Markov Switching Monetary Policy in a two-country DSGE Model.

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Abstract

Estimates from a structural VAR model for the Eurozone and the US provide evidence in favor of regime shifts in monetary policy of both the ECB and the Fed. Splitting the sample in the pre- and post-Trichet era we find that real exchange rate targeting is beneficial in only the first sample. We construct a two-country DSGE model in order to find an explanation for this effect. We find that real exchange rate targeting is always beneficial when only foreign monetary policy switches regimes. However, when we allow for markov switching in monetary policy of both countries this result does not necessarily hold. We analyze the conditions under which real exchange rate targeting is beneficial, in this case, depending on the regime that each central bank's monetary policy lies in.

Keywords: Taylor rule, real exchange rate targeting, markov-switching.

JEL Classification: E52, F41, F42.

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1 *Introduction*

There has been an extensive research on the effects of monetary policy on key macroeconomic variables. Moreover, there seems to be a consensus on the view that its effects vary over time. However, there is not a consensus as far as the sources of the variation of those effects are concerned. There is a large empirical literature leading to conflicting conlcusions. Some researchers suggest that it is the Central Bank's change in the views for the economy leading to time-varying effects on the economy. In particular, Clarida, Gali and Gertler (2000) estimating a simple interest rate rule for the US economy for the pre- and post-Volcker period suggest that monetary policy is more successful, in the latter period, at ruling out undesired nonfundamental fluctuations. Cogley and Sargent (2001) estimating a reduced form VAR model with drifting coefficients, find similar resutls. Lubik and Shorfheide (2004) reach the same conclusion. They find that the FED's monetary policy has been accommodating inflation fluctuations in the pre-Volcker period, while being more hawkish in the post-Volcker one. All these authors find evidence in favour of changing coefficients in the interest rate rule over time. Clarida, Gali and Gerlter, for instance, find the coefficient on the inflation target to be less than one during the 1970s, whereas it being greater than one from 1980 onwards. Lubik and Shorfheide find similar estimates. Boivin and Giannoni (2006) provide further support to this argument. Estimating first a structural VAR model for thwo subperiods, before and after 1980, they find that the effects of monetary policy on output and inflation were amplified in the period staring from 1979 onwards, compared to the one before 1980.

There has been, though, wide criticism on the above results. In particular, it has been questioned the conlcusion that monetary pol-

icy has changed. The latter critisism relies on the fact that it may be that the behavior of the private sector in the period after 1980 has changed[1]. This, of course, would imply that the sensitivity of output and inflation to monetary policy changed. Boivin and Giannoni (2006) construct a stylized structural model for the US economy as in Smets and Wouters (2003, 2004) and Rotemberg and Woodford (1997) allowing for various kinds of shocks. Estimating their model for the two different periods, they conclude that the coefficient in the FED's interest rate rule have changed over time, being higher in the second period (post 1979). Some authors argue that if this is true, then this could be the main reason for the high inflation observed during the 1970s (Davig and Leeper, 2007 and Lubik and Shorfheide, 2004). In other words, according to this approach, it is the FED's interest rule that did not satisfy the Taylor principle during the 1970s, that allowed for high variations in output and inflation[2]. Additionally, Boivin and Giannoni find that changes in the private sector behaviour do not seem to affect the transmission mechanism importantly. Boivin (2005) finds similar evidence. Estimating a time varying parameters model, he finds evidence in favour of changes in the way monetary policy is conducted. Furthermore, Boivin concludes that the change in the parameters has been gradual and not discontinuous.

On the other hand, there is a number of papers arguing that it is not the change in the conduct of monetary policy, but the change in the volatility of the shocks hitting the economy. If the latter is true, it could be the case that the conduct US monetary policy has not changed at all. Stock and Watson (2003) attribute a significant part of the reduced volatility to smaller macroeconomic shocks. Sims (2001) and Stock (2001) argue that the conclusion of a change

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[1] Changes in the private sector behaviour may arise from technological progress or financial innovations, as suggested by Boivin and Giannoni (2006).

[2] The Taylor principle requires a coefficient on inflation greater than one, in order to rule out sunspot equilibria.

in the conduct of monetary policy may be highly sensitive to heteroskedasticity. Sims (1999) and Sims and Zha (2004) support the view that the majority of the changes between the pre- and the post-Volcker periods are attributed to changes in the shocks variances. Cogley and Sargent (2005), however, allowing for heteroskedasticity, find that there have been important changes in the policy parameters. Additionally, Boivin (2005) performing a similar robustness exercise, concludes that there has been a switch in the monetary policy between the two periods.

A weakness of the studies arguing in favour of changes in the policy parameters is that the conclusions are based on estimates using ex post data. Orphanides (2001) argues that estimates using ex post data can lead to the wrong conclusions about monetary policy. The reason is that those are data not available to the policy maker the time policy is decided. Moreover, Orphanides (2002), criticising Clarida, Gali and Gertler (2000), argues that if the same interest rate rule for the US is estimated with real time data instead, it is easy to observe that there has not been an important change in the conduct of monetary policy before and after 1979. Boivin (2005), however, finds that the conclusion of a change in the way monetary policy has been conducted in the US, is robust to using real time data.

Until now there is not ample evidence on the way monetary policy is conducted in the Eurozone. It difficult, though, to believe that the ECB has not decided even slighlt switches of its monetary policy. Even though the main feature of its monetary policy is to stabilize the inflation rate at a level close to 2%, we tend to believe that there were periods during which the ECB was less hawkish in battling against inflation[3].

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[3]We define hawkish the policy that has as its first priority inflation control. We will give a more detailed definition about what a hawkish behavior for the ECB and the FED, separately, is, later on when we will be presenting the model.

In a similar model, Mavromatis (2010) finds that extending a simple interest rate rule with a real exchange rate target leads to lower welfare losses and improves inflation control. Regime switching, though, is not considered. We extend this paper by allowing for regime switching. This allows us to adjust the theoretical model with the empirical results mentioned above. At first we examine the importance of a real exchange rate target for the ECB when the FED switches regimes. Then we proceed to allow for regime switches in the monetary policy of the ECB. By comparing alternative versions of the model, we show that when both central banks switch regimes, real exchange rate targeting does not always imply better control of inflation and lower output, CPI and PPI inflation volatility. Therefore, exchange rate targeting leads to a better inflation control only under certain conditions. Moreover, our analysis provides an alternative way to discuss upon the issue of monetary policy cooperation beyond welfare loss comparisons as in Pappa (2004).

The main goal, thus, of this paper is to show that exchange rate targeting is recommended for the ECB under certain conditions when there is regime switching in monetary policy. If adopted, under those conditions, the ECB is able achieve lower volatility for the key macroeconomic variables no matter what the monetary policy regime of either central bank is. If those conditions do not hold, then exchange rate targeting leads to higher volatilities regardless of the regime. Another goal is to show that even though the Central bank of a country does not change its monetary policy, domestic inflation and output volatility may increase because of a switch in the monetary policy in one of its main international trade partners. Domestic monetary policy switch may, thus, prove to be necessary. Before presenting the results from a two country DSGE model, we estimate a structural VAR model as in Christiano, Eichenbaum and Evans (2005) and Boivin and Giannoni (2002). Using monthly data

for the Eurozone and the US, we compute the impulse response functions under exchange rate targeting and under the standard Taylor rule. We then perform parameter stability tests. Those tests allow us to explore whether there were regime changes in monetary policy of both central banks during the period considered. Splitting the sample in two sub-samples, one for the pre- and one for the post-Trichet period, we find that the responses of output and inflation following different types of shocks are different. The latter could provide evidence in favour of a regime swtich in the monetary policy of either the ECB, or the FED, or both.

2 Empirical evidence and motivation

2.1 Data

We assume a two country model. US is assumed to be the foreign country and the Eurozone to be the home country.

Monthly data were gathered from the IMF International Financial Statistics for the CPI of each country, the end of period spot exchange rate of the Euro against the US dollar respectively. The Federal funds rate for the US and the interbank overnight rate for the Eurozone were used as proxies for the nominal interest rate. Output was proxied by the industrial production. The dataset spans from 1997:1 to 2009:3.

2.2 Methodology

2.2.1 The SVAR model

In this section we present the structural VAR model that is estimated. The SVAR model includes seven variables. Namely, the output gap, CPI inflation and the nominal interest rate. We also added the real exchange rate. The output gap was obtained by using the Hodrick-Prescott filter.

The way restictions are imposed and identitfication is carried out resembles to that of Boivin and Giannoni (2002). The structural VAR receives the following form

$$A_0 X_t = \sum_{j=1}^{k} A_j X_{t-j} + U_t$$

where A_j, $j = 1, \ldots, 7$ are 7×7 matrices and

$X_t = (y_{Euro,t}, \pi_{Euro,t}, i_{Euro,t}, q_t, y_{US,t}, \pi_{US,t}, i_{US,t})'$, where i_t is the nominal interest rate, q_t the real exchange rate, y_t the output gap, π_t the inflation rate and $U_t = C \times \varepsilon_t$ is the matrix of combined errors. The latter is specified as

$$U_t = \begin{bmatrix} 1 & 0 & 0 & 0 & c_{5,t}^{y_{Euro}} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ c_{1,t}^{i_{Euro}} & c_{2,t}^{i_{Euro}} & 1 & c_{4,t}^{i_{Euro}} & 0 & 0 & 0 \\ 0 & c_{2,t}^{q} & 0 & 1 & 0 & c_{6,t}^{q} & 0 \\ c_{1,t}^{y_{US}} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & c_{5,t}^{i_{US}} & c_{6,t}^{i_{US}} & 1 \end{bmatrix} \times \begin{bmatrix} \varepsilon_{y^{Euro}} \\ \varepsilon_{\pi^{Euro}} \\ \varepsilon_{i^{Euro}} \\ \varepsilon_{q} \\ \varepsilon_{y^{US}} \\ \varepsilon_{\pi^{US}} \\ \varepsilon_{i^{US}} \end{bmatrix}$$

We follow a standard identification procedure. The monetary policy shock is identified using an approach as in Sims (1992), Christiano et al., (1999, 2001) and Angeloni (2003). Looking at the first line of the coefficients matrix above, we allow for aggregate demand shocks in the US to affect contemporaneously output in the Eurozone. The same restriction was imposed on output in the US, as given in the fifth line. That is demand shocks in the Eurozone affect contemporaneously US output. This effect is captured by allowing $c_{1,t}^{y_{US}}$ to be nonzero. Both Eurozone and US CPI inflation are affected by shocks to other variables only with a lag. The real

exchange rate is affected contemporaneously by shocks to both CPI rates, apart from its own shocks. Finally, as in Boivin and Giannoni (2002) we allow output gap and CPI inflation shocks to affect the nominal rates contemporaneously. Since we are also interested in exploring whether real exchange rate tageting improves inflation control in the Eurozone, we allow the former to affect contemporaneously the nominal interest rate in the Eurozone. Therefore, the standard Taylor rule accrues once $c_{4,t}^{i_{Euro}}$ is set equal to zero.

2.2.2 *Parameter stability tests*

After estimating the model for the whole sample we perform parameter stability tests. Two tests were carried out, that of Andrews-Quandt and that of Andrews and Ploeberger. We chose those two tests because we do not need to specify a break date as is the case in the Chow test. Both tests provide support of regime switch in the parameters of the model. In the table below we present the results of the tests. In order to save space we present the result from the Andrews-Quandt test only.

Dep. vrb	Regressors						
	CPI_{Euro}	Gap_{Euro}	i_{Euro}	RER	CPI_{US}	Gap_{US}	i_{US}
CPI_{Euro}	0.109553	0.246455	0.060642	0.066691	0.911821	0.172310	0.000190
Gap_{Euro}	0.262800	0.184051	0.022814	0.351289	0.094538	0.000785	0.162539
i_{Euro}	0.144644	0.614182	0.164542	0.092285	0.990414	0.290894	0.000263
RER	0.192063	0.975479	0.243297	0.189228	0.725445	0.379115	0.008430
CPI_{US}	0.208829	0.353551	0.057318	0.071689	0.770709	0.388295	0.001146
Gap_{US}	0.099132	0.057322	0.083961	0.361355	0.021785	0.017851	0.124585
i_{US}	0.193410	0.251643	0.239196	0.122186	0.703107	0.259844	0.000974

Notes: P-values reported. Red: Significant at 5% s.l., Blue: Significant at 10% s.l.

Looking at the p-values for the coefficients in the interest rate rules, there is evidence in favour of changes in the wa each Central bank sets its policy rate over the period considered. As far as the Euro-
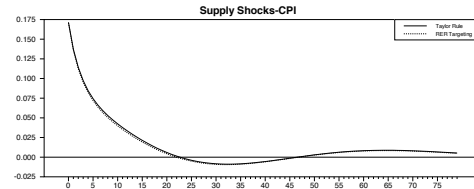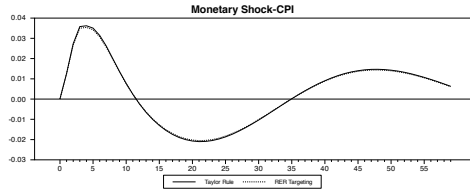
zone is concerned the p-values in the CPI inflation and the output gap equation parameters are over 0.05, implying that at 5% significance level there are not statitistically significant changes in the coefficients. This could be interpreted as evidence in favour of no swtich in the behavior of the private sector. On the other hand, results show that in the US there is some evidence in favour of switch in the bahaviour of the private sector. For two coefficients in the US output gap equation, that of the output gap of the Eurozone and that of the US, the p-values show that there has been a change over time at 1% and 5% significance level, respectively. As far as the US CPI inflation equation is concerned, the stability test shows that only the coefficient on the US output gap has changed significantly. Finally, parameters in the equation for the Federal funds rate seem to have changed at 1% significance level. The only exceptions are those of the two output gaps.
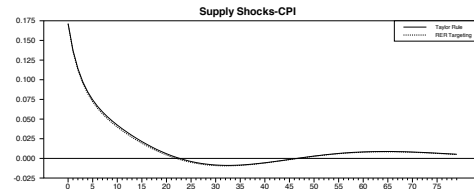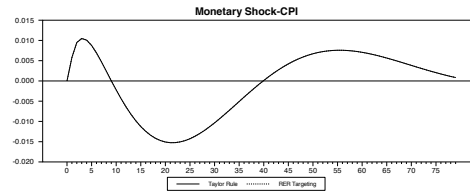
2.3 *Impulse response analysis*

One of the goals of this paper is to show that monetary policy has switched regimes during the period considered. The timing, however, of the switch is something that cannot be determined ver easily. Therefore, for simplicity we impose that time of a potential swtich in monetary policy. We split, thus, the sample into two sub-samples. The first covers the period when the president of the ECB was Wim Duisemberg, spanning from the start of the sample until Novermber 2003. The second covers the period starting from December 2003, that is the Jean Claude Trichet presidency.
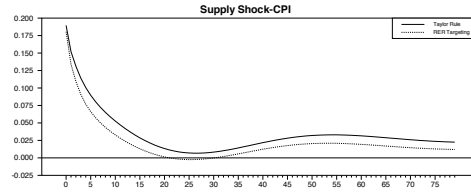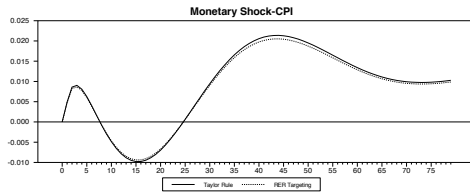
Whole sample

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Monetary Shock-CPI

0.04
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03

0   5   10   15   20   25   30   35   40   45   50   55

Taylor Rule ········ RER Targeting

Supply Shocks-CPI

0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000

Taylor Rule
RER Targeting

Wim Duisemberg

Supply Shocks-CPI

0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000

Taylor Rule
RER Targeting

Monetary Shock-CPI

0.015
0.010
0.005
0.000
-0.005
-0.010
-0.015
-0.020

0   5   10   15   20   25   30   35   40   45   50   55   60   65   70   75

Taylor Rule ········ RER Targeting

Supply Shocks-CPI

0.175
0.150
0.125
0.100
0.075
0.050
0.025

Taylor Rule
RER Targeting

Monetary Shock-CPI

0.015
0.025
0.020
0.015
0.010
0.005
0.000
-0.005
-0.010

0   5   10   15   20   25   30   35   40   45   50   55   60   65   70   75

Taylor Rule ········ RER Targeting

Monetary Shock-CPI

0.015
0.010
0.005
0.000
-0.005
-0.010

0   5   10   15   20   25   30   35   40   45   50   55   60   65   70   75

Taylor Rule ········ RER Targeting

Jean Claude Trichet

Supply Shocks-CPI

0.175
0.150
0.125

Taylor Rule
RER Targeting

Monetary Shock-CPI

0.025
0.020
0.015
0.010
0.005
0.000
-0.005
-0.010

0   5   10   15   20   25   30   35   40   45   50   55   60   65   70   75

Taylor Rule ········ RER Targeting

Supply Shock-CPI

0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
-0.025

0   5   10   15   20   25   30   35   40   45   50   55   60   65   70   75

Taylor Rule
RER Targeting

Supply Shock-CPI

0.200
0.175
0.150
0.125
0.075
0.050
0.000

Taylor Rule
RER Targeting

In the graphs above, the solid line represents inflation response under the Taylor rule, while the dotted line the response under real exchange rate targeting. Splitting the sample into two sub-samples leads to important difference in the way inflation responds to monetary and supply shocks. At first, we see no difference between the Taylor rule and real exchange rate targeting in the whole sample. The same result holds for the Duisemberg years. On the other hand, there are gains from real exchange rate targeting in the Trichet period. Another observation is that, following a monetary policy shock the inflation rate peaks at 0.01 in the Duisemberg period, whereas it peaks at 0.02 in the Trichet period. Moreover, inflation persistence in the latter period seems to be higher. It is plausible then to conlude that the ECB has been less hawkish in dampening inflation fluctu-

ations in the period after December 2003. Inflation jumps higher following a supply shock in the Trichet period, under the Taylor rule, compared to its response during the Duisemberg presidency. However, in the second sub-sample, it seems that the ECB could replicate inflation response, achieved in the first subsample after a supply shock, by simply introducing a real exchange rate target.

3   *Structure of the Model*

A stochastic model is specified as in Benigno (2004), Obstfeld and Rogoff (1998, 1999). Prices adjust in a sticky way as in Calvo (1983). Each country exports and imports goods. There are shipping costs (iceberg type) in transporting goods from one country to the other. Transaction costs are modeled as in Dumas (1992), Sercu, Van Hulle and Uppal (1995) and Coeurdacier (2006)[4].

Monetary policy is conducted by the Central Bank which uses the short term nominal interest rate as its instrument. In the present model, the Central Bank must take into account the degree to which home country is involved in international trade. The threshold behavior of the real exchange rate, implies a threshold behavior for the instrument, once the former is introduced into the rule. Consequently, the interest rate rule will be regime dependent.

3.1   *Households*

In this section, we specify the structure of the baseline, two country stochastic general equilibrium model. Each country is populated by a continuum of infinitely lived and identical households in the interval $[0, 1]$. Foreign variables are denoted with an asterisk. There are two kinds of households as in Amato and Laubach (2003). We allow ψ to denote the probability that the household is able to choose its consumption optimally, and which is independent of the house-

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[4]Coeurdacier introduces transaction costs in the price aggregator assuming that the price of the imported good will be $(1 + \tau)p_j$. We follow the same approach.

hold's history. Therefore, by the law of large numbers, in each period a fraction ψ of households will reoptimise, whereas the remaining fraction $1 - \psi$ will not. The latter will choose its consumption in period t according to the following rule of thumb

$$C_t^R = C_{t-1} \tag{1}$$

where C_t denotes aggregate per capita consumption in period t. The remaining $1 - \psi$ of households choose C_t^O so as to maximize their ultity. Thus, per capita cunsumption in preiod t is given by

$$C_t = \psi C_t^O + (1 - \psi) C_t^R \tag{2}$$

As in Laubach and Amato, this midification to the consumer's problem is based on the assumption that it is costly to reoptimise every period[5]. The households who choose consumtion optimally choose C_t^O to maximize their uitlity function. They derive utility from consumption and disutility from labor supply. The utility function, thus, is specified as

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{(C_s)^{1-\sigma}}{1 - \sigma} - \frac{(L_s)^{1+\gamma}}{1 + \gamma} \right] \tag{3}$$

where σ is the degree of relative risk aversion. C_t is a composite consumption index described as

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[5]Amato and Laubach note that Rule (1) has the important feature that rule-of-thumb consumers learn from optimizing households with one period delay. Hence, although Rule (1) is not optimal, it has three important properties. First agents are not required to compute anything. Second, rule-of-thumb households learn from optimizing ones, because lat period's decisions by the latter are part of C_{t-1}. Third, the differences between C_t^R and C_t^O are bounded, and will be zero in the steady state.

$$C_t = \left[\delta^{\frac{1}{\rho}} C_{H,t}^{\frac{\rho-1}{\rho}} + (1-\delta)^{\frac{1}{\rho}} C_{F,t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

$$\rho > 1 \qquad (4)$$

$$C_t^* = \left[(\delta^*)^{\frac{1}{\rho}} (C_{F,t}^*)^{\frac{\rho-1}{\rho}} + (1-\delta^*)^{\frac{1}{\rho}} (C_{H,t}^*)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

where ρ captures the intratemporal elasticity of substitution between home and foreign goods. $\delta \geq \frac{1}{2}$ is a parameter of home bias in preferences. C_H is the home consumption index. C_F is the foreign consumption index. Consumption indices in the home and the foreign country are defined as

$$C_{H,t} = \left[\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad C_{F,t} = \left[\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

$$(5)$$

$$C_{H,t}^* = \left[\int_0^1 c_t^*(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad C_{F,t}^* = \left[\int_0^1 c_t^*(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

Money deflator is given by the aggregate consumption price index for the home and foreign country respectively, which is specified as

$$P_t = \left[\delta(P_{H,t})^{1-\rho} + (1-\delta) \left[(1+\tau_t) P_{F,t} \right]^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

$$(6)$$

$$P_t^* = \left[\delta^*(P_{F,t}^*)^{1-\rho} + (1-\delta^*) \left[(1+\tau_t) P_{H,t}^* \right]^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

where P_H and P_F are price indices for home and foreign goods, expressed in the domestic currency and τ_t captures the time varying transaction cost assumed to follow a stationary $AR(1)$, $\tau_t = \rho_\tau \tau_{t-1} +$

ν_t, $\nu_t \backsim N(0, \sigma^2)$. The price indices for the Home and Foreign country are defined as

$$P_{H,t} = \left[\int_0^1 p_t(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \quad , \quad P_{F,t} = \left[\int_0^1 p_t(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \tag{7}$$

$$P_{H,t}^* = \left[\int_0^1 p_t^*(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \quad , \quad P_{F,t}^* = \left[\int_0^1 p_t^*(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}}$$

In each period t the economy experiences one of the finitely many events $s^t \in \Omega$ (Ω being the set of the finitely many states). Let h^t denote the history of realized states until period t included. The probability of particular state to occur is defined as $\pi(s^{t+1}|h^t)$. The initial realization s_0 is given.

Capital markets are complete. The consumers of both countries purchase state contingent bonds denominated in the domestic currency, $B(s^{t+1})$ for domestic agents and $B^*(s^{t+1})$ for foreign agents at price $Q(S^{t+1}|h^t)$. That is $B(s^{t+1})$ denotes the home agent's holdings of a one period nominal bond paying one unit of the home currency if state s^{t+1} occurs and 0 otherwise.

The home agent maximizes her utility subject to the period budget constraint

$$P_t C_t + M_t^h + \sum_{s^t \in \Omega} Q(s^{t+1}|h^t)B_{t+1} = B_t(s^t) + W_t L_t + \Pi_t \tag{8}$$

where W_t is the nominal wage, S_t are nominal transfers the individual receives from the government and i_t is the nominal interest rate.

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3.2 *First order conditions*

Maximizing the utility function (1) subject to the budget constraint (6) yields the following first order conditions

$$Q(s^{t+1}|h^t) = \frac{\beta\pi(s^{t+1}|h^t)P_t}{P_{t+1}(s_{t+1})} \left(\frac{C_t^O}{(C_{t+1}^O(s^{t+1}) + x)}\right)^{\sigma} \tag{9}$$

$$L_t = (C_t^O)^{-\sigma}w_t^{\frac{1}{\gamma}} \tag{10}$$

where the first equation is the usual Euler equation, the second determines the labor supply schedule and the third the demand for real money balances.

Individual demands for each good z produced in the home and in the foreign country respectively are expressed as

$$c_{h,t}(z) = \left(\frac{p_t^h(z)}{P_{H,t}}\right)^{-\theta} \left(\frac{P_{H,t}}{P_t}\right)^{-\rho} \delta C_t \tag{11}$$

$$c_{f,t}(z) = \left(\frac{p_t^f(z)}{P_{F,t}}\right)^{-\theta} \left(\frac{(1+\tau_t)P_{F,t}}{P_t}\right)^{-\rho} (1-\delta)C_t \tag{12}$$

3.3 *Risk sharing*

The fraction of foreign households who choose their consumption optimally (ψ^*) , maximize their utility subject to their budget contraint specified as

$$P_t^*C_t^* + M_t^* + \sum_{s_{t+1}} \frac{Q(s^{t+1}|s^t)B_{t+1}^*(s^{t+1})}{\varepsilon_t} = \frac{B^*(s^t)}{\varepsilon_t} + W_t^*L_t^* + \Pi_t^* \tag{13}$$

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Therefore, the Euler equation from the foreign agent's maximization problem is

$$Q(s^{t+1}|h^t) = \frac{\beta \pi(s^{t+1}|s^t)P_t^* \varepsilon_t}{P_{t+1}^*(s^{t+1})\varepsilon_{t+1}(s^{t+1})} \left(\frac{C_t^{O*}}{C_{t+1}^{O*}(s^{t+1})}\right)^{\sigma} \qquad (14)$$

International financial markets are complete. Domestic and foreign households trade in the state contingent one period nominal bonds denominated in the domestic currency. Therefore, combining (9) and (14) , we receive the following optimal risk sharing condition

$$\left(\frac{C_t^{O*}}{C_t^O}\right)^{-\sigma} = \varpi q_t \qquad (15)$$

where $\varpi \equiv \left(\frac{C_0^f + x}{C_0^h + x}\right)^{-\sigma} \frac{P_0}{\varepsilon_0 P_0^*}$ depends on initial conditions and $q_t = \frac{\varepsilon_t P_t^*}{P_t}$ is the real exchange rate.

3.4 *Price setting*

There are two types of firms, the backward looking and the forward looking. As a result, inflation will depend on both its lagged and forward values. Prices are sticky with a price setting behavior *à la* Calvo (1983). At each date, each firm changes its price with a probability $1 - \omega$, regardless of the time since it last adjusted its price. The probability of not changing the price, thus, is ω. The probability of not changing the price in the subsequent s periods is ω^s. Consequently, the price decision at time t determines profits for the next s periods. The price level for home goods at date t will be defined as

$$P_{H,t} = \left[\omega P_{H,t-1}^{1-\theta} + (1-\omega)\widetilde{p}_t(h)^{1-\theta}\right]^{\frac{1}{1-\theta}} \qquad (16)$$

Firms that are given the opportunity to adjust their prices will either follow a rule of thumb (backward looking firms) or will chose the price that maximizes their expected discounted profits (forward looking firms). The price $\tilde{p}_t(h)$ that will be set at date t is specified as

$$\tilde{p}_t(h) = \zeta p_t^B(h) + (1 - \zeta) p_t^F(h) \tag{17}$$

where $\zeta \in (0,1)$ is the fraction of backward looking firms, $p_t^B(h)$ and $p_t^F(h)$ is the price set by the backward and the forward looking firms, respectively. A continuum of firms is assumed for the home economy indexed by $z \in [0,1]$. Each firm produces a differentiated good, with a technology

$$Y_t(z) = A_t L_t(z) \tag{18}$$

where A_t is a country specific productivity shock at date t which is assumed to follow a log stationary process $\alpha_t = \rho_{\alpha_t} \alpha_{t-1} + \upsilon_t$, where υ_t is an *i.i.d.* process.

The strucure of productivity shocks across the two countries receives the following form

$$\begin{bmatrix} \alpha_t \\ \alpha_t^* \end{bmatrix} = \begin{bmatrix} \rho_{\alpha_t} & \rho_{\alpha_t \alpha_t^*} \\ \rho_{\alpha_t^* \alpha_t} & \rho_{\alpha_t^*} \end{bmatrix} \begin{bmatrix} \alpha_{t-1} \\ \alpha_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{\alpha,t} \\ \varepsilon_{\alpha^*,t}^* \end{bmatrix}$$

where $\begin{bmatrix} \varepsilon_{\alpha,t} \\ \varepsilon_{\alpha^*,t}^* \end{bmatrix} \sim N(0, \Sigma^2)$, with $\Sigma^2 = \begin{bmatrix} \sigma_{\varepsilon_a}^2 & 0 \\ 0 & \sigma_{\varepsilon_{\alpha^*}^*}^2 \end{bmatrix}$.

Each firm chooses a price for the home market and a price for the foreign market.

Backward looking firms.

Backward looking firms set their prices according to the following rule

$$p_t^B(h) = P_{H,t-1} + \pi_{H,t-1} \quad and \quad p_t^{B*}(h) = P_{H,t-1}^* + \pi_{H,t-1}^* \quad (19)$$

Forward looking firms.

Forward looking firms set their prices by maximing their expected discounted profits. Their maximization problem comprises of two decisions. The one concerns the price for the domestic market and the other the price charged in the foreign market, when it exports. Hence their maximization problem is described as

$$max E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} \left\{ \widetilde{p}_t(h) y_{t+s}^h(h) + \varepsilon_t \widetilde{p}_t^*(h) y_{t+s}^f(h) - W_{t+s}^h L_{t+s}^h \right\}$$
$$(20)$$

where $y_t^i(h)$, $i = h, f$ is the demand for the home good for home and foreign agents specified as

$$y_t^h(p_t(h)) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} \left(\frac{P_{H,t}}{P_t} \right)^{-\rho} \delta C_t, \quad (21)$$

$$y_t^f(p_t^*(h)) = \left(\frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} \left(\frac{(1 + \tau_t) P_{H,t}^*}{P_t^*} \right)^{-\rho} (1 - \delta^*) C_t^* \quad (22)$$

The firm maximizes its objective function (25) subject to (26) in order to find the optimal price for the Home good in the Home economy. It maximizes subject to (27), in order to find the optimal

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price for the Home good in the Foreign economy. The firm chooses a price for the Home good in the Home economy that satisfies the first order condition

$$E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}(p_t(h)) \left\{ p_t(h) - \frac{\theta}{\theta-1} MC_{t+s} \right\} = 0$$

where $MC_{t+s} = \frac{W_{t+s}}{A_{t+s}}$ denotes the nominal marginal cost and $\frac{\theta}{\theta-1}$ captures the optimal markup.

The optimal price, thus, for the Home good in the Home country is specified as

$$p_t(h) = \frac{\theta}{\theta-1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y_{t+s}^h(p_t(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}^h(p_t(h))} \tag{23}$$

Respectively, the optimal price for the Home good in the Foreign country is specified as

$$p_t^*(h) = \frac{\theta}{\theta-1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y_{t+s}^f(p_t^*(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}^f(p_t^*(h)) \varepsilon_{t+s}} \tag{24}$$

Finally, dividing (16) by $P_{H,t-1}$:

$$\Pi_{H,t}^{1-\theta} = \omega + (1-\omega) \left(\frac{\widetilde{p}_t(h)}{P_{H,t-1}} \right)^{1-\theta} \tag{25}$$

where $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$.

Similarly, for the foreign goods consumed in the home economy:

$$\Pi_{F,t}^{1-\theta} = \omega + (1-\omega) \left(\frac{\widetilde{p}_t(f)}{P_{F,t-1}} \right)^{1-\theta} \tag{26}$$

The aggregate price level dynamics are specified, thus, as

$$\Pi_t^{1-\rho} = \delta \left[\left(\frac{P_{H,t-1}}{P_{t-1}} \right) \Pi_{H,t} \right]^{1-\rho} + (1 - \delta) \left[(1 + \tau_t) \left(\frac{P_{F,t-1}}{P_{t-1}} \right) \Pi_{F,t} \right]^{1-\rho} \tag{27}$$

4 Log linearized model

A log linearized version of the relationships found in the previous section serves in providing us with a way to deal with the problem of no closed form solution. Additionally, this is a way to end up in a state space form which can be estimated using real time series data.

4.1 *Supply side*

We use a first order Taylor approximation around the steady state of zero inflation rate. Log linearized variables are denoted with a hat.

After loglinearizing the first order condition (9), the price level equations (21) and (22), the production function (23) the demand schedules faced by each firm (26) and (27) and optimal price setting rules (28) and (29), we receive the two relations describing the domestically consumed home goods inflation rate and the respective of the home goods consumed in the Foreign country as in Benigno (2004)

$$\pi_{H,t} = b_{\pi_{H,-1}} \pi_{H,t-1} + b_{\pi_{H,-1}^*} \pi_{H,t-1}^* + \beta E_t \pi_{H,t+1} + b_{\pi_H^*} \pi_{H,t}^* + b_C \hat{C}_t + \ldots$$

$$\ldots + b_T \hat{T}_t + b_{T^*} \hat{T}_t^* + b_q \hat{q}_t + b_a a_t + \varepsilon_{H,t} \tag{28}$$

$$\pi_{H,t}^* = b_{\pi_{H,-1}} \pi_{H,t-1} + b_{\pi_{H,-1}^*} \pi_{H,t-1}^* + \beta E_t \pi_{H,t+1}^* + b_{\pi_H}^* \pi_{H,t} + b_C^* \hat{C}_t + \ldots$$

$$\ldots + b_T^* \hat{T}_t + b_{T^*}^* \hat{T}_t^* + b_q^* \hat{q}_t + b_a^* a_t + \varepsilon_{H,t}^* \tag{29}$$

where $\varepsilon_{H,t}$ and $\varepsilon_{H,t}^*$ are *i.i.d.* cost push shocks. $T_t = \frac{(1+\tau_t)P_{F,t}}{P_{H,t}}$ and $T_t^* = \frac{(1+\tau_t^*)P_{H,t}^*}{P_{F,t}^*}$ captures the terms of trade for the Home and Foreign country respectively.

The log linearized aggregate price level relation (22) is specified as

$$\pi_t = \pi_{H,t} + (1-\delta)(\pi_{F,t} - \pi_{H,t} + (\rho_\tau - 1)\hat{\tau}_t) \tag{30}$$

which can be further simplified as[6]

$$\pi_t = \pi_{H,t} + (1-\delta)\Delta \hat{T}_t$$

4.2   *Demand side*

In this section we proceed to the loglinearization of the Euler equation

$$C_t^O = \kappa(i_t - E_t \pi_{t+1}) + E_t C_{t+1}^O \tag{31}$$

where $\kappa = -\frac{1}{\sigma}$, and using (2) the Euler equation receives the forward form, which includes both backward and forward looking elements

$$C_t = \frac{\kappa \psi}{2-\psi}(i_t - E_t \pi_{t+1}) + \frac{1}{2-\psi}E_t C_{t+1} + \frac{1-\psi}{2-\psi}C_{t-1} \tag{32}$$

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[6]To end up to that expression, we used equation $\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} + \hat{\tau}_t$ for the terms of trade which is reported later in the text.

Goods market clearing assumes the following two conditions

$$Y = C_H + C_H^* + G_t \quad and \quad Y^* = C_F + C_F^* + G_t^*$$

where G_t and G_t^* capture government expenditures for home and foreign country respectively, assumed to follow an exogenous stationary $AR(1)$ process $g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$ and $g_t^* = \rho_{g^*} g_{t-1}^* + \varepsilon_{g,t}^*$, $\varepsilon_{g,t} \sim N(0, \sigma_{\varepsilon_g}^2)$ and $\varepsilon_{g,t}^* \sim N(0, \sigma_{\varepsilon_g}^{*2})$.

Using the demand schedules as in (24) and (25), and then loglinearizing using the goods market equilibrium conditions, we end up to the following expressions for consumption in the Home country

$$\hat{C}_t = \frac{1}{-1+\delta+\delta^*} \left(\delta^* \hat{Y}_t - (1-\delta) \hat{Y}_t^* + (1-\delta)\rho \left(\delta^* \hat{T}_t^* - \delta \hat{T}_t \right) \right)$$

$$(33)$$

Therefore, combining equations (32) and (33), we derive the aggregate demand equation:

$$\hat{Y}_t = \eta_1 \hat{Y}_{t-1} + \eta_2 E_t \hat{Y}_{t+1} + \eta_3 (i_t - E_t \pi_{t+1}) + \eta_4 \Delta \hat{Y}_t^* + \eta_5 E_t \Delta \hat{Y}_{t+1}^* + \eta_6 \Delta \hat{T}_t +$$

$$\dots \eta_7 E_t \Delta \hat{T}_{t+1} + \eta_8 \Delta \hat{T}_t^* + \eta_9 E_t \Delta \hat{T}_{t+1}^* \qquad (34)$$

where η_i, $i = 1, .., 9$ are defined in detail in the appendix.

4.3 *Real exhange rate and relative prices*

The real exchange rate dynamics are specified by the following relationship

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$$\Delta \hat{q}_t = \Delta \varepsilon_t + \pi_t^* - \pi_t \tag{35}$$

In the Home country the price of imported goods relative to Home goods is specified as $T_t = \frac{(1+\tau_t)P_{F,t}}{P_{H,t}}$, whereas in the Foreign country the realtive price of Home exported goods to Foreign goods is specified as $T_t^* = \frac{(1+\tau_t^*)P_{H,t}^*}{P_{F,t}^*}$. Loglinearizing those two expressions we receive the following

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} + (\rho_\tau - 1)\hat{\tau}_t, \quad \hat{T}_t^* = \hat{T}_{t-1}^* + \pi_{H,t}^* - \pi_{F,t}^* + (\rho_{\tau^*} - 1)\hat{\tau}_t^*$$

4.4   *Flexible price equilibrium*

At the flexible price equilibrium firms adjust their prices at each period. Each firm will set its marginal cost equal to the optimal marginal cost (i.e. $-log\left(\frac{\theta}{\theta-1}\right)$) which is constant over time and equal across firms. Since firms adjust their prices every period, monetary policy will not have any real effects into the economy. The real marginal cost is specified by the following equations

$$mc_t = -log\left(\frac{\theta}{\theta-1}\right) = -\mu$$

$$mc_t = w_t - \alpha_t - \nu$$

where w_t is the real wage, α_t (log) productivity and ν a subsidy to labor. Solving for the case with flexible prices, we receive the following set of equations describing the equilibrium processes for

output, consumption, labor, real interest rate and real exchange rate, given by:

$$y_t^n = \psi_c \bar{c}_{t-1} + \psi_\zeta \zeta + \psi_a \alpha_t + \psi_{a^*} \alpha_t^* + \psi_\tau \tau_t + \psi_{\tau^*} \tau_t^* + \psi_g g_t + \psi_{g^*} g_t^* \qquad (36)$$

$$c_t^n = \tilde{\psi}_c \bar{c}_{t-1} + \psi_\zeta \zeta + \left(\frac{\gamma \delta^* + \sigma}{\delta(\gamma + \sigma) - \gamma(1 - \delta^*)} \right) \psi_a \alpha_t - \left(\frac{\gamma}{\sigma} \psi_{a^*} \right) \alpha_t^* - \left(\frac{\gamma}{\sigma} \psi_\tau \right) \tau_t - \left(\frac{\gamma}{\sigma} \psi_{\tau^*} \right) \tau_t^* - \left(\frac{\gamma}{\sigma} \psi_g \right) g_t - \left(\frac{\gamma}{\sigma} \psi_{g^*} \right) g_t^*$$
$$(37)$$

$$l_t^n = \tilde{\psi}_c \bar{c}_{t-1} + \psi_\zeta \zeta + \left(\frac{\gamma(\delta^*(1 - \sigma) - (1 - \delta)) - \sigma(1 - \delta)\psi_a}{\delta(\gamma + \sigma) - \gamma(1 - \delta^*)} \right) \alpha_t - \psi_{a^*} \alpha_t^* + \psi_\tau \tau_t + \psi_{\tau^*} \tau_t^* + \psi_g g_t + \psi_{g^*} g_t^*$$
$$(38)$$

$$r_t^n = \tilde{\tilde{\psi}}_c \bar{c}_{t-1} + \left(\frac{(\gamma \delta^* + \sigma)(1 - \rho_a)\psi_a}{\kappa \delta(\gamma + \sigma) - \gamma(1 - \delta^*)} \right) \alpha_t - \left(\frac{\gamma(1 - \rho_{a^*})\psi_{a^*}}{\kappa \sigma} \right) \alpha_t^* - \left(\frac{\gamma(1 - \rho_\tau)\psi_\tau}{\kappa \sigma} \right) \tau_t +$$
$$\left(\frac{\gamma(1 - \rho_{\tau^*})\psi_{\tau^*}}{\kappa \sigma} \right) \tau_t^* - \dots$$

$$\dots - \left(\frac{\gamma(1 - \rho_g)\psi_g}{\kappa \sigma} \right) g_t - \left(\frac{\gamma(1 - \rho_{g^*})\psi_{g^*}}{\kappa \sigma} \right) g_t^* \qquad (39)$$

5   Monetary Policy

Monetary policy is conducted through nominal interest rate rules by the Central Bank. As a first step we consider a simple framework as in Benigno (2004) with no markov switching in monetary policy. The Taylor rule is compared to an interest rate rule where the Central bank targets the real exchange rate as well. Since the focus in this paper is to find which policy leads to lower output and inflation volatility, comparison is made in terms of the volatilities of those two variables that each policy leads to.

Open economy monetary policy literature has often rejected the importance of the exchange rate in the interest rate feedback rules, either because it is argued that its effect is already there, indirectly

through its pass through on prices and then in inflation (Ball, 1999; Taylor, 1999), or because data do not support its significance (Clarida, Gali and Gertler, 1998). However, a weakness of many empirical studies is that they do not estimate a structural model, but, rather, they estimate an interest rate rule. This strategy is able, of course, to rpovide some information about the range of values of the coefficients, but its weakness rests on the fact that it does not take into account the interactions among the fundamental variables in the economy. An estimated, thus, structural model of inflation, output is able to account for this. Therefore, the estimated parameters in such an exercise include more information than single equation estimates[7].

Another weakness of the exsiting literature on monetary policy in open economy models is that it fails to take into account potential regime switches in the way monetary policy is conducted in either one of the two or in both countries. We allow for such changes. We first show that even though domestic monetary policy may not switch, a switch in the foreign monetary policy has effects on the volatility of domestic output and inflation, given the structure of the model. At this stage we compare the standard Taylor rule with the rule that includes a real exchange rate target when only the foreign country's monetary policy changes over time[8]. We then proceed to the case where domestic monetary policy switches as well.

In section 5.1 we present the different interest rate rules that will be considered in the calibration exercise in section 6. In each subsection of section 5.1, we explain the reasons why each different rule is considered. In section 5.2 we present a welfare loss function derived through a second order approximation of the repesentative household's utility function according to Rotemberg and Woodford

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[7]Clarida, Gali and Gertler estimate just an interest rate rule equation, instead of a structural model.

[8]In this paper when we refer to changes in monetary policy, we mean changes in the coefficients in the interest rate rule.

(1998).

5.1 Policy rules

In this section we focus on different policy rules. We characterize optimal the rule that leads to the lowest variation in output and inflation. Each rule leads to a different system of equations and, thus, different conditions that are necessary for determinacy. The rules considered will be of a standard Taylor form to more generalized ones.

5.1.1 Markov switching monetary policy rules

In this subsection we describe how Markov switching is introduced into the model. A markov swtiching interest rate rule is specified as

$$i_t = \phi_{s_t,x} x_t + \phi_{s_t,\pi} \pi_t + \varepsilon_t \tag{40}$$

where s_t captures the realized polic regime taking values 1 or 2. Regime follows a Markov process with transition probabilities $p_{ij} = P[s_{t+1} = j|s_t = i]$, where $i, j = 1, 2$. This specification implies the policy maker and the private sector observes the current regime. Therefore, private sector expectations about future inflation, for example, are specified as $E[\pi_{t+1}|\Omega_t]$, where $\Omega_t = \{s_t, s_{t-1}, \ldots, \varepsilon_t, \varepsilon_{t-1}, \ldots, \varepsilon_t^*, \varepsilon_{t-1}^*, \ldots\}$ captures its information set. Having, thus, assumed a two regime markov process for monetary policy, the transition probability matrix P receives the form

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

where p_{11} measures the probability of staying at date $t+1$ in regime 1 and p_{12} the probability of moving to regime 2 at date $t+1$ while

being in regime 1 at date t. p_{22} measures the probability of staying in regime 2 at date $t+1$ and p_{21} the probability of moving to regime 1 at date $t + 1$ while being in regime 2 at date t.

Monetary policy may switch because of various reasons. One of them could be the switch of the interests of the Central banker. There may be periods, for example, that he is more interested in fighting unemployment than inflation. As a result, the weight on inflation in the interest rate rule could be lower. A monetary policy switch may also be justified by the change of the Central banker. As already mentioned, there is a high number of papers arguing that the US monetary policy has been more accommodative as regards inflation fluctuations in the pre-Volcker period. Our emirical findings in section 2 suggest something similar, regarding the monetary policy of the ECB. We found that inflation was more volatile in the Trichet period after an either monetary or supply shock. Hence, a first thought could be that the ECB was less interested in dampening inflation volatility down in the Trichet era. However, a question that rises is whether this result is due to ECB fighting inflation volatility less, or due to a change in the US monetary policy, affecting thus eurozone inflation fluctuations, or both?

5.1.2 *Interest rate rules when only foreign monetary policy changes over time*

In this section we present the case where only the foreign Central bank changes its policy. Consequently, the coefficients in the interest rate rule of the home Central bank do not change over time. As far as the coefficients in the interest rate rule of the foreign country are concerned, we assume that there are periods where the foreign Central bank is more hawkish in controlling inflation (i.e. a coefficient greater than one) and sometimes dovish (i.e. a cofficient less than one)[9]. As for the interest rate rule in the home country we as-

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[9]Since our model is a two country one, we do not need to worry that much about potential

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sume that it may take the form of either the Taylor rule, or may be extended with a real exchange rate target. We do that because, as already mentioned, we are interested in figuring out whether a real exchange rate target allows the home central bank achieve a better control of inflation and output gap volatility. Hence, the home country interest rate rules, whose performance will be compared receive the following form

$$i_t = \phi_x x_t + \phi_\pi \pi_t \tag{41}$$

$$i_t = \phi_x x_t + \phi_\pi \pi_t + \phi_q q_t \tag{42}$$

while the interest rate rule in the foreign country receives the following form

$$i_t^* = \phi_{s_t,x} x_t^* + \phi_{s_t,\pi} \pi_t^* \tag{43}$$

5.1.3 *Interest rate rules when monetary policy changes in both countries*

In this section we present the case where both Central banks change the coefficients in their interest rate rules over time. For simplicity, we assume that the only change the home Central bank does is in the coefficient on the real exchange rate target. We do that, because our goal is to see whether and when it is beneficial for the home country to target the real exchange rate or not. The interest rate rule, thus, in both countries receive the following form.

$$i_t = \phi_x x_t + \phi_\pi \pi_t + \phi_{s_t,q} q_t \tag{44}$$

indeterminacy when the coefficient on inflation in one of the two countries is less than one. For reasonably high degrees of price rigidities in both countries, it is enough that one of the two countries attaches a weight on inflation in the interest rate rule greater than one, in order to achieve determinacy. For more details about the conditions for determinacy in two country models see Benigno and Benigno (2006) and the references therein.

$$i_t^* = \phi_{s_t,x} x_t^* + \phi_{s_t,\pi} \pi_t^* \tag{45}$$

Note that the interest rate rule (i.e. the coefficiets) for the foreign country is the same as that assumed in the previous section.

5.2 *Welfare*

The Central Bank sets the interest rate in such a way to minimize a measure of social loss derived by a second order Taylor expansion of the consumer's utility function as in Rotemberg and Woodford (1998), Amato and Laubach (2003) and Pappa (2004). It is summarized as[10]

$$W_t = -\tfrac{1}{2} u_c C \{ \lambda_1 (y_t - y_t^n)^2 + \lambda_2 (y_t - y_{t-1})^2 + \lambda_3 (y_t^* - y_t^{*n})^2 + \lambda_4 (y_t^* - y_{t-1}^*)^2 + \ldots$$

$$+ \lambda_5 \pi_{H,t}^2 + \lambda_6 (\pi_{H,t} - \pi_{H,t-1})^2 + \lambda_7 (\pi_{H,t}^*)^2 + \lambda_8 (\pi_{H,t}^* - \pi_{H,t-1}^*)^2 + \lambda_9 (q_t - q_t^n)^2 + \ldots$$

$$+ \lambda_{10}(q_t - q_{t-1})^2 + \lambda_{11}(q_t + y_t)^2 + \lambda_{12}(q_t + y_t^*)^2 + \lambda_{13}(q_{t-1} + y_t)^2 + \lambda_{14}(q_{t-1} + y_t^*)^2 + \ldots$$

$$+ \lambda_{15}(y_t + y_{t-1}^*)^2 + \lambda_{16}(y_{t-1} + y_t^*)^2 + \lambda_{17}(y_{t-1} - y_{t-1}^n)(q_{t-1} - q_{t-1}^n) + \ldots$$

$$+ \lambda_{18}(y_{t-1}^* - y_{t-1}^{*n})(q_{t-1} - q_{t-1}^n) + \lambda_{19}(y_{t-1} - y_{t-1}^n)(y_{t-1}^* - y_{t-1}^{*n}) + \ldots$$

$$+ \lambda_{20}(c_{t-1}^* - c_{t-1}^{*n})(q_{t-1} - q_{t-1}^n) + \lambda_{21}(c_t - c_t^n)(q_t - q_t^n) \} + t.i.p. + O(||\xi||^3) \tag{46}$$

where the coefficients λ_i, $i = 1, ..., 21$ are functions of the structural parameters and are defined in detail in the appendix.

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[10]The derivation of the loss function is given in detail in the Appendix.

6 *Parameterization*

In this section we proceed to the parameterization of the model in order to evaluate alternative monetary policy rules. As a first exercise we simulate the model under the simplest scenario. That is, we evaluate whether adding a real exchange rate target leads to lower welfare losses and better inflation control, when there is no markov switiching in either country. We then proceed to the case where either one or both countries change their monetary policy.

6.1 *Parameterization results*

In this section we calibrate the model to investigate how the variables of the model respond to shocks. Firstly, we want to show the importance of including the exchange rate into an interest rate rule, in general. The model is calibrated in the presence of five one standard devation shocks. Namely, a somestic monetary shock, domestic and foreign productivity shocks and domestic and foreign transaction costs shocks. In table 5 below we provide the values of the calibrated parameters.

Table 5: Parameter Values

Structural parameters		
σ	2	(Pappa, 2004 and Amato & Laubach, 2004)
θ	7.88	(Pappa, 2004, Amato &Laubach, 2003)
ρ	1	(Gali & Monacelli, 2005 and Chari, Kehoe & McGrattan, 1998)
γ	1	(Gali & Monacelli, 2002)
$\omega = \omega^*$	0.75	(Pappa, 2004)
$\delta = \delta^*$	0.8	(Pappa, 2004)
$\zeta = \zeta^*$	0.2	(Amato & Laubach, 2003)
$\psi = \psi^*$	0.6	
ϕ_π	1.5	(Taylor, 1993)
ϕ_x	0.5	(Taylor, 1993)
ϕ_q	0.45	(Svensson, 2000)
Interest rate weight		
λ_r	0.237	(Amato & Laubach, 2003)

6.1.1 *No markov switching*

Using the parameter values at table 5 we perform a simulation exercise evaluating the welfare loss when the coefficient on the real exchange rate target in the home country increases from zero to five. We assume that foreign monetary is characterized by the Taylor rule. In figure 1 below we show how welfare loss and CPI inflation volatility changes as the weight on the real exchange rate target increase. At figure 2 we present the impulse response functions for the output gap and CPI inflation following a monetary policy shock.

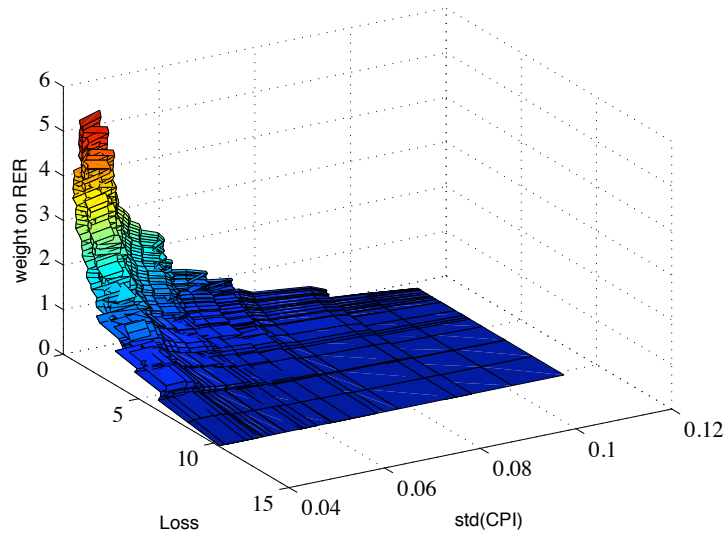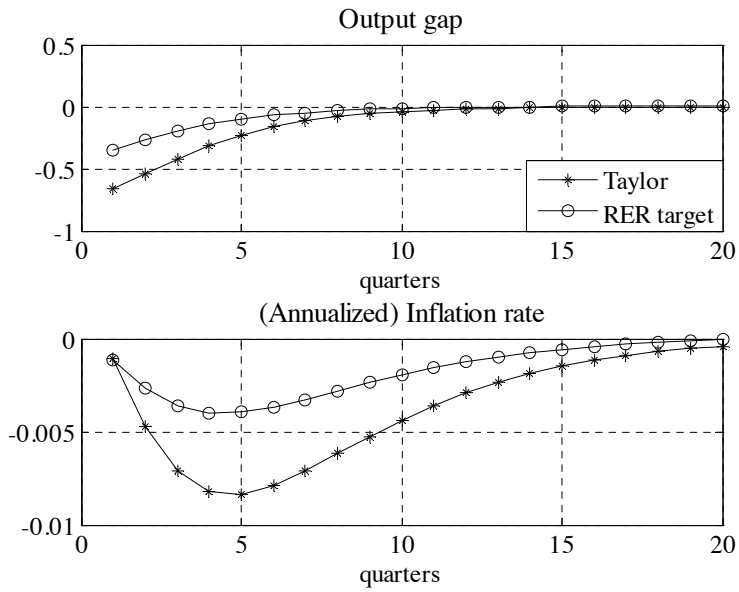Figure 1: Welfare loss and CPI volatility

Figure 2: IRFs - Monetary Policy shock

Both figures show that the domestic Central bank is able to control inflation fluctuations better by simply assigning a positive weight

on the real exchange rate target. Both CPI volatility and welfare loss fall cosiderably for even small values (i.e. between zero and one) of the coefficient. Additionally, CPI response following a monetary policy shock is amplified, leading to a lower fall than would be obtained under the Taylor rule. Finally, the output gap response is amplified too. Following an unanticipated increase in the policy rate, it falls less when the real exchange rate is also targeted.

6.1.2 *Markov swtiching in both countries*

In this section we allow for changes in the monetary policy of both countries. In the home country we assume that there are periods where the Central bank adopts a real exchange rate target and periods during which it follows a simple Taylor rule. As already mentioned, for simplicity, we assume that the coefficients on inflation and the output gap in the home Central bank's interest rate rule do not change. We allow only the coefficient on the real exchange rate to be either zero or 0.45. Monetary policy, thus, in the two country world will be characterized by the two interest rate rules, (44) for the home and (45) for the foreign country, respectively.

We split the analysis into two cases. In the first case, we assume that in regime 1 the home central bank targets the real exchange rate, while the foreign is dovish (i.e. assigns a coeffient on inflation less than one). Hence, in regime 2 the home central bank does not target the real exchange rate, while the foreign is hawkish. In the second case, we assume that in regime 1 the Home central bank targets the real exchange rate, while the foreign central bank is hawkish (i.e. it assigns a coefficient on inflation greater than one, in its interest rate rule). Hence, in regime 2, the home central bank will not target the real exchange rate, while the foreign central bank will assign a coefficient on inflation less than one

1st case

As already mentioned in this case in regime 1 the home central bank targets the real exchange rate and the foregn is dovish, while in regime 2 the former does not target the real exchange rate and the latter is hawkish. We first calibrate the model to derive the impulse responses inflation after a polic shock and then we show their standard deviations in each regime.
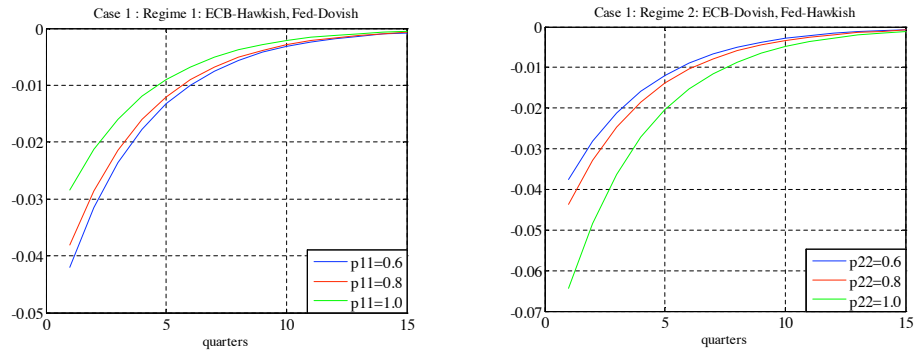

	Standard Deviations (Monetary Policy Shock)					
	Regime 1-Home Country					
	Hawkish-Dovish			**Both Hawkish (or Dovish)**		
Probs	*Output*	*CPI*	*PPI*	*Output*	*CPI*	*PPI*
P11=0.8, P22=0.8	0.2079	0.0057	0.0254	0.2390	0.0264	0.0343
P11=0.6, P22=0.8	0.2147	0.0065	0.0266	0.3373	0.0794	0.0494
P11=0.2, P22=0.8	0.2245	0.0070	0.0279	0.3467	0.3057	0.0420
	Regime 2-Home Country					
Probs	*Output*	*CPI*	*PPI*	*Output*	*CPI*	*PPI*
P11=0.8, P22=0.8	0.2625	0.0054	0.0289	0.4035	0.0903	0.0309
P11=0.8, P22=0.6	0.2504	0.0038	0.0273	0.2959	0.0166	0.0288
P11=0.8, P22=0.2	0.2376	0.0029	0.0259	0.2429	0.0063	0.0257

The impulse response functions show the responses of CPI inflation

in either regime after a monetary policy shock. The green line corresponds to the absorbing state (i.e. $p11 = 1.0$). In the left panel, it is shown that as the probability of not staying in the future in regime 1 falls, the response of inflation to an unanticipated increase in the interest rate is larger. The opposite holds in the right panel. As the probability of not staying in regime 2 in the future falls, the response of inflation is amplified. In other words, as the probability of moving tomorrow to the regime where the home central bank will target the real exchange rate increases, inflation is better controlled today. This is the so called, expectations formation effect. It is given by the distance between the green and the red (or blue) impulse response functions. It is clear, thus, that as long as the probability of staying in regime 2 tomorrow is less than one, the home central bank is able to achieve a better control of inflation before even it starts doing real exchange targeting. This is due to the expectations formation effect.

2nd case

In this case in regime 1 the home central bank targets the real exchange rate and the foregn is hawkish, while in regime 2 the former does not target the real exchange rate and the latter is dovish.
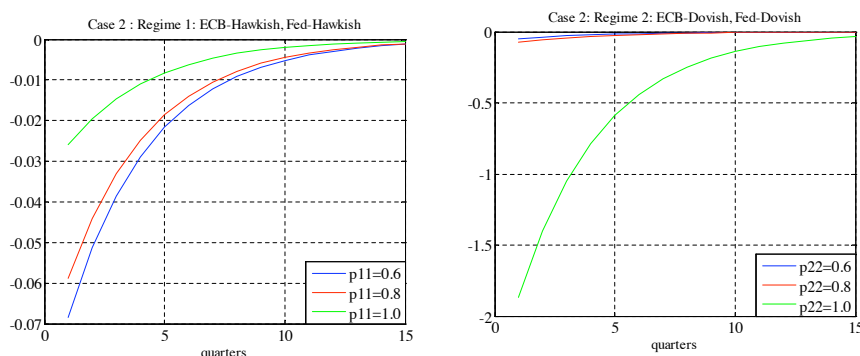
As in the previous case we observe that as the probability of not

staying in regime 1 falls, the response of inflation after a monetary policy shock is stronger. On the other hand, the expectations formation effect when the probability of moving to regime 1 tomorrow, while currently being in regime 2, increases, is very large. This is not surprising, since in regime 2 both central banks care less about inflation[11].

6.1.3 *Volatility implications*

Looking at the table of standard devations, we are able to clarify when it is benficial for the home central bank to target the real exchange rate, and when it is not. It is clear that the volatility of inflation in the second case is larger than in the the first one. In other words, when in regime 1 the home central bank targets the real exchange rate, while the foreign is hawkish, the volatility of inflation is higher. The intuition behind this result is the following. With a probability of moving to regime 2 being even slighlty nonzero, volatility increases in both regimes. Agents know that in regime 2 both central banks will care less about inflation. Hence, for nonzero probability of moving to regime 2, they adjust their expectations in such a way so that to drive inflation volatility higher. On the other hand, volatility in the first case is lower. In this case agents know that in both regimes one of the two central banks cares a lot about inflation volatility. This knowledge enables them to adjust their expectation in such a way so that not to allow for large increases in volatility, even when the home central is not targeting the real exchange rate, or the foreign is dovish. Therefore, we conclude that it is beneficial for the home central bank to change to real exchange rate targeting only when the foreign central bank is dovish.

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[11]The fact that the foreign central bank cares less about inflation in regime 2 is clear, since it is dovish (i.e. less than one weight on inflation in its interest rate rule). The argument that the home central bank cares less about inflation in regime 2 comes from the fact that in this regime it does not target the real exchange rate. We have already shown that targeting the real exchange rate amplifies the responses of inflation, after a policy shock.

7 Conclusions

We estimated a structural VAR model for the Eurozone and the US and found evidence in favor of regime shifts in monetary policy of both the ECB and the Fed. We constructed a two-country DSGE model to show that in a simple framework without markov-switching in monetary policy, real exchange rate targeting allows the home central bank to achieve a better control of inflation fluctuations. We, then, introduced markov-switching in monetary policy in order to analyze the effects of real exchange rate targeting and regime shifts on inflation volatility. When only foreign monetary policy switches regimes, it is always beneficial for the home central bank to target the real exchange rate as well. However, when we allowed for regime shifts in the domestic monetary policy, our findings showed that this is not always the case. Real exchange rate targeting is beneficial for the home country, when the foreign central bank is dovish (i.e. does not satisfy the Taylor principle). When the home central bank does real exchange rate targeting, while the foreign central bank is hawkish in the same regime, then this leads to large increases in inflation volatility, regardless of whether the home central bank targets the real exchange rate or not. The driving force for this result seems to be the way the private sector forms its expectations about inflation. Knowing that in each of the two regimes one of the two central banks cares a lot about inflation is enough in order for expectations not to allow for large future increases in inflation volatility.

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