International equity and bond positions in a DSGE model with variety risk in consumption[☆]

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Abstract

This paper analyzes equity and bond positions in a two-country DSGE model where the available number of varieties is endogenously determiend. In numerical computation of zero-order steady state portfolios, we employ the method developed by Devereux and Sutherland (2008). Households face not only nominal price but also variety risk in real exchange rate fluctuations. With such a variety risk, home biased equity positions are amplified further than the standard model in literature which incorporates investment dynamics. The result is robust with or without firm heterogeneity.

Keywords: terms of trade, Home biased equity puzzle, firm entry, firm heterogeneity
JEL classification: F12, F41, F43

1. Introduction

The last 30 years have witnessed a significant increase in both goods and financial assets exchange. For instance the share of imported goods in U.S. GDP has more than doubled, from 4.8 % in 1972 to 11.7 % in 2001. In particular, the expansion of trade has

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been driven by so called "extensive margins", the number of available varieties, contrary to "intensive margins", the quantity of a given set of goods. For the U.S. in the same period, imported extensive margins have quadrupled (Broda and Weinstein (2004), Broda and Weinstein (2006)). In parallel of such a trend, cross-border assets holdings have been developed and gross external positions of major developed countries have exceeded its value of GDP (Lane and Milesi-Ferretti (2003), Lane and Milesi-Ferretti (2005), Lane and Milesi-Ferretti (2007)). What is puzzling is that, despite such a development in cross-border assets holdings, still we continue to observe sizable home biased equity positions among industrialized countries (French and Poterba (1991), Tesar and Werner (1995), Obstfeld and Rogoff (2000)).

Cross-border assets holdings are useful for consumption smoothing, thus for mutual insurances in consumption risk among countries. Optimal portfolio positions are considered against various risks which prevent such an optimal pass of consumptions. Both fluctuations in relative nominal price of consumption baskets and those in non-financial assets can be a source of such risk. In addition to these risks, fluctuations in extensive margins should be a new source of risk in consumption. What is the implication of such a "variety risk" on international risk sharing mechanism? The present paper addresses this issue.

The recent literature has been developed around the attempts on the resolution of "the home biased equity puzzle". When equity positions are used against consumption risk induced by terms of trade fluctuations, except a special case where they contribute perfectly for risk sharing (Cole and Obstfeld (1991)), a realistic home biased position arises only when the elasticity of substitution between local and imported goods is very small, smaller than unity (Uppal (1993), Kollmann (2006a), Kollmann (2006b), Obstfeld (2007.), Civelli. (2008), Coeurdacier (2009)). The reason is that at the moment of real exchange rate appreciation (expensive consumption goods), equity returns of domestic firms rise compared to those of foreign because of strong income effect when the elasticity is low.

Although theoretically possible, it is noticed that this class of models has counterfac-

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tual aspect: between terms of trade fluctuations and equity returns there are no empirical regularities (van Wincoop and Warnock (2006)). At the next stage of research, however, this inconvenient aspect of the theoretical model has overcome by introducing nominal bonds or forward exchange positions whose returns correlate perfectly with terms of trade fluctuations, thus leaving equities to load on other risks such as labor income risk (Coeurdacier et al. (2007), Coeurdacier et al. (2010), Coeurdacier and Gourinchas (2008), Engel and Matsumoto (2005)).

Our paper is built from such a point: there are real exchange rate and labor income risk to be hedged using equities and nominal bonds. For that purpose we incorporate cross-border assets holdings in equities and nominal bonds into a model based on Ghironi and Melitz (2005) where extensive margins are determined endogenously. In our model, the available set of varieties or the way to consume the same set of varieties in consumption baskets would be different across countries depending on whether there is heterogeneity in firms' marginal costs. Such a variety risk in real exchange rate fluctuations cannot be hedged by nominal bonds only. This is because nominal bonds returns do not load on the 'welfare-based' real exchange rate fluctuations including extensive margins.

The hedging mechanism in our paper are as following. Successful home biased equity positions as in the data are principally driven by investment fluctuations and induced labor income risk. A home biased equity position can be a good position when induced labor income flows and equity returns correlate negatively (Heathcote and Perri (2004)). But at the same time such an investment shock induces fluctuations in extensive margins. When a positive investment shock induces a higher number of varieties, hence a real depreciation in welfare based, because equity returns decrease with such a positive investment shock, home biased equity positions become also good hedge against this additional variety risk in real exchange rate fluctuations. As a result, amplified home biased equity positions arise. We explore this hedging mechanism analytically relying on static budget constraint as well as numerically with the method developed by Devereux and Sutherland (2008). The result is shown to be robust for the model with and without heterogenous firms. Our paper hence adds further arguments why we continue to observe strong home biased

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equity positions among industrialized countries. Expansion of trade in extensive margins would account for that.

The paper is close to Coeurdacier et al. (2010) in the spirit which analyze zero-order steady state equity and positions (and their first-order dynamics) in a DSGE model. A major difference is that 'investments' in this paper take the form of new firm creations, not in the form of capital accumulation process. Bui (2009) also extends their framework by introducing a nominal rigidity in goods price and find numerically a home biased equity and short local bond positions in incomplete markets setting. Castello (2008) finds analytically a home biased equity position using a static budget constraint in the model including endogenous fluctuations of extensive margins.

The structure of the paper is as follows. In the next section we present the full model including firm heterogeneity. In section 3 the method to compute zero-order steady state portfolios is explained. For the purpose of comparison, the model without firm heterogeneity is briefly presented in section 4. In section 5, portfolios are calibrated. In section 6, we explore the intuition of portfolios analytically as well as numerically. At the end we conclude briefly.

2. The model

There are two countries, Home and Foreign. Foreign variables are denoted with asterisk. Each country is populated by unit mass of households who consume, work, save and invest. Holdings of financial assets cross the border. Saving is made by nominal CPI-indexed bonds, issued by each country. Investment is made in the form of firm creation. Firms are supposed to be heterogenous in marginal costs. Households in each country buy a share of mutual funds among heterogenous firms of both countries. There are four exogenous shocks which hit all firms in homogenous way: on labor productivity in goods creation (intensive margins) and that in firm creation (extensive margins) for each country.

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2.1. Households

The Home representative household maximizes the discounted utility, $E_{t-1} \sum_{s=t}^{\infty} \beta_t^{s-t} U_t$. The utility at t depends on consumption and labor supply as follows,

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}, \tag{1}$$

where $\gamma \geq 1$ denotes relative risk aversion. The parameter $\chi \, (> 0)$ represents the degree of non satisfaction from supplying labor, L_t and the parameter $\varphi \, (\geq 0)$ is Frisch elasticity of labor supply[1]. With this specification the marginal disutility in providing one additional labor is increasing.

β_t is an endogenous discount factor which evolves as,

$$\beta_{t+1} = \beta_t \Upsilon (C_{A.t}), \, \beta_0 = 1 \tag{2}$$

$C_{A.t}$ is aggregate Home consumption which coincides to C_t in our setting. We give a functional form of $\Upsilon(C_t)$ as $\Upsilon(C_t) = \bar{\beta} C_t^{-\upsilon}$ where $0 \leq \upsilon < \gamma$ and $0 < \bar{\beta} C^{-\upsilon} < 1$. Because $\Upsilon'(C_t) < 0$, this specification of endogenous discount factor guarantees the stationarity of the model including net foreign asset dynamics as explained in Schmitt-Grohe and Uribe (2003). The consumption basket is defined as:

$$C_t = \left[\alpha^{\frac{1}{\omega}} C_{H.t}^{1-\frac{1}{\omega}} + (1-\alpha)^{\frac{1}{\omega}} C_{F.t}^{1-\frac{1}{\omega}} \right]^{\frac{1}{1-\frac{1}{\omega}}}, \tag{3}$$

where $\alpha \, (> 1/2)$ is home bias in consumption. $\omega \, (> 0)$ denotes the elasticity of substitution between Home local $(C_{H.t})$ and Foreign imported goods $(C_{F.t})$. Each $C_{H.t}$ and $C_{F.t}$ basket is composed of $N_{D.t}$ number of Home and $N_{X.t}^*$ number of Foreign varieties as follows:

$$C_{H.t} = V_{H.t} \left[\int_0^{N_{D.t}} c_{hD.t}^{1-\frac{1}{\sigma}} dhD \right]^{\frac{1}{1-\frac{1}{\sigma}}}, \qquad C_{F.t} = V_{F.t}^* \left[\int_0^{N_{X.t}^*} c_{fX.t}^{1-\frac{1}{\sigma}} df X \right]^{\frac{1}{1-\frac{1}{\sigma}}}, \tag{4}$$

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[1] With $\varphi = \infty$ the marginal disutility of supplying labor becomes constant, χ. When $\varphi = 0$ the marginal disutility becomes infinite and the labor supply becomes inelastic.

where $V_{H.t} \equiv N_{D.t}^{\psi-\frac{1}{\sigma-1}}$ and $V_{F.t}^* \equiv N_{X.t}^{*\psi-\frac{1}{\sigma-1}}$. $c_{hD.t}$ is the demand for domestic variety in the Home country indexed by hD. $c_{fX.t}$ is the demand for an imported variety indexed by fX. σ (> 1) denotes the elasticity of substitution among varieties. ψ (≥ 0) represents the marginal increase of the utility which stems from consuming one additional variety in each basket. We suppose conventionally $\sigma \geq \omega$. With the above specification the preference becomes Dixit and Stiglitz (1977) when $\psi = \frac{1}{\sigma-1}$. When $\psi = 0$ there is no love for variety and the household is just satisfied by intensive margins.

The corresponding price indices are given by,

$$P_t = \left[\alpha P_{H.t}^{1-\omega} + (1-\alpha) P_{F.t}^{1-\omega}\right]^{\frac{1}{1-\omega}}, \tag{5}$$

and

$$P_{H.t} = \frac{1}{V_{H.t}} \left[\int_0^{N_{D.t}} p_{hD.t}^{1-\sigma} dhD\right]^{\frac{1}{1-\sigma}}, \qquad P_{F.t} = \frac{1}{V_{F.t}} \left[\int_0^{N_{X.t}^*} p_{fX.t}^{1-\sigma} dfX\right]^{\frac{1}{1-\sigma}}. \tag{6}$$

Observe that price indices fluctuate with extensive margins whose impact is larger the higher the love for variety, ψ. The similar expression holds in the Foreign country.

2.1.1. Budget constraint

The period-by-period budget constraint for the Home representative household is given by,

$$C_t + s_{h.t+1} x_{h.t}^s (N_{D.t} + N_{E.t}) + s_{f.t+1} x_{f.t}^s (N_{D.t}^* + N_{E.t}^*) + b_{h.t+1} x_{h.t}^b + b_{f.t+1} x_{f.t}^b$$

$$= w_t L_t + s_{h.t} N_{D.t} \left(x_{h.t}^s + \widetilde{d}_{h.t}\right) + s_{f.t} N_{D.t}^* \left(x_{f.t}^s + \widetilde{d}_{f.t}\right) + b_{h.t} \left(x_{h.t}^b + \frac{\widehat{P}_t}{P_t}\right) + b_{f.t} \left(x_{f.t}^b + \frac{\widehat{P}_t^*}{P_t}\right). \tag{7}$$

The price of Home consumption basket is taken as numeraire. The household finances all existing firms in each country including new entrants whose number are expressed by $N_{E.t}$ and $N_{E.t}^*$. $s_{h.t+1}(s_{f.t+1})$ denotes share holdings into $t+1$ and $x_{h.t}^s$ ($x_{f.t}^s$) is real share

price of Home (Foreign) mutual funds.[2] $\widetilde{d}_{h.t}$ $(\widetilde{d}_{f.t})$ is the average real dividends among heterogenous Home (Foreign) exporters and non-exporters. $b_{h.t+1}(b_{f.t+1})$ represents Home (Foreign) bond holdings into $t+1$. Nominal CPI-indexed Home (Foreign) bonds provide a nominal payoff, $\widehat{P}_t$ $(\widehat{P}_t^*)$. $x_{h.t}^b$ $(x_{f.t}^b)$ is the price of Home (Foreign) real bonds. w_t is real wage and L_t is labor supply by the household.

For the representative Foreign household, the real budget constraint expressed in terms of Foreign consumption basket becomes:

$$
C_t^* + s_{f.t+1}^* x_{f.t}^{s*} \left(N_{D.t}^* + N_{E.t}^* \right) + s_{h.t+1}^* x_{h.t}^{s*} \left(N_{D.t} + N_{E.t} \right) + b_{f.t+1}^* x_{f.t}^{b*} + b_{h.t+1}^* x_{h.t}^{b*}
$$

$$
= w_t^* L_t^* + s_{f.t}^* N_{D.t}^* \left(x_{f.t}^{s*} + \widetilde{d}_{f.t}^* \right) + s_{h.t}^* N_{D.t} \left(x_{h.t}^{s*} + \widetilde{d}_{h.t}^* \right) + b_{f.t}^* \left(x_{f.t}^{b*} + \frac{\widehat{P}_t^*}{P_t^*} \right) + b_{h.t}^* \left(x_{h.t}^{b*} + \frac{\widehat{P}_t}{P_t^*} \right).
$$

$$(8)$$

Note that asset markets clearings imply that $s_{h.t+1} + s_{h.t+1}^* = s_{f.t+1} + s_{f.t+1}^* = 1$ and $b_{h.t+1} + b_{h.t+1}^* = b_{f.t+1} + b_{f.t+1}^* = 0, \forall t$.

Also, we define the real exchange rate Q_t as:

$$
Q_t = \frac{P_t^*}{P_t}. \tag{9}
$$

We refer this to 'welfare based' because it fluctuates with extensive margins as well as nominal prices' fluctuations

2.1.2. First order conditions

In writing the first order conditions about assets holdings, it will turn out to be useful to define real returns of each asset as follows:[3]

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[2] Equities in this paper take the form of mutual funds. This is for the simplification rather than a good picture of reality. A notable exception is Martin and Rey (2004) which analyze 'extensive margins' of assets.

[3] Notice that by considering motion of firms equations that we will see,

$$
\frac{N_{D.t}}{N_{D.t-1} + N_{E.t-1}} = \frac{N_{D.t}^*}{N_{D.t-1}^* + N_{E.t-1}^*} = 1 - \delta \tag{10}
$$

$$r_{h.t}^s \equiv (1-\delta)\, \frac{x_{h.t}^s + \widetilde{d}_{h.t}}{x_{h.t-1}^s}, \quad r_{f.t}^s \equiv (1-\delta)\, \frac{x_{f.t}^s + \widetilde{d}_{f.t}}{x_{f.t-1}^s}, \tag{11}$$

and

$$r_{h.t}^b \equiv \frac{x_{h.t}^b + \frac{\widehat{P_t}}{P_t}}{x_{h.t-1}^b}, \quad r_{f.t}^b \equiv \frac{x_{f.t}^b + \frac{\widehat{P_t^*}}{P_t}}{x_{f.t-1}^b}. \tag{12}$$

$r_{h.t}^s$, $r_{f.t}^s$, $r_{h.t}^b$ and $r_{f.t}^b$ are gross equity/bond real returns between $t-1$ and t. Using these real returns the first order conditions for Home and Foreign equity holdings become,[4]

$$1 = \Upsilon(C_t)\, E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} r_{h.t+1}^s, \quad 1 = \Upsilon(C_t)\, E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} r_{f.t+1}^s, \tag{14}$$

The first order conditions for Home and Foreign bond holdings become,[5]

$$1 = \Upsilon(C_t)\, E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} r_{h.t+1}^b, \quad 1 = \Upsilon(C_t)\, E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} r_{f.t+1}^b.$$

Observe that up to first order approximation these assets are perfect substitute. This rises the indeterminacy problem of zero-order steady state portfolios as discussed in Devereux and Sutherland (2008).

The optimal consumptions of Home and Foreign goods are given by,

$$C_{H.t} = \left(\frac{P_{H.t}}{P_t} \right)^{-\omega} \alpha C_t, \qquad C_{F.t} = \left(\frac{P_{F.t}}{P_t} \right)^{-\omega} (1-\alpha)\, C_t. \tag{16}$$

Equity returns, $r_{h.t}^s$ and $r_{f.t}^s$ include investment costs in the form of exogenous destruction rate as δ fraction of firms disappear in each period. As we will see first-order "static budget constraint" fail to capture this investment dynamic.

[4]Those for the Foreign representative household become,

$$1 = \Upsilon(C_t^*)\, E_t \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} r_{h.t+1}^s \frac{Q_t}{Q_{t+1}} \ , \ 1 = \Upsilon(C_t^*)\, E_t \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} r_{f.t+1}^s \frac{Q_t}{Q_{t+1}}. \tag{13}$$

[5]Those of Foreign counterparts are,

$$1 = \Upsilon(C_t^*)\, E_t \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} r_{h.t+1}^b \frac{Q_t}{Q_{t+1}} \ , \ 1 = \Upsilon(C_t^*)\, E_t \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} r_{f.t+1}^b \frac{Q_t}{Q_{t+1}}. \tag{15}$$

Using symmetry among varieties, optimal consumptions for each individual firm's variety are given by,

$$c_{hD.t} = V_{H.t}^{\sigma-1} \left(\frac{p_{hD.t}}{P_{H.t}} \right)^{-\sigma} C_{H.t}, \qquad c_{fX.t} = V_{F.t}^{\sigma-1} \left(\frac{p_{fX.t}}{P_{F.t}} \right)^{-\sigma} C_{F.t}. \qquad (17)$$

The similar expressions hold for the Foreign country.

For notational convenience we express real prices as follows:

$$\rho_{H.t} = \frac{P_{H.t}}{P_t}, \qquad \rho_{F.t} = \frac{P_{F.t}}{P_t}, \qquad \rho_{hD.t} = \frac{p_{hD.t}}{P_t}, \qquad \rho_{fX.t} = \frac{p_{fX.t}}{P_t},$$

$$\rho_{H.t}^* = \frac{P_{H.t}^*}{P_t^*}, \qquad \rho_{F.t}^* = \frac{P_{F.t}^*}{P_t^*}, \qquad \rho_{hD.t}^* = \frac{p_{hD.t}^*}{P_t^*}, \qquad \rho_{fX.t}^* = \frac{p_{fX.t}^*}{P_t^*}.$$

2.2. Firms

2.2.1. Free entry

Supply side of the model is almost identical to Ghironi and Melitz (2005) except a difference that entry costs are paid with capital goods as well as labor in our model.[6]

We suppose that firm creation activity needs an amount of firms setting up goods, f_E. The production of such goods is done by the following Cobb-Douglas technology using labor $l_{EM.t}$ and capital goods K_t as inputs,

$$f_E = \left(\frac{Z_{E.t} l_{EM.t}}{\theta} \right)^{\theta} \left(\frac{K_t}{1-\theta} \right)^{1-\theta}, \qquad (18)$$

where $Z_{E.t}$ is the labor productivity which is specific for firm setup and identical across firms. θ $(1-\theta)$ is the share of labor (capital) in total entry costs. For simplicity, we suppose the capital goods K_t has the same composition as consumption goods, C_t.

The cost minimization problem yields the following factor demands,

$$l_{EM.t} = \left(\frac{w_t}{\mu_t} \right)^{-1} \theta f_E, \qquad K_t = \left(\frac{1}{\mu_t} \right)^{-1} (1-\theta) f_E, \qquad (19)$$

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[6]This would be more realistic specification. This types of specification of entry cost is proposed in Bilbiie et al. (2007).

where $\mu_t = (w_t/Z_{E.t})^\theta$ is real entry costs.

At the equilibrium, share price should be equal to the costs for entry providing the following free entry condition:

$$x_{h.t}^s = \left(\frac{w_t}{Z_{E.t}}\right)^\theta f_E. \tag{20}$$

The similar conditions hold for the Foreign country.

2.2.2. Motion of firms

Motion of firms follows Ghironi and Melitz (2005). This is given by,

$$N_{D.t+1} = (1-\delta)(N_{D.t} + N_{Et}). \tag{21}$$

Production of intensive margins take place only one period after the entry. New entrants need "one time-to-build". δ represents an exogenous depreciation rate. This "death shock" takes place at the very end of the period after the investment has been completed. Combined with the Euler equations about equity holdings, with the above specification investment costs enter in the form of $1-\delta$ in equity returns $r_{h.t}^s$ and $r_{f.t}^s$.

The similar condition holds for the Foreign country.

2.2.3. The average firm specific productivity

Firms are heterogenous in terms of firm specific labor productivity which they draw upon entry. This productivity is drown from the following Pareto distribution:

$$G(z_D) = 1 - \left(\frac{z_{D\,\min}}{z_D}\right)^k, \tag{22}$$

where $z_{D\,\min}$ is the minimum level productivity and k $(k > \sigma - 1)$ is a parameter which shapes the distribution. Two special "average" productivities are defined following Melitz (2003):

$$\widetilde{z}_D \equiv \left[\int_{z_{\min}}^{\infty} z_D^{\sigma-1} dG(z_D)\right]^{\frac{1}{\sigma-1}} = z_{D\,\min}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}, \tag{23}$$

and

$$\widetilde{z}_{X.t} \equiv \left[\frac{1}{1 - G(z_{X.t})} \int_{z_{X.t}}^{\infty} z_D^{\sigma-1} dG(z_D) \right]^{\frac{1}{\sigma-1}} = z_{X.t} \left[\frac{k}{k - (\sigma - 1)} \right]^{\frac{1}{\sigma-1}}, \qquad (24)$$

where $\widetilde{z}_D$ is the average firm specific productivity and $\widetilde{z}_{X.t}$ is that for exporting firms in the Home country.

The similar conditions hold for the Foreign country.

2.2.4. Average productions and profits from domestic sales

Provided the above average productivities, as Ghironi and Melitz (2005), domestic firms' behaviors are summarized by looking for the average firm which has $\widetilde{z}_D$.

Production scale of the average firm $(\widetilde{y}_{hD.t})$ is given by the following technology:

$$\widetilde{y}_{hD.t} = Z_t \widetilde{z}_D l_{D.t}, \qquad (25)$$

where Z_t denotes the productivity shock which hit all domestic firms. $l_{D.t}$ denotes the labor demand by this average firm. Operational real profits from domestic sales are expressed as:

$$\widetilde{d}_{D.t} = \left(\widetilde{\rho}_{hD.t} - \frac{w_t}{Z_t \widetilde{z}_D} \right) \widetilde{y}_{hD.t}.$$

Goods market clearing implies,

$$\widetilde{y}_{hD.t} = \widetilde{c}_{hD.t} + N_{E.t} \widetilde{k}_{hD.t}, \qquad (26)$$

where $\widetilde{c}_{hD.t}$ and $\widetilde{k}_{hD.t}$ are the consumption and capital demand for this average firms. Using optimal demand found in the previous section, $\widetilde{y}_{hD.t}$ can be rewritten as follows,

$$\widetilde{y}_{hD.t} = N_{D.t}^{\psi(\sigma-1)-1} \widetilde{\rho}_{hD.t}^{-\sigma} \rho_{H.t}^{\sigma-\omega} \alpha M_t, \qquad (27)$$

where M_t is consumption and investment goods demand in the Home country:

$$M_t = C_t + N_{E.t} K_t . \qquad (28)$$

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Note that using free entry condition (20) and factor demand (19), it is verified that $1 - \theta$ fractions of real investment costs are paid as capital goods: $K_t = (1 - \theta) x^s_{h.t}$. When $\theta = 0$, M_t coincides to the aggregated demand, aggregated consumption plus aggregated investment.

Profit maximization yields the standard pricing in monopolistic competition. Individual real price becomes the real marginal costs over markup:

$$\widetilde{\rho}_{hD.t} = \frac{\sigma}{\sigma - 1} \frac{w_t}{Z_t \widetilde{z}_D}. \tag{29}$$

Finally using the above optimal pricing and the fact that $\rho_{H.t} = N^{-\psi}_{D.t} \widetilde{\rho}_{hD.t}$, total real average profits from the average domestic sales are expressed as:

$$\widetilde{d}_{D.t} = \frac{1}{\sigma} N^{\psi(\omega-1)-1}_{D.t} \widetilde{\rho}^{1-\omega}_{h.t} \alpha M_t. \tag{30}$$

The similar conditions hold for the Foreign country.

2.2.5. Average productions and profits from exporting sales

We discuss about the average exporting firm which has the average firm specific exporting productivity, $\widetilde{z}_{X.t}$. This summarizes the exporting activity in the Home country. Production technology for the average exporting firms is given by,

$$\widetilde{y}_{hX.t} = Z_t \widetilde{z}_{X.t} l_{X.t}, \tag{31}$$

where $\widetilde{y}^*_{hX.t}$ is the average production to be exported and $l_{X.t}$ is the labor demand by this average firm.

Following Ghironi and Melitz (2005), exporting abroad supposed to be costly. Exporting firms should pay an amortized fixed costs in each period. These costs are defined in terms of effective labor, f_X. Thus operational real average profits from the average exporting sales are expressed as:

$$\widetilde{d}_{X.t} = \left(\widetilde{\rho}_{hX.t} - \frac{w_t}{Z_t \widetilde{z}_{X.t}} \right) \widetilde{y}^*_{hX.t} - \mu_{X.t} f_X, \tag{32}$$

where $\mu_{X.t} = \frac{w_t}{Z_t}$ is real amortized costs for exporting. Goods market clearing implies,

$$\widetilde{y}_{hX.t} = \widetilde{c}^*_{hX.t} + N_{E.t}\widetilde{k}^*_{hX.t}, \tag{33}$$

where $\widetilde{c}^*_{hX.t}$ and $\widetilde{k}^*_{hX.t}$ denote the average consumption and capital demand in exporting sales. Using optimal demand found in the previous section, $\widetilde{y}_{hX.t}$ can be rewritten as,

$$\widetilde{y}_{hX.t} = N_{X.t}^{\psi(\sigma-1)-1}\widetilde{\rho}^{*-\sigma}_{hX.t}\rho^{*\sigma-\omega}_{H.t}(1-\alpha)M^*_t, \tag{34}$$

where M^*_t is the consumption and investment goods demand in the Foreign country:

$$M^*_t = C^*_t + N^*_{E.t}K^*_t, \tag{35}$$

where $K^*_t = (1-\theta)\,x^{s*}_{f.t}$.

Pricing in exporting market is standard. Without any transportation cost, it becomes,

$$\widetilde{\rho}^*_{hX.t} = Q_t^{-1}\widetilde{\rho}_{hX.t}, \quad \text{where } \widetilde{\rho}_{hX.t} = \frac{\sigma}{\sigma-1}\frac{w_t}{Z_t\widetilde{z}_{X.t}}. \tag{36}$$

Finally using the above optimal pricing and the fact that $\rho^*_{H.t} = N_{X.t}^{-\psi}\widetilde{\rho}^*_{hX.t}$, total real average profits from the average exporting sales are expressed as:

$$\widetilde{d}_{X.t} = \frac{Q_t}{\sigma}N_{X.t}^{\psi(\omega-1)-1}\widetilde{\rho}^{*1-\omega}_{hX.t}(1-\alpha)M^*_t - \mu_{X.t}f_X. \tag{37}$$

At the end, the average profits of among all Home originated firms (including domestic as well as exporting firms), $\widetilde{d}_{h.t}$ as follows:

$$\widetilde{d}_{h.t} = \widetilde{d}_{D.t} + \frac{N_{X.t}}{N_{D.t}}\widetilde{d}_{X.t}. \tag{38}$$

The similar conditions hold for the Foreign country.

2.2.6. Determination of the number of exporters and cutoff productivity

Using the average firm exporting productivity and the Pareto density function, the share of exporters in domestic total firms is given by:

$$\frac{N_{X.t}}{N_{D.t}} = z^k_{\min}\left(\widetilde{z}_{X.t}\right)^{-k}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{k}{\sigma-1}}. \tag{39}$$

There exists a 'cutoff level' firm which earns just 0 profits from exporting such as,

$$d_{X.t}(z_{X.t}) = \frac{Q_t}{\sigma} N_{X.t}^{\psi(\omega-1)-1} \rho_{hX.t}^{*1-\omega}(z_{X.t})(1-\alpha) M_t^* - \mu_{X.t} f_X = 0. \tag{40}$$

Using the average profits from exporting (37), the above condition is rewritten as,

$$\widetilde{d}_{X.t} = \mu_{X.t} f_X \frac{\sigma-1}{k-(\sigma-1)}. \tag{41}$$

(39) and (41) determine the number of exporters, $N_{X.t}$ and the average productivity of exporting firms, $\widetilde{z}_{X.t}$.

The similar conditions hold for Foreign country.

2.3. Labor markets clearings

In the end of the non-portfolio part of the model, we discuss labor markets clearings. L_t units of supplied labor are used in the production for domestic sales, for exporting sales (including fixed cost for exporting) and in firm creation activity. This implies:

$$L_t = N_{D.t} l_{D.t} + N_{X.t}(l_{X.t} + l_{FX.t}) + N_{E:t} l_{EM.t}, \tag{42}$$

where $l_{FX.t}$ is labor demand for fixed costs for exporting. Noting $l_{D.t} = (\sigma-1)\frac{\widetilde{d}_{D.t}}{w_t}$, $l_{X.t} + l_{FX.t} = (\sigma-1)\frac{\widetilde{d}_{X.t}}{w_t} + \sigma l_{FX.t}$, $l_{FX.t} = \frac{\mu_{X.t} f_X}{w_t}$ and $l_{EM.t} = \theta\frac{x_{h.t}^s}{wt}$ the above labor market clearing in the Home country can be rewritten as follows:

$$L_t = (\sigma-1)\frac{N_{D.t}\widetilde{d}_{h.t}}{w_t} + \theta\frac{N_{E.t}x_{h.t}^s}{w_t} + \sigma\frac{N_{X.t}\mu_{X.t}f_X}{w_t}. \tag{43}$$

The similar condition holds for the Foreign country.

3. Zero-order steady state portfolios

The indeterminacy problem of zero-order steady state portfolios is overcome by considering the second order approximation of Euler equations about asset holdings (Devereux and Sutherland (2008) and Tille and van Wincoop (2008)). These equations provide sufficient conditions to pin down the steady state portfolios. We employe the method

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developed by Devereux and Sutherland in computing numerically zero-order steady state portfolios. In doing so we express the budget constraint of the Home representative household (7) following Devereux and Sutherland (2008). Net foreign assets for the Home country at the end of period t $(NFA_{h.t+1})$ is defined as the sum of net foreign equity and bond positions as follows,

$$NFA_{h.t+1} \equiv s_{f.t+1} x_{f.t}^s \left(N_{D.t}^* + N_{E.t}^*\right) - s_{h.t+1}^* x_{h.t}^s \left(N_{D.t} + N_{E.t}\right) + b_{h.t+1} x_{h.t}^b + b_{f.t+1} x_{f.t}^b. \quad (44)$$

Thus the budget constraint can be rewritten as follows:

$$NFA_{h.t+1} = NX_{h.t} + NFA_{h.t} r_{h.t}^b + \xi_{h.t}, \quad (45)$$

where

$$NX_{h.t} \equiv N_{D.t} \tilde{\rho}_{hD.t} \tilde{y}_{hD.t} + N_{X.t} \tilde{\rho}_{hX.t} \tilde{y}_{hX.t} - C_t - N_{E.t} K_t, \quad (46)$$

$$\xi_{h.t} \equiv s_{f.t} x_{f.t-1}^s \left(N_{D.t-1}^* + N_{E.t-1}^*\right) \left(r_{f.t}^s - r_{h.t}^b\right)$$
$$- s_{h.t}^* x_{h.t-1}^s \left(N_{D.t-1} + N_{E.t-1}\right) \left(r_{h.t}^s - r_{h.t}^b\right) + b_{f.t} x_{f.t-1}^b \left(r_{f.t}^b - r_{h.t}^b\right). \quad (47)$$

$NX_{h.t}$ are net exports of the Home country. $\xi_{h.t}$ denotes "excess returns" on net foreign assets between $t-1$ and t relative to returns on Home CPI-indexed bond, $r_{h.t}^b$.

In addition, defining real holdings at the end of period t as,[7]

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[7]Real holdings (expressed in terms of Home consumption basket) for the Foreign representative households are,

$$a_{h.t}^{s*} \equiv s_{h.t+1}^* x_{h.t}^s \left(N_{D.t} + N_{E.t}\right), \quad a_{f.t}^{s*} \equiv s_{f.t+1}^* x_{f.t}^s \left(N_{D.t}^* + N_{E.t}^*\right), \quad (48)$$

and

$$a_{h.t}^{b*} \equiv b_{h.t+1}^* x_{h.t}^b, \quad a_{f.t}^{b*} \equiv b_{f.t+1}^* x_{f.t}^b. \quad (49)$$

$$a_{h.t}^s \equiv s_{h.t+1} x_{h.t}^s \left(N_{D.t} + N_{E.t} \right), \quad a_{f.t}^s \equiv s_{f.t+1} x_{f.t}^s \left(N_{D.t}^* + N_{E.t}^* \right), \tag{50}$$

$$a_{h.t}^b \equiv b_{h.t+1} x_{h.t}^b, \quad a_{f.t}^b \equiv b_{f.t+1} x_{f.t}^b. \tag{51}$$

So the excess returns can be rewritten in vector form as follows,

$$\xi_{h.t} \equiv a_{t-1}' r_{x.t}, \tag{52}$$

$$a_{t-1}' = \begin{bmatrix} a_{f.t-1}^s & -a_{h.t-1}^{s*} & a_{f.t-1}^b \end{bmatrix}, \tag{53}$$

$$r_{x.t}' = \begin{bmatrix} r_{fx.t}^s & r_{hx.t}^s & r_{hx.t}^b \end{bmatrix}, \tag{54}$$

where $r_{fx.t}^s \equiv r_{f.t}^s - r_{h.t}^b$, $r_{hx.t}^s \equiv r_{h.t}^s - r_{h.t}^b$ and $r_{hx.t}^b \equiv r_{f.t}^b - r_{h.t}^b$. Up to the first-order approximation, $E_t r_{x.t+1}'$ is a zero mean i.i.d shock. The above form of budget constraint (45) permits to compute necessary matrices for the solution of zero-order steady state portfolios.[8] Then we can apply directly the formula developed by Devereux and Sutherland (2008). Given these steady state portfolios, finally the first-order system contains 48 equations and 48 endogenously determined variables. The whole system including the steady state and its first-order equations is summarized in appendix.

4. The model without heterogeneity

It is particularly useful to consider a model without firm heterogeneity for the purpose of comparison. Imposing the symmetry among firms, the model collapses to the one analyzed in Hamano (2009). In this case, all firms export and there is no non-tradeness arising from firms' heterogeneity. In the benchmark system, by imposing $\widetilde{z}_D = \widetilde{z}_{X.t} = \widetilde{z}_D^* = \widetilde{z}_{X.t}^* = 1$ without loss of generality, $N_{D.t} = N_{X.t}$, $N_{D.t}^* = N_{X.t}^*$ and $f_X = 0$, we do not need any more the equations about pricing for export (because $\widetilde{\rho}_{hD.t} = \widetilde{\rho}_{hX.t}$ and

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[8] R_1, R_2, and D_1, D_2 matrices in notations of Devereux and Sutherland (2008). See their papers in detail.

$\widetilde{\rho}^*_{fD.t} = \widetilde{\rho}^*_{fX.t}$), real costs for exporting, share of exporting firms and zero-profit export cutoff conditions to make the model without heterogeneity. So we have eight less variables and eight less equations compared to the full system.

5. Calibration

We calibrate zero-order portfolios with following parameters:

Table 1: Baseline parametrization

γ	constant risk aversion	2
Υ	discount factor	0.96
υ	convergence speed	0.04
φ	Frisch elasticity of labor supply	2
σ	elasticity of substitution among varieties	7
ω	between Home and Foreign goods	6
α	home bias in consumption	0.85
δ	death shock	0.10
θ	share of labor in entry cost	0.64
k	Pareto distribution	6.60
ψ	love for variety	$1/(\sigma - 1)$
f_X	fixed cost for exporting	0.0068

Constant risk aversion (γ), discount factor (Υ), Frisch elasticity of labor supply (φ), Home bias in consumption (α) come from Coeurdacier et al. (2010). They are in the rage of the standard RBC literature. The parameter υ which governs the convergence speed of net foreign asset is set to 0.04. The death shock (δ) comes from Ghironi and Melitz (2005) where it is chosen to match the U.S. empirical level of 10 percent job destruction per year. The elasticity of substitution between local and imported goods (ω) is set to 6. This choice of elasticity may be considered too high compared to the low value used in the literature which typically ranges from 0.5 to 2. However, this is well in the rage of micro founded estimation in trade literature (Romalis (2007)). Recently Imbs and

Mejean (2009) argue the conventional estimation about the elasticity of substitution has downward bias without considering the heterogeneity of these values among sectors and propose the elasticity around 7. Given this value, the elasticity of substitution among varieties (σ) is set to 7. A great ambiguity surrounds the parameter ψ, the love for variety. We set it at $1/(\sigma - 1)$, the corresponding value to the standard Dixit-Stiglitz preference. We choose the share of labor (θ) in entry cost as 0.64 from Heathcote and Perri (2002) which estimates the labor and capital share in Cobb-Douglas production function. Again this is the standard value in the model including capital and labor in its production function. The parameter which shapes the Pareto distribution (k) is given by $k = 1/1.67 + \sigma - 1$ following Ghironi and Melitz (2005). We set f_X so that the 21% of firms export at the steady state. The fact that only 21% of firms engage in exporting activity comes originally from Bernard et al. (2003).

Finally productivity processes are taken from Coeurdacier et al. (2010) where they estimate them among industrialized countries. We define the vector of AR(1) process as $\mathsf{Z}_{t+1} = \Omega \mathsf{Z}_t + \epsilon_t$ where $\mathsf{Z}_t = \begin{bmatrix} Z_t, & Z_t^* & Z_{E.t}, & Z_{E.t}^* \end{bmatrix}'$, $\varepsilon_t = \begin{bmatrix} \varepsilon_{Z.t}, & \varepsilon_{Z.t}^* & \varepsilon_{Z_E.t}, & \varepsilon_{Z_E.t}^* \end{bmatrix}'$, and the matrix of Ω and the variance covariance matrix of innovations Σ are given by,

$$
\Omega = \begin{bmatrix} 0.75 & 0 & 0 & 0 \\ 0 & 0.75 & 0 & 0 \\ 0 & 0 & 0.79 & 0 \\ 0 & 0 & 0 & 0.79 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} 1.44 & 0.65 & 0 & 0 \\ 0.65 & 1.44 & 0 & 0 \\ 0 & 0 & 2.99 & 0.57 \\ 0 & 0 & 0.57 & 2.99 \end{bmatrix}. \tag{55}
$$

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6. Steady state portfolios

Given the above parameters, zero-order steady state portfolios are found as in Table 2 (Nominal bonds). For both specifications, with and without heterogenous firm, we

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[9] As it is mentioned in Devereux and Sutherland (2008), when the number of assets and the number of shocks are identical in the model, the steady state portfolios become independent from variance-covariance matrix of shock, Σ. And this is the case for our model.

observe aggressive home biased equity positions ($s = 4.02$ and 5.91) and relatively strong long positions for domestic bonds ($b = 3.72$ and $b = 3.18$).

Table 2: Steady state portfolios

	s	b
Nominal bonds		
With hetero	4.02	3.72
Without hetero	5.91	3.18
Real bonds		
With hetero	0.81	2.87
Without hetero	0.93	1.76

6.1. Nominal price and variety risk in real exchange rate fluctuations

Why the aggressive home biased equity positions appear? A crucial difference compared to the standard model which investigates portfolio positions is the existence of extensive margins. Fluctuations of extensive margins implies that households may consume different set of varieties across countries. As a result, they should insure not only against relative nominal price but also variety risk in their consumption baskets.

It must be well understood that in the present model the real exchange rate fluctuates with relative nominal price and relative number of available extensive margins. We can decompose its' first-order deviation as,

$$\mathsf{Q}_t = \widehat{\mathsf{Q}}_t + \psi R_v$$

where $\widehat{\mathsf{Q}}_t$ ($= \widehat{\mathsf{P}}_t^* - \widehat{\mathsf{P}}_t$) is the variation of nominal real exchange rate and R_v represents variations in the number of relative available varieties. Each term takes different expressions depending on whether there is firm heterogeneity or not.

Without firm heterogeneity, it becomes,

$$\mathsf{Q}_t = (2\alpha - 1)\,\mathsf{TOT}_t + \psi\,(2\alpha - 1)\,\mathsf{N}_{D.t}^R.$$

where terms of trade are defined as the price of imported in terms of exported goods as $\mathsf{TOT}_t = \widetilde{\mathsf{p}}^*_{fX.t} - \widetilde{\mathsf{p}}_{hX.t}$.

With firm heterogeneity it takes the form of,

$$\mathsf{Q}_t = (2S_{ED} - 1)\,\mathsf{TOT}_t - S_{ED}\widetilde{\mathsf{z}}^R_{\mathsf{X}.t} + \psi\,(2S_{ED} - 1)\,\mathsf{N}^R_{D.t} + \psi\,(1 - S_{ED})\left(\mathsf{N}^R_{D.t} - \mathsf{N}^R_{X.t}\right) \quad (56)$$

where $\mathsf{N}^R_{D.t} = \mathsf{N}_{D.t} - \mathsf{N}^*_{D.t}$, $\mathsf{N}^R_{X.t} = \mathsf{N}_{X.t} - \mathsf{N}^*_{X.t}$ and $\widetilde{\mathsf{z}}^R_{\mathsf{X}.t} = \widetilde{\mathsf{z}}_{\mathsf{X}.t} - \widetilde{\mathsf{z}}^*_{\mathsf{X}.t}$. $S_{ED} = \alpha\rho_H^{1-\omega}$ denotes the steady state expenditure share on domestic goods in total expenditure.[10]

Note in both cases when there is no love for variety ($\psi = 0$), the real exchange rate fluctuations are driven only by the nominal exchange rate, $\widehat{\mathsf{Q}}_t$. In general, for consumers both nominal exchange rate and variety terms are perceived as risk in real exchange rate fluctuations to be hedged using financial assets.

6.2. A sketch of hedging intuition using the static budget constraint

The intuition of portfolios would be best described relying on the first-order "static budget constraint" as Coeurdacier et al. (2010). This method consists to find portfolio positions which support complete market allocations by plugging the first-order perfect risk sharing condition in the first-order static budget constraint. For instance, in our model using the budget constraint (7) and (8), the first-order relative static budget constraint becomes,

$$\mathsf{P}_t + \mathsf{C}_t - (\mathsf{P}^*_t + \mathsf{C}^*_t) = S_W\left(\mathsf{w}^R_t + \mathsf{I}^R_t\right) + (2s - 1)\left[S_D\left(\mathsf{N}^R_{D.t} + \widetilde{\mathsf{d}}^R_t\right) - S_I\left(\mathsf{N}^R_{E.t} + \mathsf{x}^{sR}_t\right)\right] + 2b\left(\widehat{\mathsf{P}}_t - \widehat{\mathsf{P}}^*_t\right)$$
$$(57)$$

where S_W S_D and S_I denotes the steady state labor income, dividends and investment relative to the steady state nominal expenditure (see appendix for details). $\mathsf{w}^R_t + \mathsf{I}^R_t$, $\mathsf{N}^R_{D.t} + \widetilde{\mathsf{d}}^R_t$ and $\mathsf{N}^R_{E.t} + \mathsf{x}^{sR}_t$ represent the first-order relative nominal labor income, dividends

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[10]With firm heterogeneity, further sources contribute to the real exchange rate fluctuations because the number of imported extensive margins and its imported price would be different between two countries. By shutting down firm heterogeneity, two expressions of real exchange rate fluctuations become the same.

and investment respectively. In writing we also used financial market clearing conditions: $s_{h.t} + s_{h.t}^* = 1$, $s_{f.t} + s_{f.t}^* = 1$, $b_{h.t} + b_{h.t}^* = 0$ and $b_{f.t} + b_{f.t}^* = 0$ supposing that there exists steady state symmetric portfolios such as $s = s_{h.t} = s_{f.t}^*$, and $b = b_{h.t} = b_{f.t}^*$. The left hand side of the above first-order static budget constraint is relative nominal spending and right hand is relative nominal income generated by non-fiancial and financial assets.

Unfortunately spelling out the intuition of portfolio positions relying on the above static budget constraint cannot be applied here because the above first-order static-budget constraint fails to capture the first-order "period-by-period" dynamics. The reason is that investment costs enter as $S_I \left(\mathsf{N}_{E.t}^R + \mathsf{x}_t^{sR}\right)$ in equity returns $\left(S_D \left(\mathsf{N}_{D.t} + \widetilde{d}_t\right) - S_I \left(\mathsf{N}_{E.t} + \mathsf{x}_t^s\right)\right)$ for the static budget constraint while in the original first-order period-by-period dynamic, investment costs enter in equity returns $(\mathsf{r}_{h.t}^s)$ in the form of exogenous death shock $1 - \delta$.

Nevertheless, we find the usage of the static budget constraint useful by just "sketching" hedging mechanism at work. We follow Coeurdacier and Gourinchas (2008) for that purpose in the following demonstration. We redefine the above static budget constraint as,

$$\mathsf{P}_t + \mathsf{C}_t - \left(\mathsf{P}_t^* + \mathsf{C}_t^*\right) = S_W R_W + \left(2s - 1\right) R_e + 2b R_b. \tag{58}$$

R_W, R_e and R_b represent relative nominal labor income, relative equity and relative bond returns respectively. Plugging the perfect risk sharing condition which holds under complete financial markets,

$$\left(1 - \frac{1}{\gamma}\right) \left(\mathsf{P}_t - \mathsf{P}_t^*\right) = S_W R_W + \left(2s - 1\right) R_e + 2b R_b, \tag{59}$$

which says that under complete market, households in the Home country spend more when there is inflation because $\gamma \geq 1$. Any types of perturbations in the real exchange rate $(\mathsf{Q}_t = \mathsf{P}_t^* - \mathsf{P}_t)$, labor income (R_W), equity (R_e) and bond returns (R_b) must be balanced by an appropriate equity and bond position, s and b.

In the original period-by-period dynamic, the real exchange rate, labor income, equity and bond returns are endogenously determined depending on four exogenous shocks. In sketching the general equilibrium relationship among these variables, we posit the

following first-order static relations among them,

$$R_W = (\lambda - 1) \widehat{\mathsf{Q}}_t + \phi_{w,\epsilon} \epsilon_t, \tag{60}$$

$$R_e = (\lambda - 1) \widehat{\mathsf{Q}}_t + \phi_{e,\epsilon} \epsilon_t, \tag{61}$$

$$R_b = -\widehat{\mathsf{Q}}_t, \tag{62}$$

$$R_v = \phi_{v,q} \widehat{\mathsf{Q}}_t + \phi_{v,\epsilon} \epsilon_t, \tag{63}$$

where λ roughly represents the elasticity of substitution between local and imported goods ω and ϵ_t represents "investment shock" as discussed in Coeurdacier and Gourinchas (2008). Finally ϕ_{ij} denotes conditional covariances such as $\phi_{ij} \equiv cov(i, j)/var(j)$.

Provided these relations, it is easy to solve the optimal portfolio positions which replicates the complete market allocation. Plugging the above relations in (59), we find the following optimal equity and bond positions.

$$s = \frac{1}{2} \left[1 - \psi \frac{\phi_{v,\epsilon}}{\phi_{e,\epsilon}} \left(1 - \frac{1}{\gamma} \right) - S_w \frac{\phi_{w,\epsilon}}{\phi_{e,\epsilon}} \right], \tag{64}$$

$$b = \frac{1}{2} \left\{ \left(1 - \frac{1}{\gamma} \right) (1 + \psi \phi_{v,q}) + (\lambda - 1) \left[S_w \left(1 - \frac{\phi_{w,\epsilon}}{\phi_{e,\epsilon}} \right) - \psi \frac{\phi_{v,\epsilon}}{\phi_{e,\epsilon}} \left(1 - \frac{1}{\gamma} \right) \right] \right\}. \tag{65}$$

As it has been mentioned, particularities of the portfolio in this paper are related to the variety risk in real exchange rate. This variety risk disappear when there is no love for variety ($\psi = 0$) or when there exist "welfare-based CPI-indexed real bonds" which load perfectly on the (welfare-based) real exchange rate including variety risk. For both cases, the portfolio positions are identical to those presented in Coeurdacier and Gourinchas (2008).

For the equity position, what may induce a home biased position is the third term in the square blanket. When $\phi_{w,\epsilon} > 0$ and $\phi_{e,\epsilon} < 0$, in word, when labor income rises and

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equity returns decrease with a positive investment shock, the third term drives the home biased equity position ($s > 1/2$). This mechanism is identical to the one originally explored in Heathcote and Perri (2004). In addition, with love for variety, the second term also may contribute to the home biased equity position. This is the case when $\phi_{v,\epsilon} > 0$ provided $\phi_{e,\epsilon} < 0$, the number of available extensive margins rises with positive investment shock. In the equity position, while the third term hedges against labor income risk induced by investment shock, the second term does against variety risk induced by real exchange rate fluctuations

For the bond position, again, principal characters are identical to those discussed in Coeurdacier and Gourinchas (2008). The first term in the large blanket factorized by $\left(1 - \frac{1}{\gamma}\right)$ captures the hedge against real exchange rate fluctuations. When nominal exchange rate appreciates (for example due to terms of trade appreciation), nominal bond returns R_b increases as well because they are correlated perfectly. Thus from this stand point, having a long position ($b > 0$) becomes a good hedge against the nominal exchange rate risk. But the same nominal exchange rate appreciation may be accompanied by fluctuations in extensive margins. When the number of extensive margins rises with nominal appreciation, $\phi_{v,q} < 0$, because a higher number of available extensive margins means a real deflation, taking a short position ($b < 0$) becomes a good hedge. When $\phi_{v,q} > 0$, the opposite is true: taking a long position ($b > 0$) becomes a good hedge against this variety risk. Coeurdacier et al. (2007) analyze the similar term in the form of exogenous preference shock ("i-pod shock"). The second terms factorized by $\lambda - 1$ is the hedge against labor income and fluctuations in equity returns induced by nominal exchange rate fluctuations. As Coeurdacier and Gourinchas (2008), this term is sensitive to the elasticity of substitution between local and imported goods captured by λ. Note in particular, when $\phi_{v,\epsilon} > 0$ and $\phi_{e,\epsilon} < 0$, with relatively high λ (> 1), the second term provides a further long position for domestic bonds from variety risk.

6.3. The original period-by-period dynamic

Although the above analysis using the static budget constraint remains just a sketch, it should be a nice sketch To confirm, in Table 3, we report the conditional covariances ϕ_{ij} in

the original period-by-period dynamic. These are computed with benchmark parameters in terms of relative original investment shocks ($\epsilon = \mathsf{Z}_{E.t} - \mathsf{Z}_{E.t}^*$), nominal labor income ($R_W = \mathsf{w}_t + \mathsf{L}_t - (\mathsf{Q}_t + \mathsf{w}_t^* + \mathsf{L}_t^*)$), nominal equity returns ($R_e = \mathsf{r}_{hx.t+1}^s - \mathsf{r}_{fx.t+1}^s$), extensive margins ($R_v = S_{ED}\mathsf{N}_{D.t}^R - (1 - S_{ED})\mathsf{N}_{X.t}^R$), and nominal exchange rate ($\widehat{\mathsf{Q}}_t$) for both specifications, with and without heterogenous firms.[11]

Table 3: Conditional covariances in the model

	$\phi_{w,\epsilon}$	$\phi_{v,\epsilon}$	$\phi_{e,\epsilon}$	$\phi_{v,q}$
With hetero	0.23	0.16	-0.94	-0.16
Without hetero	0.24	0.10	-0.93	0.04

As it has been discussed in the previous subsection, home biased equity positions appear when $\phi_{w,\epsilon} > 0$ and variety risk adds further home biased position when $\phi_{v,\epsilon} > 0$ provided $\phi_{e,\epsilon} < 0$. The signs of conditional covariances in the original period-by-period dynamics are matched to these ones. For the bond position, the sign of $\phi_{v,q}$ is different depending on the model. This means that the impact of variety risk in real exchange rate hedging becomes also model dependant (first term in (65)) while overall impact of variety risk depends also on the hedge against labor income and equity returns (second term in (65)).

The above conditional covariances are also intuitively be captured by looking for the impulse response functions. Figure 1 shows the impulse response functions with 1% rise of $\varepsilon_{Z_E.t} - \varepsilon_{Z_E^*.t}$, for R_W, R_e, R_v, and $\widehat{\mathsf{Q}}_t$. With such a positive investment shock, new entry rises on impact. Equity returns decrease to finance their costs of creation while labor demand increases by these new entrants bringing wage and nominal real exchange rate into appreciation. Along the time, the number of firms rises gradually. The welfare based real exchange rate, Q_t follows this dynamic because it largely reflects variations in extensive margins. Net export decreases on impact as well because of investment shock

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[11]Conditional covariances are computed using frequency-domain technique proposed in Uhlig (1998). Variables are filtered by Hodrick-Prescott filter. Smoothing parameter is set to 100 with which we compute also the following impulse response functions.

but become positive after by exporting new extensive margins. In spite of such a recovery in net export, net foreign asset positions remain declined and take a long time to get back to its initial steady state value.

The above numerical results would show the validity of hedging intuition at work described using the static budget constraint.

6.4. Portfolio positions with real bonds

We find aggressive home biased equity positions because nominal bonds cannot load on variety risk. This is to say, if there exist counter-factual welfare-based CPI indexed real bonds which load perfectly on the real exchange rate risk (Q_t), these additional home biased equity positions should disappear. Indeed, this is the case: under benchmark parameters, equity and bond positions become, $s = 0.81$ and $b = 2.87$ with real bonds. With the version of the model without heterogenous firms, they become $s = 0.93$ and $b = 1.76$. Both home biased equity and long bond positions are boiled down.

The point is seen also clearly with a sensitivity analysis. As it is analyzed for instance in Coeurdacier (2009), hedging against real exchange rate risk using equity position means excessive sensitivity of equity position against certain parameters, in particular the elasticity of substitution between local and imported goods, ω. Through Figure 2 to Figure 3, we show the comparison of portfolio positions with two types of bonds, nominal and real bonds for each model with and without heterogeneity against various values of ω. Loading on welfare-based real exchange rate risk, equity positions show sensitivities against ω for both specifications, with and without firm heterogeneity.

7. Conclusion

We analyze zero-order steady state equity and bond portfolios in a two-country DSGE model where the number of available varieties is endogenous. With variety risk in real exchange rate fluctuations, home biased equity positions are amplified further. The result is shown to be robust with or without firm heterogeneity.

For future research, it would be interesting to incorporate another source of disturbances (for instance, a monetary shock) and investigate portfolios in incomplete markets

setting. One another direction is to calibrate the model with higher order approximation, hence to analyze portfolio rebalancing and valuation effect (Gourinchas and Rey (2007), Devereux and Sutherland (2010)). Because extensive margins might have impact on current account adjustment problem as well (Corsetti et al. (2008)), consider the problem with 'valuation effect with extensive margins' would be very interesting.
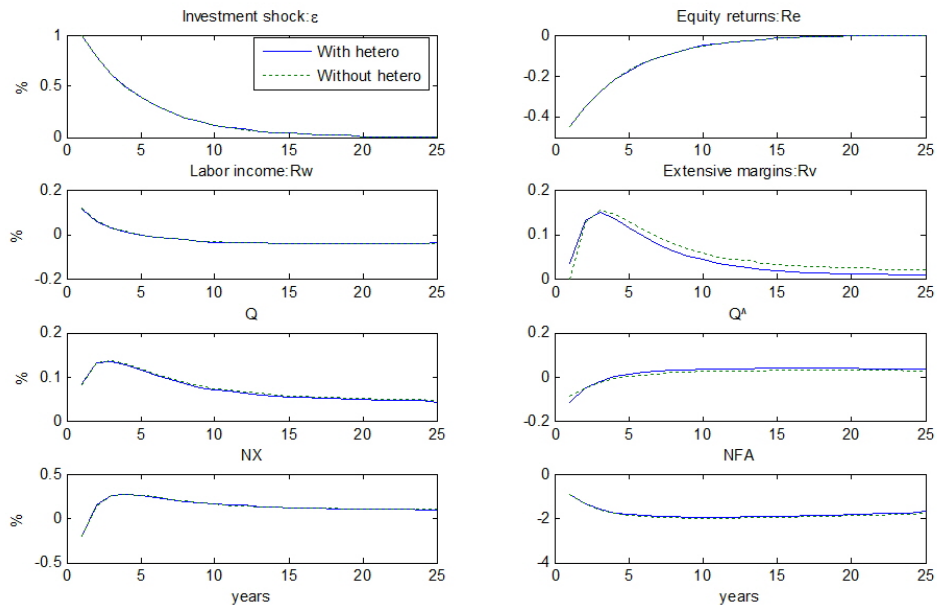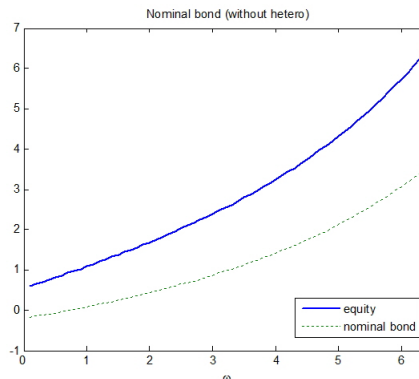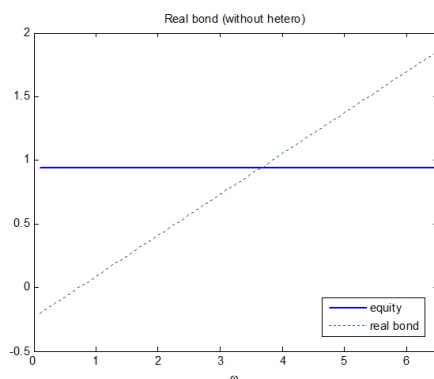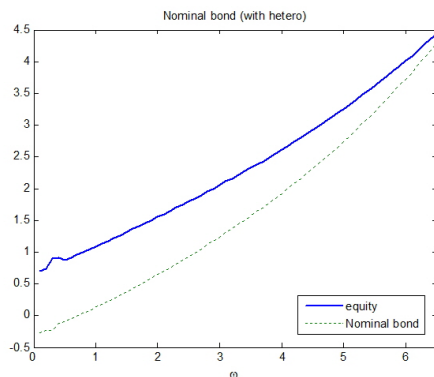
Figure 1
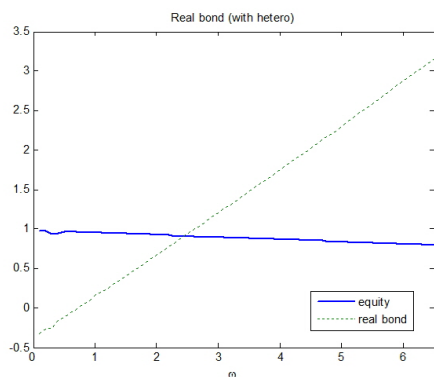
Figure 2   Figure 3

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Appendix A. System

Price indices

$$\alpha \rho_{H.t}^{1-\omega} + (1-\alpha) \rho_{F.t}^{1-\omega} = 1 \tag{A.1}$$

$$\rho_{H.t} = N_{D.t}^{-\psi} \widetilde{\rho}_{hD.t}, \qquad \rho_{F.t} = N_{X.t}^{*-\psi} \widetilde{\rho}_{fX.t} \tag{A.2}$$

$$\alpha \rho_{F.t}^{*1-\omega} + (1-\alpha) \rho_{H.t}^{*1-\omega} = 1 \tag{A.3}$$

$$\rho_{F.t}^* = N_{D.t}^{*-\psi} \widetilde{\rho}_{fD.t}^*, \qquad \rho_{H.t}^* = N_{X.t}^{-\psi} \widetilde{\rho}_{hX.t}^* \tag{A.4}$$

Pricing

$$\widetilde{\rho}_{hD.t} = \frac{\sigma}{\sigma-1} \frac{w_t}{Z_t \widetilde{z}}, \qquad \widetilde{\rho}_{hX.t} = \frac{\sigma}{\sigma-1} \frac{w_t}{Z_t \widetilde{z}_{X.t}} \qquad \widetilde{\rho}_{hX.t}^* = Q_t^{-1} \widetilde{\rho}_{hX.t} \tag{A.5}$$

$$\widetilde{\rho}_{fD.t}^* = \frac{\sigma}{\sigma-1} \frac{w_t^*}{Z_t^* \widetilde{z}_D^*}, \qquad \widetilde{\rho}_{fX.t}^* = \frac{\sigma}{\sigma-1} \frac{w_t^*}{Z_t^* \widetilde{z}_{X.t}^*} \qquad \widetilde{\rho}_{fX.t} = Q_t \widetilde{\rho}_{fX.t}^* \tag{A.6}$$

Profits

$$\widetilde{d}_{D.t} = \frac{1}{\sigma} N_{D.t}^{\psi(\omega-1)-1} \widetilde{\rho}_{hD.t}^{1-\omega} \alpha M_t \tag{A.7}$$

$$\widetilde{d}_{X.t} = \frac{Q_t}{\sigma} N_{X.t}^{\psi(\omega-1)-1} \widetilde{\rho}_{hX.t}^{*1-\omega} (1-\alpha) M_t^* - \mu_{X.t} f_X \tag{A.8}$$

$$N_{D.t} \widetilde{d}_{h.t} = N_{D.t} \widetilde{d}_{hD.t} + N_{X.t} \widetilde{d}_{hX.t} \tag{A.9}$$

$$\widetilde{d}_{D.t}^* = \frac{1}{\sigma} N_{D.t}^{*\psi(\omega-1)-1} \widetilde{\rho}_{fD.t}^{*1-\omega} \alpha M_t^* \tag{A.10}$$

$$\widetilde{d}^*_{X.t} = \frac{Q_t^{-1}}{\sigma} N_{X.t}^{*\psi(\omega-1)-1} \widetilde{\rho}_{fX.t}^{1-\omega} (1-\alpha) M_t - \mu^*_{X.t} f^*_X \tag{A.11}$$

$$N^*_{D.t} \widetilde{d}^*_{f.t} = N^*_{D.t} \widetilde{d}^*_{D.t} + N^*_{X.t} \widetilde{d}^*_{X.t} \tag{A.12}$$

Definition of M

$$M_t = C_t + (1-\theta) N_{E.t} x^s_{h.t} \tag{A.13}$$

$$M^*_t = C^*_t + (1-\theta) N^*_{E.t} x^{s*}_{f.t} \tag{A.14}$$

Real cost for exporting

$$\mu_{X.t} = \frac{w_t}{Z_t} \tag{A.15}$$

$$\mu^*_{X.t} = \frac{w^*_t}{Z^*_t} \tag{A.16}$$

Free entry

$$x^s_{h.t} = \left(\frac{w_t}{Z_{E.t}} \right)^\theta f_E \tag{A.17}$$

$$x^{s*}_{f.t} = \left(\frac{w^*_t}{Z^*_{E.t}} \right)^\theta f^*_E \tag{A.18}$$

Number of firms

$$N_{D.t+1} = (1-\delta)(N_{D.t} + N_{E.t}) \tag{A.19}$$

$$N^*_{D.t+1} = (1-\delta)(N^*_{D.t} + N^*_{E.t}) \tag{A.20}$$

Optimal labor supply

$$\chi (L_t)^{\frac{1}{\psi}} = w_t C_t^{-\gamma} \tag{A.21}$$

$$\chi \left(L_t^*\right)^{\frac{1}{\psi}} = w_t^* C_t^{*-\gamma} \tag{A.22}$$

Labor Market clearing conditions

$$L_t = (\sigma - 1) \frac{N_{D.t} \widetilde{d}_{h.t}}{w_t} + \theta \frac{N_{E.t} x_{h.t}^s}{w_t} + \sigma \frac{N_{X.t} \mu_{X.t} f_X}{w_t} \tag{A.23}$$

$$L_t^* = (\sigma - 1) \frac{N_{D.t}^* \widetilde{d}_{f.t}^*}{w_t^*} + \theta \frac{N_{E.t}^* x_{f.t}^{s*}}{w_t^*} + \sigma \frac{N_{X.t}^* \mu_{X.t}^* f_X^*}{w_t^*} \tag{A.24}$$

Share of exporting firms

$$\frac{N_{X.t}}{N_{D.t}} = z_{\min}^k \left(\widetilde{z}_{X.t}\right)^{-k} \left[\frac{k}{k - (\sigma - 1)}\right]^{\frac{k}{\sigma - 1}} \tag{A.25}$$

$$\frac{N_{X.t}^*}{N_{D.t}^*} = z_{\min}^k \left(\widetilde{z}_{X.t}^*\right)^{-k} \left[\frac{k}{k - (\sigma - 1)}\right]^{\frac{k}{\sigma - 1}} \tag{A.26}$$

Zero-profit export cutoff

$$\widetilde{d}_{X.t} = \mu_{X.t} f_X \frac{\sigma - 1}{k - (\sigma - 1)} \tag{A.27}$$

$$\widetilde{d}_{X.t}^* = \mu_{X.t}^* f_X^* \frac{\sigma - 1}{k - (\sigma - 1)} \tag{A.28}$$

Definition of real returns (expressed in Home consumption basket)

$$r_{h.t}^s \equiv (1 - \delta) \frac{x_{h.t}^s + \widetilde{d}_{h.t}}{x_{h.t-1}^s}, \tag{A.29}$$

$$r_{f.t}^s \equiv (1 - \delta) \frac{x_{f.t}^s + \widetilde{d}_{f.t}}{x_{f.t-1}^s}, \tag{A.30}$$

$$r_{h.t}^b \equiv \frac{x_{h.t}^b + \frac{\widehat{P}_t}{P_t}}{x_{h.t-1}^b}, \tag{A.31}$$

$$r_{f.t}^b \equiv \frac{x_{f.t}^b + \frac{\widehat{P_t^*}}{P_t}}{x_{f.t-1}^b}. \tag{A.32}$$

Asset markets clearing conditions

$$a_{h.t}^s + a_{h.t}^{s*} = x_{h.t}^s \left(N_{D.t} + N_{E.t}\right) \tag{A.33}$$

$$a_{f.t}^s + a_{f.t}^{s*} = x_{f.t}^s \left(N_{D.t}^* + N_{E.t}^*\right) \tag{A.34}$$

$$a_{h.t}^b + a_{h.t}^{b*} = 0 \tag{A.35}$$

$$a_{f.t}^b + a_{f.t}^{b*} = 0 \tag{A.36}$$

Euler Home and Foreign

$$C_t^{\gamma-\upsilon} E_t C_{t+1}^{-\gamma} = C_t^{*\gamma-\upsilon} E_t C_{t+1}^{*-\gamma} \frac{Q_t}{Q_{t+1}} \tag{A.37}$$

Euler shares

(Home)

$$C_t^{\upsilon-\gamma} = \overline{\beta} C_t^{-\upsilon} E_t C_{t+1}^{-\gamma} r_{h.t+1}^s \tag{A.38}$$

$$C_t^{\upsilon-\gamma} = \overline{\beta} C_t^{-\upsilon} E_t C_{t+1}^{-\gamma} r_{f.t+1}^s \tag{A.39}$$

Euler equation (bond)

(Home)

$$C_t^{\upsilon-\gamma} = \overline{\beta} C_t^{-\upsilon} E_t C_{t+1}^{-\gamma} r_{h.t+1}^b \tag{A.40}$$

$$C_t^{\upsilon-\gamma} = \overline{\beta} C_t^{-\upsilon} E_t C_{t+1}^{-\gamma} r_{f.t+1}^b \tag{A.41}$$

Definition of the expected excess returns

$$E_t r_{fx.t+1}^s = E_t \left[r_{f.t+1}^s - r_{h.t+1}^b\right] \tag{A.42}$$

$$E_t r^s_{hx.t+1} = E_t \left[r^s_{h.t+1} - r^b_{h.t+1} \right] \tag{A.43}$$

$$E_t r^b_{fx.t+1} = E_t \left[r^b_{f.t+1} - r^b_{h.t+1} \right] \tag{A.44}$$

Net foreign asset dynamics of Home

$$NFA_{h.t+1} = NX_{h.t} + NFA_{h.t} r^b_{h.t} + a^s_{f.t-1} r^s_{fx.t} - a^{s*}_{h.t-1} r^s_{hx.t} + a^b_{f.t-1} r^b_{fx.t} \tag{A.45}$$

Net export for Home

$$NX_{h.t} = \sigma \left(N_{D.t} \widetilde{d}_{h.t} + N_{X.t} \mu_{X.t} f_X \right) - M_t \tag{A.46}$$

Appendix B. Steady state

At the symmetric steady state, it is noticed that $NFA_h = NX_h = 0$. We choice the parameter χ so that the steady state labor supply $L = 1$. The parameter $\overline{\beta}$ is chosen as it guarantees $\Upsilon(C) = 0.96$. Following Ghironi and Melitz (2005) we determine the steady state fixed cost for exporting f_X so that it gives 21% share of exporting firms as documented in Bernard et al. (2003). As it will be clear the solution procedure is a little bit more complicated thanGhironi and Melitz (2005) because here $1 - \theta$ fraction of entry cost is paid in terms of consumption goods. For the purpose of comparison I follow the solution procedure presented in technical appendix of Ghironi and Melitz (2005) which is available on authors' web site. The goal is to reduce the model into 3 equations and 3 variables.

At the symmetric steady state, we suppose without loss of generality $Z = Z^* = f_E = f_E^* = z_{\min} = z^*_{\min} = 1$. Free entry condition gives $x^s = w^\theta f_E$ and motion of the firm gives $N_D = \frac{1-\delta}{\delta} N_E$. Knowing $\widetilde{d}_h = \widetilde{d}_D + \frac{N_X}{N_D} \widetilde{d}_X$ combined with Euler share equation,

$$\widetilde{d}_D + \frac{N_X}{N_D} \widetilde{d}_X = \frac{1 - \Upsilon(1-\delta)}{\Upsilon(1-\delta)} w^\theta f_E \tag{B.1}$$

The first equation will be given by rewriting the above equation. Knowing $\mu_X = wf_X$ from zero-profits-cut-off condition,

$$\widetilde{d}_X = wf_X \frac{\sigma - 1}{k - (\sigma - 1)} \tag{B.2}$$

With above expression using the steady state average domestic and exporting profits $\widetilde{d}_D$ and $\widetilde{d}_X$ I can write

$$\widetilde{d}_D = \frac{S_{ED}}{S_{EX}} \frac{N_X}{N_D} \left[\frac{\sigma - 1}{k - (\sigma - 1)} + 1 \right] wf_X \tag{B.3}$$

where $S_{ED} \equiv \alpha \rho_H^{1-\omega}$ and $S_{EX} \equiv (1 - \alpha) \rho_F^{1-\omega}$, which are the steady state share on domestic and imported goods in total consumption. Noting $\widetilde{\rho}_H = N_D^{-\psi} \widetilde{\rho}_{hD}$ and $\widetilde{\rho}_F = N_X^{-\psi} \widetilde{\rho}_{fX}^*$ and further $\widetilde{\rho}_{hD} = \frac{\sigma}{\sigma - 1} \frac{w}{\widetilde{z}_D}$ and $\widetilde{\rho}_{fX}^* = \frac{\sigma}{\sigma - 1} \frac{w}{\widetilde{z}_X}$, S_{ED}/S_{EX} is turned out to be

$$\frac{S_{ED}}{S_{EX}} = \frac{\alpha}{1 - \alpha} \left(\frac{N_D}{N_X} \right)^{-\psi(1-\omega)} \left(\frac{\widetilde{z}_X}{\widetilde{z}_D} \right)^{1-\omega} \tag{B.4}$$

Plugging (B.3) and (B.2) into (B.1) :

$$\frac{\alpha}{1 - \alpha} \left(\frac{N_X}{N_D} \right)^{1-\psi(\omega-1)} \left(\frac{\widetilde{z}_X}{\widetilde{z}_D} \right)^{1-\omega} \left[\frac{k}{k - (\sigma - 1)} \right] + \frac{N_X}{N_D} \frac{\sigma - 1}{k - (\sigma - 1)}$$
$$= \frac{1 - \Upsilon (1 - \delta)}{\Upsilon (1 - \delta)} \frac{f_E}{f_X} w^{\theta - 1} \tag{B.5}$$

The next equation is constructed from aggregated identity, zero-profits-cut-off condition and price index. At the symmetric steady state we have $C + N_E x^s = w + N_D \widetilde{d}_h$. Eliminating C by $M = C + (1 - \theta) N_E x^s$ and using $x^s = w^\theta f_E$ and (B.1) the aggregated identity can be expressed as:

$$\frac{M}{w} = 1 + \frac{1 - \Upsilon + \Upsilon \delta (1 - \theta)}{\Upsilon (1 - \delta)} N_D w^{\theta - 1} f_E \tag{B.6}$$

Next using zero-profits-cut-off condition (B.2) and the steady state expression of $\widetilde{d}_X$,

$$\frac{M}{w} = \frac{1}{1 - \alpha} N_X^{1-\psi(\omega-1)} \left(\frac{\sigma}{\sigma - 1} \frac{w}{\widetilde{z}_X} \right)^{\omega - 1} \frac{k\sigma}{k - (\sigma - 1)} f_X \tag{B.7}$$

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Dividing both side of (B.6) and (B.7) by $N_D^{1-\psi(\omega-1)}$ and equating it yields

$$\frac{1}{1-\alpha}\left(\frac{N_X}{N_D}\right)^{1-\psi(\omega-1)}\left(\frac{\sigma}{\sigma-1}\frac{w}{\widetilde{z}_X}\right)^{\omega-1}\frac{k\sigma}{k-(\sigma-1)}f_X$$
$$= \frac{1}{N_D^{1-\psi(\omega-1)}}+\frac{1-\Upsilon+\Upsilon\delta(1-\theta)}{\Upsilon(1-\delta)}N_D^{\psi(\omega-1)}w^{\theta-1}f_E \quad \text{(B.8)}$$

At the end using (B.4) from the price index,

$$\frac{\left(\frac{\sigma}{\sigma-1}\frac{w}{\widetilde{z}_X}\right)^{\omega-1}}{N_D^{\psi(\omega-1)}}=\alpha\left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{1-\omega}+(1-\alpha)\left(\frac{N_X}{N_D}\right)^{\psi(\omega-1)} \quad \text{(B.9)}$$

There are 3 equations, (B.5), (B.8) and (B.9). $\widetilde{z}_D$ is given by Pareto distribution. $\frac{N_X}{N_D}$ is set to 0.21. This requires $\widetilde{z}_X = 1.89$ with baseline parameters. These non-linear equations are solved (using computer) for N_D, w and f_X. Especially the share of fixed cost for exporting in amortized entry cost becomes 5.9%.

Other variables are easy to be found.

$$M = w + \frac{1-\Upsilon+\Upsilon\delta(1-\theta)}{\Upsilon(1-\delta)}N_D w^\theta f_E \quad \text{(B.10)}$$

$$N_X = \frac{21}{100}N_D \quad \text{(B.11)}$$

$$N_E = \frac{\delta}{1-\delta}N_D \quad \text{(B.12)}$$

$$v = w^\theta f_E \quad \text{(B.13)}$$

$$C = M - (1-\theta)N_E v \quad \text{(B.14)}$$

$$\widetilde{\rho}_{hD} = \frac{\sigma}{\sigma-1}\frac{w}{\widetilde{z}} \text{ and } \widetilde{\rho}_{fX} = \frac{\sigma}{\sigma-1}\frac{w}{\widetilde{z}_X} \quad \text{(B.15)}$$

$$\rho_H = N_D^{-\psi}\widetilde{\rho}_{hD} \text{ and } \rho_F = N_X^{*-\psi}\widetilde{\rho}_{fX} \quad \text{(B.16)}$$

Finally the value of parameter χ is set by $\chi = wC^{-\gamma}$.

Appendix B.1. Steady state ratios

The expenditure share on domestic and imported goods are

$$S_{ED} = \alpha \rho_H^{1-\omega} \text{ and } S_{EX} = 1 - S_{ED} = (1 - \alpha) \rho_F^{1-\omega} \tag{B.17}$$

The steady state ratio of fixed exporting cost relative to M is

$$S_{FX}^M = \frac{N_X w f_X}{M} \tag{B.18}$$

The share of dividends on domestic sales is

$$S_{DD}^M = \frac{N_D \widetilde{d}_D}{M} = \frac{1}{\sigma} S_{ED} \tag{B.19}$$

That of exporting sales is

$$S_X^M = \frac{N_X \widetilde{d}_X}{M} = \frac{1}{\sigma} S_{EX} - S_{FX}^M \tag{B.20}$$

The share of total dividends relative to M is

$$S_D^M = \frac{N_D \widetilde{d}}{M} = \frac{1}{\sigma} - S_{FX}^M \tag{B.21}$$

Note in passage the share of dividends from domestic and exporting sales relative to total dividends are expressed respectively

$$\frac{S_{DD}^M}{S_D^M} = \kappa S_{ED} \text{ and } \frac{S_X^M}{S_D^M} = \kappa S_{EX} \tag{B.22}$$

where

$$\kappa \equiv \frac{\frac{1}{\sigma}}{\frac{1}{\sigma} - S_{FX}^M} \tag{B.23}$$

The share of investment relative to M is

$$S_I^M = \frac{N_E x^s}{M} = \frac{\Upsilon \delta}{1 - \Upsilon (1 - \delta)} S_D^M \tag{B.24}$$

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That of wage is given by

$$S_W^M = \frac{w}{M} \tag{B.25}$$

That of consumption is given by

$$S_C^M = \frac{C}{M} = 1 - (1 - \theta) S_I^M \tag{B.26}$$

Steady state ratios relative to consumption are

Noting $M = C + (1 - \theta) N_E x_h^s$ using the above steady state ratios defined relative to M, the steady state ratios relative to the consumption becomes

$$S_I = \frac{N_E x^s}{C} = \frac{S_I^M}{1 - (1 - \theta) S_I^M}$$

$$S_D = \frac{N_D \widetilde{d}}{C} = S_D^M \left[1 + (1 - \theta) S_I\right] \tag{B.27}$$

and from aggregated identity at the symmetric steady state

$$S_W = \frac{w}{C} = 1 + S_I - S_D$$

Finally using labor market clearing

$$S_{FX} = \frac{N_X w f_X}{C} = \frac{1}{\sigma} \left[S_W - (\sigma - 1) S_D - \theta S_I\right] \tag{B.28}$$

Appendix C. First-order system

<u>Price indices</u>

$$\alpha \rho_H + (1 - \alpha) \rho_{F.t} = 0 \tag{C.1}$$

$$\rho_{H.t} = -\psi \mathsf{N}_{D.t} + \widetilde{\rho}_{hD.t}, \qquad \rho_{F.t} = -\psi \mathsf{N}_{X.t}^* + \widetilde{\rho}_{fX.t} \tag{C.2}$$

$$\alpha \rho_{F.t}^* + (1 - \alpha) \rho_{H.t}^* = 0 \tag{C.3}$$

$$\rho_{F.t}^* = -\psi N_{D.t}^* + \widetilde{\rho}_{fD.t}^*, \qquad \rho_{H.t}^* = -\psi N_{X.t} + \widetilde{\rho}_{hX.t}^* \tag{C.4}$$

Pricing

$$\widetilde{\rho}_{hD.t} = w_t - Z_t, \qquad \widetilde{\rho}_{hX.t} = w_t - Z_t - \widetilde{z}_{X.t} \qquad \widetilde{\rho}_{hX.t}^* = -Q_t + \widetilde{\rho}_{hX.t} \tag{C.5}$$

$$\widetilde{\rho}_{fD.t}^* = w_t^* - Z_t^*, \qquad \widetilde{\rho}_{fX.t}^* = w_t^* - Z_t^* - \widetilde{z}_{X.t}^* \qquad \widetilde{\rho}_{fX.t} = Q_t + \widetilde{\rho}_{fX.t}^* \tag{C.6}$$

Profits

(Home)

$$N_{D.t} + \widetilde{d}_{D.t} = \psi\left(\omega - 1\right) N_{D.t} + \left(1 - \omega\right)\widetilde{\rho}_{hD.t} + M_t \tag{C.7}$$

$$N_{X.t} + \widetilde{d}_{X.t} = Q_t + \psi\left(\omega - 1\right) N_{X.t} + \left(1 - \omega\right)\widetilde{\rho}_{hX.t}^* + M_t^* \tag{C.8}$$

$$N_{D.t} + \widetilde{d}_{h.t} = \kappa S_{ED}\left(N_{D.t} + \widetilde{d}_{D.t}\right) + \left(1 - \kappa S_{ED}\right)\left(N_{X.t} + \widetilde{d}_{X.t}\right) \tag{C.9}$$

(Foreign)

$$N_{D.t}^* + \widetilde{d}_{D.t}^* = \psi\left(\omega - 1\right) N_{D.t}^* + \left(1 - \omega\right)\widetilde{\rho}_{fD.t}^* + M_t^* \tag{C.10}$$

$$N_{X.t}^* + \widetilde{d}_{X.t}^* = -Q_t + \psi\left(\omega - 1\right) N_{X.t}^* + \left(1 - \omega\right)\widetilde{\rho}_{fX.t} + M_t \tag{C.11}$$

$$N_{D.t}^* + \widetilde{d}_{f.t}^* = \kappa S_{ED}\left(N_{D.t}^* + \widetilde{d}_{D.t}^*\right) + \left(1 - \kappa S_{ED}\right)\left(N_{X.t}^* + \widetilde{d}_{X.t}^*\right) \tag{C.12}$$

Definition of M

$$M_t = S_C^M C_t + \left(1 - \theta\right) S_I^M \left(N_{E.t} + x_{h.t}^s\right) \tag{C.13}$$

$$M_t^* = S_C^M C_t^* + \left(1 - \theta\right) S_I^M \left(N_{E.t}^* + x_{f.t}^{s*}\right) \tag{C.14}$$

Real cost for exporting

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$$\mu_{X.t} = w_t - Z_t \tag{C.15}$$

$$\mu_{X.t}^* = w_t^* - Z_t^* \tag{C.16}$$

Free entry

$$x_{h.t}^s = \theta\left(w_t - Z_{E.t}\right) \tag{C.17}$$

$$x_{f.t}^{s*} = \theta\left(w_t^* - Z_{E.t}^*\right) \tag{C.18}$$

Number of firms

$$N_{D.t+1} = (1-\delta)\, N_{D.t} + \delta N_{E.t} \tag{C.19}$$

$$N_{D.t+1}^* = (1-\delta)\, N_{D.t}^* + \delta N_{E.t}^* \tag{C.20}$$

Optimal labor supply

$$L_t = \varphi\left(w_t - \gamma C_t\right) \tag{C.21}$$

$$L_t^* = \varphi\left(w_t^* - \gamma C_t^*\right) \tag{C.22}$$

Labor Market clear

$$S_W^M\left(w_t + L_t\right) = (\sigma-1)\, S_D^M\left(N_{D.t} + \widetilde{d}_{h.t}\right) + \theta S_I^M\left(N_{E.t} + x_{h.t}^s\right) + \sigma S_{FX}^M\left(N_{X.t} + \mu_{X.t}\right) \tag{C.23}$$

$$S_W^M\left(w_t^* + L_t^*\right) = (\sigma-1)\, S_D^M\left(N_{D.t}^* + \widetilde{d}_{f.t}^*\right) + \theta S_I^M\left(N_{E.t}^* + x_{f.t}^{s*}\right) + \sigma S_{FX}^M\left(N_{X.t}^* + \mu_{X.t}^*\right) \tag{C.24}$$

Share of exporting firms

$$N_{X.t} - N_{D.t} = -k\widetilde{z}_{X.t} \tag{C.25}$$

$$N^*_{X.t} - N^*_{D.t} = -k\widetilde{z}^*_{X.t} \tag{C.26}$$

Zero profit cutoff

$$\widetilde{d}_{X.t} = \mu_{X.t} \tag{C.27}$$

$$\widetilde{d}^*_{X.t} = \mu^*_{X.t} \tag{C.28}$$

Expected real returns

$$E_t r^s_{h.t+1} \equiv \Upsilon\left(1-\delta\right) E_t x^s_{h.t+1} + \left[1-\Upsilon\left(1-\delta\right)\right] E_t \widetilde{d}_{h.t+1} - x^s_{h.t} \tag{C.29}$$

$$E_t r^s_{f.t+1} \equiv \Upsilon\left(1-\delta\right) E_t x^{s*}_{f.t+1} + \left[1-\Upsilon\left(1-\delta\right)\right] E_t \widetilde{d}^*_{f.t+1} - x^{s*}_{f.t} + E_t Q_{t+1} - Q_t \tag{C.30}$$

$$E_t r^b_{h.t+1} \equiv \Upsilon E_t x^b_{h.t+1} + \left(1-\Upsilon\right)\psi\left[S_{ED}N_{D.t+1} + \left(1-S_{ED}\right)E_t N^*_{X.t+1}\right] - x^s_{h.t} \tag{C.31}$$

$$E_t r^b_{f.t+1} \equiv \Upsilon E_t x^{b*}_{f.t+1} + \left(1-\Upsilon\right)\psi\left[S_{ED}N^*_{D.t+1} + \left(1-S_{ED}\right)E_t N_{X.t+1}\right] - x^{s*}_{f.t} + E_t Q_{t+1} - Q_t \tag{C.32}$$

Euler Home and Foreign

$$\gamma E_t\left(C_{t+1} - C^*_{t+1}\right) - E_t Q_{t+1} = \left(\gamma - \upsilon\right)\left(C_t - C^*_t\right) - Q_t \tag{C.33}$$

Euler shares

(Home)

$$E_t r^s_{h.t+1} = \gamma E_t C_{t+1} - \left(\gamma - \upsilon\right) C_t \tag{C.34}$$

$$E_t r^s_{f.t+1} = \gamma E_t C_{t+1} - (\gamma - \upsilon) C_t \tag{C.35}$$

Euler equation (bond)

(Home)

$$E_t r^b_{h.t+1} = \gamma E_t C_{t+1} - (\gamma - \upsilon) C_t \tag{C.36}$$

$$E_t r^b_{f.t+1} = \gamma E_t C_{t+1} - (\gamma - \upsilon) C_t \tag{C.37}$$

Realized excess returns

$$r^s_{fx.t+1} = \Upsilon (1-\delta) \left(x^{s*}_{f.t+1} - E_t x^{s*}_{f.t+1} \right) + [1 - \Upsilon (1-\delta)] \left(\widetilde{d}^*_{f.t+1} - E_t \widetilde{d}^*_{f.t+1} \right)$$
$$- \Upsilon \left(x^b_{h.t+1} - E_t x^b_{h.t+1} \right) + Q_{t+1} - E_t Q_{t+1} - (1-\Upsilon) \psi (1 - S_{ED}) \left(N^*_{X.t+1} - E_t N^*_{X.t+1} \right) \tag{C.38}$$

$$r^s_{hx.t+1} = \Upsilon (1-\delta) \left(x^s_{h.t+1} - E_t x^s_{h.t+1} \right) + [1 - \Upsilon (1-\delta)] \left(\widetilde{d}_{h.t+1} - E_t \widetilde{d}_{h.t+1} \right)$$
$$- \Upsilon \left(x^b_{h.t+1} - E_t x^b_{h.t+1} \right) - (1-\Upsilon) \psi (1 - S_{ED}) \left(N^*_{X.t+1} - E_t N^*_{X.t+1} \right) \tag{C.39}$$

$$r^b_{fx.t+1} = \Upsilon \left(x^{b*}_{f.t+1} - E_t x^{b*}_{f.t+1} \right) - \Upsilon \left(x^b_{h.t+1} - E_t x^b_{h.t+1} \right) + Q_{t+1} - E_t Q_{t+1}$$
$$+ (1-\Upsilon) \psi (1 - S_{ED}) \left[(N_{X.t+1} - E_t N_{X.t+1}) - \left(N^*_{X.t+1} - E_t N^*_{X.t+1} \right) \right] \tag{C.40}$$

Net Foreign asset for Home

$$NFA_{h.t+1} = NX_{h.t} + \frac{1}{\Upsilon} NFA_{h.t} + \widetilde{a}^s_f r^s_{fx.t+1} - \widetilde{a}^{s*}_h r^s_{hx.t+1} + \widetilde{a}^b_f r^b_{fx.t+1} \tag{C.41}$$

where by symmetry,

$$\widetilde{a}^s_f = \widetilde{a}^{s*}_h = \frac{a^s_f}{\Upsilon M} \quad \text{and} \quad \widetilde{a}^b_f = -\widetilde{a}^b_h = \frac{a^b_f}{\Upsilon M} \tag{C.42}$$

Net export for Home

$$NX_{h.t} \equiv \sigma S^M_D (N_{D.t} + d_{h.t}) + \sigma S^M_{FX} (N_{X.t} + \mu_{X.t}) - M_t \tag{C.43}$$