

Strategic Interactions and Contagion Effects under Monetary Unions*

Paolo Canofari

University of Rome Tor Vergata

Giovanni Di Bartolomeo

Sapienza University of Rome

Giovanni Piersanti

University of Rome Tor Vergata and University of Teramo

April, 2013

*Corresponding author: Di Bartolomeo Giovanni, Sapienza Università di Roma, Facoltà di Economia, via del Castro Laurenziano 9, 00161, Roma, Italy (giovanni.dibartolomeo@uniroma1.it). The authors thank Barbara Annichiarico, Giancarlo Marini, Alessandro Piergallini and participants at the XXI International Conference on Money, Banking and Finance (Luiss University, Rome).

Abstract. This paper applies game theory and a cost-benefit analysis to study voluntary exits and contagion effects in countries joined to a monetary union. The paper looks at two non-core, or periphery countries of a large union and examines the role of structural asymmetries and strategic interactions as determinants of equilibrium outcomes, following both country-specific and common shocks. The paper finds that under almost symmetry between countries, country-specific shocks are never associated to multiple equilibria and, if large enough, can spread to other countries leading to contagion. By contrast, common shocks are seen to sustain multiple equilibria if almost-symmetric countries are considered, and to have implications similar to those found in the country-specific case if large structural asymmetries are admitted.

JEL codes: F30, F31, F41, G01.

Keywords: Shadow exchange rate, currency crisis, monetary unions, contagion, Nash equilibria.

“Although the European Monetary Union has now survived for 11 years, the current strains within the euro zone show why it may not last for another decade without at least some of its members leaving.[...] Leaving the euro zone would be an attractive alternative for Greece because it would allow Greece to devalue its currency. That would boost Greece’s exports and reduce its imports. The resulting increase in production would offset the decline in GDP caused by the tax rise and the cuts in government spending.” (Martin Feldstein, *The Economist*, June 2, 2010).

1 Introduction

The dramatic sovereign debt-crisis surge in some European countries following the onset of the global financial crash in 2007-08, and the perceived risk of contagion to other EMU countries have been the focus of a number of recent papers analyzing the root causes of the current financial turbulence in the Euro Zone (EZ).

Drawing from the sizable literature on exchange rate crises, for example, Arghyrou and Tsoukalas (2011) and Arghyrou and Kontonikas (2012) propose a model of EZ crisis that built around the Obstfeld (1996)- and Krugman (1998)-style model of currency crises, also known in the literature as second- and third-generation models of crises, respectively.¹ They find evidence of contagion to the majority of EMU countries from Greece, and of a striking shift in market pricing behavior from a model applying the same risk premium on government bonds of all EMU countries before 2007, despite intra-European imbalances, to one applying huge spreads thereafter, to reflect both currency risk and default risk on a country-by-country basis.²

Making use of a second-generation approach, De Grauwe (2011) and De Grauwe and Ji (2012) provide a self-fulfilling theoretical explanation of sovereign debt crisis in the Euro Zone. They argue that because members of a monetary union issue debt in a currency over which they have no control (i.e., they are borrowing in a ‘foreign’ currency), government bond markets are fatally exposed to self-fulfilling liquidity crises that can degenerate into a

¹A comprehensive and detailed analytical discussion of the existing theoretical literature on currency and financial crises is to be found in Piersanti (2012).

²Similar results are found, e.g., in Schuknecht *et al.* (2011), Gibson *et al.* (2012), Borgy *et al.* (2012), De Grauwe and Ji (2012). Evidence of contagion effects from Greece, Ireland and Portugal to other Euro Zone countries are also given, e.g., in Arezki *et al.* (2011), De Sanctis (2012), and Metiu (2012).

solvency crisis through unsustainable rise in the interest rates and deep recessions. In addition, as in a currency union financial markets become more and more integrated, spillover effects and contagion from a ‘ground-zero’ country (the first country to undergo a crisis) to other Euro Zone countries are more likely to take place.

Combining the features of both first and second generation approaches, Canofari *et al.* (2012) advance a simple theoretical framework of speculative attacks and crisis in currency unions that includes the main channels for contagion across member countries. They derive a sustainability index for countries operating in hard peg regimes (such as currency unions, currency boards or full dollarization) that builds on cost benefit analysis. The index exploits the relationship between the shadow exchange rate and the output gap required to remain in a hard monetary system, and the model implies that a monetary union is viable when the index shows the capability of the member countries to remain in the hard peg arrangement. This is possible as long as the divergence between the costs of staying relative to the benefits does not exceed a threshold value.

By applying their index to EZ countries in order to evaluate the sustainability of the Euro after the global financial crisis, Canofari *et al.* (2012) show that tensions do exist, particularly for Greece and Portugal who show a severe loss of competitiveness against Germany. However, these tensions appear not so far of such entity as to necessarily cause a breakdown of the common currency, although self-fulfilling speculative attacks, starting in countries with weaker fundamentals, might well take place if the EZ governments failed to send clear signals indicating their strong political willingness to sustain the common currency. Should this uncertain scenario persist, Canofari *et al.* (2012) also predicts that the survival of the Euro might be seriously threat-

ened through the spillover and contagion effects that would inevitably trigger among EZ markets and countries.

This paper extends Canofari *et al.* (2012) by employing game theory and a cost-benefit analysis to investigate voluntary exits and contagion effects in a monetary union.³ Specifically, we look at an asymmetric monetary union consisting of a "core" and a "periphery", and focus on interactions between peripheral economies following a random shock on aggregate demand, abstracting from interactions with the core, for simplicity. The cases of both common and country specific shocks are investigated.

The rest of paper is organized as follows. Section 2 briefly illustrates the Canofari *et al.* (2012) setup and its extension to a game framework to analyse the relative incentive that one or more countries in the periphery face to voluntarily exit the union. Section 3 discusses contagion effects under a country-specific shock. Section 4 scrutinizes equilibrium solutions under a common shock. Finally, section 5 concludes.

2 The basic model

Our model describes an asymmetric three-country monetary union consisting of a "core" or leader country and two small "periphery" economies or "no-core" countries (A and B).⁴ In order to focus on contagion phenomena in the periphery, we abstract from possible interactions between the core and non-core countries, letting policy decisions in the "periphery" have little or

³Woo and Vamvakidis (2012) use a similar approach to provide a ranking of countries having the most incentive to exit the euro area.

⁴The basic setup builds around Canofari *et al.* (2012) to which we refer for more details. For a similar approach, see also Masson (1999), Buiters *et al.* (2001), and Berger and Wagner (2005).

no impact on the "core".⁵ We also take intra-trade between the two small peripheral countries to be negligible, assuming that most of trade occur with the core.

Each economy produces only one good and these goods are imperfect substitutes for one another. Output is a function of the real wage and nominal rigidities exist in the form of a one period wage contract. For simplicity, we let the union-wide inflation rate be equal to zero.

Measuring all variables in logs, we can describe the basic model by the following equations.

The aggregate supply for country i is

$$y_t^i = a_i(s_t^i - \bar{s}^i) + \tilde{y}^i \quad i \in \{A, B\}, \quad (1)$$

where y_t^i is date t output, \tilde{y}^i is the worker's output desired level, s_t^i is the (shadow) nominal exchange rate for country i at time t , and \bar{s}^i the relative entry currency parity.⁶ The nominal exchange rate is defined as the price of the union common currency in terms of the local currency of country i .

The international demand for the goods produced in country i , d_t^i , depends on the real effective exchange rate, q_t^i :

$$d_t^i = \sigma_i q_t^i - u_t^i \quad (2)$$

where u_t^i is an i.i.d. random shock described by a continuous, bell-shaped and symmetric (around zero) probability density function.

⁵This assumption is meant to capture gaming aspects of real world monetary unions (e.g. EMU) where a leader country or a "core" can impose its rules on the whole system. See, e.g., De Grauwe (2012).

⁶The shadow exchange rate is here the floating rate that would prevail at any date t in country i conditional on exit from the monetary union. The key role this variable plays in the theory of exchange rate crises is described in Piersanti (2012).

Assuming that prices are fixed for simplicity, the real effective exchange rate in country i can be written as:

$$q_t^i \equiv s_t^i - \beta s_t^j \quad i, j \in \{A, B\} \quad i \neq j, \quad (3)$$

where β measures the impact of a devaluation in country j on competitiveness of country i .

Equilibrium in the goods market of country i implies:

$$y_t^i = \sigma_i (s_t^i - \beta s_t^j) - u_t^i \quad i, j \in \{A, B\} \quad i \neq j, \quad (4)$$

which we can express as

$$\begin{aligned} y_t^i &= \sigma_i (s_t^i - \bar{s}^i) + y_t^{i,F} \\ y_t^{i,F} &\equiv \sigma_i (\bar{s}^i - \beta s_t^j) - u_t^i, \quad i, j \in \{A, B\} \quad i \neq j \end{aligned} \quad (5)$$

where $y_t^{i,F}$ denotes the output for country i required to stay in the currency union. This equation discloses that, once in the monetary union, the incentive to exit for each country comes from the increase in output with respect to $y_t^{i,F}$ which can be obtained by a realignment of the exchange rate.

The exit/no exit game

Let now the policymaker in country i minimize a loss function defined over the output gap and inflation (measured by the change in nominal exchange rate). Consistent with the second generation approach, let also a linear term, measuring the cost the policymaker incurs if he chose to exit from the monetary union, be added in the loss function, namely

$$L_t^i = (y_t^i - \bar{y}^i)^2 + \theta_i (s_t^i - \bar{s}^i)^2 + \delta \mathbb{C}^i, \quad (6)$$

where \bar{y}^i is the policymaker's output target, θ_i is the inflation aversion coefficient, \mathbb{C}^i is the cost of opting out, assumed to be fixed for simplicity, and δ is a dummy variable defined as $\delta = 0$ if country i remains in the monetary union, so that $\Delta s_t^i \equiv s_t^i - \bar{s}^i = 0$, and $\delta = 1$ if it exits and $\Delta s_t^i \neq 0$. For simplicity, we assume a common inflation aversion coefficient between the two countries (i.e., $\theta_A = \theta_B = \theta$) and focus, instead, on the possible differences in the opting out costs. This cost may have several sources and could reflect, for example, the loss of anti-inflation credibility and international reputation and the effects on (foreign) debt accumulation and other financial variables that need not be linked to the size of devaluation rate.⁷

In order to analyze voluntary exits and contagion effects under a monetary union, we now focus on a 'currency-union' game where each country can choose between two actions, either choosing to remain in the monetary union and set $\Delta s = 0$, denoted by *No exit*, or choosing to leave it and set $\Delta s \neq 0$, denoted by *Exit*. We let \bar{y}^i be equal to $y_t^{i,F}$ in absence of shocks. Hence, $y_t^{i,F} - \bar{y}^i = E_{t-1} \Delta s_t^i = \Delta s_t^i = 0$ and both countries decide to stay in the monetary union if no shock occurs at time t .⁸ Conditional on the realization of a shock that causes deviations of output from its desired level, the policymaker in country i decides to leave or not the monetary union by comparing the welfare losses arising from alternative policy regimes. Thus, the policymaker's problem is to identify the threshold value of the shock at which it is optimal to operate a regime change. As policy decisions are not independent in our model, an interaction between the policymakers optimizing behavior necessarily develops, and this can lead to multiple equilibria and contagion effects across countries.

⁷See, e.g., Obstfeld (1994, 1997), Jeanne (1997), Piersanti (2012, chap.3).

⁸A proof is given in Appendix A, where the nominal fixed parities required to sustain a monetary union agreement is also computed.

The solutions to this game arise from strategy profiles that form a Nash equilibrium. This requires solving first the model to obtain the optimal policies and corresponding losses for each country given the strategy followed by the other country, and then finding the incentive to deviate from it to check if it is or not a Nash equilibrium.

In order to simplify discussion and make mathematical expressions less blurred, we now normalize the output target levels and the nominal fixed parities to one, and assume that the elasticity of aggregate demand to the real exchange rate be the same in the two external countries. Accordingly, henceforth we set $\tilde{y}^A = \bar{y}^A = \tilde{y}^B = \bar{y}^B = 0$, $\bar{s}^A = \bar{s}^B = 0$, and $\sigma_A = \sigma_B = \sigma$.

The structure of the game identifies four regimes: no countries exit (regime N); only country A exits (regime D^A); only country B exits (regime D^B); both countries exit (regime E).

Losses associated to the above regimes are described in the following payoff matrix

A / B	<i>No exit</i>	<i>Exit</i>
<i>No Exit</i>	L_N^A, L_N^B	$L_{D^B}^A, L_{D^B}^B$
<i>Exit</i>	$L_{D^A}^A, L_{D^A}^B$	L_E^A, L_E^B

where L_h^i denotes the loss for policymaker i in regime h . This matrix can be used to find the unique or multiple Nash equilibrium solutions of the game.

These equilibria are stated formally in the following

Proposition 1 *Regime E is a Nash equilibrium if and only if $L_{D^B}^A - L_E^A > 0$ and $L_{D^A}^B - L_E^B > 0$. Regime N is a Nash equilibrium if and only if $L_{D^A}^A - L_N^A > 0$ and $L_{D^B}^B - L_N^B > 0$. Regime D^B is a Nash equilibrium if and only if $L_E^A - L_{D^B}^A > 0$ and $L_N^B - L_{D^B}^B > 0$. Regime D^A is a Nash equilibrium if and only if $L_N^A - L_{D^A}^A > 0$ and $L_E^B - L_{D^A}^B > 0$.*

Proof. The proof is straightforward. ■

To compute the losses shown in the above matrix, we need the policy-makers' problem in all the regimes to be solved. Optimal policies for country $i \in \{A, B\}$ are obtained by minimizing (6) with respect to s_t^i subject to (5) and $\delta = 1$ under the *Exit* option; otherwise $s_t^i - \bar{s}^i = \delta = 0$ under the *No Exit* choice.

The reaction function for country i is given by

$$\begin{cases} s_t^i = \frac{(u_t^i + \sigma \beta s_t^j) \sigma}{\sigma^2 + \theta} & \text{for } \delta = 1 \\ s_t^i = 0 & \text{otherwise} \end{cases} \quad i, j \in \{A, B\} \quad i \neq j, \quad (7)$$

and optimal policies in all the regimes are:

$$\text{Regime } N : \{s_t^A = 0, s_t^B = 0\} \quad (8)$$

$$\text{Regime } D^B : \left\{ s_t^A = 0, s_t^B = \frac{\sigma u_t^B}{\sigma^2 + \theta} \right\} \quad (9)$$

$$\text{Regime } D^A : \left\{ s_t^A = \frac{\sigma u_t^A}{\sigma^2 + \theta}, s_t^B = 0 \right\} \quad (10)$$

$$\text{Regime } E : \left\{ s_t^A = \frac{\sigma[(\sigma^2 + \theta) u_t^A + \beta u_t^B \sigma^2]}{(\sigma^2 + \theta)^2 - \beta^2 \sigma^4}, s_t^B = \frac{\sigma[(\sigma^2 + \theta) u_t^B + \beta u_t^A \sigma^2]}{(\sigma^2 + \theta)^2 - \beta^2 \sigma^4} \right\}. \quad (11)$$

Finally, using (8)-(11), (5) and (6), the corresponding losses for country $i \in \{A, B\}$ required to find Nash equilibria are given by:

$$L_N^i = (u_t^i)^2 \quad (12)$$

$$L_{D^i}^i = \frac{\theta}{\sigma^2 + \theta} (u_t^i)^2 + \mathbb{C}^i \quad (13)$$

$$L_{D^j}^i = \left(\frac{\beta \sigma^2 u_t^j}{\sigma^2 + \theta} + u_t^i \right)^2 \quad (14)$$

$$L_E^i = \frac{(\sigma^2 + \theta) \theta [(\sigma^2 + \theta) u_t^i + \sigma^2 \beta u_t^j]^2}{[(\sigma^2 + \theta)^2 - \beta^2 \sigma^4]^2} + \mathbb{C}^i, \quad i \neq j \in \{A, B\}, \quad (15)$$

where L_N^i indicates the loss of country i when both countries decides to stay in, $L_{D^i}^i$ is the loss of country i when it choices to exit, $L_{D^j}^i$ is the loss of

country i when country j decides to exit, and L_E^i is the loss of country i when both countries opt out.

3 Country-specific shock and contagion

To investigate the impact of a country-specific shock and contagion effects across countries, we now let, with no loss of generality, $u_t^B = 0$, $u_t^A \equiv v_t$, and $\mathbb{C}^A = \mathbb{C}^B = \mathbb{C}$.⁹ Solving for v_t using (12)-(15) we find that the threshold values of the shock at which the governments in country A and B are indifferent between opting out and remain in the union are:

$$v^* = \frac{1}{\sigma} \sqrt{(\sigma^2 + \theta) \mathbb{C}} \quad (16)$$

$$v^{**} = \frac{1}{\sigma} Q_S \sqrt{(\sigma^2 + \theta) \mathbb{C}} = Q_S v^* , \quad (17)$$

where $Q_S = \sqrt{\frac{(\sigma^4 + \theta \sigma^2)}{\sigma^4 \beta^2} \frac{[(\sigma^2 + \theta)^2 - \sigma^4 \beta^2]^2}{[(\sigma^2 + \theta)^2 - \sigma^4 \beta^2]^2 - \theta(\sigma^2 + \theta)^3}} > 1$; thus, $v^{**} > v^*$.¹⁰

The following proposition summarizes the main implications of the model under a country-specific shock.

Proposition 2 *Country A exits the monetary union and devalues if and only if $v_t > v^*$; both policymakers exit and devalue (contagion) if $v_t > v^{**}$.*

Proof. Write the incentive to move from one regime to another in a compact form as: $a_1 = L_{DB}^A - L_E^A$, $a_2 = L_{DA}^A - L_N^A$, $b_1 = L_{DA}^B - L_E^B$, $b_2 = L_{DB}^B - L_N^B$.

⁹The symmetry hypothesis $\mathbb{C}^A = \mathbb{C}^B = \mathbb{C}$ is here only to isolate the effect of the shock. Asymmetries are taken up below under common shocks.

¹⁰Notice that $Q_S > 1$ as long as β is smaller than one: the first term under the root is larger than one; the second term is always larger than one for admissible values of parameters.

For instance, if $a_2 > 0$ country A has no incentive to leave the monetary union. Letting $u_t^B = 0$, it is easy to check that

$$a_1 > 0 \iff v_t > v^{***} \quad (18)$$

$$b_1 > 0 \iff v_t > v^{**} \quad (19)$$

$$a_2 > 0 \iff v_t < v^* \quad (20)$$

$$b_2 > 0 \text{ always} \quad (21)$$

where $v^{**} > v^{***} = \frac{\beta\sigma^2}{\sigma^2+\theta}v^{**}$. Therefore, from Proposition 1 and conditions (18)-(21) it follows that: ■

Proposition 3 (i) *Regime E is a Nash equilibrium if and only if a_1 and b_1 are both positive, i.e. $v_t > v^{**}$.*

(ii) *Regime N is a Nash equilibrium if and only if $a_2 > 0$ and $b_2 > 0$, i.e. $v_t < v^*$.*

(iii) *Regime D^A is a Nash equilibrium if $a_2 < 0$ and $b_1 < 0$, i.e. $v^* < v_t < v^{**}$.*

(iv) *Regime D^B is never a Nash equilibrium as it requires $b_2 < 0$.*

The intuition behind this result is simple. When country-specific shocks are small enough, no country would find it profitable to opt out and the stability of the monetary union is preserved. By contrast, for large value of the shocks two events can be discerned: 1) a value at which only the country dealing with the shock may find it optimal to exit and devalue; 2) a higher value at which both countries optimally choose to exit, so giving rise to contagion. Proposition 2 also makes clear that both multiple equilibria and the (perverse) event where only the country not hit by the shock moves out can never occur.

4 Common shocks and multiple equilibria

We now focus on common shocks and possible asymmetries in the opting out costs between countries. We study the effects of a common shock by setting $u_t^A = u_t^B = u_t$, and those of possible asymmetries by letting, with no loss of generality, $\mathbb{C}^A \geq \mathbb{C}^B$.¹¹

Solving for u_t using (18)-(21), we can identify the following critical values for the shock

$$u_t^{**} = \frac{1}{\sigma} \sqrt{(\sigma^2 + \theta) \mathbb{C}^B} \quad (22)$$

$$u_t^* = \frac{Q_C}{\sigma} \sqrt{(\sigma^2 + \theta) \mathbb{C}^A}, \quad (23)$$

where $Q_C = \sqrt{\frac{(\sigma^2 + \theta - \sigma^2 \beta)^2 (\sigma^2 + \theta) \sigma^2}{[(\sigma^2 + \theta)^2 - \sigma^4 \beta^2]^2 - \theta (\sigma^2 + \theta)^3}} \in (0, 1)$.¹² These values allowed us to establish two cases according to the relative size of \mathbb{C}^A with respect to \mathbb{C}^B : 1) $\mathbb{C}^A \in [\mathbb{C}^B, \mathbb{C}^B/Q_C^2)$, implying $u^{**} > u^*$; 2) $\mathbb{C}^A > \mathbb{C}^B/Q_C^2$, implying $u^* > u^{**}$. We refer to the former as the case of no or small asymmetries¹³ and to the latter as that of large asymmetries. We describe the effects of a common shock in the following propositions.

Proposition 4 (no or small asymmetries) *Under $\mathbb{C}^A \in [\mathbb{C}^B, \mathbb{C}^B/Q_C^2)$, there are thresholds $u^* < u^{**}$ such that:*

- (i) *if $u_t < u^*$, no country devalue (Regime N);*
- (ii) *if $u_t^A \in (u^*, u^{**})$, multiple equilibria arise (Regime N or E);*

¹¹Some numerical simulations based on the present theoretical framework with large asymmetries and heterogeneity are given in Canofari *et al.* (2012a).

¹²See Appendix B.

¹³Notice that if $\mathbb{C}^A = \mathbb{C}^B$, $u^{**} > u^*$.

(iii) if $u_t^A > u^{**}$, both policymakers devalue (Regime E).

Proposition 5 (large asymmetries) Under $\mathbb{C}^A > \mathbb{C}^B/Q_C^2$, there are thresholds $u^* > u^{**}$ such that:

(i) if $u_t < u^{**}$, no country devalue (Regime N);

(ii) if $u_t \in (u^{**}, u^*)$, only country B devalue;

(iii) if $u_t > u^*$, both policymakers devalue (Regime E).

Proof. Under $u_t^A = u_t^B = u_t$, the alternative regimes can be identified as follows:

$$a_1 > 0 \iff u_t > u^* = \frac{Q_C}{\sigma} \sqrt{(\sigma^2 + \theta) \mathbb{C}_A} \quad (24)$$

$$b_1 > 0 \iff u_t > u^{***} = \frac{Q_C}{\sigma} \sqrt{(\sigma^2 + \theta) \mathbb{C}_B} \quad (25)$$

$$a_2 > 0 \iff u_t < u^{****} = \frac{\sqrt{(\sigma^2 + \theta) \mathbb{C}_A}}{\sigma} \quad (26)$$

$$b_2 > 0 \iff u_t < u^{**} = \frac{\sqrt{(\sigma^2 + \theta) \mathbb{C}_B}}{\sigma}. \quad (27)$$

■

Take up the no or small asymmetry case. From Proposition 1 and (19)-(20), we find that: a) E is a Nash equilibrium iff a_1 and b_1 are both positive, i.e. $u_t > \max(u^*, u^{***}) = u^*$; b) N is a Nash equilibrium iff a_2 and b_2 are both positive, i.e. $u_t < \min(u^{**}, u^{****}) = u^{**}$; c) D^A is never a Nash equilibrium as it would require a_2 and b_1 to be both negative; d) D^B is never a Nash equilibrium as it would require a_1 and b_2 to be both negative.

Consider now the large asymmetry case. We can see that: a) E is a Nash equilibrium iff a_1 and b_1 are both positive, i.e. $u_t > \max(u^*, u^{***}) = u^*$; b) N is a Nash equilibrium iff a_2 and b_2 are both positive, i.e. $u_t < \min(u^{**}, u^{****}) = u^{**}$; c) D^A is never a Nash equilibrium as it would require

a_2 and b_1 to be both negative; d) D^B is a Nash equilibrium iff a_1 and b_2 are both negative, i.e. $u^{**} < u_t < u^*$. These restrictions make propositions 3 and 4 straightforward.

Notice that under almost symmetry, multiple equilibria can arise when both countries either stay in or exit. Thus, by emphasizing countries differences, asymmetries lead to results that are similar to those found in the country-specific shock scenario, as the behavior of the country with the lower opting out cost appear to echo that of a country dealing with an idiosyncratic shock.

5 Conclusions

This paper applied game theory and a cost-benefit analysis to study voluntary exits and contagion effects in countries joined to a monetary union. The paper looked at two non-core, or periphery countries of a large monetary union and examined the role of structural asymmetries and strategic interactions in determining the set of equilibrium solutions under both country-specific and common shocks.

The main implications are as follows. Country-specific shocks are never associated to multiple equilibria under almost symmetry between countries. If large enough, however, they can beget the country's exit and be transmitted across the boards (contagion effects), thus posing a serious threat to union's stability. By contrast, common shocks may sustain multiple equilibria if almost-symmetric countries are considered, and have implications similar to those found in the country-specific case if large structural asymmetries are admitted.

Appendix A – Monetary union parities

In this appendix we show that countries have no incentive to leave the monetary union in absence of shocks, and compute the nominal parities consistent with a monetary union agreement.

For convenience, let the basic theoretical framework be rewritten as the following multi-country AD/AS model

$$y_t^i = a_i(s_t^i - \bar{s}^i) + \bar{y}^i \quad (\text{A.1})$$

$$d_t^i = \sigma_i(s_t^i - \beta s_t^j) - u_t^i \quad (\text{A.2})$$

$$d_t^i = y_t^i, \quad i, j \in \{A, B\} \text{ and } i \neq j, \quad (\text{A.3})$$

setting $\tilde{y}^i = \bar{y}^i$, $i \in \{A, B\}$, for simplicity. From (6), it is easy to check that a monetary union is sustainable if $s_t^i = \bar{s}^i$ implies $y_t^i = \bar{y}^i$ for $i \in \{A, B\}$. This condition is sufficient and also necessary if $\mathbb{C}^i = \mathbb{C}^j = 0$. As \mathbb{C} is the cost of breaking the agreement, it is sensible to assume that $\mathbb{C} = 0$ initially, when the union is framed.

Equilibrium under $u_t^i = u_t^j = 0$ implies

$$\bar{y}^A = \sigma_A [s_t^A - \beta s_t^B] \quad (\text{A.4})$$

$$\bar{y}^B = \sigma_B [s_t^B - \beta s_t^A]. \quad (\text{A.5})$$

Using (A.4) and (A.5), the nominal fixed parities (\bar{s}^A and \bar{s}^B) that imply $y_t^A = \bar{y}^A$ and $y_t^B = \bar{y}^B$ are¹⁴

$$\bar{s}^i = \frac{\bar{y}^i / \sigma_i + \beta \bar{y}^j / \sigma_j}{1 - \beta^2}, \quad (\text{A.6})$$

for $i, j \in \{A, B\}$ and $i \neq j$.

¹⁴To obtain (A.6), simply set $s_t^A = \bar{s}^A$ and $s_t^B = \bar{s}^B$, and solve the system (A.4)-(A.5) for \bar{s}^A and \bar{s}^B .

Finally, letting $\sigma_i = \sigma_j = \sigma$, $\bar{y}^i = \bar{y}^j = \bar{y}$, (A.6) becomes

$$\bar{s}^A = \bar{s}^B = \bar{s} = \frac{1}{1 - \beta} \frac{\bar{y}}{\sigma}.$$

Under these restrictions, if no shock occurs, countries will have no incentive to exit and deviate from the agreed policy rule. Accordingly, if $u_t^i = u_t^j = 0$, a rational expectation equilibrium implies $\Delta s_t^i \equiv s_t^i = \bar{s}^i = \Delta s_t^j = E_{t-1} \Delta s_t^i = E_{t-1} \Delta s_t^j = 0$, and $y_t^{i,F} = y_t^i = y_t^j = y_t^{j,F} = \bar{y}$.

Appendix B – The value of Q_C

Figure A1 plots $Q = 1 - Q_C = 1 - \sqrt{\frac{(\sigma^2 + \theta - \sigma^2 \beta)^2 (\sigma^2 + \theta) \sigma^2}{[(\sigma^2 + \theta)^2 - \sigma^4 \beta^2]^2 - \theta (\sigma^2 + \theta)^3}}$, for $\sigma = 0.01..100$, $\theta = 0..100$ and $\beta = 1/2$, and shows that Q is always positive.

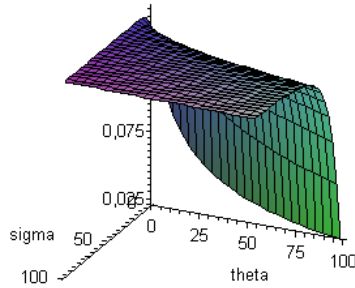


Figure A1

The same result obtains letting the value for β to vary from 0 to 1/2 by a step of 0.1. Results are available upon request.

Positive values for Q implies $Q_C < 1$. Thus, as Q_C is always greater than zero, $Q_C \in (0, 1)$.

References

- Arezki, R., B. Candelon, and A.N.R. Sy (2011), "Sovereign Rating News and Financial Markets Spillovers: Evidence from the European Debt Crisis," *IMF Working Paper Series*, No.11/68.
- Arghyrou, M.G. and J.D Tsoukalas (2011), "The Greek Debt Crisis: Likely Causes, Mechanics and Outcomes," *The World Economy*, 34: 173-191.
- Arghyrou, M.G. and A. Kontonikas (2012), "The EMU Sovereign-Debt Crisis: Fundamentals, Expectations and Contagion," *Journal of International Financial Markets, Institutions & Money*, 22: 658-677.
- Berger, W. and H. Wagner (2005), "Interdependent Expectations and the Spread of Currency Crises," *IMF Staff Papers*, 52: 41-54.
- Borgy V., T. Laubach, J.-S. Mésonnier and J.-P. Renne (2012), "Fiscal Sustainability, Default Risk and Euro Area Sovereign Bond Spreads," *Banque de France Documents de Travail*, No. 350.
- Buiter, W.H., G. Corsetti and P.A. Pesenti (2001), *Financial Markets and European Monetary Cooperation: The Lessons of the 1992-93 Exchange Rate*, Cambridge University Press, Cambridge U.K.
- Canofari, P., G. Marini and G. Piersanti (2012), "The Sustainability of Monetary Unions. Can the Euro Survive?," *CEIS Tor Vergata Research Paper Series*, No. 226.
- De Grauwe, P. (2011), "The Governance of a Fragile Eurozone," *CEPS Working Documents*, No. 346.
- De Grauwe, P. (2012), "In Search of Symmetry in the Eurozone," *CEPS Policy Briefs*, No. 268.
- De Grauwe P. and Y. Ji (2012), "Self-Fulfilling Crises in the Eurozone: An Empirical Test," *CEPS Working Documents*, No. 366.
- De Santis, R.A. (2012), "The Euro Area Sovereign Debt Crisis: Safe Haven, Credit Rating Agencies and the Spread of the Fever from Greece, Ireland and Portugal," *ECB Working Paper Series*, No. 1419.
- Gibson, H.D., S.G. Hall and G.S. Tavlas (2012), "The Greek Financial Crisis: Growing Imbalances and Sovereign Spreads," *Journal of International Money and Finance*, 31: 498-516.

- Jeanne, O. (1997). "Are Currency Crises Self-Fulfilling? A Test". *Journal of International Economics*, 43 (November), 263-86.
- Krugman, P.R. (1998), "What Happened to Asia?". Available on-line at: <http://web.mit.edu/krugman/www/disinter.html>
- Masson, P.R. (1999), "Contagion: Macroeconomic Models with Multiple Equilibria," *Journal of International Money and Finance*, 18: 587-602.
- Metiu, N. (2012), "Sovereign Risk Contagion in the Eurozone," *Economics Letters*, 117: 35-38.
- Obstfeld, M. (1994). "The Logic of Currency Crises". *Cahiers Économiques et Monétaires*, No. 43, Banque de France, 189-213.
- Obstfeld, M. (1996), "Models of Currency Crises with Self-Fulfilling Features," *European Economic Review*, 40: 1037-1047.
- Obstfeld, M. (1997). "Destabilizing Effects of Exchange-Rate Escape Clauses". *Journal of International Economics*, 43 (August) 61-77.
- Piersanti, G. (2012), *The Macroeconomic Theory of Exchange Rate Crises*, Oxford University Press, Oxford.
- Schuknecht, L., J. von Hagen and G. Wolswijk (2011), "Government Bond Risk Premiums in the EU Revisited: The Impact of the Financial Crisis," *European Journal of Political Economy*, 27: 36-43.
- Woo, D. and A. Vamvakidis (2012). "Game Theory and Euro Breakup Risk Premium". Bank of America Merrill Lynch Research Report, July.