions, that purchasing power parity holds and that the consumption real exchange rate is constant. Gali and Monacelli (2005) extend this approach by applying Calvo sticky pricing and analysing the policy effects of either a Taylor rule or an exchange rate peg. Engels (2009) extend this open economy model by incorporating local-currency pricing and allowing for differences in domestic and foreign household preferences. Including the

exchange rate in the monetary policy analysis not only takes account of a large and important financial market, the exchange rate could also be viewed as a policy objective itself. For example, Leitemo and Söderström (2005) include exchange rate uncertainty into the framework of a NK model and analyse different monetary policy rules. They find evidence that an interest rate reaction function in the form of a Taylor rule works particularly well. Similarly, Wang and Wu (2012) report that in their analysis of a group of exchange rate models for 10 OECD countries, the Taylor rule performs best empirically as a monetary policy rule. Taylor (2001) generally discusses the role of the exchange rate in monetary policy rules. Reflecting these empirical and theoretical findings, we model the policy reaction function as a Taylor rule.

Our paper makes various contributions to the extant literature. First, we include exchange rates as well as stock prices in the monetary policy equation. Second, we explicitly address the issue of simultaneity between monetary policy and financial markets by incorporating three financial markets (i.e., markets for foreign exchange, bonds, and stocks) as an additional set of equations in our baseline model. Third, we undertake formal mathematical analysis of the extended model consisting of two additional financial market equations in a continuous-time framework. We transform the NK model in stochastic differential equations, apply numerical algorithms to derive stable solutions, and study the evolution of various variables over time. Our reading of the literature is that the combination of applying Taylor rules to financial markets and extending the NK model with two dynamic financial market equations is unique.

Our approach shows some similarity to previous papers by Asada et al. (2006); Chen et al. (2006a,b). These authors transform the Keynesian AS/AD model into a disequilibrium model with a wage-price spiral and include two Phillips curves, one targeting wages, the other targeting prices. The model is transformed into five differential equations-explaining real wages, real money balances, investment climate, labour intensity, and inflationary climate-and its dynamics are extensively analysed Malikane and Semmler (2008b) extend this framework by including the exchange rate and Malikane and Semmler (2008a)

consider asset prices. However, inclusion of both asset markets and exchange rates has not yet been undertaken, particularly not in a framework controlling for the simultaneity between monetary policy and financial markets.

In the next section, we develop the theoretical model. Section 2 contains a brief description and analysis of the baseline model. In Section 3, we study this model further by making various assumptions about size and degree of openness of the simulated economies. Section 4 concludes.

2 The Baseline Model

We base our model on the New-Keynesian three equation model of Clarida, Gali, and Gertler (1999). Aggregate supply is represented by the NK Phillips curve (NKPC), as is done, for example, by Blanchard (2007, 2008). It evolves from a Calvo pricing equation, which mirrors price rigidities by allowing only a fraction of firms to implement price changes at one point in time. The literature proposes several extensions and variations of the NKPC. Roberts (1995) applies an expectation-augmented Phillips Curve. Woodford (2005) studies the NKPC by including firm-specific capital. A general overview of recent achievements is provided by Turnovsky (2011). However, since our focus is on the integration of financial markets, we employ the original NKPC to keep the model tractable. Aggregate demand is modelled by a (dynamic) investment/saving (IS) curve. Households' optimal saving decisions lead to the consumption Euler equation and log-linearisation of it yields the new IS curve, where current output is driven by expected future output and interest rates (Allsopp and Vines 2000; Clarida, Gali, and Gertler 1999).

Due to our concerns about globalisation and the importance of studying open economies, we follow the recent literature and adjust the NK model. Extending aggregate demand to an open economy model is relatively easy. We follow Bofinger, Mayer, and Wollmershäuser (2005) and add net exports and the exchange rate to the IS curve. Regarding aggregate supply, we follow Gali and Monacelli (2005); Mihailov, Rumler, and Scharler (2011) and model an open economy NKPC. Basically, this implies including the exchange rate. Con-

cerning the Taylor rule, we include influences of domestic and foreign financial markets as well as the exchange rate. This allows us to study spillover effects more thoroughly. Generally, our main open economy approach is similar to that of Leitemo and Söderström (2005); Lubik and Schorfheide (2007); Svensson (2000). However, for reasons of brevity, we omit the full derivation of each equation and refer the interested reader to Gali and Gertler (2007); Woodford (2005).

Our core model consists of four equations. In line with Lubik and Schorfheide (2007), we include the natural logarithm of the exchange rate in the Taylor rule equation (1) and in the aggregate demand equation (3). Similarly, we follow Leitemo and Söderström (2005); Lubik and Schorfheide (2007) and include these changes in the Phillips curve (2) as well. The exchange rate equation (4) follows Lubik and Schorfheide (2007); Svensson (2000). Both Leitemo and Söderström (2005); Lubik and Schorfheide (2007) and Svensson (2000) employ a mix of forward- and backward-looking equations. To simplify the model, we rely on the forward-looking New Classical approach and set the backward-looking parameter equal to zero. The parameter i_t denotes the monetary policy (interest) rate, π_t the inflation rate, y_t the output gap, and e_t the exchange rate at time t. Parameters belonging to the interest rate equation are denoted by α; β, δ, and γ refer to the NKPC, the exchange rate, and the IS curve, respectively.

$$i_t = \alpha_i i_{t-1} + \alpha_\pi \pi_t + \alpha_y y_t + \alpha_e \log(e_t) \tag{1}$$

$$\pi_t = \beta_\pi \pi_{t+1} + \beta_y y_t + \beta_e \log(e_t) \tag{2}$$

$$y_t = y_{t+1} - \gamma_i(i_t - \pi_{t+1}) + \gamma_e \log(e_t) + \gamma_{y^*} y_t^* \tag{3}$$

$$\log(e_t) = \delta_e \log(e_{t-1}) - \delta_i i_{t-1} + \delta_\pi \pi_t - \delta_i^* i_{t-1}^* + \delta_\pi^* \pi_t \tag{4}$$

In a first step, we add three financial markets to the baseline model (1-4). In addition to the exchange rate equation, we model the financial sector by simulating bond and stock markets for each country. To the best of our knowledge, we are the first to choose a continuous-time framework, which greatly facilitates applying common modelling tools from the field of finance,

e.g., Brownian motion processes. In a second step, we interact financial sector, monetary policy (Equation 1), and the real sector (Equations 2 and 3) derived above. We follow our assumption of simultaneity and highly interacted markets and augment the standard representation of stocks and bonds, such as shown by (Merton 1969), by incorporating the monetary policy rate, the foreign bond yield, and the log-linearised domestic stock price to the bond yield equation. Similarly, we add the bond yield, the domestic monetary policy rate, and the foreign stock price to the stock price equation. Since we consider stock prices as the primary link between real and monetary economy, we add the output gap to the stock market equation. We denote v_t as the bond yield at time t and x_t as the stock price at time t. Variables marked with an asterisk indicate foreign variables and parameters.

$$i_t = \alpha_i i_{t-1} + \alpha_\pi \pi_t + \alpha_y y_t + \alpha_e \log(e_t) + \alpha_v v_t + \alpha_x \log(x_t) \qquad (5)$$

$$v_t = \eta_v v_{t-1} + \eta_v^* v_t^* + \eta_x \log(x_t) + \eta_i i_t \qquad (6)$$

$$\log(x_t) = \mu_x \log(x_{t-1}) - \mu_x^* \log(x_{t-1}^*) + \mu_v v_t + \mu_y y_t + \mu_i i_t \qquad (7)$$

The relationship between monetary policy and the financial sector is not unidimensional (Rigobon 2003; Rigobon and Sack 2003). In line with Belke and Klose (2010), we extend the Taylor rule with a financial market term. Following Ramón and Vázquez (2006), we employ disturbances ϵ as autoregressive processes in the NKPC, the IS curve and the exchange rate. Our core model in discrete time is shown in the Appendix Model (Model 10a).

In a last step, we construct the continuous time model (Model 11a). In line with the relevant finance literature (Barndorff-Nielsen and Shephard 2001; Fama 1965; Malkiel and Fama 1970), we model stock prices as geometric Brownian motion processes and bond yields as Ornstein-Uhlenbeck processes. Since disturbances are specified as AR(1) processes, we can interpret the main macroeconomic variables as standard Brownian motions.

In general, stability is an important aspect of differential equations. Loosely speaking, stable solutions are those that vary only very little after small changes in the initial values. In economic terms, this means that if an econ-

omy drifts away from its steady state, it will return or, alternatively, it will not move very far away. To analyse the stability of Model (11a), we rely on Lyapunov techniques (Khasminskii 2012). A brief overview of this concept is given in the Appendix. Khasminskii (ibid.) provide a much more thorough overview. We apply the following Lyapunov function

$$V(x) = \|x\|_2^2 = \left(\sqrt{\sum |x_i|^2} \right)^2 \tag{8}$$

where $\|\|_2$ denotes the Euclidean Norm. Since (since $\alpha_i^f \alpha_i^h \delta_e (\sigma_e + \sigma_\pi^f + \sigma_\pi^h + \sigma_v^f + \sigma_v^h + \sigma_y^f + \sigma_y^h) > 0$), the zero solution is unstable, there is no global stable rest point, but only parameter-dependent partial solutions. In the following section, we thoroughly analyse stability for each set of parameters we derive.

Please note, however, that we concentrate our dynamic analysis on the short-run adjustment. Within this time frame, there is no guarantee that the variables will return to their starting values. Thus, although the system is stable in a Lyapunov sense, we find that some variables remain on a clear trend within our window of analysis.

3 Baseline Scenario

To analyse different policies and scenarios, we delineate various stylised economies both in the context of deriving a stable solution as well as in the context of obtaining empirical evidence. As far as we know, the literature contains no empirical evidence for such a highly interacted model. To gain some preliminary insight, we apply an ad hoc econometric approach. Ignoring issues of simultaneity, we estimate the model equation-by-equation via OLS. As there is no unique solution to the system of stochastic differential equations (SDEs), we approximate trajectories by applying the Euler-Maruyama scheme (Saito and Mitsui 2001) and obtain estimators based on 1,000 replications. We normalise the time period to lie between 0 and 1 and impose a step size of 0.01.

To justify our approach, we graph the core model of Clarida, Gali, and Gertler (1999) and control for its well-established characteristics. For the prac-
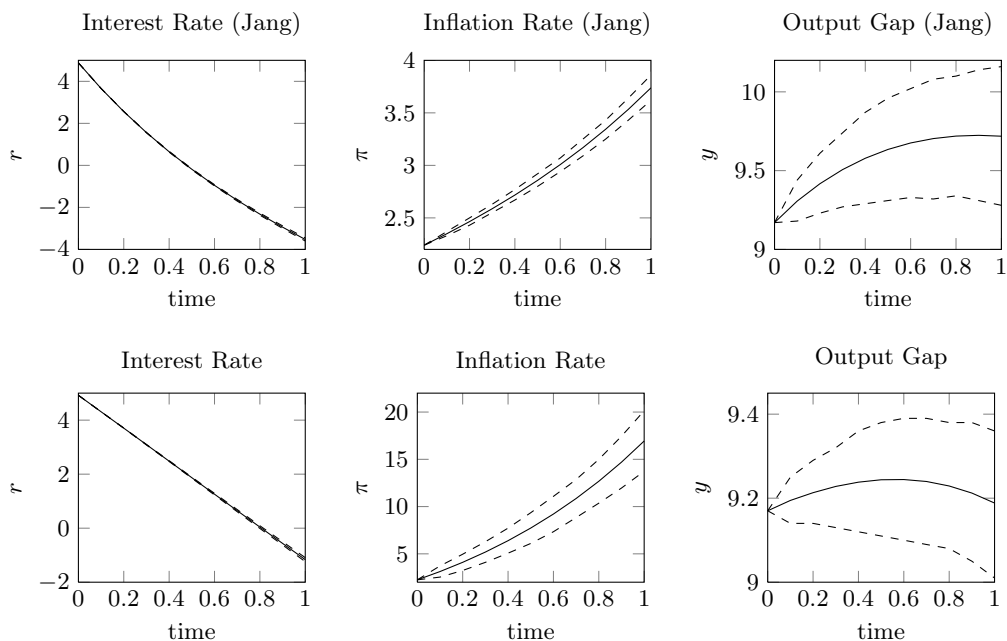
Figure 1: **Three-Equation NK Model**

tical implementation, we rely on values from Jang (2012) and, moreover, utilise our point estimates. Afterward, we solve the SDEs for both sets of parameters. Trajectories are shown in Figure 1. Both approaches provide similar solutions and are in line with economic theory: after conducting restrictive monetary policy, output increases above potential, causing upward pressure on prices, which causes a rise in inflation.

3.1 Economic Analysis of the NK Model with Two Financial Markets

As noted above, there is no unique steady state (stable solution); therefore, we rely on parameter calibration to analyse key characteristics of our continuous-time NK model. As pointed out in Gardiner (2009) calibration of the parameters is rather arbitrary and different parameters lead to different stable solutions. However, we compensate for this by applying economically reasonable parameters. We generate three stable scenarios to describe stylised

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versions of closed and open economies. First, we extend the conventional model of an (almost) closed economic environment by including financial markets. Specifically, we set the parameters in the model so that the influence of the exchange rate is almost zero. Second, we fix the parameters in a way such that we model two equally-sized open economies. Moreover, we ensure that the financial sector and the real sector of each economy are integrated with each other. Third, we consider the case of a big and a small open economy by allowing for an asymmetric impact of foreign shocks, that is, they affect one economy but not the other.

In principle, the choice of parameters in the theoretical model is arbitrary. Our approach is as follows. First, we derive an economically motivated steady state. Second, we choose parameters such that the ratios of important macroeconomic variables fit to standard economic theory and/or empirical stylised facts and provide a stable model solution. Third, we apply initial values which are different from our steady state values and solve for the corresponding SDEs. In this manner, we can analyse those circumstances of the model which (i) provide established trajectories, (ii) destabilise the system, or (iii) contradict mainstream theory.

In accordance with the idea behind purchasing power parity, the steady state nominal exchange rate takes the value one. Furthermore, in line with the intertemporal budget constraint, we assume the long-run trade balance to be zero. Additionally, we assume a stable monetary environment and choose small values for the inflation rate and short-term interest rate, which is the monetary policy instrument. To implement the *ex post* Fisher effect, we set bond yields equal to the sum of inflation rate and interest rate. For simplicity, we choose the value one for stock prices.

There are various ways to fix the initial values. For example, we can choose 'normal times' or more 'turbulent times'. We start with 'normal times' and develop an economy with a slightly negative trade balance and some distress in its bond market. Parameters are chosen in line with standard New Keynesian theory and detailed parameter information is given in Table (1) in the Appendix. All scenarios start with the same initial values and, by construction,

share the same rest point, which facilitates comparison of scenarios. We change as few parameters as possible, e.g., in the case of an (almost) closed economy, we mainly reduce the influence of the exchange rate and exclude all international interaction terms. For instance, the Taylor rate in the open-economy scenario for Country A is:

$$di = i - \frac{1}{3} \left(i + 2.5\pi - \log(i - 3v + 2x + 0.7x^* + 1.5y) + 0.4v + 2y + \log(e) + 0.6\log(x) \right) dt$$

In line with Taylor-rule-related econometric research, we give the greatest weight to the interest rate, whereas the parameters associated with the exchange rate or stock and bond markets, which are commonly excluded, receive rather small weights. We arrange the remaining parameters in a similar way. All analyses of the different stylised economies start by conducting expansionary monetary policy in Country A. Studying the interaction of monetary policy and financial markets is the main purpose of our simulations. Note that all scenarios are stable in the Lyapunov sense (the real part of the function varies between -2.0 and -100).

The first scenario in Figure (2) illustrates the dynamic adjustment in two (almost) closed economies. It enriches the three-equation New Keynesian model by taking into account the working of financial markets and allows the exchange rate to have only a small influence. A classic example of a big closed economy is the United States, which, however, is embedded in a globalised world. Furthermore, one can argue, for example, that the economic interaction between Finland and Panama might be of little importance for either economy, although both countries are rather open. In the following, a dashed line represents Country A and a solid line Country B.

Country A's central bank engages in expansive monetary policy by decreasing the short-term interest rate. This makes an adjustment in the term structure necessary, causing long-term interest rates, such as bond yields, to decline by the same amount (as inflation has not yet increased and we consider the *ex post* Fisher effect). Demand for interest-rate sensitive expenditures rises, leading to a hike in firms' expenditure on investment goods. As a result of increasing investment, the net worth of Country A's firms increases, which is reflected in a stock market boom. A decrease of the real interest rate by

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5 basis points within one year leads to a corresponding increase of 20 basis points in stock prices within the same period. Due to the lower interest rate, households' saving incentives decline and consumer demand increases.

Because short-term aggregate demand determines output, the demand increase in Country A slowly pushes aggregate output above potential output. The capital outflow following the fall in interest rates leads to a steady depreciation of Country A's currency and a fall in the terms of trade, as prices do not rise sufficiently to offset the exchange rate effect. However, due to the poor linkage between the countries, the deterioration in the terms of trade merely depresses the inflation rate in Country B. Its interest rate increases slightly within two years, a timid reaction to the depreciation of its currency. As interest rate and inflation rate are almost stable, bond yields also remain on their growth path. In contrast, Country B's stock market deteriorates with time, as the fall in competiveness and in the inflation rate reduces firms' profitability and, over time, negatively affects the output gap.

The second scenario (see Figure 4) reflects two open economies under the assumption of equal-sized countries. A real-world example is Germany and the UK. Suppose again that Country A's central bank engages in expansive monetary policy by decreasing the short-term interest rate. Like in the closed-economy scenario, long-term interest rates start to fall. Due to enhanced borrowing conditions in Country A, firms' investment increases and thus their net worth rises and stock prices experience a hike. Moreover, as households and firms spend more money, aggregate demand increases and as does real income. The drop in interest rates makes deposits in Country A's currency less attractive and capital outflows depreciate its currency. Since prices of domestic goods expand only slowly, improved terms of trade help boost net exports to Country B. Higher foreign demand stimulates the economy further, so that output rises strongly over time. As imports increase, prices for goods in Country B decline. Reflecting purchasing power parity, the decrease in inflation in Country B is similar in size to the increase in Country A. The time frame underlying this adjustment suggests that the monetary policy transmission process needs almost one year to noticeable affect inflation.

Figure 2: **Trajectories for the (almost) closed economy**
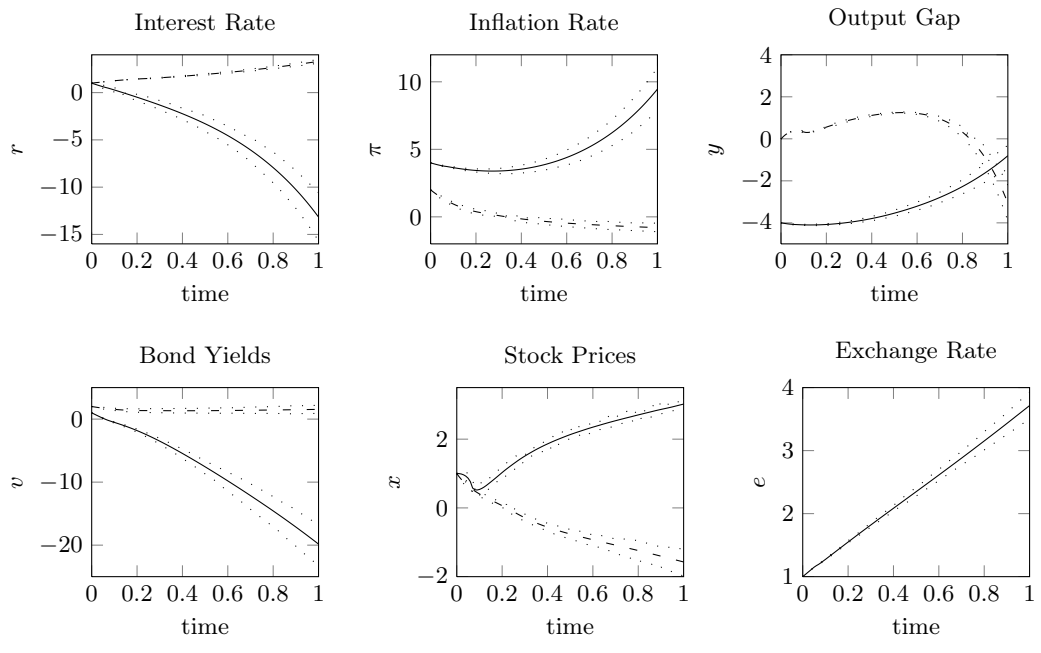
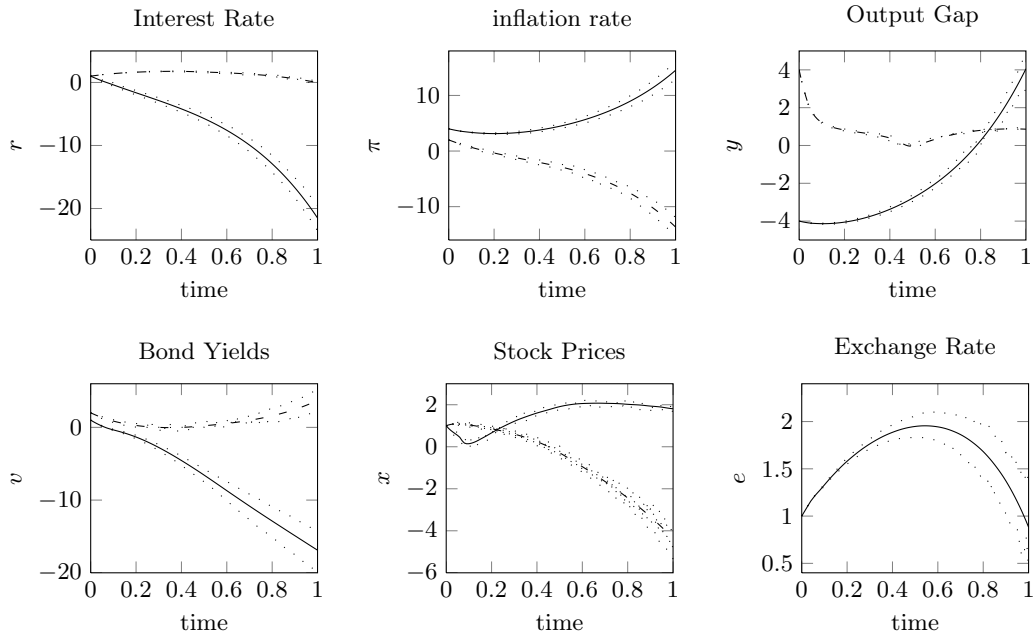Solid lines show trajectories for Country A, dashed lines for Country B

Figure 3: *
Solid lines show trajectories for Country A, dashed lines for Country B

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As expected, cross-country interactions in the open economies are much more pronounced than in the case of the (almost) closed economies. For example, the decrease in Country B's stock prices after the change in interest rate in Country A is twice as large in the former scenario compared to the latter. Contrary to the (almost) closed economy scenario, the drop in the inflation rate causes Country B's central bank to lower its target rate after about eight months, leading to lower long-run interest rates. However, the economic impact is quite small: a target rate change of 5 basis points in Country A causes a decrease in Country B's interest rate of only 0.2 basis points. Finally, the decreasing price level improves Country B's competitiveness, with the consequence that after about one year, the output gap starts to slowly recover. The increasing interest rate differential causes an outflow of capital from Country A to Country B and the former's currency begins to depreciate. Not surprisingly, we detect a greater effect of domestic policy on a country's economic development than of foreign policy. Yet, the international transmission of changes from Country A to Country B affects economic variables very differently. For example, the negative effect of Country A's policy on Country B's output is reversed after some time, but there are extraordinarily large differences in financial market reactions. Although exchange rate and bond yields in Country B move back to their starting value, stock prices in Country B continue to fall within our time window. Thus, Country A's expansionary monetary policy move has caused a stock market crisis in Country B.

Our third scenario comprises a small economy and a big economy, such as Canada and the United States. We shed light on this case by first conducting policy in the big country (Country A in Figure (5)) and then conducting policy in the small country (as shown in Figure 7 in the Appendix). In general, the model results are in line with our expectations, namely, that the small country reacts to the big country but not vice versa. To economise on space, we focus on differences from the first two scenarios and refrain from repeating the transmission process.

The first scenario simulates an expansionary monetary policy in the big country (see Figure (5)). We observe two notable differences: first, inflation

Figure 4: **Trajectories for the open economy**

Solid lines show trajectories for Country A, dashed lines for Country B

in the small country (Country B) increases and this effect is larger than in the equal-sized country open economy scenario. Second, some financial markets react differently compared to the case of equal-sized countries. (i) Country A's exchange rate continues to depreciate, but its rate of growth is slightly lower after interest rates start to rise modestly in Country B. (ii) While we observe a stock market crash in the equal-sized country case, Country B, the small country, actually experiences a stock market boom after about one year. (iii) Moreover, since stock prices follow Country A's growth path, we now have a positive international stock market correlation. (iv) Finally, Country B's stock prices react even more sensitively to Country A's monetary policy than does Country A's own stock market. Thus, at least in the case of small countries, domestic asset markets could be highly influenced by foreign monetary policy actions.

The second case, where the monetary policy action occurs in the small country, is in line with the (almost) closed economy scenario; trajectories are pro-

Figure 5: **Trajectories for the big and small country**
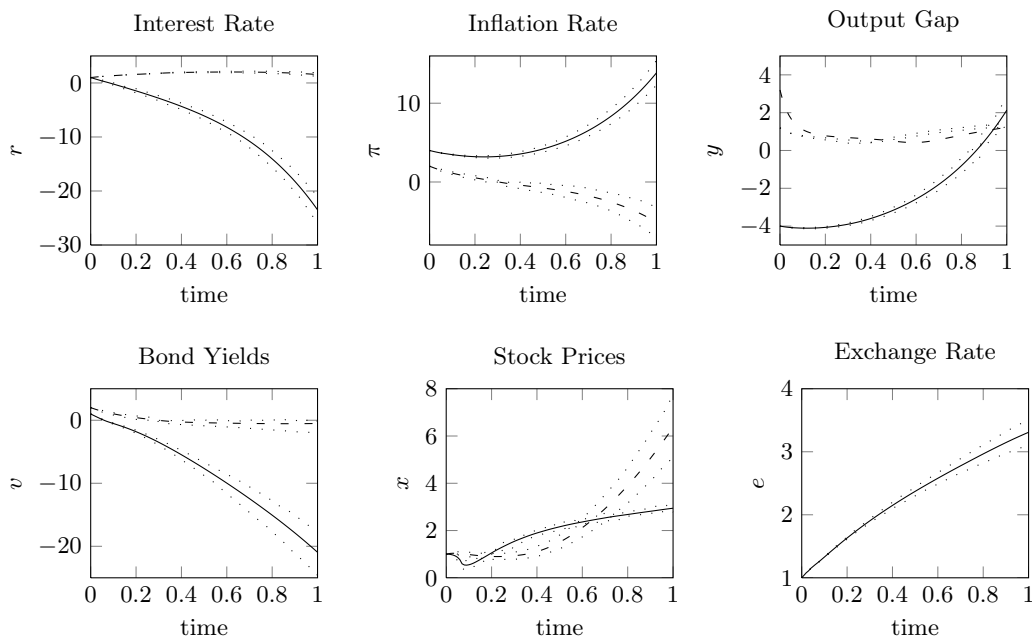
Solid lines show trajectories for Country A, dashed lines for Country B

vided in the Appendix in Figure (7).

3.2 Employing Empirically Estimated Parameters

Our second approach to analysing economic behaviour is based on using econometric methods to determine model parameters. Thus, through the use of macroeconomic data, we obtain point estimates of each parameter in each equation of the time-adjusted model given in the Appendix, Equation (10a).Therefore, we need to rearrange the estimated parameters to fit the discrete core model. Rearranged coefficients are available on request.

Data are obtained from the Bureau of Economic Analysis, the Federal Reserve, the U.S. Bureau of Labor Statistics, and Statistics Canada. We choose Canada and the United States as benchmark countries, as doing so allows us to obtain estimates of a bigger and a smaller economy which are highly integrated. We employ data for the last 30 years, starting in Q3:1983 up to

Q2:2012, such that we have 120 observations. We use the de-trended difference of log GDP as output gap (de-trended via HP-filter), change in log CPI as inflation rate, and change in the Federal Funds rate or the cash rate for the Bank of Canada (in quarterly averages) as monetary policy target rates. Regarding financial variables, we employ S&P 500 and TSX, respectively, as stock prices and yields on five-year T-Bills as bond returns. OLS estimation results are given in Table (2). As initial values for the SDEs, we choose the average of each variable and diffusion terms are randomly distributed between 0 and 0.5. We standardise values in the simulations to avoid creating artificial differences in the size of coefficients due to different units of measurement.

To avoid unnecessary repetition, we do not simulate any policy in this estimation scenario. However, the trajectories displayed in Figure (6) reveal some interesting economic adjustment processes. First, the case study fits the purely theoretical model of the small and big economy in many ways. In line with our expectations, the output gaps of the two countries move quite differently. Reflecting the higher inflation rate in the United States than in Canada, the Fed starts to significantly tighten monetary policy. The Bank of Canada is forced to follow, albeit not necessarily to exactly the same extent. Although the increase in the US output gap continues for about a year, contractionary monetary policy brings it under control, despite further rising prices. However, the increase in prices and the decrease in output cause a depreciation of the exchange rate.

Focussing on the effects of different financial markets suggests several things. First, both US as well as Canadian bond yields basically stay constant, despite a notable increase in short-term interest rates, particularly in the United States. Second, in line with our theoretical scenario depicting asymmetric open economies, Canadian stock prices react very sensitively to US economic developments. In fact, they react much more strongly than do US stock prices. Finally, we observe that the exchange rate is more influenced by US than by the Canadian markets, which again reflects the size difference.

To analyse steady-state properties, we take means of all variables during the observation period. They result in a positive Lyapunov function, which implies

Figure 6: **Results for the OLS estimation**
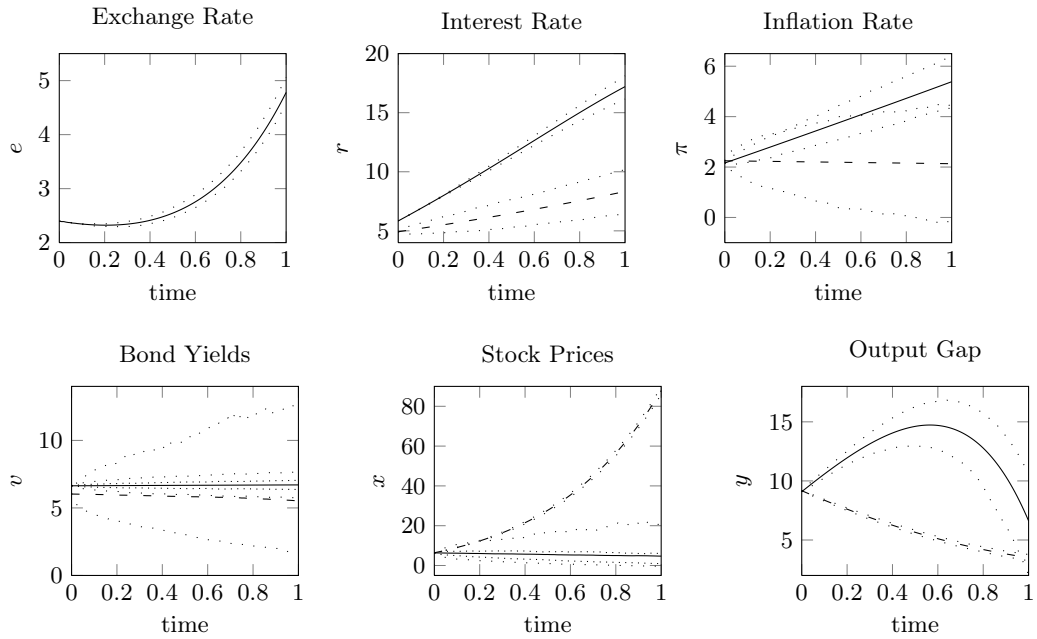
Solid lines show trajectories for the USA, dashed lines for Canada

that in spite of covering 30 years, the solutions are dynamically unstable. This does not imply that the United States and Canada are suffering unstable times, but that markets, in this case financial markets, converge to a non-steady state. One explanation might be changes in financial markets occurring over the last 30 years, e.g., an increase in liquidity due to deregulation, such that the steady state had to be adjusted over time.

4 Conclusions

In this paper, we extend the classical New Keynesian model of Clarida, Gali, and Gertler (1999) and Lubik and Schorfheide (2007) by, first, including a financial sector and, second, applying stochastic differential equations and thereby switching into a continuous-time framework. We employ prior research from finance and model the financial sector by including exchange rate market, bond market, and stock market, thereby acknowledging that these markets are

driven by different aspects of the economy. For example,, bonds are strongly affected by sovereign debt, whereas stock markets are heavily influenced by the real economy. Applying stochastic differential equations allows us to rely on research starting with Merton (1969, 1970) and model the financial sector as Brownian motion or Ohrnstein-Uhlenbeck processes. Furthermore, we employ Lyapunov techniques to analyse stability of the solutions and steady-state properties. Thus, in our analysis, we combine New Keynesian macroeconomic analysis, classic finance research, and standard mathematical procedures.

Our main purpose is to analyse the effects of monetary policy on financial markets and the feedback from financial markets to the real economy. In line with economic theory and empirical evidence, we start with a steady-state solution. We assume balanced trade and consider the influence of purchasing power parity as well as of the Fisher equation. In a second step, we distinguish between an (almost) closed economy, an equally-sized one, and an unequally-sized one. We base the model parameters on empirical findings from the Taylor rule, New Keynesian Phillips curve, and the IS curve; our financial equations rely on findings by Merton (1969) and Black and Scholes (1973). However, we extend all equations by accounting for spillover effects from monetary variables to real variables and vice versa. Finally, we solve the model for an initial value, which is commonplace, but deviate from the steady-state point and observe the development of the trajectories of the solutions.

All simulation scenarios start by conducting expansive monetary policy. Following the change in short-term interest rates, we study the transmission channel to analyse spillover effects between monetary and real variables, as well as between domestic and foreign markets. Specifically, based on market size and financial market structure, we identify differences in the influence of monetary policy. In the case of an (almost) closed economy, by construction, there are few spillover effects. In the case of equally-sized economies, we identify transmission effects on real variables, such as the output gap, monetary variables, such as the interest rate, and financial markets. Given the rather closed nature of the simulation framework, domestic effects prevail. However, when studying the case of a big and a small economy, we find evidence that foreign

developments tend to dominate domestic financial markets, in spite of very limited international transmission channels. In particular, a small country's stock market and exchange rate react sensitively to foreign monetary policy. In contrast, domestic changes affect primarily the bond market.

To illustrate the usefulness of our findings, we use actual data from the United States and Canada as an example of unequally-sized economies. Covering 30 years, we use point estimates based on OLS regressions to derive the model's parameters. The simulation results support our findings from the purely theoretically parameterized model. We find spillover effects from monetary policy for both countries. Moreover, Canadian stock markets react sensitively to US monetary policy and, similar to the theoretical model, they even exceed domestic changes after one year. However, we detect potential market inefficiencies (defined as unstable solutions), particularly in respect to the bond market, which seem to be somewhat stronger in the United States than in Canada. This result is supported by computing the Lyapunov function for this model, which is highly positive and suggests inherent instability in the financial markets. Our model emphasises that accounting for differences in financial markets helps explain economic spillover effects across countries.

Our study also has some interesting policy implications. We find evidence that monetary policy actions spill over to other countries. The impact and size of the effect depend on, first, the size of the economy, second, the linkage between the markets and, third, the structure of the markets. Policymakers, particularly those in small- or medium-sized countries, should take into account that spillovers from big countries could have effects that are even stronger than domestic policies. In contrast, we find evidence supporting the conventional wisdom that big countries and relatively closed economies can design and engage in policy primarily based on domestic factors.

Moreover, monetary policy affects stock markets and markets for foreign exchange more strongly than it does bond markets. Cursory evidence from recent monetary policy actions supports these findings. For example, the purchase of Greek sovereign bonds by the European Central Bank decreased Greek interest rates only for a short time. Investors of sovereign bonds look more

closely at fiscal policy than at monetary policy. In contrast, stock markets, especially in unequally-sized economies, appear to react notably to monetary policy changes.

There are various fruitful ways to extend this analysis. First, analysing spillover effects from the demand side to both financial markets and monetary policy might provide new insights. Moreover, we started the analysis by conducting policy and observing the transmission mechanism. However, the opposite effect might be just as interesting. In particular, it would be interesting to analyse how variations in financial markets influence the outcome of monetary policy. Moreover, considering the recent financial and sovereign debt crisis, a specific analysis of crises could be interesting. The model would also allow varying the monetary policy reaction function by assessing the influence of financial markets differently.

In this paper, we did not account for either a fiscal sector or employment. Thus, a second extension of our model by including government fiscal policy targets could yield interesting insights, especially as we observe that bond markets react very little to monetary policy. Our previous analysis suggests that bonds markets are relatively more driven by fiscal policy and this could be tested within the context of a differently specified model. Furthermore, the model can be extended by accounting for even more financial markets. Inclusion of the Black-Scholes formula or advanced option pricing techniques might result in an almost complete model of the financial system. Third, and more technical, inclusion of jump-diffusion processes would enlarge the model by adding volatility and some more capricious market actions.

5 Appendix

A: Mathematical Definitions

Definition We consider the system

$$dX_t = f(X_t)dt + \sigma(X_t)dW_t$$
$$X_s = \eta$$

for $t \in [s, T]$ and $0 \leq s < T$. A real function $V \in C^2(\mathbb{R})$ is called Lyapunov function if, for some constant $K > 0$, it holds

1. $V(x) > 0$,

2. $LV = V_x(x)f(x) + \frac{1}{2}tr'\sigma(x)V_{xx}(x)\sigma(x) \leq KV(x)$, $x \in \mathbb{R}^n$ and

3. $\lim_{|x| \to \infty} V(x) = +\infty$

Theorem 5.1. *Let V be a non-negative C^2 Lyapunov function such that $\lim_{|x| \to \infty} LV(x) = -\infty$. Assume that there exists a regular solution to the SDE. Then there exists a stationary solution.*

(Khasminskii 2012).

Theorem 5.2. *Let $V(t, x)$ be non-negative such that $LV(x) \to -\infty$ for $|x| \to \infty$. Assume that there exists a regular solution to the SDE. Then there exists a stationary solution*

B: Model Specifics

$$e_{t-1} = \beta_e^e e_t + \beta_e^i i_t + \beta_e^{i^*} i_t^* + \beta_e^\pi \pi_t + \beta_e^{\pi^*} \pi_t^* + \beta_e^v v_t + \beta_e^{v^*} v_t^* + \beta_e^x x_t + \beta_e^{x^*} x_t^* + \beta_e^y y_t + \beta_e^{y^*} y_t^*$$
$$i_{t-1} = \beta_i^e e_t + \beta_i^i i_t + \beta_i^\pi \pi_t + \beta_i^v v_t + \beta_i^x x_t + \beta_i^y y_t$$
$$\pi_{t-1} = \beta_\pi^e e_t + \beta_\pi^i i_t + \beta_\pi^\pi \pi_t + \beta_\pi^v v_t + \beta_\pi^x x_t + \beta_\pi^y y_t + \beta_\pi^{y^*} y_t^* \qquad\qquad (9a)$$
$$v_{t-1} = \beta_v^i i_t + \beta_v^v v_t + \beta_v^{v^*} v_t^* + \beta_v^x x_t$$
$$x_{t-1} = \beta_x^i i_t + \beta_x^v v_t + \beta_x^x x_t + \beta_x^{x^*} x_t^* + \beta_x^y y_t$$
$$y_{t-1} = \beta_y^e e_t + \beta_y^i i_t + \beta_y^\pi \pi_t + \beta_y^v v_t + \beta_y^x x_t + \beta_y^y y_t + \beta_y^{y^*} y_t^*$$

$$
\begin{aligned}
\log(e_t) &= \frac{1}{\delta_e}((\log(e_{t+1}) + \delta_\pi \pi_{t+1} + \delta_\pi^* \pi_{t+1}^*)) \\
&\quad + \frac{1}{\alpha_i \delta_e}(\delta_i(\alpha_\pi \pi_{t+1} - \log(i_{t+1}\mu_i f - \mu_v v_{t+1} + \mu_x x_{t+1} + \mu_x^* x_{t+1}^* + \mu_y y_{t+1}) \\
&\qquad\qquad - i_{t+1} + \alpha_v v_{t+1} + \alpha_y y_{t+1} + \alpha_e \log(e_{t+1}) + \alpha_x \log(x_{t+1}))) \\
&\quad + \frac{1}{\alpha_i^* \delta_e}(\delta_i^*(\alpha_\pi^* \pi_{t+1}^* - \log(i_{t+1}^* \mu_i^* - \mu_v^* v_{t+1}^* + \mu_x^* x_{t+1}^* + \mu_x^* x_{t+1} + \mu_y^* y_{t+1}^*) \\
&\qquad\qquad - i_{t+1}^* + \alpha_v^* v_{t+1}^* + \alpha_y^* y_{t+1}^* + \alpha_e^* \log(e_{t+1}) + \alpha_x^* \log(x_{t+1}^*))) \\
i_t &= \frac{i_{t+1}}{\alpha_i} - \frac{1}{\alpha_i}(\alpha_\pi \pi_{t+1} - \log(i_{t+1} * \mu_i - \mu_v v_{t+1} + \mu_x x_{t+1} + \mu_x^* x_{t+1}^* + \mu_y y_{t+1}) \\
&\qquad\qquad + \alpha_v v_{t+1} + \alpha_y y_{t+1} + \alpha_e \log(e_{t+1}) + \alpha_x \log(x_{t+1})) \\
\pi_t &= (\beta_\pi + \beta_y \gamma_i (\frac{\alpha_\pi}{\alpha_i} + 1))\pi_{t+1} + \beta_y(y_{t+1} + \gamma_y fh * y_{t+1}^* + \gamma_e f * \log(e_{t+1})) \\
&\quad + \frac{\beta_y}{\alpha_i}(\gamma_i f * (\alpha_v f * v_{t+1} - \log(i_{t+1} * \mu_i f - \mu_v v_{t+1} + \mu_x x_{t+1} + \mu_x^* * x_{t+1}^* + \mu_y y_{t+1}) \\
&\qquad\qquad - i_{t+1} + \alpha_y y_{t+1} + \alpha_e \log(e_{t+1}) + \alpha_x \log(x_{t+1}))) + \beta_e \log(e_{t+1}) \\
&\qquad\qquad\qquad (10a) \\
\log(x_t) &= \mu_x x_{t+1} + i_{t+1}\mu_i f - \mu_v v_{t+1} + \mu_x^* x_{t+1}^* + \mu_y y_{t+1} \\
v_t &= \eta_v v_{t+1} + \eta_v^* v_{t+1}^* - i_{t+1}\eta_i f - \eta_x \log(x_{t+1}) \\
&\quad - \log(i_{t+1}\mu_i f - \mu_v v_{t+1} + \mu_x x_{t+1} + \mu_x^* x_{t+1}^* + \mu_y y_{t+1}) \\
y_t &= (\frac{\alpha_y \gamma_i}{\alpha_i} + 1)y_{t+1} + \gamma_y^* * y_{t+1}^* + \gamma_e * \log(e_{t+1}) + \gamma_i \pi_{t+1} \\
&\quad + \frac{\gamma_i}{\alpha_i}(\alpha_\pi * \pi_{t+1} - \log(i_{t+1}\mu_i - \mu_v v_{t+1} + \mu_x x_{t+1} + \mu_x^* x_{t+1}^* + \mu_y y_{t+1}) - i_{t+1} \\
&\qquad\qquad + \alpha_v v_{t+1} + \alpha_e \log(e_{t+1}) + \alpha_x \log(x_{t+1}))
\end{aligned}
$$

$$de = (\log(e) - \frac{1}{\delta_e}((\log(e) + \delta_\pi^f \pi^f + \delta_\pi^h \pi^h))$$

$$+ \frac{\delta_i^f}{\delta_e \alpha_i^f}(\alpha_\pi^f \pi^f - i^f + \alpha_v^f v^f + \alpha_y^f y^f + \alpha_e^f \log(e) + \alpha_x^f \log(x^f))$$

$$+ \frac{\delta_i^h}{\delta_e \alpha_i^h}(\alpha_\pi^h \pi^h - i^h + \alpha_v^h v^h + \alpha_y^h y^h + \alpha_e^h \log(e) + \alpha_x^h \log(x^h))$$

$$- \frac{\delta_i^h}{\delta_e \alpha_i^h} \log(i^h \mu_i^h - \mu_v^h v^h + \mu_x^h x^h + \mu_x^{hf} x^f + \mu_y^h y^h)$$

$$- \frac{\delta_i^h}{\delta_e \alpha_i^h} \log(i^f \mu_i^f - \mu_v^f v^f + \mu_x^f x^f + \mu_x^{fh} x^h + \mu_y^f y^f))dt + \sigma_e dW_t^e$$

$$di^f = ((1 - \frac{1}{\alpha_i^f})i^f + \frac{1}{\alpha_i^f}(\alpha_\pi^f \pi^f + \alpha_v^f v^f + \alpha_y^f y^f + \alpha_e^f \log(e) + \alpha_x^f \log(x^f))\frac{-1}{\alpha_i^f} \log(i^f \mu_i^f$$

$$- \mu_v^f v^f + \mu_x^f x^f + \mu_x^{fh} x^h + \mu_y^f y^f))dt$$

$$di^h = ((1 - \frac{1}{\alpha_i^h})i^h + \frac{1}{\alpha_i^h}(\alpha_\pi^h \pi^h + \alpha_v^h v^h + \alpha_y^h y^h + \alpha_e^h \log(e) + \alpha_x^h \log(x^h))\frac{-1}{\alpha_i^h} \log(i^h \mu_i^h$$

$$- \mu_v^h v^h + \mu_x^h x^h + \mu_x^{hf} x^f + \mu_y^h y^h))dt$$

$$d\pi^f = ((1 - \beta_y^f \gamma_i^f (\frac{\alpha_\pi^f}{\alpha_i^f} + 1) - \beta_\pi^f)\pi^f - \beta_y^f(y^f + \gamma_y^{fh} y^h + \gamma_e^f \log(e)$$

$$+ \frac{1}{\alpha_i^f}(\gamma_i^f(\alpha_v^f v^f - i^f + \alpha_y^f y^f + \alpha_e^f \log(e) + \alpha_x^f \log(x^f)))) - \beta_e^f \log(e)$$

$$- frac1\alpha_i^f \log(i^f \mu_i^f - \mu_v^f v^f + \mu_x^f x^f + \mu_x^{fh} x^h + \mu_y^f y^f))dt + \sigma_{\pi^f} dW_t^{\pi^f}$$

$$d\pi^h = ((1 - \beta_y^h \gamma_i^h (\frac{\alpha_\pi^h}{\alpha_i^h} + 1) - \beta_\pi^h)\pi^h - \beta_y^h(y^h + \gamma_y^{hf} y^f + \gamma_e^h \log(e)$$

$$+ \frac{1}{\alpha_i^h}(\gamma_i^h(\alpha_v^h v^h - i^h + \alpha_y^h y^h + \alpha_e^h \log(e) + \alpha_x^h \log(x^h)))) - \beta_e^h \log(e)$$

$$- \frac{1}{\alpha_i^h} \log(i^h \mu_i^h - \mu_v^h v^h + \mu_x^h x^h + \mu_x^{hf} x^f + \mu_y^h y^h))dt + \sigma_{\pi^h} dW_t^{\pi^h}$$

$$dv^f = ((mean(v^f) - (1 - \eta_v^f)v^f + \log(i^f \mu_i^f - \mu_v^f v^f + \mu_x^f x^f + \mu_x^{fh} x^h + \mu_y^f y^f) + i^f \eta_i^f$$

$$- \eta_v^{fh} v^h + \eta_x^f \log(x^f)))dt + \sigma_{v^f} dW_t^{v^f}$$

$$dv^h = ((mean(v^h) - (1 - \eta_v^h)v^h + \log(i^h \mu_i^h - \mu_v^h v^h + \mu_x^h x^h + \mu_x^{hf} x^f + \mu_y^h y^h) + i^h \eta_i^h$$

$$- \eta_v^{hf} v^f + \eta_x^h \log(x^h)))dt + \sigma_{v^h} dW_t^{v^h}$$

$$dx^f = ((1 - \mu_x^f)x^f + \mu_v^f v^f - i^f \mu_i^f - \mu_x^{fh} x^h - \mu_y^f y^f)dt + \sigma_{x^f} x^f dW_t^{x^f} \qquad (11a)$$

$$dx^h = ((1 - \mu_x^h)x^h + \mu_v^h v^h - i^h \mu_i^h - \mu_x^{hf} x^f - \mu_y^h y^h)dt + \sigma_{x^h} x^h dW_t^{x^h}$$

$$dy^f = ((-(\alpha_y^f \gamma_i^f)/\alpha_i^f)y^f - \gamma_y^{fh}y^h - \gamma_e^f \log(e)$$
$$- \gamma_i^f(\pi^f + \frac{1}{\alpha_i^f}(\alpha_\pi^f \pi^f + \alpha_e^f \log(e) - i^f + \alpha_v^f v^f + \alpha_x^f \log(x^f)))$$
$$+ \frac{\gamma_i^f}{\alpha_i^f} \log(i^f \mu_i^f - \mu_v^f v^f + \mu_x^f x^f + \mu_x^{fh} x^h + \mu_y^f y^f))dt + \sigma_{y^f} dW_t^{y^f}$$
$$dy^h = ((-(\alpha_y^h \gamma_i^h)/\alpha_i^h)y^h - \gamma_y^{hf}y^f - \gamma_e^h \log(e)$$
$$- \gamma_i^h(\pi^h + \frac{1}{\alpha_i^h}(\alpha_\pi^h \pi^h + \alpha_e^h \log(e) - i^h + \alpha_v^h v^h + \alpha_x^h \log(x^h)))$$
$$+ \frac{\gamma_i^h}{\alpha_i^h} \log(i^h \mu_i^h - \mu_v^h v^h + \mu_x^h x^h + \mu_x^{hf} x^f + \mu_y^h y^h))dt + \sigma_{y^h} dW_t^{y^h}$$

Figure 7: **Big and small country (Policy in the small country)**
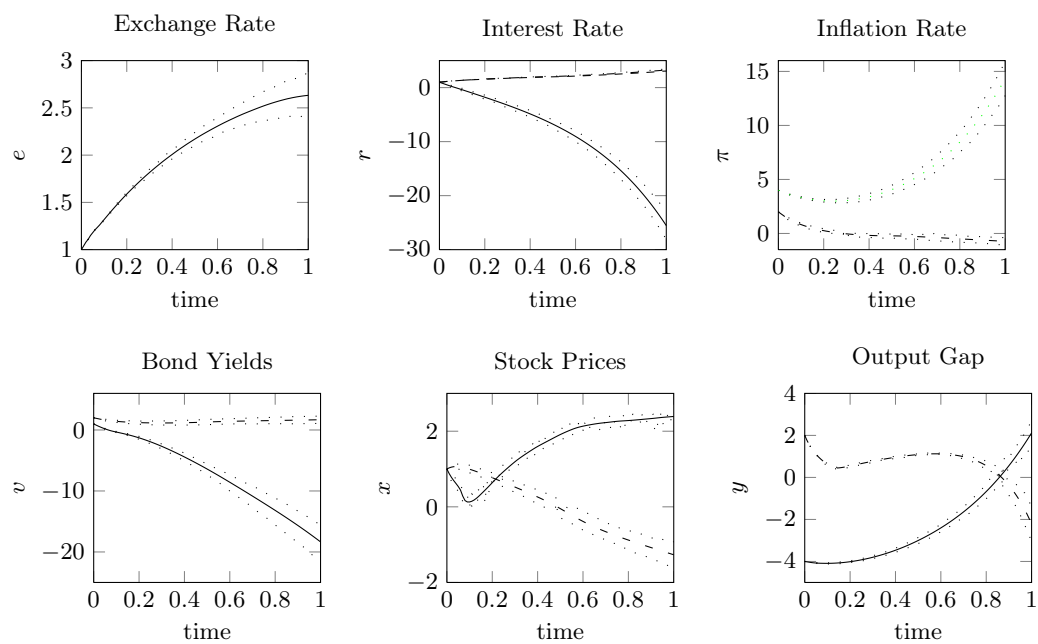
Solid lines show trajectories for Country A, dashed lines for Country B

C: Tables and Figures

Rest point $(1,\ 1,\ 2,\ 1,\ 2,\ 3,\ 4,\ 1,\ 1,\ 0,\ 0)'$

Initial Value $(1,\ 1,\ 1,\ 2,\ 4,\ 2,\ 1,\ 1,\ 1,\ 4,\ -4)'$

	Country A		Country B	
	Connected Countries	Disconnected Countries	Connected Countries	Disconnected Countries
δ_e	3.0	3.0	—	—
δ_i	0.5	0.5	1.0	1.0
δ_π	1.2	1.2	1.8	1.8
α_e	2.0	0.5	2.0	0.5
α_i	3.0	3.0	3.0	3.0
α_π	1.5	1.5	1.5	1.5
α_v	0.4	0.4	0.4	0.4
α_x	0.8	0.8	0.8	0.8
α_y	2.0	2.0	2.0	2.0
β_e	2.0	0.5	2.0	0.5
β_π	-2.5	-2.5	-2.5	-2.5
β_y	1.2	1.2	1.2	1.2
μ_i	1.0	1.0	1.0	1.0
μ_v	2.0	2.0	-10.0	-10.0
μ_x	2.0	2.0	2.0	2.0
μ_x^*	-0.7	—	0.9	—
μ_y	1.5	1.5	1.5	1.5
η_i	2.0	2.0	2.0	2.0
η_v	-5.0	-5.0	-5.0	-5.0
η_x	2.0	2.0	2.0	2.0
η_v^*	1.0	—	1.0	—
γ_e	1.0	0.1	1.0	0.1
γ_i	1.5	1.5	1.5	1.5
γ_y^*	1.0	—	1.0	—

Table 1: Parameters

	const	e	r_{us}	r_{ca}	π_{us}	π_{ca}	v_{us}	v_{ca}	x_{us}	x_{ca}	y_{us}	y_{ca}
e [a]	-36,50	0,81	0,01	-0,06	-0,05	0,01	-0,25	0,30	-0,21	0,85	9,97	-6,45
	10,16	0,05	0,04	0,03	0,04	0,03	0,07	0,07	0,18	0,26	2,62	1,56
r_{us}	17,95	-0,09	0,90	-	0,14	-	-0,22	-	-0,65	-	-1,29	-
	6,91	0,07	0,07	-	0,08	-	0,13	-	0,54	-	0,90	-
r_{ca}	3,48	0,03	-	1,03	-	0,04	-	-0,11	-	-0,36	-	-0,07
	9,06	0,06	-	0,07	-	0,07	-	0,09	-	0,34	-	1,16
π_{us}	-39,74	-0,36	0,13	-	0,75		-0,29	-	-1,40	-	13,23	-7,71
	22,20	0,13	0,06	-	0,07		0,11	-	0,52	-	5,61	3,20
π_{ca}	-17,44	0,01	-	0,15	-	0,71	-	-0,12	-	-0,85	3,20	-0,65
	21,06	0,12	-	0,07	-	0,08	-	0,11	-	0,37	5,63	3,36
v_{us}	10,65	-	0,23	-	-	-	0,18	0,34	-1,37	-	-	-
	1,79	-	0,04	-	-	-	0,07	0,11	0,24	-	-	-
v_{ca}	6,09	-	-	0,23	-	-	0,70	-0,09		-0,74	-	-
	1,43	-	-	0,05	-	-	0,09	0,07		0,18	-	-
x_{us}	-2,03	-	0,01	-	-	-	0,01		0,01	0,82	0,32	-
	0,61	-	0,01	-	-	-	0,01		0,03	0,06	0,08	-
x_{ca}	-4,65	-	-	0,01	-	-	-	0,002	0,93	-0,12	-	0,63
	1,30	-	-	0,01	-	-	-	0,01	0,04	0,06	-	0,18
y_{us}	0,00[b]	0,00	0,00	-	0,00	-	0,00	-	-0,01	-	-0,03	1,03
	0,06	0,00	0,00	-	0,00	-	0,00	-	0,00	-	0,01	0,01
y_{ca}	-0,01	-	0,00	0,00	-	0,00	-	0,00	-	0,00	0,98	0,02
	0,03	-	0,00	0,00	-	0,00	-	0,00	-	0,00	0,01	0,00

[a]   The first line is the coefficient, the second line the standard errors
[b]   Values displayed as "'0,00"' are non-zero but relatively small. In the computations of the solutions we apply the correct values.

Table 2: Estimates

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