The Zero Lower Bound and the Dual Mandate*

William T. Gavin†
Federal Reserve Bank of St. Louis

Benjamin D. Keen‡
University of Oklahoma

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Abstract

This article uses a DSGE framework to evaluate the role of monetary policy in determining the likelihood of encountering the zero lower bound. We find that the probability of experiencing episodes of being at zero lower bound depends almost exclusively on the monetary policy rule. A policy rule, such as the one proposed by Taylor (1993) which is based on the dual mandate is highly likely to lead to episodes of zero short-term interest rates if the central bank is not committed to its inflation target. Our results on nominal interest rate and inflation dynamics do not depend on the particular mechanism that makes monetary policy have real effects. The key and necessary assumption is that expectations are forward looking. The bottom line in models in which monetary policy can influence the real economy is that a central bank must be committed to a long-run average-inflation objective if it wishes to achieve a dual mandate while avoiding the zero lower bound.

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†William T. Gavin, Vice President and Economist, Research Department, Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63166; Tel: (314) 444-8578; E-mail: gavin@stls.frb.org.

‡Benjamin D. Keen, Associate Professor, Department of Economics, University of Oklahoma, 729 Elm Ave., 329 Hester Hall, Norman, OK 73019; Tel: (405) 325-5900; E-mail: ben.keen@ou.edu.

1 Introduction

In over 300 years of central banking history, there are few extended episodes with the market interest rate at zero. Two well-known examples are in the United States during the Great Depression in the 1930s and in Japan since 1995. Japan appears to have settled into a regime with steady state inflation rate near or slightly below zero, the money market rate effectively at zero, and long-term government bonds around two percent. We investigate the factors that determine the probability of having extended periods of a zero money market interest rate when both the inflation target and the steady state real interest rate are positive. Except when explicitly testing for the sensitivity of our results to the level of the inflation target, we assume that the target is two percent.

Why is hitting the zero lower bound (ZLB) a bad thing. According to many economic models this may be the optimal monetary policy because it promotes an optimal level of real cash balances. But that is not the case examined in this paper. Friedman (1969) argued that the optimum quantity of money would be associated with a zero nominal interest rate. Kocherlakota (2009) argues that this result is common in most monetary models. The optimal inflation rate in such models is the negative of the steady state risk free real interest rate. Japan comes close to meeting this condition.

Williams (2009) argues that central banks should 'embrace' the zero lower bound as it shows that monetary policymakers are doing all they can to stimulate an under-employed economy. He suggests that the failure to hit the zero lower bound in the past was a sign of suboptimal policy response. Even if one believes that an aggressive interest rate policy is necessary to stabilize the economy, it is not clear that a good policy rule would result in regular episodes at the zero lower bound. There is little or no consideration that being at the zero lower bound for an extended period may have harmful effects on the structure and performance of the economy.

Benhabib, Schmidt-Grohe, and Uribe (2001) show that when the central bank is using a Taylor-type policy rule, hitting the zero lower bound may be a sign that the economy has moved to an undesirable low output, low inflation equilibrium. When there is uncertainty about the central bank's inflation objective, low interest rates may be interpreted as a sign of a low inflation target. Attempts to stimulate the economy by promising to keep the interest rate at zero my backfire as inflation expectations fall rather than rise.[1]

The most common objection to being at the zero lower bound is that it constrains the Fed's ability to achieve the mandate for full employment. Reifschneider and Williams (2000), Chung et al. (2011), Adam and Billi (2006, 2007), and Coibion, Gorodnichenko, and Vieland (2010) discuss the optimal policy when zero lower bound events are possible and provide analysis of the welfare losses during ZLB events. Chung et al. (2011) argue that the literature understates the probability of hitting

[1]See Bullard (2010) for problems with the ZLB associated with inflation expectations.

the zero lower bound because past analyses have not taken proper account of model uncertainty, including uncertainty about the shock processes hitting the economy. They attribute the ZLB event to factors exogenous to the model. In this paper, we show that the systematic aspects of monetary policy are the prime determinants of ZLB events. A key premise of this paper is that the Federal Reserve policy has evolved into a regime in which zero lower bound events are likely to occur.

Before turning to a model based analysis, it is useful to review some U.S. history. Figure 1 shows a history of two interest rates: the yield on 10-year U.S. Treasury bonds and the overnight interest rate on bank reserves—the federal funds rate. We can divide the period into three subperiods: The period before Paul Volcker became Fed Chairman (in August 1978) when the Fed lost control of inflation and inflation expectations; the period in which Volcker ended the acceleration of inflation and began a gradual disinflation, and the most recent period after Volcker was replaced by Alan Greensapn.[2] The first period was characterized by a steady upward march in the 10-year yield; it was between 2 and $2\frac{1}{2}$ percent in 1955 and rose in fits and starts to over 10 percent in 1979. In this early period, the federal funds rate was often above the 10-year rate. The Fed was regularly 'fighting inflation.' As inflation became a problem the Fed would have to raise rates very high to discourage borrowing and spending. Aggregate demand and inflation would fall for a time, but the inflation trend kept rising. There were two periods before October 1979 in which the Fed kept the federal funds rate relatively low as the expansion proceeded, in 1971 to 1973 and again in 1976 to 1978. In both of these episodes the Fed kept the rate low in an attempt to speed up the recovery. In both instances, the economy did not grow faster, but inflation did. The lesson learned then was that the Fed could not conduct output stabilization policy using low interest rates because doing so would quickly lead to accelerating inflation. As we will see, that lesson was not quite correct.

Between October 1979 and October 1982, the Federal Reserve implemented monetary policy by focusing open market operations on a short-run target for bank reserves. This led to high and volatile interest rates, but it also caused people to change their views about future monetary policy and the inflation objective. From 1982, interest rates and inflation followed a fluctuating but downward trend. In contrast to the earlier period, the federal funds rarely traded at a rate higher than the 10-year bond yield. In the period from 1992 to 1994, the Fed held the federal funds rate well below the 10-year bond yield well into the recovery from the 1991 recession. But there was no subsequent acceleration in inflation. Again, following the 2001 recession, the Fed held the overnight rate well below the 10-year yield and there was little acceleration in inflation or inflation expectations. Both episodes were characterized as "jobless recoveries" and the federal funds rate was held down in an attempt to stimulate faster real growth. During this period, speeches by Federal Reserve officials indicated that, although the Federal Reserve did not have an explicit inflation target,

[2]See Lindsey, Orphanides and Rasche (2005) for a detailed analysis of the policy reform that was implemented in October 1979.

they wanted inflation to be low and would do whatever was necessary to prevent a 1970s rerun of high and rising inflation. Interest rates remained relatively low, and inflation appeared to remain under control.

The Federal Reserve lowered the interest rate so much in 2002 and 2003 to stimulate higher inflation in an apparently successful attempt to avoid hitting the zero lower bound. Why did the federal funds rate go to zero in 2008:Q4? The obvious answer is that the Federal Reserve injected several hundred billion dollars of excess reserves into the banking system which had been operating with less than $10 billion. This zero lower bound event was caused by the central bank assistance to large financial institutions. One lesson is that ZLB events will follow the rescue of large financial firms. Another lesson (revised from the 1970s) is that the inflation response to low interest rates depends on the credibility of monetary policy.

The change in the behavior of these two interest rates between the first and third periods is at least indirect evidence that the Fed has some influence over real interest rates. The data suggest the Fed has the ability to change the nature of the term structure of interest rates over cyclical frequencies without changing the long run expected inflation rate. During the past 15 years, the economics profession has come to characterize monetary policy using Taylor-type rules. The purpose of this paper is to characterize the regime that has evolved in the United States, to show why this regime is likely to produce ZLB events and to show how policy can be modified to achieve the Fed's dual mandate without hitting the ZLB on a regular basis. In Section 2, we briefly describe the model framework that we use. We begin with a typical New Keynesian model with nominal rigidities. Here, the monetary transmission mechanism works through nominal rigidities. In Section 3, we describe the computational experiments in which we calculate the probability of ZLB events occurring under alternative policy regimes. We also present sensitivity analysis to parameter values and the shock processes.

2 The Model

The model is a typical New Keynesian specification with sticky prices. Households purchase consumption and investment goods from the firms. Firms, on the other hand, employ labor and rent capital from the households. Monetary policy is conducted through lump-sum monetary transfers which are determined by the monetary authority's nominal interest rate rule.

2.1 Households

Households are infinitely-lived agents who prefer consumption, c_t, and real money balances, m_t, but dislike work, n_t. Those preferences are represented by the following

expected utility function:

$$U = E_t \left[\sum_{j=0}^{\infty} \beta^j a_{t+j} \left(\ln(c_{t+j}) - \phi_n \frac{n_{t+j}^{1+\zeta} - 1}{1+\zeta} + \phi_m \ln\left(m_{t+j}\right) \right) \right], \tag{1}$$

where E_t is the expectations operator at time t, β is the discount factor which is between 0 and 1, $1/\zeta$ is the labor supply elasticity, $\phi_n > 0$, and $\phi_m > 0$. The preference parameter a_t is an aggregate demand shock which follows an autoregressive process of order one:

$$\ln(a_t) = \rho_a \ln(a_t) + \sigma_a \varepsilon_{a,t},$$

where $1 > \rho_a \geq 0$, $\sigma_a > 0$, and $\varepsilon_{a,t} \sim N(0,1)$.

Each period, households purchase consumption and investment goods, i_t, and acquire real bonds, b_t, and real money holdings, m_t. Those outlays are funded by the real value of bond payments from last period, $R_{t-1}b_{t-1}/\pi_t$, the real value of last period's real money balances, m_{t-1}/π_t, labor income, $w_t n_t$, capital rental income, $q_t k_t$, real dividends from the firms, d_t, and real lump-sum transfers from the monetary authority, t_t, where R_{t-1} is the nominal interest rate from period $t-1$ to t, π_t is the inflation rate, w_t is the real wage, q_t is the real capital rental rate, and k_t is the capital stock. The following budget constraint describes the households' flow of funds:

$$c_t + i_t + b_t + m_t = R_{t-1}b_{t-1}/\pi_t + m_{t-1}/\pi_t + w_t n_t + q_t k_t + d_t + t_t. \tag{2}$$

Households own the capital which they rent to the firms. The capital accumulation equation is

$$k_{t+1} = \left[1 - S\left(\frac{i_t}{i_{t-1}} \right) \right] i_t + (1-\delta) k_t, \tag{3}$$

where $S(\cdot)$ is the functional form for the investment adjustment costs and δ is the depreciation rate. The investment adjustment costs, $S(i_t/i_{t-1})i_t$, denote the resources lost in the conversion of investment to capital which depend on how much the level of investment adjusts. Formally, the households select values for c_t, i_t, k_{t+1}, n_t, m_t, and b_t that maximize its utility, (1), subject to its budget constraint, (2), and capital accumulation equation, (3).

2.2 Firms

Firms are monopolistically competitive producers of differentiated goods. Specifically, firm f produces its differentiated product, $y_{f,t}$, by combining its firm-specific labor, $n_{f,t}$, and capital, $k_{f,t}$, inputs with the aggregate level of technology, z_t, such that

$$y_{f,t} = z_t (k_{f,t})^{\alpha} (n_{f,t})^{1-\alpha}, \tag{4}$$

where $1 > \alpha > 0$. The technology parameter, z_t, follows an autoregressive process:

$$\ln(z_t/z) = \rho_z \ln(z_t/z) + \sigma_z \varepsilon_{z,t},$$

where z is the steady-state value of z_t, $1 > \rho_z \geq 0$, $\sigma_z > 0$, and $\varepsilon_{z,t} \sim N(0,1)$. Firm f's labor and capital inputs are rented from perfectly competitive markets at the prevailing real wage, w_t, and capital rental rate, q_t, respectively. Given those input prices, firm f seeks to minimize its production costs:

$$w_t n_{f,t} + q_t k_{f,t}, \tag{5}$$

subject to (4).

The differentiated output of each firm is then aggregated using the Dixit and Stiglitz (1977) method to calculate total output, y_t:

$$y_t = \left[\int_0^1 {y_{f,t}}^{(\epsilon_t - 1)/\epsilon_t} df \right]^{\epsilon_t/(\epsilon_t - 1)}. \tag{6}$$

The price elasticity of demand for a differentiated good, $-\epsilon_t$, follows an autoregressive process:

$$\ln(\epsilon_t/\epsilon) = \rho_\epsilon \ln(\epsilon_t/\epsilon) + \sigma_\epsilon \varepsilon_{\epsilon,t}, \tag{7}$$

where ϵ is the steady-state value of ϵ_t, $1 > \rho_\epsilon \geq 0$, $\sigma_\epsilon > 0$, and $\varepsilon_{\epsilon,t} \sim N(0,1)$. Each differentiated good, $y_{f,t}$, sells at a price $P_{f,t}$. Cost minimization on the part of households implies that the demand schedule for $y_{f,t}$ is a decreasing function of its relative price:

$$y_{f,t} = \left(\frac{P_{f,t}}{P_t} \right)^{-\epsilon_t} y_t, \tag{8}$$

where P_t is a nonlinear price aggregate index of a continuum of differentiated goods:

$$P_t = \left[\int_0^1 {P_{f,t}}^{1-\epsilon_t} df \right]^{1/(1-\epsilon_t)}.$$

The price-setting behavior of firm f is based on Calvo (1983). In each period, the probability that firm f can select a new price, P_t^*, is $(1-\eta)$, while the probability that it can raise its price only by the steady-state inflation rate, π, is η. When a price-setting opportunity exists, firm f selects a price, P_t^*, which maximizes the present value of expected future profits to the households given the probability of future adjustment opportunities:

$$\max_{P_t^*} E_t \left[\sum_{j=0}^{\infty} \beta^j \eta^j \lambda_{t+j} [(\pi^j P_t^* / P_{t+j}) y_{f,t+j} - w_{t+j} n_{f,t+j} - q_{t+j} k_{f,t+j}] \right], \tag{9}$$

subject to the firm's demand schedule, (8), and the input factor demands from the firm's cost minimization problem, (5). The value of $\beta^j \lambda_{t+j}$ characterizes the value of profits to households j periods in the future, whereas η^j represents the probability that another price-setting opportunity will not take place in the next j periods.

When the first-order condition for P_t^* from (9) is linearized around its steady state, the following New Keynesian Phillips curve is obtained:

$$\ln(\pi_t/\pi) = [(1-\eta)(1-\beta\eta)/\eta]\ln(\psi_t/\psi) + \beta E[\ln(\pi_{t+1}/\pi)] + \ln(e_t/e), \tag{10}$$

where ψ_t is the real marginal cost of producing an additional unit of output and e_t, which resembles a cost-push shock, is a transformation of the price elasticity parameter, ϵ_t. That is, $e_t = [(1-\eta)(1-\beta\eta)/(\eta(\epsilon-1))]\epsilon_t$, where the parameters from the shock process in (7) are re-specified as follows: $\rho_e = \rho_\epsilon$, $\sigma_e = [(1-\eta)(1-\beta\eta)/(\eta(\epsilon-1))]\sigma_\epsilon$, and $\varepsilon_{e,t} = \varepsilon_{\epsilon,t}$.

2.3 Monetary Authority

The monetary authority utilizes a generalized Taylor (1993) style nominal interest rate rule. Specifically, the monetary authority adjusts the nominal interest rate in response to percentage deviations from the steady state for the inflation rate, π_t, the growth rate of output, dy_t, output, y_t, and the price level, p_t, such that

$$\ln(R_t/R) = \theta_\pi \ln(\pi_t/\pi) + \theta_{dy}\ln(dy_t/dy) + \theta_y \ln(y_t/y) + \theta_p \ln(p_t/p) + \nu_{R,t},$$

where the variables without time subscripts are steady-state values and the policy parameters, $\theta_\pi, \theta_{dy}, \theta_y, \theta_p$ are all assumed to be greater than or equal to zero. Finally, the monetary policy shock, $\nu_{R,t}$, follows an autoregressive process of order one:

$$\nu_{R,t} = \rho_R \nu_{R,t-1} + \sigma_R \varepsilon_{R,t},$$

where $1 > \rho_R \geq 0$, $\sigma_R > 0$, and $\varepsilon_{R,t} \sim N(0,1)$.

3 Equilibrium and Estimation

The first-order conditions, identity equations, and exogenous shocks form a system of difference equations. Since all of the variables are stationary, the model converges to a steady-state equilibrium in the absence of exogenous shocks.[3] The system of equations is linearized around that nonstochastic steady state and then standard techniques are utilized to obtain its rational expectations solution.[4] By transforming the rational expectations solution into a state-space framework, the Kalman filter can calculate the optimal linear projection of the observed variables which is used to obtain the sample likelihood function. Since the rational expectations solution is a function of the model's parameters, we estimate key parameters by maximizing the model's likelihood function with respect to the estimated parameters.

[3] The nonstationary variables P_t and P_t^* are eliminated from the model when first-order conditions from the firms' price-setting problem is transformed into the New Keynesian Phillips curve, (10).

[4] Our model is solved and estimated using the techniques embedded in the Dynare software.

Our model is estimated using quarterly U.S. data on output, inflation, the nominal interest rate, and the capital rental rate over the sample period 1983:Q1-2007:Q4.[5] Output is expressed as real gross domestic product in chained 2005 dollars divided by the civilian, noninstitutionalized population, age 16 and over. The inflation rate is the percent change in the GDP implicit price deflator, while the nominal interest rate is the effective federal funds rate. The capital rental rate is the annualized 3-month rate of return on capital constructed in Gomme, Ravikumar, and Rupert (2011) from the National Income and Product Accounts data. Since the model assumes that all variables move around their steady states, we eliminate the long-run upward trend in output by passing the output data through the Hodrick-Prescott filter.

Some of the model's parameters are set prior to estimation because either the data contains little information about them or the parameter is not central to our analysis. Since our focus is on determining the frequency and duration of episodes where the nominal interest rate hits the zero lower bound, we restrict our estimation to the parameters in the monetary policy rule and those parameters related to the exogenous sources of variation in the model. We begin by setting the steady-state gross inflation rate, π, to 1.0064, which is equal to the average 2.56 percent annual inflation rate observed in the sample period data. Since $\beta = \pi/R$ in the steady state, the discount rate, β, is set to the average ratio of the gross inflation rate to the gross nominal interest rate over the sample period which is 0.9931. The labor supply elasticity, $1/\zeta$, is set to 3, while ϕ_n is selected so that the steady-state value of labor, n, is 1/3. We do not need to specify a value for ϕ_M because m_t and the first-order condition for m_t are easily dropped from any model in which the monetary authority follows a nominal interest rate rule and money is additively separable in the utility function. Capital's share of output, α, is 0.33, steady-state technology, z, is 1, and the depreciation rate, δ, is set to a quarterly rate of 2.5 percent. The investment adjustment costs parameters are specified consistent with Christiano, Eichenbaum, and Evans (2005), so that investment adjustment costs are only binding on the second derivative with respect to the change in investment ($S(1) = S'(1) = 0$ and $S''(1) = 2.5$). The steady-state price elasticity is 6, so that the average markup of price over marginal cost is 20 percent. Lastly, the probability of price adjustment, $(1 - \eta)$, is set equal to 0.25 which implies that firms adjust their prices on average once a year. The estimated values for the remaining ten parameters are displayed in Table 1.

4 Computation Experiments

In each of these experiments, we assume that the central bank uses a Taylor-type rule. The baseline inflation target is assumed to be 2 percent. In each case we run

[5]Since our model contains four sources of exogenous disturbances and no measurement error, estimating the model with more than four observed variables causes the covariance matrix of the data to be singular.

the experiments for 25000 years or 100,000 quarters. We then record the number of times that the model predicts zero interest rates and also the number of ZLB events which we define as episodes lasting at least 2 quarters.[6] Our computational model is linear so that the interest rates can be negative. We record all nonpositive interest rates as zero. Results for the ZLB events are displayed in histograms, which report the number of events in each bin on the vertical axis and the length of the episodes on the horizontal axis.[7] Note that the current episode in the United States, which began in December 2008, is already over 3 years old. Japan has been at or near the zero lower bound since 1995.

The Dual Mandate. Figure 2 shows the effect of putting more or less weight on the output—trying to fulfill the full employment part of the dual mandate more or less aggressively. The first bin, labeled 4 quarters, depicts the number of ZLB events that last 2, 3 or 4 quarters. The number of events lasting 4 quarters or less rises from 0.3 percent when there is no weight on output in the policy rule, to 1.4 percent when the weight is 0.5, as suggested by Taylor (1993). The frequency of events lasting 1 to 2 years fall to about 0.3 percent for events lasting between for 5 to 8 quarters and to half that for events lasting 9 to 12 quarters. The last bin shows the number of events lasting more than 40 quarters. If a central bank were following a Taylor rule with a 2 percent inflation target, the odds of having a 10 year-long ZLB event would be very rare, but still possible in this New Keynesian model.

The accompanying Table 2, provides more results from this set of experiments. The second column indicates the percentage of results in which the interest rate was recorded as zero, including episodes lasting just one period. The third column lists the length of the longest ZLB event. The final 2 columns report the standard deviation of the output and inflation deviations from the steady state value.

Given a Taylor-type rule with 1.5 on the inflation gap, the best we can do to avoid the zero lower bound is to ignore the dual mandate and put zero weight on output. With no weight on the output there are almost no ZLB events of being at the zero lower bound in simulations of 100,000 quarters. Interest rates went to zero in 0.3 percent of the quarters, but the rate almost always became positive in the following quarter so the likelihood of having a ZLB event was a miniscule 0.003 percent. As Table 2 shows, among these versions of the Taylor rule, putting no weight on output results in the least number of ZLB events and the minimum variance for inflation. Putting more weight on output increases the likelihood of a ZLB event and increases the variance of inflation but decreases the variance of output.

As the central bank begins to pay attention to output, the likelihood of hitting the zero lower bound rises. If the weight is raised to 0.2, the likelihood of having an ZLB event that lasts between 2 and 4 quarters rises to 0.6 percent, with the longest ZLB event lasting 30 quarters, about the length of the current ZLB event in the United

[6] We use a common seed in all experiments. Detailed results are available on request.

[7] The frequency of one qurter episodes is so large that including it obscures the differences among longer episodes in the histograms.

States. Using Taylor's (1993) suggestion, which puts 0.5 weight on the output, raises the likelihood of having short ZLB events to 1.4 percent and results in the longest ZLB event lasting 145 quarters. With the Taylor rule, the likelihood of being at the zero lower bound rises to 16.3 percent.

But many researchers and business economists have estimated Taylor rules in which the reaction to output is substantially larger than the value recommended by Taylor. Doubling or tripling the weight on both output and inflation has little effect on the results. However, increasing the weight on output without also increasing the weight on inflation dramatically increases the likelihood of hitting the zero lower bound. The resulting volatility of inflation, however, becomes unrealistically large.

The last two columns of Table 2 show that there is a trade-off of the type noted by Taylor (1979). The central bank can reduce the volatility of the output, but only at the expense of higher inflation variability. Going from a strict inflation target with no weight on the output to the Taylor rule lowers variability of the output from 6.05 percent to 5.63 percent. It raises inflation variability from 0.30 percent to 1.48 percent. Although researchers continue to use deviation of output from some ad hoc trend as a measure of the output gap, and to use ad hoc objective functions which assume that all declines in output volatility are good, such practice is inconsistent with the New Keynesian model. In this model, the policy maker is attempting to reduce distortions caused by sticky prices. Were it not for the distortion coming from monopolistic competition in the market for intermediate goods, the optimal path for output would be the path in a similar model with flexible prices. In this case the theoretically correct measure of the output would be the deviation of output in the distorted economy from the path of output in the flexible price economy. In our model, the standard deviation of output in flexible price equilibrium with a Taylor rule is just a bit above 6 percent.

Commitment to an Inflation Target. The problem with the Taylor rule is that it targets the short-run inflation rate. When the central bank misses the target for any reason, the target miss is forgiven and the expected price level has a random walk component. The reason the dual mandate causes such wide swings in the interest rate and the inflation rate is because output and employment are much more volatile than inflation. By putting more weight on output, the policy transmits the fluctuations in output and employment into wide swings in the inflation rate and interest rates. The Taylor rule offers no remedy for these fluctuations. In an analysis of the effect of discretionary inflation targeting, Adam and Billi (2007) show that if the central could commit to the inflation objective, there would be a significant welfare gain and losses associated with the zero lower bound would be essentially eliminated.

Kydland and Prescott (1977) argued that it can be difficult, if not impossible, for governments to commit to good long-run policies if the optimal short-run policy runs counter to it. As long as people believe that there is a trade-off between full employment and inflation, they will always want more inflation when there is some unemployment. The Federal Reserve's dual mandate makes it easy for policymakers

to ignore the inflation objective when it appear to conflict with the full employment objective.

Svensson (1999) offers a solution for policymakers who cannot commit. He shows that a central bank can mimic the inflation targeting regime with commitment by adopting a price level path targeting regime with discretion. He finds that the discretion solution to a model with a price path target is isomorphic to the commitment solution when the central bank targets inflation. Next, we consider what happens when the central bank follows a Taylor rule, but also pays some attention to the path for the price level that would occur if the inflation target were perfectly achieved year in and year out. Under this rule, the central bank gradually corrects the price level for the net accumulation of past deviations from the inflation target. This policy works for three reasons: First, shocks to the price level of often transitory and self correcting. Second, if the shock is not transitory, then the price gap grows and, for a given reaction coefficient, θ_p, the policy has more bite. Third, long-run expectations are concentrated by the cointergration relationship that exists between the price level and the target for the price path.

In the experiments reported in Figure 3, the policymaker is pursuing the dual mandate by following the Taylor rule, but is trying to mimic the commitment equilibrium by putting some weight on a pedetermined path for the price level. The results for the Taylor rule are reproduced in Figure 3 to show how much policy improves when the central bank puts some weight on the price level path target. Setting $\theta_p = 0.1$ substantially reduces the likelihood of being at the zero lower bound. The frequency of long ZLB events goes to zero and the frequency of the shortest ZLB events drops dramatically. Table 3 shows that the likelihood of hitting zero drops from 16.3 percent to 2.2 percent. Choosing a higher weight resulted in even fewer occurrences—with $\theta_p = 0.2(0.3)$ there was only a 0.6 (0.3)percent chance of hitting the zero lower bound.

Note that a similar result can be obtained by pursing the inflation target more aggressively. Figure 4 and Table 4 show that increasing the response to the inflation gap can eliminate ZLB events of the zero lower bound, but does not deliver as much of a moderating effect on output volatility that you get with the price path target. There is also the a practical implementation problem with regimes that have very high reaction coefficients. The general equilibrium result of putting high weight on the inflation gap is that inflation gaps become small and interest rates become less volatile. But this depends on the public believing that the central bank has a large weight. In practice this may involve a period of learning and volatile inflation and interest rates. With a price path target, the reaction is much more modest and can be demonstrated without taking extreme actions while the public is learning about the rule.

Blanchard, Dell'Ariccia and Mauro (2010) and, much earlier, Summers (1991) have recommended that the Fed accept a higher than otherwise optimal inflation target as a means of avoiding the zero lower bound. Figure 5 and Table 5 show how

the sensitivity of ZLB events to the level of the inflation target. In the model, changing the inflation target just moves the steady state; it does not change variation around the steady state. Reducing the target to 1 percent raises the number of quarters spent at the zero lower bound from 16.3 percent to 20.4 percent. Raising the target to 3 percent, reduces the likelihood of being at zero to 12.8 percent. In all cases, the policy of changing the inflation target is dominated by committing to a explicit inflation target or by targeting the 2 percent inflation target more aggressively. Studies of the optimal inflation rate in New Keynesian models (See, for example, Coibion, et al., 2010) or in models with imperfectly indexed tax systems (See, for example, Bullard and Russell, 2004) suggests that closer to zero inflation is better for social welfare.

In this section we have examined the likelihood of being at the zero lower bound under alternative specifications of the monetary policy rule. The monetary policy shocks and the persistence of those shocks are, to some extent, a part of the policy regime. Central banks may be more or less explicit about their inflation target and they may react more or less to events that are not embedded in the output and inflation gaps. Sensitivity tests show that the likelihood of hitting the zero lower bound is not at all sensitive to estimated uncertainty about the persistence or size of the monetary policy shocks. Monetary policy has important effects on the likelihood of hitting the zero lower bound, but those effects are caused by the systematic part of monetary policy, not the driving process for policy shocks.

To summarize the key monetary policy results from our computational experiments, we find that a central bank can avoid the zero lower bound by giving up the dual mandate, something that is not considered politically feasible, one can respond more aggressively to the inflation gap, or one can commit to an inflation target by putting some weight on a target for a price level path. In forward-looking macroeconomic models, committing to an inflation target is clearly the best way to avoid the zero lower bound and achieve the dual mandate. So why hasn't the Federal Reserve adopted a price level path target (or equivalently, a long-run average inflation target)? One reason is a widespread belief about price path targeting that it would increase the chances of having bouts of deflation (See Fisher 1994). This is just not true. In all our experiments, we also saved the results for inflation. As we did with the zero lower bound on interest rates, we recorded ZLB events of negative inflation—deflation. Figure 5 shows the Taylor rule as well as three cases of the Taylor rule increasing weight on a price path. For values of the weight on inflation less than 0.3, there is a slightly higher incidence of deflation, but there is also a dramatic drop in volatility of inflation. Putting weight of 0.26 or higher on the price gap reduces the incidence of deflation. The intuition is simply that the price path concentrates long run inflation expectations at the target rate.

Sensitivity Tests for Shock Processes and Model Structure. In this section we test the sensitivity of the model to the size of parameters in the model structure and in the driving processes for the shocks. The most important shock is the technology shock because it drives the volatility of output. Figures 6 shows that, for the estimate range

of reasonable parameter values, the persistence of the shock is important, but the results are more complicated. Reducing the persistence by two standard deviations results in fewer ZLB events. However, after some point, increasing the persistence also reduces the likelihood of a ZLB event because, as Table 6 shows, after some point, the higher persistence increases the volatility of output, inflation (and the interest rate) so much that we get many more one period instances of hitting the ZLB which we do not include as ZLB events–the nominal interest rate is at zero more than a third of the time. Changing the standard deviation of the technology shock causes corresponding changes in the volatility of the economy, but practically no change in the likelihood of a zero lower bound event. Demand, labor supply and monetary policy shocks just do not matter for the likelihood of hitting the zero lower bound—at least for the range of uncertainty that we estimated for the variance and persistence of these shocks.

We examined the model parameters. The only one that mattered much at all for the likelihood of hitting the zero lower bound was the degree of price stickiness. Figure 7 depicts the likelihood of hitting the ZLB for different degrees of price stickiness. At the baseline setting, $\eta = 0.25$, prices are fixed on average for about 4 quarters. When we lower the probability of a firm getting to change its price to 0.1, the firm may expect to be stuck with the same price for several years. The likelihood of hitting the zero lower bound is much lower, as is the volatility of output. But the economy is also very far away from the flexible price optimum.[8] Raising the value of η to 0.5 results in an economy that looks very much like a flexible price economy. Except for an extreme and unrealistic degree of price stickiness, this parameter does not matter much for the likelihood of hitting the zero lower bound. The difference in the likelihood of hitting the zero lower bound is small for any setting of the Calvo parameter between our baseline and completely flexible prices.

We also checked the sensitivity of our results to variation in the cost of adjustment for investment and the labor supply elasticity. The variance decompositions show that demand shocks explain an important share of output fluctuations, but attempts to stabilize demand shocks also stabilize interest rates and do not affect the likelihood of hitting the zero lower bound.

4.1 Conclusion

A key result in this paper is that the monetary policy regime is the primary factor determining the likelihood of zero lower bound events. It is the systematic part of the monetary policy regime that matters. The likelihood of a zero lower bound event is not sensitive to the preference and production parameters that are typically used in DSGE models. The same is true for the persistence and variance of the driving processes for shocks to aggregate demand, monetary policy and markups. Of the

[8] Putting a weight of 2 on a price gap tacked onto a Taylor rule brings this very sticky price economy close to the flexible price path for output in this model.

shocks we considered, only the technology shocks matter for the likelihood of a zero lower bound event.

We have not discussed the case of the financial crisis as a shock to the economy. Chung et al. (2011) look for the answer in empirically estimated models with little or no financial structure. In policy models, the shock is represented as a surprising reduction in output. The recent output shock coincided with the financal panic of September 2008 in which the interest rate went to zero because the Fed stopped sterilizing loans to big banks. A different version of the Taylor rule will not prevent future financial crises. The rate will naturally go to zero if the central bank floods the eocnomy with liquidity in response to such crises.

Our results imply that the current policy regime with a short run inflation objective aimed at dual objectives for price stability and full employment is likely to experience zero lower bound events, even in the absence of financial crises. Looking back to Figure 1, inflation and the interest rate appeared to be headed on a downward trend from since 1981. Policymakers appear to have had as much of a problem stopping the decline in inflation as their predecessors had in stopping the acceleration of inflation during the 1970s. Once the interest rate hits zero, a policymaker using the Taylor rule has a problem managing expectations about future inflation. As Chung et al. (2010) recommend, policymakers seem to have embraced the zero lower bound.

The best way to achieve the dual mandate in the forward-looking models used at central banks is to commit to a clear inflation objective. Managing expectations is the key to successful monetary policy. But the key is to manage expectations about the long-run average inflation rate, not the short-term interest rate. We show that trying to pursue a dual mandate for price stability and full employment will likely lead to the zero lower bound if the central bank is not committed to an inflation objective.

Although we have not worked through all the models currently in use to analyze monetary policy, we speculate that our results hold generally in all dynamic stochastic general equilibrium models with forward-looking agents. Different models will have different implications for real variables, but implications for inflation and the nominal interest rate are quite robust across a wide range of New Keynesian and New Classical specifications.

4.2 References

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Table 1: Maximum Likelihood Estimates and Standard Errors

Parameter	Description	Estimate	Standard Error
θ_π	Policy response to inflation	1.9211	0.2784
θ_{dy}	Policy response to output growth	0.3467	0.1018
ρ_z	Persistence of technology shock	0.9801	0.0091
ρ_a	Persistence of aggregate demand shock	0.8728	0.0258
ρ_e	Persistence of cost-push shock	0.5819	0.0730
ρ_R	Persistence of money policy shock	0.1586	0.0611
σ_z	Std. dev. of technology shock	0.0077	0.0005
σ_a	Std. dev. of aggregate demand shock	0.0045	0.0007
σ_e	Std. dev. of cost-push shock	0.0011	0.0001
σ_R	Std. dev. of monetary policy shock	0.0055	0.0005

Table 2. Volatility Implications of Having a Dual Mandate
(Baseline Model)

Weight on output	% of quarters with R = 0	Longest Epidsode	Std Dev Output	Std Dev Inflation
0	0.3%	42	6.12%	0.30%
0.1	1.2%	42	6.03%	0.47%
0.2	4.0%	30	5.95%	0.72%
0.3	8.2%	79	5.86%	0.98%
0.4	12.4%	124	5.78%	1.25%
0.5	16.3%	145	5.69%	1.50%

Table 3. Volatility Implications of Targeting the Price Level
(Baseline Model with weight 1.0 on output)

Weight on price gap	% of quarters with $R = 0$	Longest ZLB event	% of quarters with $\pi < 0$	Longest deflation episode	Std dev output	Std dev inflation
0	16.3%	145	16.3%	325	5.69%	1.50%
0.1	2.2%	18	23.0%	74	5.72%	0.68%
0.2	0.6%	32	15.7%	53	5.80%	0.50%
0.3	0.3%	64	11.4%	83	5.85%	0.41%

Table 4. Volatility Implications of Changing Weight on the Inflation Gap
(Baseline Model--Taylor Rule)

θ_π	% of quarters with $R = 0$	Longest ZLB event	Std dev output	Std dev inflation
1.5	16.3%	145	5.69%	1.50%
2	5.5%	45	5.91%	0.81%
2.5	2.3%	23	5.99%	0.56%
3	1.2%	31	6.04%	0.44%

Table 5. Volatility Implications of the Level of Inflation Target
(Baseline Model--Taylor Rule)

π target	% of quarters with R = 0	Longest SLB event	Std dev output	Std dev inflation
1%	20.4%	161	5.69%	1.50%
2%	16.3%	145	5.69%	1.50%
3%	12.8%	137	5.69%	1.50%

Table 6. Volatility Implications of Persistence of Technology Shocks
(Baseline Model--Taylor Rule)

ρ_Z	% of quarters with R = 0	Longest ZLB event	Std dev output	Std dev inflation
0.9983	42.6%	2247	19.33%	4.86%
.0.9801	16.3%	145	5.69%	1.50%
0.9619	10.0%	66	3.96%	1.10%

Table 7. Volatility Implications of size of output shocks
(Baseline Model--Taylor Rule)

ρ_Z	% of quarters with R = 0	Longest Epidsode	Std Dev Output gap	Std Dev Inflation
0.0087	18.9%	157	6.42%	1.69%
0.0077	16.3%	145	5.69%	1.50%
0.0067	13.5%	137	4.96%	1.32%

Figure 1. US Interest Rates: 1955 to 2010

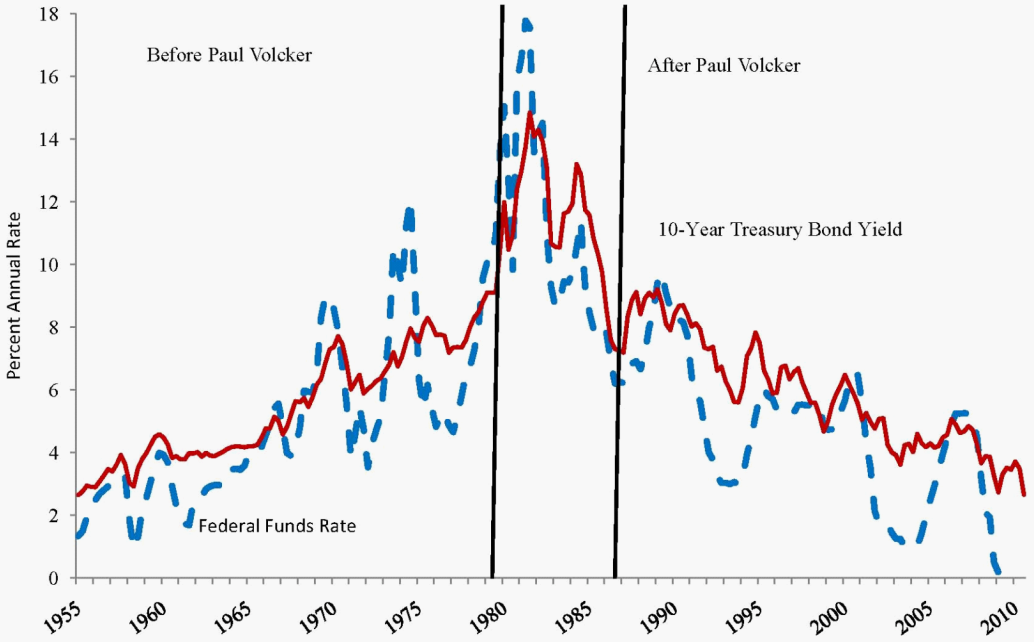
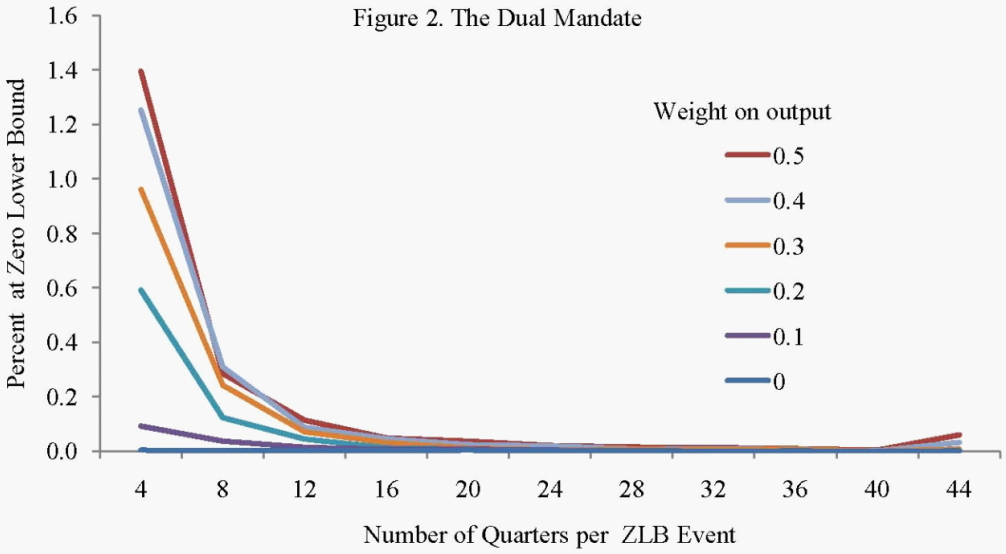
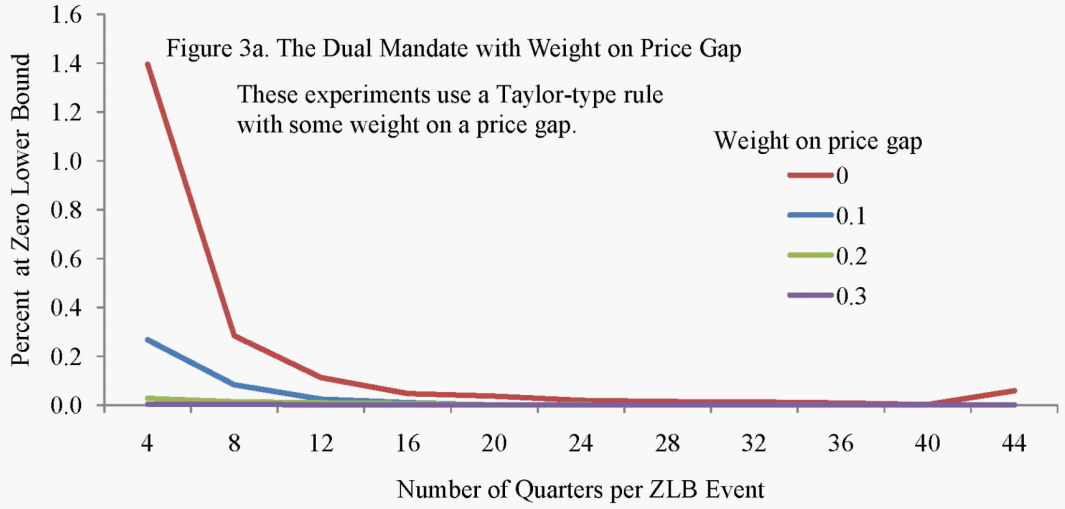


Figure 2. The Dual Mandate



A ZLB event is defined as having at least 2 consecutive quarters of zero nominal interest rate



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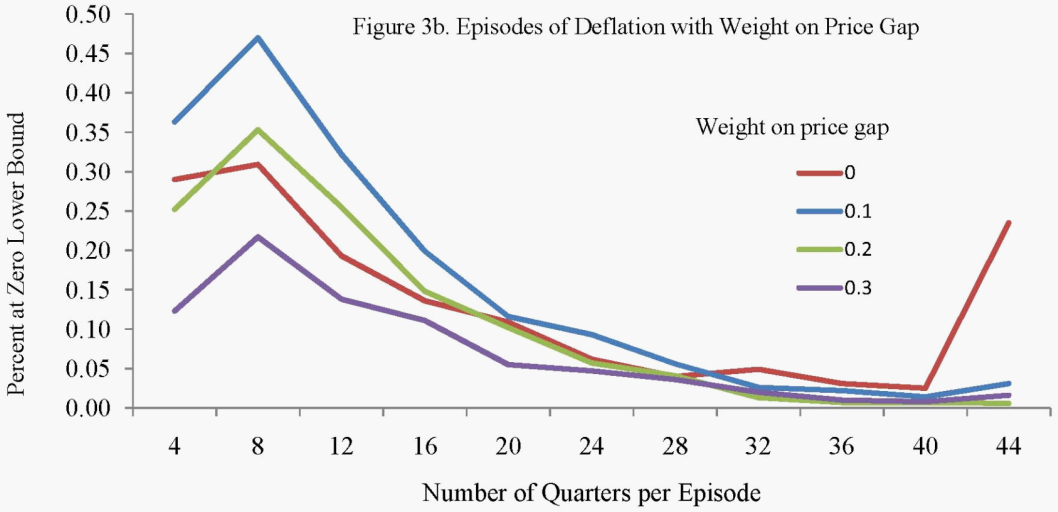
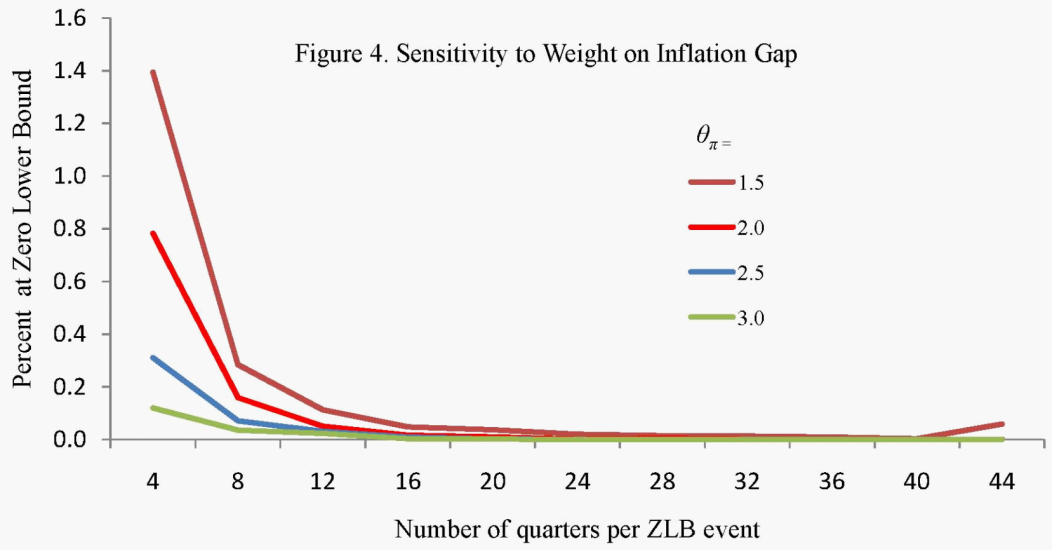
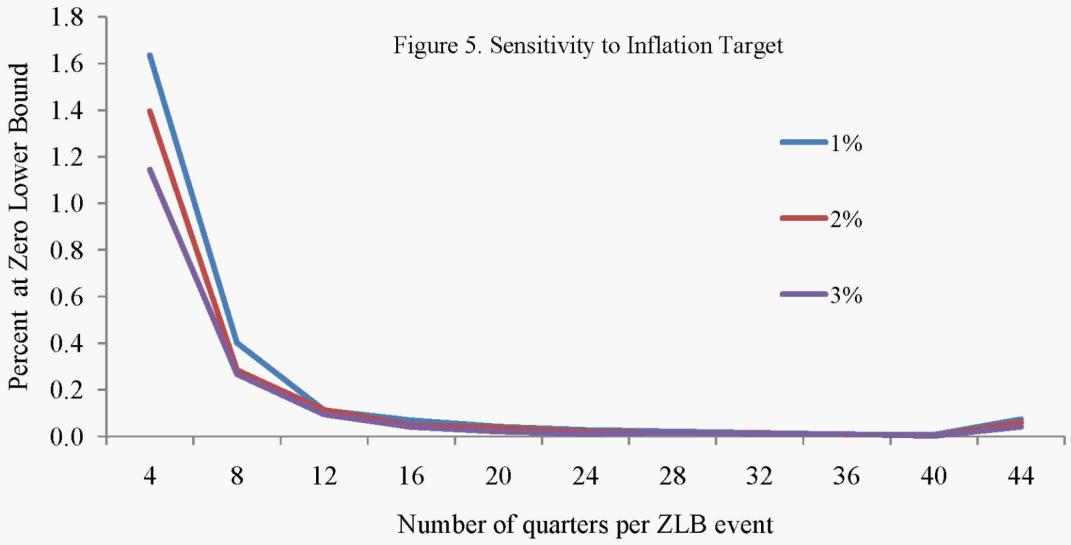


A deflation episode defined as having at least 4 consecutive quarters of deflation



ZLB event defined as having at least 2 consecutive quarters of zero nominal interest rate



ZLB event defined as having at least 2 consecutive quarters of zero nominal interest rate

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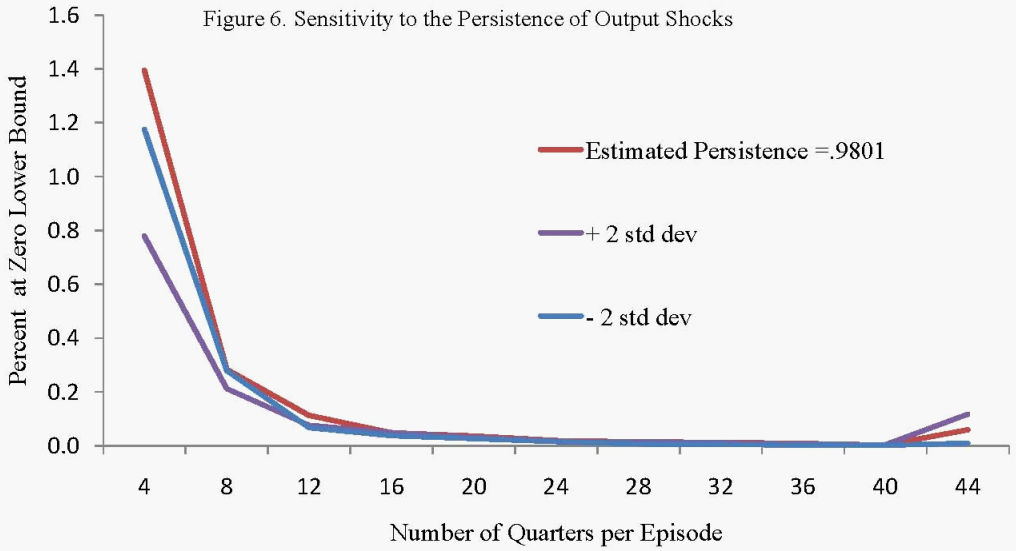
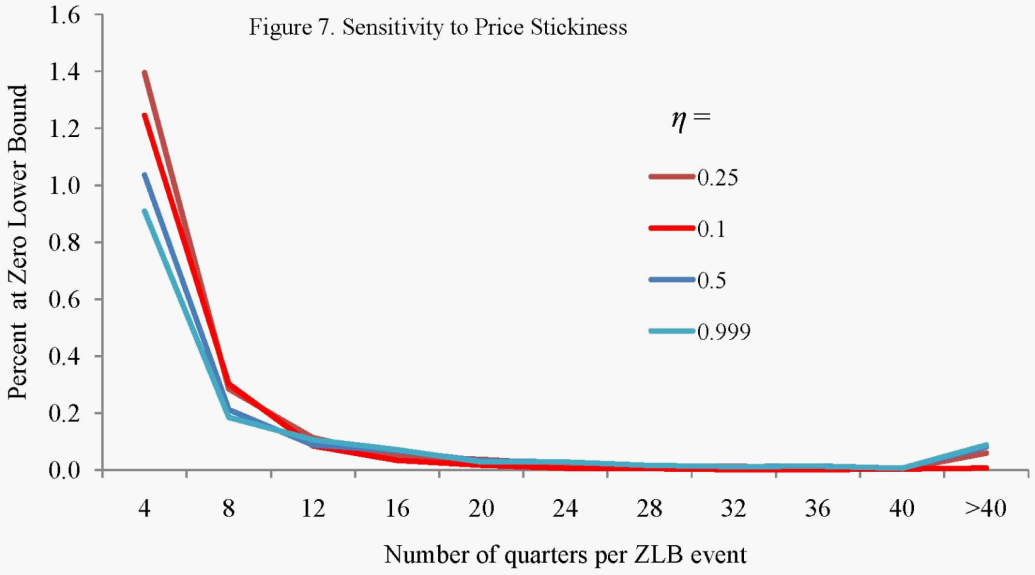
Figure 6. Sensitivity to the Persistence of Output Shocks



Episode defined as having at least 2 consecutive quarters of zero nominal interest rate

Figure 7. Sensitivity to Price Stickiness



ZLB event defined as having at least 2 consecutive quarters of zero nominal interest rate