A general theory of controllability and expectations anchoring for small-open economies[*]

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Abstract. Rational expectations are often used as an argument against policy activism, as they may undermine or neutralize the policymaker's actions. Although this sometimes happens, rational expectations do not always imply policy invariance or ineffectiveness. In fact, in certain circumstances rational expectations can enhance our power to control an economy over time. In those cases, policy announcements can be used to extend the impact of conventional policy instruments. We present a general forward-looking policy framework and use it to provide a formal rationale for testing when policymakers can and cannot expect to be able to manage expectations. To describe the relevance of our results applications are shown for policy design in small open economies. Those are the cases where domestic policies are at their weakest and our ability to influence expectations most constrained.

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1. Introduction.

Since the work of Barro (1974), Sargent and Wallace (1975) and Lucas (1976), rational expectations have been regarded as placing severe limits on what can be achieved in a world of policy conflicts; and as requiring strong policy commitments to get even that

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far. Rational expectations are often said to imply that such commitments cannot be considered credible and to lead inevitably to Pareto inferior outcomes.

This argument however does not allow for policymakers who actively engage in managing expectations by making policy announcements, alongside policy interventions, for the express purpose of shifting the expectations path itself.[1] If they can do that, private expectations will be exactly consistent with what the private sector/policymakers expect the outcomes to be; and no one will be required to move off their expected path (make expectation errors) for the policies to work. The literature has often used this idea formally and informally in debates over the feasibility and desirability of trying to anchor inflation expectations for monetary policy, or in arguments over the desirability of publishing interest rate forecasts.[2] It is also an idea in the minds of the policy makers; see, for example, the European Central Bank's concern that long term policies introduced to combat the current financial crisis (greater transparency, new regulation, reduced pro-cyclicality, planned liquidity withdrawals) should have their effects now (Trichet, 2008); equally the announcement of new fiscal stimulus or credit guarantee packages. But what the literature has not done is identify the conditions under which we can expect to be able to manage expectations in this way, and their effect on the scope for policy, as opposed to pointing to the possibility and importance of managing expectations.

Several recent papers highlight the relevance of these questions. Mertens and Ravn (2010) show, in the context of a specific model, that the impact of any fiscal expansion is part the result of anticipation effects and part genuine impact in the sense normally meant by policy multipliers. But how much, in any particular case, is anticipations and how much is genuine causal impact? Our analysis allows us to answer that question for the general case using the partitioned matrix in (7) below; that is, for any model and without additional estimation uncertainties. In a similar vein, Eusepi and Preston (2010) show that different communication strategies matter in this context—mainly because different strategies have different short and long term effects. Our dynamic analysis allows the

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[1] By "actively manage" we make a distinction between cases where we study the outcomes and stability of rationally expected policies *given* the behaviour of the system (the conventional case: Blanchard and Khan 1980); and the case where the policy authorities try to influence the behaviour of the system itself. In this paper, we are concerned with the latter case. The distinction itself was made and analysed in Hughes Hallett *et al*. (2010).

[2] See Woodford (2005), Blinder *et al*. (2008) or Rudebusch and Williams (2008); and more formal models will be found in Soderlind (1999), Woodford (2003).

policymaker to pick out which strategy, if communicated properly, would have an impact and at which horizon(s)—again for a general model without the parameter restrictions of the original paper; and, conversely, by showing what parameter restrictions must not hold if any communication is to be used successfully. Somewhat more indirectly, Canova and Gambetti (2010) show that expectations can and do get anchored in the sense that their influence does not vary over time even if inflation is changing. But the question remains, how can that happen and what expectations are implied?

This paper investigates the circumstances under which policy announcements, if properly communicated, can be used to supplement or extend the impact of conventional policy instruments. The idea is that rational expectations may, in certain cases, enhance the power to control an economy over time. Hence, contrary to conventional wisdom, rational expectations may, but do not always, neutralize the policymaker's action.

Specifically, we consider the design of economic policy within a general rational expectations framework and show that policy invariance can only arise in specific cases (where the unit root or rank conditions specified below fail). In all other cases policy announcements may be used to help steer economic behaviour, and certain targets will become reachable in reduced time. The rationale for this result can be understood by using the concept of controllability, introduced in the classical theory of economic policy by Tinbergen (1952), and its dynamic extensions. If a policymaker is able to achieve any desired vector of targets given some exogenous expectations, then he will also be able to do it with endogenous expectations.[3] If nothing else, he could exploit the endogenous expectations to achieve his targets in a shorter time.

To make use of this property of rational expectations, however, another ingredient must be present. The policymakers must be able to communicate, in a clear and effective manner, the intent and purpose of their policies and how exactly these policies will work. This will be necessary to convince the private sector that the policy measures will in fact be undertaken when it comes to the point; and that it is reasonable to expect that the planned outcomes will be achieved. Otherwise there is no reason to suppose the private

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[3] Like most other papers with policy announcements, including those just cited, this paper supposes a single policymaker. The extension to multiple policymakers is set out in Acocella *et al*. (2009).

3

sector would shift, or anchor, their expectations of future outcomes as a result of these announcements of future policy actions.

Our approach differs from the recent trend in the literature on communication. In our framework, the crucial element is to reaffirm the targets and why the chosen policies can be expected to reach them; as emphasised by Eggertson and Pugsley (2006), Moessner and Nelson (2008), Ferrero and Secchi (2009), Libich (2009), and by Woodford's (2003) observation that policy trade-offs will be eased when expectations fall into line with the chosen policies. By contrast, much of the recent literature on communication has focused on the quality of forecasts, on the degree of divergence or agreement among policy makers, and on transcripts or voting records from policy committees (Ehrmann and Fratzscher, 2005, 2007; Jansen and de Haan, 2006; Visser and Swank, 2007).

The rest of the paper is organized as follows. Section 2 poses the communications problem by means of an example drawn from a simple small-open economy (SOE) model with a New Keynesian structure. Section 3 puts this example into a general framework, deriving the reduced and final form of a model with a single policy maker and rational forward-looking expectations. In this section we deal with the conditions for static and dynamic controllability and demonstrate that, contrary to conventional wisdom, dynamic controllability can be enhanced by rational expectations. Our purpose is to identify the circumstances in which controllability is possible; and the conditions when it is not. Section 4 then describes how announcements of future policies can help to ensure static or dynamic controllability of a simple SOE model with forward-looking markets, as described in section 2, under various assumptions. Section 5 contains a specific illustration of our main point in the context of monetary policy in more complex SOE models. Section 6 concludes.

2. Controllability in a simple SOE macro-model.

Consider an example of a simple SOE model with a New Keynesian structure[4] based on a forward-looking Phillips curve (1), an Euler equation (2), the aggregate demand (3) and

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[4] The model is a simple SOE based on Clarida *et al.* (2001, 2002), Benigno and Benigno (2003), Galì and Monacelli (2005), where we also include indexation to past inflation to obtain both forward and backward inflation components in the price adjustment equation (Mishkin 2002). Specifically, the model (1)-(3) is based on a perfect pass through assumption (Clarida *et al.*, 2001). Later on, we introduce a more complex model which removes this assumption.

augmented by trade policy; finally, equation (4) relates the household's demand for home versus foreign goods to the terms of trade:

(1) $\qquad \pi_t = (1 - \lambda)\beta E_t \pi_{t+1} + \lambda \pi_{t-1} + \tilde{\kappa} mc_t$

(2) $\qquad c_t = E_t c_{t+1} - \tilde{\sigma}\left(i_t - E_t \pi_{t+1} - \gamma E_t \Delta s_{t+1}\right) + \tilde{\chi} m_t + \mu_t^p$

(3) $\qquad y_t = c_t + \gamma s_t \quad$ with $\ s_t = y_t - y_t^w$

where π is the inflation rate, c is consumption, y is output, i is the nominal interest rate, s are the terms of trade; y^w is the rest of the world demand; m is an index of trade policy; $mc = vx + \mu^m + \tilde{\phi}m$ is the marginal cost of production and $x = y - \bar{y}$ is an output gap, relative to the natural rate $\bar{y}$, that is inversely related to the rest of the world demand. Finally, $\bar{y} = -v_0 y^w$; and y^w, μ^p and μ^m are stochastic terms—representing rest of the world demand, preference and mark-up shocks. In this model monetary policy is rather standard. But our focus is on the effects of trade policies on the supply side ($\tilde{\phi}$) via changes in marginal costs, e.g., subsidies; and on the demand side ($\tilde{\chi}$), via variations in domestic public expenditures or tariffs, or both.

If we want to analyze the role of policy announcements, and the importance of communication as a way to enhance the effectiveness of policy instruments, we must first discuss the conditions under which the policymaker can control the system given by equations (1)-(3). To create a general theory to do that, we need to generalise.

3. A general model for analyzing controllability.

All dynamic models as (1)-(3) can be expressed in the following general form:

(4) $\qquad Y_t = AY_{t-1} + BY_{t+1/t} + CX_t + u_t \qquad\qquad$ for $\ t = 1.....T$

where T is a finite, but possibly large number; and where $Y_{t+1/t} = E(Y_{t+1}|\Omega_t)$ denotes the mathematical expectation of Y_{t+1} conditional on Ω_t (the information set available at t). In our set up, Y_t is a vector of n endogenous variables at time t; X_t is a vector of m potential policy instruments; and u_t is a vector of exogenous shocks or other influences which have a known mean, but otherwise come from unspecified probability distributions. The

matrices *A*, *B* and *C* are constant and of order *n*, *n*, and $n \times m$ respectively, and have at least some elements which are non-zero.

Model (1)-(3) can now be written in the general form (4). After a certain amount of algebra, the model can be reduced to the following compact, but general form in terms of the output gap and the inflation rate:

$$(5) \quad \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} (1-\lambda)\beta + \kappa\sigma & \kappa \\ \sigma & 1 \end{bmatrix} E_t \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} + \begin{bmatrix} \lambda & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \varphi + \kappa\chi & -\kappa\sigma \\ \chi & -\sigma \end{bmatrix} \begin{pmatrix} m_t \\ i_t \end{pmatrix} + u_t$$

where $\kappa = v\tilde{\kappa}$; $\phi = \tilde{\kappa}\tilde{\phi}$; $\sigma = \tilde{\sigma}\delta^{-1}$; $\chi = \tilde{\chi}\delta^{-1}$; $\delta = 1 + (\tilde{\sigma} - 1)\gamma$; $u_t = [v_t, \kappa v_t + \varepsilon_t]'$; $v_t = \tilde{\kappa}\mu_t^m$; $\varepsilon_t = [\mu_t^d - (\tilde{\sigma} - 1)\gamma E_t \Delta y_{t+1}^w]\delta^{-1}$. The definition of matrices *A*, *B* and *C* in (4) is straightforward.

Model (4) can also be solved from the perspective of any particular period, say $t = 1$, by putting it into final form conditional on the information available in that period:[5]

$$(6) \quad \begin{pmatrix} Y_{1/1} \\ : \\ : \\ : \\ Y_{T/1} \end{pmatrix} = \begin{bmatrix} I & -B & 0 & . & 0 \\ -A & I & & & . \\ 0 & & . & & 0 \\ . & . & & . & -B \\ 0 & . & 0 & -A & I \end{bmatrix}^{-1} \left\{ \begin{bmatrix} C & 0 & . & . & 0 \\ 0 & . & & & . \\ . & & . & & . \\ . & & & . & 0 \\ 0 & . & . & 0 & C \end{bmatrix} \begin{pmatrix} X_{1/1} \\ . \\ . \\ . \\ X_{T/1} \end{pmatrix} + \begin{pmatrix} u_{1/1} \\ . \\ . \\ . \\ u_{T/1} \end{pmatrix} + \begin{pmatrix} AY_0 \\ 0 \\ . \\ . \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ . \\ . \\ 0 \\ BY_{T+1/1} \end{pmatrix} \right\}$$

In the above representation, y_0 is a known initial condition for $t = 1$; and $Y_{T+1/1}$ is some assumed or projected terminal condition – most likely the one that describes the economy's expected long run equilibrium state as part of Ω_t. Although (4) has been solved from the point of view of Ω_1, it must be understood that it could have been derived for each Ω_t, $t = 1....T$, in turn (where $Y_{j/t} = E_t(Y_j)$ if $j \geq t$, but $Y_{j/t} = Y_j$ if $j < t$; and similarly for *X* and *u*). However, for simplicity, we will consider the Ω_1 case only in what follows. The generalisation to any other value of *t* is then obvious.

Second, the equation to which (6) is the solution makes it clear that *neither* the policymakers, *nor* the private sector are required to move off their expected paths (make expectational errors) for the policies to work. In fact equations (7) and (8) below show precisely the opposite; that those expectations are exactly consistent with what the private

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[5] Hughes Hallett and Fisher (1988). Equation (6) ties the expectations of $y_{t/1}$ down as a function of announced policies, while those for each $x_{t/1}$ are tied down by either (8) or proposition 1 below.

sector/policymakers expect the outcomes to be. The only question is whether policies, or policy announcements, can be found that will shift the expectations path by the required amount. The task of this paper is to find the conditions under which that can be done; that is, when it is possible to shift expectations in such a way that the economy's anticipated outcomes reach certain target values at certain points of time, and when it is not possible. It is easy to see that (4) always exists if the matrix inverse, T_T^{-1} in (6), exists. This is demonstrated in Hughes Hallett *et al.* (2008); the condition for T_T^{-1} to exist being that the matrix product AB shall not contain a unit root.[6] Given that, we rewrite (6) as:

(7)
$$\begin{pmatrix} Y_{1/1} \\ . \\ . \\ . \\ Y_{T/1} \end{pmatrix} = \begin{bmatrix} R_{11} & . & . & . & R_{1T} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ R_{T1} & . & . & . & R_{TT} \end{bmatrix} \begin{pmatrix} X_{1/1} \\ . \\ . \\ . \\ X_{T/1} \end{pmatrix} + \begin{pmatrix} b_{1/1} \\ . \\ . \\ . \\ b_{T/1} \end{pmatrix} \qquad \text{or} \quad Y = RX + b$$

where $R = T_T^{-1}(I \otimes C)$, $b = T_T^{-1}\{E(u/\Omega_1) + (A':0)'Y_0 + (0:B')'Y_{T+1/1}\}$, and "$\otimes$" denotes a Kronecker product. In this representation, each $R_{t,j} = \partial y_{t/1} / \partial x_{j/1}$ is an $n \times m$ matrix of policy multipliers for t, $j = 1...T$. But notice that R is *not* block triangular. That is, $R_{t,j} \neq 0$ even if $t < j$. Hence equation (7) implies $R_{t,j}$ is a matrix of conventional policy multipliers between $y_{t/1}$ and $x_{j/1}$, with a delay of $t{-}j$ periods between implementation and realization, if $t \geq j$. But $R_{t,j}$ represents a matrix of anticipatory effects, on $y_{t/1}$, of an announced or anticipated policy change $x_{j/1}$ at some point in the future when $t{<}j$.[7]

We now proceed in two steps:

a) We consider **static controllability** first. Static or Tinbergen controllability is normally defined (see Holly and Hughes Hallett, 1989) as the set of conditions which must hold if an arbitrary set of target values is to be reached for the endogenous variables Y_t in each period—at least in expectation, given that the original model is stochastic. Define those target values to be $Y_{t/1}^d$, where superscript d denotes a desired value from the perspective of

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[6] A weaker condition, if $T{\to}\infty$, would be the usual saddle point property (Hughes Hallett and Fisher, 1988). Notice that this result automatically implies that the traditional Phillips curve model would not be controllable in the long run ($T{\to}\infty$), but might be in the short run, since T_T would be lower triangular with A having a unit root as $T{\to}\infty$. It was our purpose to collect conditions when the system is not controllable, as well as when it is. The unit root condition on A (or on AB in our more general formulation) is one; the other, a failure of the rank condition in proposition 1.

[7] A conventional "backward-looking" model will have $R_{t,j}{=}0$ for all $t{<}j$; *and* constant multipliers $R_{t,j}{=}R_{t,-j}$ for $t-j = 0...T{-}1$, if the model at (3) is linear. Neither of these things is true in (7).

period 1. We then define Y^d to be a stacked vector of those desired values across all time periods. Static controllability, in each period in turn, therefore requires the matrix R in (7) to possess an inverse:

(8) $X = R^{-1}(Y^d - b)$

where Y, X and b are all understood to be expectations conditioned on the current information set Ω_t, for each $t = 1 \ldots T$ in turn, as noted in (7). But since $R = T_T^{-1} C_T$, where $C_T = I_T \otimes C$, we can see $R_T^{-1} = (T_T^{-1} C_T)^{-1} = C_T^{-1} T_T$ exists if and only if $C_T^{-1} = I_T \otimes C^{-1}$ exists, since we already know that T_T^{-1} exists. But the instrument coefficient matrix, C, can only possess an inverse if $n=m$ and C has full rank. Thus $n=m$ is a necessary condition for static controllability; and linear independence in the impacts of the instruments on the targets is sufficient. There is therefore no change to the traditional static controllability conditions when there are rational forward looking expectations.

b) Next we consider **dynamic controllability.**[8] An economy is said to controllable dynamically if a sequence of instrument values $X_1, \ldots, X_t$ can be found to reach any arbitrary value, Y_t^d, for the target variables in period t, at least in expectation, given an arbitrary starting point Y_0 (Holly and Hughes Hallett, 1989). In that case, we are no longer concerned with the period-by-period controllability of target variables between periods 1 and $t-1$. Viewed from period 1, dynamic controllability therefore requires a sequence of intended instrument values $X_{1/1}, \ldots, X_{T/1}$ that can guarantee $Y_{t/1}^d$ is reached in period t. Given (7), this will be possible only if the sequence of policy multipliers and anticipatory effects, $R_{t,1} \ldots R_{t,T}$, is of full rank: $rank(R_{t,1} \ldots R_{t,T}) = n$, given an arbitrary initial state y_0 and a specified terminal condition $y_{T+1/1}$. This follows because $Y_{t/1}^d = (R_{t,1} \ldots R_{t/T}) X + b_{t/1}$ is reachable over (1, t), using a Moore-Penrose generalized left inverse, if $rank(R_{t,1} \ldots R_{t,T}) = n$. But if $T \geq n$, $rank(R_{t,1} \ldots R_{t,T}) = rank(R_{t,1} \ldots R_{t,n}) = n$ which provides the result: an economy represented by equation (3) is dynamically controllable over (1, t), with $T \geq n$, if $rank(R_{t,1} \ldots R_{t,n}) = n$.

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[8] Dynamic controllability conditions are well known for conventional models; but not for rational expectations models.

These controllability conditions contain an important generalization over the traditional case with a backward looking model. If $n > t$, which is entirely possible for small values of t, then dynamic controllability will be available through the reactions of $y_{t/1}$ to the implemented policy choices $X_{1/1}...X_{t/1}$; *and through the anticipatory effects of announced or anticipated policy interventions* that still lie in the future, $X_{t+1/1}....X_{n/1}$. That implies the policymaker can use policy announcements, in addition to policy interventions, to guide the course of the economy. In a conventional model that would not be possible since $R_{t,j} = 0$ for all $j > t$. In effect, the policymaker has a greater number of policy "instruments" at his disposal than in an economy without anticipations.

As a consequence, $Y_{1/1}$ itself is controllable from the first period, even if there are too few instruments ($m < n$), provided that $T \geq n$ and $r(R_{t,1}....R_{t,n}) = n$. The astute policy-maker will therefore realise that good communication lies at the heart of the policy problem if he/she wants to reach their policy targets in the early periods or at lower cost—a fact that has not been lost on central bank policymakers in their attempts to control or anchor private sector expectations of future inflation in such a way as to make interest rate policies more effective (Woodford 2005; Blinder *et al*. 2008; Rudebusch and Williams, 2008).

Second, dynamic controllability is possible with a much reduced instrument set compared to static controllability. There are two parts to this reduction: a) the ability to use one or more instruments sequentially rather than a group of several instruments used once and in parallel; and b) the ability to augment or replace parts of an existing instrument set with announcements of future policy changes.

Third, it is important to note that, while $X_{1/1}$ will represent implemented decisions when it comes to controlling $Y_{t/1}$, the $X_{2/1}....X_{n/1}$ values, being policy announcements, may never be carried out. However, because they are decisions that lie in the future, the possibility of time inconsistency plays no role in the controllability of $Y_{t/1}$ so long as they are *genuinely* held expectations at this point. And they will be genuine because they are the values that deliver the first best outcomes. Policymakers have no incentive, strategic advantage or interest in choosing to make themselves worse off than they need to be:

Proposition 1. Forward looking rational expectations enhance the power to control an economy over time[9]: policy announcements may be used to supplement and extend the impact of conventional policy instruments, and that controllability is now available with a reduced instrument set from $t=1$, if $rank(R_{11}...R_{1n}) = n$.

Corollary. All $Y_{t/1}$ values, including the first period's targets $Y_{1/1}$, are now dynamically controllable if the rank condition in proposition 1 holds. This is an important extension over conventional dynamic controllability where period $t=n$ is the earliest date at which we can guarantee controllability if there is a single policy instrument; or $t=n/2$ if there are two instruments, and so on. Thus $Y_{1/1}$ is controllable from the first period, even if there are insufficient instruments ($m < n$), provided that both $T \geq n$ and proposition 1 holds.

***Comments.* i)** It is important to see why time inconsistent behaviour will not appear here. Controllability at period t means that, barring unforeseen shocks, the policymaker will be able to reach his first best values for Y_t (in expectation). Hence, $Y_{t/t}=Y_{t/1}=Y_t^d$ are fixed and known quantities. But $Y_{1/t}=Y_{1/1}$ is fixed by history as pointed out below (6); and $X_{1/t}=X_{1/1}$ likewise. It is then easy to see, by (6), that if nothing else changes $X_{t/t}=X_{t/1}$. The policy maker is of course free to set $X_{t/t}\neq X_{t/1}$. But he would never do so because Y_t^d is a first best value which can be reached given no further information changes or unforeseen shocks. So to assert time inconsistency in this case is to claim that rational policymakers would deliberately choose to make themselves worse off, given the chance, and that the private sector should expect them to try to do so.

ii) It is important to examine the conditions that permit effective signaling and commitment, but it is equally important to recognize that there is a class of problems for which they are neither necessary nor relevant (although clear communication, that the necessary policy changes will be made, is still required). And it will typically be a *large class*, given that we can get down to cases with just one instrument and many targets if $T \geq n$ is large enough. Hence, time inconsistency is an exception rather than the rule.

iii) Put differently, our results explain when commitment is needed. So long as the no unit root condition on *AB* and the rank condition in Proposition 1 are both satisfied, the

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[9]A possibility recognized by the Fed, which argues that "forward guidance" is now key to its policies (Williams, 2011).

private sector knows that policymakers have controllability and can reach their desired target values in all cases. There is no point in expecting anything else: the system is fully tied down by (6) and proposition 1. But if the controllability conditions are not satisfied, or if private expectations can offset the policymaker's actions, or if the cost to the policy maker of the necessary actions is too large, then expectations are not tied down and commitment by the policy authorities will be necessary.

iv) The importance of providing credible, convincing explanations of future policies and target values, and the ability to reach them, is now obvious. If the policymakers and the private sector share information sets, or the policymakers are thought to possess superior information, then the economy will reach its first best outcomes. Thus credibility requires private agents to check if the policymakers' announcement/explanations match their own projections, in so far as they have firm projections. But if they do not share information, or if the private sector has access to better information, then the policymakers' announced outcomes will not be reached. Instead the outcomes will be what private agents expect to happen given the announced policies. The policymakers then have an incentive to adjust their policies to regain their preferred target values. But that is the normal process of correcting an implementation error (wrong information), not time inconsistency.

4. Applications of the general theory to problems of controllability

We now go back to our simple SOE macro-model introduced in section 2, and examine its controllability under different assumptions about the managing of monetary and trade policies in an open economy context. In the next section we will concentrate on monetary and financial policies, using a richer set of examples where an additional open economy monetary policy channel is present and pass-through might be incomplete.

We consider two cases to buttress our expectation anchoring argument: the simple backwards-looking case ($\beta = 0$) without rational expectations, and a more general case with forward-looking variables ($\beta \neq 0$) based on rational expectations.

Case 1. *Controllability without rational expectations.* If $\beta = 0$ we have a recursive system in place of (1) and (2), i.e.:

$$(9) \qquad \pi_t = \lambda \pi_{t-1} - \sigma \kappa i_t + \left(\chi \kappa + \phi \right) m_t + \kappa \varepsilon_t + v_t,$$

(10) $x_t = -\sigma i_t + \chi m_t + \varepsilon_t$

This system has static controllability in the classic sense since the policy multipliers in

(11) $\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} -\kappa\sigma & \chi\kappa+\phi \\ -\sigma & \chi \end{bmatrix} \begin{bmatrix} i_t \\ m_t \end{bmatrix} + \begin{bmatrix} v_t + \lambda\pi_{t-1} + \kappa\varepsilon_{t-1} \\ \varepsilon_t \end{bmatrix}$

form a non-singular matrix so long as $\phi \neq 0$. Consequently any required values for π_t and x_t can be reached in expectation.

However **if** only one policy instrument, i_t, is available and no trade policy $(\phi = \chi = 0)$ [10], **or** if trade policy has just one independent channel of influence $(\phi = 0)$, then static controllability is lost and the desired values for π_t and x_t cannot be reached in each (or indeed the current) period. The multiplier matrix in (11) is singular in either case. Nevertheless the system is still dynamically controllable over two periods. Consider the first case: $\phi = \chi = 0$. Back-substituting one period in (11), we get

(12) $\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{pmatrix} \lambda^2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{bmatrix} -\kappa\sigma & -\lambda\kappa\sigma \\ -\sigma & 0 \end{bmatrix} \begin{bmatrix} i_t \\ i_{t-1} \end{bmatrix} + \begin{pmatrix} 1 & \kappa \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_t \\ \varepsilon_t \end{pmatrix} + \begin{pmatrix} \lambda & \lambda\kappa \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v_{t-1} \\ \varepsilon_{t-1} \end{pmatrix}$

The policy multiplier matrix is now non-singular for *all* parameter values. We can reach any desired values for π_t and x_t after two periods using monetary policy alone.

The same is true if trade policy has only one channel of influence: $\phi = 0$. In this case the multiplier matrix in (11) is singular and the static controllability property is lost. But back-substituting one period implies

(13) $\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{pmatrix} \lambda^2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{bmatrix} -\kappa\sigma & -\lambda\kappa\sigma \\ -\sigma & 0 \end{bmatrix} \begin{bmatrix} i_t \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} \kappa\chi & \lambda\kappa\chi \\ \chi & 0 \end{bmatrix} \begin{bmatrix} m_t \\ m_{t-1} \end{bmatrix} +$ errors

in which both multiplier matrices are non-singular for all nonzero parameter values. So, to reach desired values for π_t and x_t after two periods, we can either use interest rates twice; or trade policy twice; or first interest rates and then trade policy, or trade policy and then interest rates in an "asynchronous game". In the latter two cases, the multiplier

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[10] As is the case in most of the monetary policy literature since Barro and Gordon (1983) and Rogoff (1985).

matrices would become $\begin{bmatrix} -\kappa\sigma & -\lambda\kappa\chi \\ -\sigma & 0 \end{bmatrix}\begin{bmatrix} i_t \\ m_{t-1} \end{bmatrix}$ and $\begin{bmatrix} \kappa\chi & -\lambda\kappa\sigma \\ \chi & 0 \end{bmatrix}\begin{bmatrix} m_t \\ i_{t-1} \end{bmatrix}$ respectively. But

it would still take two periods to reach our target values.

Case 2. *Controllability with rational, forward-looking expectations.* Allowing for the full effects of both policy instruments takes us back to (4); and hence to a model of the form of (3) with parameter matrices given by (5). That model has a policy multiplier matrix C, which is non-singular if $\phi \neq 0$. Hence we have static controllability given arbitrary initial conditions, and arbitrary values for the terminal conditions.

So far the story is the same as in the case with no rational expectations. However, things change if we have only one instrument (monetary policy) available. To see that, we write our policy problem as a two-period problem with both policy instruments:

$$(14) \quad \begin{bmatrix} \pi_t \\ x_t \\ \pi_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} I & -B \\ -A & I \end{bmatrix}^{-1}\begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix}\begin{bmatrix} f_t \\ i_t \\ E_t(f_{t+1}) \\ E_t(i_{t+1}) \end{bmatrix} + \begin{bmatrix} I & -B \\ -A & I \end{bmatrix}^{-1}\begin{pmatrix} w_t \\ w_{t+1} \end{pmatrix} \quad \text{say,}$$

as a particular case of (6). We are interested in only the first two rows of (14) for static controllability. But we will need to use the whole system if we are to go to dynamic controllability in the second period. The policy multiplier matrix implied by (14) is then:

$$(15) \quad \begin{bmatrix} \phi+\kappa\chi & -\kappa\sigma & [(1-\Delta)\phi/\lambda+1]\kappa\chi & -[(1-\Delta)/\lambda+1]\kappa\sigma \\ \chi\Delta+\sigma\lambda(\phi+\kappa\chi) & -\sigma[\Delta+\kappa\lambda\sigma] & \chi[\sigma\lambda\kappa+\Delta+\sigma\phi\kappa(\lambda(1-\Delta)+\Delta)] & -\sigma[\sigma\lambda\kappa+\Delta+\sigma\kappa(\lambda(1-\Delta)+\Delta)] \\ \lambda(\phi+\kappa\chi) & -\kappa\sigma\lambda & (\phi+\lambda)\kappa\chi & -(1+\lambda)\kappa\sigma \\ 0 & 0 & \chi & -\sigma \end{bmatrix}\Delta^{-1}$$

where $\Delta = 1-[\lambda(1-\lambda)+\lambda\kappa\sigma] \neq 0$. The one period static controllability policy multiplier matrix (the top left 2x2 sub-matrix) is non-singular as noted above. But if there is just one policy (monetary policy) available, $\phi = \chi = 0$, then instead of (15) we get

$$(16) \quad \begin{bmatrix} 0 & -\kappa\sigma & 0 & -(1-\Delta+\lambda)\kappa\sigma/\lambda \\ 0 & -\sigma(\kappa\sigma\lambda+\Delta) & 0 & -\sigma[\sigma\kappa(1-\lambda)+1]\Delta \\ 0 & -\kappa\sigma\lambda & 0 & -\sigma\kappa(1+\lambda) \\ 0 & 0 & 0 & -\sigma \end{bmatrix}\Delta^{-1}.$$

Alternatively, if trade policy is used but has one channel of influence $(\phi = 0)$, we get:

$$
(17) \quad
\begin{bmatrix}
\kappa\chi & -\kappa\sigma & \kappa\chi & -(1-\Delta+\lambda)\kappa\sigma/\lambda \\
\chi(\Delta+\kappa\sigma\lambda) & -\sigma(\kappa\sigma\lambda+\Delta) & \lambda\kappa\chi & -\sigma[\sigma\kappa(1-\lambda)+1]\Delta \\
\lambda\kappa\chi & -\kappa\sigma\lambda & \lambda\kappa\chi & -\sigma\kappa(1+\lambda) \\
0 & 0 & \chi & -\sigma
\end{bmatrix}
.\Delta^{-1}.
$$

Hence, for the first case, we can rewrite the one period, static controllability problem by inserting (16) into (14) to yield

$$
(18) \quad
\begin{bmatrix} \pi_t \\ x_t \end{bmatrix}
= -\Delta^{-1}
\begin{bmatrix}
\kappa\sigma & \kappa\sigma(1-\Delta+\lambda)/\lambda \\
\sigma(\kappa\sigma\lambda+\Delta) & \sigma[\sigma\kappa(1-\lambda)+1]\Delta
\end{bmatrix}
\begin{bmatrix} i_t \\ E_t(i_{t+1}) \end{bmatrix}
+ w_t
$$

whose multiplier matrix is easily seen to be non-singular. In other words, we have static controllability from the point of view of period t, despite having a single instrument and two targets. This is achieved by using i_t and policy announcements of what will happen in period $t+1$. So, in this case we have static controllability even if there are insufficient policy instruments.

The same thing happens when trade policy is announced but limited to one channel of influence. If we use trade policy, (18) will be replaced by inserting (17) in (14) to yield:

$$
(19) \quad
\begin{bmatrix} \pi_t \\ x_t \end{bmatrix}
= \Delta^{-1}
\begin{bmatrix}
\kappa\chi & \kappa\chi \\
\chi(\Delta+\kappa\sigma\lambda) & \lambda\kappa\chi
\end{bmatrix}
\begin{bmatrix} f_t \\ E_t(f_{t+1}) \end{bmatrix}
+ w_t
$$

whose multiplier matrix is also non-singular. So again we have static controllability at period t, an option we didn't have in the non-rational expectations case. And for the same reason: credible policy announcements become a second instrument. In addition, we can easily generate examples which imply static controllability using mixed policies: for example, monetary policy and announcements about future trade policies (a permanent policy change); or trade policy plus announcements of future monetary policies.

The crucial point, however, is that when the economy's dynamics are forward looking (case 2) policymaking requires us to exercise good communication skills in order to make the policy announcements credible and have some effect—that is to make the private sector shift or anchor their expectations of the likely outcomes to values that suit the policymakers' purpose. In other words, the private sector has to be made to believe that the necessary policy changes will actually take place and achieve the said outcomes.

By contrast, when the economy dynamics is purely backward looking (case 1) policymaking does not require the same communication skills since there are no expectations to be anchored. It is worth to noticing that these examples have no lagged expectations terms in them; that is, expectations of current, earlier or future variables conditioned on earlier information sets, such as you might get in trade agreements, financial contracts, wage contracts or wage indexation agreed to earlier. This is because current policies or policy announcements cannot be used to steer/anchor past expectations that may now be locked into existing agreements or contracts. Our theory is good for any arbitrary initial position.

5. Communication: Should policymakers try to manage expectations?

5.1 Announcement and expectations in a SOE

One of the great debates of monetary policy is whether central banks should allow forecasts of future interest rates to be published. Rudebusch and Williams (2008) and Eusepi and Preston (2010) argue this can be used to strengthen economic policy.[11] But others argue that to do so may imply a stronger consensus or more certainty about future policies than actually exists; or that it may propagate errors and make it more difficult to adjust policies again later in the face of unexpected shocks. In addition, private agents may overreact to noisy public signals, but under-react to more accurate private information (see, Faust and Svensson, 2002; Amato *et al.*, 2003; Walsh, 2007).

We investigate this argument by using a stylized well-known open-economy model, summarized by a two-equation system:

(20) $\quad y_t = \rho y_{t-1} + \alpha(\pi_t - E_t \pi_{t+1}) - \beta(i_t - E_t \pi_{t+1}) + \varepsilon_t$

(21) $\quad i_t = c_0 + c_1(\pi_t - \pi^*) + c_2 y_t + v_t$

where y_t is the real output; π_t is the rate of inflation; $E_t p_{t+1}$ the private sector's current expectation for the rate of inflation; and π^* is the government or central bank's target inflation rate; i_t is the nominal rate of interest, ε_t is a supply shock with mean zero and

---

[11] See Archer (2005), Moessner and Nelson (2008), Ferrero and Secchi (2009) for evidence to support this view.

constant variance, and v_t is a monetary policy shock with mean zero and constant variance. All parameters are positive.

Equation (20) is an a dynamic open economy Phillips curve,[12] which consists of a short-run Phillips curve with persistence ($\rho \neq 0$), set within a standard forward looking Lucas supply function (a long run Phillips curve) and elaborated to include the effects of real interest rate changes on output. The only policy instrument in this example will be i_t. Equation (21) is therefore a Taylor rule: c_0 is a constant term reflecting the equilibrium rate of interest; v_t the possible control errors; π^* is the inflation target; and determinacy (the Taylor principle) suggests $c_1 > 1$. Finally, notice that β measures the strength of the open economy channel for monetary policy transmission. When β is zero, the model is reduced to a conventional closed economy model.

To obtain a reduced form for (20)-(21), corresponding to (3), we renormalize (21) on π_t, set $u_t = p^* - c_1^{-1}(c_0 - v_t)$ and solve (20) and (21). This transforms our system to:

$$(22) \quad \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \frac{\rho}{\Delta} \begin{bmatrix} 1 & 0 \\ -c_2 c_1^{-1} & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \end{bmatrix} + \frac{\beta - \alpha}{\Delta} \begin{bmatrix} 0 & 1 \\ 0 & -c_2 c_1^{-1} \end{bmatrix} \begin{bmatrix} y_{t+1/1} \\ \pi_{t+1/1} \end{bmatrix} +$$

$$+ \frac{1}{c_1 \Delta} \begin{bmatrix} \alpha - \beta c_1 \\ \beta c_2 + 1 \end{bmatrix} i_t + \frac{1}{\Delta} \begin{bmatrix} 1 & \alpha \\ -c_2 c_1^{-1} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ u_t \end{bmatrix},$$

where $\Delta = (1 + \alpha c_1^{-1} c_2) \neq 0$, which does not allow static controllability. However, we can write the two period policy problem, as we did before, using (6):

$$(23) \quad \begin{bmatrix} y_t \\ \pi_t \\ y_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} I & -B \\ -A & I \end{bmatrix}^{-1} \left(\begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} i_t \\ E_t i_{t+1} \end{bmatrix} + \begin{bmatrix} A \begin{bmatrix} y_0 \\ \pi_0 \end{bmatrix} \\ B \begin{bmatrix} E_t y_{t+2} \\ E_t \pi_{t+2} \end{bmatrix} \end{bmatrix} + \frac{1}{\Delta} \begin{bmatrix} \varepsilon_t + \alpha u_t \\ u_t - c_2 c_1^{-1} \varepsilon_t \\ \varepsilon_{t+1} + \alpha u_{t+1} \\ u_{t+1} - c_2 c_1^{-1} \varepsilon_{t+1} \end{bmatrix} \right)$$

where A, B and C are the first, second and third coefficient matrices in (22). The policy multiplier matrix for this model is then:

---

[12] The model can be seen as a simplified but forward-looking version of Ball (1999), where foreign inflation and interest rates are taken as given or as stochastic factors (and hence omitted).

$$(24) \quad \frac{1}{\Phi} \begin{bmatrix} (\alpha - \beta c_1)\Delta & (\beta - \alpha)(1 + \beta c_2) \\ \dfrac{\rho(\beta - \alpha)(\alpha - \beta c_1)c_1^{-1}c_2^2 + (1 + \beta c_2)\Phi}{c_1\Delta} & -c_2 c_1^{-1}(\beta - \alpha)(1 + \beta c_2) \\ \rho(\alpha - \beta c_1) & \dfrac{\rho(\beta - \alpha)(1 + \beta c_2)c_1 + (\alpha - \beta c_1)\Phi}{c_1\Delta} \\ -c_2 c_1^{-1}\rho(\alpha - \beta c_1) & (1 + \beta c_2)\Delta \end{bmatrix} \begin{bmatrix} i_t \\ E_t i_{t+1} \end{bmatrix}$$

where $\Phi = \Delta^2 c_1 + \rho(\beta - \alpha)c_2$. Putting (24) back into (23), this standard model tells us that (y_t, π_t) and (y_{t+1}, π_{t+1}) are both statically (and immediately) controllable using current policies and announcements or projections of future actions. That is because both the upper and lower partitions of the multiplier matrix in equation (24) are non-singular.[13] And since (y_t, π_t) and (y_{t+1}, π_{t+1}) are current and expected future target values from the perspective of period t, this means that the policymakers can control not only current inflation and output (in expectation at least); *but also* inflation expectations and growth forecasts for the next period. Thus they have the ability to control inflation and output by anchoring those expectations, as claimed.

This example is therefore in line with our theory of dynamic controllability under rational expectations, and makes the case for publishing conditional forecasts of future interest rates as a normal part of the monetary policy framework. Indeed it would become more difficult to achieve successful outcomes if inflation expectations could not be tied down by forecast announcements at the same time as inflation itself. And this goes for open economies just as for closed economies. There may be other ways of controlling the outcomes for (y_{t+1}, π_{t+1}) in period $t+1$ of course; for example using current or past interest rates as implied by (24). But that is a conventional backwards looking use of dynamic controllability, and may still fail to tie down/anchor expectations in the terminal period.

5.2. Managing expectations in an open economy with low pass through

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[13] Unless the underlying parameters satisfy $\alpha = \beta$ exactly in the upper sub-matrix (for current outcomes); or $\alpha = \beta c_1$ in the lower one (future expected outcomes). These are sufficient conditions; and they cannot both hold simultaneously since we require $c_1 > 1$ for the Taylor principle. So, even in these cases, we guaranteed to control either future expectations or current outcomes through policy announcements/forecasts. Beyond that, there is one particular value in the dynamics, ρ, which leads to singularity for the whole system—corresponding to the unit root condition identified in section 3.

We now consider a SOE model with low pass through[14] developed by Monacelli (2005), which overcomes the simple closed-economy isomorphic model described in section 4. The introduction of incomplete pass-through renders the analysis of monetary policy of an open economy fundamentally different from that of a closed economy because a short-run trade-off between the stabilization of inflation and the output gap emerges.[15]

The model is described by the following equations:

(25) $\pi_t = (1-\lambda)\pi_t^h + \lambda\pi_t^f$ (domestic CPI inflation)

(26) $\pi_t^h = \beta E_t \pi_{t+1}^h + \kappa x_t + \chi\varphi_t^f$ (domestic Phillips curve)

(27) $x_t = E_t x_{t+1} - \sigma(i_t - \pi_{t+1}^h - \bar{r}_t) + \mu\left(E_t\varphi_{t+1}^f - \varphi_t^f\right)$ (domestic IS curve)

(28) $\pi_t^f = \beta E_t \pi_{t+1}^f + \lambda\varphi_t^f$ (imported local currency inflation)

(29) $i_t - i_t^* = E_t\left(\pi_{t+1}^h - \pi_{t+1}^f\right) + E_t\varphi_{t+1}^f - \varphi_t^f$ (uncovered interest parity)

where x_t is the output gap; π_t is the inflation rate, π_t^h and π_t^f are domestic producer and (local currency) imported inflation, respectively; φ_t^f denotes deviation of the world price $(e_t + p_t^*)$ from the domestic currency price of imports (p_t^f), a measure of the deviations from the law of one price. Then i_t and i_t^* are domestic and foreign interest rates; $\bar{r}_t$ is the natural real rate of interest, which depends on domestic productivity and expected growth in world output. We assume that both i_t^* and $\bar{r}_t$ follow some known stochastic processes.

Equation (25) just defines domestic CPI inflation. Equation (26) is the domestic aggregate supply assuming price stickiness of the Calvo or Rotemberg kind; equation (27) represents the demand side of the economy (a standard IS curve obtained from the consumer's Euler equation); equation (28) describe the dynamics of imports domestic currency inflation; and equation (29) closes the model by imposing the uncovered interest parity. Such an equation results from combining efficiency conditions for an optimal portfolio of bonds held by both domestic and foreign residents.

After some detailed algebra, the model (25)-(29) can be reduced to the form of (4):

---

[14] Campa and Goldberg (2005) estimate import pass-through elasticities for a range of OECD countries. They find that the degree of pass-through is partial in the short-run, but it becomes gradually complete in the long-run.

[15] As shown by Monacelli (2005), in such a context a nominal depreciation determines the wedge between the price paid by the importers in world markets and the local currency price in the domestic markets. This wedge acts as an increase in the real marginal cost and therefore increases foreign goods inflation.

$$(30) \quad \begin{bmatrix} x_t \\ \pi_t \\ \pi_t^f \\ \varphi_t \end{bmatrix} = \begin{bmatrix} -\kappa & \frac{1}{1-\lambda} & \frac{-\lambda}{1-\lambda} & -\chi \\ 1 & 0 & 0 & \mu \\ 0 & 0 & 1 & -\lambda \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \left(\begin{bmatrix} 0 & \frac{\beta}{1-\lambda} & \frac{-\lambda\beta}{1-\lambda} & 0 \\ 1 & \frac{\sigma}{1-\lambda} & \frac{-\lambda\sigma}{1-\lambda} & \mu \\ 0 & 0 & \beta & 0 \\ 0 & \frac{1}{1-\lambda} & \frac{-1}{1-\lambda} & 1 \end{bmatrix} \begin{bmatrix} x_{t+1} \\ \pi_{t+1} \\ \pi_{t+1}^f \\ \varphi_{t+1} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \sigma \\ 1 \end{bmatrix} i_t + \begin{bmatrix} 0 \\ \sigma\bar{r}_t \\ 0 \\ i_t^* \end{bmatrix} + errors \right)$$

where $\left[x_t, \pi_t, \pi_t^f, \varphi_t^f \right]' = Y_t$; $i_t = X_t$. Definitions for *A*, *B* and *C* are now straightforward:

$A=0$;

$$B = \begin{bmatrix} 1 & \frac{\sigma-\mu}{1-\lambda} & \frac{\mu-\lambda\sigma}{1-\lambda} & 0 \\ \kappa(1-\lambda) & \frac{\lambda^2+(1-\lambda)[\chi+\beta+\kappa(\sigma-\mu)]}{1-\lambda} & \lambda & \lambda^2+\chi(1-\lambda) \\ 0 & \frac{\lambda}{1-\lambda} & \beta-\frac{1}{1-\lambda} & \lambda \\ 0 & \frac{1}{1-\lambda} & -\frac{1}{1-\lambda} & 1 \end{bmatrix} ; \quad C = \begin{bmatrix} \mu \\ (1-\lambda)(\mu\kappa-\chi)-\lambda(\sigma+\lambda) \\ -(\sigma+\lambda) \\ -1 \end{bmatrix} .$$

Thus the model's final form, (6), implies an endogenous variable coefficient (Toeplitz) matrix which is triangular; of full rank (note that $det(B)=\beta^2$), and with an inverse that is "full" (all entries non-zero). Consequently, (6) implies that matrix *R* in (7) is given by

$$(31) \quad R = \begin{bmatrix} C & BC & B^2C & ... & B^{T-1}C \\ 0 & C & BC & . & . \\ . & . & C & . & . \\ . & & . & . & BC \\ 0 & . & . & 0 & C \end{bmatrix} .$$

Here $R_{t,j}= 0$ for all $t>j$; but $R_{t,j}=B^{t-j}C$ if $j \geq t$. However, even if we do not assign specific numerical values to the parameters/partial derivatives in the model, we can still determine if the policy targets are controllable, and if expectations can be managed to achieve certain specified targets in period *t*. We can do that by evaluating the appropriate rows in *R* and using proposition 1.

The first thing to notice is that $rank(C)=rank(BC)=1$; so that if we take *T*=2 (a two period problem) for illustration, $R = \begin{bmatrix} C & BC \\ 0 & C \end{bmatrix}$ and $rank(R)=2$. This means that we can achieve two of the policy goals at most in the two periods available: a zero output gap, x_t, and low inflation, π_t, in the current period (but not in the future) perhaps; **or** next year's output gap and inflation targets, but not those in the current period; **or** low inflation this year and a stabilised output gap next year (or vice versa). Or, if we want to achieve all

four targets this year, we need both to set current interest rates and make announcements or interest rate forecasts of the next three years[16] (which, given the natural uncertainty of economic events, may stretch credibility too far). But what we cannot do is achieve all four targets—inflation, output, imported inflation and competiveness on world markets—in less than four policy years; or more than one of them in any one policy year; or more than one of them without needing to make (credible) policy announcements about future policies, next year and in the following years.

These conclusions are *radically* different from those in the traditional monetary policy literature because they imply there is no short term trade-off between inflation and output stabilisation when the rate of exchange rate pass through is taken into account. To get even as far as that trade-off, let alone resolve it, some interest rate forecasts/forward guidance will be necessary, rather than an option – unless carefully coordinated packages of fiscal-monetary policies can be brought to bear. And additional targets mean extending the period of the package of announcements yet further. These results bring a whole new perspective to the importance of the pass-through problem specifically for performance and policy design, which has yet be to acknowledged in the literature.

Another point: all of these results can be derived directly, without any need for complicated numerical simulations or analytic calculations. Just as well because such calculations typically yield expressions that are too complicated to allow us to learn anything. The appendix shows the simplest case to make the point: an evaluation of (31) when $T=2$. On the other hand, the size of the immediate impact *and* announcement effects of any policy changes has to be checked by computing the matrices C, BC, B^2C….etc numerically.

6. Concluding remarks.

This paper has examined the ability of policymakers to control a small open economy, and their ability to steer or anchor private expectations in support of their policies, when agents and financial markets have forward-looking (rational) expectations. We find that the policymakers' ability to systematically exploit expectations leads to an extension of their policy capability—subject to certain conditions. We provide examples from trade

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[16] From the rank of R in (31) when $T=4$.

and monetary policy to show when that is possible, and when it is not. Perhaps the most radical departure from conventional wisdom comes with imperfect pass-through; a signal of the importance of that issue for performance and policy design in open economies.

Rational expectations have a twofold nature. Since the Lucas critique, the emphasis has been on the implication that the policymaker cannot fool a private sector that correctly anticipates the policies and the equilibrium. This has been done by investigating policy invariance, underlining the role of rational expectations in offsetting the policy interventions. This paper shows that, because policy invariance can only emerge when there is a conflict between the public and the private sector, this will not always be what happens. Policymakers may be able to induce private agents to shift their expectations.

Second, rational expectations are also a powerful mechanism, in combination with the chosen policy values, for influencing the natural dynamics of the economy. There has been much less interest in this second aspect of rational expectations which implies that policy announcements may often be used to systematically increase the power of the policymaker's interventions. A notable exception is Woodford's *timeless perspective,* and the use of interest rate forecasts or inflation targets to anchor inflation expectations.

In proposition 1, we have obtained a rank condition that defines the circumstances under which rational expectations can be used to increase a policymaker's power to control an economy over time. It shows how communication and policy announcements can be exploited to supplement and extend the impact of conventional policy instruments. This gives us a formal justification for using policies designed to manage expectations, and defines the circumstances in which expectations cannot be anchored.

One implication of this condition is that, in the absence of changes in information, once a policy sequence has been announced there can be no question of revising an announcement for strategic reasons, or of expecting it to be revised, because the policy maker can and is known to be able to reach his first best outcomes. So they have no incentive or interest in not following through on their announcements since it would make them worse off than they need be. In that sense, these policy announcements are all implementable.

A second implication is that the quality and credibility of communication by policy makers is a key part of the problem. It is important to examine the conditions that permit

effective signalling and commitment, but it is equally important to recognize that there is a whole class of problems for which they are neither necessary nor relevant (although clear communication is still required so that agents can check the consistency of the announced policies and target values). And it will typically be a large class given that we can get down to one instrument and still have controllability if the policy horizon is long enough.

This last point then leads four corollaries: a) Successful communication decomposes into two parts; consistency with the announced targets, and clear priorities to signal credibility when there are insufficient instruments or time periods. In the past the literature has concentrated on the second element, neglecting the first. b) Policymakers who are patient can achieve, and be expected to achieve, what goals they want (their first best targets) if they have the time, $t \geq n$, or enough instruments. c) Time inconsistent behaviour is therefore a limited phenomenon; but will become a problem if policy makers find themselves under pressure and want a quick fix by trying to reach too many targets in too short a time. d) Time inconsistency is not a general or widespread problem. That is a useful conclusion because policymakers appear not make extensive use of it in practice, except perhaps during elections (point c)), and we need to be able to explain why.

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Appendix

The *R* matrix in (31), for *T*=2 and an open economy with incomplete pass-through is

$$R = \begin{bmatrix} \mu & -\mu\lambda + \mu - \sigma\chi + \sigma\kappa\mu - \mu\sigma + \mu\chi - \kappa\mu^2 \\ -\lambda\sigma + \kappa\mu - \kappa\mu\lambda - \lambda^2 - \chi + \chi\lambda & \Delta \\ -\sigma - \lambda & \lambda^2 - \chi\lambda - \lambda - \lambda\beta + \kappa\mu\lambda + \lambda\sigma - \beta\sigma \\ -1 & \lambda - 1 + \sigma - \chi + \kappa\mu \\ 0 & \mu \\ 0 & -\lambda\sigma + \kappa\mu - \kappa\mu\lambda - \lambda^2 - \chi + \chi\lambda \\ 0 & -\sigma - \lambda \\ 0 & -1 \end{bmatrix}$$

where

$$\Delta = -\sigma\kappa\mu - \sigma\chi\lambda + 2\kappa\mu\chi + \sigma\chi - 2\chi\lambda^2 - \lambda^2\beta - \chi\beta + \kappa^2\sigma\mu + \kappa^2\mu^2\lambda - \beta\lambda\sigma$$
$$- \kappa^2\sigma\mu\lambda + \kappa\sigma\chi\lambda - 2\kappa\mu\chi\lambda - \kappa^2\mu^2 - \chi^2 + \chi^2\lambda + \beta\kappa\mu + \lambda^2\sigma - \chi + \lambda^3 - \lambda^2$$
$$+ \kappa\mu + 2\chi\lambda - 2\kappa\mu\lambda + 2\kappa\mu\lambda^2 + \chi\lambda\beta - \kappa\sigma\chi - \beta\kappa\mu\lambda + \sigma\kappa\mu\lambda$$