Robust monetary policy in the micro-founded two-country model of the currency union

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For two-country model of currency union with sticky prices the problem of model uncertainty is analyzed. By methods of robust control we derive robust monetary policy that works reasonably well even in the worst-case of model perturbations. We find some violation of Brainard principle and show that central bank's optimal reaction to the economic shocks becomes more aggressive with an increase in its fear of misspecification.

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Introduction

A usual way of elaborating optimal monetary policy is to design it in the context of a particular model of the economy. However nobody knows the true and extremely complex structure of the economy. In attempt to capture at least some of the most important economic regularities by means of tractable model a certain degree of simplification is needed. The resulting analytical framework appears to be more or less stylized. Consequently, nobody can be absolutely confident about the predicting power of any particular model employed for the monetary policy analysis. Thus, the problem of dealing with this model uncertainty or uncertainty about the true structure of the economy arises.

There are two common approaches to this problem. The first one implies policymaker's learning about the true structure of the economy. But this process requires a lot of time and possibly the second approach is more appropriate to tackle uncertainty quickly. This second method is a searching for the *robust monetary policy* that works reasonably well across a certain set of models or model specifications. According to the type of the model set and specific type of uncertainty the analysis can be made in different ways.

The main question which is usually in the centre of attention in the robust literature concerns the comparison of robust policies and simple optimal ones, designed for the particular model. The seminal result called Brainard conservatism assumes that robust policy is less aggressive in the reaction to the economic shocks than a policy, constructed for a single model without taking into account model uncertainty.

There are many examples of such analysis for the USA and for the euro area and some authors agree with this general finding while the others do not. All the models used in such analysis in the application to the euro area represent the area-wide models which operate with aggregated data. But it seems to be not the best approach, as a euro area represents a set of interacting heterogeneous countries.

So in our analysis we adopt for this problem two-country micro-founded model of currency area. We derive a micro-founded welfare criterion on which a policy evaluation is based. It is shown that terms of trade between these regions matter for the population welfare and so using a one-country model for analysis of such constructions as monetary union is unjustified.

Then we calibrate this model for the European Union taking into account the differences between regions.

After that we construct a robust to model uncertainty monetary policy under commitment. This construction is based on the robust control methodology.

We illustrate how the robust control techniques initiated by Hansen, Sargent (2001) can be applied for multi-country models with rational expectations. We show, contrary to Bihan (2002), Zacovic, Wieland, Rustem (2005), that the main characteristics of robust policy and economic outcome depends crucially on the particular parameter – a willingness of the central bank to construct a robust policy or a fear of model misspecification.

To choose a proper extent of model misspecification we adopt an error detection probability approach firstly initiated by Hansen, Sargent.

The rest of the paper is organized as follows: Chapter 1 outlines existing literature on the robust questions. The two-country model is presented in Chapter 2 where the central bank's micro-founded loss function is also derived. In Chapter 3 we apply robust control techniques for this model and derive the characteristics of the robust policy. The last section concludes and outlines the possible directions for the future research.

Chapter 1. Literature

1.1 Types of uncertainty

The notion of uncertainty can be referred to some empiric variables or to the model structure as a whole. To clearly understand the application area of robust theory accurate distinction of different sources of uncertainty is needed. Knight (1921) contrasted risk with uncertainty. Risk refers to the processes which have known distributions, while uncertainty arises from unknown factors and processes or from the processes which hardly can be described by statistic terms. Speaking about empiric variables knightian notion of risk corresponds to the aleatory or statistical uncertainty, while the term ‘uncertainty’ refers to the epistemic or systemic uncertainty.

Uncertainty about the model structure or model uncertainty can arise both from aleatory (possible estimation errors, par example) and epistemic sources (unknown factors or interactions of economic variables). In both cases robust techniques can be implemented, but the practical method differs with the particular type of model uncertainty. All possible cases of model uncertainty can be roughly divided into two broad classes.

The first class can be named ‘more or less parametric’ uncertainty. In this case the overall structure of the model is taken as true but the values of specific parameters are uncertain. This very class can be divided into 3 types according to the extent to which model uncertainty is counted as a parametric (Tetlow, Muehlen 2004):

- *Bayesian uncertainty*, the most parametric case. Under this type of model uncertainty parameters of the model are assumed to have known distributions.

- *Structured Knightian uncertainty*. In this case uncertainty is assumed to be located in some particular parameters of the model. The true values of these parameters are supposed to lie between the known minimum and maximum possible levels.

- *Unstructured Knightian uncertainty*, the less parametric variant. Under such uncertainty neither its location nor its nature are specified. This case can be analyzed as a game played by the central banker against a "malevolent nature" (Onatski, Stock 2000).

The second class takes into account much more severe extent of uncertainty than the first one – so called *specification uncertainty*. Not only the precise parameters' values but even core structural model characteristics are treated as unknown. In the macroeconomic models such core characteristics can represent a character of expectations (forward- or backward-looking), adherence to microfoundations or, par example, lag structure. In all cases this class of uncertainty admits the true economy to be considerably different from the given model, and not only by the value of some parameters.

1.2 Methods of analysis

Like possible kinds of model uncertainty, all analyzing robust techniques can also be divided in two classes. The first one refers to the fist class of uncertainty – the parametric one and implies that the policymaker or economist has only one reference model. The real structure of the economy may differ from this model and the policymaker believes that the true economy lies in the 'specified neighborhood' of a baseline model (Brainard, 1967). This neighborhood includes all possible deviations from the reference framework and one can interpret this approach as analysis of a set of similar but not identical models (Giannoni, 2002).

The second approach to the analysis implies a much more serious extent of uncertainty – *specification uncertainty*. In this case such important characteristics as a kind of expectations (backward- or forward-looking or even a hybrid case) or lag structure of variables are taken as uncertain. Thus the risk of specification error is so high that the use of a single reference model is not appropriate. Moreover, monetary policy rules based only on a single model may cause very bad performance if the true economy differs from the baseline model by character of expectations implemented (Levin, Williams 2003). That's why the followers of this approach propose the analysis of a set of distinct or 'competing' models with very different core characteristics.

1.3 Choice criteria

Despite of general approach chosen we need to have criteria according to which one can evaluate the robustness of monetary policy rule (a rule which works reasonably well in all models from the model set (either similar or competing models)).

The fist possible way is *model or Bayesian averaging* (Brock, Durlauf, West (2004)) when the parameters of final rule minimize a weighted sum of policymaker's losses under distinct models from the using set. The weights used are posterior probabilities of each model to be the true one. There are several problems concerning this method. The first one is the difficulty with computing these probabilities. The second concerns the fact that large difficult models fit data better than the smaller ones only due to the inclusion of large set of variables, not because of their 'better' quality. But in this case the weights of these large models will be unreasonably high in comparison with simpler constructions.

Both problems concerned here are not a question if *equal weights* are chosen as in Levin, Williams (2003). This method differs from the Bayesian averaging because model weights do not correspond to posterior probabilities.

Minimax criterion implies minimization of policymaker's losses in the worst variant from the set of possible outcomes. This choice allows expecting a reasonably well performance of this robust rule in all possible cases.

Some authors try to combine advantages of different choice variants and some *hybrid criteria* arise. Kuester, Wieland (2008) utilizes an *ambiguity aversion principle* when model weights are attached like in the Bayesian method but to the worst possible outcomes extra weights are given.

Performance maximization criterion is very similar to the min-max one. In this case the problem is represented as a zero-sum game between a policymaker and a 'malevolent nature', which is aimed to maximize the losses (Giannoni, 2002).

Under *stability maximization criterion* (for example in Onatski and Stock (2002)) policy is chosen to maximize a set of models, which are stable under such a rule.

Finally, *fault tolerance approach* initiated by Levin, Williams (2003) implies firstly constructing the optimal policy rule for each model from the using set. Then a researcher assesses a model's *fault tolerance* by analyzing deviations of one of the policy parameters from its optimal value, holding the other parameters fixed. If the loss function is relatively insensitive to changes in all parameters of the policy rule, the model is considered as a *fault tolerant* one. Then zones of *mutual tolerance* are computed. For these zones a "compromise" policy can be constructed. And this policy is considered as a robust monetary policy under model uncertainty.

1.4 General results

The core characteristics of obtained robust rules crucially depend on the choice criteria applied. Many authors try to compare these different rules in order to recognize some regularity in their performance. The first result was so called 'Brainard's principle' according to which a policy under uncertainty must be less aggressive than a certain one. This principle is confirmed by some authors (Bihan (2002), Zacovic, Wieland, Rustem (2005), etc) and rejected by others (Onatski, Stock (2002), Leitemo and Söderström (2008)), who have found a more aggressive than a certain policy robust rules. Generally speaking model averaging techniques lead to comparatively moderate policy reaction while min-max approach implies more aggressive monetary rules (Zacovic, Wieland, Rustem, 2005). But even this very cautious statement is not always the truth. Leitemo and Söderström (2005) state that the extent of policy reaction depends on the shock type, the source of uncertainty and even on the willingness of policymaker to implement robust policy (his robust preferences).

1.5 Robust analysis for euro area

The extent of model uncertainty for euro area and consequently the pertinence of robust techniques are very high. There are several reasons for it. First of all in comparison with the USA models, constructing models of euro area is relatively new and unelaborated domain. Secondly the true interactions between European authorities are really complex. While monetary policy is decided uniquely by the

European Central Bank the fiscal measures are undertaken in each country independently which in turn provoke additional questions when constructing monetary policy. Monetary authority cannot monitor entirely the fiscal motives and measures, so that is an additional source of uncertainty. Thirdly the question of data aggregation arises. There is also a more normative problem of necessary central bank's reactions on the country specific shocks or what is the true welfare criterion for these models.

As the first ECB's president Willem Duisenberg once said, "*this is not a trivial task given the large uncertainties that we are facing due to the establishment of a multi-country monetary union. Not only can we expect some of the historical relationships to change due to this shift in regime, but also, in many cases, there is a lack of comparable and cross-country data series that can be used to estimate such relationships.*"

So, many authors deal with the problem of model uncertainty relative to the euro area. Altavilla, Ciccarelli (2008) shows on the models of euro area that model uncertainty really represents a danger for the monetary policy and model averaging can be a useful tool for the policy conduct.

The whole domain of analysis concerns monetary policy under inflation uncertainty. Ungueloni, Coenen, Smets (2003), Jaaskela (2005), Adalid, Coenen, McAdam, Siviero (2005) and Coenen (2007) found that under high degree of inflation the optimal monetary policy rules are more aggressive and moreover, such rules are more robust. Some of these articles are based on the sole baseline model while others examine different sets of competing frameworks. The main general recommendation for the monetary authority is to assume higher extent of inflation persistence when constructing monetary policy. So, optimal robust policy should imply relatively high degree of inertia.

The similar result is obtained by Bihan, Sahuc (2002) who examine parameter uncertainty and find that for the model of euro area ‘Brainard’ principle holds. Similar result is obtained by Zcovic, Wieland, Rustem (2005) in spite of min-max criterion applied.

Kuester, Wieland (2008) search for the robust policy using a hybrid approach which combines Bayesian and min-max features. The weights attached to the competitive euro models are computed according to the Bayesian principle but the extra weights are added to the worst possible variants.

All the papers concerning robust policy characteristics in the euro zone deal with area-wide models. But Benigno (2004) showed that terms of trade between different countries also matter and thus must be in the optimal policy construction. In our paper we illustrate a robust policy construction for the two-country model of monetary union.

Chapter 2. Reference model of monetary union

In this paper we assume a unique central bank that decides for the monetary policy in a two-country currency union. This bank has in its possession a single micro-founded model with sticky prices that is taken as reference but there are some doubts concerning its quality, thus the monetary authority tackles with a model uncertainty problem. So, the central bank needs to construct not simply an optimal policy for this model but a robust policy for the possible misspecification.

Possible model distortions are introduced as choices of some malevolent agent which aims at the central bank's losses maximization. So the main task is a resolution of min-max problem by means of robust control techniques initiated by Hansen, Sargent (2001).

In this paper we apply the two-country optimizing model with sticky prices described in Benigno (2004). The difference of this paper from the base model of Benigno is the choice of α - parameter of price inertia. While Benigno doesn't specify its value for the future calculations, in our analysis we need to have some specific numbers. So on a base of European data in divide countries of EMU in two regions according to their price rigidity and compute the parameter α for each of regions. Then these values are used to derive the robust monetary policy for the union.

The whole derivation of the final equations is presented in the **Appendix A** and here we introduce only the main features of the model.

The currency union consists of two countries or regions (H and F). The population of this union represents a unit-continuum where agents from [0, n) interval belong to the H-country and the rest [n, 1] are F-country's inhabitants.

Each country has an independent local government, which determines fiscal policy in a corresponding region. Monetary measures are defined by a single central bank.

Each agent is simultaneously producer of a single differentiated good and consumer of all good types manufactured in the union. So there is an inter-regional trade while migration of labor force is absent. Number of goods, produced in the H region, equals to n, so this parameter represents also an economic size of this region or the share of union GDP produced in the region H.

2.1 Consumer problem

$$\max U_t^j = E_t\left\{\sum_{s=t}^{\infty}\beta^{s-t}\left[U\left(C_s^j\right)+L\left(\frac{M_s^j}{P_t^i};\varepsilon_s^i\right)-V\left(y_s^j;z_s^i\right)\right]\right\} \quad (1)$$

$$\begin{aligned} s.t.E_t\left\{q_t^i B_t^{i,j}\right\}+\frac{B_t^j}{P_t\left(1+R_t\right)}+\frac{M_t^j}{P_t}\le \\ B_{t-1}^{j,i}+\frac{M_{t-1}^j+B_{t-1}^j}{P_t}+\left(1-\tau^i\right)\frac{p_t\left(j\right)y_t\left(j\right)}{P_t\left(1+R_t\right)}-C_t^j+\frac{Q_t^{i,j}}{P_t}\forall t \end{aligned} \quad (2)$$

where $j\in\left[0,1\right]$ is agent's index, i - country's index (H or F). β - intertemporal discount rate. Utility function of agent j at period s positively depends on his consumption C_s^j, his stock of real money balances $\frac{M_s^j}{P_s^i}$ (this can be interpreted as some liquidity preferences of agents) and negatively on the $j's$ supply of a differentiated good (y_s^j), as y_s^j is a function of labor hours and a term $V\left(\bullet\right)$ represents labor disutility.

ε_s^i is a country-specific liquidity preference shock, while z_s^i represents a productivity shock in country i.

$E_t X_{t+k}$ stands for the expected in the period t value of variable X in the period $t+k$; E_t is an operator of rational expectations.

Every agent consumes home and foreign good bundles, which are substitutes. So index C_t^j is a composition of consumption indices for home and foreign goods:

$$C^j = \frac{\left(C_H^j\right)^n \left(C_F^j\right)^{1-n}}{n^n \left(1-n\right)^{1-n}} \tag{3}$$

Within each bundle distinct products are substitutes with an elasticity of substitution σ. Indices of j agent's consumption of home and foreign goods are represented by the following relations:

$$C_H^j = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c^j\left(h\right)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{1-\sigma}} \tag{4}$$

$$C_F^j = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c^j\left(f\right)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{1-\sigma}} \tag{5}$$

where $c_t^j(h)$ is a quantity of a good $h \in [0,n)$ from the H country consumed by the j agent and $c_t^j(f)$ is the same for the good $f \in [n,1]$ produced in the F region.

Corresponding price indices are:

Region i's price index: $P^i \equiv \left(P_H^i\right)^n \left(P_F^i\right)^{1-n}$

We also introduce terms of trade in the following way: $T_t = \frac{P_t^F}{P_t^H}$.

Price indices of H and F bundles sold on the i region's market:

$$P_H^i \equiv \left[\left(\frac{1}{n} \right) \int_0^n p^i(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}}, \quad P_F^i \equiv \left[\left(\frac{1}{1-n} \right) \int_n^1 p^i(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}} \tag{6}$$

where $p^i(h)$ is a price of a good h sold in the region i, the same is assumed for the good f. Under assumption of zero transaction costs every good j must be sold at equal prices in both regions, or $p^H(j) = p^F(j) \forall j$.

Consumer's budget constraint (2) includes:

$B_t^{j,i}$	the real value at time t of the agent j's portfolio of contingent securities issued in region i and denominated in units of the consumption-based price index with one-period maturity
q_t^i	the vector of the security prices
B_t^j	the agent j's holding of the nominal one-period non-contingent bond denominated in the union currency
R_t	the nominal interest rate, monetary authority's instrument
τ^i	a regional proportional tax on nominal income
$Q_t^{j,i}$	nominal lump-sum transfers from the fiscal authority of region i to the agent j

2.2 Fiscal policy

Fiscal policies are determined by the local governments in each country separately. Each government collects taxes τ^i, determines transfers $Q_t^{j,i}$ and purchases only products produced in its own country. We don't deal with the problem of fiscal policy determination, so we don't solve any programs for the transfers or taxes. We only take these values as given under assumption that the intertemporal budget constraint is held:

$$E\sum_{t=0}^{\infty}\frac{\tau^i Y_t^i - Q_t^i - G_t^i}{\prod_{s=0}^{t}(1+R_s)} = 0$$

So, the whole sums of government expenditures are given by G_t^H and G_t^F respectively.

It should be noted that we assume a central bank which takes the fiscal measures as given and autonomous and so there is no space for the fiscal-monetary interactions and even for the optimal construction of fiscal policy. But one can incorporate an additional assumptions concerning the fiscal policy and even the relations between fiscal and monetary authorities interactions to expand an analysis to the optimal construction of fiscal measures or taking into account game aspects of policy determination. For example, the similar analysis can be made for the model of Beetsma, Jensen (2005), where policy interactions can be taken into account.

2.3 Total demand

The total demand for each good is a sum of all private agents' demands and the demand of the corresponding government:

$$\begin{aligned} y^d(h) &= \left(\frac{p(h)}{P_H}\right)^{-\sigma}\left[T^{1-n}\int_0^1 C^j dj + G^H\right] \\ y^d(f) &= \left(\frac{p(f)}{P_F}\right)^{-\sigma}\left[T^{-n}\int_0^1 C^j dj + G^F\right] \end{aligned} \tag{7}$$

Or, naming a union-wide index of consumption

$$C^{W} = \int_0^1 C^{j} dj$$

$$y^{d}(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma}\left[T^{1-n}C^{W} + G^{H}\right]$$
$$y^{d}(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma}\left[T^{-n}C^{W} + G^{F}\right] \tag{8}$$

Or on the region level:

$$Y^{H} = \frac{\left[T^{1-n}C^{W} + G^{H}\right]}{P^{H}}$$
$$Y^{F} = \frac{\left[T^{-n}C^{W} + G^{F}\right]}{P^{F}} \tag{9}$$

2.4 Firms

Producers in the model are monopolists on their products' markets. They set prices according to the Calvo (1983). Each seller faces a probability $(1-\alpha)$ of adjusting his price.

This type of nominal rigidity implies that dynamics of price level in every country i can de represented by the following link:

$$P_{J,t}^{1-\sigma} = \alpha^{J} P_{J,t-1}^{1-\sigma} + \left(1-\alpha^{J}\right)\tilde{p}_t\left(i\right)^{1-\sigma} \tag{10}$$

So, the term α can be interpreted as an extent of nominal rigidity in the corresponding region. With a help of this parameter heterogeneity between regions can be introduced into analysis (like in Brissimis, Skotida, 2008).

2.5 Deterministic steady state

Unfortunately, this model cannot be solved in the closed-form. Consequently, the analysis is made in the terms of deviations from the deterministic equilibrium. Deterministic steady state without any shocks and zero inflation rate is taken as a base for the analysis. In the following analysis a notation $\bar{X}$ corresponds to the value of variable X in the deterministic steady state. So we assume that

$$\bar{\varepsilon} = \bar{G} = \bar{z} = \bar{\pi} = 0 \text{ and } \begin{matrix} \bar{C} = const \\ \bar{T} = const \\ \bar{Y} = const \end{matrix}$$

We simplify the future analysis restricting our attention only to the case of equal tax rates $\tau^H = \tau^F$ and

$\bar{T} = 1$ and $\bar{Y}^F = \bar{Y}^H = \bar{C}$

2.6 Equilibrium with sticky prices

In the following analysis only equilibria which are close to the deterministic case described earlier are discussed. So the solution can be made in terms of small fluctuations from the deterministic case and an appropriate log-linearization of the model is usually conducted.

In the log-linearized equilibrium conditions by $\tilde{X}_t$ we denote the deviation of logarithmic of variable X from the steady state when prices are flexible, while $\hat{X}_t$ is the same deviation under sticky prices. In other words

$$\tilde{X} \equiv \ln \frac{X(fl.prices)}{\bar{X}}$$
$$\hat{X} \equiv \ln \frac{X(st.prices)}{\bar{X}}$$

Differences between these two deviations are named by small letters:

$$x_t = \hat{X}_t - \tilde{X}_t$$

In some cases it is useful to determine a union-wide variable X^W, which represents a weighted average of specific countries values: $X^W = nX^H + (1-n)X^F$, or a relative variable, which is simply the difference between some values for different countries: $X^R \equiv X^F - X^H$.

So, the law of motion of the economy is presented by the following equations:

$$E_t\hat{C}^W_{t+1} = \hat{C}^W_t + \rho^{-1}\left(\hat{R}_t - E_t\pi^W_{t+1}\right) \tag{11}$$

$$\hat{Y}^H_t = (1-n)\hat{T}_t + \hat{C}^W_t + g^H_t \tag{12}$$

$$\hat{Y}^F_t = -n\hat{T}_t + \hat{C}^W_t + g^F_t \tag{13}$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi^F_t - \pi^H_t \tag{14}$$

$$\pi^H_t = (1-n)k^H_T\left(\hat{T}_t - \tilde{T}_t\right) + k^H_C y^W_t + \beta E_t\pi^H_{t+1} \tag{15}$$

$$\pi^F_t = -nk^F_T\left(\hat{T}_t - \tilde{T}_t\right) + k^F_C y^W_t + \beta E_t\pi^F_{t+1} \tag{16}$$

where $k^i_C = \left[\frac{(1-\alpha^i\beta)(1-\alpha^i)}{\alpha^i}\right]\left[\frac{\rho+\eta}{1+\sigma\eta}\right]$, $k^i_T = k^i_C\left[\frac{1+\eta}{\rho+\eta}\right]$ and $\rho \equiv -\frac{U_{CC}\bar{C}}{U_C}$, $\eta \equiv \frac{V_{yy}\bar{C}}{V_y}$, $\bar{Y}_t \equiv -\frac{V_{Y\varepsilon}\varepsilon}{V_{yy}\bar{C}}$

The first three equations determine the relations between consumption, government spending, output gap, expected future inflation and the value of nominal interest rate. To simplify future calculations we assume that a fiscal policy is autonomous from any monetary decisions and is treated as known with certainty, so $g^i_{t+1} = E_t g^i_{t+1}$. In this case we can rearrange equations (11-13) into (17):

$$E_t y_{t+1}^W = y_t^W + \frac{1}{\rho}\left[\hat{R} - E_t \pi_{t+1}^W\right] \quad (17)$$

This equation is a usual form of IS-curve for the hole currency area, which determines an output gap, which depends positively on its future expected value, expected future inflation and negatively – on the nominal interest rate.

Equations (15-16) describe supply side of the union economy and stand for the New Keynesian Phillips curves. From these equations inflation rates in the union regions are determined by the union-wide output gap, expectations of future inflation and the union terms of trade. Usually inside terms of trade are omitted from such type of analysis, based on the union-wide models, so the optimal policy is constructed for the aggregate levels of inflation and output. But from the equations (15-16) is clear that taking into account trade flows between regions can be important for policy constructing. For this very purpose we offer a more-than-one-country model for analyzing robustness characteristics.

Equation (14) goes explicitly from the definition of terms of trade and represents a dynamic of this variable determined by its past value and the current inflation rates in both countries.

So the central bank's task has to set a nominal rate minimizing its welfare function subject to the equations (14-17). Thus, in the model there are 4 forward-looking variables (T_t π_t^H, π_t^F, y_t^W) and one policy control variable (R_t).

2.7 Welfare criterion

We assume that central bank is benevolent and tries to maximize social welfare W given by $W = E_0\left\{\sum_{t-0}^{+\infty} \beta^t w_t\right\}$ - an expected weighted sum of all future values of average utility in the union $w_t \equiv U(C_t) - \int_0^1 V\left(y_t(j), z_t^i\right) dj$.

The second-order approximation of the welfare function is based on the Beetsma (2005) and gives the following form of welfare criterion (see Appendix for details):

$W = -E_0 \left\{ \sum_{t-0}^{+\infty} \beta^t L_t \right\}$, where one period loss is given by

$$L_t = \Lambda \left[y_t^W \right]^2 + n(1-n)\Gamma \left[\hat{T}_t - \tilde{T}_t \right]^2 + \gamma_H \left(\pi_t^H \right)^2 + \gamma_F \left(\pi_t^F \right)^2 + t.i.p + o\left(\|\varepsilon\|^3 \right) \quad (18)$$

Where t.i.p. stands for the terms independent from policy and the last part of this relation $\|\varepsilon\|^3$ includes all parameters of more-than-second order of approximation

and $$\Lambda = \frac{1/\sigma}{n/k_C^H + (1-n)/k_C^F} \quad (19)$$

$$\Gamma = \frac{(1+\eta)/\sigma}{(n/k_C^H + (1-n)/k_C^F)(\rho+\eta)} \quad (20)$$

$$\gamma_H = \frac{n/k_C^H}{n/k_C^H + (1-n)/k_C^F} \quad (21)$$

$$\gamma_F = \frac{(1-n)/k_C^F}{n/k_C^H + (1-n)/k_C^F} \quad (22)$$

We can see that in this model the loss function L_t has a usual quadratic form. But contrary to the seminal representation where the loss value is defined by the output gap and inflation rates, terms of trade are also present in this function. It can be explained by the reference model structure, as the Phillips curves structure includes trade between regions and takes into account not only a currency area as a whole but also relationships within a union.

2.8 Calibration

In our calibration we partly follow Benigno (2004). Thus we chose a value of elasticity of producing differentiated goods η equal to 0.67. Parameter of intertemporal substitution β equals to 0.99. The degree of monopolistic competition σ is taken to be equal to 7.66. Risk-aversion coefficient ρ is assumed to be 1/6.

Moreover, it is assumed that the shock $\tilde{T}_t$ follows the auto-regressive process of the kind $\tilde{T}_t = 0.95\tilde{T}_{t-1} + \varepsilon_t$, where the term ε_t stands for the white-noise process with variance 0.0086.

The main difficulty concerns the choice of price inertia parameter α^i. In this section we don't follow Begnino (2004) who allows these parameters to vary across a wide range of possible values. In contrast, our choice of these values is based on the estimations of by Vermeulen at al (2007).

***Table 1 Frequency of price changes and country weights in Euro GDP** (%)*

	(2)	(3)
Belgium	0.24	4.0
France	0.25	22.3
Germany	0.22	34.3
Italy	0.15	17.5
Portugal	0.23	1.5
Spain	0.21	8.5
Euro area	0.22	

Note:

(2): Frequency of price changes $(1-\alpha)$ *Source: Vermeulen at al (2007)*

(3): Country weight in Euro GDP (%)

We take a frequency of price changes as a proxy for the probability to change a price $(1-\alpha)$.

We divide countries in two groups according to the following scheme: if a frequency of price changes is lower or equal to 0.22 (average frequency for the union), the country belongs to the H region. If this frequency is higher than 0.22, the country is a part of F region. Thus for the countries with available data region H consists of Germany, Spain and Italy, while region F consists of France, Belgium and Portugal.

According to the Table 1 H region produces around 70% of union output, so we calibrate the region size to the 0.7. According to the corresponding weights we assume that an average frequency of price change in the region H equals to 0.17, while the same ratio for the region F comes to 0.23.

These values correspond to the model parameters $\alpha^H = 0.83$ and $\alpha^F = 0.77$

Now we compute the corresponding weights in the welfare function:

$\Lambda = 0.0058$

$\Gamma = 0.0024$

$$\gamma_H = 0.82$$
$$\gamma_F = 1 - \gamma_H = 0.18$$

So, we can see that the weight attached to the stickier region (H) in the loss function is higher than a weight of a more flexible one which corresponds to the main findings of papers studying questions of nominal inertia in the models of euro-area (see, for example, Adalid, Coenen, McAdam, Siviero (2005)).

2.9 Central bank optimization problem

According to the made assumptions and the calibration from the previous section we can compute values of the main parameters of the model. So, the central bank's program can be rewritten as:

$$\max E_0\left\{\sum_{t-0}^{+\infty}\beta^t L_t\right\}$$
$$L_t = 0.0058\left[y_t^W\right]^2 + 0.0024\left[\hat{T}_t - \tilde{T}_t\right]^2 + 0.82\left(\pi_t^H\right)^2 + 0.18\left(\pi_t^F\right)^2 \tag{23}$$

$$s.t.\begin{cases} E_t y_{t+1}^W = y_t^W + 6\left[R - E_t\pi_{t+1}^W\right] \\ \pi_t^H = 0.03\left(\hat{T}_t - \tilde{T}_t\right) + 0.005 y_t^W + 0.99 E_t \pi_{t+1}^H \\ \pi_t^F = -0.014\left(\hat{T}_t - \tilde{T}_t\right) + 0.01 y_t^W + 0.99 E_t \pi_{t+1}^F \\ \hat{T}_t - \tilde{T}_t = \hat{T}_{t-1} - \tilde{T}_{t-1} + \pi_t^F - \pi_t^H + e_t \\ e_t = 0.95 e_{t-1} + \varepsilon_t \end{cases}$$

This specification represents the problem of search for the optimal monetary policy on the base of one reference model. From here is seen that the main concern of the central bank is inflation in the region with the highest price stickiness.

In the following sections we discuss a policy of central bank which takes this model as reference one but fears that reality can be rather different from this base framework. In the next section we state the program of robust policy construction and then we find parameters of the monetary policy under model uncertainty.

2.10 State space form of the model

For the convenience of further analysis the program (23) can be rewritten in the usual state space form.

For this we firstly expand the low of motion of terms of trade to the period $t+1$ and take the rational expectation:

$$\hat{T}_{t+1} - \tilde{T}_{t+1} = \hat{T}_t - \tilde{T}_t + \pi_{t+1}^F - \pi_{t+1}^H + 0.95 e_{t+1}$$
$$E_t\left(\hat{T}_{t+1} - \tilde{T}_{t+1}\right) = \hat{T}_t - \tilde{T}_t + E_t\pi_{t+1}^F - E_t\pi_{t+1}^H + 0.95 e_t \tag{24}$$

From this, using (14), we obtain:

$$E_t\left(\hat{T}_{t+1}-\tilde{T}_{t+1}\right)=\hat{T}_t-\tilde{T}_t+1.01\pi_t^F+1.01*0.014\left(\hat{T}_t-\tilde{T}_t\right)-1.01*0.01y_t^W$$
$$-1.01\pi_t^H+1.01*0.03\left(\hat{T}_t-\tilde{T}_t\right)+1.01*0.005y_t^W+0.95e_t \qquad (25)$$
$$=1.044\left(\hat{T}_t-\tilde{T}\right)+1.01\pi_t^F-1.01\pi_t^H-0.01515y_t^W+0.95e_t$$

From other equations of (23) we obtain expressions for the all expectations of forward-looking variables:

$$E_t y_{t+1}^W=-0.012\left(\hat{T}_t-\tilde{T}\right)+1.04y_t^W-4.24\pi_t^H-1.82\pi_t^F+6R$$
$$E_t\pi_{t+1}^H=-0.003\left(\hat{T}_t-\tilde{T}\right)-0.005y_t^W+1.01\pi_t^H \qquad (26)$$
$$E_t\pi_{t+1}^F=0.014\left(\hat{T}_t-\tilde{T}\right)-0.0098y_t^W+1.01\pi_t^F$$

So the state space form of our model is the following:

$$e_{t+1}=0.95e_t+\varepsilon_{t+1}$$
$$E_t\left(\hat{T}_{t+1}-\tilde{T}_{t+1}\right)=1.044\left(\hat{T}_t-\tilde{T}\right)-0.01515y_t^W-1.01\pi_t^H+1.01\pi_t^F+0.95e_t$$
$$E_t y_{t+1}^W=-0.012\left(\hat{T}_t-\tilde{T}\right)+1.04y_t^W-4.24\pi_t^H-1.82\pi_t^F+6R \qquad (27)$$
$$E_t\pi_{t+1}^H=-0.003\left(\hat{T}_t-\tilde{T}\right)-0.005y_t^W+1.01\pi_t^H$$
$$E_t\pi_{t+1}^F=0.014\left(\hat{T}_t-\tilde{T}\right)-0.0098y_t^W+1.01\pi_t^F$$

Or in the brief form:

$$\begin{bmatrix} e_{t+1} \\ E_t z_{t+1} \end{bmatrix} = A\begin{bmatrix} e_t \\ z_t \end{bmatrix} + BR + C\varepsilon_{t+1} \qquad (28)$$

Where $z_t=\begin{bmatrix} \tilde{T}_t-\hat{T}_t \\ y_t^W \\ \pi_t^H \\ \pi_t^F \end{bmatrix}$ - a vector of forward-looking variables, $E_t z_{t+1}$ is an expected in the period t future value of vector z. A is matrix of size 5×5 of corresponding coefficients from (27). $B=\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $C=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, showing that only a predetermined variable e_t is allowed to be affected be shocks.

Minimizing loss function can be also rewritten in the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t (x'_t Q x_t) \tag{29}$$

where $x_t = \begin{bmatrix} e_t \\ z_t \end{bmatrix}$ and Q is matrix of size 5×5:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & n(1-n)\Gamma & 0 & 0 & 0 \\ 0 & 0 & \Lambda & 0 & 0 \\ 0 & 0 & 0 & \gamma_H & 0 \\ 0 & 0 & 0 & 0 & 1-\gamma_H \end{bmatrix} \tag{30}$$

So, the problem (53) can be rewritten as:

$$\begin{aligned} &\min_R E_0 \sum_{t=0}^{\infty} \beta^t (x'_t Q x_t) \\ &s.t. \begin{bmatrix} e_{t+1} \\ E_t z_{t+1} \end{bmatrix} = A \begin{bmatrix} e_t \\ z_t \end{bmatrix} + BR + C\varepsilon_{t+1} \end{aligned} \tag{31}$$

Chapter 3. Robust control specification

3.1 Model uncertainty specification

In modeling the robust program we implement a Hansen-Sargent's approach which is also called robust control.

We assume that the central bank has (57) as a reference model describing the economy. But at the same time this monetary authority fears that this reference construction doesn't correspond to the real state of nature properly – there is a risk of misspecification. In other words some perturbations of modeled economy from the real one are allowed. The possible sources of these perturbations are some unknown variables or processes.

To account this possible misspecification monetary authority analyses only a class of alternative models, which cannot be distinguished from the reference one with help of statistical methods. In other words a set of possible perturbations is limited - it includes only such perturbations, which with some fixed probability will not be discovered. The reason to impose this restriction on the possible misspecification is quite clear – for great perturbations, when the real economy differs considerably from the reference one, there is no any reason to take any decision on the base of this concrete model and adaptation of the model to reality is needed.

So the task for the central bank is to construct not an optimal for this model policy but a policy, which performs reasonably well even if there is any perturbation. For this purpose a min-max criterion is applied – a robust policy is such one that produces the smallest losses in the case of the worst model perturbation.

For this worst-case criterion we can suppose that perturbations from the reference model take the form of some additional shocks v_{t+s} which are added to the standard ε_{t+s} in the model (57) and are induced by so called 'malevolent nature' or 'evil agent', which tries to maximize losses of the central bank. So the robust program can be represented by simultaneous two-agents' game, where the evil agents chooses a perturbation from the reference model υ_{t+s} , the central bank decides the values of its instrument – interest rate. The set of possible perturbations is modeled by the restriction on the evil agent's measures υ_{t+s}.

In general there are three possibilities for the central bank to interact with private agents who form expectations in the economy:

1) Commitment to the optimal policy
2) Commitment to some simple policy rule (for example, seminal Taylor rule)
3) Discretion

Here we assume commitment case. In this situation several assumptions concerning the private perceptions of the possible misspecification can be made:

1) Population has the same fear of misspecification as the central bank
2) Population is more cautious than monetary authority
3) Population is more careless

In our analysis we consider the first simplest case, when population and central bank have the same reference model and the same ideas concerning possible misspecifications. In the rest two cases the population's reference model and its set of model deviations should be specified.

Moreover, we need decide if the evil agent commits or not. Following Hansen, Sargent we analyze a case when both optimizing agent commit. The main reason for this assumption is the fact that our malevolent agent is only a metaphor used to construct and solve the min-max problem. The central bank under uncertainty derives its policy as if this evil nature existed. That's why we suppose that evil agent arrives and solves its program only when the central bank optimizes.

3.2. Constraint robust control problem

As we've already discussed in the previous section, there is some restriction on the evil agent's measures. We assume the following intertemporal constraint of the malevolent nature:

$$E_0 \sum_{t=0}^{\infty} \beta^t \upsilon_{t+1}' \upsilon_{t+1} \leq \eta \tag{32}$$

where υ_t is a vector of disturbances initiated by the malevolent nature in the economy. And η is a possible total extent of model misspecification. In other words, (32) represents the allowed set of perturbations discussed earlier. If we imagine all possible perturbed models as a cloud around the reference one and η is its radius.

This representation of possible model set corresponds to the *constraint robust control problem* of the central bank in the following form:

$$\begin{aligned} &\min_R \max_\upsilon E_0 \sum_{t=0}^{\infty} \beta^t (x'_t Q x_t) \\ &s.t. \begin{bmatrix} e_{t+1} \\ E_t z_{t+1} \end{bmatrix} = A \begin{bmatrix} e_t \\ z_t \end{bmatrix} + BR + C\left(\varepsilon_{t+1} + \upsilon_{t+1}\right) \\ &E_0 \sum_{t=0}^{\infty} \beta^t \upsilon_{t+1}' \upsilon_{t+1} \leq \eta \end{aligned} \tag{33}$$

Size of possible perturbations, η is defined by the central bank's fear of misspecification: higher fear allows higher possibilities of the malevolent agent and bigger deviations of the real economy from the reference one and is reflected by higher η in (32). Higher fear of misspecification at the same time stands for higher *preferences for the robustness.*

For example, if we assume that the central bank has no any fear of misspecification or it has no preferences for robustness, $\eta = 0$, and we obtain the program equivalent to (31).

Notice that we analyze policy decisions for the moment t and evil actions for the moment $t+1$. This means that the value of this strategic shock is known at the time t. (Alternative assumption supposes that the evil agents' actions are not known at the previous moment or are hidden by the stochastic errors of this period).

For a given law of motion of economy the total welfare depends only on the policymaker's and malevolent agent's steps and the economic shocks. So we rewrite problem (33) in the following form:

$$\begin{aligned} &\min_R \max_\upsilon \sum_{t=0}^{\infty} \beta^t L_t(R, \upsilon) \\ &\sum_{t=0}^{\infty} \beta^t D_{t+1}(\upsilon) \le \eta \end{aligned} \qquad (34)$$

where R is a sequence of central bank's decisions and υ is a sequence of strategic shocks initiated by malevolent agent: $D_{t+1}(\upsilon) = \upsilon_{t+1}'\upsilon_{t+1}$

3.3. Multiplier or penalty robust control problem

Another possible representation of robust control program is so called multiplier or penalty problem. The program (33) takes the following form:

$$\begin{aligned} &\min_R \max_\upsilon E_0 \sum_{t=0}^{\infty} \beta^t (x'_t Q x_t - \theta \upsilon_{t+1}'\upsilon_{t+1}) \\ &s.t. \begin{bmatrix} e_{t+1} \\ E_t z_{t+1} \end{bmatrix} = A \begin{bmatrix} e_t \\ z_t \end{bmatrix} + BR + C(\varepsilon_{t+1} + \upsilon_{t+1}) \end{aligned} \qquad (35)$$

where θ represents a set of possible deviations of reference model from the real economy. When this term θ is low this set is very large, when θ is high, there are only very limited distortions. The case of $\theta \to \infty$ corresponds to the usual optimal control program when no account of possible misspecifications is taken.

Rewriting it in the same form as (34), we obtain:

$$\min_{R} \max_{\upsilon} \sum_{t=0}^{\infty} \beta^{t} L_{t}(R,\upsilon) - \theta \sum_{t=0}^{\infty} \beta^{t} D_{t+1}(\upsilon) \qquad (36)$$

3.4. Relations between two robust programs

In this section we derive relations between two robust programs presented earlier.

First of all we rename the overall losses of the constraint robust program as

$$\begin{aligned} K(\eta) &= \min_{R} \max_{\upsilon \in \mathrm{H}(\eta)} \sum_{t=0}^{\infty} \beta^{t} L_{t}(R,\upsilon) \\ \mathrm{H}(\eta) &= \left\{ \upsilon : \sum_{t=0}^{\infty} \beta^{t} D_{t+1}(\upsilon) \le \eta \right\} \end{aligned} \qquad (37)$$

The resulting level of optimized function for the penalty problem is named:

$$V(\theta) = \min_{R} \max_{\upsilon} \sum_{t=0}^{\infty} \beta^{t} L_{t}(R,\upsilon) - \theta \sum_{t=0}^{\infty} \beta^{t} D_{t+1}(\upsilon) \qquad (38)$$

Now we reformulate constraint robust problem (34) in the Lagrangian form:

$$\min_{R} \max_{\upsilon} \min_{\theta} \sum_{t=0}^{\infty} \beta^{t} L_{t}(R,\upsilon) - \theta \left[\sum_{t=0}^{\infty} \beta^{t} D_{t+1}(\upsilon) - \eta \right] \qquad (39),$$

where θ is a Lagrange multiplier for the malevolent agent's constraint.

Changing the order of optimization (we can do it if we consider only cases when equilibrium exists) we obtain

$$\begin{aligned} &\min_{\theta} \min_{R} \max_{\upsilon} \sum_{t=0}^{\infty} \beta^{t} L_{t}(R,\upsilon) - \theta \left[\sum_{t=0}^{\infty} \beta^{t} D_{t+1}(\upsilon) - \eta \right] = \\ &= \min_{\theta} V(\theta) + \theta\eta \end{aligned} \qquad (40)$$

From (40) it is seen that for a given θ the last term of this expression ($\theta\eta$) doesn't influence the choice of R and υ. So the following statement is true:

Statement 1. For the given η R^* and υ^* are solutions of constraint robust program. Then exists a θ^* (equal to the optimal Lagrange multiplier from the constraint robust program) such that the corresponding penalty robust program has the same solution and

$$K\left(\eta^*\right)=\min_{\theta} V\left(\theta\right)+\theta\eta^* \tag{41}$$

Now we consider that for some θ^* the penalty program is resolved. From (41) it follows that for any η and θ $K\left(\eta\right)\le V\left(\theta\right)+\theta\eta$, so for any η

$$V\left(\theta^*\right)\ge K\left(\eta\right)-\theta^*\eta \tag{42}$$

From here we conclude that

$$V\left(\theta^*\right)\ge \max_{\eta} K\left(\eta\right)-\theta^*\eta \tag{43}$$

Or, using the definition of $K\left(\eta\right)$,

$$V\left(\theta^*\right)\ge \max_{\eta}\left[\min_{R}\max_{\upsilon\in \mathrm{H}(\eta)}\left[\sum_{t=0}^{\infty}\beta^t L_t(R,\upsilon)\right]-\theta^*\eta\right] \tag{44}$$

We maximize the right side of the expression if we put $\sum_{t=0}^{\infty}\beta^t D_{t+1}(\upsilon)$ instead of η, as the constraint in (64) holds.

$$\begin{aligned}&\max_{\eta}\min_{R}\max_{\upsilon\in \mathrm{H}(\eta)}\sum_{t=0}^{\infty}\beta^t L_t(R,\upsilon)-\theta^*\sum_{t=0}^{\infty}\beta^t D_{t+1}(\upsilon)=\\&\max_{\eta}\min_{R}\max_{\upsilon\in \mathrm{H}(\eta)}\sum_{t=0}^{\infty}\beta^t L_t(R,\upsilon)-\theta^*\eta\end{aligned} \tag{45}$$

We rewrite the penalty robust program

$$V\left(\theta^*\right) = \min_R \max_\upsilon \sum_{t=0}^{\infty} \beta^t L_t(R,\upsilon) - \theta^* \sum_{t=0}^{\infty} \beta^t D_{t+1}(\upsilon) =$$

$$= \max_\upsilon \min_R \sum_{t=0}^{\infty} \beta^t L_t(R,\upsilon) - \theta^* \sum_{t=0}^{\infty} \beta^t D_{t+1}(\upsilon) = \quad (46)$$

$$= \max_\eta \max_{\upsilon \in \mathrm{H}(\eta)} \min_R \sum_{t=0}^{\infty} \beta^t L_t(R,\upsilon) - \theta^* \sum_{t=0}^{\infty} \beta^t D_{t+1}(\upsilon)$$

Taking into account (45) and (46) we rewrite the definition of $V\left(\theta^*\right)$:

$$V\left(\theta^*\right) = \max_\eta \max_{\upsilon \in \mathrm{H}(\eta)} \min_R \sum_{t=0}^{\infty} \beta^t L_t(R,\upsilon) - \theta^* \eta = \quad (47)$$

$$= \max_\eta K\left(\eta\right) - \theta^* \eta$$

For given η the last part of expression, $\theta^*\eta$ doesn't influence the choice of R and υ, so the solution of penalty robust corresponds to the solution of the constrain robust program.

Statement 2. For some θ^* R^* and υ^* are solutions of penalty robust program. Then the constraint robust program with $\eta^* = \sum_{t=0}^{\infty} \beta^t D_{t+1}(\upsilon^*)$ has the same solution.

So, we've showed that two robust programs are tightly related. The size of possible uncertainty is given by η in the constraint robust program and by θ in the penalty program. These two measures are interconnected in the way presented by Statements 1 and 2. From (41) and (47) we can see that higher θ means lower corresponding value of η and vice versa. So when we speak about higher uncertainty we mean higher η and lower θ.

3.5. Robust control program solution

In this and the next sections we tackle with the penalty robust program remembering that all these results can be easily represented as the solution of corresponding constraint problem. This problem is solved numerically and here we present some technical details of solution proposed in Giordani, Soderlind (2004).

Giordani, Soderlind (2004) assume the following robust policy program:

$$\min_{\{u\}_0^\infty} \max_{\{v\}} E_0 \sum_{t=0}^{\infty} \beta^t (x'_t Qx_t + u_t{}' Ru_t + 2x_t^{'} Uu_t - \theta v_{t+1}{}' v_{t+1})$$
$$s.t. \begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + Bu_t + C\left(\varepsilon_{t+1} + v_{t+1}\right)$$
$$C = \begin{bmatrix} C_1 \\ O_{n_2 \times n_1} \end{bmatrix} \qquad \text{, where}$$

x_{1t} is a $n_1 \times 1$ vector of predetermined or backward-looking variables, x_{2t} is $n_2 \times 1$ vector of forward-looking variables, vector $x_t^{'}$ is a vector of all variables of interest equal to $\left(x_{1t}^{'}, x_{2t}^{'}\right)$. u_t is a vector of central bank instruments of size $(k \times 1)$. ε_{t+1} is an iid $n_1 \times 1$ vector of shocks and v_{t+1} is an $n_1 \times 1$ vector of strategic shocks, initiated by evil agent. Q and R are symmetric matrices.

Our program (35) is a private case of such of Giordani, Soderlind (2004) with zero-matrix and the sole policy instrument – interest rate, so $k = 1$. $n_1 = 1$, as we have only one predetermined variable e_t and there are 4 forward-looking variables in the model, so our vector of forward-looking variables is of size $n_2 = 4$. Because the core structure of our model and such proposed in Giordani, Soderlind (2004) are identical we now pass to the solution algorithm of our specific program.

First of all we define the error of expectations for the period $t+1$:

$$\xi_{t+1}^z = z_{t+1} - E_t z_{t+1} \qquad (48)$$

Then we note the total error of the model as:

$$\xi_{t+1} = \begin{bmatrix} \varepsilon_{t+1} \\ \xi_{t+1}^{z} \end{bmatrix}_{5\times 1} \tag{49}$$

So the program (36) can be rewritten as follows:

$$\begin{aligned} &\min\max E_0 \sum_{t=0}^{\infty} \beta^t (x'_t Q x_t - \theta \upsilon_{t+1}{}' \upsilon_{t+1}) \\ &s.t. x_{t+1} = A x_t + BR + \xi_{t+1} + C\upsilon_{t+1} \end{aligned} \tag{50}$$

The Lagrangian of this representation is:

$$L_0 = E_0 \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} (x'_t Q x_t - \theta \upsilon_{t+1}{}' \upsilon_{t+1}) + \\ +2\rho'_{t+1}\left(A x_t + B R_t + \xi_{t+1} + C\upsilon_{t+1} - x_{t+1} \right) \end{bmatrix} \tag{51}$$

where ρ_{t+1} is a vector of size 5×1 of corresponding langrangian multipliers.

The first-order conditions with respect to ρ_{t+1}, x_t, R_t and υ_{t+1} are:

$$\begin{bmatrix} I_5 & O_{5\times1} & O_{5\times1} & O_{5\times5} \\ O_{5\times5} & O_{5\times1} & O_{5\times1} & \beta A'_{5\times5} \\ O_{1\times5} & O_{1\times1} & O_{1\times1} & B'_{1\times5} \\ O_{1\times5} & O_{1\times1} & O_{1\times1} & C'_{1\times5} \end{bmatrix} \begin{bmatrix} x_{t+1} \\ R_{t+1} \\ \upsilon_{t+2} \\ E_t \rho_{t+1} \end{bmatrix} = \begin{bmatrix} A_{5\times5} & B_{5\times1} & C_{5\times1} & O_{5\times5} \\ -\beta Q_{5\times1} & O_{5\times1} & O_{5\times1} & I_5 \\ O_{1\times5} & O_{1\times1} & O_{1\times1} & O_{1\times5} \\ O_{1\times5} & O_{1\times1} & \theta I_1 & O_{1\times5} \end{bmatrix} \begin{bmatrix} x_t \\ R_t \\ \upsilon_{t+1} \\ \rho_t \end{bmatrix} + \begin{bmatrix} \xi_{t+1} \\ O_{5\times1} \\ O_{1\times1} \\ O_{1\times1} \end{bmatrix} \tag{52}$$

where $O_{n\times k}$ stands for zero-matrix of $n \times k$ size.

Or if we rename the matrices of corresponding coefficients in (52) we obtain

$$G \begin{bmatrix} x_{t+1} \\ R_{t+1} \\ \upsilon_{t+2} \\ E_t \rho_{t+1} \end{bmatrix} = D \begin{bmatrix} x_t \\ R_t \\ \upsilon_{t+1} \\ \rho_t \end{bmatrix} + \begin{bmatrix} \xi_{t+1} \\ O_{5\times1} \\ O_{1\times1} \\ O_{1\times1} \end{bmatrix} \tag{53}$$

Then we take conditional expectation of (53) and noting that ρ_t^e is a shadow price corresponding to the predetermined variable e_t and ρ_t^z is a vector (4×1) of shadow prices for the z_t we obtain:

$$GE_t\begin{bmatrix} e_{t+1} \\ \rho^z_{t+1} \\ z_{t+1} \\ R_{t+1} \\ \upsilon_{t+2} \\ \rho^e_{t+1} \end{bmatrix} = D\begin{bmatrix} e_t \\ \rho^z_t \\ z_t \\ R_t \\ \upsilon_{t+1} \\ \rho^e_t \end{bmatrix}$$, or posing $k_t = \begin{bmatrix} e_t \\ \rho^z_t \end{bmatrix}$ and $\lambda_t = \begin{bmatrix} z_t \\ R_t \\ \upsilon_{t+1} \\ \rho^e_t \end{bmatrix}$ we obtain:

$$GE_t\begin{bmatrix} k_{t+1} \\ \lambda_{t+1} \end{bmatrix} = D\begin{bmatrix} k_t \\ \lambda_t \end{bmatrix} \quad (54)$$

Vector $k_t = \begin{bmatrix} e_t \\ \rho^z_t \end{bmatrix}$ has initial conditions: the first term is the predetermined variable with some initial condition e_0 and the second represents shadow prices for the forward-looking variables and thus can be chosen freely at the initial period to be equaled to zero, so that $\rho^z_0 = 0$.

Then we use decomposition proposed in the Giordani, Soderlind (2004).

Square matrices G and D can be represented by the following decomposition:

$$G = VSZ^H$$
$$D = VTZ^H$$
, when Z^H is a transpose of conjugate of Z.

Then, $V^H V = Z^H Z = I$ and S and T are upper triangular.

Hence, (54) can be rewritten as:

$$VSZ^H E_t\begin{bmatrix} k_{t+1} \\ \lambda_{t+1} \end{bmatrix} = VTZ^H\begin{bmatrix} k_t \\ \lambda_t \end{bmatrix} \quad (55)$$

Premultiply (55) by V^H

$$SZ^H E_t\begin{bmatrix} k_{t+1} \\ \lambda_{t+1} \end{bmatrix} = TZ^H\begin{bmatrix} k_t \\ \lambda_t \end{bmatrix} \quad (56)$$

As S and T are upper triangular, (56) can be rewritten so that at the first places stable solutions go. Stability of the solution is checked by the corresponding eigenvalue – its modulus should be less than 1. If we denote now

$$\begin{bmatrix}\theta_t\\ \delta_t\end{bmatrix}=Z^H\begin{bmatrix}k_t\\ \lambda_t\end{bmatrix}, \qquad (57)$$

where θ corresponds to the stable roots and δ - to the unstable ones, we obtain:

$SE_t\begin{bmatrix}\theta_{t+1}\\ \delta_{t+1}\end{bmatrix}=T\begin{bmatrix}\theta_t\\ \delta_t\end{bmatrix}$, or, as S and T are upper triangular,

$$\begin{bmatrix}S_{\theta\theta} & S_{\theta\delta}\\ \text{O} & S_{\delta\delta}\end{bmatrix}E_t\begin{bmatrix}\theta_{t+1}\\ \delta_{t+1}\end{bmatrix}=\begin{bmatrix}T_{\theta\theta} & T_{\theta\delta}\\ \text{O} & T_{\delta\delta}\end{bmatrix}\begin{bmatrix}\theta_t\\ \delta_t\end{bmatrix}, \qquad (58)$$

where J_{ij} comes for the corresponding part of J matrix.

For every stable solution $\delta_t=0$, so $S_{\theta\theta}E_t\theta_{t+1}=T_{\theta\theta}\theta_t$ or $E_t\theta_{t+1}=S_{\theta\theta}{}^{-1}T_{\theta\theta}\theta_t$ (59)

Then, using $e_{t+1}-E_te_{t+1}=C_e\varepsilon_{t+1}$ and from Giordani (2004) $\rho^z_{t+1}-E_t\rho^z_{t+1}=0$, we have

$$\begin{bmatrix}e_{t+1}\\ \rho^z_{t+1}\end{bmatrix}-E_t\begin{bmatrix}e_{t+1}\\ \rho^z_{t+1}\end{bmatrix}=k_{t+1}-E_tk_{t+1}=\begin{bmatrix}C_e\varepsilon_{t+1}\\ \text{O}\end{bmatrix} \qquad (60)$$

From (42) $\begin{bmatrix}k_t\\ \lambda_t\end{bmatrix}=Z\begin{bmatrix}\theta_t\\ \delta_t\end{bmatrix}=\begin{bmatrix}Z_{k\theta} & Z_{k\delta}\\ Z_{\lambda\theta} & Z_{\lambda\delta}\end{bmatrix}\begin{bmatrix}\theta_t\\ \delta_t\end{bmatrix}$ (61)

And as $\delta_t=0$,

$$\begin{bmatrix}k_t\\ \lambda_t\end{bmatrix}=\begin{bmatrix}Z_{k\theta}\\ Z_{k\delta}\end{bmatrix}\theta_t \qquad (62)$$

Hence, $k_{t+1}-E_tk_{t+1}=Z_{k\theta}(\theta_{t+1}-E_t\theta_{t+1})=\begin{bmatrix}C_e\varepsilon_{t+1}\\ \text{O}\end{bmatrix}$ (63)

In assumption that $Z_{k\theta}$ is invertible and using (44), we obtain:

$$\theta_{t+1}=S_{\theta\theta}{}^{-1}T_{\theta\theta}\theta_t+Z^{-1}_{k\theta}\begin{bmatrix}C_e\varepsilon_{t+1}\\ \text{O}_{4\times 1}\end{bmatrix} \qquad (64)$$

Then

$$k_{t+1}=\begin{bmatrix}e_{t+1}\\ \rho^z_{t+1}\end{bmatrix}=Z_{k\theta}\theta_{t+1}=Z_{k\theta}S_{\theta\theta}{}^{-1}T_{\theta\theta}\theta_t+\begin{bmatrix}C_e\varepsilon_{t+1}\\ \text{O}_{4\times 1}\end{bmatrix} \qquad (65)$$

Or $\begin{bmatrix}e_{t+1}\\ \rho^z_{t+1}\end{bmatrix}=Z_{k\theta}S_{\theta\theta}{}^{-1}T_{\theta\theta}Z^{-1}_{k\theta}\begin{bmatrix}e_t\\ \rho^z_t\end{bmatrix}+\begin{bmatrix}C_e\varepsilon_{t+1}\\ \text{O}_{4\times 1}\end{bmatrix}=M\begin{bmatrix}e_t\\ \rho^z_t\end{bmatrix}+\begin{bmatrix}C_e\varepsilon_{t+1}\\ \text{O}_{4\times 1}\end{bmatrix}$ (66)

And $\lambda_t = \begin{bmatrix} z_t \\ R_t \\ \upsilon_{t+1} \\ \rho_t^e \end{bmatrix} = Z_{\lambda\theta} Z_{k\theta}^{-1} \begin{bmatrix} e_t \\ \rho_t^z \end{bmatrix} = N \begin{bmatrix} e_t \\ \rho_t^z \end{bmatrix}$ (67)

From the last equation (67) one can derive an optimal robust policy, which is constructed as some reaction on the values of predetermined variables and shadow prices of forward-looking variables:

$$R_t = \left(Z_{\lambda\theta} Z_{k\theta}^{-1} \right)_R \begin{bmatrix} e_t \\ \rho_t^z \end{bmatrix} \quad (68)$$

Equilibrium dynamics of the model depends crucially on the actions taken by the evil agent in reality. And here we distinguish two variants: worst-case model and approximating one.

The worst case takes place when the evil agent's action realizes in reality. So such a model is described by (66-67). The opposite situation, when the central bank constructs the robust policy but the evil agent doesn't act refers as an approximating model. We obtain it from approximating model by putting all strategic shocks in (67) equal to zero.

Surely, the resulting dynamics of the economy depends crucially on the value of misspecification fears, θ. The next section is devoted to the definition of reasonable value for this parameter.

3.6. Definition of possible model set θ

Following Dennis (2009), we choose the value of this parameter on the basis of error detection probability method. The main idea under this approach is that model from the possible model set cannot be easily distinguished using the available data. In other words, the central bank cannot decide if real data are generated with the actions of malevolent agent or in the reality there is no any anti-authority measures.

The first situation, when the malevolent nature takes all possible resources to influence the central bank's losses, is named 'worst case' model (W). The second case, when the central bank insure against this agent by robust policy but the evil nature doesn't takes any measures is treated as 'approximating' model (A).

So, according to the error detection method, the central bank should not be able to distinguish between this to models, (W) and (A), using all available information.

In the opposite case, when he can truly decide if there is any model detection, a policymaker has no any need of robust policy, he simply adopts the model to the reality.

Probability of error $\pi(\theta)$:

$$\pi(\theta) = \Pr(L_A > L_W | W)/2 + \Pr(L_W > L_A | A)/2 \tag{69}$$

Where L is a likelihood function. The first part of right-hand expression stands for the probability to treat the model as an approximating case while in reality malevolent nature interrupts the data generating process and the second part is probability to take the model as a worst-case one while there are no any nature actions.

This probability is computed with a help of simulations and depends crucially on the value of θ. We need to choose allowed error probability and find corresponding size of misspecification.

For example, 50% error probability corresponds to zero robustness (the case of standard optimal policy in the model with rational expectations). Lower $\pi(\theta)$ means higher misspecification fear and higher robust preferences (or lower θ).

3.7. Some computational results

We've analyzed several variants of policy robustness. Corresponding results of computations are summarized in ***Appendix B,*** where notions M and N stand for the worst-case, M_a and N_a describe law of motion of approximating model. F_v represents a malevolent agent reaction on the predetermined variables.

What we are interested in are the coefficients of the robust policies, which are represented by $R_t = \left(Z_{\lambda\theta} Z_{k\theta}^{-1}\right)_R \begin{bmatrix} e_t \\ \rho_t^z \end{bmatrix}$

The monetary policy reactions under different preferences for robustness are summarized in ***Table2***.

Table 2. Parameters of robust monetary policy

$$R_t = \left(Z_{\lambda\theta} Z_{k\theta}^{-1}\right)_R \begin{bmatrix} e_t \\ \rho_t^z \end{bmatrix}$$

Error detection probability	θ	r_1	r_2	r_3	r_4	r_5
20%	1.11	0.0025	-0.6147	-29.1285	0.0394	0.1278
30%	1.8797	0.0023	-0.6154	-29.1285	0.0394	0.1278
40%	1.8816	0.0022	-0.6154	-29.1285	0.0394	0.1278
50%	2000	0.0020	-0.6166	-29.1285	0.0394	0.1278

The most important is the first coefficient r_1, which represents reaction on the terms of trade. As this parameter is subject to the shock influence, this coefficient also reflects policy reaction to the shocks. If we tract the module of the corresponding coefficient as a degree of policy aggressiveness, we can see that aggressiveness of policy reaction on the predetermined variable e_t rises with increase in the preferences for robustness. This relation can be explained by the fact that with higher fear of misspecification the central bank supposes that there is higher possibility that the shock in the economy is not simply i.i.d process but is initiated by the malevolent agent, and he reacts more aggressively. Here we see some violation of Brainard principle for the monetary union and result similar to the standard min-max strategy of more aggressive reaction to any shocks.

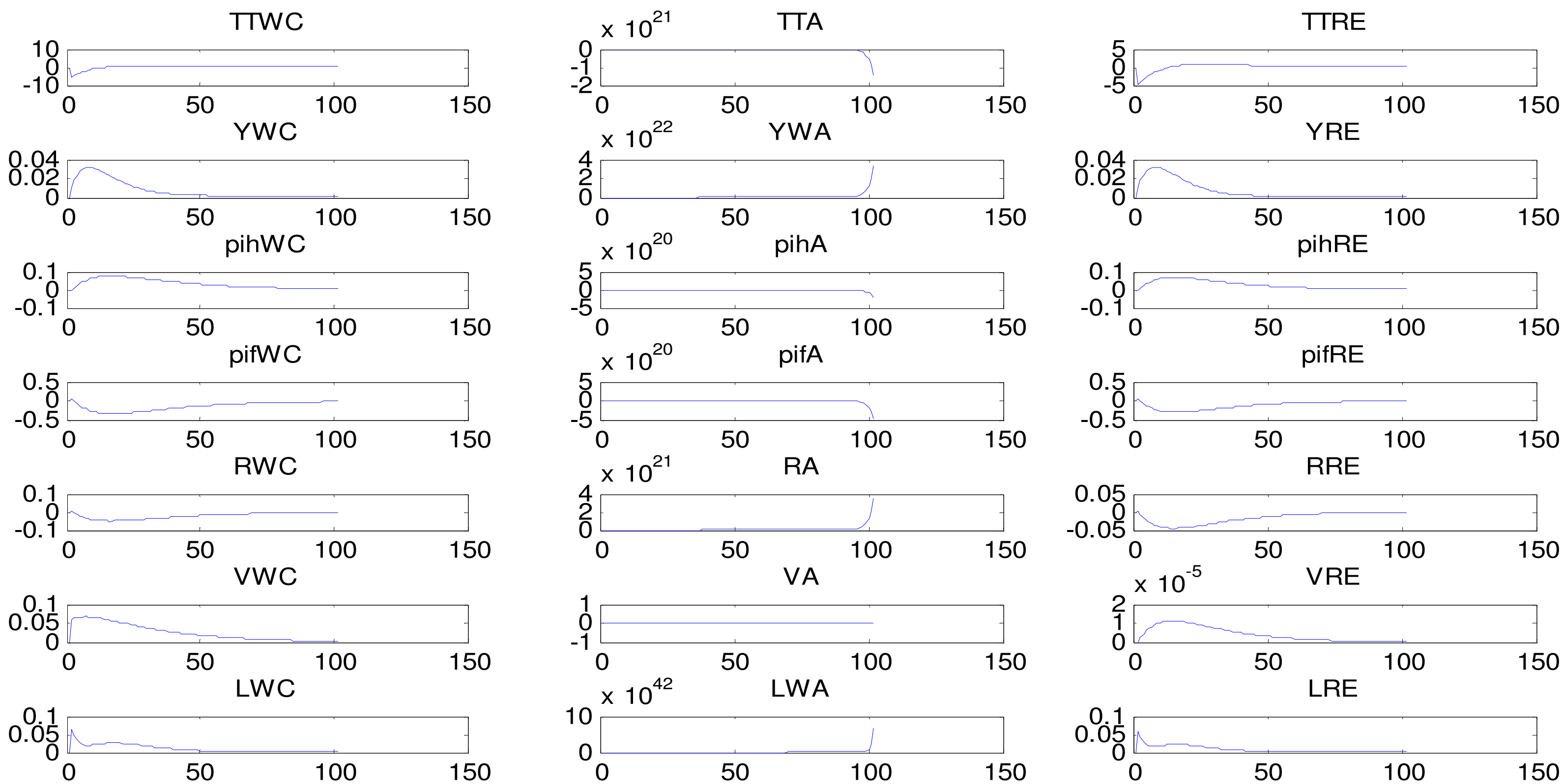
Another interesting point is the sign of policy reaction on the shock: if there is a positive shock, the central bank raises the interest rate. There is a clear intuition under this fact: according to (23) this shock increases inflation in the H region and decreases inflation in the second part of the union. But according to the coefficients in the policy loss function, the thing of its most concern is inflation in the region with higher inflation persistence – H, as its weight in losses is much more important than other variables. So in response to such shock the reasonable response of central bank is an increase in interest rate what decreases total output in the union and thus stabilizes inflation in the region of the highest interest.

Now we simulate conduct of economy under a shock of union's terms of trade, ε_t

Impulse-response functions are summarized in the ***Graph 1***, where the following notations are used:

TT	Terms of trade	V	Value of strategic evil shock
Y	Output gap	L	Value of loss function
pih	Inflation in the H region	WC	Worst-case model
pif	Inflation in the F region	A	Approximating model
R	Monetary policy instrument	RE	Rational expectations without robustness

Graph 1. Terms of trade shock in the worst-case, approximating model and rational expectations without robustness



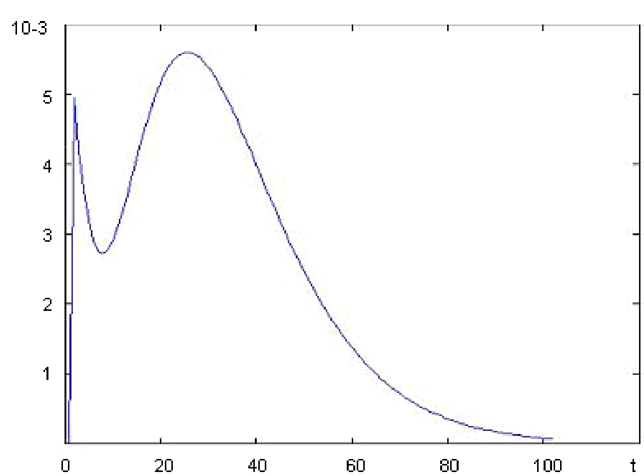
We can see from this picture that malevolent nature reacts on this shock by editing some 'strategic' shock (worst-case model). Knowing this, the central bank increases interest rate. The result of this measure is a moderation of the H's inflation rise.

Result of policy-nature game in the worst case is a rise of output gap and inflation in the H region rise, while region F 's inflation decreases. The last graph in the first column represents losses associated with the worst-case model. We can see, that malevolent actions cause sufficient welfare losses.

When there is no malevolent action, the losses of nation are practically nil (see column 2), even if robust policy is applied. So, in this case the robust policy can properly counterattack any external shock. If we consider RE case, when central bank takes the possibility of misspecification as zero, the losses are higher than in the case of a simple robust policy.

We compare these two policies defining the difference between total welfare under robust policy and under non-robust one. Result is presented in the following graph:

Graph 2. Welfare benefits from robust policy application



We can see that robust policy entails some welfare benefits in comparison with the alternative. So there is an evident basis for robust policy construction.

Conclusion

For the micro-founded two-country model of currency union we have constructed a robust policy under commitment. We've found that the central bank reacts on the shocks more aggressively when higher extent of possible misspecification is admitted, thus Brainard principle is violated.

We've shown technical details of robust control optimization in the adaptation for two-country models. There are many possible applications of this method for the construction of monetary policy in the currency area.

First of all we've considered only the case of terms of trade shock. The analysis can be easily extended to other economic shocks. For example, technological changes can be taken into account.

Secondly, we've constructed a full commitment policy. But the control of such policy is rather difficult, so the problem of policy inconsistency can arise. Control of simple monetary rules (for example, of a Taylor type) is easier and so such rules can contribute to the population confidence to the central bank measures. So the fist possible extension is construction of robust monetary policy rule.

Thirdly, in our model there is no stance for monetary and fiscal policy interactions. The influence of central bank preferences for robustness on the economy under assumption of strategic interactions between monetary and fiscal authorities can be analyzed, for example, on the base of two-country model of Beetsma, Jensen (2005).

Then, our analysis can be easily extended to the case of tree or more countries. For example, representation of EMU as a union of Germany, France and Italy, the largest European economies, might provide the research with some important extensions.

Finally, we've discussed only a case of two-country world. Nevertheless, interactions with the rest of the world via international trade and capital flows can be analyzed.

Appendix A. Derivation of the reference model

1) Solution of consumer problem

Solution of the consumer problem consists of 3 stages:

1) Solving a problem (1) subject to (2) one determines an optimal value of consumption index C_t^j, an optimal real balances stock $\frac{M}{P}$,an optimal output supplied.

   This problem can be easily rewritten:

$$\max E_t \sum_{s=0}^{\infty} \beta^s U\left(C_{t+s}^j\right) + L\left(\frac{M_{t+s}^j}{P_{t+s}}, \varepsilon_{t+s}^i\right) - V\left(y_{t+s}^j, z_{t+s}^i\right) +$$

$$+E_t \sum_{s=0}^{\infty} \lambda_{t+s} \begin{bmatrix} B_{t+s-1}^{i,j} + \frac{M_{t+s-1}^j + B_{t+s-1}^j}{P_{t+s}} + \left(1-\tau^i\right) \frac{p_{t+s}(j)\, y_{t+s}(j)}{P_{t+s}} - C_t^j + \frac{Q_{t+s}^{i,j}}{P_{t+s}} \\ -E_{t+s}\left\{q_{t+s}^i B_t^{i,j}\right\} - \frac{B_{t+s}^j}{P_{t+s}\left(1+R_{t+s}\right)} - \frac{M_{t+s}^j}{P_{t+s}} \end{bmatrix}$$

From where we obtain the relevant first-order conditions:

$$L_{\frac{M}{P}}\left(\frac{M_t^i}{P_t}, \varepsilon_t^i\right) = \frac{R_t}{1+R_t} U_C\left(C_t^i\right) \qquad \text{(A.1)}$$

$$U_C\left(C_t^i\right) = \left(1+R_t\right) \beta E_t \left\{ U_C\left(C_{t+1}^i\right) \frac{P_t}{P_{t+1}} \right\} \qquad \text{(A.2)}$$

$$E_t U_C\left(C_{t+s}^j\right) = E_t \frac{\lambda_{t+s}}{P_{t+s}} \qquad \text{(A.3)}$$

(A.1) tells us that in the optimal consumer choice marginal rate of substitution between consumption and real money balances holdings must coincide with the price of real money balances in terms of consumption index. (A.2) represents a usual Euler equation for the forward-looking model. According to (A.2) the marginal utility of consumption in the period t must equals to the expected marginal utility of the consumption in the next period with regard to the relative prices and the time preference.

2) Given a decision of the first stage C_t^j an individual must optimally allocate this value between H and F bundles so to minimize expenditures. In other words, the second stage problem is:

$$\min\left\{P_H^j C_H^j + P_F^j C_F^j\right\} \quad s.t.(3) \qquad \text{(A.4)}$$

Solution of this stage is the following:

$$C_H^j = C^j n\left(\frac{P_F}{P_H}\right)^{1-n}$$

$$C_F^j = C^j (1-n)\left(\frac{P_H}{P_F}\right)^{n}$$

Or, naming $T \equiv \dfrac{P_F}{P_H}$ terms of trade between two regions,

$$C_H^j = C^j n\left(T\right)^{1-n}$$
$$C_F^j = C^j (1-n)\left(T\right)^{-n} \qquad \text{(A.5)}$$

3) Given the optimal values C_H^j and C_F^j from the previous stage an individual must optimally allocate these values between distinct goods from the corresponding bundles. The corresponding problems are:

$$\min \int_0^n p(h)c^j\left(h\right)dh \quad s.t.(4) \qquad \text{and} \qquad \min \int_n^1 p(f)c^j\left(f\right)df \quad s.t.(5) \qquad \text{(A.6)}$$

Solutions of this stage are given by:

$$c^j(h) = \frac{C_H^j (P_H)^\sigma}{np(h)^\sigma}$$
$$c^j(f) = \frac{C_F^j (P_F)^\sigma}{(1-n)\, p(f)^\sigma}$$
, or, using a solution of the previous stage,

$$c^j(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} T^{1-n} C^j$$
$$c^j(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma} T^{-n} C^j \qquad \text{(A.7)}$$

2) Governments

Given these values the problem of the choice for the distinct goods is similar to the problem (A.6) of a private agent who allocates optimally G_t^H and G_t^F between different goods. So the problem of fiscal authorities can be rewritten in the following manner:

$$\min \int_0^n p(h) g^H(h)\, dh$$
$$s.t.G^H = \left[\int_0^n g(h)^{\frac{\sigma-1}{\sigma}} dh\right]^{\frac{\sigma}{\sigma-1}}$$

for the government H and

$$\min \int_n^1 p(f) g^F(f)\, df$$
$$s.t.G^F = \left[\int_n^1 g(f)^{\frac{\sigma-1}{\sigma}} df\right]^{\frac{\sigma}{\sigma-1}}$$
. Corresponding solutions are:

$$g(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} G^H$$
$$g(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma} G^F \qquad \text{(A.8)}$$

3) Firms program

If a firm changes a price at period t, it sets a price $\tilde{p}_t(j)$ which has to maximize the following function:

$$E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \left[\lambda_{t+k} \left(1-\tau^i\right) \tilde{p}_t(j)\, \tilde{y}_{t,t+k}(j) - V\left(\tilde{y}_{t,t+k}(j), z^i_{t+k}\right) \right], \qquad \text{(A.9)}$$

Where $\lambda_{t+k} = \dfrac{U_c\left(C_{t+k}\right)}{P_{t+k}}$ represents the marginal utility of nominal income from (A.3) and $\tilde{y}_{t,t+k}(j)$ is a total demand for the good j at period $t+k$ if $\tilde{p}_t(j)$ is applied.

From (7) demand constraint for the firms can be rewritten as:

$$\tilde{y}_{t,t+k}(h) = \left(\frac{\tilde{p}_t(h)}{P_{H,t+k}}\right)^{-\sigma} \left[T_{t+k}{}^{1-n} C_{t+k} + G_{t+k}{}^{H} \right]$$
$$\tilde{y}_{t,t+k}(f) = \left(\frac{\tilde{p}_t(f)}{P_{F,t+k}}\right)^{-\sigma} \left[T_{t+k}{}^{-n} C_{t+k} + G_{t+k}{}^{F} \right] \qquad \text{(A.10)}$$

So the firm's problem can be rewritten in the following form:

$$\max E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \begin{bmatrix} \lambda_{t+k} \left(1-\tau^i\right) \tilde{p}_t(j) \left(\dfrac{\tilde{p}_t(j)}{P_{J,t+k}}\right)^{-\sigma} \left[T_{t+k}{}^{1-n} C_{t+k} + G_{t+k}{}^{J} \right] \\ -V\left(\left(\dfrac{\tilde{p}_t(j)}{P_{J,t+k}}\right)^{-\sigma} \left[T_{t+k}{}^{1-n} C_{t+k} + G_{t+k}{}^{J} \right], z_{t+k}^{J} \right) \end{bmatrix}$$

$FOC:$

$$\begin{aligned} &\left(1-\tau^i\right)(1-\sigma) E_t \sum_{k=0}^{\infty} \lambda_{t+k} \left(\alpha^i \beta\right)^k \tilde{y}_{t,t+k}(j) + \\ &+\sigma \frac{1}{\tilde{p}_t(j)} E_t \sum_{k=0}^{\infty} V_y{}'\left(\tilde{y}_{t,t+k}(j), z_{t+k}^{J}\right) \tilde{y}_{t,t+k}(j) = 0 \end{aligned} \quad \text{(A.11)}$$

$$\tilde{p}_t(j) = \frac{\sigma}{\left(1-\tau^i\right)(\sigma-1)} \frac{E_t \sum_{k=0}^{\infty} V_y{}'\left(\tilde{y}_{t,t+k}(j), z_{t+k}^{J}\right) \tilde{y}_{t,t+k}(j)}{E_t \sum_{k=0}^{\infty} \lambda_{t+k} \left(\alpha^i \beta\right)^k \tilde{y}_{t,t+k}(j)}$$

4) Deterministic equilibrium

From the first-order conditions of consumer problem it can be seen that this stationary equilibrium can be implemented if the monetary policy instrument, interest rate is set so to countervail the intertemporal discount rate: $1+\overline{R} = \dfrac{1}{\beta}$.

From (A.11) using (A.3) the following expression can be obtained:

$$\begin{aligned} \overline{p}^i &= \frac{\sigma}{\left(1-\tau^i\right)(\sigma-1)} \frac{\overline{y}(j) V_y\left(\overline{y}(j),0\right) \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k}{\overline{\lambda}\, \overline{y}(j) \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k} \\ \overline{p}^i &= \frac{\sigma}{\left(1-\tau^i\right)(\sigma-1)} \frac{\overline{y}(j) V_y\left(\overline{y}(j),0\right) \overline{P}}{U_C \overline{y}(j)} \end{aligned} \quad \text{(A.12)}$$

Taking into account that from (A.12) in the deterministic equilibrium the following equation should be accomplished: $\begin{matrix} \overline{p}_H = \overline{P}_H \\ \overline{p}_F = \overline{P}_F \end{matrix}$. From the definition of terms of trade $P = P_H (T)^{1-n} = P_F T^{-n}$

Also $\bar{y}^H = T^{1-n}\bar{C}$
$\bar{y}^F = T^{-n}\bar{C}$

Hence, FOC of consumer problem can be rewritten in the following form:

$$\left(1-\tau^H\right)U_C\left(\bar{C}\right)=\frac{\sigma}{\left(\sigma-1\right)}\bar{T}^{1-n}V_y\left(\bar{T}^{1-n}\bar{C},0\right)$$
$$\left(1-\tau^F\right)U_C\left(\bar{C}\right)=\frac{\sigma}{\left(\sigma-1\right)}\bar{T}^{-n}V_y\left(\bar{T}^{-n}\bar{C},0\right) \qquad \text{(A.13)}$$

5) Linearization of sticky-prices equilibrium

To obtain a linearization under flexible prices we derive an approximation of (A.13) around the steady state:

$$-\rho\tilde{C}_t=\left(1-n\right)\tilde{T}_t+\eta\left[\left(1-n\right)\tilde{T}_t+\tilde{C}_t+\tilde{g}_t^H\right]-\eta\bar{Y}_t^H$$
$$-\rho\tilde{C}_t=\left(-n\right)\tilde{T}_t+\eta\left[\left(-n\right)\tilde{T}_t+\tilde{C}_t+\tilde{g}_t^F\right]-\eta\bar{Y}_t^F \qquad \text{(A.14)}$$

Where $\bar{Y}^i$ is supply shock in the region i, and $\tilde{g}^i$ is a corresponding shock of government purchases. Summing (A.14) with weights n and (1-n) we obtain:

$$-\rho\tilde{C}_t=\eta\left[\tilde{C}_t+\tilde{g}^W\right]-\eta\bar{Y}_t^W$$

Or $\tilde{C}_t^W = \frac{\eta}{\eta+\rho}\left(\bar{Y}_t^W-\tilde{g}^W\right)$ (A.15)

Subtracting (A.14) we obtain $0=\tilde{T}_t+\eta\left[\tilde{T}_t+\tilde{g}_t^R\right]-\eta\bar{Y}_t^R$

$$\tilde{T}_t=\frac{\eta}{1+\eta}\left[\tilde{g}_t^R-\bar{Y}_t^R\right] \qquad \text{(A.16)}$$

And also, taking into account that $\tilde{Y}^W = \tilde{C}^W + g_t^W$, we can rewrite:

$$\tilde{Y}_t^W=\frac{\eta}{\rho+\eta}\bar{Y}_t^W+\frac{\rho}{\rho+\eta}g_t^W \qquad \text{(A.17)}$$

Now we obtain a linearized equilibrium conditions under sticky prices. Firstly we take first order condition from the consumer problem (A.2) to derive:

$$E_t\hat{C}_{t+1}^W=\hat{C}_t^W+\rho^{-1}\left(\hat{R}_t-E_t\pi_{t+1}^W\right) \qquad \text{(A.18)}$$

Then, log-linearizing (9) around the steady state we obtain:

$$\hat{Y}_t^H = (1-n)\hat{T}_t + \hat{C}_t^W + g_t^H \tag{A.19}$$

$$\hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t^W + g_t^F \tag{A.20}$$

Then, using definition of terms of trade, we have

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H \tag{A.21}$$

To obtain the log-linearized Phillips curves, we need to rewrite FOCs from (A.11):

$$E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \left[\lambda_{t+k}\left(1-\tau^i\right) p_t(j) - V_y\left(y_{t,t+k}(j), z_{t+k}^i\right)\right] y_{t,t+k}(j) = 0 \tag{A.22}$$

Taking from (A.2) we have

$$\lambda_{t+k} = \frac{U_c\left(C_{t+k}\right)}{P_{t+k}}$$

$$E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \left[U_C\left(C_{t+k}\right)\left(1-\tau^i\right)\frac{p_t(j)}{P_{H,t+k}} T_{t+k}^{n-1} - V_y\left(y_{t,t+k}(j), z_{t+k}^i\right)\right] y_{t,t+k}(j) = 0 \tag{A.23}$$

Linearization of (A.23) gives:

$$E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \left[\hat{p}_{t,t+k} - (1-n)\hat{T}_{t+k} - \rho\hat{C}_{t+k} - \eta\hat{y}_{t+k}\right] = 0 \tag{A.24}$$

with $\hat{p}_{t,t+k} = \ln \frac{p_t}{P_{i,t+k}}$.

Now we log-linearise $\tilde{y}_{t,y+k}(j)$. For example, for the region H from (8)

$$\hat{y}_{t+k} = -\sigma\hat{p}_{t,t+k} + (1-n)\hat{T}_{t+k} + \hat{C}_{t+k} \tag{A.25}$$

Subtracting (A.25) into (A.24) yields:

$$E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \left[\hat{p}_{t,t+k} - (1-n)\hat{T}_{t+k} - \rho\hat{C}_{t+k} - \eta\left\{-\sigma\hat{p}_{t,t+k} + (1-n)\hat{T}_{t+k} + \hat{C}_{t+k} - \bar{Y}_{t+k}^i\right\}\right] = 0$$

$$E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \left[(1+\eta\sigma)\hat{p}_{t,t+k} - (1-n)(1+\eta)\hat{T}_{t+k} - (\eta+\rho)\hat{C}_{t+k} + \eta\bar{Y}_{t+k}^i\right] = 0 \tag{A.26}$$

And reminding that $\hat{p}_{t,t+k} = \hat{p}_{t,t} + \sum_{s=1}^{k} \pi_{t+s}^{i}$,

$$E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \left[(1+\eta\sigma) \left\{ \hat{p}_{t,t} + \sum_{s=1}^{k} \pi_{t+s}^{i} \right\} - (1-n)(1+\eta)\hat{T}_{t+k} - (\eta+\rho)\hat{C}_{t+k} + \eta \bar{Y}_{t+k}^{i} \right] = 0$$

$$\begin{aligned} &(1+\eta\sigma)\hat{p}_{t,t} E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k + (1+\eta\sigma) E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \sum_{s=1}^{k} \pi_{t+s}^{i} - \\ &-E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \left[(1-n)(1+\eta)\hat{T}_{t+k} + (\eta+\rho)\hat{C}_{t+k} - \eta \bar{Y}_{t+k}^{i} \right] = 0 \end{aligned} \quad \text{(A.27)}$$

Or,

$$\frac{(1+\eta\sigma)\hat{p}_{t,t}}{1-\alpha^i \beta} = (1+\eta\sigma) E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \sum_{s=1}^{k} \pi_{t+s}^{i} +$$

$$+E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \left[(1-n)(1+\eta)\hat{T}_{t+k} + (\eta+\rho)\hat{C}_{t+k} - \eta \bar{Y}_{t+k}^{i} \right],$$ from where

$$\begin{aligned} &\frac{\hat{p}_{t,t}}{1-\alpha^i \beta} = E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \sum_{s=1}^{k} \pi_{t+s}^{i} + \\ &+\frac{1}{(1+\eta\sigma)} E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \left[(1-n)(1+\eta)\hat{T}_{t+k} + (\eta+\rho)\hat{C}_{t+k} - \eta \bar{Y}_{t+k}^{i} \right] \end{aligned} \quad \text{(A.28)}$$

After log-linearisation of (10) we obtain that $\hat{p}_{t,t} = \frac{\alpha^i}{1-\alpha^i} \pi_t^i$, and so

$$\begin{aligned} &\frac{\pi_t^i}{1-\alpha^i \beta} \frac{\alpha^i}{1-\alpha^i} = E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \frac{\pi_{t+k}^{i}}{1-\alpha^i \beta} + \\ &+\frac{1}{(1+\eta\sigma)} E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \left[(1-n)(1+\eta)\hat{T}_{t+k} + (\eta+\rho)\hat{C}_{t+k} - \eta \bar{Y}_{t+k}^{i} \right] \end{aligned} \quad \text{(A.29)}$$

$$\pi_t^i = \frac{1-\alpha^i}{\alpha^i} E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \pi_{t+k}^{i} +$$

Or $$+\frac{1-\alpha^i}{\alpha^i} \frac{1-\alpha^i \beta}{(1+\eta\sigma)} E_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \left[(1-n)(1+\eta)\hat{T}_{t+k} + (\eta+\rho)\hat{C}_{t+k} - \eta \bar{Y}_{t+k}^{i} \right] \quad \text{(A.30)}$$

Taking the same for $t+1$ period and taking expectation for the previous period, we have:

$$E_t\pi^i_{t+1} = \frac{1-\alpha^i}{\alpha^i} E_t \sum_{k=1}^{\infty} \left(\alpha^i\beta\right)^k \pi^i_{t+k} +$$
$$+\frac{1-\alpha^i}{\alpha^i}\frac{1-\alpha^i\beta}{(1+\eta\sigma)} E_t \sum_{k=1}^{\infty} \left(\alpha^i\beta\right)^k \left[(1-n)(1+\eta)\hat{T}_{t+k} + (\eta+\rho)\hat{C}_{t+k} - \eta\overline{Y}^i_{t+k}\right] \quad \text{(A.31)}$$

Then we multiply (A.30) by β and subtracting this value from (A.29) we obtain:

$$\pi^i_t - \beta E_t\pi^i_{t+1} = \frac{1-\alpha^i}{\alpha^i} E_t \sum_{k=0}^{\infty} \left(\alpha^i\beta\right)^k \pi^i_{t+k} +$$
$$+\frac{1-\alpha^i}{\alpha^i}\frac{1-\alpha^i\beta}{(1+\eta\sigma)}\left[(1-n)(1+\eta)\hat{T}_t + (\eta+\rho)\hat{C}_t - \eta\overline{Y}^i_t\right] \quad \text{(A.32)}$$

From (A.15-17):

$$-(1+\eta)(1-n)\tilde{T}_t - (\rho+\eta)\tilde{C}_t = -\eta\overline{Y}^H_t \quad \text{(A.33)}$$

So, (A.31) can be rewritten:

$$\pi^H_t = (1-n)k^H_T\left(\hat{T}_t - \tilde{T}_t\right) + k^H_C y^W_t + \beta E_t\pi^H_{t+1} \quad \text{(A.34)}$$

Analogically

$$\pi^F_t = -nk^F_T\left(\hat{T}_t - \tilde{T}_t\right) + k^F_C y^W_t + \beta E_t\pi^F_{t+1} \quad \text{(A.35)}$$

Where k^i_T and k^i_C are combinations of model coefficients stated earlier in the following form:

$$k^i_C = \left[\frac{\left(1-\alpha^i\beta\right)\left(1-\alpha^i\right)}{\alpha^i}\right]\left[\frac{\rho+\eta}{1+\sigma\eta}\right]$$
$$k^i_T = k^i_C\left[\frac{1+\eta}{\rho+\eta}\right] \quad \text{(A.36)}$$

6) Derivation of micro-founded loss function

Derivation of micro-founded loss function is based on the Beetsma (2005) method, applied for our model.

Welfare criterion for the central bank, as it was shown in the paper, is a weighted expected sum of future one-period welfare ratios w_t:

$$W = E_0 \left\{ \sum_{t-0}^{+\infty} \beta^t w_t \right\} \quad \text{(A.37)}$$

$$w_t \equiv U\left(C_t\right) - \int_0^1 V\left(y_t\left(j\right), z_t^i\right) dj$$

To obtain a loss function of a usual type the second-order expansion of welfare function around a steady state is needed. We remind briefly the main characteristics of this steady-state: $\bar{C} = \bar{Y}$ and $\bar{T} = 1$

A welfare criterion can be rewritten as:

$$w_t \equiv U\left(C_t\right) - \int_0^n V\left(y_t\left(h\right), z_t^H\right) dh - \int_n^1 V\left(y_t\left(f\right), z_t^F\right) df \quad \text{(A.38)}$$

a) $\underline{U\left(C_t\right)}$

We take the second-order expansion of $U\left(C_t\right)$ around the steady-state value $\bar{C}$.

$$U\left(C_t\right) = U\left(\bar{C}\right) + U_C\left(C_t - \bar{C}\right) + \frac{1}{2} U_{CC}\left(C_t - \bar{C}\right)^2 + O\left(\left\|\varepsilon\right\|^3\right), \quad \text{(A.39)}$$

$O\left(\left\|\varepsilon\right\|^3\right)$ - terms of higher-than-second order.

Taking into account that $\hat{C} = \ln \frac{C}{\bar{C}}$, the second-order expansion of C_t around $\bar{C}$ implies:

$$C_t = \bar{C}\left[1+\hat{C}_t+\frac{1}{2}\hat{C}_t^2\right]+O\left(\|\varepsilon\|^3\right) \tag{A.40}$$

Substituting (A.40) into (A.39) yields:

$$U(C_t)=U\left(\bar{C}\right)+U_C\bar{C}\left[\hat{C}_t+\frac{1}{2}\hat{C}_t^2\right]+\frac{1}{2}U_{CC}\bar{C}^2\left(\hat{C}_t\right)^2+O\left(\|\varepsilon\|^3\right) \tag{A.41}$$

And hence

$$\begin{aligned} U(C_t) &= U_C\bar{C}\left[\hat{C}_t+\frac{1}{2}\hat{C}_t^2+\frac{1}{2}\frac{U_{CC}\bar{C}}{U_C}\left(\hat{C}_t\right)^2\right]+t.i.p+O\left(\|\varepsilon\|^3\right)= \\ &= U_C\bar{C}\left[\hat{C}_t+\frac{1}{2}(1-\rho)\hat{C}_t^2\right]+t.i.p+O\left(\|\varepsilon\|^3\right) \end{aligned} \tag{A.42}$$

Where $t.i.p.$ stand for terms independent of policy

b) $V\left(y_t(j),z_t^i\right)$

For the H region (all steps are to repeated for the F region):

$$\begin{aligned} V\left(y_t(j),z_t^H\right) &= V(\bar{Y};0)+V_y\left(y_t(h)-\bar{Y}\right)+V_z z_t^H+\frac{1}{2}V_{yy}\left(y_t(h)-\bar{Y}\right)^2+ \\ &+V_{yz}z_t^h\left(y_t(h)-\bar{Y}\right)+\frac{1}{2}\left(z_t^i\right)'V_{zz}z_t^H+O\left(\|\varepsilon\|^3\right) \end{aligned} \tag{A.43}$$

Second-order log-approximation for y :

$$y_t(h)=\bar{Y}\left[1+\hat{y}_t(h)+\frac{1}{2}\hat{y}_t^2(h)\right]+O\left(\|\varepsilon\|^3\right) \tag{A.44}$$

Substituting (A.44) into (A.43) yields:

$$V\left(y_t(h), z_t^H\right) = V_y\bar{Y}\left[\hat{y}_t(h) + \frac{1}{2}\hat{y}_t^2(h) + \frac{1}{2}\frac{V_{yy}\bar{Y}}{V_y}\hat{y}_t^2(h) + \frac{V_{yz}}{V_y}z_t^H\hat{y}_t(h)\right] + t.i.p. + \mathrm{O}\left(\|\varepsilon\|^3\right) \tag{A.45}$$

Or, as $\eta = \frac{V_{yy}\bar{Y}}{V_y}$,

$$V\left(y_t(h), z_t^H\right) = V_y\bar{Y}\left[\hat{y}_t(h) + \frac{1+\eta}{2}\hat{y}_t^2(h) + \frac{V_{yz}}{V_y}z_t^H\hat{y}_t(h)\right] + t.i.p. + \mathrm{O}\left(\|\varepsilon\|^3\right) \tag{A.46}$$

Then as $V_{yy}\overline{YY_t} = -V_{yz}z_t$

$$V\left(y_t(h), z_t^H\right) = V_y\bar{Y}\left[\hat{y}_t(h) + \frac{1+\eta}{2}\hat{y}_t^2(h) - \frac{V_{yy}\overline{YY}_t^H}{V_y}\hat{y}_t(h)\right] + t.i.p. + \mathrm{O}\left(\|\varepsilon\|^3\right) =$$
$$= V_y\bar{Y}\left[\hat{y}_t(h) + \frac{1+\eta}{2}\hat{y}_t^2(h) - \eta\bar{Y}_t^H\hat{y}_t(h)\right] + t.i.p. + \mathrm{O}\left(\|\varepsilon\|^3\right) \tag{A.47}$$

Recalling that $(1-\phi)U_C\left(\bar{C}\right) = V_y\left(\bar{Y}, 0\right)$, where $\phi \equiv \frac{1}{\sigma}$, we can rewrite (A.47):

$$V\left(y_t(h), z_t^H\right) = V_C\bar{Y}\left[(1-\phi)\hat{y}_t(h) + \frac{1+\eta}{2}\hat{y}_t^2(h) - \eta\bar{Y}_t^H\hat{y}_t(h)\right] + t.i.p. + \mathrm{O}\left(\|\varepsilon\|^3\right) \tag{A.48}$$

Now we integrate (A.48) over the H region population:

$$\frac{1}{n}\int_0^n V\left(y_t(h), z_t^H\right)dj = V_C\bar{Y}\left[(1-\phi)E\hat{y}_t(h) + \frac{1+\eta}{2}E\hat{y}_t^2(h) - \eta\bar{Y}_t^H E_t\hat{y}_t(h)\right] + t.i.p. + \mathrm{O}\left(\|\varepsilon\|^3\right) =$$
$$= V_C\bar{Y}\left[(1-\phi)E\hat{y}_t(h) + \frac{1+\eta}{2}\left(Var\hat{y}_t(h) + E^2\hat{y}_t(h)\right) - \eta\bar{Y}_t^H E_t\hat{y}_t\right] + t.i.p. + \mathrm{O}\left(\|\varepsilon\|^3\right) \tag{A.49}$$

A second-order log-expansion of Y^H:

$$\hat{Y}_t^H = E\hat{y}_t(h) + \frac{1}{2}\frac{\sigma-1}{\sigma}Var\hat{y}_t(h) + \mathrm{O}(\|\xi\|^3) \tag{A.50}$$

Taking a term $E\hat{y}_t(h)$ from (A.50) a relation (A.49) becomes

$$\frac{1}{n}\int_0^n V\left(y_t\left(h\right),z_t^H\right)dj = U_C\bar{Y}\begin{bmatrix}\left(1-\phi\right)\hat{Y}_t^H - \frac{1}{2}\frac{\sigma-1}{\sigma}Var\hat{y}_t\left(h\right)+ \\ +\frac{1+\eta}{2}\left(Var\hat{y}_t(h)+\left(\hat{Y}_t^H\right)^2\right)-\eta\bar{Y}_t^H\hat{Y}_t^H\end{bmatrix}+t.i.p.+\mathrm{O}\left(\left\|\varepsilon\right\|^3\right)= \quad \text{(A.51)}$$

$$=U_C\bar{Y}\left[\left(1-\phi\right)\hat{Y}_t^H+\frac{1}{2}\left[\sigma^{-1}+\eta\right]Var\hat{y}_t\left(h\right)+\frac{1+\eta}{2}\left(\hat{Y}_t^H\right)^2-\eta\bar{Y}_t^H\hat{Y}_t^H\right]+t.i.p.+\mathrm{O}\left(\left\|\varepsilon\right\|^3\right)$$

The same expression for the second region takes the following form:

$$\frac{1}{n}\int_n^1 V\left(y_t\left(f\right),z_t^F\right)dj = U_C\bar{Y}\left[\left(1-\phi\right)\hat{Y}_t^F+\frac{1}{2}\left[\sigma^{-1}+\eta\right]Var\hat{y}_t\left(f\right)+\frac{1+\eta}{2}\left(\hat{Y}_t^F\right)^2-\eta\bar{Y}_t^F\hat{Y}_t^F\right] \quad \text{(A.52)}$$
$$+t.i.p.+\mathrm{O}\left(\left\|\varepsilon\right\|^3\right)$$

Substituting (A. 42,51,52) into (A.38) subject to the equality of $\bar{C}$ and $\bar{Y}$ yields:

$$w_t = U_C\bar{C}\left\{\begin{matrix}\left[\hat{C}_t+\frac{1}{2}\left(1-\rho\right)\hat{C}_t^2\right]- \\ -\left[\begin{bmatrix}\left(1-\phi\right)\hat{Y}_t^W+\frac{1}{2}\left[\sigma^{-1}+\eta\right]\left[nVar\hat{y}_t\left(h\right)+\left(1-n\right)Var\hat{y}_t\left(f\right)\right]+ \\ \frac{1+\eta}{2}\left(n\left(\hat{Y}_t^H\right)^2+\left(1-n\right)\left(\hat{Y}_t^F\right)^2\right)-\eta n\bar{Y}_t^H\hat{Y}_t^H-\eta\left(1-n\right)\bar{Y}_t^F\hat{Y}_t^F\end{bmatrix}\right]\end{matrix}\right\} \quad \text{(A.53)}$$
$$+t.i.p+\mathrm{O}\left(\left\|\xi\right\|^3\right)$$

c) ***<u>Expansion of</u>*** $\hat{Y}_t$

Now we define a function $W\left(Y_t^H\right)=\hat{Y}_t^H=\ln\left(Y_t^H/\bar{Y}\right)$. Thus, an approximation yields:

$$\hat{Y}_t^H = W\left(\bar{Y}\right)+W'\left(\bar{Y}\right)\left(Y_t^H-\bar{Y}\right)+\frac{1}{2}W''\left(\bar{Y}\right)\left(Y_t^H-\bar{Y}\right)^2+\mathrm{O}\left(\left\|\varepsilon\right\|^3\right)=$$

$$=\left(\frac{\left(Y_t^H-\bar{Y}\right)}{\bar{Y}}\right)-\frac{1}{2}\left(\frac{\left(Y_t^H-\bar{Y}\right)}{\bar{Y}}\right)^2+\mathrm{O}\left(\left\|\varepsilon\right\|^3\right)= \quad \text{(A.54)}$$

$$=\frac{T_t^{1-n}C_t+G_t^H-\left(\bar{T}^{1-n}\bar{C}+\bar{G}^H\right)}{\bar{Y}}-\frac{1}{2}\left[\frac{T_t^{1-n}C_t+G_t^H-\left(\bar{T}^{1-n}\bar{C}+\bar{G}^H\right)}{\bar{Y}}\right]^2+\mathrm{O}\left(\left\|\varepsilon\right\|^3\right)$$

Reminding, $\bar{C} = \bar{Y}, \bar{T} = 1$, we obtain from (A.54)

$$\hat{Y}_t^H = \frac{T_t^{1-n}C_t + G_t^H - \left(\bar{Y}+0\right)}{\bar{Y}} - \frac{1}{2}\left[\frac{T_t^{1-n}C_t + G_t^H - \left(\bar{Y}+0\right)}{\bar{Y}}\right]^2 + \mathrm{O}\left(\|\varepsilon\|^3\right) \tag{A.55}$$

Now we define $Z\left(T_t, C_t\right) \equiv T_t^{1-n}C_t$ and expose it around the steady state:

$$\begin{aligned} Z\left(T_t, C_t\right) &= Z\left(\bar{T}, \bar{C}\right) + Z_T\left(T_t - \bar{T}\right) + \frac{1}{2}Z_{TT}\left(T_t - \bar{T}\right)^2 + Z_C\left(C_t - \bar{C}\right) + \\ &+ \frac{1}{2}Z_{CC}\left(C_t - \bar{C}\right)^2 + Z_{TC}\left(T_t - \bar{T}\right)\left(C_t - \bar{C}\right) + \mathrm{O}\left(\|\varepsilon\|^3\right) = \\ &= \bar{C} + (1-n)\bar{C}\left(T_t - 1\right) + \bar{C}\frac{C - \bar{C}}{\bar{C}} - \frac{1}{2}(1-n)n\bar{C}\left(T_t - 1\right)^2 + \\ &+ (1-n)\left(T_t - 1\right)\bar{C}\frac{C - \bar{C}}{\bar{C}} + \mathrm{O}\left(\|\varepsilon\|^3\right) \end{aligned} \tag{A.56}$$

$$\begin{aligned} \frac{T_t^{1-n}C_t - \bar{Y}}{\bar{Y}} &= (1-n)\left(T_t - 1\right) - \frac{1}{2}(1-n)n\left(T_t - 1\right)^2 + \left(\frac{C_t - \bar{C}}{\bar{C}}\right) + \\ &+ (1-n)\left(T_t - 1\right)\left(\frac{C_t - \bar{C}}{\bar{C}}\right) + \mathrm{O}\left(\|\varepsilon\|^3\right) \end{aligned} \tag{A.57}$$

Expansion for T_t:

$$T_t = \left[1 + \hat{T}_t + \frac{1}{2}\hat{T}_t^2\right] + \mathrm{O}\left(\|\varepsilon\|^3\right) \tag{A.58}$$

Substituting (A.40) and (A.58) into (A.57) yields:

$$\begin{aligned} \frac{Y_t^H - \bar{Y}}{\bar{Y}} &= (1-n)\left(\hat{T}_t + \frac{1}{2}\hat{T}_t^2\right) - \frac{1}{2}(1-n)n\left(\hat{T}_t + \frac{1}{2}\hat{T}_t^2\right)^2 + \left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right) + \\ &(1-n)\left(\hat{T}_t + \frac{1}{2}\hat{T}_t^2\right)\left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right) + \mathrm{O}\left(\|\varepsilon\|^3\right) = (1-n)\hat{T}_t + \frac{1}{2}(1-n)^2\hat{T}_t^2 + \hat{C}_t + \frac{1}{2}\hat{C}_t^2 \\ &+ (1-n)\hat{T}_t\hat{C}_t + \mathrm{O}\left(\|\varepsilon\|^3\right) \end{aligned} \tag{A.59}$$

$$\left(\frac{Y_t^H - \bar{Y}}{\bar{Y}}\right)^2 = (1-n)^2\hat{T}_t^2 + \hat{C}_t^2 + 2(1-n)\hat{T}_t\hat{C}_t + \mathrm{O}\left(\|\varepsilon\|^3\right) \tag{A.60}$$

$$\hat{Y}_t^H = \frac{Y_t^H - \overline{Y}}{\overline{Y}} - \frac{1}{2}\left(\frac{Y_t^H - \overline{Y}}{\overline{Y}}\right)^2 + \mathrm{O}\left(\|\varepsilon\|^3\right) = (1-n)\hat{T}_t + \hat{C}_t + \mathrm{O}\left(\|\varepsilon\|^3\right) \quad \text{(A.61)}$$

In a similar way:

$$\hat{Y}_t^F = (-n)\hat{T}_t + \hat{C}_t + \mathrm{O}\left(\|\varepsilon\|^3\right) \quad \text{(A.62)}$$

And

$$\hat{Y}_t^W = n\hat{Y}_t^H + (1-n)\hat{Y}_t^F = \hat{C}_t \quad \text{(A.63)}$$

Further,

$$\left(\hat{Y}_t^H\right)^2 = \left[(1-n)\hat{T}_t + \hat{C}_t\right]^2 + \mathrm{O}\left(\|\varepsilon\|^3\right) \quad \text{(A.64)}$$

$$\left(\hat{Y}_t^F\right)^2 = \left[(-n)\hat{T}_t + \hat{C}_t\right]^2 + \mathrm{O}\left(\|\varepsilon\|^3\right) \quad \text{(A.65)}$$

$$\begin{aligned} &n\left(\hat{Y}_t^H\right)^2 + (1-n)\left(\hat{Y}_t^F\right)^2 = n\left[(1-n)^2\hat{T}_t^2 + 2(1-n)\hat{T}_t\hat{C}_t + \hat{C}_t^2\right] + \\ &(1-n)\left[n^2\hat{T}_t^2 - 2n\hat{T}_t\hat{C}_t + \hat{C}_t^2\right] + \\ &+\mathrm{O}\left(\|\varepsilon\|^3\right) = n(1-n)\hat{T}_t^2 + \hat{C}_t^2 + \mathrm{O}\left(\|\varepsilon\|^3\right) \end{aligned} \quad \text{(A.66)}$$

Further,

$$\eta\left[n\overline{Y}_t^H\hat{Y}_t^H + (1-n)\overline{Y}_t^F\hat{Y}_t^F\right] = -\eta(1-n)n\hat{T}_t\overline{Y}_t^R + \eta\hat{C}_t\overline{Y}_t^W + \mathrm{O}\left(\|\varepsilon\|^3\right) \quad \text{(A.67)}$$

d) Welfare derivation

Now we can substitute (A.63,66 and 67) into A.53:

$$
\begin{aligned}
&\frac{w_t}{U_C \overline{C}} = \phi \hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 - n\frac{1+\eta}{2}\hat{C}_t^2 - (1-n)\frac{1+\eta}{2}\hat{C}_t^2 - \frac{1}{2}(1+\eta)n(1-n)\hat{T}_t^2 \\
&+\eta\left[n\overline{Y}_t^H \hat{Y}_t^H + (1-n)\overline{Y}_t^F \hat{Y}_t^F\right] - \frac{1}{2}\frac{1+\eta\sigma}{\sigma}\left[nVar\hat{y}_t(h) + (1-n)Var\hat{y}_t(f)\right] \\
&+t.i.p. + \mathrm{O}\left(\|\varepsilon\|^3\right) = \phi\hat{C}_t - \frac{1}{2}(\rho+\eta)\hat{C}_t^2 - \frac{1}{2}(1+\eta)n(1-n)\hat{T}_t^2 + \\
&+\eta\left[n\overline{Y}_t^H \hat{Y}_t^H + (1-n)\overline{Y}_t^F \hat{Y}_t^F\right] \\
&-\frac{1}{2}\frac{1+\eta\sigma}{\sigma}\left[nVar\hat{y}_t(h) + (1-n)Var\hat{y}_t(f)\right] + t.i.p. + \mathrm{O}\left(\|\varepsilon\|^3\right) = \\
&\phi\hat{C}_t - \frac{1}{2}(\rho+\eta)\hat{C}_t^2 - \frac{1}{2}(1+\eta)n(1-n)\hat{T}_t^2 - \eta(1-n)n\hat{T}_t\overline{Y}_t^R + \eta\hat{C}_t\overline{Y}_t^W \\
&-\frac{1}{2}\frac{1+\eta\sigma}{\sigma}\left[nVar\hat{y}_t(h) + (1-n)Var\hat{y}_t(f)\right] + t.i.p. + \mathrm{O}\left(\|\varepsilon\|^3\right)
\end{aligned}
\tag{A.68}
$$

Now we rewrite (A.68) in terms of gaps:

$$
\begin{aligned}
&\frac{w_t}{U_C \overline{C}} = \phi\hat{C}_t - \frac{1}{2}(\rho+\eta)\left(\hat{C}_t - \tilde{C}_t\right)^2 - \frac{1}{2}(1+\eta)n(1-n)\left(\hat{T}_t - \tilde{T}_t\right)^2 - (\rho+\eta)\hat{C}_t\tilde{C}_t \\
&-n(1-n)(1+\eta)\hat{T}_t\,\tilde{T}_t - \eta(1-n)n\hat{T}_t\overline{Y}_t^R + \eta\hat{C}_t\overline{Y}_t^W - \\
&-\frac{1}{2}\frac{1+\eta\sigma}{\sigma}\left[nVar\hat{y}_t(h) + (1-n)Var\hat{y}_t(f)\right] \\
&+t.i.p. + \mathrm{O}\left(\|\varepsilon\|^3\right)
\end{aligned}
\tag{A.69}
$$

From Beetsma (25)

$$
\tilde{C}_t^W = \frac{\eta}{\eta+\rho}\overline{Y}_t^W \tag{A.70}
$$

Then,

$$
\eta\overline{Y}_t^W - (\rho+\eta)\tilde{C}_t = 0 \tag{A.71}
$$

Under substitution of (A.71) into (A.69) we obtain the following expression:

$$
\begin{aligned}
&\frac{w_t}{U_C \overline{C}} = -\frac{1}{2}(\rho+\eta)\left(\hat{C}_t - \tilde{C}_t\right)^2 - \frac{1}{2}(1+\eta)n(1-n)\left(\hat{T}_t - \tilde{T}_t\right)^2 - \\
&-\frac{1}{2}\frac{1+\eta\sigma}{\sigma}\left[nVar\hat{y}_t(h) + (1-n)Var\hat{y}_t(f)\right] + t.i.p. + \mathrm{O}\left(\|\varepsilon\|^3\right)
\end{aligned}
\tag{A.72}
$$

Now we derive the last two parameters: $Var\hat{y}_t \_and \_Var\hat{y}_t\left(f\right)$:

$$\mathrm{var}\left[\log\, y_t\, \left(h\right)\right] = \sigma^2\, \mathrm{var}\left[\log\, p_t\left(h\right)\right] + \mathrm{O}\left(\left\|\varepsilon\right\|^3\right) \tag{A.73}$$

Taking $\overline{p}_t \equiv E\left[\log p_t\left(h\right)\right]$, we have:

$$\mathrm{var}\left[\log p_t\left(h\right)\right] = \mathrm{var}\left[\log p_t\left(h\right) - \overline{p}_{t-1}\right] = \alpha^H\, \mathrm{var}\left[\log p_{t-1}\left(h\right)\right] +$$
$$\left(1-\alpha^H\right)\left[\log \tilde{p}_t\left(h\right) - \overline{p}_{t-1}\right]^2 - \left(\Delta\overline{p}_t\right)^2$$

Then, $\overline{p}_t - \overline{p}_{t-1} = \left(1-\alpha^H\right)\left[\log \tilde{p}_t\left(h\right) - \overline{p}_{t-1}\right]$ (A.74)

Using $\overline{p}_t = \log P_{H,t} + \mathrm{O}\left(\left\|\varepsilon\right\|^3\right)$ we obtain:

$$\begin{aligned}\mathrm{var}\left[\log p_t\left(h\right)\right] &= \alpha^H\, \mathrm{var}\left[\log p_{t-1}\left(h\right)\right] + \frac{\alpha^H}{1-\alpha^H}\left(\pi_t^H\right)^2 + \mathrm{O}\left(\left\|\varepsilon\right\|^3\right) = \\ &= \sum_{s=0}^{t}\left(\alpha^H\right)^{t-s}\frac{\alpha^H}{1-\alpha^H}\left(\pi_s^H\right)^2 + t.i.p. + \mathrm{O}\left(\left\|\varepsilon\right\|^3\right)\end{aligned} \tag{A.75}$$

Thus,

$$\sum_{t=0}^{\infty}\beta^t\, \mathrm{var}\left[\log\, p_t\left(h\right)\right] = \frac{\alpha^H}{\left(1-\alpha^H\beta\right)\left(1-\alpha^H\right)}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^H\right)^2 + t.i.p + \mathrm{O}\left(\left\|\varepsilon\right\|^3\right) \tag{A.76}$$

And for the second region:

$$\sum_{t=0}^{\infty}\beta^t\, \mathrm{var}\left[\log\, p_t\left(f\right)\right] = \frac{\alpha^F}{\left(1-\alpha^F\beta\right)\left(1-\alpha^F\right)}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^F\right)^2 + t.i.p + \mathrm{O}\left(\left\|\varepsilon\right\|^3\right) \tag{A.77}$$

So, the second-order welfare approximation gives:

$$\sum_{t=0}^{\infty} \beta^t E_0 w_t^C$$

$$\frac{w_t^C}{U_C \overline{C}} = -\frac{1}{2}(\rho+\eta)\left(\hat{C}_t - \tilde{C}_t\right)^2 - \frac{1}{2}(1+\eta)n(1-n)\left(\hat{T}_t - \tilde{T}_t\right)^2 - \tag{A.78}$$

$$-\frac{1}{2}\frac{1+\eta\sigma}{\sigma}\left[\begin{array}{l} n\sigma^2 \dfrac{\alpha^H}{\left(1-\alpha^H\beta\right)\left(1-\alpha^H\right)}\left(\pi_t^H\right)^2 + \\ +(1-n)\sigma^2 \dfrac{\alpha^F}{\left(1-\alpha^F\beta\right)\left(1-\alpha^F\right)}\left(\pi_t^F\right)^2 \end{array}\right]$$

Or in terms of losses the welfare function can be rewritten as:

$$L = \sum_{t=0}^{\infty} \beta^t E_0 L_t$$

$$L_t = \frac{1/\sigma}{n/k_C^H + (1-n)/k_C^F}\left(y^W\right)^2 + \frac{n(1-n)(1+\eta)/\sigma}{(n/k_C^H + (1-n)/k_C^F)(\rho+\eta)}\left(\hat{T}_t - \tilde{T}_t\right)^2 \tag{A.79}$$

$$+\frac{n/k_C^H}{n/k_C^H + (1-n)/k_C^F}\left(\pi_t^H\right)^2 + \frac{(1-n)/k_C^F}{n/k_C^H + (1-n)/k_C^F}\left(\pi_t^F\right)^2$$

Appendix B. Computational results

Economy under robust monetary policy, $\theta = 1.11$, error detection probability 0.2.

The worst-case model:

$$M = \begin{bmatrix} 0.9557 & 0.0356 & -0.0001 & 0.0001 & -0.0004 \\ 0.0121 & 0.8848 & -0.0034 & 0.0002 & -0.0016 \\ 0 & 0 & 0.001 & 0 & 0 \\ 0.0183 & 0.0514 & -0.6626 & 0.9290 & 0.0292 \\ -0.0221 & -0.0948 & -0.2675 & 0.0066 & 0.8842 \end{bmatrix}$$

$$N = \begin{bmatrix} -5.1321 & 36.5837 & 2.1954 & 1.0273 & -4.4182 \\ 0.0101 & 2.1954 & 173.1234 & 0.8125 & 1.5149 \\ -0.0076 & 1.0273 & 0.8125 & 0.0879 & -0.0379 \\ 0.0559 & -4.4182 & 1.5149 & -0.0379 & 0.6573 \\ 0.0025 & -0.6147 & -29.1285 & 0.0394 & 0.1278 \\ 0.0618 & 0.3827 & -0.0006 & 0.0007 & -0.0048 \\ 0.7056 & 5.1321 & -0.0101 & 0.0076 & -0.0559 \end{bmatrix}$$

Approximating model:

$$Ma = \begin{bmatrix} 0.9500 & -0.0000 & 0.0000 & -0.0000 & -0.0000 \\ 0.1586 & 1.7919 & -0.0048 & 0.0018 & -0.0131 \\ -0.0039 & -0.0242 & 0.0000 & -0.0000 & 0.0003 \\ -1.2604 & -7.8692 & -0.6497 & 0.9153 & 0.1293 \\ 0.8978 & 5.6037 & -0.2768 & 0.0164 & 0.8122 \end{bmatrix}$$

$$Na = \begin{bmatrix} -5.1321 & 36.5837 & 2.1954 & 1.0273 & -4.4182 \\ 0.0101 & 2.1954 & 173.1234 & 0.8125 & 1.5149 \\ -0.0076 & 1.0273 & 0.8125 & 0.0879 & -0.0379 \\ 0.0559 & -4.4182 & 1.5149 & -0.0379 & 0.6573 \\ 0.0025 & -0.6147 & -29.1285 & 0.0394 & 0.1278 \\ 0 & 0 & 0 & 0 & 0 \\ 0.7056 & 5.1321 & -0.0101 & 0.0076 & -0.0559 \end{bmatrix}$$

Malevolent nature reaction:

$$Fv = \begin{bmatrix} 8.7708 & 1.8734 & -0.0500 & -16.3588 & 11.7701 \end{bmatrix}$$

Losses in the worst case=0.6103

Losses in the approximating model =0.6070

Optimal policy without model uncertainty:

$$M = \begin{bmatrix} 0.9500 & 0.0000 & -0.0000 & -0.0000 & 0.0000 \\ 0.9500 & 1.0440 & -0.0151 & -1.0100 & 1.0100 \\ 48.7227 & 10.7279 & -0.3678 & -83.0632 & 70.4103 \\ -0.0000 & -0.0030 & -0.0050 & 1.0100 & -0.0000 \\ -0.0000 & 0.0140 & -0.0098 & 0.0000 & 1.0100 \end{bmatrix}$$

$$N = \begin{bmatrix} 8.1204 & 1.7900 & -0.2346 & -13.1372 & 12.0384 \\ 59.9670 & 13.0517 & -0.3487 & -114.0189 & 82.0922 \\ 13.0517 & 2.8706 & -0.0768 & -25.0869 & 18.0723 \\ -0.3487 & -0.0768 & 0.0083 & 0.6056 & -0.5016 \\ -114.0189 & -25.0869 & 0.6056 & 231.5947 & -157.0743 \\ 82.0922 & 18.0723 & -0.5016 & -157.0743 & 115.3894 \end{bmatrix}$$

Rational-expectation case, the central bank doesn't put any attention to the malevolent actions, error probability detection 50%:

$\theta = 2000$

$$M = \begin{bmatrix} 0.9500 & 0.0000 & -0.0000 & 0.0000 & -0.0000 \\ 0.0117 & 0.8831 & -0.0034 & 0.0002 & -0.0016 \\ 0 & 0 & 0.001 & 0 & 0 \\ 0.0173 & 0.0482 & -0.6626 & 0.9290 & 0.0292 \\ -0.0208 & -0.0907 & -0.2675 & 0.0066 & 0.8841 \end{bmatrix}$$

$$N = \begin{bmatrix} -4.9307 & 37.2998 & 2.1951 & 1.0291 & -4.4315 \\ 0.0102 & 2.1951 & 173.12 & 0.8125 & 1.5149 \\ -0.0070 & 1.0291 & 0.8125 & 0.0879 & -0.0380 \\ 0.0513 & -4.4315 & 1.5149 & -0.0380 & 0.6576 \\ 0.0020 & -0.6161 & -29.1285 & 0.0394 & 0.1278 \\ 0.0000 & 0.0002 & -0.0000 & 0.0000 & -0.0000 \\ 0.6213 & 4.9307 & -0.0102 & 0.0070 & -0.0513 \end{bmatrix}$$

$$Ma = \begin{bmatrix} 0.9500 & -0.0000 & 0.0000 & -0.0000 & -0.0000 \\ 0.0116 & 0.8829 & -0.0034 & 0.0002 & -0.0016 \\ 0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 \\ 0.0177 & 0.0504 & -0.6626 & 0.9290 & 0.0292 \\ -0.0211 & -0.0923 & -0.2675 & 0.0066 & 0.8841 \end{bmatrix}$$

$$Na = \begin{bmatrix} -4.9307 & 37.2998 & 2.1951 & 1.0291 & -4.4315 \\ 0.0102 & 2.1951 & 173.1234 & 0.8125 & 1.5149 \\ -0.0070 & 1.0291 & 0.8125 & 0.0879 & -0.0380 \\ 0.0513 & -4.4315 & 1.5149 & -0.0380 & 0.6576 \\ 0.0020 & -0.6161 & -29.1285 & 0.0394 & 0.1278 \\ 0 & 0 & 0 & 0 & 0 \\ 0.6213 & 4.9307 & -0.0102 & 0.0070 & -0.0513 \end{bmatrix}$$

$$Fv = \begin{bmatrix} -0.0024 & -0.0005 & 0.0000 & 0.0046 & -0.0033 \end{bmatrix}$$

Losses worst-case = 0.5373

Losses approximating model=0.5373

Error detection probability 30%, θ =1.8797

$$M = \begin{bmatrix} 0.9532 & 0.0206 & -0.0000 & 0.0000 & -0.0003 \\ 0.0119 & 0.8841 & -0.0034 & 0.0002 & -0.0016 \\ 0 & 0 & 0.001 & 0 & 0 \\ 0.0179 & 0.0500 & -0.6626 & 0.9290 & 0.0292 \\ -0.0215 & -0.0930 & -0.2675 & 0.0066 & 0.8841 \end{bmatrix}$$

$$N = \begin{bmatrix} -5.0414 & 36.8941 & 2.1953 & 1.0281 & -4.4241 \\ 0.0102 & 2.1953 & 173.1234 & 0.8125 & 1.5149 \\ -0.0073 & 1.0281 & 0.8125 & 0.0879 & -0.0380 \\ 0.0538 & -4.4241 & 1.5149 & -0.0380 & 0.6575 \\ 0.0022 & -0.6154 & -29.1285 & 0.0394 & 0.1278 \\ 0.0345 & 0.2215 & -0.0004 & 0.0004 & -0.0028 \\ 0.6666 & 5.0414 & -0.0102 & 0.0073 & -0.0538 \end{bmatrix}$$

$$Ma = \begin{bmatrix} 0.9500 & 0.0000 & 0.0000 & -0.0000 & 0.0000 \\ -0.3189 & -1.2414 & 0.0002 & -0.0034 & 0.0248 \\ 0.0088 & 0.0567 & -0.0001 & 0.0001 & -0.0007 \\ 2.9077 & 18.6130 & -0.6935 & 0.9605 & -0.2013 \\ -2.1013 & -13.4526 & -0.2453 & -0.0161 & 1.0501 \end{bmatrix}$$

$$Na = \begin{bmatrix} -5.0414 & 36.8941 & 2.1953 & 1.0281 & -4.4241 \\ 0.0102 & 2.1953 & 173.1234 & 0.8125 & 1.5149 \\ -0.0073 & 1.0281 & 0.8125 & 0.0879 & -0.0380 \\ 0.0538 & -4.4241 & 1.5149 & -0.0380 & 0.6575 \\ 0.0022 & -0.6154 & -29.1285 & 0.0394 & 0.1278 \\ 0 & 0 & 0 & 0 & 0 \\ 0.6666 & 5.0414 & -0.0102 & 0.0073 & -0.0538 \end{bmatrix}$$

$$Fv = \begin{bmatrix} -20.7672 & -4.4688 & 0.1193 & 39.0286 & -28.0881 \end{bmatrix}$$

Error detection probability 40%, $\theta = 1.8816$

$$M = \begin{bmatrix} 0.9532 & 0.0206 & -0.0000 & 0.0000 & -0.0003 \\ 0.0119 & 0.8841 & -0.0034 & 0.0002 & -0.0016 \\ 0 & 0 & 0.001 & 0 & 0 \\ 0.0179 & 0.0500 & -0.6626 & 0.9290 & 0.0292 \\ -0.0215 & -0.0930 & -0.2675 & 0.0066 & 0.8841 \end{bmatrix}$$

$$N = \begin{bmatrix} -5.0413 & 36.8946 & 2.1953 & 1.0281 & -4.4241 \\ 0.0102 & 2.1953 & 173.1234 & 0.8125 & 1.5149 \\ -0.0073 & 1.0281 & 0.8125 & 0.0879 & -0.0380 \\ 0.0538 & -4.4241 & 1.5149 & -0.0380 & 0.6575 \\ 0.0022 & -0.6154 & -29.1285 & 0.0394 & 0.1278 \\ 0.0344 & 0.2212 & -0.0004 & 0.0004 & -0.0027 \\ 0.6665 & 5.0413 & -0.0102 & 0.0073 & -0.0538 \end{bmatrix}$$

$$Ma = \begin{bmatrix} 0.9500 & 0.0000 & -0.0000 & 0.0000 & 0.0000 \\ -0.3162 & -1.2236 & 0.0001 & -0.0034 & 0.0246 \\ 0.0088 & 0.0563 & -0.0001 & 0.0001 & -0.0007 \\ 2.8833 & 18.4577 & -0.6932 & 0.9602 & -0.1994 \\ -2.0838 & -13.3409 & -0.2454 & -0.0159 & 1.0487 \end{bmatrix}$$

$$Na = \begin{bmatrix} -5.0413 & 36.8946 & 2.1953 & 1.0281 & -4.4241 \\ 0.0102 & 2.1953 & 173.1234 & 0.8125 & 1.5149 \\ -0.0073 & 1.0281 & 0.8125 & 0.0879 & -0.0380 \\ 0.0538 & -4.4241 & 1.5149 & -0.0380 & 0.6575 \\ 0.0022 & -0.6154 & -29.1285 & 0.0394 & 0.1278 \\ 0 & 0 & 0 & 0 & 0 \\ 0.6665 & 5.0413 & -0.0102 & 0.0073 & -0.0538 \end{bmatrix}$$

$$Fv = \begin{bmatrix} -20.5938 & -4.4315 & 0.1183 & 38.7031 & -27.8538 \end{bmatrix}$$

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