Extensive Margin of Trade and Business Cycle Correlations

J.-S. PENTECOTE, J.-C. POUTINEAU, F. RONDEAU*
CREM CNRS 6211 - University of Rennes 1

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Abstract

Trade, finance and specialization are often used to explain business cycle correlations. This paper investigates a new determinant: the extensive margin of trade. According to Kose and Yi (2006), Corsetti, Martin and Pesenti (2007) and Galstyan and Lane (2008), the extensive margin has effects on the terms of trade. In this case, the intensive margin of trade, the volume of trade and specialization can be affected. Thus, the extensive margin can have direct and indirect effects on business cycle synchronization. First, this article present a two-country model to show the effect of the extensive margin on business cycle correlation. Then, an econometric application is applied for 11 European countries that are supposed to have significant bilateral extensive margin as significant business cycle correlations. European bilateral extensive margins appear to be unstable over time and space. Then, an econometric application shows a negative effect of the extensive margin on business cycle correlations. As potential endogenous effects are possible between extensive margin, specialization and synchronization, three stages least square estimator is used. Furthermore, a cross section data application is completed by a panel data estimation and both show a significant and negative effect of extensive margin on synchronization. This paper shows that extensive margin and specialization have direct and indirect negative effects on business cycle synchronization. Finally, distinguish between extensive and intensive margin is crucial to evaluate effects of trade integration on business cycle synchronization.

Keywords: Trade, Business cycles, Integration, Specialization.
JEL: F4; F44; F15

*Corresponding author, fabien.rondeau@univ-rennes1.fr, 7 place Hoche 35000 Rennes, France. The authors are grateful to Nathalie Colombier for helpful assistance.

1 Introduction

In the context of a monetary union, as in Europe, synchronization is a special issue. Indeed, optimality depends on the degree of synchronization between members. In the litterature, many determinants are tested to explain degrees of business cycles synchronization as trade, finance, specialization, intra-industry,... But many of these determinants can evolve due to the instauration of the monetary union. In the seminal article of Frankel and Rose (1998), "countries with closer trade links tend to have more tightly correlated business cycles" and so the optimum currency area criteria is endogenous. According to Bergin and Lin (2008, 2010),, currency unions in general and the European Monetary Union (EMU) in particular, raise absolute and relative extensive margin.

In this article, to explain business cycle correlations in Europe, we try to identify the role played by a new endogenous variable: the extensive margin of trade.

First, a two-country model describing a monetary union shows a negative effect of .

Then, using the BACI database with 5,000 varieties of products from 1995 to 2007, the extensive margin is defined as the value of new exports between two countries for one year divided by the total bilateral trade.

2 General framework

We propose a model to study the relationship between the margins of international trade and macroeconomic convergence in a monetary union.

We describ a two-country world that forms a monetary union. It is populated by a great number of household (normalised to 1) and a great number of firms, each specialised in the production of a good that is imperfectly substitutable in the function of consumption. The number of firms fluctuates.

2.1 Representative household

The representative consumer j of the domestic countries maximes,

$$\underset{c_t, n_t, m_t^s, m_t^d, b_{t+1}, k_{t+1}}{Max} \qquad E_t \sum_{j=0}^{\infty} \beta^j \left[\ln c_{t+j} - \Xi \frac{(l_{t+j})^{1+\kappa}}{1+\kappa} \right]$$

subject to

$$\begin{aligned} E_t \beta^j \left[w_{t+j} l_{t+j} + b_{t+j} - c_{t+j} - (1+r_{t+j})^{-1} b_{t+1+j} \right] &= 0, \\ j &= 0, 1, 2, ..., \ (\lambda_{t+j}) \end{aligned}$$

The set of first order conditions of the household can be written as follows,

$$c_t^{-1} - \lambda_t = 0,$$
$$-\Xi\,(l_t)^{\kappa} + \lambda_t w_t = 0,$$
$$-\lambda_t(1+r_t)^{-1} + \beta E_t \lambda_{t+1} = 0,$$

We can combine these first order conditions to get an euler bond equation, an euler share equation and a labour supply curve according to,

$$c_t^{-1} = \beta(1+r_t)E_t c_{t+1}^{-1},$$
$$\Xi\,(l_t)^{\kappa} = c_t^{-1} w_t,$$

At a given period t the consumer must allocate its total consumption between (tradable and non tradable) home goods and (imported) foreign goods. The consumption and the consumption price indexes are defined accroding to the CES aggregators,

$$c_t(i) = \left(\int_0^{n_{D,t}} c_{D,t}(\omega,i)^{\frac{\theta-1}{\theta}} d\omega + \int_0^{n_{X,t}} c_{X,t}(\omega,i)^{\frac{\theta-1}{\theta}} dj + \int_0^{n^*_{X,t}} c_{M,t}(\omega,i)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}}$$

$$P_{c,t} = \left(\int_0^{n_{D,t}} p_{D,t}(\omega)^{1-\theta} d\omega + \int_0^{n_{X,t}} p_{X,t}(\omega)^{1-\theta} d\omega + \int_0^{n^*_{X,t}} p_{M,t}(\omega)^{1-\theta} d\omega \right)^{\frac{1}{1-\theta}}$$

where, $c_{D,t}(\omega,i), c_{X,t}(\omega,i), c_{M,t}(\omega,i)$ represent the individual demand of national non traded goods, traded domestic goods and foreign imports, where $p_{D,t}(\omega), p_{X,t}(\omega), p_{M,t}(\omega)$ are the associate nominal prices. and $n_{D,t}$ and $n_{X,t}$,are respectively the number of domestic tradable and non tradable goods and $n^*_{X,t}$ is the number of imported good from the foreign economy. Defining the real price of good ω of sector $j = \{D, X, M\}$ as $\rho_{j,t}(\omega) = \frac{p_{D,t}(\omega)}{P_{c,t}}$, we can write the demand of a representative good of each segment of the domestic goods market as,

$$\begin{aligned} c_{D,t}(\omega,i) &= \rho_{D,t}(\omega)^{-\theta} c_t(i), \\ c_{X,t}(\omega,i) &= \rho_{X,t}(\omega)^{-\theta} c_t(i), \\ c_{M,t}(\omega,i) &= \rho_{M,t}(\omega)^{-\theta} c_t(i). \end{aligned}$$

2.2 Firms

2.2.1 The dynamic motion of firms

The repartition of firms between sectors is variable. At period t there are n_{Dt} firms that operate in the non traded goods sector and $n_{X,t}$ firms that operate in the traded sector. In the economy the repartition of firms between the two sectors come from differences in productivity. The most productive firms operate on the traded goods segment while the less productive serve the non traded goods segment. Following the litterature (to be cited) we assume that firm heterogeneity comes from a specific shock z that corrects the total productivity shock. The production function of the representative firm that produces the good of type ω is,

$$y_t(\omega) = z(\omega) A_t \ell_t^d(\omega),$$

We assume that productivity $z(\omega)$ has a Pareto distribution with lower bound $z_{\min}$ and shape parameter $k > (\theta - 1)$. the pdf and cdf of z are $g(z) = k z_{\min}^k / z^{k+1}$ and $G(z) = 1 - (z_{\min}/z)^k$. A_t is a productivity shock homogeneous to all firms producing final goods such that,

$$\log A_{t+1} = \rho \log A_t + \xi_{A,t},$$

2.2.2 Firms decisions when prices are sticky

We assume that prices are chosen before the beginning of the production period so that firms must pay to change it according to a Rotemberg (1996) technology. The representative firm ω must pay a quadratic cost $\Gamma_t(\omega)$ to adjust its selling price,

$$\Gamma_t(\omega) = \frac{\psi}{2}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right)^2 \rho_t(\omega) y_t^d(\omega).$$

The adjustment cost is paid in terms of consumption goods.

If the firm operates on the domestic segment, so the demand faced by the representative firm is, $y_{D,t}^d(\omega) = \rho_{D,t}(\omega)^{-\sigma}[c_t + \Gamma_t]$ with $c_t = \int_0^1 c_t(i)\, di$, $\Gamma_t = \int_0^{n_t} \Gamma_t(\omega)\, d\omega$. In period t, the representative firm ω chooses $p_{D,t}(\omega)$ to maximise $d_{D,t}(\omega) + v_{D,t}(\omega)$,

$$v_{D,t}(\omega) = E_{t s=t+1}^{\infty} \beta^{s-t} \left(\Lambda_{t,s} d_{D,s}(\omega)\right),$$

$$d_{D,t}(\omega) = \rho_{D,t}(\omega) y_{D,t}^d(\omega) - \frac{\psi}{2}\left(\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)} - 1\right)^2 \rho_{D,t}(\omega) y_{D,t}^d(\omega) - \frac{w_t}{z(\omega) A_t} y_{D,t}^d(\omega).$$

Defining, $\pi_{D,t}(\omega) = \left(\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)} - 1\right)$ we get the optimal selling price on the non traded segment of the domestic goods market,

$$
\begin{aligned}
\rho_{D,t}(\omega) &= \mu_{D,t}\frac{w_t}{zA_t} \\
\mu_{D,t}(\omega) &= \frac{\sigma}{(\sigma-1)\left(1-\frac{\psi}{2}\pi_{D,t}(\omega)^2\right)+\psi\Psi_{D,t}(\omega)} \\
\Psi_{D,t}(\omega) &= \pi_{D,t}(\omega)\left(1+\pi_{D,t}(\omega)\right)-E_t\left\{\beta(1-\delta)\Lambda_{t,t+1}\left(\pi_{D,t+1}(\omega)\frac{\left(1+\pi_{D,t+1}(\omega)\right)^2}{(1+\pi_{C,t+1})}\frac{y_{X,t+1}(\omega)}{y_{X,t}(\omega)}\right)\right\}
\end{aligned}
$$

Thus we can write, the dividends according to,

$$
d_{D,t}(\omega)=\rho_{D,t}(\omega)^{1-\theta}\left[c_t+\Gamma_t\right]\left[1-\frac{1}{\mu_{D,t}}-\frac{\psi}{2}\pi_{D,t}(\omega)^2\right]
$$

If the firm operates on the export segment, so the demand faced by the representative firrm is, $y^d_{X,t}(\omega)=\rho^{-\theta}_{X,t}\left[(c_t+\Gamma_t)+\phi q_t^{1-\theta}\left(c^*_t+\Gamma^*_t\right)\right]$ with $c^*_t=\int_0^1 c^*_t(j)\,dj$, $\Gamma_t=\int_0^{n_t}\Gamma^*_t(\omega)\,d\omega$, where q_t is the real exchange rate defined as, and where τ is the iceberg transportation cost (ie, to consume one unit of the domestic traded good, foreign consumer must buy $(1+\tau)$ units of this good, since a proportion τ disapears during the travel between the two economies. Furthermore, we assume that the domestic firm that exports must pay an entry cost $\frac{w_t}{A_t}f_X$ (in terms of the efficient real wage) on the export segment of the economy. Thus the representative domestic firm maximizes,

$$
v_{D,t}(\omega)=E_{t}{}_{s=t+1}^{\infty}\left[(1-\delta)\beta\right]^{s-t}\left(\Lambda_{t,s}d_{D,s}(\omega)\right),
$$

$$
d_{X,t}(\omega)=\rho_{X,t}(\omega)\,y^d_{X,t}(\omega)-\frac{\psi}{2}\left(\frac{p_{X,t}(\omega)}{p_{X,t-1}(\omega)}-1\right)^2\rho_{X,t}(\omega)\,y^d_{X,t}(\omega)-\frac{w_t}{z(\omega)A_t}y^d_{X,t}(\omega)-\frac{w_t}{A_t}f_X.
$$

$$
d_{X,t}(\omega)=\rho_{X,t}(\omega)\,y^d_{D,t}(\omega)-\frac{w_t}{z(\omega)Z_t}y^d_t(\omega)-\frac{w_t}{A_t}f_X
$$

where $\rho_{X,t}(\omega)=\frac{p_{X,t}(\omega)}{P_{c,t}}$ andi where $y^d_{D,t}(\omega)=\rho^{-\theta}_{X,t}\left[c_t+\Gamma_t\right]+(1+\tau)q_t\rho^{*-\theta}_{M,t}\left[c^*_t+\Gamma^*_t\right]=\rho^{-\theta}_{X,t}\left[\left[c_t+\Gamma_t\right]+(1+\tau)^{1-\theta}\right.$ $\rho^{-\theta}_{X,t}\left[\left[c_t+\Gamma_t\right]+\phi q_t^{1+\theta}\left[c^*_t+\Gamma^*_t\right]\right]$. The firms located on the exporting segment must set their selling price at

$$
\begin{aligned}
\rho_{X,t}(\omega) &= \mu_{X,t}\frac{w_t}{z(\omega)A_t} \\
\mu_{X,t}(\omega) &= \frac{\sigma}{(\sigma-1)\left(1-\frac{\psi}{2}\pi_{X,t}(\omega)^2\right)+\psi\Psi_{X,t}(\omega)} \\
\Psi_{X,t}(\omega) &= \pi_{X,t}(\omega)\left(1+\pi_{X,t}(\omega)\right)-E_t\left\{\beta\Lambda_{t,t+1}\left(\pi_{X,t+1}(\omega)\frac{\left(1+\pi_{X,t+1}(\omega)\right)^2}{(1+\pi_{C,t+1})}\frac{y_{X,t+1}(\omega)}{y_{X,t}(\omega)}\right)\right\}
\end{aligned}
$$

The corresponding real price of the same good in the forign economy in terms of the foreign consumption basket is given by,

$$\rho_{M,t}^{*}=(1+\tau)\frac{\rho_{X,t}}{q_t}$$

and the dividend of period t given the optimal relative price is,

$$d_{X,t}(\omega)=\rho_{X,t}(\omega)^{1-\theta}\left[[c_t+\Gamma_t]+\phi q_t^{1+\theta}\left[c_t^{*}+\Gamma_t^{*}\right]\right]\left[1-\frac{1}{\mu_{X,t}}-\frac{\psi}{2}\pi_{X,t}(\omega)^2\right]-\frac{w_t}{A_t}f_X$$

To determine the repartition of firms between traded and non traded goods, one first have to express the number of exporting firms in terms of the cut off point z_X. The cut off point between the two sectors is determined by the last firm that enters the traded segment. Its level of individual productivity $z(\omega)=z_X$ is such that it breaks even. Thus, the cut off point is determined by $d_{X,t}(\omega)=d_{D,t}(\omega)$.

$$\begin{aligned}
&\rho_{D,t}(\omega,z_X)^{1-\theta}\left[c_t+\Gamma_t\right]\left[1-\frac{1}{\mu_{D,t}(\omega,z_X)}-\frac{\psi}{2}\pi_{X,t}(\omega,z_X)^2\right]\\
=\;&\rho_{X,t}(\omega,z_X)^{1-\theta}\left[[c_t+\Gamma_t]+\phi q_t^{1+\theta}\left[c_t^{*}+\Gamma_t^{*}\right]\right]\left[1-\frac{1}{\mu_{X,t}(\omega,z_X)}-\frac{\psi}{2}\pi_{X,t}(\omega,z_X)^2\right]-\frac{w_t}{A_t}f_X,
\end{aligned}$$

where $\rho_{D,t}(\omega,z_X)=\mathrm{T}\rho_{X,t}(\omega,z_X)$, $\mu_{D,t}(\omega,z_X)=\mu_{X,t}(\omega,z_X)$, and $\pi_{D,t}(\omega,z_X)=\pi_{X,t}(\omega,z_X)$. Thus the cut off point is such that, the marginal gain of exporting (ie, to have acces to the foreign demand) is equal to the marginal cost of entering the export segment of the goods market, ie,

$$\rho_{,t}(\omega,z_X)^{1-\theta}\left[\phi q_t^{1+\theta}\left[c_t^{*}+\Gamma_t^{*}\right]\right]\left[1-\frac{1}{\mu_t(\omega,z_X)}-\frac{\psi}{2}\pi_t(\omega,z_X)^2\right]=\frac{w_t}{A_t}f_X,$$

$$z_X=\left(\frac{\mu_t(\omega,z_X)^{\theta}\,\frac{w_t}{A_t}f_X}{\left[\phi q_t^{1+\theta}\left[c_t^{*}+\Gamma_t^{*}\right]\right]\left[\mu_t(\omega,z_X)\left(1-\frac{\psi}{2}\pi_t(\omega,z_X)^2\right)-1\right]}\right)^{\frac{1}{\theta-1}}\frac{w_t}{A_t},$$

There should be a problem with regard to the previous expression with flexible prices. Indeed, to get the cut of point we need to know two characteristics of the last firm that enters the traded segment : $\mu_{X,t}(\omega,z_X)$ and $\pi_{X,t}(\omega,z_X)$.

2.3 Aggregation

The first step is to aggregate the behaviour of the firms sector by sector. The first step is to compute the average levels $\tilde{z}_{D,t},\tilde{z}_{X,t},\tilde{z}_{D,t}^{*},\tilde{z}_{X,t}^{*}$.Using the characteristics of the pareto distribution, defining $\nabla=\left(\frac{k}{k-(\theta-1)}\right)$, we get,

$$
\begin{aligned}
\tilde{z}_t &= \left[\int_{z_{\min}}^{\infty} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}} = \nabla^{\frac{1}{\theta-1}} z_{\min}, \\
\tilde{z}_{X,t} &= \left[\frac{1}{1-G(z)} \int_{z_X}^{\infty} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}} = \nabla^{\frac{1}{\theta-1}} z_{X,t}
\end{aligned}
$$

the average productivity level of the non traded sector can be obtained from the genenral aggregation scheme under the pareto distribution, ie, given the pareto distribution, the relative weight of exporting firms is determined by the cut off point, since,

$$
n_{X,t} = 1 - G(z_X)
$$

since, given the pareto distribution, $G(z) = 1 - (z_{\min}/z)^k$, we get,

$$
n_{X,t} = (z_{\min}/z_{X,t})^k \, n_t
$$

$$
\tilde{z}_t = \left(1 - (z_{\min}/z_{X,t})^k\right) \tilde{z}_{D,t} + (z_{\min}/z_{X,t})^k \, \tilde{z}_{X,t}
$$

thus,

$$
\tilde{z}_{D,t} = \nabla^{\frac{1}{\theta-1}} \left(\frac{z_{\min} - z_{\min}^k z_{X,t}^{1-k}}{1 - z_{\min}^k z_{X,t}^{-k}}\right)
$$

thus, we have,

$$
\begin{aligned}
\tilde{z}_t = \nabla^{\frac{1}{\theta-1}} z_{\min}, \tilde{z}_{X,t} = \nabla^{\frac{1}{\theta-1}} z_{X,t} \ , \tilde{z}_{D,t} = \nabla^{\frac{1}{\theta-1}} \left(\frac{z_{\min} - z_{\min}^k z_{X,t}^{1-k}}{1 - z_{\min}^k z_{X,t}^{-k}}\right) \\
\tilde{z}_t^* = \nabla^{\frac{1}{\theta-1}} z_{\min}^*, \tilde{z}_{X,t}^* = \nabla^{\frac{1}{\theta-1}} z_{X,t}^*, \tilde{z}_{D,t}^* = \nabla^{\frac{1}{\theta-1}} \left(\frac{z_{\min}^* - z_{\min}^{*k} z_{X,t}^{*1-k}}{1 - z_{\min}^{*k} z_{X,t}^{-*k}}\right)
\end{aligned}
$$

the characteristics of these averages are such that following Melitz, The productivity averages $\tilde{z}_{D,t}$, $\tilde{z}_{X,t}$, and z2dc*X,t are constructed in such a way that d2dc D,t dD,t(z2dcD) (d2dc *D,t d*D,t(z2dcD)) represents the average firm profit earned from domestic sales for all home (foreign) producers; and d2dc X,t dX,t(z2dcX,t) (d2dc *X,t d*X,t(z2dc*X,t)) represents the average firm export profits for all home (foreign) exporters. Thus, d2dc t d2dc D,t [1 G(zX,t)]d2dc X,t and d2dc*t d2dc *D,t [1 G(z*X,t)]d2dc *X,t represent the average total profits of home andforeign firms, since 1 G(zX,t) and 1 G(z*X,t) represent theproportion of home and foreign firms that export and earn export profitsThen we use the fact that firms are symemetric onde corrected by the relative level of productivity, ie, $\frac{\mu_t(\omega,z_X)}{z_{X,t}} = \frac{\tilde{\mu}_{X,t}}{\tilde{z}_{X,t}}$ and $\frac{\pi_t(\omega,z_X)}{z_{X,t}} = \frac{\tilde{\pi}_{X,t}}{\tilde{z}_{X,t}}$, so that, $\mu_t(\omega, z_X) = \nabla^{-\frac{1}{\theta-1}} \tilde{\mu}_{X,t}$ and $\pi_t(\omega, z_X) = \nabla^{-\frac{1}{\theta-1}} \tilde{\pi}_{X,t}$. Thus, the cut off point writes,

$$z_X = \left(\frac{\nabla^{-\frac{1}{\theta-1}} \tilde{\mu}_{X,t}^{\theta} \frac{w_t}{A_t} f_X}{\left[\phi q_t^{1+\theta} \left[c_t^* + \Gamma_t^*\right]\right] \left[\nabla^{-\frac{1}{\theta-1}} \tilde{\mu}_{X,t} \left(1 - \frac{\psi}{2} \nabla^{-\frac{1}{\theta-1}} \tilde{\pi}_{X,t}^2\right) - 1\right]} \right)^{\frac{1}{\theta-1}} \frac{w_t}{A_t},$$

Thus the number of firms that operate on each segment of the goods≤ market is given by,

$$\begin{aligned} n_{X,t} &= \left(\frac{\nabla^{-\frac{1}{\theta-1}} \tilde{\mu}_{X,t}^{\theta} \frac{w_t}{A_t} f_X}{\left[\phi q_t^{1+\theta} \left[c_t^* + \Gamma_t^*\right]\right] \left[\nabla^{-\frac{1}{\theta-1}} \tilde{\mu}_{X,t} \left(1 - \frac{\psi}{2} \nabla^{-\frac{1}{\theta-1}} \tilde{\pi}_{X,t}^2\right) - 1\right]} \right)^{-\frac{k}{\theta-1}} \left(\frac{w_t}{z_{\min} A_t} \right)^{-k} \\ n_{D,t} &= 1 - \left(\frac{\nabla^{-\frac{1}{\theta-1}} \tilde{\mu}_{X,t}^{\theta} \frac{w_t}{A_t} f_X}{\left[\phi q_t^{1+\theta} \left[c_t^* + \Gamma_t^*\right]\right] \left[\nabla^{-\frac{1}{\theta-1}} \tilde{\mu}_{X,t} \left(1 - \frac{\psi}{2} \nabla^{-\frac{1}{\theta-1}} \tilde{\pi}_{X,t}^2\right) - 1\right]} \right)^{-\frac{k}{\theta-1}} \left(\frac{w_t}{z_{\min} A_t} \right)_t^{-k} \end{aligned}$$

Thus we can get the average real price of goods for the different sectors of the model as,

$$\tilde{\rho}_{D,t} = \rho_{D,t}(\tilde{z}_{D,t}) = \frac{\tilde{p}_{D,t}}{P_{c,t}} = \nabla^{\frac{1}{\theta-1}} \tilde{\mu}_{D,t} \frac{w_t}{\tilde{z}_{D,t} Z_t}$$

$$\tilde{\rho}_{X,t}(\omega) = \rho_{X,t}(\tilde{z}_{X,t}) = \frac{\tilde{p}_{X,t}}{P_{c,t}} = \nabla^{\frac{1}{\theta-1}} \frac{\tilde{\mu}_{X,t} w_t}{\tilde{z}_{X,t} \ Z_t}$$

and, for the foreign economy

$$\tilde{\rho}_{D,t}^* = \nabla^{\frac{1}{\theta-1}} \tilde{\mu}_{D,t}^* \frac{w_t^*}{\tilde{z}_{D,t}^* Z_t^*}$$

$$\tilde{\rho}_{X,t}^* = \nabla^{\frac{1}{\theta-1}} \frac{\tilde{\mu}_{X,t}^* w_t^*}{\tilde{z}_{X,t}^* \ A_t^*}$$

average prodcution is then obtained by computing,

$$\tilde{y}_{D,t} = \tilde{\rho}_{D,t}^{-\sigma} \left[c_t + \Gamma_t\right],$$

$$\tilde{y}_{X,t} = \tilde{\rho}_{X,t}^{-\sigma} \left[(c_t + \Gamma_t) + q_t^{1+\theta} \phi \left(c_t^* + \Gamma_t^*\right)\right],$$

$$\tilde{y}_{D,t}^* = \tilde{\rho}_{D,t}^{*-\sigma} \left[c_t^* + \Gamma_t^*\right],$$

$$\tilde{y}_{X,t}^* = \tilde{\rho}_{X,t}^{*-\sigma} \left[(c_t^* + \Gamma_t^*) + q_t^{-(1+\theta)} \phi \left(c_t + \Gamma_t\right)\right],$$

Second, national variables are obtained by using the characteristics of the pareto distribution

$$\begin{aligned} Y_t &= \int_0^1 \rho_t(\omega) \, y_t(\omega) \, d\omega = \tilde{\rho}_t \tilde{y}_t, \\ Y_t^* &= \int_0^1 \rho_t^*(\omega) \, y_t^*(\omega) \, d\omega = \tilde{\rho}_t^* \tilde{y}_t^*, \end{aligned}$$

with

$$
\begin{aligned}
\tilde{\rho}_t \tilde{y}_t &= \frac{n_{D,t}}{n_t}\tilde{\rho}_{D,t}\tilde{y}_{D,t} + \frac{n_{X,t}}{n_t} q_t \tilde{\rho}_{X,t}\tilde{y}_{X,t} \\
&= (1-(z_{\min}/z_{X,t})^k)\tilde{\rho}_{D,t}\tilde{y}_{D,t} + (z_{\min}/z_{X,t})^k\, q_t\tilde{\rho}_{X,t}\tilde{y}_{X,t}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\rho}_t^* \tilde{y}_t^* &= \frac{n_{D,t}^*}{n_t^*}\tilde{\rho}_{D,t}^*\tilde{y}_{D,t}^* + \frac{n_{X,t}^*}{n_t^*}\frac{\tilde{\rho}_{X,t}^*}{q_t}\tilde{y}_{X,t}^* \\
&= (1-(z_{\min}/z_{X,t})^k)\tilde{\rho}_{D,t}^*\tilde{y}_{D,t}^* + (z_{\min}/z_{X,t})^k\,\frac{\tilde{\rho}_{X,t}^*}{q_t}\tilde{y}_{X,t}^*
\end{aligned}
$$

consumption price indexes

$$
\begin{aligned}
1 &= n_{D,t}\tilde{\rho}_{D,t}^{1-\theta} + n_{X,t}\tilde{\rho}_{X,t}^{1-\theta} + n_{x,t}^*\tilde{\rho}_{M,t}^{1-\theta} \\
1 &= n_{D,t}^*\tilde{\rho}_{D,t}^{*1-\theta} + n_{X,t}^*\tilde{\rho}_{X,t}^{*1-\theta} + n_{x,t}\tilde{\rho}_{M,t}^{*1-\theta}
\end{aligned}
$$

and since we solve the model under the equilibrium of the current account, we get,

$$
q_t = \frac{n_{X,t}^*\tilde{\rho}_{X,t}^{*1-\theta} c_t}{n_{X,t}\tilde{p}_{X,t}^{1-\theta} c_t^*}
$$

Finally, we assume symetry in asset holdings, so that, at the aggregate level, $x_t = x_{t-1} = 1$.and the average value of dividends that is used to compute the average value of shares is given by,

$$
\tilde{d}_t = \frac{n_{D,t}}{n_t}\tilde{d}_{D,t} + \frac{n_{X,t}}{n_t}\tilde{d}_{X,t}
$$

$$
\tilde{d}_t = (1-(z_{\min}/z_{X,t})^k)\tilde{d}_{D,t} + (z_{\min}/z_{X,t})^k\,\tilde{d}_{X,t}
$$

aggegation of dividends by sector

$$
\tilde{d}_{D,t}(\omega) = \frac{\tilde{\rho}_{D,t}(\omega)^{1-\sigma}}{\tilde{\mu}_{D,t}}\left[c_t + \Gamma_t\right]\left[\tilde{\mu}_{D,t}\left(1 - \frac{\psi}{2}\tilde{\pi}_{D,t}(\omega)^2\right) - 1\right]
$$

$$
\tilde{d}_{X,t}(\omega) = \frac{\tilde{\rho}_{X,t}(\omega)^{1-\sigma}}{\tilde{\mu}_{X,t}}\left[(c_t + \Gamma_t) + \phi q_t^{1-\theta}(c_t^* + \Gamma_t^*)\right]\left[\tilde{\mu}_{X,t}\left(1 - \frac{\psi}{2}\tilde{\pi}_{X,t}(\omega)^2\right) - 1\right] - \frac{w_t}{Z_t} f_X
$$

2.4 Monetary Policy and Genaral Equilibrium

We assume that there is one central bank at the union level that implement monetary policy by following a simple taylor rule,

$$TR$$

In this setting, a competitive equilibrium is defined as a sequence of quantities

$$\{Q_t\}_{t=0}^{\infty} = \left\{y_t, c_t, l_t, l_t^d, m_t, m_t^d, n_t, n_{e,t}\right\}_{t=0}^{\infty},$$

and a sequence of real prices,

$$\{P_t\}_{t=0}^{\infty} = \left\{\rho_t, w_t, r_t^b, v_t, d_t, R_{t,}^b R_t^{IB}, R_{t,}^L, R_t^T, \pi_t, \pi_t^C\right\}_{t=0}^{\infty},$$

such that, for a given sequence of prices $\{P_t\}_{t=0}^{\infty}$, the realization of shocks $\{S_t\}_{t=0}^{\infty} = \{A1_t, A2_t, A3_t, \varepsilon_t\}_{t=0}^{\infty}$, the sequence $\{Q_t\}_{t=0}^{\infty}$ respects first order conditions for households and maximizes firm profits and for a given sequence of quantities $\{Q_t\}_{t=0}^{\infty}$, the realization of shocks $\{S_t\}_{t=0}^{\infty} = \{A1_t, A2_t, A3_t, \varepsilon_t\}_{t=0}^{\infty}$, the sequence $\{P_t\}_{t=0}^{\infty}$, guarantees labour and goods market equilibrium as follows,

$$\int_0^1 l_t^s(j)\, dj = \int_0^{n_t} l_t^d(\omega)\, d\omega,$$

$$\int_0^1 c_t(j)\, dj = [1 - \frac{\psi}{2}\pi_{C,t}] n_t \tilde{\rho}_t \tilde{y}_t$$

to be completed

3 The model in log deviation

Here, we define the steady state, and provide the dynamics expression of the model befor proceeding to simulations.

3.1 Computing the steady state of the model

The model can be solved in steady state by assuming that all the exogenous sources of variability are fixed at their expected value. we assume that the two country are perfectly symmetric. To compute the steady state, we assume that all shocks are zero, so that $A = 1$; prices are constant, $\pi = 0$; furthermore, since there is no money nominal prices are undetermined so that nominal prices are equal to 1. we assume this to the nominal wage so that the real wage is equal to 1, which is also the marginal productivity of labour in the steady state. Since there is no inflation, and since the current account is balanced, $c = Y$.

First, combining the FOC on labour supply with the production function and the real wage in steady state, we get,

$$-\Xi\left(c\right)^{\kappa} + c^{-1} = 0$$

thus,

$$\Xi^{\frac{-1}{(1+\kappa)}} = c$$

The cut off point is then obtained as,

$$z_X = \left(\frac{\nabla^{-\frac{1}{\theta-1}}\left(\frac{\theta}{\theta-1}\right)^{\theta} f_X}{\phi c\left[\nabla^{-\frac{1}{\theta-1}}\left(\frac{\theta}{\theta-1}\right) - 1\right]}\right)^{\frac{1}{\theta-1}},$$

so, that,

$$n_X = \left(\frac{\nabla^{-\frac{1}{\theta-1}}\left(\frac{\theta}{\theta-1}\right)^{\theta} f_X}{\phi c\left[\nabla^{-\frac{1}{\theta-1}}\left(\frac{\theta}{\theta-1}\right) - 1\right]}\right)^{-k} n_t$$

and,

$$n_{D,t} = \left[1 - \left(\frac{\nabla^{-\frac{1}{\theta-1}}\left(\frac{\theta}{\theta-1}\right)^{\theta} f_X}{\phi c\left[\nabla^{-\frac{1}{\theta-1}}\left(\frac{\theta}{\theta-1}\right) - 1\right]}\right)^{-k}\right] n_t$$

steady state relative prices are (since $\tilde{z}_D = \nabla^{\frac{1}{\theta-1}}\left(\frac{1-z_X^{1-k}}{1-z_{X,}^{-k}}\right)$)

$$\tilde{\rho}_{D,t} = \left(\frac{\theta}{\theta-1}\right)\nabla^{-\frac{1}{\theta-1}}\left(\frac{1-z_X^{-k}}{1-z_{X,}^{1-k}}\right)$$

$$\tilde{\rho}_X = \left(\frac{\theta}{\theta-1}\right) z_X^{-1}$$

$$\tilde{\rho}_M = (1+\tau)\left(\frac{\theta}{\theta-1}\right) z_X^{-1}$$

the total number of firms comes from the variety effect on consumption prices,

$$
\begin{aligned}
1 &= n_D \left[\left(\frac{\theta}{\theta-1} \right) \nabla^{-\frac{1}{\theta-1}} \left(\frac{1-z_X^{-k}}{1-z_{X,}^{1-k}} \right) \right]^{1-\theta} + n_X (1+\phi) \left[\left(\frac{\theta}{\theta-1} \right) z_X^{-1} \right]^{1-\theta} \\
1 &= \left(1-z_X^{-k}\right) n \left[\left(\frac{\theta}{\theta-1} \right) \nabla^{-\frac{1}{\theta-1}} \left(\frac{1-z_X^{-k}}{1-z_{X,}^{1-k}} \right) \right]^{1-\theta} + z_X^{-k} n (1+\phi) \left[\left(\frac{\theta}{\theta-1} \right) z_X^{-1} \right]^{1-\theta} \\
n &= \frac{1}{\left(1-z_X^{-k}\right) \left[\left(\frac{\theta}{\theta-1} \right) \nabla^{-\frac{1}{\theta-1}} \left(\frac{1-z_X^{-k}}{1-z_{X,}^{1-k}} \right) \right]^{1-\theta} + z_X^{-k} (1+\phi) \left[\left(\frac{\theta}{\theta-1} \right) z_X^{-1} \right]^{1-\theta}}
\end{aligned}
$$

Thus,

$$
\begin{aligned}
n_X &= \frac{\left(1-z_X^{-k}\right)}{\left(1-z_X^{-k}\right) \left[\left(\frac{\theta}{\theta-1} \right) \nabla^{-\frac{1}{\theta-1}} \left(\frac{1-z_X^{-k}}{1-z_{X,}^{1-k}} \right) \right]^{1-\theta} + z_X^{-k} (1+\phi) \left[\left(\frac{\theta}{\theta-1} \right) z_X^{-1} \right]^{1-\theta}} \\
n_D &= \frac{z_X^{-1}}{\left(1-z_X^{-k}\right) \left[\left(\frac{\theta}{\theta-1} \right) \nabla^{-\frac{1}{\theta-1}} \left(\frac{1-z_X^{-k}}{1-z_{X,}^{1-k}} \right) \right]^{1-\theta} + z_X^{-k} (1+\phi) \left[\left(\frac{\theta}{\theta-1} \right) z_X^{-1} \right]^{1-\theta}} \\
n_E &= \frac{\delta}{(1-\delta)} \frac{1}{\left(1-z_X^{-k}\right) \left[\left(\frac{\theta}{\theta-1} \right) \nabla^{-\frac{1}{\theta-1}} \left(\frac{1-z_X^{-k}}{1-z_{X,}^{1-k}} \right) \right]^{1-\theta} + z_X^{-k} (1+\phi) \left[\left(\frac{\theta}{\theta-1} \right) z_X^{-1} \right]^{1-\theta}}
\end{aligned}
$$

the steady state ratio that will be needed below are,

$$
\begin{aligned}
\frac{\tilde{v}}{\left(\tilde{d}+\tilde{v}\right)} &= \beta \\
\frac{\tilde{d}}{\left(\tilde{d}+\tilde{v}\right)} &= [1-\beta]
\end{aligned}
$$

3.2 Dynamic equations

All equation in log deviation is expressed in average terms (either sectorial, national or union wide). We drop the tild to make the notation lighter.

The consumer FOC

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - (r_t - r)$$

$$\widehat{c}_t^* = E_t \widehat{c}_{t+1}^* - (r_t^* - r)$$

$$\kappa \widehat{l}_t = \widehat{w}_t - \widehat{c}_t$$

$$\kappa \widehat{l}_t^* = \widehat{w}_t^* - \widehat{c}_t^*$$

dividends

$$\hat{d}_{D,t} = (1-\theta)\,\hat{\rho}_{D,t} + \hat{c}_t + (\theta-1)\,\hat{\mu}_{D,t}$$

$$\hat{d}^*_{D,t} = (1-\theta)\,\hat{\rho}^*_{D,t} + \hat{c}^*_t + (\theta-1)\,\hat{\mu}^*_{D,t}$$

$$\tilde{d}_{X,t}(\omega) = (1-\theta)\,\hat{\rho}_{X,t} + \hat{c}_t + (1+\theta)\,\hat{q}_t + \hat{c}^*_t + (\theta-1)\,\hat{\mu}_{X,t} - \hat{w}_t + \hat{A}_t$$

$$\tilde{d}^*_{X,t}(\omega) = (1-\theta)\,\hat{\rho}^*_{X,t} + \hat{c}^*_t - (1+\theta)\,\hat{q}_t + \hat{c}_t + (\theta-1)\,\hat{\mu}_{X,t} - \hat{w}^*_t + \hat{A}^*_t$$

equations for the motion of firms :free entry condition

$$\hat{z}_X = \theta\left(\hat{w}_t - \hat{A}_t\right) - (1+\theta)\,\hat{q}_t - \hat{c}^*_t$$

$$\hat{z}^*_X = \theta\left(\hat{w}^*_t - \hat{A}^*_t\right) + (1+\theta)\,\hat{q}_t - \hat{c}_t$$

$$\hat{n}_{X,t} = -k\hat{z}_{X,t}$$

$$\hat{n}^*_{X,t} = -k\hat{z}^*_{X,t}$$

$$\hat{n}_{D,t} = -\frac{kz_X^{-k}}{\left(1-z_X^{-k}\right)}\hat{z}_{X,t}$$

$$\hat{n}^*_{D,t} = -\frac{kz_X^{-k}}{\left(1-z_X^{-k}\right)}\hat{z}^*_{X,t}$$

Inflation dynamic equation (New Keynesian Phillips curve)

$$(\pi_{Dt} - \pi_D) = \beta E_t(\pi_{D,t+1} - \pi_D) - \tfrac{\sigma-1}{\psi}\widehat{\mu}_{Dt},$$

$$(\pi^*_{Dt} - \pi_D) = \beta E_t(\pi^*_{D,t+1} - \pi_D) - \tfrac{\sigma-1}{\psi}\widehat{\mu}^*_{Dt},$$

$$(\pi_{Xt} - \pi_X) = \beta E_t(\pi_{X,t+1} - \pi_X) - \tfrac{\sigma-1}{\psi}\widehat{\mu}_{Xt},$$

$$(\pi^*_{Xt} - \pi_X) = \beta E_t(\pi^*_{X,t+1} - \pi_X) - \tfrac{\sigma-1}{\psi}\widehat{\mu}^*_{Xt},$$

$$\widehat{\mu}_{Xt} = \widehat{\rho}_{X,t} - \widehat{w}_t + \widehat{A}_t + \hat{z}_{X,t},$$

$$\widehat{\mu}^*_{Xt} = \widehat{\rho}^*_{X,t} - \widehat{w}^*_t + \widehat{A}^*_t + \hat{z}^*_{X,t},$$

$$\widehat{\mu}_{Dt} = \widehat{\rho}_{D,t} - \widehat{w}_t + \widehat{A}_t + \hat{z}_{D,t}$$

$$\widehat{\mu}^*_{Dt} = \widehat{\rho}^*_{D,t} - \widehat{w}^*_t + \widehat{A}^*_t + \hat{z}^*_{D,t}$$

with,

$$\hat{z}_{D,t} = -\left(\frac{\left(1-z_{X,t}^{1-k}\right)kz_{X,t}^{-k} + \left(1-z_{X,t}^{-k}\right)(k-1)\,z_{X,t}^{1-k}}{\left(1-z_{X,t}^{1-k}\right)\left(1-z_{X,t}^{-k}\right)}\right)\hat{z}_{X,t}$$

$$\hat{z}^*_{D,t} = -\left(\frac{\left(1 - z^{1-k}_{X,t}\right) k z^{-k}_{X,t} + \left(1 - z^{-k}_{X,t}\right)(k-1)\, z^{1-k}_{X,t}}{\left(1 - z^{1-k}_{X,t}\right)\left(1 - z^{-k}_{X,t}\right)}\right) \hat{z}^*_{X,t}$$

labor supply curveindividual production function

$y^d_{D,t}(\omega) = \rho_{D,t}(\omega)^{-\sigma}\left[c_t + \Gamma_t\right]$
$y^d_{X,t}(\omega) = \rho^{-\theta}_{X,t}\left[(c_t + \Gamma_t) + \phi q^{1-\theta}_t (c^*_t + \Gamma^*_t)\right]$

$$\widehat{y}_{X,t} = -\theta\widehat{\rho}_{X,t} + \widehat{c}_t. + (1-\theta)\widehat{q}_t + \widehat{c}^*_t$$

$$\widehat{y}^*_{X,t} = -\theta\widehat{\rho}^*_{X,t} + \widehat{c}^*_t. - (1-\theta)\widehat{q}_t + \widehat{c}_t$$

$$\widehat{y}_{D,t} = -\theta\widehat{\rho}_{D,t} + \widehat{c}_t$$

$$\widehat{y}^*_{D,t} = -\theta\widehat{\rho}^*_{X,t} + \widehat{c}^*_t$$

$$\widehat{y}_{X,t} = \widehat{l}_{X,t} + \widehat{A}_t. + \hat{z}_{X,t}$$

$$\widehat{y}^*_{X,t} = \widehat{l}^*_{X,t} + \widehat{A}^*_t. + \hat{z}^*_{X,t}$$

$$\widehat{y}_{D,t} = \widehat{l}_{D,t} + \widehat{A}_t + \hat{z}_{D,t}$$

$$\widehat{y}^*_{D,t} = \widehat{l}^*_{D,t} + \widehat{A}^*_t + \hat{z}^*_{D,t}$$

$$\widehat{l}_t = (1 - z_X{}^{-k})\widehat{l}_{D,t} + z_X{}^{-k}\widehat{l}_{X,t}$$

$$\widehat{l}^*_t = (1 - z_X{}^{-k})\widehat{l}^*_{D,t} + z_X{}^{-k}\widehat{l}^*_{X,t}$$

consumption and consumption price index price indexes

$$\hat{n}_{D,t} + \hat{n}_{X,t} + \hat{n}^*_{X,t} = (\theta - 1)\left(\hat{\rho}_{D,t} + \hat{\rho}_{X,t} + \hat{q}_t + \hat{\rho}^*_{X,t}\right)$$

$$\hat{n}^*_{D,t} + \hat{n}^*_{X,t} + \hat{n}_{X,t} = (\theta - 1)\left(\hat{\rho}^*_{D,t} + \hat{\rho}^*_{X,t} - \hat{q}_t + \hat{\rho}_{X,t}\right)$$

nominal interest rate of bonds

$$(r_t - r) = \left(R^W_t - R\right) - E_t\left(\pi^C_{t+1} - \pi\right),$$

$$(r^*_t - r) = \left(R^W_t - R\right) - E_t\left(\pi^{*C}_{t+1} - \pi\right),$$

consumption inflation we define :

$$S_{D,t} = \frac{c_{D,t}}{c_t} = \rho^{1-\theta}_{D,t}$$

$$S_{X,t} = \frac{c_{X,t}}{c_t} = \rho^{1-\theta}_{X,t}$$

$$S_{M,t} = \frac{c_{M,t}}{c_t} = \rho^{1-\theta}_{M,t} = q^{\theta}_t \phi^{\frac{-\theta}{1-\theta}} \rho^{*1-\theta}_{X,t}$$

thus, in steady state,

$$S_D = \left(\left(\frac{\theta}{\theta - 1} \right) \nabla^{-\frac{1}{\theta-1}} \left(\frac{1 - z_X^{-k}}{1 - z_{X,}^{1-k}} \right) \right)^{1-\theta}$$

$$S_X = \left(\left(\frac{\theta}{\theta - 1} \right) z_X^{-1} \right)^{1-\theta}$$

$$S_M = \phi \left(\left(\frac{\theta}{\theta - 1} \right) z_X^{-1} \right)^{1-\theta}$$

$$S_M \hat{q}_t = (\theta - 1) \left(S_D \hat{\rho}_{D,t} + S_X \hat{\rho}_{X,t} + S_M \hat{\rho}^*_{X,t} \right)$$

$$S_M \hat{q}_t = -(\theta - 1) \left(S_D \hat{\rho}^*_{D,t} + S_X \hat{\rho}^*_{X,t} + S_M \hat{\rho}_{X,t} \right)$$

so that, we get

$$(\pi_{C,t} - \pi) = S_D(\pi_{Dt} - \pi_D) + S_X(\pi_{Xt} - \pi_X) + S_M \left[(\pi^*_{Xt} - \pi_X) \right]$$

$$\left(\pi^*_{C,t} - \pi \right) = S_D(\pi^*_{Dt} - \pi_D) + S_X(\pi^*_{Xt} - \pi_X) + S_M \left[(\pi_{Xt} - \pi_X) \right]$$

production inflation :

$$(\pi_t - \pi) = (1 - z_X{}^{-k})(\pi_{Dt} - \pi_D) + z_X{}^{-k}(\pi_{Xt} - \pi_X)$$

$$(\pi^*_t - \pi) = (1 - z_X{}^{-k})(\pi^*_{Dt} - \pi_D) + z_X{}^{-k}(\pi^*_{Xt} - \pi_X)$$

and since we solve the model under the equilibrium of the current account, we get,

$$-(\theta - 1) \hat{\rho}_{X,t} + (\hat{q}_t + \hat{c}^*_t) = -(\theta - 1) \hat{\rho}^*_{X,t} + \hat{c}_t$$

interbank nominal interest rate from a Taylor (1993)-type monetary policy rule

$$\left(R^W_t - R \right) = \mu_3 \left(R^W_{t-1} - R \right) + (1 - \mu_3) \left[\mu_1 \left(\frac{1}{2} (\pi_t + \pi^*_t)^{TARGET} - \pi \right) + \tfrac{\mu_2}{4} \frac{1}{2} \left(\widehat{c}_t + \widehat{c}^*_t \right) \right] + \varepsilon_t, \tag{L18}$$

The set of 58 dynamic equations, may provide solution paths for the following variables : 54 (46) national variables, $\widehat{c}_t$,$(r_t - r)$,$\widehat{l}_t$,$\widehat{w}$, $\widehat{d}_{D,t}$, $\widehat{\mu}_{D,t}$, $\widehat{\rho}_{D,t}$, $\widehat{d}_{X,t}$, $\widehat{\mu}_{X,t}$,$\widehat{\rho}_{X,t}$,$\hat{z}_{X,t}$, $\widehat{n}_{X,t}$, $\widehat{n}_{D,t}$, $(\pi_{D,t} - \pi)$, $(\pi_{X,t} - \pi)$, $(\pi_{C,t} - \pi)$, $(\pi_t - \pi)$, $\hat{y}_{X,t}$, $\hat{y}_{D,t}$, $\widehat{l}_{X,t}$, $\widehat{l}_{D,t}$,, $\widehat{A}_t$, $\widehat{c}^*_t$,$(r^*_t - r)$,$\widehat{l}^*_t$,$\widehat{w}^*_t$, $\widehat{v}^*_t$, $\widehat{d}^*_t$, $\widehat{d}^*_{D,t}$, $\widehat{\mu}^*_{D,t}$, $\widehat{\rho}^*_{D,t}$, $\widehat{d}^*_{X,t}$, $\widehat{\mu}^*_{X,t}$,$\widehat{\rho}^*_{X,t}$,$\hat{z}^*_{X,t}$, $\widehat{n}^*_t$, $\widehat{n}^*_{E,t}$, $\widehat{n}^*_{X,t}$, $\widehat{n}^*_{D,t}$, $\left(\pi^*_{D,t} - \pi \right)$, $\left(\pi^*_{X,t} - \pi \right)$, $\left(\pi^*_{C,t} - \pi \right)$, $(\pi^*_t - \pi)$, $\hat{y}^*_{X,t}$, $\hat{y}^*_{D,t}$, $\widehat{l}^*_{X,t}$, $\widehat{l}^*_{D,t}$, $\hat{y}^*_t$, $\widehat{\rho}^*_t$, $\widehat{A}^*_t$,1 international variable, $\hat{q}_t$, 1 union wide variable, $\left(R^W_t - R \right)$.

regarding the status of variables we have : 5 predetermined variables $\widehat{n}_{t-1}$, $\widehat{n}_{E,t-1}$, $\widehat{n}^*_{X,t-1}$, $\widehat{n}^*_{D,t-1}$, $\left(R^W_{t-1} - R \right)$, 2 exogenous variables : $\xi_{A,t}$, $\xi^*_{A,t}$

4 Data

Data are extracted from different databases. In this section, we present data used and their sources. Data concerns 11 countries: Belgium-Luxembourg, France, Germany, Ireland, Italy, Portugal, Spain, the Netherlands, Finland and Austria and 13 years: from 1995 to 2007.

4.1 Business Cycles Synchronization

The dependent variable:

$$C_t = \frac{1}{2} ln \left(\frac{(1+C)}{(1-C)} \right)$$

where C is the pairwise correlation coefficient for each country pair. Correlations between European GDP are used (industrial production indexes are used as an alternaltive measure of activity).

4.2 Trade

To calculate the extensive margin of trade, we used the BACI database from the CEPII. The OECD database are used in order to measure openness.

- Bilateral Extensive Margin

  Total Trade:

  $$TT_{ij,t} = \sum_k X_{ij,k,t}$$

  with $X_{ij,k,t}$ the value of exports for the good k at the period t .
  Extensive Trade:

  $$ET_{ij,t} = \sum_k EX^n_{ij,k,t}$$

  with $EX^n_{ij,t}$ the value of exports at the period t if there is no export from i to j for the good k at the period *t-1*.
  The relative bilateral extensive margin:

  $$EM = \frac{ET_{ij,t}}{TT_{ij,t}}$$

- Trade Intensity

$$Trade_{ij,t} = \frac{X_{ij,t}+M_{ij,t}}{Y_{i,t}}$$

4.3 Gravity variables

As stressed by Clark and van Wincoop (2001), output correlations among countries (or regions) can also be influenced by distance factors. Dummy variables from the CEPII bilateral distance database are used to control for contiguity (border effect) and common language. Economic distance between pairs of countries is proxied by the log of the distance (in km) between their capital cities (respectiveley *contig*, *lang* and *dist* variables).
Finally, *pibprod* measures the effect of size on trade:

$$pibprod_{ij,t} = log(Y_{i,t} \times Y_{j,t})$$

4.4 Sectoral Specialization

Data come from the OECD database and concern 27 sectors.

$$specia = \sum_s |V_{is} - V_{js}|$$

V_{is} denotes the GDP share of industry s in country i.

4.5 Financial closeness

We also account for financial linkages between country pairs as suggested by Otto, Voss, and Willard (2001). Various measures are considered in our regressions to control for bilateral financial integration.

Firstly, we compute yearly averages of monthly real interest rate differentials. The latter are built from nominal interest rates and consumer price indices. Following Otto et al. (2001), ex post real interest rates in country i for year t are calculated according to:
$r_{i,t} = i_{i,t} - 100\frac{(P_{i,t+1}-P_{i,t})}{P_{i,t}}$
with $i_{i,t}$ the (yearly average of) the nominal average interest rate, $P_{i,t}$ is the (yearly average of) consumer price index in country i in year t. We consider two version of this index. Financial dependencies in the short run are captured through interest rates on three-month treasury bills (*IFI1*), while a similar index is constructed with the rates on ten-year maturity government bonds to control for financial linkages at a longer horizon (*IFI2*). The corresponding time series

were extracted from the OECD database from 1996:01 to 2007:12.

The degree of financial closeness is measured by the real interest rate spread which is calculated as the natural of log of the annual standard-deviation :
$rs_{ij,t} = ln\left(\sigma\left(r_{i,t} - r_{j,t}\right)\right)$

Secondly, real equity returns are calculated on the basis of monthly nominal stock market indices and consumer price indices (*IFI3*). We use OECD data from the same sample period as before. From these, levels are averaged annually so that real returns are computed as the relative variation of the stock index in real terms:
$s_{i,t} = \frac{\frac{S_{i,t}}{P_{i,t}}}{\frac{S_{i,t-1}}{P_{i,t-1}}} - 1$ with $S_{i,t}$ the stock index in country i in year t, $P_{i,t}$ the consumer price index described above. Like the real interest rates, estimations are based on the logarithm of the standard deviation of the spread on real equity returns. Consider countries i and j, this amounts to :
$ss_{ij,t} = ln\left(\sigma\left(ss_{i,t} - s_{j,t}\right)\right)$

Thirdly, we also take the logarithm of the standard deviation of the difference of real bilateral exchange rates into account (*rxrvol*).
$es_{ij,t} = ln\left(\sigma\left(e_{i,t} - e_{j,t}\right)\right)$ with $e_{i,t}$ the US dollar rate for the domestic currency in country i. There are taken from the Pacific Retrieval Interface of the British Colombia University. These are monthly averages over 1996-2007.
Fourthly, we include the absolute difference between the net foreign asset (NFA) positions of a country pair as it is done in Imbs (2004) and Inklaar et al. (2008). This last variable is used as an index of (bilateral) capital restrictions (*IFI4*). The NFA annual data series were dowloaded from the last version of Lane and Milesi-Feretti's (2009) database. To get comparable results with Inklaar et al. (2008), we consider absolute differences between the GDP ratios of the cumulated current accounts for each country pair.
Finally, as Inklaar et al. (2008), cyclically adjusted government primary balance (as a percentage of potential GDP, from the OECD database) are used as exogenous variable (*fisc)*.

5 Descriptive results

The first two columns of table 1 report the total number of new exported goods on a year-by-year basis in the UE-15 area and in France, respectively. It ranges from 21 to almost 30 thousands new products each year exported from one EU Member State to (at least) another. New varietes of French exports accounts for seven percents of this total on average over the

Table 1: Number of products exported to all countries j

	UE	France	France %UE	Delta UE	Delta Fra
1995–1996	26,509	2,007	7.57		
1996–1997	25,291	1,945	7.69	-4.59	-3.09
1997–1998	23,891	1,8	7.53	-5.54	-7.46
1998–1999	24,929	1,802	7.23	4.34	0.11
1999–2000	22,378	1,661	7.42	-10.23	-7.82
2000–2001	23,3	1,683	7.22	4.12	1.32
2001–2002	25,351	1,678	6.62	8.80	-0.30
2002–2003	21,906	1,574	7.19	-13.59	-6.20
2003–2004	22,455	1,705	7.59	2.51	8.32
2004–2005	29,665	2,002	6.75	32.11	17.42
2005–2006	21,939	1,647	7.51	-26.04	-17.73
2006–2007	21,502	1,502	6.99	-1.99	-8.80

sample period (see column 3). As it is apparent from the last two columns of table 1, the extensive margin exhibits substantial variability. There seems to be greater variations in the extensive margins after the European monetary unification than before. However, these figures may also capture the effect of the EU enlargement to the Eastern New Member States since May 2004.

Some basic descriptive statistics on the extensive margin (labelled EM) are provided for each of the eleven founder members of the Euro area in table 2 below. Consolidated data are are only available for Belgium and the Luxemburg.

The extensive margin seems to be quite low on average in all countries over the whole sample period. It reaches its highest level in Austria (3.24%) while it is the lowest in Germany (less than 0.5%).

Things look different when looking at rows 2–4 of table 2. There may be substantial extensive margins in a specific year and with respect to a given trade partner. For example, the gains from new traded varietes culminated at 13.85% of the total gains in France in 2001. In the French case, the highest extensive margin concerns exports to Finland (country 9).

The three next rows of table 2 show the distribution of the highest recorded MEr. Taking Ireland as an example, the extensive margin accounts for at least 1% of the total gains in trade

Table 2: Extensive margins: country-based descriptive statistics

EM Exports	Belg.-Lux.	France	Germany	Ireland	Italy	Portugal	Spain	Neth.	Finland	Austria
Country index	1	2	3	4	5	6	7	8	9	10
Average in % **(1)**	1.06	0.90	0.42	1.80	0.97	2.53	1.86	1.01	2.85	3.24
Max MEr %	18.12	13.85	5.40	9.83	10.57	18.01	25.95	17.22	27.13	38.43
Max. (destination country)	4	9	4	6	9	4	9	4	4	6
Max. (Year)	1999	2001	1997	1999	2006	1996	2001	1997	2001	2007
Nb EM $\geq 1\%$	43	30	13	69	25	83	50	34	90	77
Nb EM $\geq 5\%$	2	4	2	9	6	16	9	4	14	24
Nb EM $\geq 10\%$	1	3	0	0	1	5	4	1	4	8
Average (%) to 4+6+9 **(2)**	2.58	2.45	1.02	2.25	2.28	4.52	4.45	2.17	4.07	7.89
Ratio **(2)**/**(1)**	2.44	2.72	2.43	1.25	2.35	1.79	2.39	2.15	1.43	2.44

in more than half of the total cases (9 possible destinations to trade over 12 years). But this figure shrinks to less than 10% of the occurrences (9 over 108 exactly) for the extensive margin to exceed 5%.

The third row of table 2 shows that the greatest extensive margin is usually observed against three trade partners, namely Ireland, Portugal, and Finland. According to this, we have calculated the mean percentage of the extensive margin against these three importers. As expected, figures are noticeably higher than the whole average: the corresponding ratio in the last row varies from 1.43 (in Finland) to 2.72 in France.

5.1 Coherence functions between EM and GDP correlations

Average coherences (at high frequencies) with HP filter. If the values of coherences are bigger than 0.41, 0.51 and 0.61, coherences are significant at 10, 5 and 1% respectively.

	Lux	Fra	Ger	Ire	Ita	Por	Spa	Net	Fin	Aus	Average
Lux		0.701	0.945	0.577	0.806	0.620	0.792	0.495	0.885	0.610	0.714
Fra	0.666		0.520	0.709	0.499	0.496	0.687	0.565	0.551	0.679	0.597
Ger	0.729	0.479		0.694	0.459	0.611	0.746	0.419	0.839	0.837	0.646
Ire	0.964	0.479	0.687		0.428	0.799	0.630	0.923	0.606	0.804	0.702
Ita	0.456	0.678	0.677	0.851		0.667	0.693	0.401	0.867	0.670	0.662
Por	0.459	0.668	0.746	0.780	0.520		0.932	0.968	0.719	**0.382**	0.686
Spa	0.562	0.848	0.439	0.848	0.857	0.781		0.916	0.815	0.771	0.760
Net	0.516	0.686	0.814	0.647	**0.331**	0.701	0.672		0.982	0.747	0.677
Fin	0.939	0.592	0.821	0.505	0.557	0.598	0.703	0.600		0.874	0.688
Aus	0.745	0.833	0.865	0.856	0.690	0.771	0.642	0.694	0.792		0.765
Average	0.671	0.663	0.724	0.719	0.572	0.672	0.722	0.664	0.784	0.708	

In bold, no significant at 10%.

Average coherences (at high frequencies) with BK filter.

	Lux	Fra	Ger	Ire	Ita	Por	Spa	Net	Fin	Aus	Average
Lux		0.719	0.906	0.494	0.889	0.661	0.842	0.645	0.899	0.562	0.735
Fra	0.656		0.458	0.684	0.628	0.539	0.663	**0.373**	0.562	0.650	0.579
Ger	0.827	0.595		0.648	0.502	0.535	0.864	0.429	0.809	0.748	0.662
Ire	0.965	0.449	0.645		0.499	0.833	0.673	0.746	0.571	0.910	0.699
Ita	0.570	0.543	0.620	0.922		0.553	0.574	**0.387**	0.872	0.444	0.609
Por	0.436	0.580	0.616	0.751	0.414		0.865	0.959	0.699	**0.256**	0.620
Spa	0.665	0.863	**0.286**	0.760	0.854	0.832		0.933	0.721	0.725	0.738
Net	0.550	0.652	0.887	0.588	**0.364**	0.896	0.715		0.972	0.683	0.701
Fin	0.894	0.515	0.765	0.445	0.460	0.532	0.614	0.468		0.633	0.592
Aus	0.668	0.627	0.795	0.887	0.445	**0.377**	0.567	0.674	0.453		0.610
Average	0.692	0.616	0.664	0.686	0.562	0.640	0.709	0.624	0.729	0.623	

In bold, no significant at 5%.

6 Econometric results

6.1 Cross Section for the year 2007

6.1.1 with trade

$$C_{i,j} = \alpha_0 + \alpha_1 trade_{i,j} + \alpha_2 specia_{i,j} + \alpha_3 fisc_{i,j} + \alpha_4 rxrvol_{i,j} + \alpha_5 dist_{i,j} + \alpha_6 contig_{i,j} \quad (1)$$

$$+\alpha_7 lang_{i,j} + \alpha_8 pibprod_{i,j} + \alpha_9 ifi1_{i,j} + \alpha_{10} ifi2_{i,j} + \alpha_{11} ifi3_{i,j} + \alpha_{12} ifi4_{i,j} + \epsilon_{i,j} \quad (2)$$

		OLS			SUR			3SLS	
	synchro	trade	specia	synchro	trade	specia	synchro	trade	specia
trade	0.034		-10.349	0.024		-16.312	0.012		-39.569
	(0.19)		(0.46)	(0.14)		(0.77)	(0.04)		(0.07)
specia	-0.006	-0.000		-0.005	-0.000		-0.004	0.000	
	(5.20)***	(0.32)		(4.62)***	(0.59)		(1.94)*	(0.11)	
fisc	0.003	-0.001		0.004	-0.001		0.005	-0.001	
	(0.41)	(0.29)		(0.53)	(0.32)		(0.61)	(0.36)	
ifi1	0.094			0.095			0.099		
	(3.89)***			(4.10)***			(2.97)***		
ifi2	-0.040	0.007	6.915	-0.045	0.008	6.862	-0.049	0.005	6.941
	(2.26)**	(0.84)	(4.60)***	(2.70)***	(1.00)	(4.82)***	(2.05)**	(0.57)	(1.70)*
rxrvol	0.001	0.001		0.001	0.001		0.001	0.001	
	(0.36)	(0.49)		(0.39)	(0.54)		(0.37)	(0.46)	
dist		-0.000	0.005		-0.000	0.005		-0.000	0.004
		(1.75)*	(2.86)***		(1.78)*	(3.05)***		(1.87)*	(0.43)
contig		0.057	9.822		0.058	9.899		0.054	10.752
		(3.09)***	(2.74)***		(3.35)***	(2.94)***		(2.93)***	(0.35)
lang		0.022	-2.189		0.021	-1.648		0.023	-0.818
		(1.55)	(0.80)		(1.58)	(0.64)		(1.67)*	(0.08)
pibprod		-0.038	37.717		-0.040	41.950		-0.045	44.409
		(0.91)	(4.05)***		(1.00)	(4.79)***		(1.09)	(1.78)*
ifi3			-3.122			-2.928			-2.930
			(1.30)			(1.30)			(0.50)
ifi4			9.467			9.577			9.535
			(3.57)***			(3.85)***			(3.86)***
Constant	1.409	0.424	-299.103	1.356	0.444	-336.743	1.336	0.470	-357.555
	(7.34)***	(1.12)	(3.49)***	(7.42)***	(1.24)	(4.19)***	(4.13)***	(1.28)	(1.46)
Observations	90	90	90	90	90	90	90	90	90
R-squared	0.43	0.46	0.46						

Absolute value of t statistics in parentheses

* significant at 10%; ** significant at 5%; *** significant at 1%

6.1.2 with EM

$$C_{i,j} = \alpha_0 + \alpha_1 EM_{i,j} + \alpha_2 specia_{i,j} + \alpha_3 fisc_{i,j} + \alpha_4 rxrvol_{i,j} + \alpha_5 dist_{i,j} + \alpha_6 contig_{i,j} \quad (3)$$

$$+\alpha_7 lang_{i,j} + \alpha_8 pibprod_{i,j} + \alpha_9 ifi1_{i,j} + \alpha_{10} ifi2_{i,j} + \alpha_{11} ifi3_{i,j} + \alpha_{12} ifi4_{i,j} + \epsilon_{i,j} \quad (4)$$

	OLS			SUR			3SLS		
	synchro	EM	specia	synchro	EM	specia	synchro	EM	specia
EM	-0.004		-0.158	-0.003		-0.338	-0.001		-5.657
	(1.96)*		(0.81)	(1.86)*		(1.83)*	(0.19)		(4.28)***
specia	-0.007	-0.064		-0.006	-0.113		-0.007	-0.174	
	(7.30)***	(1.10)		(7.02)***	(2.05)**		(5.04)***	(6.24)***	
fisc	0.008			0.009			0.007		
	(1.10)			(1.27)			(1.06)		
ifi1	0.108			0.110			0.116		
	(4.70)***			(4.99)***			(4.35)***		
rxrvol	0.001	0.076		0.002	0.061		0.002	0.001	
	(0.61)	(0.49)		(0.74)	(0.42)		(0.84)	(0.02)	
distw		0.003	0.005		0.004	0.006		0.004	0.022
		(3.49)***	(3.05)***		(3.92)***	(3.69)***		(4.59)***	(3.66)***
ifi2		1.106	6.505		1.389	6.270		1.513	8.557
		(1.19)	(4.46)***		(1.56)	(4.52)***		(1.87)*	(1.92)*
contig		0.829	10.410		1.407	10.425		2.271	13.036
		(0.41)	(3.19)***		(0.73)	(3.37)***		(1.29)	(1.33)
lang		0.518	-1.580		0.305	-1.316		0.026	0.201
		(0.34)	(0.59)		(0.21)	(0.52)		(0.02)	(0.03)
pibprod		0.520	38.842		2.038	41.782		6.162	37.078
		(0.11)	(4.20)***		(0.45)	(4.76)***		(1.46)	(1.56)
ifi4			9.245			9.211			0.345
			(3.47)***			(3.66)***			(0.20)
Constant	1.669	-0.301	-300.650	1.662	-11.521	-328.619	1.719	-46.834	-284.651
	(11.83)***	(0.01)	(3.53)***	(12.25)***	(0.28)	(4.06)***	(9.65)***	(1.21)	(1.31)
Observations	90	90	90	90	90	90	90	90	90
R-squared	0.42	0.19	0.45						

Absolute value of t statistics in parentheses

* significant at 10%; ** significant at 5%; *** significant at 1%

6.2 Panel data

	3SLS			3SLS FE			3SLS + RE		
	synchro	EM	specia	synchro	EM	specia	synchro	EM	specia
EM	-0.021		-3.599	-0.035		-7.216	-0.023		-3.992
	(2.97)***		(1.71)*	(3.73)***		(1.52)	(3.03)***		(1.69)*
specia	-0.002	0.020		0.002	0.065		-0.001	0.025	
	(1.34)	(0.71)		(0.82)	(1.82)*		(0.92)	(0.85)	
fisc	-0.016	0.032		-0.009	0.022		-0.015	0.028	
	(3.38)***	(0.84)		(1.82)*	(1.04)		(3.10)***	(0.80)	
ifi1	-0.039	0.088	1.602	-0.037	-0.084	0.940	-0.039	0.062	1.534
	(10.05)***	(1.51)	(3.55)***	(8.97)***	(1.49)	(1.86)*	(9.94)***	(1.06)	(3.34)***
rxrvol	-0.008	0.069		-0.010	0.052		-0.009	0.066	
	(3.75)***	(2.52)**		(4.35)***	(1.83)*		(3.93)***	(2.39)**	
pibpro		-0.115	31.228		-5.538	6.157		-0.915	29.262
		(0.10)	(4.68)***		(4.36)***	(0.82)		(0.77)	(4.50)***
dist		0.002	0.008		0.001	0.013		0.002	0.009
		(8.11)***	(2.18)**		(5.91)***	(1.68)*		(7.81)***	(2.13)**
lang		0.050	-5.212		0.560	-0.718		0.163	-4.492
		(0.15)	(3.84)***		(1.88)*	(0.23)		(0.50)	(3.20)***
contig		-0.215	12.004		-0.314	11.388		-0.226	11.686
		(0.41)	(6.92)***		(0.64)	(4.80)***		(0.43)	(6.50)***
ifi4			9.622			8.988			9.284
			(7.78)***			(5.25)***			(7.21)***
Constant	0.550	0.820	-242.306	0.429	46.924	-23.184	0.467	6.737	-198.717
	(9.76)***	(0.08)	(4.13)***	(6.09)***	(4.45)***	(0.35)	(9.06)***	(0.77)	(3.94)***
Observations	1080	1080	1080						

Absolute value of z statistics in parentheses

* significant at 10%; ** significant at 5%; *** significant at 1%

7 Conclusion

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A Model

The solution to this problem is as follows : taking into account goods market equilibrium and the production technology, so that,

$$d_t(\omega) = \left(\frac{p_t(\omega)}{P_t}\right)^{1-\sigma}[c_t+\Gamma_t] - \frac{\psi}{2}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)}-1\right)^2\left(\frac{p_t(\omega)}{P_t}\right)^{1-\sigma}[c_t+\Gamma_t] - \frac{w_t}{z(\omega)Z_t}\left(\frac{p_t(\omega)}{P_t}\right)^{-\sigma}[c_t+\Gamma_t],$$

we can write, the function to be maximized wrt the selling price, as,

$$\begin{aligned} d_{D,t}(\omega) + v_{D,t}(\omega) \quad = \quad & \left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{1-\sigma}[c_t+\Gamma_t] - \frac{\psi}{2}\left(\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)}-1\right)^2\left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{1-\sigma}[c_t+\Gamma_t] \\ & -\frac{w_t}{z(\omega)Z_t}\left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{-\sigma}[c_t+\Gamma_t] \\ & +E_t\left\{\beta(1-\delta)\Lambda_{t,t+1}\left(-\frac{\psi}{2}\left(\frac{p_{D,t+1}(\omega)}{p_{D,t}(\omega)}-1\right)^2\left(\frac{p_{D,t+1}(\omega)}{P_{c,t+1}}\right)^{1-\sigma}[c_{t+1}+\Gamma_{t+1}]\right)\right\} + tip \end{aligned}$$

The FOC wrt $p_t(\omega)$ is,

$$\begin{aligned} & (1-\sigma)\left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{-\sigma}\left(\frac{1}{P_t}\right)[c_t+\Gamma_t] - \kappa\left(\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)}-1\right)\frac{1}{p_{t-1}(\omega)}\left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{1-\sigma}[c_t+\Gamma_t] \\ & -\frac{\kappa}{2}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)}-1\right)^2(1-\sigma)\left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{-\sigma}\left(\frac{1}{P_{c,t}}\right)[c_t+\Gamma_t] \\ & +\sigma\frac{w_t}{zZ_t}\left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{-\sigma-1}\left(\frac{1}{P_{c,t}}\right)[c_t+\Gamma_t] \\ & +E_t\left\{sc\beta(1-\delta)sc\Lambda_{t,t+1}\left(\kappa\left(\frac{p_{D,t+1}(\omega)}{p_{D,t}(\omega)}-1\right)\left(\frac{p_{D,t+1}(\omega)}{p_{D,t}(\omega)}\right)\left(\frac{1}{p_{D,t}(\omega)}\right)\left(\frac{p_{D,t+1}(\omega)}{P_{c,t+1}}\right)^{1-\sigma}[c_{t+1}+\right.\right. \end{aligned}$$

$+tip= 0.$

In the case of flexible prices, $\psi = 0$, so that,

$$(1-\sigma)\left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{-\sigma}\left(\frac{1}{P_t}\right)[c_t+\Gamma_t] + \sigma\frac{w_t}{zZ_t}\left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{-\sigma-1}\left(\frac{1}{P_t}\right)[c_t+\Gamma_t] = 0,$$

ie,

$$p_{D,t}(\omega) = \frac{\sigma}{(\sigma-1)}\frac{P_t w_t}{zZ_t}.$$

In the more general case with $\psi \neq 0$, we can write,

$$
\begin{aligned}
& (\sigma-1)\left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{-\sigma}\left(\frac{1}{P_{c,t}}\right)[c_t+\Gamma_t]+\psi\left(\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)}-1\right)\frac{1}{p_{Dt-1}(\omega)}\left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{1-\sigma}[c_t+\Gamma_t] \\
& +\frac{\psi}{2}\left(\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)}-1\right)^2(1-\sigma)\left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{-\sigma}\left(\frac{1}{P_{c,t}}\right)[c_t+\Gamma_t] \\
& -E_t\left\{\beta(1-\delta)\Lambda_{t,t+1}\left(\psi\left(\frac{p_{D,t+1}(\omega)}{p_{D,t}(\omega)}-1\right)\left(\frac{p_{D,t+1}(\omega)}{p_{D,t}(\omega)}\right)\left(\frac{1}{p_{D,t}(\omega)}\right)\left(\frac{p_{D,t+1}(\omega)}{P_{c,t+1}}\right)^{1-\sigma}[c_{t+1}+\Gamma_{t+1}]\right)\right\} \\
= \; & \sigma\frac{w_t}{zA_t}\left(\frac{p_{D,t}(\omega)}{P_{cnt}}\right)^{-\sigma-1}\left(\frac{1}{P_{c,t}}\right)[c_t+\Gamma_t].
\end{aligned}
$$

Multiplying both sides by $P_{c,t}$ and using the definition, that in equilibrium, $y_t(\omega)=y_t^d(\omega)=\left(\frac{p_{D,t}(\omega)}{P_t}\right)^{-\sigma}[c_t+\Gamma_t]$, we get,

$$
\begin{aligned}
& (\sigma-1)\,y_{D,t}(\omega)+\kappa\left(\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)}-1\right)\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)}y_{D,t}(\omega)+\frac{\psi}{2}\left(\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)}-1\right)^2(1-\sigma)\,y_{D,t}(\omega) \\
& -E_t\left\{\beta(1-\delta)\Lambda_{t,t+1}\left(\psi\left(\frac{p_{D,t+1}(\omega)}{p_{D,t}(\omega)}-1\right)\left(\frac{p_{D,t+1}(\omega)}{p_{D,t}(\omega)}\right)\left(\frac{P_{c,t}}{p_{D,t}(\omega)}\right)\left(\frac{p_{D,t+1}(\omega)}{P_{c,t+1}}\right)y_{D,t+1}(\omega)\right)\right\} \\
= \; & \sigma\frac{w_t}{zA_t}\left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{-1}y_{D,t}(\omega).
\end{aligned}
$$

Rearranging,

$$
\begin{aligned}
& (\sigma-1)\,y_{D,t}(\omega)\left(1-\frac{\psi}{2}\left(\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)}-1\right)^2\right)+\psi\left(\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)}-1\right)\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)}y_{D,t}(\omega) \\
& -\kappa E_t\left\{\beta(1-\delta)\Lambda_{t,t+1}\left(\left(\frac{p_{D,t+1}(\omega)}{p_{D,t}(\omega)}-1\right)\left(\frac{p_{D,t+1}(\omega)}{p_{D,t}(\omega)}\right)^2\left(\frac{P_{c,t}}{P_{c,t+1}}\right)y_{D,t+1}(\omega)\right)\right\} \\
= \; & \sigma\frac{w_t}{z_t}\left(\frac{p_t(\omega)}{P_{c,t}}\right)^{-1}y_{D,t}(\omega),
\end{aligned}
$$

so that finally, we write,

$$
\begin{aligned}
\rho_t(\omega) &= \left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)=\frac{\sigma y_{D,t}(\omega)}{(\sigma-1)\,y_{D,t}(\omega)\left(1-\frac{\psi}{2}\left(\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)}-1\right)^2\right)+\psi\Psi_{Dt}}\frac{w_t}{zA_t} \\
\Psi_{D,t} &= \left(\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)}-1\right)\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)}y_{D,t}(\omega) \\
& \quad -E_t\left\{\beta(1-\delta)\Lambda_{t,t+1}\left(\left(\frac{p_{D,t+1}(\omega)}{p_{D,t}(\omega)}-1\right)\left(\frac{p_{D,t+1}(\omega)}{p_{D,t}(\omega)}\right)^2\left(\frac{P_{c,t}}{P_{c,t+1}}\right)y_{D,t+1}(\omega)\right)\right\}
\end{aligned}
$$

$$
\tilde{\pi}_t=(1-z_{X,t}^{-k})\tilde{\pi}_{D,t}+z_{X,t}^{-k}\tilde{\pi}_{X,t}
$$

thus the economy philips curve writes

$$\tilde{\pi}_{D,t} = (1-\delta)\beta E_t(\tilde{\pi}_{D,t+1}) - \tfrac{\sigma-1}{\psi}\widehat{\mu}_{D,t}, \tag{L8}$$

$$\tilde{\pi}_{X,t} = (1-\delta)\beta E_t(\tilde{\pi}_{X,t+1}) - \tfrac{\sigma-1}{\psi}\widehat{\mu}_{X,t}, \tag{L8}$$

$$d_{D,t}(\omega) = \left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{1-\theta}[c_t+\Gamma_t] - \frac{\psi}{2}\left(\frac{p_{D,t}(\omega)}{p_{D,t-1}(\omega)}-1\right)^2\left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{1-\theta}[c_t+\Gamma_t] - \frac{w_t}{z(\omega)Z_t}\left(\frac{p_{D,t}(\omega)}{P_{c,t}}\right)^{-\theta}[c_t + \Gamma$$

$$d_{X,t}(\omega) = \rho_{X,t}(\omega)^{1-\theta}\left[c_t^W + \Gamma_t^W\right] - \frac{\psi}{2}\left(\frac{p_{X,t}(\omega)}{p_{X,t-1}(\omega)}-1\right)^2\rho_{X,t}(\omega)^{1-\theta}\left[c_t^W+\Gamma_t^W\right] - \frac{1}{\mu_{X,t}}\rho_{X,t}(\omega)^{1-\theta}\left[c_t^W+\Gamma_t^W\right] -$$