# How Okun's law is non-Linear in Europe: A Semi-Parametric Approach.

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**Abstract**

This article presents evidence of a non-linear Okun's law in a panel of 16 European countries. We use a semi-parametric approach in a short-run framework without imposing any assumption on the Okun's law functional form. Using quarterly data from 1984 to 2009, our estimations support that unemployment responds strongly to output in the early recession phases (downturn) and during expansions. By contrast, in the middle of recessions and during recoveries, the impact of output on unemployment tends to be weaker. This kind of asymmetry has never been empirically stated for European countries. Our results could explain the current European jobless recovery and can be rationalized by theoretical arguments.

*J.E.L classification: C14, E32, J00*

*Keywords: Okun's law, asymmetry, semi-parametric regression*

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# 1 Introduction

European countries are recovering from the deepest economic crisis since World War II. According to IMF (2010), some countries are facing persistently high unemployment level despite the ongoing recovery. This jobless recovery contrasts with the sharp rise of unemployment caused by the huge fall of output during the Great Recession. Besides, the response of unemployment to output during the crisis was different across economies in Europe. For instance, the unemployment rate increase of about 7 percentage points in Spain was associated with an output decrease of 3.75%. Whereas the slight unemployment increase of 0.4 percentage points in Germany occured despite an output fall of more than 7.5%. These facts puzzle economists and challenge the empirical relationship between output and unemployment.

Connection between labor market and business cycle can be studied using Okun's law framework which describes the inverse relationship between changes in real output and unemployment rate. The relation is intuitive: growth is associated with falling unemployment while growth slowdowns are accompanied by rising unemployment. As Knotek (2007) suggests, the appeal of this statistical relationship is its simplicity ands its regularity. Having received a large empirical support, it belongs therefore to core beliefs in practical macroeconomics. According to Huang and Chang (2005), this rule of thumb can be used for both theoretical and empirical reasons. First, Okun's law is a key element in construction of macroeconomic model like aggregate supply (by combining with Phillips curve). Second, the relationship has important implications for macroeconomic policy, because it provides a benchmark to measure the cost of unemployment in terms of output (e.g. effectiveness of disinflationary policy) and, therefore indicates the desirable growth rate. There is today a consensus about the qualitative nature of Okun's law in industrialized countries. However, we note discrepancies among results on the relationship quantitative magnitude. Estimates of Okun's coefficient vary in fact, both across countries and time periods (Freeman (2001) and Lee (2000)).

As shown by Neftçi (1984), macroeconomic time series exhibit non-linear or asymmetric behavior over business cycle phases. The output-unemployment relationship could therefore possibly be non-linear. Testing for asymmetry in Okun's law is of great interest for several reasons. First, asymmetry would provide an explanation for the varying effectiveness of structural and stabilization policies. It could this way assist in discriminating among alternatives theories of joint labor and goods market behavior. Then, this asymmetry could lead to forecast errors if ignored when present, and would have important implications for aggregation issues especially in the context of EMU (as advocated by Virén (2001)). Asymmetry in the Okun's relation framework means the response of unemployment to output is different when the economy is expanding or contracting. This obviously differs from a linear specification which implicitly states that the unemployment response to output is the same during expansions and recessions. A potential Okun's law asymmetry can be understood in two different manners. The negative relation can indeed be either stronger or weaker during recessions than during expansions. In an extensive survey, Silvapulle et al. (2004) provide alternative theoretical explanations of the two possible Okun's law asymmetry patterns.

On one hand, unemployment could respond more strongly to growth when economy is contracting than expanding. Indeed, according to the firm's risk aversion argument, bad news are believed more quickly than good news. So as the economy goes into a recession, firms respond very quickly by laying off workers. By contrast, during the recovery phase, firms fear the possibility that the recovery may not last and are consequently reluctant to hire new employees. Yet, employers prefer to adjust their intensive margin (number of hours per employee) rather than their extensive margin (number of employees). This asymmetry feature could also be linked with hysteresis literature (Lang and De Perreti (2009) and Schorderet (2001) for instance). Hysteresis can in fact arise from human capital depreciation or because of an insiders-outsiders effect, and can explain why unemployment does not return to its initial level immediately after a recession. This asymmetry pattern will be called *Firms's Risk Aversion Hypothesis* hereafter.

On the other hand, unemployment could respond more strongly to output when the economy is expanding than contracting. Silvapulle et al. (2004) propose this time, an explanation based on the labor market rigidity and the investment in the workers training. The idea (originally advanced by Oi (1962)), is that labor is a quasi-fixed factor due to costs associated with firing and training employees. In this context, firms are reluctant to dismiss workers during recessions because of the institutional restrictions on layoffs and their investment in staff training. Employers also want to retain their skilled workers to avoid looking for trained labor force when economic conditions improve. Conversely, during recoveries and because there are few restrictions on hiring additional employees, they would hire more workers. Therefore, firms tend to decrease their intensive margin rather than extensive margin using "just in time" employment practices during recessions (such as reduction of overtime and utilization of part-time workers). These features explain that output effect on unemployment can be close to zero during contractions. Finally, this asymmetry pattern will be called *Labor Hoarding Hypothesis* hereafter.

Macroeconomic literature offers a large body of empirical studies devoted to the Okun's law asymmetry. For USA, numerous papers like Beaton (2010), Crespo Cuaresma (2003), Palley (1993), Silvapulle et al. (2004) or Huang and Chang (2005) for Canada show that the effect of growth on unemployment is asymmetric and stronger during contraction than during expansions. These results are in line with the *Firm's Risk Aversion Hypothesis*. On the contrary, asymmetry evidences for the European case, are rather mixed. For instance, Lee (2000), uses a model where output is determined by positive or negative changes in unemployment using First-Difference and Gap version on annual data. For the Gap version, Okun's coefficients are higher in absolute values during expansions than contractions; confirming this way the *Labour Hoarding Hypothesis*. But for the Firts-Difference version, results depend upon the country considered. Therefore, Lee (2000) concludes that the asymmetry type and so the verified hypothesis depends on the methodology used. Virén (2001) presents a model whereby changes in unemployment depend of an error-correction term (which represents the long-run unemployment level related to working-age population and labour market structure) and on output changes in a short-run perspective. He uses different two-thresholds regimes to study the effect of output on unemployment. When output changes are split into positive and negative values, he finds a higher effect of growth on unemployment when the economy is contracting than expanding, findings which are in line with the *Firms's Risk Aversion Hypothesis*. By contrast, results differ when output growth is associated with cyclical output (calculated using Hodrick-Prescott filter) or cyclical unemployment (calculated as the difference between the observed and the long-run unemployment rate). He shows indeed, that output growth has a strong effect on unemployment when the output is high (above the long-run trend) or unemployment is low (below the long-run level). Thus, he concludes that unemployment is particularly sensitive to output when output turns a downward trend and that output changes only have a minor effect on unemployment in the middle of a depression.

Empirical studies on asymmetry on Okun's law were usually performed on individual countries, Fouquau (2008) goes beyond this approach, investigating asymmetry to an aggregate level. Applying a non-dynamic panel thresholds model for OECD countries with quarterly data, Fouquau (2008) points out asymmetry implies the existence of four regimes, regardless of the Okun's law First-Difference or Gap version used. He underscores a strong correlation between output and unemployment for contractions and fast expansions periods (called extreme regimes), whereas this relation tends to weaken when economy stands at intermediate regimes. He concludes that Okun's law apply well only in extreme regimes. We have to note, however, that the great majority of his observations are in the intermediate regimes whereas extreme regimes are characterized by very few observations.

This article aims at (i) proposing a semi-parametric empirical framework to estimate Okun's law with potential non-linearity (ii) providing a good description of European labor market over the business cycle at an aggregate level. To do so, we confront the traditional linear specification (used as a benchmark) to a more flexible approach and underscore the linear specification inappropriateness. Applied to a balanced panel of 16 European countries with quarterly data from 1984 to 2009, our paper clearly estblishes a non-linear Okun's relationship. In addition, combining insights from the two usual Okun's law versions (i.e. Gap and First Difference versions) we are able to reconcile the two asymmetry patterns presented above. Our findings support that the output - unemployment relationship strength varies along the business cycle, and that both the *Firms's Risk Aversion Hypothesis* and the *Labor Hoarding Hypothesis* are successively relevant for the European case.

The paper is organized as follows. Section 2 outlines our methodology through the presentation of the two Okun's law versions and the semi-parametric specification. Our sample is described in Section 3. Section 4 presents and discusses our results while Section 5 concludes.

## 2 Methodology

In his seminal paper, Okun (1962) suggests two ways to estimate the output - unemployment relationship: a First-Difference and a Gap version. In our article and like Lee (2000), we consider both versions to provide a balanced treatment of the issue and to assess the robustness of our results. In addition, the two versions do not provide exactly the same kind of information since the Gap version focuses on the cyclical activity aspect while the First-Difference version, by nature, mixes cyclical and structural dimensions. In the First-

Difference version, unemployment and output variables are expressed in first differences (GDP growth and changes in unemployment rate) whereas in the Gap version they are expressed in terms of their cyclical components or deviations from long-run trend. Focusing on a short-run perspective, we regress unemployment on output as recommended by Barreto and Howland (1993). The First-Difference version is of the following form:

$$\Delta u_t = \alpha + \gamma \Delta y_t + \epsilon_t \tag{1}$$

Where $\Delta u_t$ and $\Delta y_t$ are the quarterly changes in the unemployment rate and logarithm of real GDP, $\alpha$ is a constant term and $\epsilon_t$ is a disturbance term. The parameter $\gamma$ captures the short-run effect of real GDP on unemployment, i.e. the Okun's coefficient. The Gap version of Okun's law is naturally based on the notion of gap between observed and natural unemployment rate or between the observed and potential output. The Gap version specification is given by the expression:

$$u_t^c = \alpha + \gamma y_t^c + \eta_t \ , \tag{2}$$

$$u_t^c \equiv u_t - u_t^n$$

$$y_t^c \equiv y_t - y_t^n$$

Where $u_t$ denotes the observed unemployment rate; $u_t^c$ denotes the cyclical unemployment (unemployment gap) rate; $u_t^n$ denotes the natural unemployment rate; $y_t^c$ denotes the logarithm of cyclical output (output gap); $y_t$ denotes the logarithm of observed output; $y_t^n$ denotes the logarithm of potential output;$\alpha$ is a constant term;$\eta_t$ is a disturbance term and the parameter $\gamma$ captures Okun's coefficient. For both versions, we expect to find a negative correlation between unemployment and output ($\gamma < 0$).

In contrast to First-Difference version, Gap version requires knowledge about natural unemployment and potential output which are not directly observable. Several approaches exist for the estimation of these two trend values. Simple methods like deterministic linear trend or quadratic trend often fail to account for the stochastic component of output or unemployment in determining their equilibrium values. More sophisticated methods can be used to take into accounts potential stochastic trends in output and unemployment: Hodrick-Prescott (HP) filter, Beveridge-Nelson decomposition, band-pass filter and Harvey's structural time series, for instance. The HP filter and the Beveridge-Nelson decomposition are described by Lee (2000) as being "ad hoc" in nature. Particularly, the HP filter has received severe criticism as it might create biased estimates of the cyclical components and inducing spurious results. In addition, band-pass filter often display similar results to HP filter as shown by Huang and Chang (2005). In this study, we choose the univariate version of Harvey (1989) structural time series to extract the cyclical components of unemployment and output[1]. The rationale for this choice is motivated by Silvapulle et al. (2004) which argue as follows:

> "On one hand, Harvey structural time series provide the most useful framework within which to present stylized facts on time series, since it is explicitly based on the stochastic properties of the data. On the other hand, it provides useful information and serves as a basis for exposing the limitations of ARIMA models and models based on deterministic trends with single breaks."

We propose a two-step estimation approach to investigate whether the Okun's law is linear in Europe or not. In this prospect, we estimate an Okun's relation using two different frameworks. The first is a traditional (linear) fixed-effect model (i.e. the Least Square Dummy Variable, LSDV hereafter)[2]. We then use this estimation as a benchmark to discuss the outcome of a semi-parametric specification which allows the output/unemployment relation to be nonlinear. We run these two different kinds of estimation for both the First-Difference and the Gap version of Okun's law. The equation to be estimated in both frameworks (presented here for the First Difference version) is:

$$\Delta u_{it} = \alpha + m(\Delta y_{it}) + \beta_j(X_j) + \epsilon_{it} \tag{3}$$

$i = 1, \ldots, n, \quad t = 1, \ldots, T \;$ and $\quad j = 1, \ldots, n-1$

We assume that $\epsilon_{it}$ is an independent random error with $E\left[\epsilon_{it}|\Delta y_{it}, X_j\right] = 0$ and $V\left[\epsilon_{it}|\Delta y_{it}, X_j\right] = \sigma^2\left(\Delta y_{it}, X_j\right)$. Where $\Delta u_{it}$ and $\Delta y_{it}$ are respectively the endogenous and exogenous variables defined in equation 1, and $X_j$ is a vector of $(n-1)$ dummy variables (United Kingdom as the

[1] Interested readers can find the extraction details in appendix A

[2] We use the classical Hsiao tests procedure (Hsiao (1986)) in order to choose the LSDV model against the between, the pooled and the random effect model.

reference, one dummy is dropped in order to avoid perfect multicollinearity issue). These are individual fixed effects allowing to tackle issues related to institutional differences among countries (labor market institutions, demographics ...). For the LSDV estimation as well as for the semi-parametric one, $\beta_j$ is a vector of parameters to be estimated traditionally (by OLS). The treatment of $m(\Delta y_{it})$, on the contrary, differs from one framework to the other. In the fixed effect model, $m$ is considered as another parameter to be traditionally estimated, implying that the growth - unemployment change relation is linear. By contrast, there are no a priori assumptions on the functional form of $m(\Delta y_{it})$ in the semi parametric specification, allowing a potential non linearity to appear. Like in Huang and Lin (2006), this method proposes a consistent estimation of what the actual functional relation looks like. Equation 3 is very similar to the non-dynamic panel threshold model proposed by Fouquau (2008). However our approach is not constrained by the number of regimes and allows smooth transition between regimes. Moreover, equation 3 remains on purpose, very close to the original equations used by Okun (1962) in his seminal paper. Following Robinson (1988), estimating such a semi-parametric model requires following the steps exposed hereafter. Consider:

$$E\left[\Delta u_{it}|\Delta y_{it}\right] = E\left[X_j|\Delta y_{it}\right]\beta_j + m(\Delta y_{it})$$

And therefore

$$\Delta u_{it} - E\left[\Delta u_{it}|\Delta y_{it}\right] = X_j\beta_j - E\left[X_j|\Delta y_{it}\right]\beta_j + \epsilon_{it} \tag{4}$$

We estimate $E\left[\Delta u_{it}|\Delta y_{it}\right]$ and $E\left[X_j|\Delta y_{it}\right]$ by a set of bivariate non parametric regressions, and $\beta_j$ by a linear regression of $\Delta u_{it} - \hat{E}\left[\Delta u_{it}|\Delta y_{it}\right]$ on $X_j - \hat{E}\left[X_j|\Delta y_{it}\right]$. We then have:

$$m(\Delta y_{it}) = E\left[\Delta u_{it}|\Delta y_{it}\right] - E\left[X_j|\Delta y_{it}\right]\beta_j$$

and an estimate:

$$\hat{m}(\Delta y_{it}) = \hat{E}\left[\Delta u_{it}|\Delta y_{it}\right] - \hat{E}\left[X_j|\Delta y_{it}\right]\beta_j$$

$E\left[\Delta u_{it}|\Delta y_{it}\right]$ and $E\left[X_j|\Delta y_{it}\right]$ of equation 4 are estimated using a kernel based approach. We use both the local constant kernel and the local linear kernel estimators but will only discuss the results of the local linear estimator since both estimators give almost the same outcome. The kernel used is Gaussian (Epanechnikov has been tested as well to asses robustness) and the bandwidth is chosen using a back-fitting algorithm combining several selection methods (such as cross validation or "plug-in" method). This class of semi-parametric model has been proved to be as good as a well-specified parametric model (with $m$ known, Robinson (1988)). Besides, $\beta$ parameters interpretation is alike one of a standard linear regression.

# 3 Data and results

## 3.1 Data

The present study uses data for 16 European countries[3] from 1984 to 2009 using quarterly data. We use harmonized level of unemployment rate and logarithm of real GDP[4] (In Millions of US dollars, fixed PPP, OECD reference year and seasonally adjusted) multiplied by 100, both extracted from OECD.stat. As stated in Freeman (2001), using harmonized data is crucial because criteria for assembling data are not the same among countries. We use a panel of countries to highlight potential non-linearity that could not be taken into account country by country due to the insufficient number of observations, as advocated by Fouquau (2008). Moreover, the use of a European countries panel is particularly appropriate in the European economic integration and Euro area context (Freeman (2001)). Our dataset and variables are presented in Table 1. Besides, the four main variable empirical densities are displayed in Figure 1. From these densities and the Jarque-Berra statistics displayed in Table 1, none of these variables appears to be normally distributed. Recall that a linear (parametric) regression implicitly assumes that the regressed variables are normally distributed; a fact pleading again in favor of a non parametric estimation.

[3]Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and United Kingdom.

[4]Panel unit root test (Im et al. (2003) test with data corrected for cross-sectionnal dependence) establishe that the null-hypothesis of unit root in the series in level cannot be rejected at the 5% level.

Table 1: Data and Variables

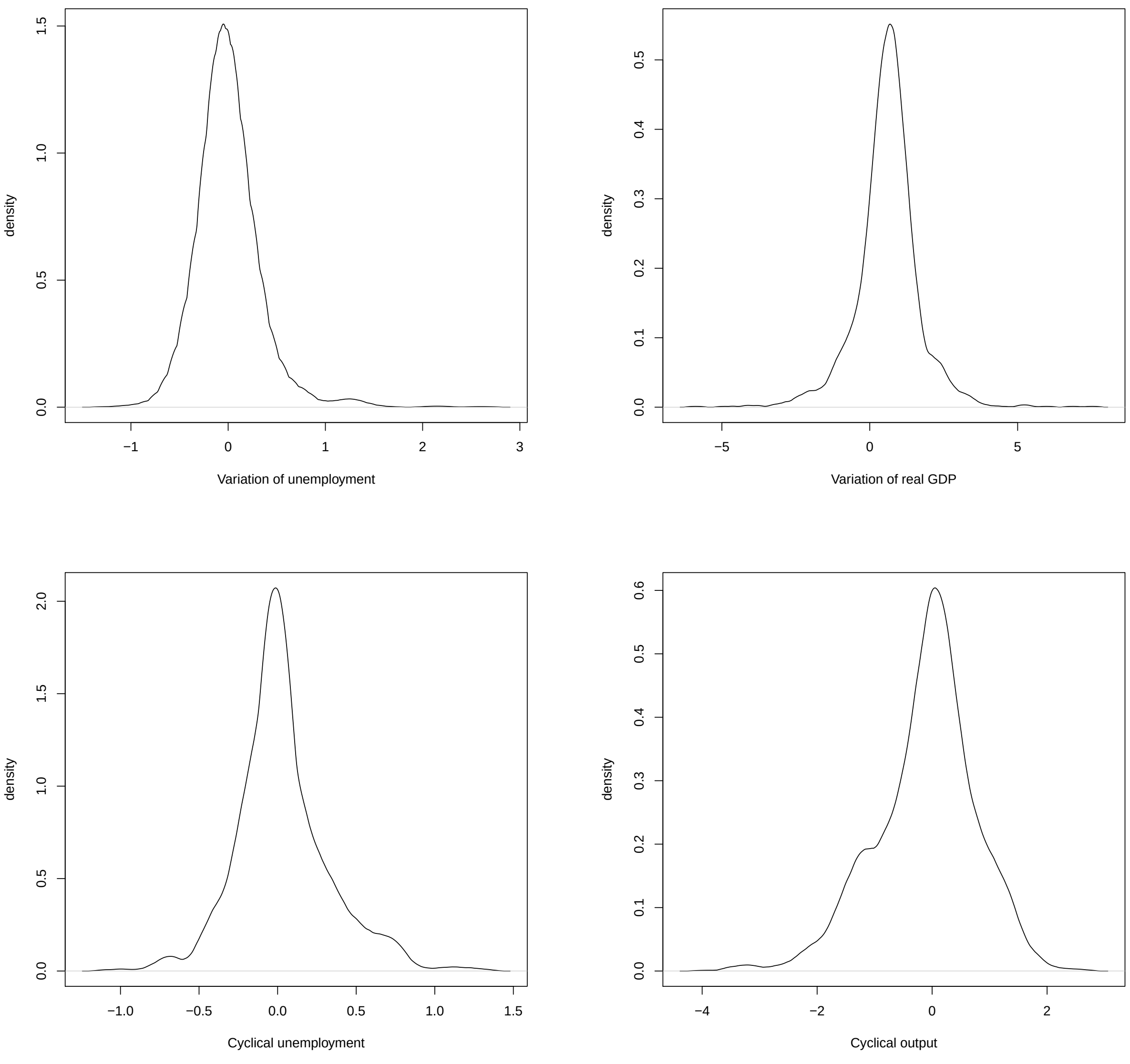
| Name | Definition | Summary Statistics | | | | |
|---|---|---|---|---|---|---|
| | | Min | Mean | Max | St. Error | Jarque-Bera |
| $\Delta u$ | Change in unemployment rate measured in percentage points | -1.2 | 0.003 | 2.6 | 0.32 | 3651.11*** |
| $\Delta y$ | Change in log. of real GDP in percentage (i.e. growth) | -5.883 | 0.601 | 7.518 | 0.60 | 1844.64*** |
| $y^c$ | Cyclical Output, as the gap between the observed output and the long-run trend output in percentage (using Harvey's structural time series) | -3.852 | -0.105 | 2.525 | 0.86 | 133.20*** |
| $u^c$ | Cyclical Unemployment as the gap between the observed unemployment rate and the equilibrium unemployment rate in percentage points (using Harvey's structural time series ) | -1.081 | 0.031 | 1.317 | 0.29 | 304.57*** |

Note: *** for the Jarque-Bera statistics means the hypothesis of normality is rejected at the 1% level.

To complete our sample description, we present in Table 2, the estimates of equations 1 and 2 for each country. The most interesting results are as follows: (i) all coefficients are negative, confirming that Okun's law performs well in European countries, (ii) the relation holds regardless of the methodology used, (iii) coefficients tend to be lower for the Firts-Difference version than for the Gap version, (iv) the elasticity of unemployment to output generally differs only slightly among countries. We can note however, that it is rather low for Norway and Luxembourg but quite high for Spain and United Kingdom in both versions. As in Moosa (1997), we explain these results by institutional differences that determine the labor market relative flexibility. For instance, United Kingdom has the least regulated labor market according to the EPL[5] index.

[5]This index measures the Employment Protection Legislation strictness and is computed by OECD.

Figure 1: Non parametric density estimates of Var. of Unemployment, Var. of Real GDP, Cyclical Unemployment and Cyclical Output




*We obtain these densities using non parametric kernel estimations*

Table 2: Individual regressions

| Okun's law version | **First-Difference** | | **Gap** | |
|---|---|---|---|---|
| Countries | Coefficient | *t*-stat | Coefficient | *t*-stat |
| Austria | -0.100** | -2.39 | -0.093*** | -6.67 |
| Belgium | -0.110** | -2.89 | -0.280*** | -11.49 |
| Denmark | -0.083** | -3.26 | -0.286*** | -10.05 |
| Finland | -0.223** | -6.61 | -0.184*** | -4.11 |
| France | -0.221** | -6.74 | -0.246*** | -6.30 |
| Germany | -0.127** | -5.03 | -0.235*** | -6.99 |
| Ireland | -0.135** | -5.66 | -0.263*** | -8.97 |
| Italy | -0.063* | -1.84 | -0.219*** | -5.94 |
| Luxembourg | -0.014 | -1.44 | -0.093*** | -4.55 |
| Netherlands | -0.097** | -3.45 | -0.266*** | -19.66 |
| Norway | -0.009 | -0.47 | -0.036*** | -3.16 |
| Portugal | -0.147** | -5.46 | -0.255*** | -22.09 |
| Spain | -0.402** | -7.21 | -0.362*** | -15.04 |
| Sweden | -0.168** | -5.02 | -0.150*** | -13.81 |
| Switzerland | -0.142** | -5.80 | -0.246*** | -12.27 |
| United Kingdom | -0.243** | -8.03 | -0.310*** | -5.44 |
| Average | -0.143 | | -0.220 | |

Note: *, ** and *** stands for Significant at the 10%, 5% and 1% level respectively.

## 3.2 Results

The estimates of the parametric specification (LSDV) are reported in Table 3. As seen in the First-Difference version column, results suggest that a 1 percent fall in real GDP is associated with a 0.11 percentage point increase of unemployment rate in the short-run. For Gap version, results point out that a 1 percentage fall in cyclical output is correlated with a 0.21 percentage point rise in cyclical unemployment. Both coefficients are significantly different from zero at the 1% confidence level according to t-statistics. If the relation nature is similar among versions, the correlation magnitude depends on the methodology used. Our Okun's coefficients for Europe are also lower in absolute value than those generally found for United States. Indeed, using First-Difference version, Okun (1962) obtains a coefficient of -0.30 over 1947-1960, Beaton (2010) finds a coefficient of -0.21 over 1990-2009, while Crespo Cuaresma (2003) using Gap version with Harvey's structural time series, points out a coefficient of -0.24 over 1965-1999. These findings are in accordance with Beaton (2010) and Lee (2000) who find a stronger correlation between output and unemployment for United States compared to Europe. Besides, it seems that country individual effects are more relevant for Gap version than for First-Difference version of Okun's law. To summarize, these first parametric estimation results are in line with previous references and can be considered as a relevant benchmark to discuss our semi parametric estimation.

Table 3: Estimation results

| | **First-Difference version** | | | | **Gap version** | | | |
|---|---|---|---|---|---|---|---|---|
| | LSDV | | Semi-parametric | | LSDV | | Semi-parametric | |
| Variable | Coef. | *t*-stat | Coef. | *t*-stat | Coef. | *t*-stat | Coef. | *t*-stat |
| Intercept | 0.03 | 1.22 | - | - | 0.05** | 2.39 | - | - |
| Change in Real GDP | **-0.11*** ** | **-16.75** | - | - | - | - | - | - |
| Cyclical output | - | - | - | - | **-0.21*** ** | **-32.64** | - | - |
| Austria | 0.03 | 0.92 | 0.01 | 0.56 | -0.03 | -1.03 | 0.04** | 1.98 |
| Belgium | 0.00 | -0.11 | -0.02 | -0.92 | 0.00 | 0.02 | 0.08*** | 3.89 |
| Denmark | 0.00 | 0.08 | -0.04 | -1.56 | -0.05* | -1.83 | 0.02 | 0.97 |
| Finland | 0.05 | 1.28 | 0.02 | 0.77 | -0.08*** | -2.75 | 0.00 | -0.37 |
| France | 0.02 | 0.55 | 0.00 | 0.09 | -0.08*** | -2.58 | 0.00 | 0.15 |
| Germany | 0.00 | 0.18 | -0.02 | -0.79 | -0.08*** | -2.77 | -0.02 | -1.21 |
| Ireland | 0.07* | 1.75 | 0.00 | 0.32 | -0.06* | -1.89 | 0.02 | 1.00 |
| Italy | 0.00 | 0.19 | -0.01 | -0.58 | -0.05* | -1.86 | 0.02 | 1.05 |
| Luxembourg | 0.11*** | 2.69 | 0.05* | 1.79 | -0.01 | -0.42 | 0.06*** | 2.73 |
| Netherlands | -0.01 | -0.28 | -0.03 | -1.19 | 0.02 | 0.65 | 0.10*** | 4.46 |
| Norway | 0.03 | 0.84 | -0.01 | -0.45 | -0.09*** | -2.81 | -0.02 | -1.10 |
| Portugal | 0.05 | 1.18 | 0.01 | 0.47 | -0.02 | -0.63 | 0.04** | 1.96 |
| Spain | 0.06 | 1.59 | 0.04 | 1.43 | -0.05* | -1.73 | 0.02 | 0.90 |
| Sweden | 0.07* | 1.84 | 0.05* | 1.71 | -0.04 | -1.46 | 0.03 | 1.29 |
| Switzerland | 0.04 | 1.03 | 0.01 | 0.54 | -0.05* | -1.76 | 0.01 | 0.68 |
| Number of obs. | 1616 | | 1616 | | 1616 | | 1616 | |
| Li and Wang stat. | | 1.48* | | | | 2.16** | | |

Note: *, **, *** means Significant at the 10%, the 5% and the 1% level, respectively. United Kingdom is used as the reference group.

Figures 2 and 3 present the Okun's law semi-parametric representation (solid black curve) through the estimated function $m$ of equation 3 for each version and with the confidence intervals at the 95% level[6]. In a comparison purpose, we add for each version the parametric counterpart (straight dashed line) of the Okun's law estimation. We do find a negative relation between output and unemployment; providing a qualitative support for Okun's law in our panel. Overall, Figures 2 and 3 exhibit in both cases a non-linear relation and shows that there is not a unique tradeoff between output and unemployment. It confirms the

[6] In order to asses robustness, we also regressed unemployment on lagged output (because of potential time-lag variations, Beaton (2010)) or focusing on euro area countries only; our results remains unchanged.

inappropriateness of a linear specification. Indeed, an important part of the parametric line lies outside the semi-parametric confidence intervals for both specifications. In addition, we use the Li and Wang (1998) specification test to compare the parametric and the semi parametric estimated model. We can see, at the bottom of Table 3, that this test rejects for both First-Difference and Gap version the parametric model against the semi parametric one[7] of the present article. We can therefore conclude that the semi parametric model provides a better approximation of the data than the traditional parametric one.

For the First-Difference version ((Figure 2), our estimation shows the existence of at least three regimes. Indeed, it can be seen that the curve slope for intermediate growth level (about between -1% and 1%) is steeper than the curve slope corresponding to high level of negative or positive growth (approximately below -2% or above 2 %). Most of all, the curve exhibit a sharper slope when the growth is negative (even in case of severe recession) than when the growth is positive. The *Firms's Risk Aversion Hypothesis* seems to be verified in Europefor the Okun's law First-Difference version, as employers respond very quickly by laying off workers when the economy goes into recession.

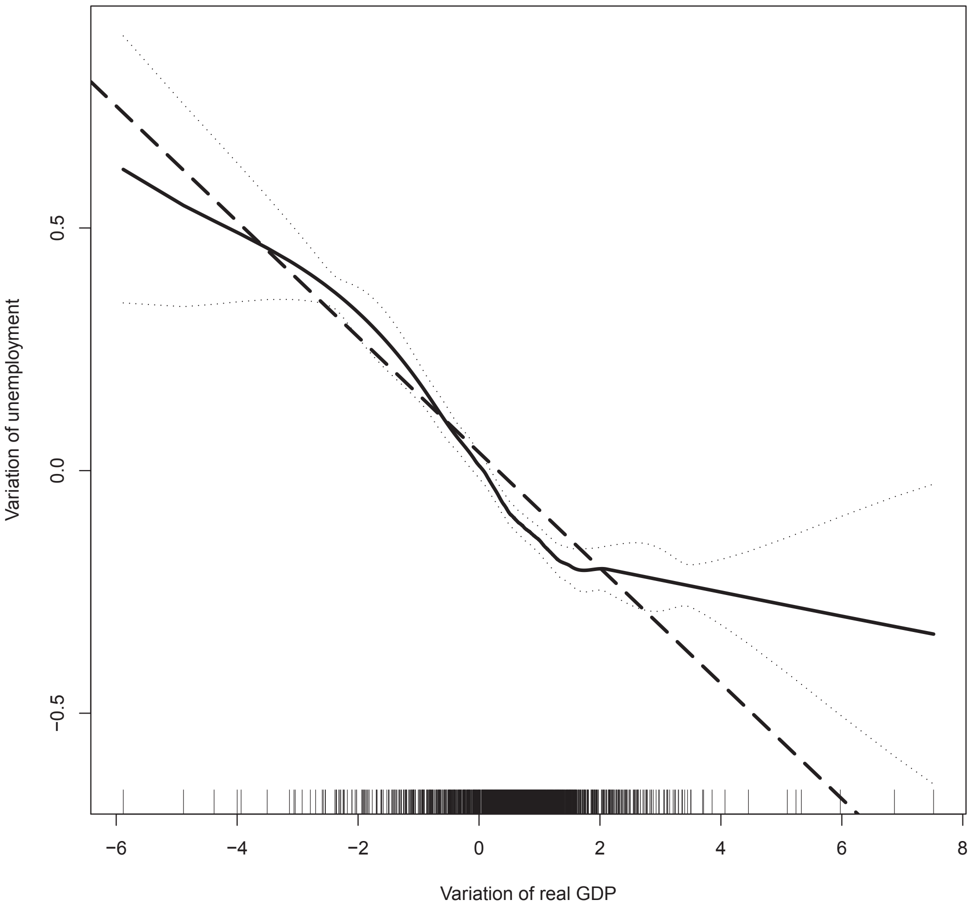
Gap version (Figure 3) also confirms the nonlinear relationship between unemployment and output and reveals the existence of broadly two regimes. The curve slope corresponding to low values of cyclical output (approximately below -1%) is indeed smaller than the curve slopes corresponding to intermediate (approximately between -1% and 0%) or even cyclical output positive values. In other words, when cyclical output stands below its long run value and reaches its trough, the impact of output on unemployment tends to be weakened. The *Labor Hoarding Hypothesis* may somehow be relevant for Gap version, since as the recession last, employers are more and more reluctant to layoff their most costly trained employees.

Figure 2 and 3 highlights overall that the trade off between unemployment and output depends crucially on the cyclical output situation and on the real GDP growth level. Moreover, by combining the informations given by the two graphs, we are able to draw the Okun's relation dynamics through recession and recovery. First-Difference and Gap versions give, indeed, complementary insights. We have the feeling the development along the business cycle is as follows.

When economy goes into downturn (i.e. around zero cyclical output with negative growth), unemployment is very sensitive to output. By contrast, when economy is in the middle of

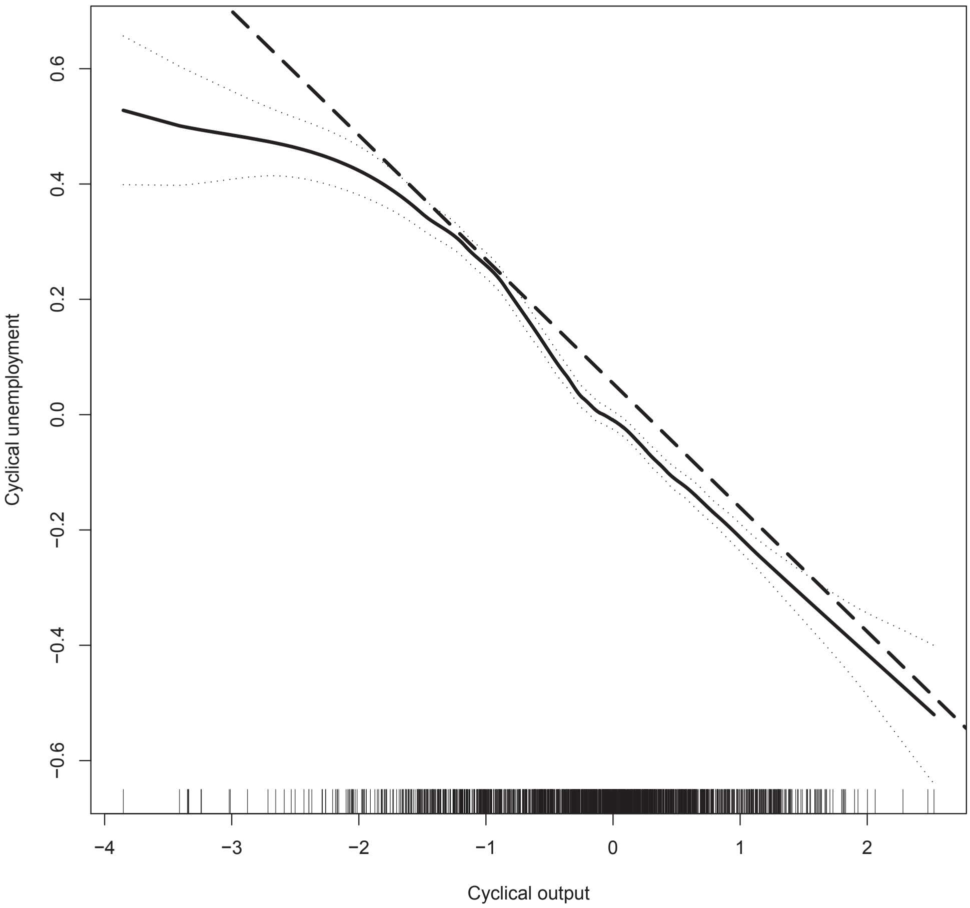
[7]The test statistic computation is detailed in the appendix B

Figure 2: Semi-parametric estimates of Okun's law First-Difference version




*The solid curve is the Okun's law semi-parametric representation. The straight dashed lines correspond to the equivalent parametric specification. The dashed curves are the confidence intervals at the 95% level*

Figure 3: Semi-parametric estimates of Okun's law Gap version




*The solid curve is the Okun's law semi-parametric representation. The straight dashed lines correspond to the equivalent parametric specification. The dashed curves are the confidence intervals at the 95% level* a recession and when output reaches its trough (i.e. with very low cyclical output and negative growth), the impact of output on unemployment becomes smaller. In the same way, during the recovery (i.e. with always cyclical output negative values but positive growth), the impact of output on unemployment remains weak. This finding implies that we should expect jobless growth periods during recoveries. Finally, when economy goes into expansion (i.e. with positive cyclical output and positive growth), Okun's relation becomes stronger again. These results can at least partly explain why previous researches found contrasted results in European countries about the asymmetry nature of the Okun's law. Our work support the proposition of Lee (2000) that inferences based on a single model may lead to inaccurate asymmetry interpretations; we have therefore to consider both versions of Okun's law.

Our findings reconcile in a way the two potential asymmetry patterns we exposed in introduction. We show indeed that combining two different information sources allows to understand how *Firm's Risk Aversion* and *Labour Hoarding Hypotheses* can successively be valid along the business cycle.

# 4 Conclusion

Using a semi-parametric approach, we described the Okun's law for European countries. Our estimation show an asymmetric Okun's relation, suggesting that the linear specification traditionally employed may leads to inaccurate forecasts. The parametric estimation method fails indeed to catch the several regimes nature of the output-unemployment relation. The evidence of Okun's law varying over the business cycle recalls that the relationship is more a rule of thumb than an economy structural feature.

Combining the two versions of Okun's law, we reconcile the two asymmetry patterns proposed by Silvapulle et al. (2004). We find indeed, that unemployment responds more strongly to output when the economy is contracting than expanding, as firms quickly react by firing workers which haveweak training or with few restrictions on layoffs. However, the impact of output on unemployment tends to be weakened, when output reaches its trough, as firms prefer to keep their key personnel costly trained.

This asymmetry has to be taken into account when designing structural and stabilization policies. During downturn, the high Okun's coefficient advocates management policies of short-run demand to reduce unemployment rate. Conversely, these policies will be inadequate in the middle of recessions and during recoveries because the coefficient is very low, rather justifying active labour market programs.

This paper is a contribution to a better understanding of the European labour market at an aggregate level. Nevertheless, a lot of work still has to be done on how the non linearity of Okun's law can be explained. This work also provides a good illustration of how non-parametric methods can highlight features of the data that parametric techniques fail to uncover.

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# A The Structural Times Series Approach

In this article, the structural times series model of Harvey (1989) is applied to quarterly data to decompose the observed output and unemployment series into unobserved trend and cyclical component. Specifically, the model may be written, following the presentation of Silvapulle et al. (2004), as:

$$z_t = \mu_t + \varphi_t + \epsilon_t \ ,$$

where $z_t$ is the observed value of the series; $\mu_t$, $\varphi_t$ are the unobserved trend and cyclical component, respectively and $\epsilon_t$ is an irregular component. The cyclical and trend components are assumed to be uncorrelated while $\epsilon_t$ is assumed to be a white noise. The trend component represents the long-run movement of a series and thus may represent the potential value of the underlying variable. This component is assumed to be stochastic and linear and can be represented by

$$\mu_t = \mu_{t-1} + \beta_t + \eta_t$$

$$\beta_t = \beta_{t-1} + \zeta_t$$

where $\eta_t \hookrightarrow NID\ (0, \sigma_\eta^2)$, and $\zeta_t \hookrightarrow NID\ (0, \sigma_\zeta^2)$. $\mu_t$ is a random walk with a drift factor $\beta_t$, which follows a first-order autoregressive process. This process collapses to a simple random walk with drift if $\sigma_\zeta^2 = 0$ and to a deterministic linear trend if $\sigma_\eta^2 = 0$ as well. If, on the other hand, $\sigma_\eta^2 = 0$ while $\sigma_\zeta^2 \neq 0$, the process will have a trend that changes relatively smoothly.

The cyclical component is assumed to be a stationary linear process and may be written:

$$\varphi_t = a \cos \theta_t + b \sin \theta_t \ ,$$

where $t$ is time and the amplitude of the cycle is given by $(a^2 + b^2)^{1/2}$. In order to make the cycle stochastic, the parameters $a$ and $b$ are allowed to evolve over time, while preserving continuity is achieved by writing down a recursion for constructing $\varphi$ before introducing the stochastic components. By introducing disturbances and a damping factor, we have:

$$\varphi_t = \rho \left(\kappa_{t-1} \cos\theta + \varphi^*_{t-1} \sin\theta\right) + \omega_t$$

$$\varphi^*_t = \rho \left(-\varphi_{t-1} \sin\theta + \varphi^*_{t-1} \cos\theta\right) + \omega^*_t$$

where $\varphi^*_t$ appears by construction such that $\omega_t$ and $\omega^*_t$ are uncorrelated white noise disturbances with variances $\sigma^2_\omega$ and $\sigma^2_{\omega^*}$ respectively. The parameters $0 \leq \rho \leq 1$ and $0 \leq \theta \leq \pi$ are the frequencies of the cycle and the damping factors on the amplitude, respectively. In order to make numerical optimization easier, the constraint $\sigma^2_\omega = \sigma^2_{\omega^*}$ is imposed.

The above equations can be cast into state-space form to calculate the likelihood function using the Kalman filter and so use the maximum likelihood approach. As a result, the extracted cyclical components of output and unemployment are used to estimate the Gap version of Okun's law.

# B Li and Wang (1998) specification test

In order to discriminate which of the parametric or non parametric specification we estimate fits the data best, we propose to use the Li and Wang (1998) specification test as a criteria. This test is especially designed to help choosing among models when comparing parametric and non parametric approaches. In this appendix, we follow the presentation of Azomahou et al. (2010).

The test statistics proposed in Li and Wang (1998) is:

$$I_n = \frac{1}{n^2 h} \sum_{i=1}^{n} \sum_{j=1_{j \neq i}}^{n} U_{in} U_{jn} K_{ij}$$

where $U_{in} = g_i - \hat{\delta} x_{0,i} - Z_i' \hat{\gamma}$ are the residuals, $\hat{\gamma}$ is the parameters vector estimated with the alternative model ($H_1$) and $\hat{\delta}$ is the OLS estimator of $\delta$ based on the parametric model ($H_0$). Under $H_0$, $J_n = nh^{1/2} I_n / \sqrt{\hat{\Omega}} \hookrightarrow N(0,1)$ in distribution when $n \to \infty$,
where $\hat{\Omega} = \left(\frac{1}{n^2 h}\right)^{-1} \sum_{i=1}^{n} \sum_{j=1_{j \neq i}}^{n} U_{in}^2 U_{jn}^2 K_{ij}^2$ is a consistent estimator of

$$\Omega = 2 \left[ \int K^2(v) dv \right] E \left\{ f_{x_0}(x_0) \left[ E(\sigma^2(x_0, Z) | x_0) \right]^2 \right\}$$

with $f_{x_0}(x_0)$ being the non parametric density of $x_0$ and $\sigma^2(x_0, Z) = E(U^2 | x_0, Z)$.

Under $H_1$, Prob$[J_n \geq S_n] \to 1$, when $n \to \infty$, where $S_n$ is a nonstochastic bounded sequence. Note that the test is unilateral. We reject $H_0$ if $J_n > c_\kappa$ for the significance level $\kappa$, where $c_\kappa$ stands for the $\kappa^{th}$ upper percentile of the standard normal distribution. Interestingly, $\hat{\gamma}$ et $\hat{\delta}$ does not affect the asymptotic distribution of $J_n$ under the null hypothesis.

Li and Wang (1998) have studied the test properties in limited distance and showed that a normal approximation is less relevant for small samples. They suggest in this case, to use of a bootstrapped version of their test. In our study, the sample size (1616 observations), is big enough to allow the use of the asymptotic test version