Technical change in a two-sector model of optimal growth

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Abstract

This paper investigates into the consequences of sector-specific technological progress in a two-sector, optimal growth model. In accordance with existing theory, we find that consumption-specific Hicks-neutral technical shocks increase consumption but leave other parameters unchanged. Hicks-neutral, investment-specific technical shocks increase the wage-rental ratio, and increase steady-state consumption by a factor equal to the macroeconomic ratio of capital share to labor share. If the elasticity of substitution is equal to one in the long run, the growth regime with only investment-specific technical change is sustainable and asymptotically balanced.

Keywords: Sector-specific technical change, optimal growth, capital accumulation, golden rule, two-sector models.
JEL codes: C62, E22, O39, O41.

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Introduction

Since the seminal study by Greenwood, Hercowitz and Krusell (1997), considerable effort has been devoted by growth economists to distinguish between *consumption*-specific and *investment*-specific technical change. Figure 1 represents the post-war evolution of the ratio of real investment to real consumption in the USA. It emerges clearly from this picture that real investment has grown more rapidly than real consumption on average for more than sixty years[1].

The case for predominant investment-specific technical progress is even more obvious for the last thirty years. As figure 2 shows, with the beginning of the 'Great Moderation' the deflator of investment has started to continuously decrease relatively to the deflator of consumption.

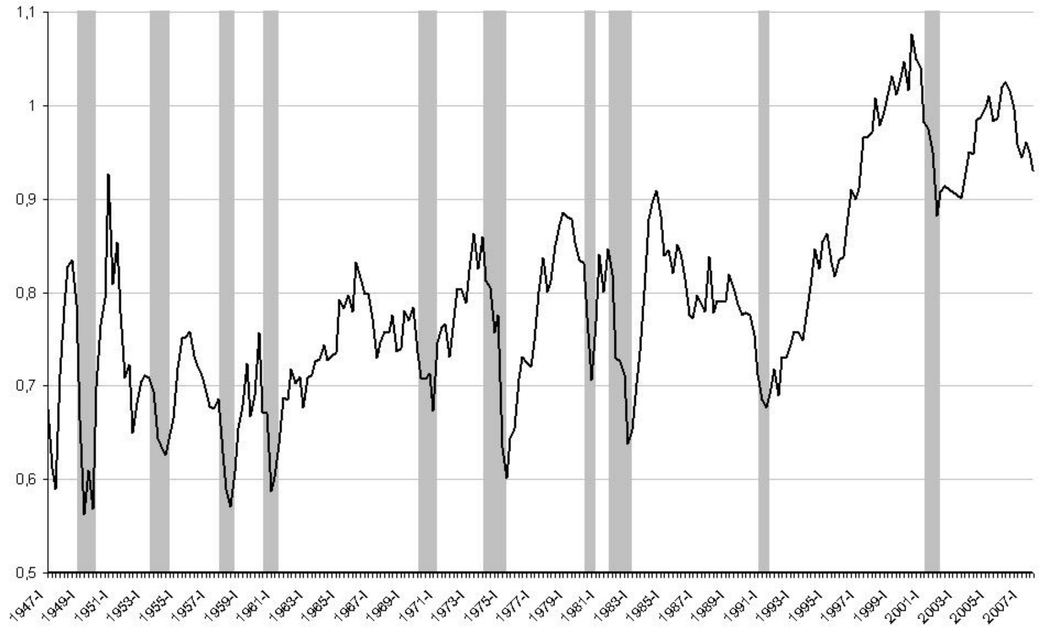

Figure 1: Macroeconomic ratio of real investment to real private consumption, 1947-I to 2007-IV (source: Bureau of Economic Analysis, NIPA database). Grey areas represent recessions according to NBER.

Consistently with these observations, several empirical studies have concluded that investment-specific technical change was responsible for a part of GDP growth that far outweighs the share of investment in GDP[2]. Firm theoretical foundations about the consequences of sector-specific technical change are however rare. Meade (1962) describes the growth path of an economy enjoying steady consumption-specific and investment-specific technological progress when production functions are of Cobb-Douglas type. Kimball (1994) has shown that, in an economy governed by a social planner with a logarithmic or power utility function, the optimal path was independent of the path of consumption-specific productivity. This neutrality result is also evoked in Fisher (1997) and Basu, Fernald, Fisher and Kimball (2010). Still, there does not exist – to our knowledge – any theoretical study of the consequences of investment-specific technical change. As figures 1 and

[1] Still, notably, the ratio of nominal investment to nominal consumption is stationary. See Whelan (2006).

[2] See Gort, Greenwood and Rupert (1999), Cummins and Violante (2002), Pakko (2002) and Ngai and Samaniego (2009) among others. On the quantitative role of investment-specific technical change in the business cycle, see Greenwood, Hercowitz and Krusell (2000) and Fisher (2006). Whelan (2006) estimates a two-sector, structural VAR model and identifies a marked investment-specific productivity growth boom starting in the early 1980's.

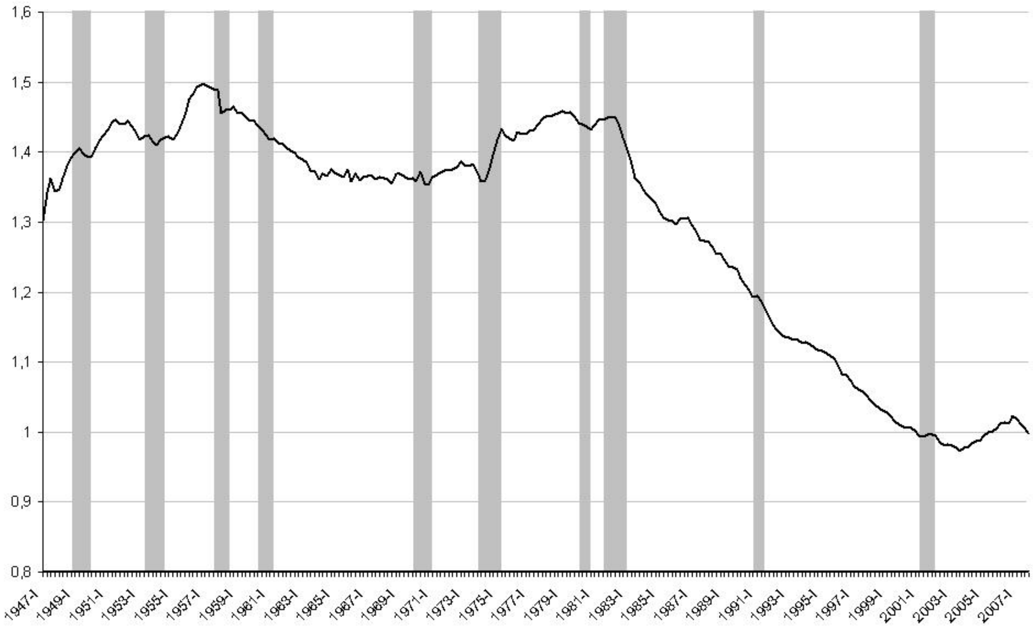

Figure 2: Ratio of investment deflator to private consumption deflator, 1947-I to 2007-IV (source: Bureau of Economic Analysis, NIPA database). Grey areas represent recessions according to NBER.

2 show, this analysis is crucial in understanding the growth pattern that prevailed in the USA at least for the last thirty years.

This paper is an attempt to partially bridge this gap. We analyze the consequences of technical shocks on the two-sector golden rule steady state. We presume this method is best suited to analyze first-order effects of technical progress, that stem from the expansion of the production possibilities of the economy. Indeed, when technical change is continuously *expected*, many effects intertwine. For example, suppose that an economy is expecting some positive technical shock (in any of the industries) to happen in the future. Then, in general, the agents will attempt to consume some of the future consumption benefits *today*. To do this, they must (temporarily) allocate more inputs to the consumption sector for some time. This strategy does not reflect optimal long-term calculation, it just merely represents some inter-temporal substitution of consumption. At least theoretically, these are second-order effects. Here we concentrate on the first-order effects which are the only ones that we observe when the economy moves from one steady state to another[3].

Consistently with existing results, we show that consumption-specific technical shocks are neutral; they do not influence the wage-rental ratio, the macroeconomic capital-labor ratio, nor the distribution of inputs between sectors: they merely make consumption increase one-for-one. Investment-specific technical shocks have richer consequences. They are shown to *increase* the steady-state wage-rental ratio and to increase final consumption by a factor equal to the macroeconomic ratio of capital share to labor share. The sustainability of a growth regime led by investment-specific productivity growth thus crucially depends on the elasticity of substitution between capital and labor inputs in the whole economy. When the elasticity is strictly greater than one, the con-

[3] Kimball (1994) shows that those effects cancel out in the dynamic framework if utility is logarithmic.

tribution of investment-specific productivity growth to consumption growth increases over time, even if investment-specific productivity growth is constant. On the contrary, when this elasticity is strictly less than one, this contribution eventually vanishes as time tends to infinity, and consumption growth cannot be sustained by investment-specific technical change alone.

The rest of the paper is organized as follows. Section 1 presents the two-sector golden rule in the absence of technical progress. Section 2 analyzes the consequences of consumption-specific technical shocks. Section 3 does the same for investment-specific technical shocks. Section 4 discusses the issue of the sustainability of the growth path with only investment-specific technical change. Section 5 concludes.

1 The two-sector golden rule

1.1 Technology and optimization problem

We suppose in this section that there is no technical change. Time is discrete. Population is constant at L and the economy is initially endowed with a stock of capital K_0. Capital depreciates by a factor δ from one period to another.

At each period, a benevolent social planner allocates available capital (K) and labor (L) resources in the consumption sector and the investment sector: $\left(K^C, L^C\right)$ goes to the former and $\left(K^I, L^I\right)$ to the latter. Inputs are costlessly transferable from one sector to another at each date. Consumption technology is represented by the production function $C_t = F\left(K_t^C, L_t^C\right)$ and investment technology by $I_t = G(K_t^I, L_t^I)$. Both production functions satisfy usual neoclassical properties: F and G are $\mathcal{C}^2$, show constant returns to scale, strictly decreasing marginal productivity with respect to both inputs and verify Inada conditions.

The planner's intertemporal preferences are summarized by a well-behaved instantaneous utility function $v(C)$ and by the rate of preferences for the present $\beta = 1/(1+\theta)$, where $\theta > 0$ denotes the subjective rate of impatience. The problem he solves is:

$$\left| \begin{array}{lrl} & \max_{\left(K_t^C, K_t^I, L_t^C, L_t^I\right)_{t\geq 0}} & \sum_{t=0}^{\infty} \frac{1}{(1+\theta)^t} v\left(C_t\right) \\ \text{s.t.} & C_t & \leq F\left(K_t^C, L_t^C\right) \\ & I_t & \leq G\left(K_t^I, L_t^I\right) \\ & K_t^C + K_t^I & \leq K_t \\ & L_t^C + L_t^I & \leq L \\ & K_{t+1} & = (1-\delta)K_t + I_t \end{array} \right. \tag{1}$$

Let's denote by $(p_t^C, p_t^I, r_t, w_t, q_t)_{t\geq 0}$ the vector of lagrangian multipliers. This optimization problem we treated in Senouci (2011). When both goods are produced, first-order equations are:

$$\begin{cases} u'(c_t) & = & p_t^C \\ q_t & = & p_t^I \\ p_t^C \frac{\partial F}{\partial K_t^C} & = & r_t \\ p_t^I \frac{\partial G}{\partial K_t^I} & = & r_t \\ p_t^C \frac{\partial F}{\partial L_t^C} & = & w_t \\ p_t^I \frac{\partial G}{\partial L_t^I} & = & w_t \\ q_t & = & \frac{1}{1+\theta}(r_{t+1} + (1-\delta)q_{t+1}) \end{cases} \tag{2}$$

The exposition is made easier with per-capita variables. Let $k = K/L$, $c = C/L$, $i = I/L$, $k^C = K^C/L^C$, $k^I = K^I/L^I$, $u(c) = v(Lc)/L$, $f\left(k^C\right) = F\left(k^C, 1\right)$, $g\left(k^I\right) = G\left(k^I, 1\right)$ and let $\omega = w/r$ denote the wage-rental ratio. Static efficiency requires that, at all dates:

$$\omega = \frac{f\left(k^C\right)}{f'\left(k^C\right)} - k^C = \frac{g\left(k^I\right)}{g'\left(k^I\right)} - k^I \tag{3}$$

Because functions $\frac{f(x)}{f'(x)} - x$ and $\frac{g(x)}{g'(x)} - x$ are both strictly increasing, equation (3) determines each sector's capital per worker ratio as two strictly increasing functions of the wage-rental ratio, $k^C(\omega)$ and $k^I(\omega)$:

$$\frac{\partial k^C}{\partial \omega} = -\left(\frac{f'^2}{ff''}\right)\left(k^C\right) > 0 \tag{4}$$

And:

$$\frac{\partial k^I}{\partial \omega} = -\left(\frac{g'^2}{gg''}\right)\left(k^I\right) > 0 \tag{5}$$

Notice that the relationships $k^C(\omega)$ and $k^I(\omega)$ are not altered by Hicks-neutral technology shocks.

Let's define $l^C = L^C/L$ and $l^I = L^I/L$ the fractions of the labor force respectively employed in the consumption and in the investment sector. Since resources are fully employed, it holds that:

$$\begin{cases} l^C k^C + l^I k^I & = k \\ l^C + l^I & = 1 \end{cases} \tag{6}$$

Thus, if $k^C \neq k^I$, we have that:

$$\begin{array}{lll} l^C & = \frac{k-k^I}{k^C-k^I} & = l^C(k,\omega) \\ l^I & = \frac{k^C-k}{k^C-k^I} & = l^I(k,\omega). \end{array} \tag{7}$$

1.2 The quasi-golden rule

Because of decreasing marginal productivity of capital in both industries, consumption cannot grow asymptotically in the absence of technical progress[4].

We have shown elsewhere[5] that, under a few conditions, the economy governed by a social planner eventually converges to a "quasi-golden rule" steady state independent of the initial conditions and of the instantaneous utility function, but dependent on the subjective discount factor, and characterized by[6]:

$$\left\{\begin{array}{rcl} g'\left(k^I\right) & = & \delta+\theta \\ \frac{f\left(k^C\right)}{f'\left(k^C\right)}-k^C & = & \frac{g\left(k^I\right)}{g'\left(k^I\right)}-k^I=\omega \\ l^I g\left(k^I\right) & = & \delta k \quad \Leftrightarrow \quad \frac{k^C-k}{k^C-k^I} g\left(k^I\right)=\delta k \end{array}\right. \tag{8}$$

We call ω_θ^* the quasi-golden rule wage-rental ratio, and k_θ^* the quasi-golden rule capital-labor ratio, for each $\theta>0$.

The first equation in (8) has the following interpretation. Suppose that the optimal steady state is reached, so that the social planner plans to set $k^C,\ k^I,\ l^C,\ l^I$ (and, so, k) constant at their quasi-golden rule value for the infinite future. Suppose further that the social planner unexpectedly receives a free unit of capital at date $t=\tau$. If he installs this marginal unit in the consumption sector from $t=\tau$ on, the resulting intertemporal benefit is of:

$$\sum_{t=\tau}^{\infty}\left(\frac{1}{1+\theta}\right)^{t-\tau}(1-\delta)^{t-\tau} f'\left(k^C(\omega_\theta^*)\right) u'\left(c_\theta^*\right)=\frac{1+\theta}{\delta+\theta} f'\left(k^C(\omega_\theta^*)\right) u'\left(c_\theta^*\right) \tag{9}$$

Another strategy is to put this additional unit of capital in the investment sector at $t=\tau$ and, then, to install the windfall quantity of capital $\left(1-\delta+g'\left(k^I\right)\right)$ in the consumption sector for $t\geq\tau+1$. The increase in the objective function of the social planner seen from $t=\tau$ is:

$$\begin{array}{r}\left(1-\delta+g'\left(k^I(\omega_\theta^*)\right)\right)\sum_{t=\tau+1}^{\infty}\left(\frac{1}{1+\theta}\right)^{t-\tau}(1-\delta)^{t-\tau-1} f'\left(k^C(\omega_\theta^*)\right) u'\left(c_\theta^*\right) \\ =\frac{1-\delta+g'\left(k^I(\omega_\theta^*)\right)}{\delta+\theta} f'\left(k^C(\omega_\theta^*)\right) u'\left(c_\theta^*\right)\end{array} \tag{10}$$

Because of intertemporal optimality, these two strategies must yield the same benefits. Equality of (9) and (10) then reduces to the first equation in (8). The quasi-golden rule is thus centered on the technology of the investment sector. Notably, ω_θ^* is independent of the production function for consumption f. This is logical, as investment can be thought as a production roundabout, which profitability only depends on the productivity of the investment sector.

[4]Notice that because production function G satisfies the Inada conditions, there exists a maximum sustainable capital stock defined by $G\left(\overline{K},L\right)=\delta\overline{K}$.

[5]Senouci (2011).

[6]See Phelps (1961, 1965) and Cass (1965) for the original one-sector formulation of the golden rules. Srinivasan (1964), Uzawa (1965), Shell (1967), Haque (1970), Galor (1992) and Cremers (2006) among others present the golden rule in two-sector frameworks.

The second equation in (8) reflects equalization of technical marginal rates of substitution across the two sectors, and the third equation in (8) reflects the condition that net investment must be zero at steady state.

1.3 The pure golden rule

When the rate of impatience θ tends to zero, steady-state approaches the pure golden rule defined by:

$$\begin{cases} g'\left(k^I\right) & = & \delta \\ \frac{f\left(k^C\right)}{f'\left(k^C\right)} - k^C & = & \frac{g\left(k^I\right)}{g'\left(k^I\right)} - k^I = \omega \\ \frac{k^C - k}{k^C - k^I} g\left(k^I\right) & = & \delta k \end{cases} \tag{11}$$

which also corresponds to the maximization of steady-state consumption problem. Let's call ω^* and k^* the pure golden rule values of ω and k. Clearly, for all $\theta > 0$, $\omega^* > \omega^*_\theta$ and $k^* > k^*_\theta$.

Some remarkable identities, already presented in Cremers (2006), hold at the golden rule steady state:

Theorem 1.1. *At the golden rule allocation:*

$$l^C = \frac{f\left(k^C\right) - k^C f'\left(k^C\right)}{f\left(k^C\right)} \tag{12}$$

$$l^I = \frac{k^C f'\left(k^C\right)}{f\left(k^C\right)} \tag{13}$$

$$\frac{l^C k^C}{k} = \frac{g\left(k^I\right) - k^I g'\left(k^I\right)}{g\left(k^I\right)} \tag{14}$$

$$\frac{l^I k^I}{k} = \frac{k^I g'\left(k^I\right)}{g\left(k^I\right)} \tag{15}$$

where all variables are at their golden rule level – $k = k^*$, $\omega = \omega^*$, $k^I = k^I\left(\omega^*\right)$, $k^C = k^C\left(\omega^*\right)$, $l^I = l^I\left(k^*, \omega^*\right)$ *and* $l^C = l^C\left(k^*, \omega^*\right)$.

Proof. Proof is straightforward by combining equations (3), (6) and (11). □

That is, the fraction of total labor supply employed in the consumption sector is equal to the labor share in the consumption sector; the fraction of total labor supply employed in the investment sector is equal to the capital share in the consumption sector; the fraction of total capital supply employed in the consumption sector is equal to the labor share in the investment sector; and the fraction of total capital supply employed in the investment sector is equal to the capital share in the investment sector.

Further, it is immediate from first-order conditions (see equations (2) in footnote 8) and from equations (12) to (15) that consumption is equal to the macroeconomic labor share of income and that investment is equal to the macroeconomic capital share of income at the golden rule steady state[7]:

$$c = l^C f\left(k^C\right) = \frac{w}{p^C} \tag{16}$$

$$i = l^I g\left(k^I\right) = \frac{rk}{p^I} \tag{17}$$

And so:

$$(y \equiv)\; p^C c + p^I i = w + rk \tag{18}$$

In extensive form, we can write (18) as:

$$(Y \equiv)\; p^C C + p^I I = wL + rK,$$

which reflects the fact that labor plus capital income equals product at the macroeconomic level.

Consequently, the golden rule property that the saving rate is equal to the macroeconomic relative share of capital[8] also holds:

$$\begin{aligned} s &\equiv \tfrac{p^I i}{p^C c + p^I i} \\ &= \tfrac{rk}{w + rk} \end{aligned} \tag{19}$$

$$s = \frac{k/\omega}{1 + k/\omega}. \tag{20}$$

2 The neutrality of technological shocks in the consumption goods sector

Imagine that the economy is initially on its steady state characterized by (8) if $\theta \neq 0$ or by (11) if $\theta = 0$. What happens when the economy experiences a permanent technological shock in the consumption-sector? If this shock is Hicks-neutral – i.e. if it turns the technology $F\left(K^C, L^C\right)$ into $BF\left(K^C, L^C\right)$ – then basically nothing. It is apparent in (8) and in (11), that quasi-golden rule and pure golden rule steady-state values of ω, k^C, k^I, l^C, l^I, and k are not affected by a Hicks-neutral, consumption-specific technological shock. Therefore, steady-state consumption increases (if the shock is positive) but the distribution of inputs, as well as the capital-labor ratio, remain unaltered. In particular, the economy does not react to consumption-specific technical progress by capital accumulation[9].

Therefore, we have the following neutrality theorem:

[7] This proposition notably makes the model that suppose that wages are consumed and profits are invested not so absurd, even if it is unpleasant to take the shortcut to make arbitriraly this kind of assumption. It is not really clear why this double duality (wages–consumption, capital–investment) prevails, but it is a tenacious conclusion of the golden rule. Srinivasan (1964) shows that this property is no longer true when the social planner has a rate of preference for the present strictly less than unity; in that case, savings are *less* than non-wage income.

[8] See Phelps (1961) and (1965) and Robinson (1962) for the original one-sector formulation, and Srinivasan (1964) on the two-sector version.

[9] As steady-state ω is not altered by a shock on B, w and r increase proportionally.

Theorem 2.1. *Let B denote total factor productivity in the consumption sector and c denote quasi-golden rule (or pure golden rule) steady-state consumption per capita level. Then:*

$$\frac{B}{c}\frac{\partial c}{\partial B} = 1. \tag{21}$$

This important property was missed by the models focusing on economy-wide technical change versus investment-specific technical change, like Greenwood et al. (1997, 2000) or Pakko (2000).

To our knowledge, Kimball (1994) was the first to prove, in a dynamic framework with specific utility functions (i.e. logarithmic and power), the result of the neutrality of consumption-specific technical change[10]. The result presented here supplements Kimball's theorems in a static framework and in this case, the result does not hinge on any specific assumption about preferences. Coupling both approaches naturally yields the following interpretation: consumption-specific productivity only influences the optimal growth path of the economy through the choices of intertemporal substitution of consumption, and not via any 'first-order' effect.

More intriguing is the fact that the *shape* of the production function of the consumption good does not influence the wage-rental ratio[11]. A direct consequence is that capital-augmenting or labor-augmenting technical change can happen in the consumption industry, without any influence on the wage-rental ratio[12].

The relative price of investment and consumption goods $p \equiv Bf'\left(k^C(\omega)\right)/g'\left(k^I(\omega)\right)$ also notably increase one-for-one with B, as shocks on consumption-specific productivity do not influence the steady-state wage-rental ratio.

3 Technological shocks in the investment goods sector

What happens to steady-state consumption after a (permanent) positive technological shock on the *investment* technology? Such a shock makes the maintenance of the existing stock of capital less resource-demanding, and thus expands steady-state production possibilities and steady-state consumption.

So, after an investment-specific technical shock, final consumption increases through capital accumulation in the consumption sector. The quantitative response depends on the planner's subjective discount rate: the more patient the planner is, the more we can expect him to take advantage of investment-specific technical progress. We escape from this difficulty by deriving comparative statics properties only for the pure golden rule steady state. This makes algebra easy and gives insight into the effect of investment-specific technical progress on the production possibilities of the economy.

[10] See also Fisher (1997, footnote 8) and Basu et al. (2010).

[11] Still, the shape of function f matters for equilibrium capital-labor ratio in the consumption sector $\left(k^C\right)$ as can be seen in (11). Consequently, steady-state values of l^I, l^C, and k depend on the exact shape of the consumption production function.

[12] As is apparent in equations (8) and (11), the distribution of inputs across sectors as well as the overall steady-state capital-labor ratio will adjust to a non-Hicks neutral technological shock within the consumption industry.

3.1 Some general properties

When the TFP of the investment sector is $A > 0$, the golden rule steady-state values of ω, k^I, k^C, and k are determined through the following system[13]:

$$\left\{ \begin{array}{rcl} Ag'\left(k^I\right) & = & \delta \\ \frac{f\left(k^C\right)}{f'\left(k^C\right)} - k^C & = & \frac{g\left(k^I\right)}{g'\left(k^I\right)} - k^I = \omega \\ \frac{k^C - k}{k^C - k^I} Ag\left(k^I\right) & = & \delta k \end{array} \right. \tag{22}$$

Notice that $Ag/Ag' = g/g'$, which reflects the invariance of the distribution of income in one particular industry to a Hicks-neutral technological shock. The wage-rental ratio, here, increases because the overall capital-labor ratio increases, and the two sectoral capital intensity ratios increase as well.

Theorem 3.1.

$$\frac{\partial \omega}{\partial A} = \frac{g\left(k^I\right)}{Ag'\left(k^I\right)} = \frac{g\left(k^I\right)}{\delta} > 0 \tag{23}$$

and so the golden rule wage-rental ratio ω is a strictly increasing function of A. Also $\omega \xrightarrow{A \to 0} 0$ and $\omega \xrightarrow{A \to \infty} \infty$.

Proof. The value of the derivative of ω with respect to A is obtained by differentiating the first equation in (22) with respect to A, and using (5):

$$\begin{array}{rl} g'\left(k^I\right) = \frac{\delta}{A} & \Rightarrow \frac{\partial \omega}{\partial A} \frac{\partial k^I}{\partial \omega} g''\left(k^I\right) = -\frac{\delta}{A^2} \\ & \Rightarrow \frac{\partial \omega}{\partial A} = \frac{g\left(k^I\right)}{\delta} \end{array}$$

Limits of $\omega(A)$ directly derive from the Inada conditions on g.

□

The effect of Hicks-neutral, investment-specific technical shock on golden rule steady-state consumption is summarized by the following theorem:

Theorem 3.2. *Let c denote the pure golden rule steady-state consumption per capita level. Then:*

$$\frac{A}{c} \frac{\partial c}{\partial A} = \frac{k}{\omega} \tag{24}$$

where k is the macroeconomic golden rule capital-labor ratio and where ω is the golden rule wage-rental ratio.

Proof. In virtue of (12), we have that $c = l^C f\left(k^C\right) = f\left(k^C\right) - k^C f'\left(k^C\right)$ at golden rule steady state. Therefore:

$$\frac{\partial c}{\partial A} = -k^C \frac{\partial k^C}{\partial \omega} \frac{\partial \omega}{\partial A} f''\left(k^C\right) \tag{25}$$

Differentiation of the first equation in (22) with respect to A yields:

$$-\frac{\partial \omega}{\partial A} \frac{\partial k^I}{\partial \omega} g''\left(k^I\right) = \frac{\delta}{A^2} \tag{26}$$

In view of (4) and (23) and of the golden rule identities (13) and (14), we can successively re-write (25) as:

[13] Since the rest of this section only deals with the golden rule value of all variables, we omit the star subscript.

$$\begin{aligned}\frac{\partial c}{\partial A} &= \frac{1}{A}\left(k^C \frac{f'^2\left(k^C\right)}{f(k^C)}\right)\frac{g\left(k^I\right)}{g'(k^I)}\\ &= \frac{1}{A}\frac{l^I g\left(k^I\right)}{g'(k^I)} f'\left(k^C\right)\\ &= \frac{1}{A} k f'\left(k^C\right)\end{aligned}$$

Finally, remark that, in virtue of (3) and (12):

$$\frac{f'\left(k^C\right)}{l^C f\left(k^C\right)} = \frac{1}{\omega},$$

which gives the final result:

$$\frac{A}{c}\frac{\partial c}{\partial A} = \frac{k}{\omega}.$$

□

From equations (16) and (17), the ratio $\frac{k}{\omega}$ is also equal to the ratio between investment and consumption, when expressed in the same unit:

$$\begin{aligned}\frac{k}{\omega} &= \frac{rK}{wL} = \frac{p^I i}{p^C c} = \frac{p^I I}{p^C C}\\ &= \frac{s}{1-s}.\end{aligned}$$

Intuitively, maybe one would tend to think that, because of decreasing marginal returns to capital in the consumption sector, consumption cannot increase asymptotically solely as result of technical progress in the investment sector, and so the ratio $\frac{k}{\omega}$ should tend to zero as A tends to infinity. Under this scenario, investment-specific technical progress barely makes the existing stock of capital less costly to maintain, freeing resources for the consumption industry, until the consumption sector asymptotically employs the whole of capital and labor inputs. According to this intuition, notably, l^I shall tend to zero as investment-specific TFP goes to infinity. But this intuition is not always true. In principle, k/ω, can either increase, decrease, or remain constant when A tends to infinity, and consequently the ratio does not need to tend to zero.

The evolution of the relative price of investment and consumption goods $p = \frac{f'\left(k^C(\omega)\right)}{Ag'(k^I(\omega))}$ after an investment-specific productivity shock is such that:

Theorem 3.3.

$$\frac{A}{p}\frac{\partial p}{\partial A} = -\frac{k}{k^C} \tag{27}$$

Proof. The denominator in the expression of p is constant and equal to δ at steady state. Then, using (4) and (23):

$$\begin{aligned}\frac{\partial \ln(p)}{\partial A} &= \frac{\partial \ln\left(f'\left(k^C\right)\right)}{\partial A}\\ &= \frac{\partial \omega}{\partial A}\frac{\partial k^C(\omega)}{\partial \omega}\frac{f''\left(k^C\right)}{f'(k^C)}\\ &= -\frac{g(k^I)}{Ag'(k^I)}\frac{f'(k^C)}{f(k^C)}\end{aligned}$$

And so, from golden rule identities (13) and (15):

$$\begin{aligned}\frac{A}{p}\frac{\partial p}{\partial A} &= \frac{1}{l^I}\frac{f'(k^C)}{f(k^C)} - \frac{Al^I g(k^I)}{Ag'(k^I)}\\ &= -\frac{k}{k^C}\end{aligned}$$

□

Thus, the price of investment goods in terms of consumption goods always decreases after an investment-specific technological shock. If the consumption sector is relatively more capital-intensive than the investment sector – that is, if $k^C(\omega) > k^I(\omega)$ for all $\omega > 0$ – then the elasticity of p with respect to A is greater than -1. In the contrary, if it is the investment sector that is capital-intensive, then this elasticity is less than -1 and relative price overshoots the size of the shock.

3.2 Investment-specific technological shocks and the distribution of inputs

How do investment-specific technical shocks affect the steady-state distribution of inputs across sectors? Let's first define the sectoral elasticities of substitution by:

$$\sigma^C(\omega) = -\frac{f'\left(f - k^C f'\right)}{k^C f f''} \tag{28}$$

$$\sigma^I(\omega) = -\frac{g'\left(g - k^I g'\right)}{k^I g g''} \tag{29}$$

We can reasonably suppose that $\sigma^C(\omega)$ and $\sigma^I(\omega)$ are bounded.

Remark that those elasticities are invariant to Hicks-neutral technical shocks. Further, it is immediate by combining (3), (4) and (5) with (28) and (29) that:

$$\frac{\partial k^C}{\partial \omega} = \sigma^C \frac{k^C}{\omega} \tag{30}$$

$$\frac{\partial k^I}{\partial \omega} = \sigma^I \frac{k^I}{\omega} \tag{31}$$

So, consequently:

$$\frac{\partial \ln \frac{k^C}{\omega}}{\partial \omega} = \frac{\sigma^C - 1}{\omega} \tag{32}$$

$$\frac{\partial \ln \frac{k^I}{\omega}}{\partial \omega} = \frac{\sigma^I - 1}{\omega} \tag{33}$$

An increase in ω will increase (resp. decrease) the relative share of capital in industry X if the elasticity of substitution between capital and labor in industry X is greater (resp. less) than unity.

By combining (3) and the golden rule identities from (12) to (15), it is clear that the following identities hold at steady state:

$$\frac{l^I}{l^C} = \frac{k^C}{\omega} \tag{34}$$

$$\frac{l^I k^I}{l^C k^C} = \frac{k^I}{\omega} \tag{35}$$

Consequently we can state the following proposition:

Proposition 3.1. *After a positive Hicks-neutral investment-specific technical shock:*

- *labor is transferred from the investment industry to the consumption industry if* $\sigma^C < 1$*;*

- *labor is transferred from the consumption industry to the investment industry if* $\sigma^C > 1$*;*

- *capital is transferred from the investment industry to the consumption industry if* $\sigma^I < 1$*;*

- *capital is transferred from the consumption industry to the investment industry if* $\sigma^I > 1$.

4 The sustainability of growth regimes led by investment-specific technical change

As noted previously, investment-specific technological progress makes final consumption grow solely as a result of capital accumulation within the consumption sector. Intuition suggests that this pattern must stop somewhere: if the Inada conditions are fulfilled, doesn't marginal productivity of capital in the consumption sector fall to zero, and so marginal benefits from investment-specific productivity growth?

We already know from the existing literature that this intuition is wrong. Meade (1962) and Whelan (2006) have shown that, when both production functions F and G are of Cobb-Douglas type, consumption eventually *does* increase asymptotically with only investment-specific technical change. In this case, real consumption and investment grow at constant, however different, rates; and nominal consumption and investment grow at the same constant rate[14].

4.1 The macroeconomic elasticity of substitution

The results presented here suggest that whether investment-specific productivity growth will bring less and less consumption benefits depend on the evolution of the k/ω ratio, that is on the ratio of capital to labor share at the macroeconomic level. When investment-specific productivity growth makes this ratio *decrease* – that is, when the macroeconomic share of capital decrease relatively to the macroeconomic share of labor – consumption growth will continuously slow down. When this

[14] In the appendix, we present a golden rule adjusted for *steady* investment-specific technical change in the case the production functions are both of Cobb-Douglas type, through which we derive the same growth rates the ones found by Meade and Whelan.

ratio *increases*, consumption growth will tend to accelerate[15].

Like shown in (30) and (31), the evolution of the sectoral ratios k^C/ω and k^I/ω depend on the sectoral elasticities of substitution between labor and capital. The ratio k/ω is a weighted average of those two quantities.

Whether the ratio k/ω goes up or down as the wage-rental ratio increases depends on the economy-wide elasticity of substitution σ, which governs the distribution of value added between labor and capital at the macroeconomic level. We define $\sigma(\omega)$ by:

$$\frac{\partial k}{\partial \omega} = \sigma \frac{k}{\omega}. \tag{36}$$

Around the golden rule steady state, we can link σ to the value of the sectoral elasticities of substitution.

Proposition 4.1.

$$\sigma = l^C(\sigma^C - 1) + \frac{l^I k^I}{k}(\sigma^I - 1) + 1 \tag{37}$$

Proof. From the golden rule identities (13) and (15) we can write the golden rule value of k as:

$$k = \frac{k^C f'(k^C)}{f(k^C)} \frac{g(k^I)}{g'(k^I)} \tag{38}$$

Log-differentiating this expression with respect to ω and using (4), (5), (6), (13), (15), (30), (31), (34) and (35) yields:

$$\begin{aligned}
\frac{\partial \ln k}{\partial \omega} &= \frac{1}{k^C}\frac{\partial k^C}{\partial \omega} + \frac{\partial k^I}{\partial \omega}\frac{g'(k^I)}{g(k^I)} - \frac{\partial k^I}{\partial \omega}\frac{g''(k^I)}{g'(k^I)} - \frac{\partial k^C}{\partial \omega}\frac{f'(k^C)}{f(k^C)} + \frac{\partial k^C}{\partial \omega}\frac{f''(k^C)}{f'(k^C)} \\
&= \frac{\sigma^C}{\omega} + \sigma^I \frac{k^I}{\omega}\frac{g'(k^I)}{g(k^I)} + \frac{g'(k^I)}{g(k^I)} - \sigma^C \frac{k^C}{\omega}\frac{f'(k^C)}{f(k^C)} - \frac{f'(k^C)}{f(k^C)} \\
&= \frac{\sigma^C}{\omega} + \frac{\sigma^I}{\omega}\frac{l^I k^I}{k} + \frac{l^I}{k} - \frac{\sigma^C}{\omega} l^I - \frac{l^I}{k^C} \\
&= \frac{\sigma^C}{\omega} + \frac{\sigma^I}{\omega}\frac{l^I k^I}{k} + \frac{1}{\omega}\frac{l^C k^C}{k} - \frac{\sigma^C}{\omega} l^I - \frac{l^C}{\omega} \\
&= \frac{1}{\omega}\left(l^C(\sigma^C - 1) + \frac{l^I k^I}{k}(\sigma^I - 1) + 1\right).
\end{aligned}$$

□

Remark that, if σ^C and σ^I are both greater (resp. less) than 1, then σ is greater (resp. less) than 1 as well.

If $\sigma > 1$ then k/ω increases as A increases, and consecutive investment-specific productivity shocks bring increasing consumption rewards. If $\sigma < 1$ then k/ω decreases and consumption growth diminishes.

[15] Notice that this property has something of a paradox, as consumption at steady state is exactly equal to the macroeconomic share of labor.

4.2 'Long-term' consequences of investment-specific technical change

When both production functions satisfy Inada conditions

It is often believed that the long-run elasticity of substitution is equal to one[16]. Formally, Barelli and Pessôa (2003) and Litina and Palivos (2008) have shown that, for all production function that fulfills the Inada conditions, the elasticity of substitution must be equal to one in the long run. As an application of this property, we can state the following theorem:

Theorem 4.1. (Unit long-run elasticity of substitution in both sectors) *If functions F and G satisfy the Inada conditions, then:*

$$\sigma^C(\omega) \xrightarrow{\omega\infty} 1$$

$$\sigma^I(\omega) \xrightarrow{\omega\infty} 1$$

Because l^C and $l^I k^I/k$ are bounded, σ also tends to 1 as A tends to infinity. So, if both production functions satisfy the Inada conditions, then:

$$\frac{\omega}{k}\frac{\partial k}{\partial \omega} \xrightarrow{A\infty} 1,$$

which means that, asymptotically, ω and k increase proportionally after an investment-specific shock, so that k/ω tends to some constant, like in the Cobb-Douglas case, and consumption grows asymptotically with investment-specific technical change[17].

When the production function of the consumption does not satisfy Inada conditions

We have supposed throughout the paper that both production functions verified Inada conditions, but the existence of the golden rule only needs Inada condition to hold in the investment sector. In virtue of the Barelli-Pessôa-Litina-Palivos theorem, this implies that the elasticity of substitution of production function G must be asymptotically one[18].

Let's drop the Inada conditions for the consumption sector.

First, suppose that σ^C is less than unity, in the precise sense that there exists some $\varepsilon > 0$ such that, for all ω, $\sigma^C < 1 - \varepsilon$. In this case, function $f(k^C)$ tends to a constant (and no more to infinity) as k^C tends to infinity[19], and so investment-specific technical change alone can never increase consumption asymptotically.

It is immediate from (30) that k^C/ω tends to 0 as investment-specific productivity A tends to infinity. That is, the relative share of capital *in the consumption sector* eventually vanishes. But from (34), we see that $l^I \to 0$ and $l^C \to 1$, and so the macroeconomic elasticity of substitution in (37) is such that:

$$\sigma(\omega) - \sigma^C(\omega) \xrightarrow{\omega\infty} 0$$

In this regime, consumption does not increase asymptotically as investment-specific productivity tends to infinity. But the economy takes advantage of investment-specific TFP growth by

[16] See Jones(2003).
[17] See appendix.
[18] Litina and Palivos (2008) show that this does *not* imply that function g must be asymptotically equivalent to some Cobb-Douglas function.
[19] See de La Grandville (2009, chapter 4).

freeing up labor resources from the investment to sector to allocate them to the consumption sector[20]. But as the macroeconomic ratio k/ω tends to zero, the relative share of capital tends to nil. When capital and labor are complements in the consumption sector, productivity increases within the investment sector ultimately impoverish those who own capital. In this regime, investment-specific technical change does not contribute to growth asymptotically but is a powerful engine of equality.

If σ^C is greater than unity, then k/ω tends to infinity, and so the macroeconomic relative labor share vanishes as A tends to infinity. Also $l^I \to 1$ and $l^C \to 0$, and so the asymptotic macroeconomic elasticity of substitution tends to 1. In this case, despite the fact that the elasticity of substitution is always greater than one, the observed macroeconomic elasticity of substitution tends to one exactly.

5 Conclusion

This article put in regards the consequences of consumption- and investment-specific technical change in a neoclassical, optimal growth framework.

Consumption-specific and investment-specific technical change are largely orthogonal. Consumption-specific shocks barely affect consumption, and virtually anything else, while the dynamic response of the economy to investment-specific shocks depends on the sectoral elasticities of substitution between labor and capital inputs. Different regimes exist according to whether elasticities are greater or less than one. Unfortunately, to our knowledge there does not exist any estimation of the sectoral elasticities of substitution[21].

What are the real-world insights that one could gain from this results with regards to modern growth phenomenon? The results we have presented yield some predictions about the links between technology and input and output prices and quantities, which could lead to some interesting empirical view on current and past growth regimes. In the perspective given in the paper, one can speculate that China has grown so fastly for several decades because productivity growth was faster in the investment industry than in the consumption industry – and it seems that it has, at the global level – while concomitantly the Chinese economy has continuously showed a high ratio of domestic investment to domestic consumption, thus accentuating the effect of investment-biased technical change on product and income relatively to what occurred in other countries.

The first-best results presented may also serve as a benchmark for further theoretical research. In particular, when it comes to the deep nature of investment in the growth process[22], it would constitute an interesting extension to analyze the dynamical consequences of investment-specific technical progress when some friction distorts the investment decisions.

[20]Remark that, from (13) and (15), the allocation of capital input across sectors stays exactly the same. The investment sector is, then, ultimately more capital-intensive than the consumption sector.

[21]Basu et al. (2010) provide estimates of the capital intensities of the consumption sector and of the investment sector in the USA for the period from 1961 to 2005. De La Grandville (2009, chapter 5), using a one-sector framework, shows that the elasticity of substitution has tended to during the last decades in most industrialized countries.

[22]See the discussion in the present introduction.

Appendix: steady growth and the golden rule in the Cobb-Douglas case

In this appendix we build on the description of the steady growth path by Meade (1962) and Whelan (2006)

We suppose that the production technologies are now:

$$c = Bl^C \left(k^C\right)^b \tag{39}$$

$$i = Al^I \left(k^I\right)^a \tag{40}$$

with $0 < a < 1$ and $0 < b < 1$. The effect of investment-specific technical progress on golden rule steady-state consumption is now characterized through a very simple equation.

Proposition 5.1. *At the golden rule steady state, with Cobb-Douglas production functions, one has that:*

$$\left(\frac{k}{\omega} =\right) \frac{A}{c}\frac{\partial c}{\partial A} = \frac{b}{1-a}. \tag{41}$$

Proof. Efficiency equation (3) is now an equation of proportionality:

$$\omega = \frac{1-a}{a} k^I \tag{42}$$

From equation (13), we have that $l^I = b$ at the golden rule, and so equation (14) with Cobb-Douglas production functions yields:

$$k = l^I \frac{\left(k^I\right)^a}{a\left(k^I\right)^{a-1}} = \frac{b}{a}k^I = \frac{b}{1-a}\omega.$$

□

Therefore, when production functions are of Cobb-Douglas type the elasticity of final consumption with regards to investment-specific TFP is constant. Unsurprisingly, this figure is the higher when the production functions F and G are more capital-intensive.

It is beyond the scope of this paper to give a complete characterization of the optimal path of allocation of resources for any initial conditions. Instead, we show a much more pleasant result that supports previous conclusions: if technologies are of Cobb-Douglas type in equations (39) and (40), then there exists a rule of thumb for the allocation of resources that, for *some* initial capital-labor ratio k_0, makes consumption grow at the gross rate of:

$$1 + z_c = (1 + z_B)(1 + z_A)^{\frac{b}{1-a}}, \tag{43}$$

where $1 + z_B = \frac{B_{t+1}}{B_t}$ and $1 + z_A = \frac{A_{t+1}}{A_t}$. This rule of thumb we characterize through the golden rule identities from (12) to (15). Suppose that the social planner allocates resources according to the following rule, at all dates[23]:

$$\begin{cases} l^C &= 1-b \\ l^I &= b \\ \frac{l^C k^C}{k} &= 1-a \\ \frac{l^I k^I}{k} &= a \end{cases} \tag{44}$$

[23] Notice that this allocation is always feasible, as it obviously verifies (6).

which corresponds to sectoral capital intensities:

$$\left\{ \begin{array}{ll} k^C & = \frac{1-a}{1-b} k \\ k^I & = \frac{a}{b} k \end{array} \right. \tag{45}$$

The accumulation equation, when inputs are allocated according to this rule, becomes:

$$k_{t+1} = (1-\delta)k_t + bA_0(1+z_A)^t \left(\frac{a}{b}k_t\right)^a \tag{46}$$

Let's first prove the following lemma:

Lemma 5.1. *If the initial capital-labor ratio satisfies the following adjusted golden rule:*

$$A_0 g'\left(\frac{a}{b}k_0\right) = \delta + (1+z_A)^{\frac{1}{1-a}} - 1 \tag{47}$$

and if the allocation of resources follows (44) for all dates, a similar adjusted golden rule is verified at all dates:

$$\forall t, \quad A_t g'\left(\frac{a}{b}k_t\right) = \delta + (1+z_A)^{\frac{1}{1-a}} - 1 \tag{48}$$

Proof. If (48) is satisfied for some $t \geq 0$:

$$A_0(1+z_A)^t a \left(\frac{a}{b}k_t\right)^{a-1} = \delta + (1+z_A)^{\frac{1}{1-a}} - 1.$$

Then, from (46):

$$\begin{array}{ll} A_{t+1}g'\left(\frac{a}{b}k_{t+1}\right) & = A_0(1+z_A)^{t+1} a \left(\frac{a}{b}\right)^{a-1} \left\{(1-\delta)k_t + bA_0(1+z_A)^t \left(\frac{a}{b}k_t\right)^a\right\}^{a-1} \\ & = A_0(1+z_A)^{t+1} a \left(\frac{a}{b}\right)^{a-1} \left\{(1-\delta)k_t + A_0(1+z_A)^t a \left(\frac{a}{b}k_t\right)^{a-1} k_t\right\}^{a-1} \\ & = A_0(1+z_A)^{t+1} a \left(\frac{a}{b}\right)^{a-1} \left\{\left(1-\delta+\delta+(1+z_A)^{\frac{1}{1-a}}-1\right) k_t\right\}^{a-1} \\ & = A_0(1+z_A)^t a \left(\frac{a}{b}\right)^{a-1} (k_t)^{a-1} \\ & = \delta + (1+z_A)^{\frac{1}{1-a}} - 1. \end{array}$$

Therefore, if the property holds for $t = 0$, then it holds for all $t \geq 0$.

□

We now prove the following existence theorem:

Theorem 5.1. *If the production functions of the two sectors are of Cobb-Douglas type like in (39) and (40) and if technical progress in each sector is Hicks-neutral and increases at steady rate, with* $\frac{B_{t+1}}{B_t} = 1 + z_B$ *and* $\frac{A_{t+1}}{A_t} = 1 + z_A$ *for all* $t \geq 0$*, then there exists a growth path where investment and the capital stock grow at the same constant gross rate of* $(1+z_A)^{\frac{1}{1-a}}$*, and where consumption grows at the constant gross rate of* $(1+z_B)(1+z_A)^{\frac{b}{1-a}}$.

Proof. If initial capital-labor ratio is at the level given by equation (47), and if the golden rule of thumb of allocation of resources is followed, (48) holds and so equation (46) can be reduced to:

$$\begin{array}{ll} k_{t+1} & = (1-\delta)k_t + aA_0(1+z_A)^t \left(\frac{a}{b}k_t\right)^{a-1} k_t \\ & = \left(1-\delta+\delta+(1+z_A)^{\frac{1}{1-a}}-1\right) k_t \\ & = (1+z_A)^{\frac{1}{1-a}} k_t. \end{array}$$

Consequently, for all t:

$$k_t = k_0 \left((1+z_A)^{\frac{1}{1-a}} \right)^t . \tag{49}$$

Investment per capita, in this case, reduces to:

$$\begin{aligned} i_t &= A_t l_t^I \left(\tfrac{a}{b} k_t\right)^a \\ &= A_0 (1+z_A)^t \left(\tfrac{a}{b} k_0 \left((1+z_A)^{\frac{1}{1-a}} \right)^t \right)^a \\ &= i_0 \left((1+z_A)^{\frac{1}{1-a}} \right)^t . \end{aligned}$$

From (39), (44), (45) and (49), we can finally write that:

$$\begin{aligned} c_t &= (1-b) B_0 (1+z_B)^t \left\{ \tfrac{1-a}{1-b} k_0 \left(\left((1+z_A)^{\frac{1}{1-a}} \right)^t \right) \right\}^b \\ &= \left\{ (1-b) B_0 \left(\tfrac{1-a}{1-b} k_0 \right)^b \right\} \left\{ (1+z_B)(1+z_A)^{\frac{b}{1-a}} \right\}^t \\ &= c_0 (1+z_c)^t . \end{aligned}$$

□

Paradoxically, the growth rate of real consumption and investment, but also of the stock of capital, do not depend on the depreciation rate δ.

Along this 'pseudo-balanced' path, with only investment-specific technical progress ($z_A > 0$ and $z_B = 0$), the ratio of real consumption to real investment and the ratio of real consumption to real capital-stock both tend to zero when as time goes on. With both types of technical progress, consumption and investment do not generally grow at the same rate. Indeed, the only balanced path eventually arises when:

$$(1+z_A)^{1-b} = (1+z_B)^{1-a} . \tag{50}$$

Perhaps surprisingly, if we let p denote the relative price of the investment good in terms of the consumption good, the quantities pi and pk grow at the same rate than c. Equality of technical marginal rates of substitution across sectors implies that the relative price can be written as[24]:

$$p = \frac{B f'\left(k^C\right)}{A g'\left(k^I\right)} . \tag{51}$$

Along the growth path of theorem 5.1, the gross rate of growth of p is constant and equal to:

$$1 + z_p = (1+z_B)(1+z_A)^{-\frac{1-b}{1-a}} , \tag{52}$$

which can be more or less than unity, and is exactly one if and only if technical progress in both sectors satisfy condition (50)[25]. One important conclusion is that the relative price of the investment good may rise without technical progress in the investment industry being steadier, and conversely.

[24]See (2).

[25]See equation (3.3).

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