Precautionary price stickiness

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Abstract

We propose a model in which price stickiness arises endogenously although firms are free to change their prices at zero physical cost. Firms are subject to idiosyncratic and aggregate shocks, and to the risk of errors in pricing. The latter is modelled as firms playing a dynamic logit equilibrium in which big mistakes are less likely than small ones. When a firm's price is sufficiently close to the optimum, it prefers to leave it unchanged, avoiding the risk of accidentally choosing a worse price. Although the decision to adjust is of the Ss type, many small price changes coexist with large ones. The hazard of price changes is downward sloping, since firms readjust quickly after detecting a mistake. And the size of price changes is largely independent of the time since adjustment. The real effects of nominal shocks are twice as large as in the Golosov-Lucas model, but still smaller than in the Calvo setup. A generalized framework in which both the price, and the decision of whether or not to adjust, are subject to logit errors, delivers macro responses more similar to the Calvo model.

Keywords: Price stickiness, logit equilibrium, state-dependent pricing, (S,s) adjustment

JEL Codes: E31, D81

1 Introduction[1]

Economic conditions change continually. A firm that attempts to maintain an optimal price in response to these changes faces at least two costly managerial challenges. First, it must repeatedly post new prices. Second, for each price update, it must choose what new price to post. The most widely adopted models of nominal rigidity have focused on the first issue, assuming that price adjustments can only occur intermittently, either with exogenous frequency (as in Calvo, 1983) or with endogenous frequency (e.g. Golosov and Lucas, 2007; Dotsey et al. 2009). Likewise, in recent work, we assumed that a firm's probability of adjusting its price is a smoothly increasing function of the value of price adjustment (Costain and Nakov 2008a, b). Our setup was motivated by bounded rationality, since it was designed to nest frictionless, fully rational behavior as a special case, and allowed us to estimate firms' degree of rationality from microdata. However, as in the Calvo model and the menu cost literature, we assumed

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that adjusting firms always set the optimal price (optimal taking into account the future costs associated with intermittent adjustment).

In this paper, we instead study the second, complementary, aspect of bounded rationality. We assume firms are able to adjust their prices costlessly in any period, but they might make mistakes when they adjust. We write the probability of setting any given price as a smoothly increasing function of the discounted present value of setting that price; full rationality is nested as the limiting case in which the firm sets the optimal price with probability one every period. Concretely, we model the probability distribution over price choices as a logit function. The general equilibrium of our economy is therefore a logit equilibrium (McKelvey and Palfrey 1995, 1998): the probability of each firm's choice is a logit function which depends on the value of each choice; moreover, the value of each choice is determined, in equilibrium, by the logit choice probabilities of other firms.

While our model does not directly impose price stickiness, due to the error-prone nature of choice we immediately derive a certain degree of endogenous price stickiness. Since decisions are error-prone, decision-making is risky. In the face of this risk, firms may refrain, on precautionary grounds, from making new decisions. When a firm's price is sufficiently close to the optimum, it prefers to "leave well enough alone", thus avoiding the risk of making a costly mistake. Thus, a risk of errors implies behavior with an (S,s) band structure, in which adjustment occurs only if the current price is sufficiently far from the optimum.

Our framework for modeling bounded rationality, logit equilibrium, has been widely applied in experimental game theory, where it has very successfully explained play in a number of games where Nash equilibrium performs poorly, such as the centipede game and Bertrand competition games (McKelvey and Palfrey 1998; Anderson, Goeree, and Holt 2002). It has been much less frequently applied in other areas of economics; we are unaware of any application of logit equilibrium inside a dynamic general equilibrium macroeconomic model.[2] Since logit equilibrium is just a one-parameter generalization of fully rational choice, it imposes much of the discipline of rationality on the model.[3]

One possible reason why macroeconomists have so rarely considered error-prone choice is that errors imply heterogeneity; the computational simplicity of a representative agent model may be lost if agents differ because of small, random mistakes. However, when applied to state-dependent pricing, this problem is less relevant, since it has long been argued that it is important to allow for heterogeneity in order to understand the dynamics of "sticky" adjustment models (see for example Caplin and Spulber 1987, Caballero 1992, and Golosov and Lucas 2007). Moreover, we have shown (Costain and Nakov 2008b) how distributional dynamics can be tractably characterized in general equilibrium, without relying on special functional forms or questionable numerical aggregation assumptions. The same numerical method we used in that paper (Reiter 2009) can be applied to a logit equilibrium model; in fact, the smoothness of the logit case makes it even easier to compute than the fully rational case. We therefore find that logit equilibrium opens the door to tractable models that can be compared quite directly and

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[2]The logit choice function is probably the most standard econometric framework for discrete choice, and has been applied to a huge number of microeconometric contexts. But logit *equilibrium*, in which each player makes logit decisions, based on payoff values which depend on other players' logit decisions, has to the best of our knowledge rarely been applied outside of experimental game theory.

[3]Haile, Hortaçsu, and Kosenok (2008) have shown that quantal response equilibrium, which has an infinite number of free parameters, is impossible to reject empirically. However, this criticism does not apply to logit equilibrium (the special case of quantal response equilibrium which has been most widely applied in practice) since it has only one free parameter.

successfully both to macroeconomic data and microeconomic data.

Summarizing our main findings, logit equilibrium fits micro price adjustment data well, in spite of the fact that we estimate only one free parameter. Our model matches well several stylized facts which are hard to reproduce with standard models of price setting. It implies that many large and small price changes coexist (see fig.1), in contrast to the implications of a fixed menu cost model (Midrigan, 2010; Klenow and Kryvtsov, 2008; Klenow and Malin, 2009). It also implies that the probability of price adjustment decreases rapidly over the first few months, and then remains essentially flat (Nakamura and Steinsson, 2008; Klenow and Malin, 2009). The empirical finding of negative duration dependence has been partially attributed to heterogeneity among price setters, but nonetheless it has been a persistent puzzle, even in models that have allowed for substantial heterogeneity. Furthermore, we find that the standard deviation of price changes is approximately constant, independent of the time since last adjustment (Klenow and Malin, 2009). Most alternative frameworks, including the Calvo and the menu cost model, instead imply that price changes are increasing in the time since last adjustment.

Finally, we calculate the general equilibrium effects of money supply shocks in our framework. Given the degree of rationality that best fits microdata, the effect of a money shock on consumption is roughly twice as large as in the Golosov-Lucas (2007) menu cost setup. The effect is much weaker than in the Calvo model because of a selection effect: all the firms that require substantial price adjustments do in fact adjust. Thus, a model in which price adjustment is slowed down by mistakes fits microdata much better than a fixed menu cost model, while implying an intermediate level of monetary nonneutrality. An extended framework in which both the price, and the decision of whether or not to adjust, are subject to logit errors, delivers macro responses which are more similar to the Calvo model.

1.1   Relationship to the literature

Early sticky price frameworks based on the notion of a "menu cost" were studied by Barro (1972), Sheshinski and Weiss (1977), and Mankiw (1985). General equilibrium solutions of these models have only been attempted more recently, at first by ignoring idiosyncratic shocks (Dotsey, King, and Wolman 1999), or by strongly restricting the distribution of such shocks (Danziger 1999; Gertler and Leahy 2006). Golosov and Lucas (2007) were the first to include explicitly frequent large idiosyncratic shocks in a quantitative model of state-dependent pricing, and approximately calculated the general equilibrium dynamics. Yet, while menu costs have been an influential idea in macroeconomics, the fact that price changes come in a great variety of sizes, including some very small, is hard for the menu cost framework to explain.

In particular, Klenow and Kryvtsov (2008) have shown that the distribution of price changes remains puzzling even if we allow for many sectors with different menu costs. As a possible explanation for the presence of small adjustments, Lach and Tsiddon (2007) and Midrigan (2010) proposed economies of scope in the pricing of multiple goods: a firm that pays to correct one large price misalignment might get to change other, less misaligned, prices on the same menu costlessly.

Rather that assuming a menu cost, our model delivers price stickiness as the result of near-rational behavior. In this it is similar in spirit to the setup of Akerlof and Yellen (1985) in which firms sometimes make mistakes if they are not very costly. Our setup shares also some features with the "rational inattention"literature (Sims, 2003; Mackowiak and Wiederholt, 2009). In this literature there is a constraint on the flow of information from the economic environment to

the decision-maker, which can give rise to sluggishness in the reaction to shocks. One possible interpretation of our setup is that the decision-maker (e.g. the CEO of a large corporation) is fully rational and has complete information about the environment, but he is subject to an implementation constraint which prevents him from perfectly communicating and enforcing his decisions throughout the organization. Given this imperfect implementation, the decisionmaker sometimes rationally prefers "not to call a meeting" and leave prices as they are, while the firm is doing reasonably well.

2 Sticky prices in partial equilibrium

In this section, we describe the partial equilibrium decision of a monopolistically competitive firm, under the assumption that the firm makes small mistakes governed by a logit probability function. In Section 3, we incorporate the firm's problem into an otherwise standard dynamic stochastic general equilibrium.

2.1 The monopolistic competitor's decision

Suppose that each firm i produces output Y_{it} under a constant returns technology, with labor N_{it} as the only input, and faces idiosyncratic productivity shocks A_{it}:

$$Y_{it} = A_{it} N_{it}$$

The idiosyncratic shocks A_{it} are given by a time-invariant Markov process, *iid* across firms. Thus A_{it} is correlated with $A_{i,t-1}$ but is uncorrelated with other firms' shocks. For numerical purposes, we assume A_{it} is drawn from a finite grid of possible values $\Gamma^a \equiv \left\{ a^1, a^2, ..., a^{\#a} \right\}$.[4]

Firms are monopolistic competitors, facing the demand curve $Y_{it} = \vartheta_t P_{it}^{-\epsilon}$, where ϑ_t represents aggregate demand. A firm's control variable is its price; we assume firms must fulfill all demand at the price they set. They hire in competitive labor markets at wage rate W_t, so period t profits are

$$P_{it} Y_{it} - W_t N_{it} = \left(P_{it} - \frac{W_t}{A_{it}} \right) Y_{it} = \left(P_{it} - \frac{W_t}{A_{it}} \right) \vartheta_t P_{it}^{-\epsilon}$$

At each point in time, a firm must decide whether or not to adjust its price. To make this decision, it must compare the value of maintaining its previous price with the value of choosing a new one. Let the value of a firm that produces with price P_{it} and productivity A_{it} at time t be $V_t(P_{it}, A_{it})$. The time subscript on the value function denotes all dependence on aggregate conditions, such as aggregate shocks or deterministic trends.[5]

If a firm chooses to adjust its price, it faces a risk of error in the new price it sets. For numerical purposes, we constrain the choice of the price P_{it} to a finite discrete grid $\Gamma^P \equiv \left\{ P^1, P^2, ... P^{\#P} \right\}$. The probability of choosing any given price is a smoothly increasing function of the value of choosing that price. This is the key assumption of our model.

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[4]Theoretically, our model would be well-defined with a continuum of possible values of productivity A_{it} and also a continuum of possible prices P_{it}. However, our numerical method for solving the model requires us to approximate the continuous case by a finite grid of possible productivities and prices. Therefore, for notational convenience, we define the model in terms of discrete grids from the beginning.

[5]We could instead write $V_t(P_{it}, A_{it})$ as $V(P_{it}, A_{it}, \Omega_t)$, where Ω_t represents the aggregate state of the economy. For concise notation we write V with a time subscript instead of including the extra argument Ω_t.

In order to treat the probability function as a primitive of the model, we define its argument in units of labor time. While we do not intend to model the computation process explicitly, we assume that the costs of decision-making are related to the labor effort (in particular, managerial labor) associated with obtaining new information and/or attempting to compute an optimal price. Therefore, when we write the values of all possible prices that might be chosen, we divide by the wage rate in order to convert them to time units. Let the probability of choosing price $P^j \in \Gamma^P$ at time t, conditional on productivity A_{it}, be $\pi_t^\xi(P^j|A_{it})$. We focus on the logit class of probabilities:

$$\pi_t^\xi(P^j|A_{it}) \equiv \frac{\exp\left(\frac{\xi V_t(P^j, A_{it})}{W_t}\right)}{\sum_{k=1}^{\#P} \exp\left(\frac{\xi V_t(P^k, A_{it})}{W_t}\right)} \tag{1}$$

The parameter ξ in the logit function can be interpreted as representing the degree of rationality; in the limit as $\xi \longrightarrow \infty$ it converges to the policy function under full rationality, in which the optimal price is chosen with probability one.[6]

We will use the notation E_t^ξ to indicate an expectation taken under the logit probability (1). The firm's expected value, conditional on adjusting to a new price $P' \in \Gamma^P$, is then

$$E_t^\xi V_t(P', A_{it}) \equiv \sum_{j=1}^{\#P} \pi_t^\xi(P^j|A_{it})V(P^j, A_{it}) = \sum_{j=1}^{\#P} \frac{\exp\left(\frac{\xi V_t(P^j, A_{it})}{W_t}\right) V(P^j, A_{it})}{\sum_{k=1}^{\#P} \exp\left(\frac{\xi V_t(P^k, A_{it})}{W_t}\right)} \tag{2}$$

The expected value of adjustment is

$$D_t(P_{it}, A_{it}) \equiv E_t^\xi V_t(P', A_{it}) - V_t(P_{it}, A_{it}) \tag{3}$$

We assume the firm adjusts its price if there is an expected gain from adjustment. That is, the probability of adjustment can be written as

$$\lambda(D_t(P_{it}, A_{it})) = \mathbf{1}\left(D_t(P_{it}, A_{it}) \geq 0\right) \tag{4}$$

We can now state the Bellman equation that governs a firm's value of producing at any given price P. The Bellman equation in this case is:

Bellman equation in partial equilibrium:

$$V_t(P, A) = \left(P - \frac{W_t}{A}\right)\vartheta_t P^{-\epsilon}$$
$$+ E_t\left\{Q_{t,t+1}\left[\left(1 - \lambda\left(D_{t+1}(P, A')\right)\right)V_{t+1}(P, A') + \lambda\left(D_{t+1}(P, A')\right)E^\xi V_{t+1}(P', A')\right]\middle| A\right\}$$

where $Q_{t,t+1}$ is the firm's stochastic discount factor. Note that the aggregate price level is absent from the above expression; it is subsumed into ϑ_t, as we show in Section 3. On the left-hand side and in the current profits term, P refers to a given firm i's price P_{it} *at the time of production*. In the expectation on the right, P represents the price $\widetilde{P}_{i,t+1}$ *at the beginning* of period $t+1$, which is the same as P_{it}, and subsequently may (with probability λ) or may not (with probability $1 - \lambda$) be adjusted prior to time $t+1$ production.

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[6]Alternatively, logit models are often written in terms of the inverse parameter $\eta \equiv \xi^{-1}$, which can be interpreted as a measure of the degree of noise instead of the degree of rationality.

We can simplify substantially by noticing that the value on the right-hand side of the equation is just the value of continuing without adjustment, plus the expected gains from adjustment, which we call G:

$$V_t(P, A) = \left(P - \frac{W_t}{A}\right)\vartheta_t P^{-\epsilon} + E_t\left\{Q_{t,t+1}\left[V_{t+1}(P, A') + G_{t+1}(P, A')\right]\big|\, A\right\} \qquad (5)$$

where

$$G_{t+1}(P, A') \equiv \lambda\left(D_{t+1}(P, A')\right) D_{t+1}(P, A') = \mathbf{1}\left(D_{t+1}(P, A') \geq 0\right) D_{t+1}(P, A') \qquad (6)$$

2.2 Deriving logit choice from an entropy constraint

Writing the Bellman equation in the form (5) is useful because it allows us to easily nest a variety of models of adjustment by choosing an appropriate function G to describe the gains from adjustment (see also Costain and Nakov 2008a). Here we will briefly consider a model in which the firm's adjustment is limited by an entropy constraint, and we will show that it generates the logit distribution (1) we assumed above.

Thus, suppose the firm chooses the probabilities π^j associated with each possible choice P^j to maximize its adjustment gains subject to a minimum level of noise (entropy) in its decision. For the time $t+1$ gains function $G_{t+1}(P, A')$, this can be written as follows:

$$G_{t+1}(P, A') = \max_{\lambda, \pi^j} \lambda\left(\sum_j \pi^j V_{t+1}(P^j, A') - V_{t+1}(P, A')\right)$$

$$\text{subject to} \quad : \quad -\lambda \sum_j \pi^j \ln \pi^j \geq \mathcal{E} \text{ and } \sum_j \pi^j \leq 1$$

This problem is written in terms of a specific time $t+1$ and state (P, A'), but the only index that is crucial for the optimization problem is j, so we abbreviate $V^j \equiv V_{t+1}(P^j, A')$ and $V^0 \equiv V_{t+1}(P, A')$.

The first-order condition for π^j is

$$\lambda V^j - \eta\lambda(1 + \ln \pi^j) - \mu = 0,$$

where η and μ are the multipliers on the two constraints. Some rearrangement yields:

$$\pi^j = \exp\left(\frac{V^j}{\eta} - 1 - \frac{\mu}{\eta\lambda}\right).$$

The multiplier μ must be chosen so that $\sum_j \pi^j \leq 1$. Note then that this is our logit probability function (1), with $\xi \equiv 1/\eta$— the "degree of rationality" corresponds to the inverse of the multiplier on the entropy constraint.

The first-order condition on the frequency of adjustment is

$$\sum_j \pi^j V^j - V^0 - \eta \sum_j \pi^j \ln \pi^j = 0$$

but this will not generally hold in equilibrium. Instead, the fact that the objective function is linear in λ implies that the solution will either be at the corner $\lambda = 1$ or at the corner $\lambda = 0$, with

$$\lambda = \mathbf{1}\left(\sum_j \pi^j V^j - V^0 - \eta \sum_j \pi^j \ln \pi^j \geq 0\right)$$

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This indicator function is equivalent to the adjustment probability (4) we derived earlier, except that it includes an additional term, $\eta \sum_j \pi^j \ln \pi^j$, representing the cost of information processing.

3   General equilibrium

We next embed this partial equilibrium framework into a dynamic New Keynesian general equilibrium model. For comparability, we use the same structure as Golosov and Lucas (2007). In addition to firms, there is a representative household and a monetary authority that chooses the money supply.

3.1   Households

The household's period utility function is

$$u(C_t) - x(N_t) + v\left(M_t/P_t\right)$$

discounted by factor β per period. Consumption C_t is a Spence-Dixit-Stiglitz aggregate of differentiated products:

$$C_t = \left[\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$$

N_t is labor supply, and M_t/P_t is real money balances. The household's period budget constraint is

$$\int_0^1 P_{it}C_{it}di + M_t + R_t^{-1}B_t = W_tN_t + M_{t-1} + T_t + B_{t-1} + \Pi_t$$

where $\int_0^1 P_{it}C_{it}di$ is total nominal spending on the differentiated goods. B_t is nominal bond holdings with interest rate $R_t - 1$; T_t represents lump sum transfers received from the monetary authority, and Π_t represents dividend payments received from the firms. In this context, optimal allocation of consumption across the differentiated goods implies $C_{it} = (P_t/P_{it})^{\epsilon}C_t$, where P_t is the price index $P_t \equiv \left\{\int_0^1 P_{it}^{1-\epsilon}di\right\}^{\frac{1}{1-\epsilon}}$.

3.2   Monetary policy and aggregate consistency

For simplicity, we assume the central bank follows an exogenous stochastic money growth rule:

$$M_t = \mu_t M_{t-1} \tag{7}$$

where $\mu_t = \mu \exp(z_t)$, and z_t is AR(1):

$$z_t = \phi_z z_{t-1} + \epsilon_t^z \tag{8}$$

Here $0 \leq \phi_z < 1$ and $\epsilon_t^z \sim i.i.d.N(0, \sigma_z^2)$ is a money growth shock. Thus the money supply trends upward by approximately factor $\mu \geq 1$ per period on average.

Seigniorage revenues are paid to the household as a lump sum transfer, and the government budget is balanced each period. Therefore the government's budget constraint is

$$M_t = M_{t-1} + T_t$$

Bond market clearing is simply $B_t = 0$. Market clearing for good i implies the following demand and supply relations for firm i:

$$Y_{it} = A_{it}N_{it} = C_{it} = P_t^\epsilon C_t P_{it}^{-\epsilon} \tag{9}$$

Also, total labor supply must equal total labor demand:

$$N_t = \int_0^1 \frac{C_{it}}{A_{it}} di = P_t^\epsilon C_t \int_0^1 P_{it}^{-\epsilon} a_{it}^{-1} di \equiv \Delta_t C_t \tag{10}$$

Labor market clearing condition (10) also defines a weighted measure of price dispersion, $\Delta_t \equiv P_t^\epsilon \int_0^1 P_{it}^{-\epsilon} a_{it}^{-1} di$, which generalizes the dispersion measure in Yun (2005) to allow for heterogeneous productivity. As in Yun's paper, an increase in Δ_t decreases the consumption goods produced per unit of labor, effectively acting like a negative shock to aggregate productivity.[7]

Aggregate consistency also requires that the demand curve and the discount factor that appear in the firm's problem be consistent with the household's problem. That is, regardless of the price-setting mechanism, C_t, N_t, P_t, W_t, R_t, C_{it}, P_{it}, and M_t must obey equations (??) - (10). In particular, to make the firm's problem (5) consistent with the goods market clearing conditions (9), the aggregate demand shift factor must be

$$\vartheta_t = C_t P_t^\epsilon \tag{11}$$

Also, we assume that the representative household owns the firms, so the stochastic discount factor in the firm's problem must be consistent with the household's euler equation. This implies that the appropriate stochastic discount factor is

$$Q_{t,t+1} = \beta \frac{P_t u'(C_{t+1})}{P_{t+1} u'(C_t)} \tag{12}$$

To write the firm's problem in general equilibrium, we simply plug (11) and (12) into the firm's problem (5). Then the value of producing with price P_{it} and productivity A_{it} is
Bellman equation in general equilibrium:

$$V_t(P, A) = \left(P - \frac{W_t}{A}\right) C_t P_t^\varepsilon P^{-\epsilon} + \beta E_t \left\{ \frac{P_t u'(C_{t+1})}{P_{t+1} u'(C_t)} \left[V_{t+1}(P, A') + G_{t+1}(P, A')\right] \middle| A \right\} \tag{13}$$

where $G_{t+1}(P, A')$ has the form described in equation (6).

3.3 State variable

At this point, we have spelled out all equilibrium conditions: household and monetary authority behavior has been described in this section, and the firms' decision was stated in Section 2. Thus can now identify the aggregate state variable Ω_t. Aggregate uncertainty in the model relates only to the money supply M_t. But since the growth rate of M_t is $AR(1)$ over time, the latest deviation in growth rates, z_t, is a state variable too. There is also a continuum of idiosyncratic productivity shocks A_{it}, $i \in [0, 1]$. Finally, since firms cannot instantly adjust their prices, they

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[7]Dorich (2007) also introduces a heterogeneity-adjusted dispersion measure that acts like an aggregate productivity shock.

are state variables too. More precisely, the state includes the joint distribution of prices and productivity shocks at the beginning of the period, prior to adjustment.

We will use the notation $\widetilde{P}_{it}$ to refer to firm i's price at the beginning of period t, prior to adjustment; this may of course differ from the price P_{it} at which it produces, because the price may be adjusted before production. Therefore we will distinguish the distribution of production prices and productivity at the time of production, which we write as $\Phi_t(P_{it}, A_{it})$, from the distribution of beginning-of-period prices and productivity, $\widetilde{\Phi}_t(\widetilde{P}_{it}, A_{it})$. Since beginning-of-period prices and productivities determine all equilibrium decisions at t, we can define the state at time t as $\Omega_t \equiv (M_t, z_t, \widetilde{\Phi}_t)$.

3.4   Detrending

So far we have written the value function and all prices in nominal terms, but we can also rewrite the model in real terms. Thus, suppose we deflate all prices by the nominal price level $P_t \equiv \left\{ \int_0^1 P_{it}^{1-\epsilon} di \right\}^{\frac{1}{1-\epsilon}}$, defining $m_t \equiv M_t/P_t$ and $w_t \equiv W_t/P_t$. Given the nominal distribution $\Phi_t(P_i, A_i)$, let us denote by $\Psi_t(p_i, A_i)$ the distribution over real production prices $p_{it} \equiv P_{it}/P_t$. Rewriting the definition of the price index in terms of these deflated prices, we have the following restriction:

$$\int_0^1 p_{it}^{1-\epsilon} di = 1$$

Notice however that the beginning-of-period *real* price is not predetermined: if we define $\widetilde{p}_{it} \equiv \widetilde{P}_{it}/P_t$, then $\widetilde{p}_{it}$ is a jump variable, and so is the distribution of real beginning-of-period prices $\widetilde{\Psi}_t(\widetilde{p}_i, A_i)$. Therefore we cannot define the real state of the economy at the beginning of t in terms of the distribution $\widetilde{\Psi}_t$.

To write the model in real terms, the level of the money supply, M_t, and the aggregate price level, P_t, must be irrelevant for determining real quantities; and we must condition on a real state variable that is predetermined at the beginning of period. Therefore, we define the real state at time t as $\Xi_t \equiv (z_t, \Psi_{t-1})$, where Ψ_{t-1} is the distribution of *lagged* prices and productivities. Note that the distribution Ψ_{t-1}, together with the shocks z_t, is sufficient to determine all equilibrium quantities at time t: in particular, it will determine the distributions $\widetilde{\Psi}_t(\widetilde{p}_i, A_i)$ and $\Psi_t(p_i, A_i)$. Therefore Ξ_t is a correct time t real state variable.

Thus it should also be possible to define a "real" value function v, meaning the nominal value function, divided by the current price level, depending on real variables only. That is,

$$V_t(P_{it}, A_{it}) = V(P_{it}, A_{it}, \Omega_t) = P_t v \left(\frac{P_{it}}{P_t}, A_{it}, \Xi_t \right) = P_t v_t (p_{it}, A_{it})$$

Deflating in this way, the Bellman equation can be rewritten as follows:
Detrended Bellman equation, general equilibrium:

$$v_t(p, A) = \left(p - \frac{w_t}{A} \right) C_t p^{-\epsilon} + \beta E_t \left\{ \frac{u'(C_{t+1})}{u'(C_t)} \left[v_{t+1} \left(\pi_{t+1}^{-1} p, A' \right) + g_{t+1} \left(\pi_{t+1}^{-1} p, A' \right) \right] \middle| A \right\} \quad (14)$$

where

$$g_{t+1} \left(\pi_{t+1}^{-1} p, A' \right) \equiv \lambda \left(w_{t+1}^{-1} d_{t+1} \left(\pi_{t+1}^{-1} p, A' \right) \right) d_{t+1} \left(\pi_{t+1}^{-1} p, A' \right)$$

$$d_{t+1}^{\xi} \left(\pi_{t+1}^{-1} p, A' \right) \equiv E_{t+1}^{\xi} v_{t+1}(p', A') - v_{t+1} \left(\pi_{t+1}^{-1} p, A' \right)$$

4 Computation

4.1 Outline of algorithm

This model represents a considerable computational challenge, because the wage, the aggregate demand factor, the stochastic discount factor, and therefore also the value function all depend on the aggregate state of the economy. In general equilibrium, at any time t, there will be many firms i facing different idiosyncratic shocks A_{it} and stuck at different prices P_{it}. The state of the economy will therefore include the entire distribution of prices and productivities. The reason for the popularity of the Calvo model is that even though firms have many different prices, up to a first-order approximation only the average price matters for equilibrium. Unfortunately, this property does not hold in general, and in the current context, we need to treat all equilibrium quantities explicitly as functions of the distribution of prices and productivity across the economy. To calculate equilibrium, we therefore need an algorithm that takes account of the distributional dynamics.

We attack this problem by implementing Reiter's (2009) solution method for dynamic general equilibrium models with heterogeneous agents and aggregate shocks. The first step in Reiter's algorithm is to calculate the steady state general equilibrium that obtains in the absence of aggregate shocks. Idiosyncratic shocks are still active, but are assumed to have converged to their ergodic distribution, so an aggregate steady state means that $z = 0$, and Ψ, π, C, R, N, and w are all constant. To solve numerically for this steady state, we will assume that real prices p_{it} and productivities A_{it} always lie on a given two-dimensional grid $\Gamma \equiv \Gamma^P \times \Gamma^a$, where $\Gamma^p \equiv \{p^1, p^2, ...p^{\#^p}\}$ is a logarithmically-spaced grid of possible values of p_{it}, and $\Gamma^a \equiv \{a^1, a^2, ...a^{\#^a}\}$ is a logarithmically-spaced grid of possible values of A_{it}. Given this grid, we can think of the steady state value function as a matrix $\mathbf{V}$ of size $\#^p \times \#^a$ comprising the values $v^{jk} \equiv v(p^j, a^k)$ associated with the prices and productivities $(p^j, a^k) \in \Gamma$. Likewise, the price distribution can be vied as a $\#^p \times \#^a$ matrix $\mathbf{\Psi}$ in which the row j, column k element Ψ^{jk} represents the fraction of firms in state (p^j, a^k) at the time of production. After appropriately adjusting our equations for consistency with this discretized representation, we can calculate the steady state general equilibrium by guessing the aggregate price level, then solving the firm's problem by discrete backwards induction, then updating the aggregate price level, and iterating to convergence.

The second step of Reiter's method constructs a linear approximation to the dynamics of the discretized model, by perturbing it around the steady state on a point-by-point basis. The method recognizes that the large system of nonlinear equations involved in calculating the general equilibrium steady state can also be interpreted as a system of nonlinear first-order autonomous difference equations describing the aggregate dynamics. For example, away from steady state, the Bellman equation relates the $\#^p \times \#^a$ matrices $\mathbf{V}_t$ and $\mathbf{V}_{t+1}$ that represent the value function at times t and $t + 1$. The row j, column k element of $\mathbf{V}_t$ is $v_t^{jk} \equiv v_t(p^j, a^k) \equiv v(p^j, a^k, \Xi_t)$, for $(p^j, a^k) \in \Gamma$. Given this representation, we no longer need to think of the Bellman equation as a functional equation that defines $v(p, a, \Xi)$ for all possible idiosyncratic and aggregate states p, a, and Ξ; instead, we simply treat it as a system of $\#^p \#^a$ expectational difference equations that determine the dynamics of the $\#^p \#^a$ variables v_t^{jk}. We linearize this large system of difference equations numerically, and then solve for the saddle-path stable solution of our linearized model using the QZ decomposition, following Klein (2000).

The crucial thing to notice about Reiter's method is that it combines linearity and nonlinearity in a way appropriate for the model at hand. In our model, idiosyncratic shocks are likely to be

larger and more economically important for individual firms' decisions than aggregate shocks. This is true in many macroeconomic contexts (e.g. precautionary saving) and in particular Klenow and Kryvstov (2008), Golosov and Lucas (2007), and Midrigan (2008) argue that firms' pricing decisions appear to be driven primarily by idiosyncratic shocks. Therefore, to deal with large idiosyncratic shocks, we treat functions of idiosyncratic states in a fully nonlinear way, by calculating them on a grid. As we emphasized above, this grid-based solution can be regarded as a large system of nonlinear equations, with equations specific to each of the grid points. By linearizing each of these equations with respect to the aggregate dynamics, we recognize that aggregate changes are unlikely to affect individual value functions in a strongly nonlinear way. That is, we are implicitly assuming that both the aggregate shocks z_t, and changes in the distribution $\mathbf{\Psi}_t$, have sufficiently smooth impact on individual values that a linear treatment of these effects suffices. On the other hand, we need *not* start from any assumption of approximate aggregation like that required for the Krusell and Smith (1998) method, nor do we need to impose any particular functional form on the distribution $\mathbf{\Psi}$.

Describing the distributional dynamics involves defining many matrices related to quantities defined on the grid Γ. From here on, we use bold face to identify matrices, and superscripts to identify notation related to grids. Matrices associated with grid Γ are all defined so that row j relates to price $p^j \in \Gamma^p$, and column k relates to productivity $a^k \in \Gamma^a$. As we mentioned already, the value function is described by matrix $\mathbf{V}_t$ with elements $v_t^{jk} \equiv v_t(p^j, a^k) \equiv v(p^j, a^k, \Xi_t)$. We also define matrices $\mathbf{D}_t$, $\mathbf{\Lambda}_t$, and $\mathbf{G}_t$, with elements $d_t^{jk} \equiv d_t(p^j, a^k)$, $\lambda_t^{jk} \equiv \lambda\left(w_t^{-1} d_t(p^j, a^k)\right)$, and $g_t^{jk} \equiv g_t(p^j, a^k)$. The distribution at the time of production is given by $\mathbf{\Psi}_t$ with elements Ψ_t^{jk} representing the fraction of firms with real price $p_{it} \equiv P_{it}/P_t = p^j$ and productivity $A_{it} = a^k$ at the time of production. We also define the beginning of period distribution $\widetilde{\mathbf{\Psi}}_t$ with elements $\widetilde{\Psi}_t^{jk}$ representing the fraction of firms with real price $\widetilde{p}_{it} \equiv \widetilde{P}_{it}/P_t = p^j$ and productivity $A_{it} = a^k$ at the beginning of the period. Shortly we will define the transition matrices that govern the relationships between all these objects.

4.2 The discretized model

In our discretized representation, the value function $\mathbf{V}_t$ is a matrix of size $\#^p \times \#^a$ with elements $v_t^{jk} \equiv v_t(p^j, a^k) \equiv v(p^j, a^k, \Xi_t)$ where $(p^j, a^k) \in \Gamma$. Other relevant $\#^p \times \#^a$ matrices include the adjustment values $\mathbf{D}_t^{\xi}$, the probabilities $\mathbf{\Lambda}_t$, and the expected gains $\mathbf{G}_t$, with (j, k) elements given by

$$d_t^{jk} \equiv d_t(p^j, a^k) \equiv E_t^{\xi} v_t(p, a^k) - v_t(p^j, a^k) \tag{15}$$

$$\lambda_t^{jk} \equiv \lambda\left(d_t^{jk}/w_t\right) \tag{16}$$

$$g_t^{jk} \equiv \lambda_t^{jk} d_t^{jk} \tag{17}$$

Finally, we also define a matrix of logit probabilities $\mathbf{\Pi}_t^{\xi}$, which has its (j, k) element given by

$$\pi_t^{jk} = \pi_t^{\xi}(p^j | a^k) \equiv \frac{\exp\left(\xi v_t^{jk}/w_t\right)}{\sum_{n=1}^{\#^p} \exp\left(\xi v_t^{jn}/w_t\right)}$$

which is the probability of choosing real price p^j conditional on productivity a^k if the firm decides to adjust its price at time t.

We can now write the discrete Bellman equation and the discrete distributional dynamics in a precise way. First, consider how the beginning-of-period distribution $\widetilde{\boldsymbol{\Psi}}_t$ is derived from the lagged distribution $\boldsymbol{\Psi}_{t-1}$. Idiosyncratic productivities A_i are driven by an exogenous Markov process, which can be defined in terms of a matrix $\mathbf{S}$ of size $\#^a \times \#^a$. The row m, column k element of $\mathbf{S}$ represents the probability

$$S^{mk} = prob(A_{it} = a^m | A_{i,t-1} = a^k)$$

Also, beginning-of-period real prices are, by definition, adjusted for inflation. Ignoring grids, the time $t-1$ real price $p_{i,t-1}$ would deflated to $\widetilde{p}_{it} \equiv p_{i,t-1}/\pi_t \equiv p_{i,t-1}P_{t-1}/P_t$ at the beginning of t. To keep prices on the grid, we define a $\#^p \times \#^p$ Markov matrix $\mathbf{R}_t$ in which the row m, column l element is

$$R_t^{ml} \equiv prob(\widetilde{p}_{it} = p^m | p_{i,t-1} = p^l)$$

When the deflated price $p_{i,t-1}/\pi_t$ falls between two grid points, matrix $\mathbf{R}_t$ must round up or down stochastically. Also, if $p_{i,t-1}/\pi_t$ lies outside the smallest and largest element of the grid, then $\mathbf{R}_t$ must round up or down to keep prices on the grid.[8] Therefore we construct $\mathbf{R}_t$ according to

$$R_t^{ml} = prob(\widetilde{p}_{it} = p^m | p_{i,t-1} = p^l, \pi_t) = \begin{cases} 1 & \text{if } \pi_t^{-1}p^l \leq p^1 = p^m \\ \frac{\pi_t^{-1}p^l - p^{n-1}}{p^n - p^{n-1}} & \text{if } p^1 < p^m = \min\{p \in \Gamma^p : p \geq \pi_t^{-1}p^l\} \\ \frac{p^{n+1} - \pi_t^{-1}p^l}{p^{n+1} - p^n} & \text{if } p^1 \leq p^m = \max\{p \in \Gamma^p : p < \pi_t^{-1}p^l\} \\ 1 & \text{if } \pi_t^{-1}p^l > p^{\#^p} = p^m \\ 0 & \text{otherwise} \end{cases}$$

(18)

Combining the adjustments of prices and productivities, we can calculate the beginning-of-period distribution $\widetilde{\boldsymbol{\Psi}}_t$ as a function of the lagged distribution of production prices $\boldsymbol{\Psi}_{t-1}$:

$$\widetilde{\boldsymbol{\Psi}}_t = \mathbf{R}_t * \boldsymbol{\Psi}_{t-1} * \mathbf{S}'$$

where $*$ represents ordinary matrix multiplication. The simplicity of this equation comes partly from the fact that the exogenous shocks to A_{it} are independent of the inflation adjustment that links $\widetilde{p}_{it}$ with p_{it-1}. Also, exogenous shocks are represented from left to right in the matrix $\boldsymbol{\Psi}_t$, so that their transitions can be treated by right multiplication, while policies are represented vertically, so that transitions related to policies can be treated by left multiplication.

Next, consider how the time t production distribution $\boldsymbol{\Psi}_t$ is derived from the beginning-of-period distribution $\widetilde{\boldsymbol{\Psi}}_t$. Suppose a firm has beginning-of-t price $\widetilde{p}_{it} \equiv \widetilde{P}_{it}/P_t = p^j \in \Gamma^p$ and productivity $A_{it} = a^k \in \Gamma^a$. This firm will adjust its production price with probability λ_t^{jk}, or will leave it unchanged ($p_{it} = \widetilde{p}_{it} = p^j$) with probability $1 - \lambda_t^{jk}$. If adjustment occurs, the probabilities of choosing all possible prices are given by the matrix $\boldsymbol{\Pi}_t^\xi$. Therefore we can calculate distribution $\boldsymbol{\Psi}_t$ from $\widetilde{\boldsymbol{\Psi}}_t$ as follows:

$$\boldsymbol{\Psi}_t = (\mathbf{E}_{\#^p\#^a} - \boldsymbol{\Lambda}) \, .* \, \widetilde{\boldsymbol{\Psi}}_t + \boldsymbol{\Pi}_t^\xi \, .* \, (\mathbf{E}_{\#^p\#^p} * (\boldsymbol{\Lambda} \, .* \, \widetilde{\boldsymbol{\Psi}}_t)) \tag{19}$$

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[8]In other words, we assume that any nominal price that would have a real value less than p^1 after inflation is automatically adjusted upwards so that its real value is p^1. This assumption is made for numerical purposes only, and has a negligible impact on the equilibrium as long as we choose a sufficiently wide grid Γ^p. If we were to compute examples with trend deflation, we would need to make an analogous adjustment to prevent real prices from exceding the maximum grid point $p^{\#^p}$.

where (as in MATLAB) the operator $.*$ represents element-by-element multiplication, and $*$ represents ordinary matrix multiplication.

The same transition matrices $\mathbf{R}$ and $\mathbf{S}$ show up when we write the Bellman equation in matrix form. Let $\mathbf{U}_t$ be the $\#^p \times \#^a$ matrix of current payoffs, with elements

$$u_t^{jk} \equiv \left(p^j - \frac{w_t}{a^k}\right) C_t(p^j)^{-\epsilon} \tag{20}$$

for $\left(p^j, a^k\right) \in \Gamma$. Then the Bellman equation is
Dynamic general equilibrium Bellman equation, matrix version:

$$\mathbf{V}_t = \mathbf{U}_t + \beta E_t \left\{ \frac{u'(C_{t+1})}{u'(C_t)} \left[\mathbf{R}_{t+1}' * (\mathbf{V}_{t+1} + \mathbf{G}_{t+1}) * \mathbf{S}\right] \right\} \tag{21}$$

The expectation E_t in the Bellman equation refers only to the effects of the time $t+1$ aggregate shock z_{t+1}, because the shocks and dynamics of the idiosyncratic state $(p^j, a^k) \in \Gamma$ are completely described by the matrices $\mathbf{R}_{t+1}'$ and $\mathbf{S}$. Note that since the Bellman equation iterates backwards in time, its transitions are represented by $\mathbf{R}'$ and $\mathbf{S}$, whereas the distributional dynamics iterate forward in time and therefore contain $\mathbf{R}$ and $\mathbf{S}'$.

While equilibrium seems to involve a very complex system of equations, the steady state is easy to solve because it reduces to a small scalar fixed-point problem, which is the first step of Reiter's (2009) method. This first step is discussed in the next subsection. The second step of the method, in which we linearize all equilibrium equations, is discussed in subsection 3.4.

4.3 Step 1: steady state

In the aggregate steady state, the shocks are zero, and the distribution takes some unchanging value $\boldsymbol{\Psi}$, so the state of the economy is constant: $\Xi_t \equiv (z_t, \Psi_{t-1}) = (0, \boldsymbol{\Psi}) \equiv \Xi$. We indicate the steady state of all equilibrium objects by dropping the time subscript t, so the steady state value function $\mathbf{V}$ has elements $v^{jk} \equiv v(p^j, a^k, \Xi) \equiv v(p^j, a^k)$.

Long run monetary neutrality in steady state implies that the rate of nominal money growth equals the rate of inflation:

$$\mu = \pi$$

Morever, the Euler equation reduces to

$$\pi = \beta R$$

Since the interest rate and inflation rate are observable, together they determine the required parameterization of β. The steady-state transition matrix $\mathbf{R}$ is known, since it depends only on steady state inflation π.

We can then calculate general equilibrium as a one-dimensional root-finding problem: guessing the wage w, we have enough information to solve the Bellman equation and the distributional dynamics.[9] Knowing the steady state aggregate distribution, we can construct the real price level, which must be one. Thus finding a value of w at which the real price level is one amounts to finding a steady state general equilibrium.

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[9]There are other, equivalent ways of describing the root-finding problem: for example, we could begin by guessing C. Guessing w is convenient since we know that in a representative-agent, flexible-price model, we have $w = \frac{\epsilon - 1}{\epsilon}$. This suggests a good starting value for the heterogeneous-agent, sticky-price calculation.

More precisely, for any w, we can calculate

$$C = \left(\frac{\chi}{w}\right)^{1/\gamma} \tag{22}$$

and then construct the matrix $\mathbf{U}$ with elements

$$u^{jk} \equiv \left(p^j - \frac{w}{a^k}\right) C(p^j)^{-\epsilon} \tag{23}$$

We then find the fixed point of the value function:

$$\mathbf{V} = \mathbf{U} + \beta \mathbf{R}' * (\mathbf{V} + \mathbf{G}) * \mathbf{S} \tag{24}$$

together with the logit probability function $\mathbf{\Pi}^\xi$, with elements

$$\pi^{kj} = \pi_t^\xi(p^j | a^k) \equiv \frac{\exp\left(\xi v^{jk}/w\right)}{\sum_{n=1}^{\#p} \exp\left(\xi v^{jn}/w\right)}$$

We can then find the steady state distribution as the fixed point of

$$\mathbf{\Psi} = (\mathbf{E}_{\#p \#a} - \mathbf{\Lambda}) \mathrel{.*} \widetilde{\mathbf{\Psi}} + \mathbf{\Pi}^\xi \mathrel{.*} (\mathbf{E}_{\#p \#p} * (\mathbf{\Lambda} \mathrel{.*} \widetilde{\mathbf{\Psi}})) \tag{25}$$

$$\widetilde{\mathbf{\Psi}} = \mathbf{R} * \mathbf{\Psi} * \mathbf{S}' \tag{26}$$

Finally, we check whether

$$1 = \sum_{j=1}^{\#p} \sum_{k=1}^{\#a} \Psi^{jk} \left(p^j\right)^{1-\epsilon} \equiv p(C) \tag{27}$$

If so, an equilibrium value of w has been found.

4.4 Step 2: linearized dynamics

Given the steady state, the general equilibrium dynamics can be calculated by linearization. To do so, we eliminate as many variables from the equation system as we can. For additional simplicity, we assume linear labor disutility, $x(N) = \chi N$. Thus the first-order condition for labor reduces to $\chi = w_t u'(C_t)$, so we don't actually need to solve for N_t in order to calculate the rest of the equilibrium.[10] We can then summarize the general equilibrium equation system in terms of the exogenous shock process z_t, the lagged distribution of idiosyncratic states $\mathbf{\Psi}_{t-1}$, which is the endogenous component of the time t aggregate state; and finally the endogenous 'jump' variables including $\mathbf{V}_t$, $\mathbf{\Pi}_t^\xi$, C_t, R_t, and π_t. The equation systems reduces to

$$z_t = \phi_z z_{t-1} + \epsilon_t^z \tag{28}$$

$$\mathbf{\Psi}_t = (\mathbf{E}_{\#p \#a} - \mathbf{\Lambda}_t) \mathrel{.*} \widetilde{\mathbf{\Psi}}_t + \mathbf{\Pi}_t^\xi \mathrel{.*} (\mathbf{E}_{\#p \#p} * (\mathbf{\Lambda}_t \mathrel{.*} \widetilde{\mathbf{\Psi}}_t)) \tag{29}$$

$$\mathbf{V}_t = \mathbf{U}_t + \beta E_t \left\{ \frac{u'(C_{t+1})}{u'(C_t)} \left[\mathbf{R}'_{t+1} * (\mathbf{V}_{t+1} + \mathbf{G}_{t+1}) * \mathbf{S}\right] \right\} \tag{30}$$

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[10]The assumption $x(N) = \xi N$ is not essential; the more general case with nonlinear labor disutility simply requires us to simulate a larger equation system that includes N_t.

$$R_t^{-1} = \beta E_t \left(\frac{u'(C_{t+1})}{\pi_{t+1} u'(C_t)} \right) \tag{31}$$

$$1 = \sum_{j=1}^{\#^p} \sum_{k=1}^{\#^a} \Psi_t^{jk} \left(p^j \right)^{1-\epsilon} \tag{32}$$

If we now collapse all the endogenous variables into a single vector

$$\overrightarrow{X}_t \equiv \left(vec\left(\mathbf{\Psi}_{t-1}\right)', \quad vec\left(\mathbf{V}_t\right)', \quad C_t, \quad R_t, \quad \pi_t \right)'$$

then the whole set of expectational difference equations (28)-(32) governing the dynamic equilibrium becomes a first-order system of the following form:

$$E_t \mathcal{F} \left(\overrightarrow{X}_{t+1}, \overrightarrow{X}_t, z_{t+1}, z_t \right) = 0 \tag{33}$$

where E_t is an expectation conditional on z_t and all previous shocks.

To see that the variables in vector $\overrightarrow{X}_t$ are in fact the only variables we need, note that given π_t and π_{t+1} we can construct $\mathbf{R}_t$ and $\mathbf{R}_{t+1}$. Given $\mathbf{R}_t$, we can construct $\widetilde{\mathbf{\Psi}}_t = \mathbf{R}_t * \mathbf{\Psi}_{t-1} * \mathbf{S}'$ from $\mathbf{\Psi}_{t-1}$. Under linear labor disutility, we can calculate $w_t = \chi / u'(C_t)$, which gives us all the information needed to construct $\mathbf{U}_t$, with (j,k) element equal to $u_t^{jk} \equiv \left(p^j - \frac{w_t}{\exp(z_t^A)a^k} \right) C_t (p^j)^{-\epsilon}$. Finally, given $\mathbf{V}_t$ and $\mathbf{V}_{t+1}$ we can construct $\mathbf{\Pi}_t^\xi$, $\mathbf{D}_t$, and $\mathbf{D}_{t+1}$, and thus $\mathbf{\Lambda}_t$ and $\mathbf{G}_{t+1}$. Therefore the variables in $\overrightarrow{X}_t$ and z_t are indeed sufficient to evaluate the system (28)-(32).

Finally, if we linearize system $\mathcal{F}$ numerically with respect to all its arguments to construct the Jacobian matrices $\mathcal{A} \equiv D_{\overrightarrow{X}_{t+1}} \mathcal{F}$, $\mathcal{B} \equiv D_{\overrightarrow{X}_t} \mathcal{F}$, $\mathcal{C} \equiv D_{z_{t+1}} \mathcal{F}$, and $\mathcal{D} \equiv D_{z_t} \mathcal{F}$, then we obtain the following first-order linear expectational difference equation system:

$$E_t \mathcal{A} \Delta \overrightarrow{X}_{t+1} + \mathcal{B} \Delta \overrightarrow{X}_t + E_t \mathcal{C} z_{t+1} + \mathcal{D} z_t = 0 \tag{34}$$

where Δ represents a deviation from steady state. This system has the form considered by Klein (2000), so we solve our model using his QZ decomposition method.[11]

5 Results

5.1 Parameterization

We calibrate our model to match the monthly frequency of price changes in US CPI data. We set the steady state growth rate of money to 0%. This is consistent with the zero average price change in the AC Nielsen data of household product purchases (Midrigan, 2008), which we use to test our model's ability to generate a realistic size distribution of price changes. The model is one of "regular" price changes, exlcuding temporary "sales".

As in Costain and Nakov (2008a, b), we take most of our parameterization directly from Golosov and Lucas (2007). Thus we set the discount factor to $\beta = 1.04^{-1/12}$. Consumption utility is CRRA, $u(C) = \frac{1}{1-\gamma} C^{1-\gamma}$, with $\gamma = 2$. Labor disutility is linear, $x(N) = \chi N$, with $\chi = 6$. The elasticity of substitution in the consumption aggregator is $\epsilon = 7$. Finally, the utility of real money holdings is logarithmic, $v(m) = \nu \log(m)$, with $\nu = 1$.

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[11]Alternatively, the equation system can be rewritten in the form of Sims (2001). We chose to implement the Klein method because it is especially simple and transparent to program.

We assume productivity is AR(1) in logs: $\log A_{it} = \rho \log A_{it-1} + \varepsilon_t^a$, where ε_t^a is a mean-zero, normal, *iid* shock. We take the autocorrelation parameter from Blundell and Bond (2000) who estimate it from a panel of 509 US manufacturing companies over 8 years, 1982-1989. Their preferred estimate is 0.565 on an annual basis, which implies ρ around 0.95 in monthly frequency.

The variance of log productivity is $\sigma_a^2 = (1-\rho^2)\sigma_\varepsilon^2$, where σ_ε^2 is the variance of the innovation ε_t^a. We set the standard deviation of log productivity to $\sigma_a = 0.06$, which is the standard deviation of "reference costs" estimated by Eichenbaum, Jaimovich, and Rebelo (2008). Given our grid-based approximation, this implies a maximum absolute log price change of 0.48, which covers, with an extra margin of 7%, the maximum absolute log price change of 0.45 observed in the AC Nielsen dataset.

This leaves us with a single parameter to estimate: the logit rationality parameter ξ. We choose ξ to match the 10% median monthly frequency of price changes estimated by Nakamura and Steinsson (2008). The rationality parameter strongly affects the frequency of price changes.

5.2   Steady state results

Table 1 and Figure 1 summarize our main estimation results. Our estimated rationality parameter is $\xi = 23.4$, implying a relatively small deviation from fully rational behavior (i.e. the coefficient of 23.4 inside the exponential function that determines the probabilities of different actions implies low probabilities of large errors). Table 1 reports the losses implied by the errors in our model: we report the mean loss suffered relative to a fully rational firm, as a percentage of average revenues of a fully rational firm. In the baseline estimate, firms lose half a percent of revenues due to imperfect rationality. The distribution of losses can also be seen in the last panel of Figure 1, which shows the distribution both before and after firms decide whether or not to adjust.

The first column of Table 1 also shows the main statistics of price adjustment implied by our estimates. With a single free parameter, we hit our single calibration target, the 10% monthly frequency of price changes estimated by Nakamura and Steinsson (2008) (the last three columns of the table show data from several sources). The remaining moments also appear quite consistent with the distribution of price adjustments in the data, which is illustrated in the first panel of Figure 1. The histogram of price changes shows 51 equally-spaced bins representing log price changes from -0.5 to 0.5. The blue shaded bars in the figure represent the AC Nielsen data, and the black line represents the results of our estimated model. The standard deviation of price adjustments is matched relatively well: 14.5% in our model versus 13.2% in the data. As in the data, half of the price adjustments in our model are price increases. Kurtosis is somewhat lower in our model (2.6) than in the data (3.5), as can be seen from the relatively fat tails in the blue shaded data in the figure.

Notably, we are able to reproduce one of the puzzling observations in the context of the fixed menu cost model, namely, the coexistence of large and small price changes. Even though typical price adjustments are large, well over 10%, around 20% of all adjustments in our model are less than 5% in absolute value, compared with 25% in the data. In contrast, both the Calvo and the fixed menu cost model imply too many small price changes. Like the menu cost model, our framework also exhibits (S,s) behavior, as illustrated by the seventh and eight panels of Figure 1; inside the (S,s) bands firms choose not to adjust because the expected value of adjustment is not high enough to justify the risk of an error. Nonetheless, since the actual size of price adjustment is determined stochastically, a wide range of price adjustments is observed, including a number

of "small" price changes.

Another striking finding relates to the behavior of price changes as a function of the time since last adjustment. The second panel of Figure 1 shows the adjustment hazard, that is, the probability of a price change as a function of the time since last adjustment. Error-prone decisions mean that firms sometimes readjust quickly after making a change, when their decision turns out to have been a mistake. This accounts for the spike in the adjustment hazard at one month. The adjustment hazard remains more mildly decreasing over the next few months, driven by small errors subsequently compounded by unfavorable shocks; it is thereafter largely flat. This pattern is quite consistent with microdata; it fails by construction in the Calvo model and also contrasts with the increasing hazard typically generated by menu cost models. Many studies have suggested that decreasing hazards in the data may be caused by heterogeneity in adjustment frequencies. While this seems to be a reasonable explanation for part of the effect observed in the data, Nakamura and Steinsson (2008), among others, find decreasing hazards even after controlling for heterogeneity.

Finally, the third panel of Figure 1 shows the size of price changes as a function of the time since last adjustment. In the Calvo model, and menu cost models, this is typically an increasing function. In the data, it is largely flat, like we find in our model.

5.3   Effects of a money growth shock

Figures 2-3 and Table 2 illustrate the macroeconomic implications of our estimated model. The figures show impulse responses of inflation, consumption, and other variables to money supply shocks ϵ_t^z, with an AR(1) persistence parameter of $\phi_z = 0.8$ (monthly, implying quarterly persistence of 0.5). Under our baseline PPS calibration, we find a degree of monetary nonneutrality smaller than the Calvo but larger than the fixed menu cost model. As we see in Figure 2, the impulse responses from the PPS specification lie in between the Calvo and the menu cost models; in particular, the consumption response under PPS is slightly stronger and more persistent on impact than the menu cost specification, but much weaker and more transitory than Calvo.

Focusing on the baseline responses (red lines with dots), the reason for the weaker response of consumption than Calvo is that a money growth shock causes a stronger spike in inflation. As Golosov and Lucas emphasized for the menu cost case, the inflation spike is driven by a selection effect: the inflation response is relatively large because the firms that adjust are the ones whose prices deviate most with respect current conditions. In the PPS setup firms' price setting decisions are subject to error, making the aggregate inflation effect slightly smaller than that for menu costs.

Likewise, in Table 2 we report a baseline estimate of the slope of the "Phillips curve" (the contemporaneous effect of inflation on output) of 0.27. This is almost double the coefficient under menu costs (0.15), but is still quite small, less than a third of the coefficient under Calvo pricing (1.1). Thus, considering our steady-state and dynamic results together, we conclude that matching price microdata does not necessarily require a model with as large a degree of aggregate stickiness as that of the Calvo model. Allowing for small mistakes in the size of price changes is very helpful for reproducing the distribution of price changes, but generates relatively trivial real effects of nominal shocks.

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5.4 Changing the degree of rationality

Tables 1-2 and Figure 3 also show also how the behavior of the PPS model varies with the parameter ξ that controls the degree of rationality. In Table 1, doubling the degree of rationality causes the frequency of price adjustment to rise from 10% to 12.3% per month, and the average price change becomes smaller. This makes sense— with greater rationality, price adjustment is less risky, so firms become willing to adjust even when their prices are not so far out of line.

Likewise, in Figure 3, doubling the degree of rationality increases the initial spike in inflation, so that the real effect of the nominal shock is smaller than in the baseline calibration. In other words, as rationality increases the PPS model eventually converges to a fully flexible setup in which money is entirely neutral.

6 Errors in both the size and the timing of adjustment

The preceding simulations have explored the empirical implications of error-prone decision-making, and the precautionary behavior it implies, in an otherwise simple and standard DSGE framework. However, as an analysis of near-rational pricing behavior, the setup explored so far exhibits an inelegant asymmetry. Standard models of price stickiness assume frictions affect the timing of price adjustment, but assume that whenever adjustment occurs, the size of the price change is exactly optimal. Our paper has explored the opposite extreme: the timing of price adjustment is assumed optimal, but mistakes occur in choosing the size of the change.

Realism, though, suggests that we should consider mistakes on both margins; a decision maker who does not know exactly what price to set is also unlikely to know exactly when adjustment is necessary. Similarly, it is clear that our previous formulation has imposed a strong and arbitrary parameter restriction. We have allowed a degree of freedom in the accuracy of decision-making, and we have estimated the parameter governing this accuracy. But we assumed, without offering any justification, that decisions can occur once per period (meaning once per month, since this is the frequency at which we simulate our model). To avoid imposing arbitrary restrictions on our parameterization, we should instead allow a degree of freedom in the *speed* of decision-making, as well as its accuracy.

6.1 Nested logit

An obvious way of extending our model to allow for mistakes both in size and in timing is to nest the logit price changes considered so far inside a logit model of the decision whether or not to adjust. Following our previous notation, a firm that does not adjust when its current state is (P, A) obtains the value $V(P, A)$. A firm that adjusts obtains expected value $E^\xi V(P', A)$, where the new price P' is governed by the logit price distribution (1). If the firm decides at time t whether or not to adjust its price, and we model this decision as a logit choice with noise parameter ξ, then the probability of adjustment at t is

$$\frac{\exp\left(\xi E_t^\xi V_t(P', A_{it})/W_t\right)}{\exp\left(\xi V_t(P, A_{it})/W_t\right) + \exp\left(\xi E_t^\xi V_t(P', A_{it})/W_t\right)} = \left[1 + \exp\left(\frac{-\xi D_t(P, A_{it})}{W_t}\right)\right]^{-1} \quad (35)$$

$$\equiv \eta\left(\frac{D_t(P, A_{it})}{W_t}\right)$$

This function has the desirable property $\eta(0) = 0.5$, meaning that when the firm chooses whether or not to adjust, and considers each option equally valuable, it takes each choice with probability one half.

It might seem natural to use this function $\eta(x)$ in the place of our previous function $\lambda(x)$. Unfortunately, this does not provide us with a free parameter related to the speed of decision-making. Even if we were to use a different noise parameter ξ in the definition of the function $\eta(x)$ from the one used in the definition of $\pi(P|A)$, the property that $\eta(0) = 0.5$ would still hold. That is, regardless of the value of ξ, setting $\lambda(x) = \eta(x)$ would implicitly assume exactly one decision per period. Therefore varying ξ fails to provide a degree of freedom in the length of the data period relative to the theoretical period over which decisions are taken.

Therefore, to allow a degree of freedom in the speed of decision-making, we assume there exists an underlying rate of decisions per period, $\overline{\lambda}$. We treat $\overline{\lambda}$ as a Poisson arrival rate, so that over any brief interval dt, the firm gets to make a decision with probability $\overline{\lambda} dt$. When a decision opportunity arrives, the firm chooses whether or not to adjust its price according to the logit rule (35). The probability of price adjustment over brief interval dt is therefore $\overline{\lambda} \eta\left(\frac{D_t(P, A_{it})}{W_t}\right) dt$, and over one model period it becomes

$$\lambda\left(\frac{D_t(P, A_{it})}{W_t}\right) = 1 - \exp\left(-\overline{\lambda}\eta\left(\frac{D_t(P, A_{it})}{W_t}\right)\right) \tag{36}$$

In the discretized version of the model, this requires

$$\lambda_t^{jk} = \lambda\left(-\overline{\lambda}\eta(d_t^{jk}/w_t)\right) \equiv 1 - \exp\left(\frac{-\overline{\lambda}}{1 + \exp\left(-\xi d_t^{jk}/w_t\right)}\right) \tag{37}$$

In this specification, $\overline{\lambda}$ represents the speed at which decisions of accuracy ξ can be made. It resembles a Calvo parameter, but nonetheless implies that the observed probability of price adjustment is state-dependent, since actual adjustments are more likely to occur when they are more valuable. When the firm actually adjusts its price, the distribution of adjustments is given by (1). As before, the fact that we regard errors as being related to the time cost of reasoning motivates us to express the argument of the function η in time units (by dividing by the wage), and to impose the same degree of noise ξ in the two logit functions (1) and (35).

6.2 Nested logit results

The dynamics of the nested logit specification can be calculated by precisely the same method we have already discussed. Relative to our previous specification, equation (36) replaces (4), where $\eta(x)$ is defined by (35), and (37) replaces (16), leaving the remaining equations unchanged.

To parameterize this specification, we must estimate both ξ and $\overline{\lambda}$. As before, we parameterize to reproduce steady-state price adjustment behavior. First, we target the frequency of price adjustment (a 10% median monthly frequency, according to the data of Nakamura and Steinsson). Second, we also target the size of price adjustment (a mean absolute log price change of 10.4%, in Midrigan's AC Nielsen data). Setting $\xi = 7.65$ and $\overline{\lambda} = 0.232$ matches both these statistics.

The results are shown in the "Nested logit" columns of Tables 1 and 2, and also in Figures 4 and 5. The main statistics of the steady-state price distribution are similar to those we

found before, and fit the data well. Nonetheless, several aspects of the model's behavior are substantially changed. In particular, firms suffer larger losses and money shocks have larger real effects in the nested logit setup as compared with the baseline model. Another key change is that the adjustment hazard becomes nearly flat, as a function of time since last adjustment, in the nested logit case.

All of these changes in the model's behavior can be understood on the basis of one key mechanism. Unlike our baseline specification, under the nested logit model the noise parameter ξ causes mistakes both in the size and in the timing of price adjustments. Comparing Figures 1 and 4, the baseline and nested logit models impose fairly similar price distributions conditional on productivity. But the function λ governing the probability of adjustment behaves very differently in the two cases, jumping from 0 to 1 (along a pair of "S,s bands") in the baseline model, but varying smoothly between 0.05 and 0.2 in the nested logit case. This difference is also seen in the density of adjusting firms, which is concentrated along the "S,s bands" in the baseline model, and is instead very diffuse for the nested logit.

By strongly smoothing out the probabilities of adjustment in this way, the nested logit specification greatly diminishes "selection effects". As Golosov and Lucas emphasized, eliminating selection effects increases the real effects of money shocks, as we can see in the Phillips curve slopes in Table 2 and the impulse response functions in Figure 5, where the nested logit model resembles the Calvo case. The smooth, unimodal distribution of price changes in the nested logit model also resembles that in the Calvo model (comparing the first panels in Figures 1 and 4), and contrasts with the mildly bimodal distribution seen in the data. Finally, the absence of selection effects is also seen in the hazard function, which retains a mild downward slope for the nested logit model but is nonetheless quite close to a flat Calvo hazard. In other words, the spike of immediate readjustments observed in the baseline model, which results from quick correction of large errors, depends on the presence of a strong selection effect.

7 Conclusions

We have analyzed the pricing behavior of near-rational firms subject to idiosyncratic and aggregate shocks which can adjust their prices at any time, but may make mistakes. We model error-prone behavior by assuming firms play a dynamic logit equilibrium. Prices are therefore endogenously sticky: when a firm's current price is sufficiently close to the optimum, it prefers to leave well enough alone, avoiding the risk of accidentally choosing a worse price.

This way of modeling price stickiness is consistent with several observations from microdata that are hard to explain in most existing frameworks. Although the decision to adjust prices has an (S,s) structure, nonetheless many small price changes coexist with larger ones; the price adjustment hazard exhibits negative duration dependence; and the size of price changes is largely independent of the time elapsed since the last adjustment.

When we estimate the logit rationality parameter, our model matches both the frequency of adjustment and the size distribution of price changes quite successfully. Since our setup guarantees that firms making sufficiently large errors will choose to adjust, it generates a "selection effect" in response to nominal shocks that largely mitigates the strong real effects of money shocks found in the Calvo model. Although our model generates many small price changes, unlike the Golosov–Lucas setup, a large fraction of small price changes in itself does not imply that money shocks should have large real effects.

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Table 1. Model-simulated statistics and evidence (zero trend inflation)

| | Model PPS | | | Calvo | MC | Nested | Evidence | | |
	$\xi{=}23.4$	$2{\times}\xi$	$\frac{1}{2}{\times}\xi$			logit	AC	NS	KK
Freq.of price changes	10	12.3	7.4	10	10	10	20.5	10	21.9
Mean absolute price change	11.9	10.1	13.5	2.8	5.5	10.5	10.4		11.3
Std of price changes	14.5	12.3	16.5	3.7	5.6	13.1	13.2		
Kurtosis of price changes	2.6	2.7	2.5	4.2	1.2	2.8	3.5		
Percent of price increases	50	50	50	48	50.7	50	50	66	56
% of abs price changes $\leq 5\%$	19.5	23.2	17.1	83.6	42.4	25.2	25		44
Mean loss from errors (% rev.)	0.5	0.4	0.7	0.61	0.12	1.9			

Note: All statistics refer to regular consumer price changes excluding sales, and are stated in percent.
The last three columns reproduce the statistics reported by Midrigan (2008) for AC Nielsen (AC),
Nakamura and Steinsson (2008) (NS), and Klenow and Kryvtsov (2008) (KK)

Table 2. Variance decomposition and Phillips curves

| *Correlated money growth shock* | Data | Model PPS | | | Calvo | MC | Nested |
($\phi_z = 0.8$)		$\xi = 23.4$	$2 \times \xi$	$\frac{1}{2} \times \xi$			logit
% of non-zero price changes		10	12.3	7.4	10	10	10
Size of money shock (std$\times$100)		0.153	0.134	0.192	0.331	0.122	0.277
Quarterly inflation (std$\times$100)	0.246	0.246	0.246	0.246	0.246	0.246	0.246
% explained by μ shock alone		100	100	100	100	100	100
Quarterly output growth (std$\times$100)	0.510	0.310	0.220	0.460	1.08	0.195	0.874
% explained by μ shock alone		60.7	43.2	90.3	212	38.3	171
Slope coeff. of Phillips curve*		0.273	0.191	0.429	1.1	0.149	0.848
Standard error		0.006	0.008	0.009	0.070	0.012	0.035
R^2 of regression		0.987	0.949	0.987	0.892	0.832	0.952

*The "slope coefficients" are 2SLS estimates of the effect of inflation on consumption
First stage: $\pi_t^q = \alpha_1 + \alpha_2 \mu_t^q + \epsilon_t$; second stage: $c_t^q = \beta_1 + \beta_2 \hat{\pi}_t^q + \varepsilon_t$, where the instrument
μ_t^q is the exogenous growth rate of the money supply and the superscript q indicates quarterly averages.
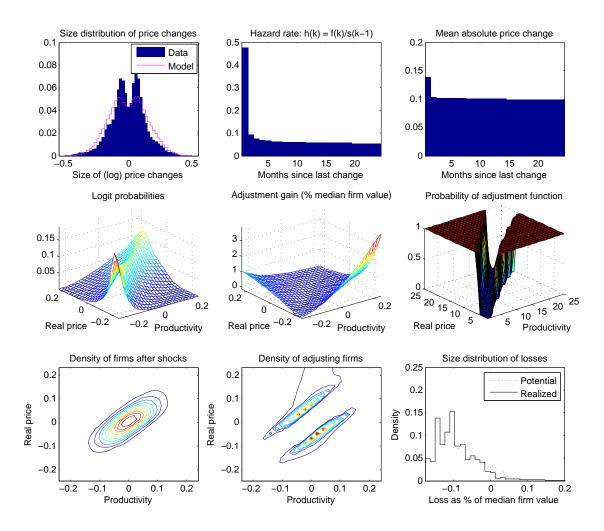
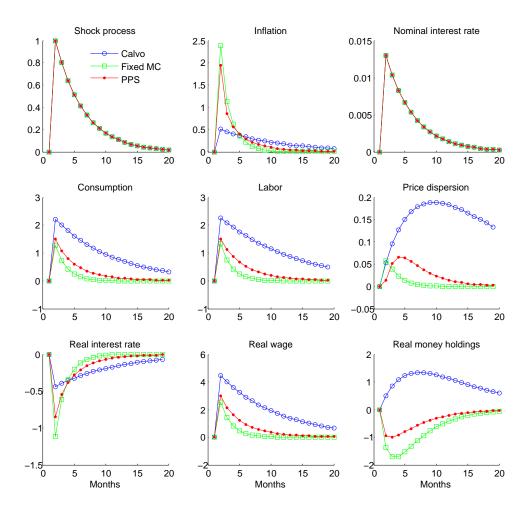Figure 1: Steady-state distributions, logit probabilities, losses

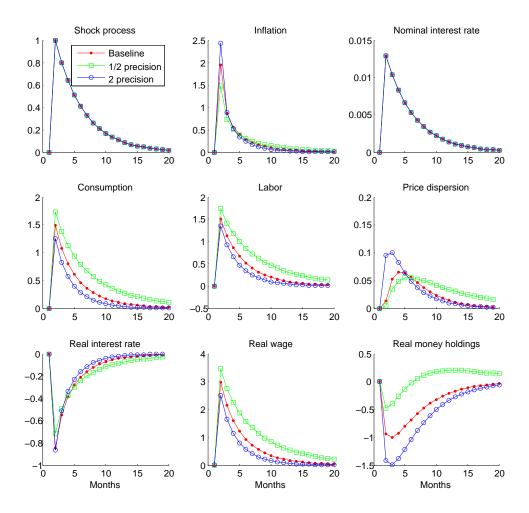Figure 2: Responses to a correlated money growth shock under alternative models

Figure 3: Responses to a correlated money growth shock for different noise values
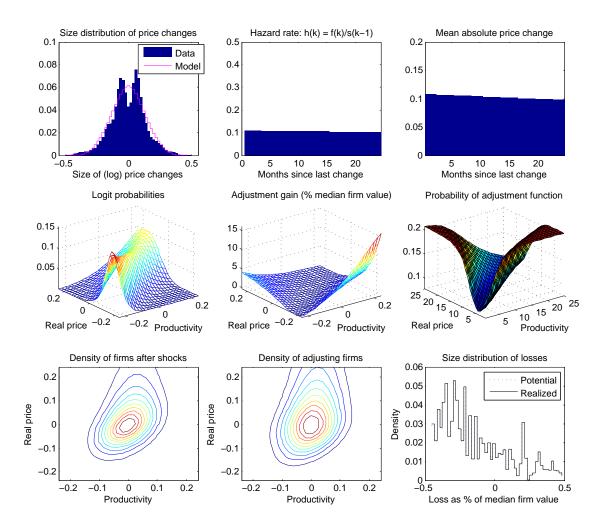
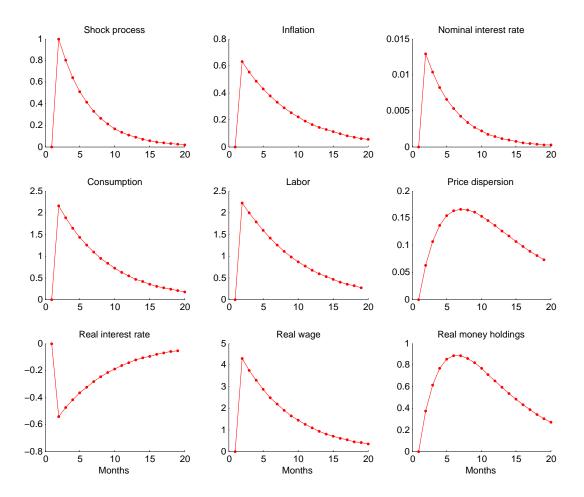Figure 4: Steady-state distributions, logit probabilities, losses (nested logit)

Figure 5: Responses to a correlated money growth shock (nested logit)