Unemployment and Portfolio Choice: Does Persistence Matter?[*]

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Abstract

We use a life cycle model to study the effects of unemployment on the portfolio choice of households in the US and in Germany. We find that, in case of short-term unemployment only, unemployment insurance offsets the negative impact of unemployment risk on households' equity holdings. When incorporating long-term unemployment, the US-equity share drops. This negative effect of unemployment is mainly driven by its high expected duration. In Germany, however, long-term unemployment does not significantly alter portfolio decisions. We show that the different responses of portfolios to unemployment can be attributed to the shapes of German and American age-income profiles.

Keywords: precautionary savings, unemployment insurance, long-term unemployment

JEL Classification: D91, E21, H31

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1 Introduction

In the aftermath of the global financial crisis, more and more people in the US are unemployed an extended period of time. While long-term unemployment is a long-standing issue on the European agenda, because of the global recession it now becomes an issue in the US as well: nearly half of the pool of unemployed are out of work for more than 27 weeks and the average duration of unemployment has increased to a long-term high of over 35 weeks (see Ilg (2010), Economist (2010)). At the same time, the need to reduce budget deficits makes it harder to provide income support by extending unemployment benefits. In some European countries, cuts in benefit payments are included in the discussions over how to reduce budget deficits.

The aim of this study is to theoretically analyze the impact of an increase in unemployment risk on the optimal portfolio decisions of households. In the presence of greater labor income risk and longer average joblessness durations, how do individuals change their share of savings invested in risky stocks and risk-free bonds? And how do these effects vary for different levels of unemployment insurance and different durations of unemployment? Studying the effects of labor market frictions and social security on the portfolio decisions of households is important for two principle reasons. On the one hand, individual portfolio choice allows agents to share consumption risks, to build up wealth and hence to smooth consumption paths over life. In terms of long-run welfare implications, it is relevant for policymakers to know how investment behavior, and thus precautionary savings and preparedness for retirement, are affected by increased unemployment risk. On the other hand, portfolio choice drives the demand for risky versus risk-free assets at the aggregate level. It thereby influences the refinancing conditions of firms and governments and in turn affects the efficient functioning of financial markets.

Our paper contributes to the literature in four main respects. First, we explicitly model the unemployment process in a life cycle model of consumption and portfolio choice using Markov-chains. The setup is similar to the one presented by Cocco,

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Gomes, and Maenhout (2005), who consider the optimal allocation of savings between riskless and risky assets over the life cycle in a calibrated model of consumption and portfolio choice. We augment their model by introducing unemployment risk following Engen and Gruber (2001) [1] and Imrohoroglu, Imrohoroglu, and Joines (1995). We show that modeling unemployment risk explicitly yields results that are similar to those obtained when imposing a small probability of a disastrous labor income shock as in Carroll (1997) and Cocco et al. (2005). That is, young agents significantly reduce the optimal share of risky assets in their portfolios if no unemployment insurance is in place. However, when receiving unemployment benefits, we find that agents' investment behavior closely resembles the case without unemployment risk.

Second, we differentiate between short- and long-term unemployment by allowing for three instead of only two employment states in the Markov-process. Even though labor market frictions are not explicitly modeled, long-term unemployment could capture frictions like bad qualification profiles in the labor force. Our results suggest that the equity share in the portfolio of households is significantly reduced until midlife even if unemployment insurance is established. We show that the high expected mean duration of the long-term unemployment state is essential for the reduction in the equity share. Assuming alternative unemployment dynamics where the distribution of different income states is independent over time but nevertheless imposing the same unconditional distribution does not have any significant impact on optimal portfolio choices.

Third, we examine how different age-income profiles affect the results. For that goal, we estimate age-income profiles using German household panel data, calibrate the fundamental parameters to the German case and compare the model implications with those found for the calibration to US-data. Our analysis reveals that the impact of unemployment risk on portfolio choice critically depends on the under-

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[1]The authors show a negative impact of unemployment insurance on asset accumulation in a life cycle framework and empirically confirm this result in a panel study for the US. However, they do not consider the optimal portfolio allocation between risky and risk-free assets.

lying income evolution: with the flatter income profiles that we get for Germany, the equity share responds much less to unemployment risk and persistence than it does with the US-specification. Using stylized piecewise-linear income profiles as inputs to our model, we show that the steepness of the income profiles during the first years of professional life is crucial for households' response to unemployment risk. This finding extends the results presented by Cocco et al. (2005) who study the sensitivity of portfolio choice to income profiles for different education groups when there is no unemployment risk. In contrast to the results *without* unemployment risk, our results with unemployment risk suggest that different income profiles alter the investment decisions of households.

Finally, we compare the model implications for the US with those for the calibration to German data. Our results point to the fact that, during professional life, US households invest a higher fraction of their savings in stocks than German households. This outcome matches the empirical facts presented in the literature (e.g. Guiso, Haliassos, and Jappelli (2002)). Using hypothetical piecewise-linear age-income profiles, we show that this pattern can be attributed to the difference in the life cycle evolution of income in the US and in Germany.

The remainder of the paper proceeds as follows. Section 2 discusses the model and section 3 the corresponding optimization problem. The calibration and parametrization is presented in section 4. Section 5 is devoted to the results: the first subsection provides the policy functions for different setups while the second subsection lays out our simulation results based on these policy functions. Section 6 concludes.

2   The Model

Our model is based on the life cycle framework with optimal consumption and portfolio choice presented in Cocco et al. (2005). We extend their model by introducing unemployment risk, which is modeled similar to that in Imrohoroglu et al. (1995).

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The model describes a partial equilibrium where households are *ex ante* homogeneous, that is they have identical preferences and are subject to the same mortality and labor income risks. *Ex post*, households differ with respect to age, employment status and wealth. They choose consumption and the share invested in risky assets endogenously, while labor supply and retirement age are assumed to be exogenous.

2.1 Preferences

The economy is inhabited by a continuum of individuals who live for a maximum of T periods, facing mortality risk in each period of life t. Let $t = 1, ..., T$ denote adult age. Each individual works up to period K when she reaches retirement age. Individual i maximizes expected discounted lifetime utility

$$E_t \sum_{t=1}^{T} \delta^{t-1} \left[\prod_{k=1}^{t} p_k \right] u(C_t) \tag{1}$$

where δ is the subjective discount factor and p_t reflects the conditional probability of survival from age t to $t + 1$.[2] Preferences are modeled by the constant relative risk aversion utility function

$$u(C_t) = \frac{C_t^{1-\gamma}}{1 - \gamma} \tag{2}$$

which positively depends on consumption at age t, C_t, while γ is the coefficient of relative risk aversion. The intertemporal elasticity of substitution is given by $1/\gamma$.

2.2 Income

Individuals earn stochastic labor income during their working life. Since labor income risk is not completely insurable against shocks, the model exhibits a certain degree of market-incompleteness. As of retirement age K agents receive a constant

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[2]By definition $p_1 = 1$ and $p_t = 0$ for $t > T$.

fraction of their last labor income in terms of retirement benefits. Thus, retirement income is stable.

2.2.1 Worker's income

During professional life, individuals face a stochastic risk of becoming unemployed. We extend the standard case of two employment states - unemployment and employment - by a third state, thus allowing for a differentiation between short- and long-term unemployment. Let $s \in S = \{e, u_s, u_l\}$ be the employment opportunities state which is assumed to follow a first-order Markov-chain. If $s = e$, the consumer is offered the opportunity to work. Whenever an individual is given the opportunity to work, he supplies labor inelastically. If $s = u_k, k = s, l$ the agent is short-term (u_s) or long-term (u_l) unemployed.

The transition matrix for the employment opportunities state is given by $\Pi(s', s) = \left[\pi_{ij}\right], i, j = e, u_s, u_l$ where each element $\pi_{ij} = Prob\{s_{t+1} = j | s_t = i\}$ reflects the probability that a particular state i is followed by state j so that

$$\Pi(s', s) = \begin{pmatrix} \pi_{ee} & \pi_{eu_s} & \pi_{eu_l} \\ \pi_{u_s e} & \pi_{u_s u_s} & \pi_{u_s u_l} \\ \pi_{u_l e} & \pi_{u_l u_s} & \pi_{u_l u_l} \end{pmatrix}. \tag{3}$$

Let $f(t, Z_{it}) = f_t$ be a deterministic function of age t and of a vector Z_{it} containing other individual characteristics which reflects the age-dependent labor income profile of agent i. Each individual's labor income can then be expressed as

$$Y_t = \begin{cases} f_t P_t \Theta_t & \text{for } t = 1, ..., K-1 \quad \text{if } s = e \\ \zeta_k f_{t-\tau} P_{t-\tau} & \text{for } t = 1, ..., K-1 \quad \text{if } s = u_k, \ k = s, l \end{cases} \tag{4}$$

where τ is the duration of the unemployment state and ζ_k is the benefit replacement ratio. In case the investor is unemployed, he receives a constant fraction ζ_k of his permanent labor income based upon the last period he worked in. Depending on the unemployment duration, the replacement ratio differs. If an agent is jobless for only a short period of time ($k = s$), they receive higher benefits than if they are long-term unemployed ($k = l$). Going back to Hall and Mishkin (1982), labor income can be decomposed into two components. On the one hand, Θ_t is a transitory shock to labor income distributed as $ln(\Theta_t) \sim N(-\sigma_\theta/2, \sigma_\theta)$, which mirrors temporary factors like one-time bonuses or sickness benefits. On the other hand, P_t is the permanent component of labor income which evolves according to

$$P_{t+1} = \begin{cases} U_{t+1}P_t & \text{for } t = 1, ..., K-1 \quad \text{if } s = e \\ P_t & \text{for } t = 1, ..., K-1 \quad \text{if } s = u_k, \ k = s, l. \end{cases} \tag{5}$$

where U_{t+1} is a log-normally distributed shock to the permanent component of labor income with $ln(U_t) \sim N(-\sigma_u/2, \sigma_u)$. Permanent shocks to labor income are, for example, job changes, chronic health problems, or pay increases. The rate of change of the age-specific deterministic component of labor income is given by $G_{t+1} = f_{t+1}/f_t$ if the agent is given the working opportunity. Overall, labor income is a serially correlated process subject to both temporary and permanent shocks as well as a positive probability of becoming unemployed in every period.

2.2.2 Income during retirement

Once agents reach the retirement age, K, they receive funding from the social security system. Similarly to unemployment benefits, retirement income is deterministic and modeled as a constant fraction λ of permanent income earned in the last period of working life

$$Y_t = \lambda f_{K-1} P_{K-1} \quad \text{for } t = K, ..., T \tag{6}$$

implying that $G_t = U_t = 1$ during retirement.

2.3 Asset market

On capital markets, the individual can either invest in bonds, B_t, or in risky assets, S_t. The riskless bond has a constant gross real return of R_f whereas stocks earn a gross real return of R_t. Excess returns are composed of the mean return on equity, μ, plus a disturbance term η:

$$R_t - R_f = \mu + \eta_t .\tag{7}$$

The expectation of the excess return is given by the mean equity-premium $E(R_t - R_f) = \mu$ and the return on equity is assumed to be independently and identically distributed as $ln(R_t) \sim N(ln(R_f + \mu) - \sigma_\eta/2, \sigma_\eta)$.

2.4 Budget constraint

Each period in his lifetime, the individual allocates his cash-on-hand, M_t, to bonds, risky assets, and consumption, C_t. Hence, cash-on-hand in period $t+1$ is defined as

$$M_{t+1} = \left[\alpha_t R_{t+1} + (1-\alpha_t)R_f\right] A_t + Y_{t+1}\tag{8}$$

where $A_t = M_t - C_t$ reflects assets after all transactions are taken in period t and thus represents the agent's savings. The variable α_t stands for the proportion of savings invested in stocks at time t.

3 Optimization problem

So far, we have two control variables, namely consumption, C_t, and the equity share, α_t, together with the four state variables M_t, P_t, f_t and s_t. Given that our optimization problem is homogeneous in the permanent components of labor income, P_t and

f_t, we normalize it by these two variables, such that the state space is reduced to two dimensions. For a detailed derivation see Appendix A. Defining $\frac{X_t}{P_t f_t} = x_t$, the normalized Bellman equation of the maximization problem can be written as

$$v_t(m_t, s_t) = \max_{c_t, \alpha_t} \left\{ u(c_t) + \delta p_t G_{t+1}^{1-\gamma} E_t \left[U_{t+1}^{1-\gamma} v_{t+1}(m_{t+1}, s_{t+1}) \right] \right\} \tag{9}$$

subject to the normalized budget constraint

$$m_{t+1} = \left[\alpha_t R_{t+1} + (1-\alpha_t) R_f \right] \frac{(m_t - c_t)}{G_{t+1} U_{t+1}} + y_{t+1} . \tag{10}$$

Writing out the expectation over the employment state s_t explicitly, the individual's dynamic programming problem can be stated as

$$v_t(m_t, s_t) = \max_{c_t, \alpha_t} \left[u(c_t) + \delta p_t G_{t+1}^{1-\gamma} \sum_{s_{t+1}} \pi(s_{t+1}|s_t) \widetilde{E}_t U_{t+1}^{1-\gamma} v_{t+1}(m_{t+1}, s_{t+1}) \right] \tag{11}$$

where he maximizes the recursive value function v_t subject to the budget constraint (10) and the non-negativity constraint $a_t \geq 0$.

The levels of the value function, consumption and all other variables can be obtained from

$$V_t(M_t, P_t, f_t, s_t) = (P_t f_t)^{1-\gamma} v_t(m_t, s_t) \quad \text{and} \tag{12}$$

$$C_t(M_t, s_t) = P_t f_t c_t(m_t, s_t) \tag{13}$$

where we multiply the normalized functions with the appropriate income-factors as in Carroll (2009).

Since no analytical solution to this finite-horizon maximization problem exists, we use numerical methods to obtain the optimal policy functions $c_t(m_t, s_t)$ and $\alpha_t(m_t, s_t)$. First, we specify a terminal decision rule and then solve the problem using backward induction. Following Carroll (2006), we discretise the state space and compute the values of the policy functions at each grid-point of possible values of the state

variables m_t and s_t. We then interpolate between the discrete points of the functions c_t and α_t to get an approximation to the optimal decision rules. Having computed the interpolated policy functions at time t, the corresponding value function can be determined. We construct the solutions for earlier periods by recursion from $t = T$ to $t = 1$.

4 Calibration

We calibrate the model to both the German and the US context. Unless otherwise stated, parameter values and functions for the US are taken from Cocco et al. (2005). The model period corresponds to one year.

[Table 1 about here.]

Table 1 summarizes the parameter values used in our benchmark simulations. Individuals in both economies enter professional life at age 20 and live up to a maximum age of 100 so that our model accounts for $T = 81$ years. We set average retirement age to $K = 62$ for Germany, according to Eurostat-data for 2008. In the US, agents stop working at age 65. Following Cocco et al. (2005), the coefficient of relative risk aversion, γ, is fixed at the value of 10 for both economies, the subjective discount rate, δ takes on a value of 0.96 which corresponds to an annual interest rate of 4 percent. Furthermore, we assume R_f, the real interest rate on the riskless asset, to be 2 percent while the mean return on stocks, μ, is set to 6 percent, hence implying an equity premium of 4 percent. The correlation between equity returns and shocks to labor income, ϕ, is set to zero as in Cocco et al. (2005).

According to OECD-data, the gross pension replacement rate, λ, i.e. pension benefits as a share of individual lifetime average earnings, is 55 percent in the US and 57 percent in Germany for 2010. Concerning the gross replacement rate for unemployment benefits, we refer to the OECD Employment Outlook (2010) where

the replacement rate for those who are unemployed for a period up to one year is $\zeta_s = 0.64$ in Germany and 0.28 in the US, whereas the replacement rate significantly drops for individuals who are long-term unemployed (five year unemployment spell, see Table 1).

The vector of conditional survival probabilities for the US and Germany, p_t, is computed from the mortality tables provided by the Human Mortality Database (http://www.mortality.org).

The transition probabilities for the Markov process are chosen such that the unconditional probability of being either short-term or long-term unemployed matches US and German data. Taking into account that the average US-unemployment rate between 2000 and 2008 was 5.1 percent with a share of long-term unemployment of roughly 10 percent of total unemployment, we calibrate the matrix Π such that the unconditional probability of being short-term unemployed amounts to 4.6 percent while the corresponding probability for long-term unemployment is 0.5 percent. We define short-term unemployment as a period of being without a job of one year whereas long-term unemployment averages six years in duration in our model.

Controlling for both unconditional probabilities as well as for the persistence of unemployment, the transition matrix we employ for the US is given by

$$\Pi(s',s) = \begin{pmatrix} 0.956 & 0.044 & 0 \\ 0.8923 & 0.091 & 0.0167 \\ 0.15 & 0 & 0.85 \end{pmatrix} \tag{14}$$

where we set $\pi_{eu_l} = 0$, because an individual is short-term unemployed first, before being counted as long-term unemployed and hence the state $s = e$ cannot be followed directly by the state $s = u_l$. Moreover, once an individual is long-term unemployed in our model, he can either stay in this state or return to work. However, it is impossible to switch from the state of long-term to short-term unemployment and

consequently we set the corresponding probability $\pi_{u_l u_s}$ equal to zero. The calibration of the employment process for Germany is done accordingly. With an average unemployment rate of 9.1 percent for the period 2000-2008 and a share of long-term unemployment of 52 percent the transition matrix is given by

$$\Pi(s', s) = \begin{pmatrix} 0.956 & 0.044 & 0 \\ 0.748 & 0.091 & 0.161 \\ 0.15 & 0 & 0.85 \end{pmatrix}. \tag{15}$$

For the scenario with two employment states, where $s \in S = \{e, u_s\}$, we adjust the transition matrix so that short-term unemployment rates of 4.6 percent and 4.4 percent for the US and Germany are achieved, respectively. Imposing an average duration of short-term unemployment of one year, we get

$$\Pi(s', s) = \begin{pmatrix} 0.956 & 0.044 \\ 0.909 & 0.091 \end{pmatrix} \tag{16}$$

for the US and

$$\Pi(s', s) = \begin{pmatrix} 0.958 & 0.042 \\ 0.909 & 0.091 \end{pmatrix} \tag{17}$$

for Germany. The deterministic part of the German labor income process, f_t, is constructed following Cocco et al. (2005). A detailed description of the estimation procedure and the data can be found in Appendix B.

We use household-data from the German Socio Economic Panel (SOEP).[3] In a first step, we regress the logarithm of real net household income on a set of age dummies and a vector Z_{it}, which contains household-specific variables such as household head gender, family status, the number of children, and household size. We control for family-specific heterogeneity using the fixed-effects estimator. In a sec-

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[3]See SOEP Group (2001) for a detailed description of the data.

ond step, the coefficients of the age dummies are regressed on a third order age-polynomial to get smoothed profiles for the model simulations.

Tables 2 and 3 show the regression results for four different specifications for Germany. First, we estimate the deterministic part of the labor income process for the whole sample. Second, the sample is subdivided into three education groups relative to high school education. Apart from the education group holding less than a high school degree, the coefficients of the age dummies are highly significant and the age-income profiles are hump-shaped over the working life. For our simulations we use the income profile for the group of households holding a high school degree (see Figure 1), since the sample size is largest for this subset.

[Table 2 about here.]

[Table 3 about here.]

The variances of the temporary and permanent shock to labor income in Germany, σ_θ^2 and σ_u^2, are taken from Fuchs-Schündeln (2008) who followed the variance decomposition procedure described in Carroll and Samwick (1997) using the original West German SOEP sample.

[Figure 1 about here.]

5 Results

We divide our analysis into three parts. First, we compare the policy functions and simulation results for the benchmark case without unemployment risk with the case of short-term unemployment for the US. In this setup, the investor may find herself in two different states in each period of her working life. If $s = e$, she is given an employment opportunity. If $s = u_s$, she is short-term unemployed. In this scenario

we consider two subcases. First, only a minimum of insurance against unemployment is available ($\zeta = 0.1$). Second, we introduce unemployment insurance with an income replacement ratio of $\zeta = 0.28$, which is in line with US data. We show that unemployment insurance, as established in the US, helps to offset the increased labor income risk. The share invested in stocks evolves thus very similarly to the benchmark case without unemployment risk. Hence, the replacement ratio seems to be important for portfolio choice.

For our second case, we consider a setup where the agent faces three possible employment states. Besides the two states $s = e, u_s$ she faces the additional risk of being long-term unemployed, i.e. $s = u_l$. In this scenario, we again differentiate between two subcases: First, we realistically calibrate the transition matrix Π, matching both the persistence of unemployment and the unconditional probabilities of being short-term or long-term unemployed to US data. When long-term unemployment is taken into account, we observe that the equity share is reduced, even in the presence of unemployment insurance. This drop is particularly important for young investors. Second, we set the conditional probabilities equal to the unconditional ones, such that the realizations of the possible states are independent over time. The model shows that the persistence of unemployment plays a key role in explaining low equity shares in the portfolio of young investors: without accounting for the average duration of unemployment optimal portfolio choice over the life cycle closely resembles the case without any unemployment risk.

As a third case, we repeat the exercises above for Germany and find that the effects observed for the US are significantly mitigated when using the German age-income profile in the model.[4] We therefore compare the benchmark scenarios for the US and Germany directly and detect that German households invest a smaller fraction of their savings in the risky asset than US-households do during profes-

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[4]Other parameter values are also adjusted to the German case (see Table 1) We run sensitivity checks in order to single out the effects of changes in different parameter values. The sensitivity checks reveal that the change in the policy functions and in the simulation results go back to the difference in the age-income profiles.

sional life. In order to analyze where this result comes from, we feed different stylized income profiles into the model. This exercise shows that it is mainly the steepness of the income profile during the early years of professional life that drives the reaction of optimal portfolio choice to unemployment.

5.1   Policy functions

In this section we discuss the policy function for the optimal share invested in stocks, $\alpha(t, m_t)$. The function $\alpha(t, m_t)$ mirrors the optimal decision rule for an investor of age t disposing of a certain amount of cash-on-hand m_t. We present the policy functions for the share invested in stocks as contour plots for each scenario studied.

The contour plots can be read in the following way. Figure 2 illustrates the optimal decision rule for the benchmark scenario in the US where we elmininate any unemployment risk. Age t is plotted at the vertical axis while the level of cash-on-hand, m_t, is on the horizontal axis. The corresponding numerical values of the associated portfolio share of stocks $\alpha(t, m_t)$ are indicated on the contour lines. The darker the area between the contour lines, the lower the associated values of α. For a given level of cash-on-hand (imagine a vertical line at $m = 4$ for example), the contour lines show that the share invested in stocks falls from close to one down to 0.56 at approximately age 48. Afterwards, α increases somewhat until retirement age $K = 65$ is reached. During the rest of her life, the investor continuously reduces the equity share as she approaches end of life T.

[Figure 2 about here.]

Looking at the plot the other way around, let us fix age at 40, for example, and examine the evolution of α_t across different levels of cash-on-hand. The contour lines reveal that the equity share is close to one up to $m = 2.5$. As wealth m increases further, α_t starts to descend, but at a diminishing rate as the contour lines lie farther

away from each other for higher levels of cash-on-hand. For $m = 10$ for example, an investor aged 40 optimally invests about 32 percent of his savings in risky assets.

5.1.1 Benchmark: no unemployment risk

We now turn to the interpretation of the baseline scenario without any unemployment risk. This scenario closely resembles the one analyzed in Cocco et al. (2005).

Let us concentrate on the retirement period first, where labor income is modeled under the simplifying assumption of being constant and certain. At any given age, the equity share decreases as cash-on-hand grows. This is explained as follows. Future retirement income can be understood as a substitute for riskless asset holdings. In other words, the stream of future retirement income reflects implicit bond holdings in the individual's asset portfolio. Agents who dispose of little wealth buy more stocks, because their future retirement income and hence their implicit risk-free asset position is larger relative to their financial wealth than for richer investors. Expressed in mathematical terms, Samuelson (1969) and Merton (1969) show that under the assumption of complete markets and absent any labor income, the fraction of wealth invested in stocks is given by

$$\alpha^* = \frac{\mu}{\gamma \sigma_\eta^2}. \tag{18}$$

Hence, the optimal equity share α^* is independent of both wealth and age in this setup. However, when introducing a constant stream of labor income, Merton (1971) and Bodie, Merton, and Samuelson (1992) reveal that investors take total wealth, that is financial wealth, M_t, plus human capital measured as the present discounted value of all future labor income, PVY_t, into account when choosing their optimal portfolio equity share, such that

$$\alpha^* = \frac{\alpha_t M_t}{M_t + PVY_t}. \tag{19}$$

where α^* denotes the fraction of *total* wealth held in stocks while α_t reflects the share of *financial* wealth invested in the risky asset. From equation (19) it follows that relative to total wealth, the portfolio equity share is constant. Since we are interested in the evolution of α_t here, let us rewrite equation (19) in the following way:

$$\alpha_t = \alpha^* \left[1 + \frac{PVY_t}{M_t} \right] . \tag{20}$$

Equation (20) illustrates the forces which drive the optimal share of financial wealth invested in stocks: it depends on the ratio of human capital, PVY_t, to financial capital, M_t. Since this ratio changes over the life cycle, α_t changes as time passes. On the one hand, for a given level of cash-on-hand, M_t, the present value of future labor income falls as the agent gets older due to (i) the shorter time-horizon, and (ii) the hump-shape of the deterministic part of labor income. Thus, the equity share α_t tends to diminish with age. On the other hand, at any given level of human capital, α_t decreases in financial capital M_t. At the limit, the share of financial wealth held in stocks converges against α^*, the optimal equity share relative to total wealth. First, at the end of life, when the present value of future labor income approaches zero, α_t converges toward α^*. Moreover, as the investor gets richer and M_t goes toward infinity his portfolio behavior increasingly resembles the optimal choice under complete markets. Consequently, these two mechanisms at work in the model imply that young agents hold a high fraction of their financial capital in the risky assets explicitly, whereas elder and richer investors tilt their portfolio toward safe assets.

Having described the evolution of the equity share during retirement, we now turn to working life, when labor income is stochastic. Holding age fixed, Figure 2 reveals that the optimal decision rule for the equity share is still decreasing in cash-on-hand. Hence, stochastic labor income also seems to be a substitute for bonds rather than stocks and thus acts as an implicit bond holding. This is due to the fact that the shocks to the labor income stream are only weakly correlated with the

disturbances to equity returns as in Cocco et al. (2005). For any given level of wealth m_t, the contour lines illustrate that during the first part of professional life, α_t falls and this happens at a slower pace for higher levels of m_t. The reduction in the equity share can be explained by the fact that the present value of future labor income is high during the first years of adult life and then eventually diminishes. As of that point, investors start to substitute for implicit bond holdings. They buy more bonds explicitly due to their precautionary savings motive: on the one hand, they built up buffers in order to insure against negative labor income shocks. On the other hand, they accumulate wealth to prepare for retirement when income falls to the constant fraction λ of labor income, aiming at a smooth consumption path over their whole life. As of age 48, the equity share begins to rise again as investors approach the retirement period where future retirement income will be certain. Moreover, they already have accumulated risk-free buffer stocks in order to protect against disturbances to labor income.

5.1.2 Scenario 1: short-term unemployment and the effects of unemployment insurance

Figures 3(a) and 3(b) show the contour lines for the scenario with unemployment risk but only very basic insurance imposing a replacement ratio of 10 percent. In comparison to the baseline scenario without unemployment risk, the following patterns appear: For high values of wealth and starting at approximately age 30, the contour plots for the optimal share invested in stocks behave similarly to those in the benchmark scenario. Unemployment risk mainly affects young investors: In the employment state (Figure 3(a)), the equity share is lower for given m_t than without unemployment risk. This tendency is amplified in the unemployment state (Figure 3(b)) where the share invested in stocks is lower for poor investors during the entire working life. The small share invested in stocks by young investors, especially while unemployed, results from the fact that young individuals start out with low levels of

labor income. When unemployed, they get only very basic benefits. Consequently, they invest a significant share of their (small amount of) savings in bonds in order to substitute for missing implicit risk-free asset holdings from labor income. During their last years in the labor force, agents quickly increase equity shares since they have accumulated a sufficient stock of wealth and approach constant and certain retirement income.

[Figure 3 about here.]

Holding age fixed, the optimal share invested in stocks starts at a low level for young investors. As m_t increases over life, the equity share increases and then decreases again. The rise in α_t kicks in at higher levels of cash-on-hand the younger the investor is, especially if being jobless. If a young person is unemployed, she will only invest in risky assets if rich. Once the investor reaches midlife, she has already accumulated a certain amount of buffer stock savings, so that even at low levels of cash-on-hand she is able to invest more in stocks than a younger person.

Having discussed the effects of unemployment risk on the optimal decision rules $\alpha(t, m_t)$ in the absence of unemployment insurance, let us now introduce unemployment insurance with a replacement ratio of 28 percent, as in the US. Figures 3(c) and 3(d) show the contour lines for $\alpha(t, m_t)$ with insurance for the employment and short-term unemployment state, respectively. When comparing with Figure 2, it is observable that the optimal policy rule for the employment state is similar to the benchmark case without any risk of becoming unemployed. Figure 3(d) indicates that if the agent is jobless, the optimal share invested in stocks is below the one in the benchmark scenario and in the employment state for the young and poor. However, the negative effect of unemployment risk is dampened if social security systems are in place: a comparison of Figures 3(b) and 3(d) shows that young and poor agents invest a greater share in stocks when granted a certain level of unemployment insurance.

[Figure 4 about here.]

5.1.3   Scenario 2: the effects of long-term unemployment

We now extend the framework with the risk of being not just short-term, but also long-term unemployed. We use the transition matrix Π that is calibrated to US data as described in Section 4. That is, we take the unconditional probabilities of becoming short-term and long-term unemployed into account and also consider the persistence of the different employment states as reflected by average durations.

Figure 4 illustrates the optimal policy functions $\alpha(t, m_t)$ for the three employment states $s = e, u_s, u_l$ allowing for persistence in the unemployment process. In all three subfigures, the portfolio share invested in stocks is less for young agents when comparing the policy functions to the benchmark case. Apart from very low levels of cash-on-hand m_t, the equity share lies below the one in the baseline scenario during the first period of working life. This tendency is reinforced going from the employment over the short-term unemployment to the long-term unemployment state. Moreover, for those individuals who are close to retirement age and endowed with very little cash-on-hand, the optimal equity share is significantly reduced. Not surprisingly, the picture is especially pronounced in the long-term unemployment state (Figure 4(c)) where the optimal equity share is heavily downsized. For example, at the age of 40 and for a given level of wealth of $m_t = 4$, the optimal share invested in stocks drops to about 24 percent in case of long-term unemployment while if employed the corresponding share is roughly 55 percent. Hence, the risk of being jobless for an extended period of time is crucial for the investment decision of a US-household.

[Figure 5 about here.]

In order to further analyze the factors responsible for the negative effect of unemployment risk on the equity share chosen by households, we change the transi-

tion matrix Π such that the unconditional probabilities of being in one of the three states are calibrated as before. However, we eliminate the persistence component of unemployment by equalizing conditional and unconditional probabilities. Consequently, the employment states do not mirror the high expected duration of unemployment displayed in the data. The resulting policy functions for the equity share α_t are presented in Figure 5. Without persistence, the policy functions look qualitatively similar to the benchmark scenario without unemployment risk apart from the dark area at very low levels of cash-on-hand. For young and poor households, the optimal decision rule resembles the case of short-term unemployment with insurance (see Figure 3(d)): While young and disposing of little wealth, investors reduce their equity share. The reduction is more pronounced the longer the average duration of unemployment is. Yet, agents respond much less to labor income risk if we do not consider the expected duration of the unemployment states.

Summing up, the following key features can be deducted from Figures 2 to 5. In all three scenarios, for a given level of cash-on-hand, the equity share decreases during retirement as t approaches the final period T. The higher the value of m_t, the slower the fall in α_t, since the reduction in future retirement income is relatively less important for wealthy agents than for poorer ones. During the working period, α_t decreases in wealth in the majority of cases, except for the unemployment states where we observe non-monotone behavior for low levels of wealth. Overall, the higher labor income risk - either presented by low unemployment benefits or by the risk of long-term unemployment - the lower the share that young investors hold in risky assets. Thus, we can state that labor income risk crowds out capital market risk for this age group. We see in the next section that our simulation results mirror this pattern when averaging the evolution of the equity share over the life cycle for a large number of investors.

5.2 Simulation results

We simulate our model 10,000 times using the Monte Carlo method and average over the 10,000 simulated investors to compute the representative evolution of the share invested in stocks over the life cycle. The following section begins with the baseline scenario abstracting from unemployment risk. Subsequently, we discuss the simulation results for the scenarios including both short- and long-term unemployment.

[Figure 6 about here.]

5.2.1 Benchmark: no unemployment risk

Figure 6 shows the evolution of consumption, income, and cash-on-hand over the life cycle for our baseline scenario. [5] The graph closely matches the results presented in Cocco et al. (2005). Income is slightly hump-shaped during working life, reaching its maximum at about age 48. A kink is observed at US average retirement age $K = 65$ when income drops to the fraction λ of the last labor income. Afterwards, during retirement, earnings are constant, as we impose the simplifying assumption that there are neither temporary nor permanent disturbances to retirement benefits.

Consumption follows a smooth path that closely matches income during the first half of adult life. Afterwards it remains largely constant. Cash-on-hand strongly increases due to the high growth rates of deterministic labor income during the first years of adult age. At about age 48, wealth is accumulated at a somewhat lower rate until the agent leaves the labor force. Once the retirement period starts, wealth is run down rapidly and at an increasing rate the closer the agent nears the end of life. This is due to mortality-enhanced impatience given that we omit bequest motives.

[Figure 7 about here.]

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[5]Consumption, income and wealth evolve similarly for all cases studied here. This is why we present the graphs only once. The only difference which appears is that wealth peaks at a somewhat lower level in case of no unemployment insurance and persistent long-term unemployment.

Figure 7 plots the share invested in stocks for the benchmark scenario together with the graphs for scenario 1 where short-term unemployment is introduced. The solid line represents the benchmark case. The graph shows that during the first years of professional life, all savings are invested in stocks. This results from the fact that the deterministic labor income profile is very steep during the first ten years of adult life and the present value of future earnings, PVY_t, is high. At the same time the level of wealth, M_t, is still low. Consequently, young investors' portfolio share held in stocks is elevated because the ratio of the expected discounted future stream of labor income to wealth, $\frac{PVY_t}{M_t}$, is high.

After the first ten years of working life, the asset share falls until approximately 55, as investors demand more and more bonds during midlife in order to assemble savings for the retirement period. Put differently, the present discounted value of future labor income decreases as the investor ages - on the one hand because the future income stream shortens, on the other hand because the age-dependent component of labor income gets flatter and eventually falls - whereas the stock of cash-on-hand grows, leading to a decrease in the ratio $\frac{PVY_t}{M_t}$ of the two variables. Approaching the end of life, the equity share rises somewhat. This can be attributed to the fact that wealth erodes at a faster rate than the present discounted value of future retirement income does just before the end of life. Thus, even though the share invested in stocks shifts in with age during this period, the net effect on α_t is positive.

5.2.2 Scenario 1: short-term unemployment and the effects of unemployment insurance

While there is no unemployment risk in the benchmark scenario, we now investigate the outcome for two employment states, namely $s \in S = \{e, u_s\}$. First, let us look at a situation where only rudimentary unemployment insurance is available with a replacement ratio ζ of 10 percent. Hence, investors' labor income is now subject to higher risk. The dashed line in Figure 7(a) reveals that under these circumstances,

the evolution of the equity share significantly changes for young investors: it lies below 0.7 at the beginning of working life compared to a value of nearly one in the benchmark scenario. The share invested in risky assets sharply rises until age 30 before it starts falling again and comes back to normal at age 35. For the remaining life-time, the curve closely matches the one associated with the benchmark scenario, given that older investors have already accumulated precautionary savings and a certain stock of wealth so that they are less affected by unemployment risk than younger investors.

Once US-unemployment insurance is introduced with a replacement ratio of $\zeta = 0.28$, the dotted line in Figure 7(a) reveals that we are basically back to the benchmark scenario with high equity shares for young investors and lower ones for older individuals. Thus, the replacement ratio seems to be of vital importance for the investment decision of households that face a certain degree of unemployment risk. The results point out that the consequences of short-term unemployment for the portfolio share held in risky assets can be compensated by a sufficient level of unemployment insurance in our model. Unemployment insurance thus acts as a substitute for safe assets in households' portfolios.

5.2.3 Scenario 2: the effects of long-term unemployment

In the second scenario, we evaluate the results for the three different employment states adding the possibility of being long-term unemployed. When an individual is short-term unemployed meaning that he is out of work for at most one year he receives 28 percent of his last income. Once he is unemployed for more that one year, he is considered being long-term unemployed and the benefit replacement ratio reduces to 10 percent.

In Figure 7(b) it can be seen that if the Markov-chain for the employment state is calibrated realistically (dashed line), that is including both unconditional probabilities and persistence, the portfolio share invested in risky assets is significantly below

what we observe without unemployment risk (solid line). As before, the cohort of young investors is mainly affected. Until the age of 40, agents invest considerably less in stocks when confronted with the risk of becoming short- *and* long-term unemployed. Under these circumstances, the social security system is unable to offset the negative impact associated with long-term unemployment. It cannot avoid that young to middle-aged individuals considerably reduce their portfolio shares held in risky assets.

The dotted line in Figure 7(b) points to the key mechanism driving our results. Once we abstract from the persistence of unemployment, the evolution of the equity share closely matches its path in the baseline scenario. Therefore we conclude that the persistence component of unemployment is crucial for the investment decision of households in the US; the high expected duration of the unemployment states thus suppresses young workers' portfolio share invested in stocks.

[Figure 8 about here.]

5.2.4 Scenario 3: comparison to the German case

Given that labor market frictions have been an issue in German labor market policies for years, we now look at the model outcome for Germany. In the following, we replicate the same exercises as for the US above.[6] We then compare the model implications for the quite generous German social security system with those from the American case. In addition, we analyze how differences in the deterministic age-income profiles impact on the model implications.

Figure 8(a) plots the evolution of the optimal equity share chosen by German households in a world with short-term unemployment. Analogously to Figure 7(a), the solid line represents the benchmark case while the dashed line shows the outcome allowing for short-term unemployment without social security. The dotted

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[6]The contour plots of the optimal decision rules for the German case are in the Appendix.

line plots the profile of the equity share under the assumption that households are covered by unemployment insurance, like in Germany, where the replacement ratio is 64 percent.

The graph reveals that, with the income profile estimated for Germany, the previous results found for the US-calibration are significantly mitigated. Even without unemployment insurance (dashed line), German households diminish their portfolio equity shares only marginally during the first years of professional life when compared to the benchmark case. This pattern is even more pronounced for the second scenario with long-term unemployment (see Figure 8(b)): here, no significant difference between persistent and non-persistent unemployment is detected.

The evolution of the equity shares resembles a world without unemployment nearly perfectly. When comparing the baseline cases for the optimal equity shares in the US and in Germany, Figure 11 reveals that German households invest less of their savings in stocks than US-households do during most of their professional lives. This model outcome qualitatively matches the empirical findings on international household portfolios (e.g. Guiso et al. (2002)): Compared to the US, European and especially German households invest less in stock markets. Not only is the participation rate lower, but also the share of financial wealth invested in stocks is about half the size of the equity share held by US-households.

What is behind the different reactions of households' equity shares to unemployment? Keeping all parameters fixed at the values consistent with the US-data but plugging the age-income profile for Germany into the model, we find that the mitigation of the response of the equity share to unemployment risk can be attributed to the income evolution over the life cycle. In order to pin down how different shapes of income profiles affect the model implications for unemployment risk, we plug stylized piecewise-linear income profiles into the model. Figure 9 plots the hypothetical income profiles that we use to study how different shapes and slopes affect portfolio choice.

[Figure 9 about here.]

[Figure 10 about here.]

As Figure 10(b) shows, the steeper the labor income profile at the beginning of professional life, the more responsive are young agent's equity shares to unemployment risk. Looking at the income profiles f_1 and f_3 that feature high growth rates of income in the first period of professional life, you observe that the corresponding equity shares start out at relatively low levels: agents who face uninsurable unemployment risk invest about 70 percent of their savings in stocks initially. In contrast to this, for flatter income profiles like f_2 and f_4, the reaction to unemployment risk is less pronounced. Figure 10(b) reveals that for these income profiles, investors start out with a higher equity share of about 85 percent. Hence, the steeper the income profile is in the twenties, the lower the starting value of the equity share α_t, no matter how the income profile is shaped toward retirement age K. This is due to the fact that with steep earnings profiles, the present discounted value of future labor income increases during the first years of working life, given that earnings are very low during this period of life, but earnings growth is high. Consequently, at young ages when labor income is low, agents significantly reduce their equity shares if unemployment is modeled. However, as soon as the present value of future earnings rises, they expand the share of savings spent on the risky asset. Investors who have a flatter age-income profile do not see the present value of income grow by much, but rather face a constant present value of income in the beginning which starts falling eventually. Thus, we find a weaker hump-shaped evolution of their equity share. Moreover, Figure 10(a) shows that the faster income grows at the beginning, the later does the portfolio share invested in stocks start to drop in the baseline scenario. For example, comparing the black solid line with the red dashed line you can see that for the steeper income profile (f_1), the equity share starts to decline later than for the flatter profile (f_2). Hence, young professionals with faster earnings growth invest more in stocks than those with flatter income profiles do.

Another point that we can take away from Figure 10 is the following. The lower deterministic income is when the agent approaches retirement age, the lower is the share invested in stocks toward the end of life (compare f_1 and f_2 with f_3 and f_4), both for the benchmark and in a world featuring unemployment. Not only does α_t drop faster for profiles f_3 and f_4, it also drops further, so that agents who have lower income when becoming retirees invest significantly less in stocks (between 20 and 30 percent for profiles f_3 and f_4) than agents who receive a hypothetical income stream f_1 or f_2. The latter invest between 40 and 50 percent of their savings in stocks. Hence, for investment behavior during retirement, only the income evolution close to retirement age matters in the model whereas income growth at the beginning of professional life does not seem to play a crucial role.

Finally, Figure 10 indicates that the equity share for income profiles f_3 and f_4 is largely constant during retirement while it slopes up for profiles f_1 and f_2. This is explained as follows: The deterministic age-income profiles f_3 and f_4 reflect considerably lower earnings over the life cycle than the stylized profiles f_1 and f_2 do. Hence, an individual who disposes of an income stream corresponding to f_1 or f_2 can accumulate much more wealth over life than an individual who earns f_3 or f_4. Toward the end of life, the present discounted value of future labor income, PVY_t, falls. At the same time, the wealth stock is run down due to mortality-enhanced impatience. The net effect on the equity share is determined by the relative importance of the reductions in PVY_t and M_t. As the wealth stock associated to income profiles f_1 and f_2 is higher than the one for f_3 and f_4, the former has to be run down at a faster pace than the latter. Consequently, if the change in PVY_t is similar in both cases, the ratio $\frac{PVY_t}{M_t}$ may increase more for the first two income profiles than for the last two. This is what we see in Figure 10.

[Figure 11 about here.]

6 Conclusion

The goal of this paper is to investigate the impact of unemployment risk on the financial investment decisions of households in the US and Germany. To this end, we use a calibrated life cycle model of consumption and portfolio choice that features unemployment risk. We allow for three employment states: besides the possibility of being employed or unemployed, we extend the state-space by explicitly differentiating between short-term and long-term unemployment. This extension is motivated by the fact that long-term unemployment plays not only an important role in describing German labor market dynamics. The 2008-09 recession made long-term unemployment an issue in the US as well.

Our main findings are summarized as follows. When only considering short-term unemployment, we theoretically show that unemployment benefits as those currently established in the US are able to countervail the negative impact of unemployment risk on the portfolio share invested in risky assets. Consequently, investors choose their equity shares as if they were facing no unemployment risk at all. Unemployment insurance thus acts as a substitute for the risk-free asset in households' portfolios.

Yet, the picture changes when taking long-term unemployment into account. In this case, even if social security systems help insure against part of the increased labor income risk, the equity share in the portfolio of young investors is significantly reduced due to enhanced precautionary savings. We show that this outcome is predominantly driven by the persistence of unemployment: When running the risk of being unemployed for an extended period of time, households' investment behavior becomes more conservative.

The results significantly change for the German case. Using the German age-income profile in the model, we show that households' reaction to an increase in unemployment risks is minimal. Since German households invest a lower share

of their savings in stocks compared to Americans in the baseline case already, they do not reduce their equity share much further when facing a higher risk of being permanently out of work. We show that this finding can be attributed to the different shapes of income profiles, namely to the relatively low growth rates of earnings in Germany at the beginning of professional life.

Summing up, unemployment benefits are important for counteracting the negative effects of increased labor income risk on portfolio choice in the US. However, the reduction of the equity share compared to a world without unemployment risk can be mitigated via benefit payments to a certain degree only. As soon as people face the risk of being unemployed for an extended period of time the equity share is depressed, even in the presence of reasonable benefit payments. Given that optimal portfolio behavior is crucial not only for individual risk sharing but also for the refinancing conditions of governments and firms on financial markets in the aggregate, our findings present one additional reason to tackle long-term unemployment. In Germany, long-term unemployment and the associated labor market frictions remain an important issue. Nevertheless, our model suggests that the established unemployment insurance systems coupled with the relatively flat average income profile are sufficient to keep optimal portfolio choice close to a world without unemployment risk. Hence, we conclude that it is the combination of the persistence of unemployment and the income profile which matters for portfolio choice.

A Appendix

Abstracting from the state variable s_t for the moment, we normalize the optimization problem with P_t and f_t in the following way.

In a first step, consider equation (8) and divide by $P_{t+1}f_{t+1}$ such that

$$\frac{M_{t+1}}{P_{t+1}f_{t+1}} = \left[\alpha_t R_{t+1} + (1-\alpha_t)R_f\right]\left(\frac{M_t}{P_t f_t} - \frac{C_t}{P_t f_t}\right)\frac{P_t f_t}{P_{t+1}f_{t+1}} + \frac{Y_{t+1}}{f_{t+1}P_{t+1}} \quad . \tag{21}$$

Defining $\frac{X_t}{P_t f_t} = x_t$, (21) can be written as

$$m_{t+1} = \left[\alpha_t R_{t+1} + (1-\alpha_t)R_f\right]\frac{(m_t - c_t)}{G_{t+1}U_{t+1}} + y_{t+1} \tag{22}$$

where U_t is the stochastic growth rate of permanent labor income and G_t reflects the growth rate of the deterministic part of the labor income process, f_t. Normalized labor income y_t is given by

$$y_t = \begin{cases} \Theta_t & \text{for } t = 1,...,K-1 \text{ if } s = e \\ \zeta_k & \text{for } t = 1,...,K-1 \text{ if } s = u_k \, k = s,l \\ \lambda & \text{for } t = K,...,T. \end{cases} \tag{23}$$

In a second step, we setup the Bellman equation for the consumer's optimization problem in the next-to-last period of life, abstracting for the moment from the employment state s_t. The consumer maximizes utility subject to equations (2)-(8) choosing C_{T-1} and α_{T-1}:

$$V_{T-1}(M_{T-1},P_{T-1},f_{T-1}) = \max_{C_{T-1},\alpha_{T-1}} \left\{u(C_{T-1}) + \delta p_{T-1}E_{T-1}V_T(M_T,P_T,f_T)\right\} . \tag{24}$$

Given that the consumer will die at the end of period T, she will consume all cash-on-hand implying that $M_T = C_T$ and hence

$$V_{T-1}(M_{T-1}, P_{T-1}, f_{T-1}) = \max_{C_{T-1}, \alpha_{T-1}} \left\{ u(C_{T-1}) + \delta p_{T-1} E_{T-1} \left[\frac{M_T^{1-\gamma}}{1-\gamma} \right] \right\} . \qquad (25)$$

Now, let us expand equation (25) by $P_t f_t$ in order to express it in lower case letters

$$V_{T-1}(\bullet) = \max_{c_{T-1}, \alpha_{T-1}} \left\{ (P_{T-1} f_{T-1})^{1-\gamma} \frac{c_{T-1}^{1-\gamma}}{1-\gamma} + \delta p_{T-1} E_{T-1} \left[\frac{(P_T f_T)^{1-\gamma} m_T^{1-\gamma}}{1-\gamma} \right] \right\}$$

$$= (P_{T-1} f_{T-1})^{1-\gamma} \underbrace{\max_{c_{T-1}, \alpha_{T-1}} \left\{ u(c_{T-1}) + \delta p_{T-1} (G_T)^{1-\gamma} E_{T-1} (U_T)^{1-\gamma} \left[\frac{m_T^{1-\gamma}}{1-\gamma} \right] \right\}}_{= v_{T-1}(m_{T-1})}$$

so that we finally have

$$V_{T-1}(M_{T-1}, P_{T-1}, f_{T-1}) = (P_{T-1} f_{T-1})^{1-\gamma} v_{T-1}(m_{T-1}) \quad . \qquad (26)$$

The same logic can be applied for all earlier periods $t = 1, ..., T-2$.

B Age-income profiles

The deterministic part of the labor income process, f_t, is constructed following Cocco et al. (2005). We use household-data from the original West German Socio Economic Panel (SOEP) data from 1992 to 2008 as a proxy for the European context. In order to allow for endogenous means of insuring against labor income risk, we take a broad measure of household labor income which includes total family income from labor earnings, private retirement income, private transfers, public transfers, and social security pensions less total family taxes[7]. As we are interested in the income evolu-

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[7]Specifically, we use Household Post-Government Income minus Asset Income from the PEQUIV-dataset of the GSOEP and deflate this measure of nominal household income using the CPI with 2006 as a base year.

tion during professional life, we include households whose head is between 22 and 63 years old in our sample. Younger and older individuals are not included because the sample size in these age groups is small and self-selection is an important feature. Focusing on the labor force, we drop household heads who are either retired or serving an apprenticeship, but keep those who are unemployed.

To construct the age-income profiles, we first regress the logarithm of net real household income on a set of age dummies and a vector Z_{it} that contains household-specific variables like gender of the head of household, marital status, the number of children, and household size. First, we estimate the deterministic part of the labor income process for the whole sample. Second, the sample is subdivided into three education groups relative to high school education. For the highest education group, we drop households with heads younger than 25 given that agents enter the labor force later than those in lower education groups. We control for family-specific effects by using the fixed-effects estimator as in Cocco et al. (2005). Table 2 shows the regression results for the four different specifications.

In a second step, the coefficients of the age dummies are regressed on a third order polynomial in age, such that we get smoothed profiles for the model simulations (see Table 3). Apart from the education group holding less than a high school degree, the coefficients of the age dummies are highly significant[8] and the age-income profiles are hump-shaped over the working life. For our simulations we use the income profile for the group of households holding a high school degree (column 3),since the sample size is largest for this subset.

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[8]For brevity, we do not show the regression results for the whole set of age dummies. The complete table for the regression results is available upon request.

C Contour plots Germany

[Figure 12 about here.]

[Figure 13 about here.]

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Parameter	Description	Value US	Value GER
T	life span (20 to 100)	81	81
K	average retirement age	65	62
γ	coefficient of relative risk aversion	10	10
δ	subjective discount factor	0.96	0.96
μ	mean return on equity ($\mu - 1$)	0.06	0.06
σ_η^2	volatility of equity log-return	0.15^2	0.15^2
R_f	real riskless rate	1.02	1.02
σ_u^2	variance of shock to permanent labor earnings	0.0106	0.012
σ_θ^2	variance of transitory shock to labor income	0.0738	0.038
ϕ	correlation between stock returns and earning shocks	0	0
ζ_s	benefit replacement rate (short term unemployment)	0.28	0.64
ζ_l	benefit replacement rate (long term unemployment)	0.1	0.36
λ	benefit replacement rate (retirement)	0.55	0.57

Table 1: Parameter values for the US and Germany

Table 2: Age-income profiles: FE-Regression

LOGINCOME	all	no high school	**high school**	more than high school
male	0.126***	-0.109	0.0708***	0.197***
	(0.0147)	(0.0682)	(0.0212)	(0.0422)
married	0.158***	0.117***	0.135***	0.131***
	(0.00746)	(0.0230)	(0.00972)	(0.0148)
children	-0.217***	-0.260***	-0.218***	-0.177***
	(0.00426)	(0.0145)	(0.00540)	(0.00820)
hhsize	0.290***	0.339***	0.293***	0.229***
	(0.00402)	(0.0124)	(0.00506)	(0.00799)
Constant	6.678***	6.810***	6.713***	6.611***
	(0.0217)	(0.0593)	(0.0282)	(0.0599)
Observations	30835	3763	18637	8009
Number of groups	3609	654	2432	999
T-bar	8.5	5.8	7.7	8.0
R-squared	0.300	0.272	0.282	0.327

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3: Age-income profiles: Coefficients in the age polynomial

	all	no high school	**high school**	more than high school
age	0.0907	0.0300	0.0530	0.362
age2/10	-0.0151	-0.00539	-0.00770	-0.0699
age3/100	0.000818	0.000255	0.000332	0.00441
Constant	-1.309	-0.334	-0.714	-5.454
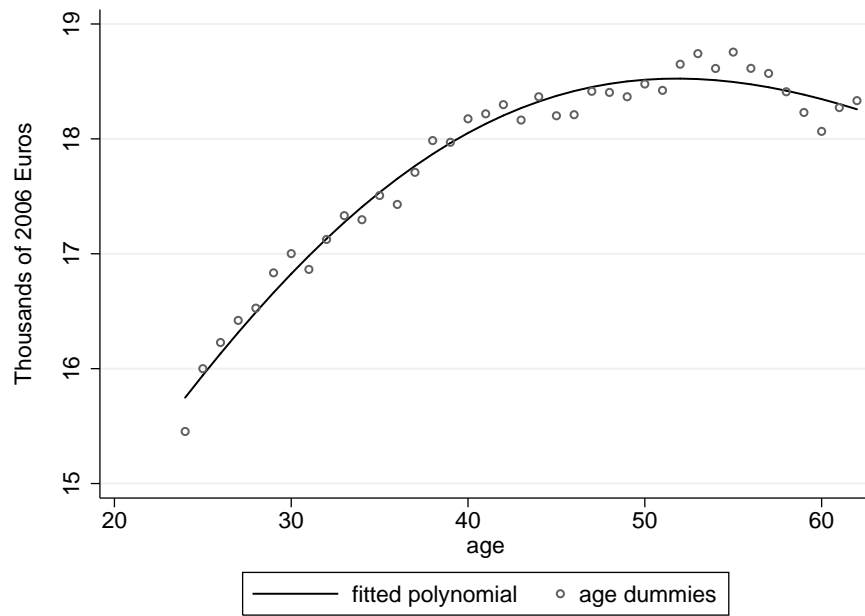
Figure 1: Age-income profile for Germany, high school education
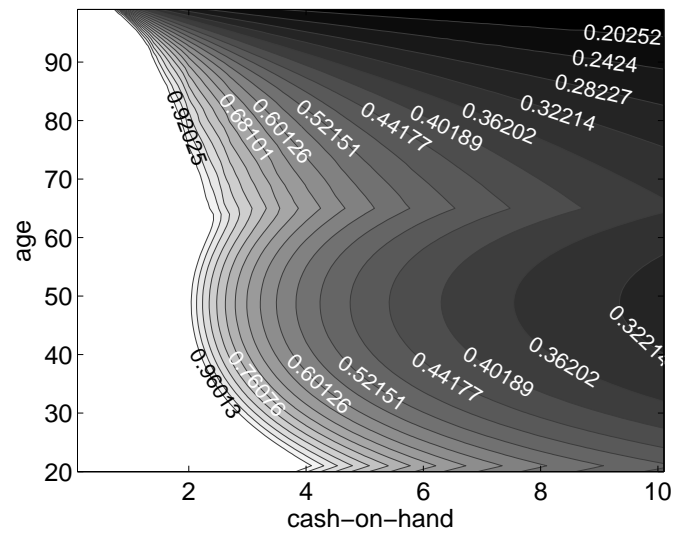
Figure 2: Contour lines for the US-equity share, no unemployment risk
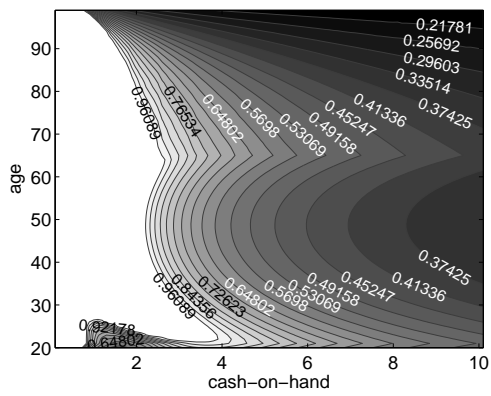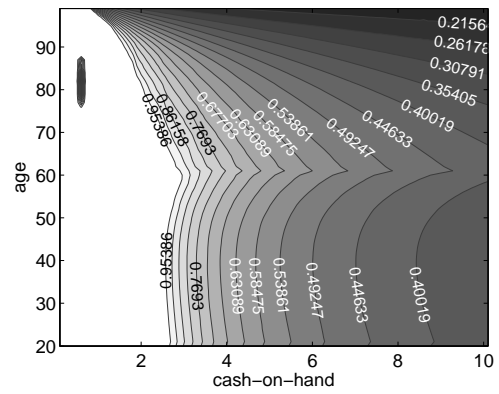
(a) Employment, no insurance
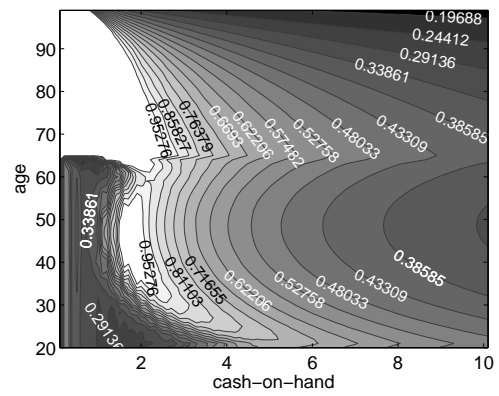
(b) Short-term unemployment, no insurance
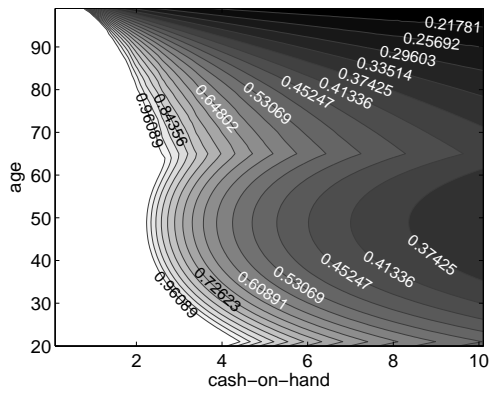
(c) Employment, insurance
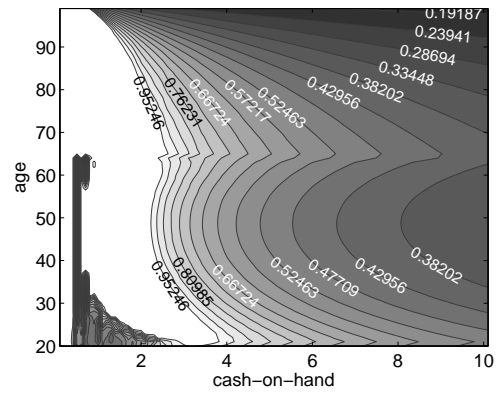
(d) Short-term unemployment, insurance

Figure 3: Contour lines for the US-equity share, short-term unemployment
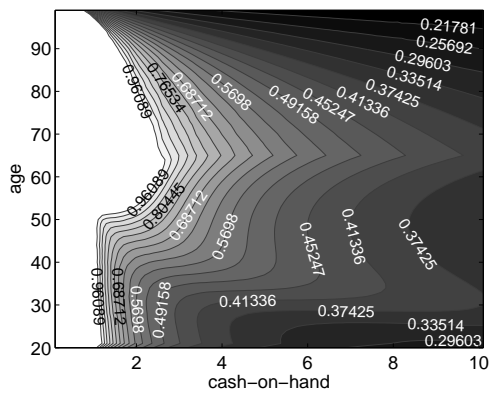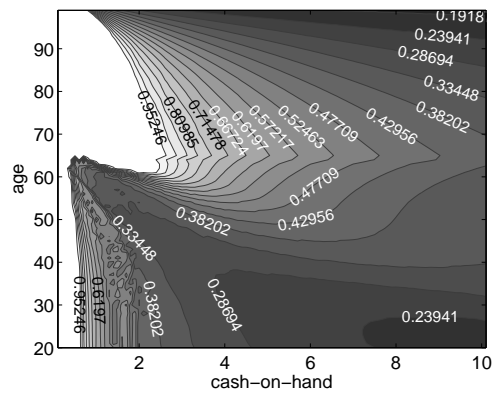
(a) Employment ($s = e$)

(b) Short-term unemployment ($s = u_s$)
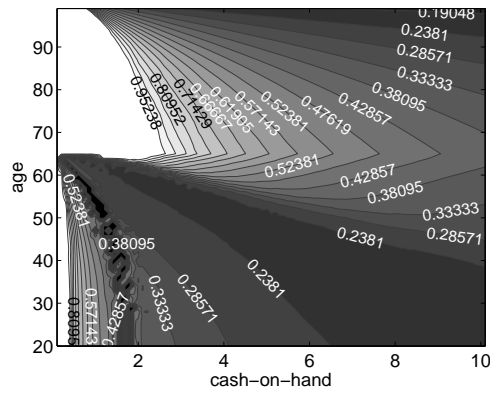
(c) Long-term unemployment ($s = u_l$)

Figure 4: Contour lines for the US-equity share, long-term unemployment
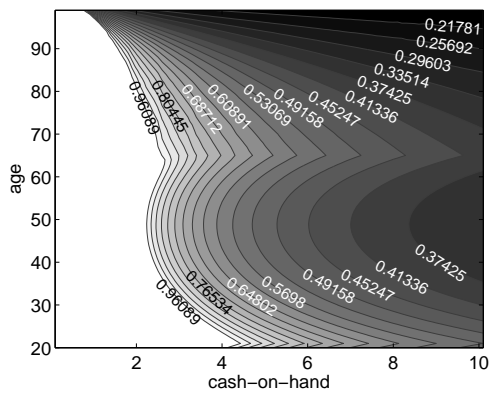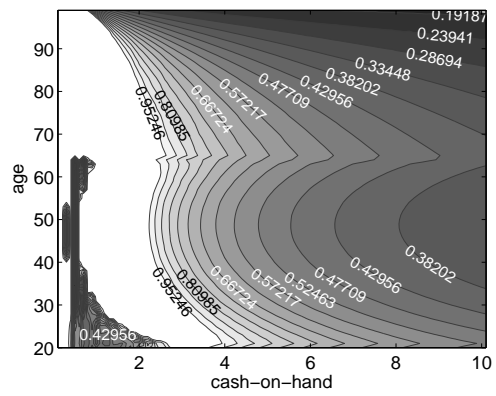
(a) Employment ($s = e$)

(b) Short-term unemployment ($s = u_s$)
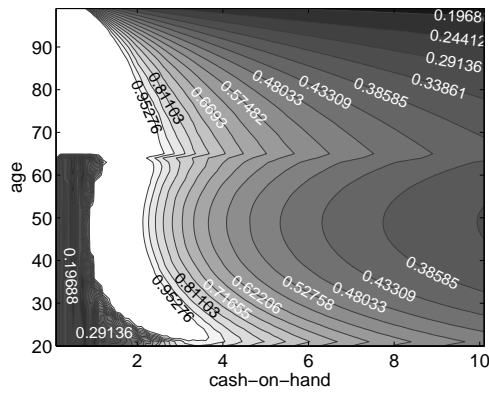
(c) Long-term unemployment ($s = u_l$)

Figure 5: Contour lines for the equity share, no persistence
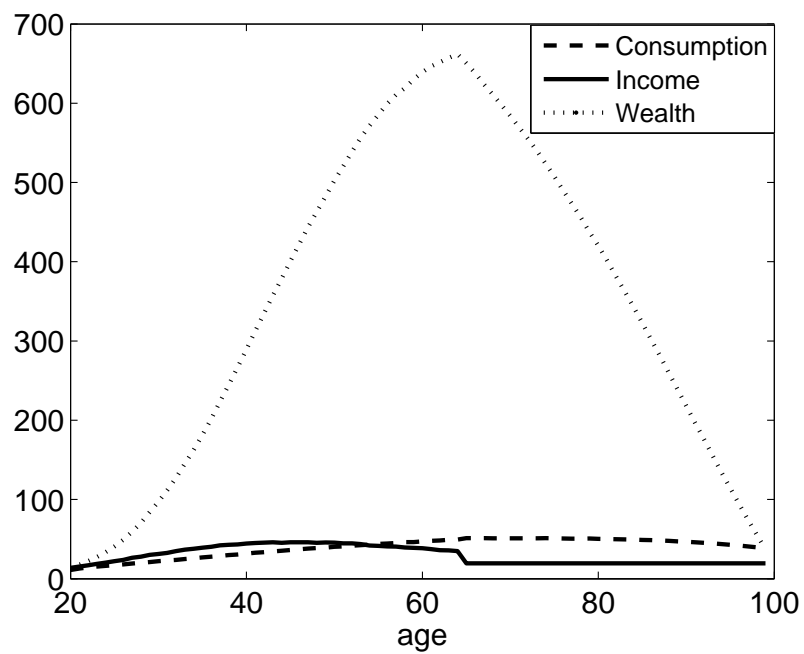
Figure 6: Simulation results for consumption, income and wealth, benchmark
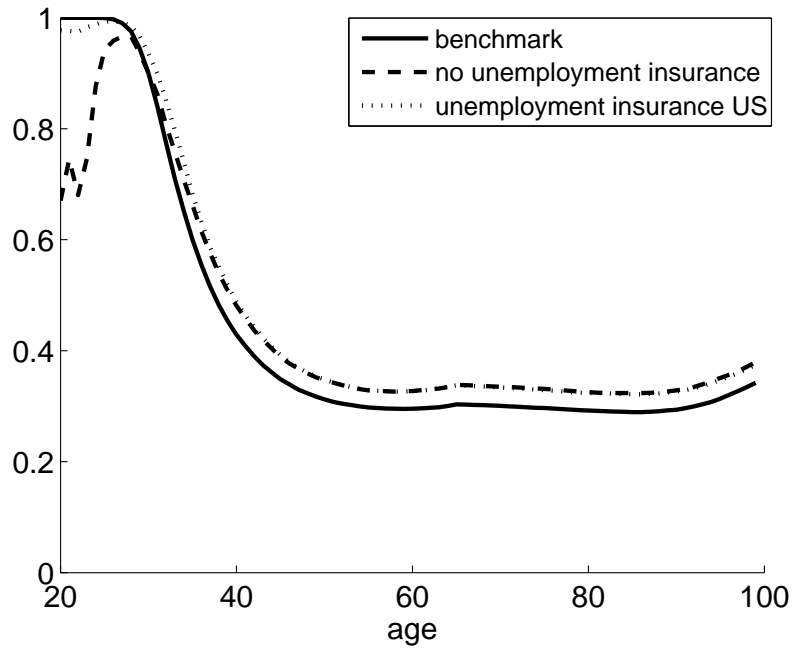
(a) With and without unemployment risk
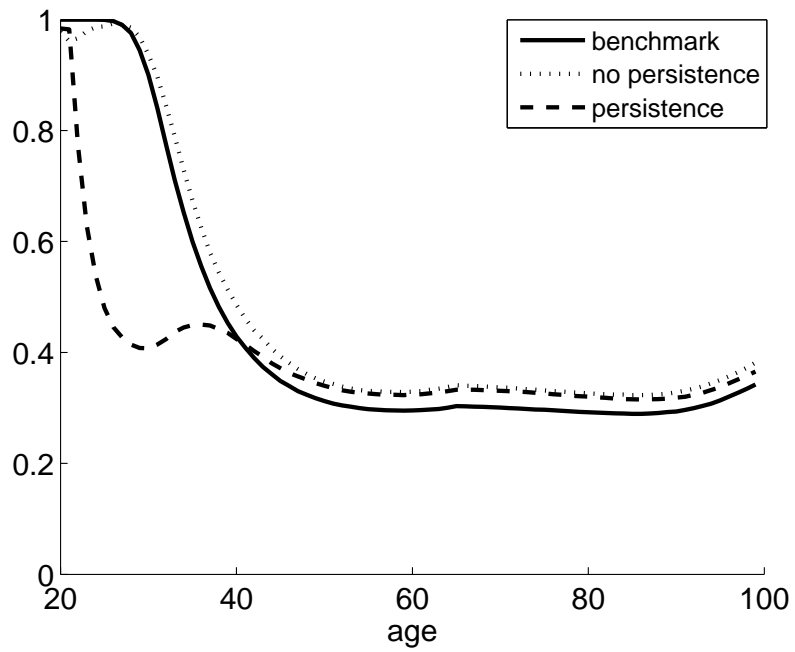
(b) with and without persistence

Figure 7: Simulation results for the US equity share
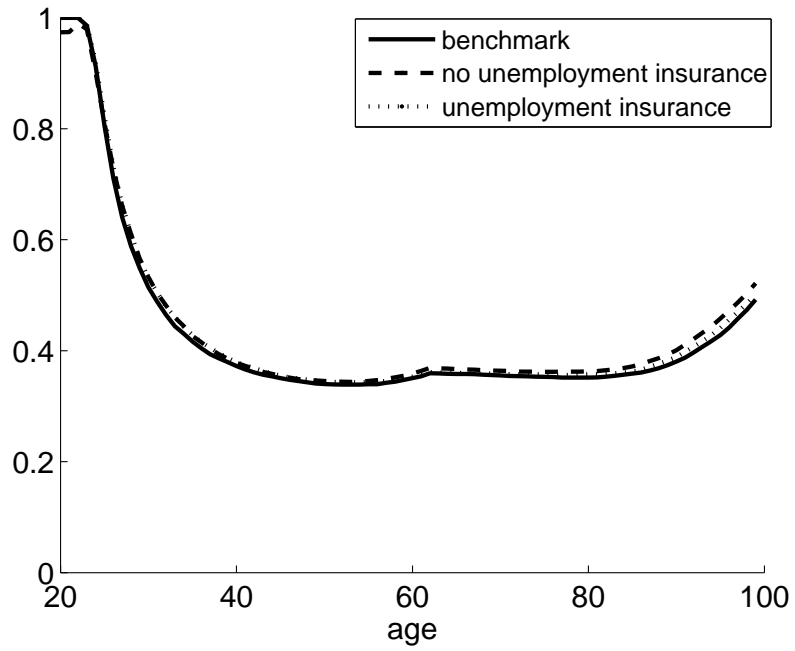
(a) With and without unemployment risk
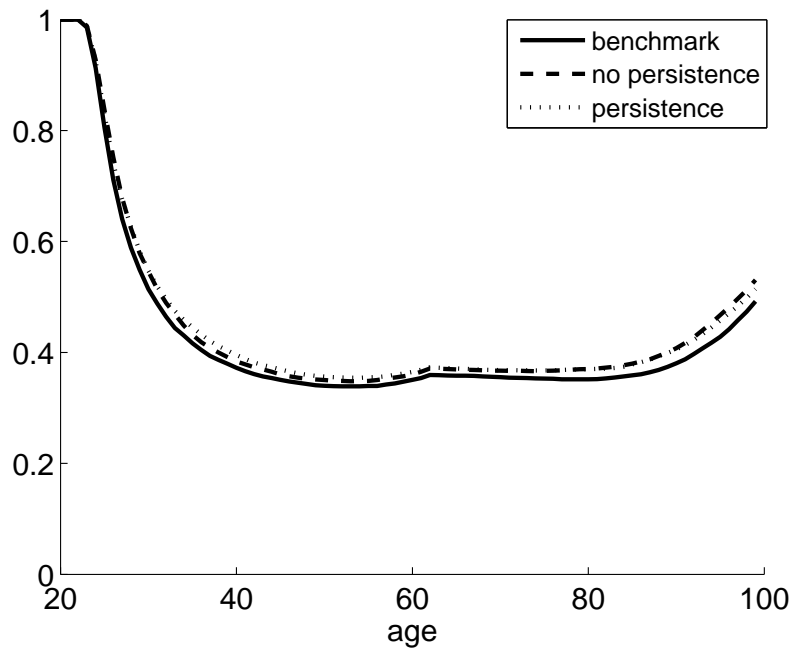
(b) with and without persistence

Figure 8: Simulation results for the German equity share
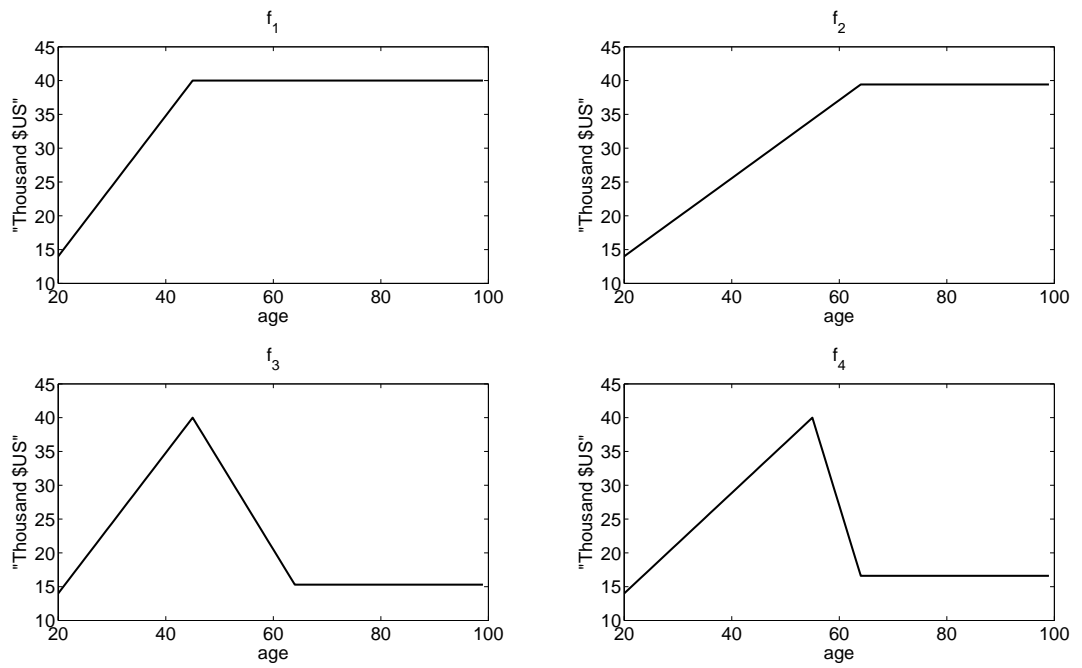
Figure 9: Hypothetic stylized age-income profiles
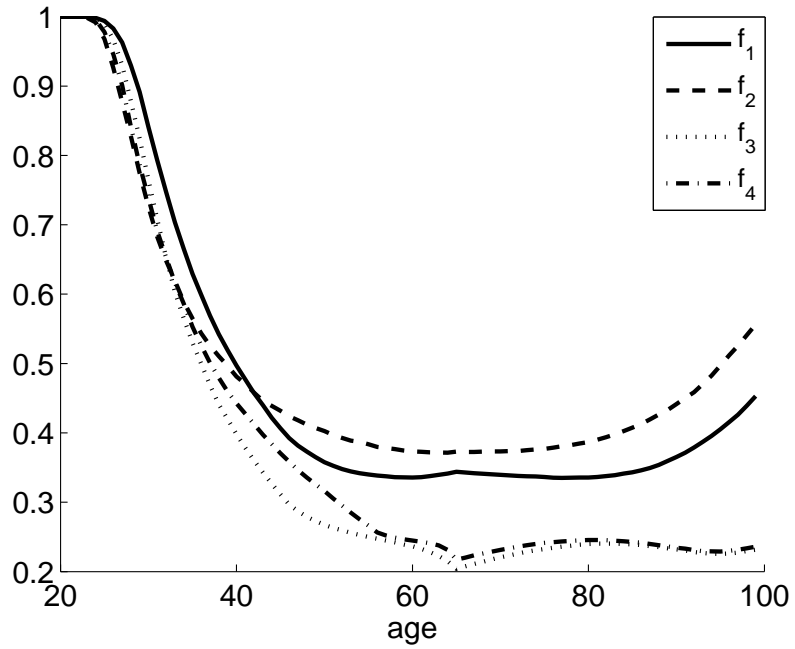
(a) Benchmark
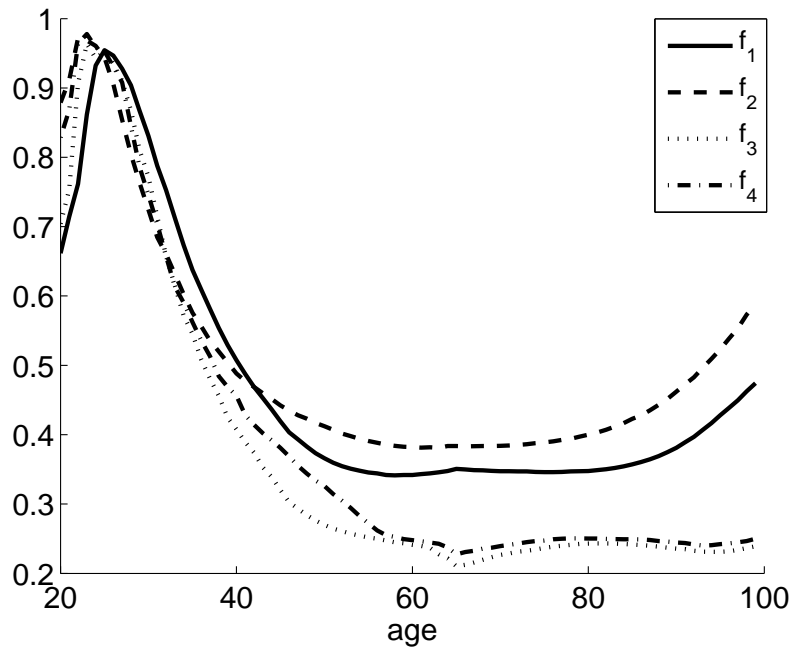
(b) Short-term unemployment, no insurance

Figure 10: Simulation results for different hypothetic age-income profiles
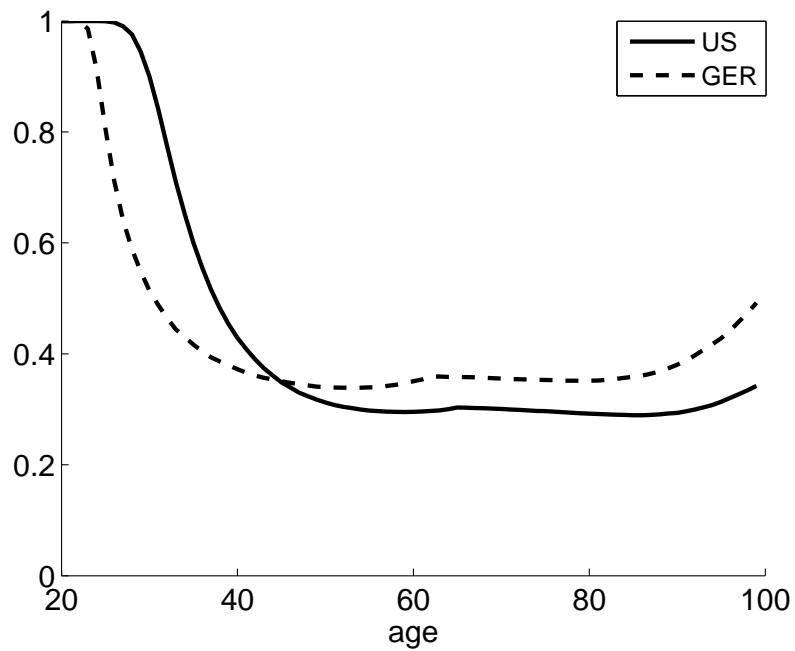
Figure 11: Simulation results for the US and German equity share, benchmark
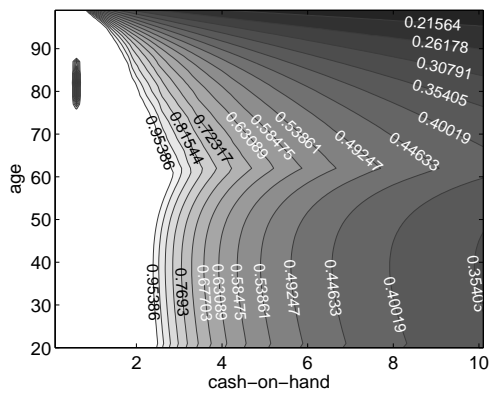
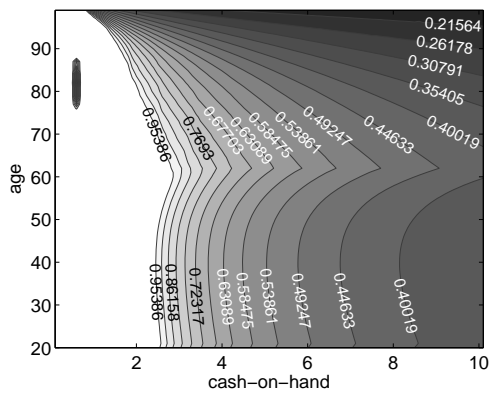(a) Employment, no insurance
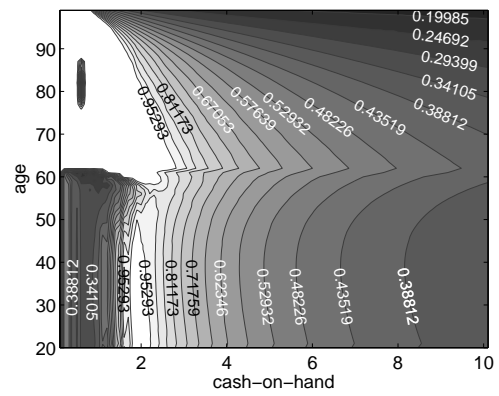
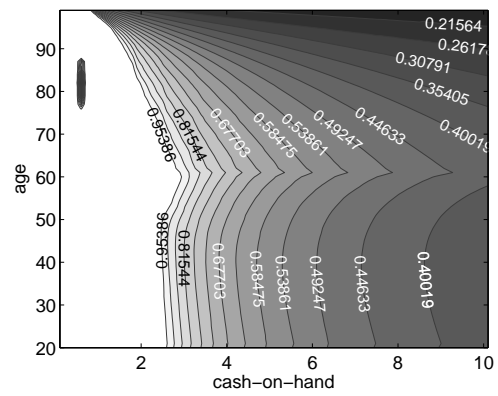(b) Short-term unemployment, no insurance
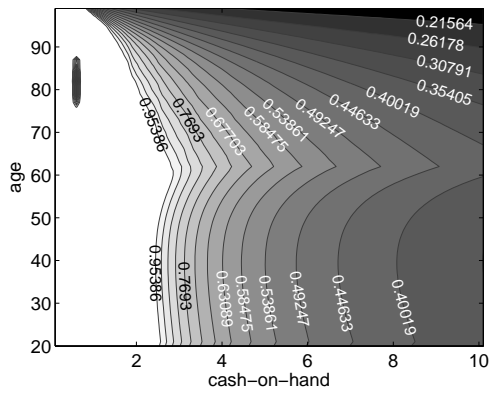
(c) Employment, insurance
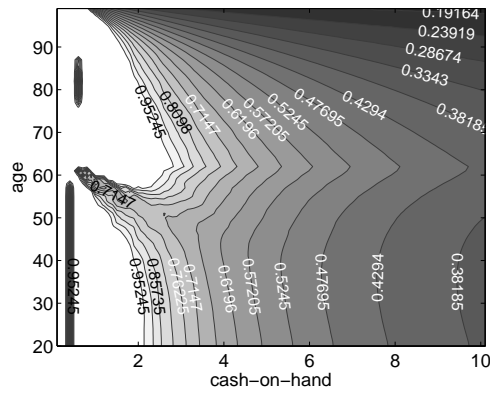
(d) Short-term unemployment, insurance

Figure 12: Contour lines for the equity share in Germany, short-term unemployment

(a) Employment ($s = e$)

(b) Short-term unemployment ($s = u_s$)

(c) Long-term unemployment ($s = u_l$)

Figure 13: Contour lines for the equity share in Germany, long-term unemployment