Unpleasant Monetarist Arithmetic
and the Sustainability of Public Debt

Sergey Pekarski

State University – Higher School of Economics
101000, Myasnitskaya str. 20, Moscow, Russia,
Tel.: +7-910-4057009; Fax: +7-495-9179858; E-mail: spekarski@hse.ru

Abstract

In the celebrated paper "Some unpleasant monetarist arithmetic", Sargent and Wallace (1981) showed that tight monetary policy is not feasible unless it is supported by appropriate fiscal adjustment. In this paper, we explore a simple forward-looking monetary model to show that a gradual or anticipated tightening of monetary policy is not necessarily characterized by "unpleasant arithmetic". This is due to possible short-run gains in seigniorage revenues during the transition period. By applying the principle of sustainable macroeconomic policy that constrains the future choices of both fiscal and monetary authorities, we show that high interest rates on public debt work for the benefit of "pleasant arithmetic." In this case permanent fiscal expansion does not necessary have inflationary consequences.

1. Introduction

One of the most important economic problem in the U.S. and other developed countries in the aftermath of 2007-2009 financial crises is the extremely high level of public debt. On one hand, there are serious doubts in the ability of necessary fiscal adjustment to keep fiscal stance sustainable. While interest rates on public debt in developed countries are relatively low nowadays, one would expect they will rise in the near future. This makes the fiscal stance even worse. On the other hand, there are concerns in the inflationary consequences of the past and projected budget deficits, as well as in the ability of monetary policy to fight inflation in these circumstances.

In the celebrated paper "Some unpleasant monetarist arithmetic" (UMA) Sargent and Wallace (1981) (SW) showed that tight monetary policy is not feasible under the regime of fiscal dominance. Without fiscal adjustment, a decrease in the growth rate of base money now requires higher seigniorage revenue (and thus a higher growth rate of base money) in the future to stabilize growing public debt. Moreover, if expectations are forward-looking, then tight money leads to higher inflation not only in the future, but immediately.

The purpose of this paper is to show that UMA is not a necessary outcome using the very simple setup of the forward-looking monetary model. In this respect, our paper is close to Buffie (2003a). He modifies the original backward-looking model developed by Drazen (1985) by allowing the inflation rate to be a jump variable and explores the equilibrium path of the real money balances and public debt "overlooked" by Drazen and SW that gives "pleasant monetarist arithmetic" for tight monetary policy.[1] This path followed by a decrease in the growth rate of base money is associated with the *transitory* gain in seigniorage revenue and thus does not require the extraction of extra revenues from money creation (or fiscal adjustment) to stabilize public debt.

In this paper, we propose a more general, though simple framework. Instead of the *ad- hoc* assumption that public debt must be stabilized by a certain date in the future, we consider the principle of sustainable macroeconomic policy that requires future budget surpluses and seigniorage revenues to provide appropriate backing for the accumulated public debt. Another important element of our analysis is that we allow for the preannouncement of future policy changes. This is important because it generates transitional dynamics in the system up to the time of the actual policy switch. We show that there is a gain in seigniorage during this transition. This is similar to Buffie's finding. However, while the existence of the special path associated with the transitory gain in seigniorage following a tightening in monetary policy in the Buffie model requires certain restrictions on preferences (money demand parameters) and the parameters of the policy switch (timing and magnitude of the change in the growth rate of base money), a preannouncement unambiguously provides a transitory gain in seigniorage.

Nevertheless, even in the case where monetary tightening is associated with a short-run gain in seigniorage, it does not automatically follow that public debt is kept sustainable. This is because tight monetary policy unambiguously leads to lower steady state seigniorage on the increasing branch ("efficient" side) of the inflation tax Laffer curve. Thus, the question is what is more important: short-term gain or long-run decrease in seigniorage revenue? In this respect, the analysis in this paper helps us to stress the crucial role of the interest rate on public debt that is present, but rather undermined in Buffie's analysis. We show that when the interest rate is relatively high, the possibility of transitory gain in seigniorage followed by a gradual or preannounced decrease in the

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[1] The backward-looking system analyzed by Drazen has an unstable steady state (for a given constant level of the growth rate of base money and budget deficit). This result falls short of the Samuelson's Correspondence Principle and may be justified only if one applies the somewhat artificial assumption that fiscal and monetary authorities can keep the economy in equilibrium in the absence of any shocks. By contrast, the forward-looking dynamics of the system are stable. This problem does not arise if one considers monetary policy in terms of money finance to bond finance ratio rather than setting the growth rate of base money. See McCallum (1984) and Liviatan (1986, 1988).

growth rate of money implies a higher present discounted value of future seigniorage that keeps public debt sustainable.[2] Thus, tight monetary policy *may* be feasible without any fiscal adjustment.

The rest of the paper is organized as follows. Section 2 presents a review of recent contribution to the literature on UMA. Section 3 contains a very simple forward-looking monetary model to analyze the interaction between fiscal and monetary policy. Special examples of the monetary policy tightening that may give rise to "pleasant monetarist arithmetic" are presented in Section 4. The role of sustainability constraint in the interaction between fiscal and monetary policy is discussed in Section 5.The final section provides concluding remark.

2. Recent literature survey

The problem of the interaction of fiscal and monetary policy sparked an interesting discussion that still continues. Liviatan (1984) and Drazen (1985) showed that UMA holds only if the demand for money is inelastic with respect to the nominal interest rate. In this case, the economy operates on the increasing branch ("efficient" side) of the inflation tax Laffer curve. Velasco (1993) arrived at similar results after modifying the Drazen model for an open economy, floating exchange rate and perfect capital mobility.

While the theoretical results were undoubtedly interesting, some economists were skeptical about how realistic the basic assumptions were. Darby (1984) considers that SW's assumption that the interest rate is greater than the growth rate of output does not hold for the economy of the USA and other developed countries. This assumption is indeed critical for all analysis of macroeconomic policy. Answering Darby's criticism, Miller and Sargent (1984) note that the UMA can (and should) be considered in a wider context, and not just literally. The growth of public debt as a result of a tightening of monetary policies can bring about an increase in the interest rate for a variety of reasons. If so, then Darby's methodology, which included the average interest rate for previous periods, could be erroneous (and subject to Lucas' critique).

Agreeing that the assumption that the interest rate is greater than the growth rate of output is not incontestable, Bhattacharya, Guzman and Smith (1998) showed that this assumption is not necessary for the existence of UMA. The authors include an extra asset in SW's model that is available to the private sector and financial intermediaries. In doing so, savings, as an additional asset, conform to the requirement of partial reservation. It was shown that, taking these additions into account, UMA can exist if the economy contains at least one asset with a rate of return that is greater than the growth rate of output. In the real world such assets, obviously, almost always exist.

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[2] This result is in sharp contrast with one obtained by Chadha and Nolan (2004), who show, that permanent budget deficits imply an upper bound on the trajectory of short-term real interest rate, which is assumed to be under indirect control of monetary authorities.

Bhattacharya et al. (1997) stress the general role of reserve requirements as an instrument that allows inflation tax base, and thus seigniorage collection, to be regulated. Bhattacharya and Haslag (2003) go further and show that in the case when a monetary tightening is realized via a higher reserves ratio, monetary arithmetic is "more pleasant". This result is based on the relationship between reserves requirements, gross real return on deposits and capital accumulation. An important implication of this paper is that while fiscal dominance imposes constraints on monetary policy as a whole, it does not do so for a particular monetary policy instrument.

Espinosa and Russell (1998a,b) and Bhattacharya and Kudoh (2002) consider the plausibility of UMA when the initial interest rate is lower than growth rate of output under different monetary policy rules (fixed bonds-money ratio rule versus money-growth rule). This avenue of research goes far beyond the original subject of UMA, considering it in a more general framework of the real effects of fiscal and monetary policy. Nikitin and Russell (2006) contribute to this literature and provide a well-structured survey.

Dornbusch (1996) suggests additional considerations that strengthen UMA. First, tight monetary policy leads to higher interest rates, and thus higher debt service and more rapid accumulation of public debt. Second, tight monetary policy may worsen the fiscal position by lowering tax revenues and increasing unavoidable government spending. Third, higher interest rates can depress economic growth, thus leading to more rapid growth of the debt to GDP ratio.[3] In fact, this means that tight money policy by worsening the set of alternatives for fiscal policy produces "unpleasant *fiscal* arithmetic" as well as UMA (see, e.g., Gokhale, 2007, and Hasko, 2007).

Apart from the discussion of the validity of UMA and its extensions on a theoretical basis, there is a wide branch of literature on the empirical implications of UMA.[4] Indeed, the main prediction of FTI is that budget deficits are inflationary either in the short-run or in the long-run.[5] The evidence is not solid, but in general, it does not support this view for major developed countries. At the same time, tight money policies are subject to UMA in highly indebted developing countries and during high inflation episodes. One possible explanation of this apparent failure is that one important assumption of FTI, namely that the economy is in a regime of fiscal dominance, may not hold in practice (at least all the time). That is, it may be the case that the government (not the central bank) adjusts its policy at times when public debt becomes high. Our analysis has important implications for this discussion. If there is indeed a possibility for tight money to

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[3] Barnett (2005) investigates consequences of tight money policy in a simple *AK* growth model with money, reserves requirements, and public debt.

[4] These are King and Plosser (1985), Viera (2000), Fischer, Sahay and Vegh (2002), Catão and Terrones (2005), Giannitsarou and Scott (2006), Kwon, McFarlane and Robinson (2009) among others.

[5] Drazen and Helpman (1990) stress that the theory does not predict the strong correlation between budget deficits and inflation that is due to uncertainty about the type and timing of future policy shifts. This, however, does not mean that a reduction in budget deficit is unnecessarily to stop high inflation.

sometimes have "unpleasant arithmetic" and "pleasant arithmetic" at other times, then FTI does not unambiguously predict the inflationary consequences of budget deficits under the assumption of fiscal dominance. In the final section of the paper we confirm that in some cases an increase in the budget deficit can be accompanied by some sort of tight (and low inflation) monetary policy. Thus, an appropriate case study is a good alternative to the time series analysis that may help clarify whether and when budget deficits are inflationary.[6]

3. The model

To simplify the exposition we employ the forward-looking model of inflation dynamics proposed by Sargent and Wallace (1973). Appendix A provides necessary micro-foundations. The demand for real money balances, $m^d = (M/P)^d$, is log-linear in the expected inflation rate, π^e:

$$m^d = e^{-\alpha \pi^e}, \tag{1}$$

where $\alpha = -(dm^d/m^d)/d\pi^e > 0$ is the semi-elasticity and the scale parameter is normalized to unity. The growth rate of real money balances is equal to the difference between the growth rate of the base money, $\mu(t) = \dot{M}^s(t)/M^s(t)$, and the actual inflation rate, π:[7]

$$\dot{x}(t) = \frac{\dot{m}(t)}{m(t)} = \mu(t) - \pi(t), \tag{2}$$

where $x = \ln m^d = -\alpha \pi^e$. Assuming perfect foresight, $\pi^e(t) = \pi(t)$, and combining equations gives us:

$$\dot{\pi}(t) = -\frac{1}{\alpha}(\mu(t) - \pi(t)). \tag{3}$$

Imposing the additional condition for the absence of a hyperinflationary bubble,

$$\lim_{t \to \infty} \pi(t) e^{-\frac{1}{\alpha}t} = 0, \tag{4}$$

we arrive at fundamental forward-looking solutions to (3) and (2):

$$\pi(t) = \frac{1}{\alpha} \int_t^\infty \mu(\tau) e^{-\frac{1}{\alpha}(\tau - t)} d\tau, \tag{5}$$

$$x(t) = -\int_t^\infty \mu(\tau) e^{-\frac{1}{\alpha}(\tau - t)} d\tau. \tag{6}$$

Seigniorage, $S = \mu m$, is thus determined by the current and future monetary policy:

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[6] Buffie (2003b) provides such a case study of inflationary episodes in Sub-Saharan African countries and describes quite different consequences of tight monetary policies. Salamon (2001) finds empirical support for UMA using a non-linear time series model to study inflation in Brazil.
[7] In general, all variables are functions of time. A dot over a variable denotes the derivative with respect to time.

$$S(t) = \mu(t)e^{-\int_t^\infty \mu(\tau)e^{-\frac{1}{\alpha}(\tau-t)}d\tau}. \tag{7}$$

Let us assume, following SW's original assumption, that the fiscal policy is dominant. That is, when monetary authorities set the trajectory of the growth rate of base money they consider future budget deficits as given. SW and subsequent authors (Drazen, 1985; Buffie, 2003) assume that monetary policy should be endogenous to the dynamics of public debt: when debt reaches some upper limit and cannot be stabilized by means of fiscal adjustment, it should be monetized. While this assumption is reasonable, it is not general. It is more convenient to consider the principle of public debt sustainability that implies a *joint* constraint on both fiscal and monetary policy: at each point in time t and for every volume of accumulated public debt $b(t)$, future trajectories of the real primary budget deficit $d(t)$ and seigniorage $S(t)$ must satisfy:

$$b(t) \leq \int_t^\infty \big(S(\tau) - d(\tau)\big)e^{-r(\tau-t)}d\tau, \tag{8}$$

where b is the real (indexed) public debt, and r is the constant real interest rate on public debt.[8] Given exogenous (dominant) fiscal policy, (8) determines the constraint on the *present discounted value* of future seigniorage, while its transitory dynamics may in fact be arbitrary. This is the crucial point in our analysis. In what follows we may characterize future monetary policy as feasible in the sense that it remains compatible with the sustainability of public debt, if it does not lead to a decrease in the present discounted value of seigniorage.

To examine whether tight monetary policy is feasible under a regime of fiscal dominance, we explore several "textbook" theoretical experiments on the forward-looking dynamics.

4. Feasibility of tight monetary policy

Permanent unexpected one-step decrease in the growth rate of base money

Consider first the case when monetary authorities conduct once and for all a decrease in μ. Prior to the date t_1, the growth rate was kept constant at $\mu = \mu_0$. Starting from date t_1, the growth rate is unexpectedly decreased and held constant at $\mu_1 < \mu_0$. It follows from equations (5)-(7) that the corresponding variables jump (without gradual transition dynamics) at time t_1 from their initial steady levels, $\pi_0 = \mu_0$, $x_0 = -\alpha\mu_0$, and $S_0 = \mu_0 e^{-\alpha\mu_0}$, to their new steady levels, $\pi_1 = \mu_1$, $x_1 = -\alpha\mu_1$, and $S_1 = \mu_1 e^{-\alpha\mu_1}$, respectively. The inflation rate (log of real money balances) becomes permanently higher (lower). We assume that the economy operates on the efficient side of the inflation tax Laffer

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[8] Equation (8) is a present-value government budget constraint. Appendix describes corresponding dynamic (flow) budget constraint (A4).

curve, where money demand is inelastic. In this case seigniorage becomes permanently smaller. Its present discounted value also decreases. If (8) holds with equality prior to time t_1, then a tightening of monetary policy leads to a violation of (8). Thus, such monetary policy is not feasible under fiscal dominance. This resembles the essence of UMA.

Permanent unexpected two-step decrease in the growth rate of base money

Surprisingly, the previous result does not hold *in general* if the growth rate of base money decreases gradually. Consider, for example, the simplest case of a two-step decrease in μ from its initial constant level $\mu = \mu_0$. At time t_1 agents learn that for the time interval $[t_1, t_2)$ μ will be set at $\mu_1 < \mu_0$, while later (for $t \geq t_2$) it will be decreased further to $\mu_2 < \mu_1$. Equations (5)-(7) determine the transitory dynamics of the variables:

$$\pi(t) = \begin{cases} \mu_0, & t < t_1, \\ \mu_1 - (\mu_1 - \mu_2)e^{-\frac{1}{\alpha}(t_2 - t)}, & t_1 \leq t < t_2, \\ \mu_2, & t \geq t_2. \end{cases} \qquad (9)$$

$$x(t) = \begin{cases} -\alpha\mu_0, & t < t_1, \\ -\alpha\mu_1 + \alpha(\mu_1 - \mu_2)e^{-\frac{1}{\alpha}(t_2 - t)}, & t_1 \leq t < t_2, \\ -\alpha\mu_2, & t \geq t_2. \end{cases} \qquad (10)$$

$$S(t) = \begin{cases} \mu_0 e^{-\alpha\mu_0}, & t < t_1, \\ \mu_1 e^{-\alpha\mu_1 + \alpha(\mu_1 - \mu_2)e^{-\frac{1}{\alpha}(t_2 - t)}}, & t_1 \leq t < t_2, \\ \mu_2 e^{-\alpha\mu_2}, & t \geq t_2. \end{cases} \qquad (11)$$

Fig. 1 illustrates the dynamics.[9] The inflation rate (the log of real money balances) has one announcement jump at time t_1 by $\Delta\pi(t = t_1) = \mu_1 - (\mu_1 - \mu_2)e^{-(1/\alpha)(t_2 - t_1)} - \mu_0 < 0$ ($\Delta x(t = t_1) = -\alpha\mu_1 + \alpha(\mu_1 - \mu_2)e^{-(1/\alpha)(t_2 - t_1)} + \alpha\mu_0 > 0$) and then it gradually declines (rises) towards its new steady state level. It follows from (11) that seigniorage is an increasing function of time for $t_1 \leq t < t_2$. It jumps twice at t_1 and t_2. The last jump is unambiguously downward: $\Delta S(t = t_2) = (\mu_2 - \mu_1)e^{-\alpha\mu_2} < 0$. The first jump at time t_1 may be either downward or upward. But even if the seigniorage experiences a downward jump at time t_1, during the transitional period $[t_1, t_2)$ it *may* become higher than its initial level. Fig. 1 depicts possible scenarios.

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[9] For the sake of space Fig. 1 and Fig. 2 present only the dynamics of μ, x and S. The time path of π is simply a regular reflection of the time path of x.

The fact that the direction of transitory dynamics of seigniorage differs from the direction of change in its steady state level has an important implication. As long as the *present discounted value* of future seigniorage is important for the sustainability of public debt, a temporary increase in seigniorage above its initial steady state during the transitional period *may* lead to a higher present discounted value. This is more likely if the interest rate is relatively high, and thus a future decrease in steady state seigniorage is heavily discounted. Putting all said together, this result implies that a gradual (two-step) decrease in the growth rate of base money *in some cases* may be consistent with the sustainability of public debt, and thus tight monetary policy conducted in this way *may* be feasible.
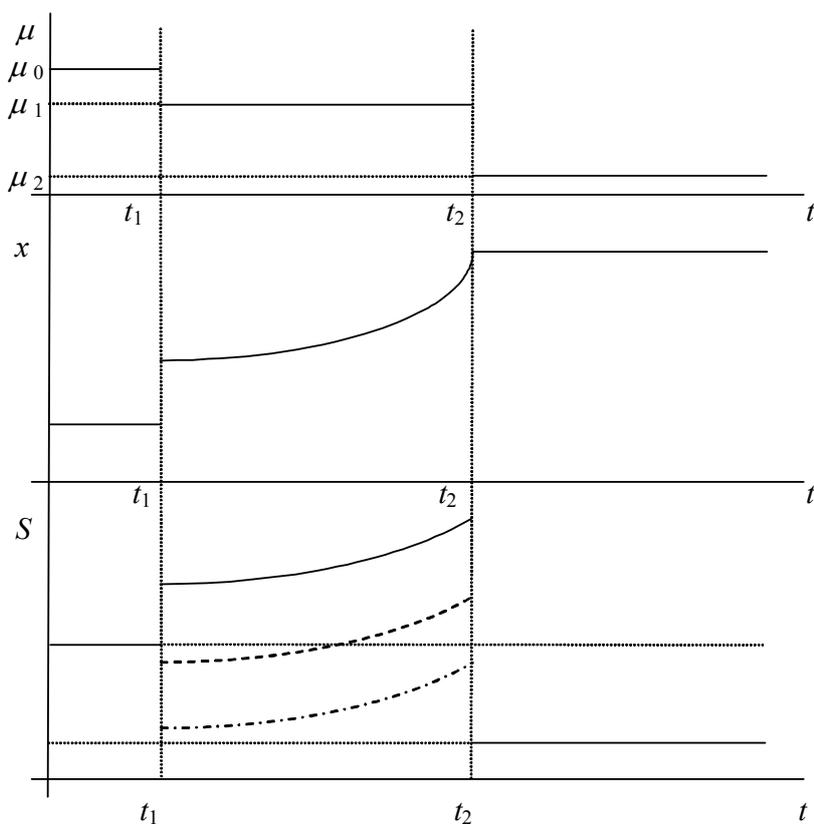

Fig. 1. The case of an unexpected two-step decrease in the growth rate of base money

A permanent anticipated decrease in the growth rate of base money

When a permanent decrease in μ is preannounced, seigniorage is always higher than its initial steady state level and rises during the transition dynamics. Consider the following example. Starting with a constant growth rate of base money, $\mu(t) = \mu_0$, at time t_A the central bank announces that in the future, starting from $t_S > t_A$, the growth rate of base money will be increased

to $\mu(t) = \mu_1 > \mu_0$.[10] The dynamics of inflation, the log of real money balances, and seigniorage are as follows:

$$\pi(t) = \begin{cases} \mu_0, & t < t_A, \\ \mu_0 - (\mu_0 - \mu_1)e^{-\frac{1}{\alpha}(t_S - t)}, & t_A \le t < t_S, \\ \mu_1, & t \ge t_S. \end{cases} \tag{12}$$

$$x(t) = \begin{cases} -\alpha\mu_0, & t < t_A, \\ -\alpha\mu_0 + \alpha(\mu_0 - \mu_1)e^{-\frac{1}{\alpha}(t_S - t)}, & t_A \le t < t_S, \\ -\alpha\mu_1, & t \ge t_S. \end{cases} \tag{13}$$

$$S(t) = \begin{cases} \mu_0 e^{-\alpha\mu_0}, & t < t_A, \\ \mu_0 e^{-\alpha\mu_0 + \alpha(\mu_0 - \mu_1)e^{-\frac{1}{\alpha}(t_S - t)}}, & t_A \le t < t_S, \\ \mu_1 e^{-\alpha\mu_1}, & t \ge t_S. \end{cases} \tag{14}$$

Prior to the announcement, the money market is in a steady state. The announcement at time $t = t_A$ leads to discrete jumps in inflation, $\Delta\pi(t = t_A) = (\mu_1 - \mu_0)e^{-(1/\alpha)(t_S - t_A)} < 0$, in the log of real money balances, $\Delta x(t = t_A) = -\alpha(\mu_1 - \mu_0)e^{-(1/\alpha)(t_S - t_A)} > 0$, and in seigniorage: $\Delta S(t = t_A) = -\mu_0 e^{-\alpha\mu_0}\left[1 - e^{-\alpha(\mu_1 - \mu_0)e^{-(1/\alpha)(t_S - t_A)}}\right] > 0$. Fig. 2 shows the time paths of these variables (see also numerical examples in the Appendix B).

Up to time t_S, when monetary policy switches, the inflation rate and the log of real money balances gradually adjust to their new steady levels (decreasing and increasing, respectively). Seigniorage gradually increases on the interval $[t_A, t_S)$ and undergoes another discrete jump, $\Delta S(t = t_S) = (\mu_1 - \mu_0)e^{-\alpha\mu_1} < 0$, at time t_S. The new steady state value of seigniorage is lower than it was initially. However, this does not unambiguously lead to a decrease in its present discounted value at time t_A. If the interest rate is relatively high, then the present value of future seigniorage revenues *may* increase. Thus, as in the previous case, tight monetary policy *may* be feasible.

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[10] Actually, the permanent preannounced change in μ can be viewed as a special case of the two-step decrease in μ when $t_1 = t_A$ and $\mu_1 = \mu_0$.
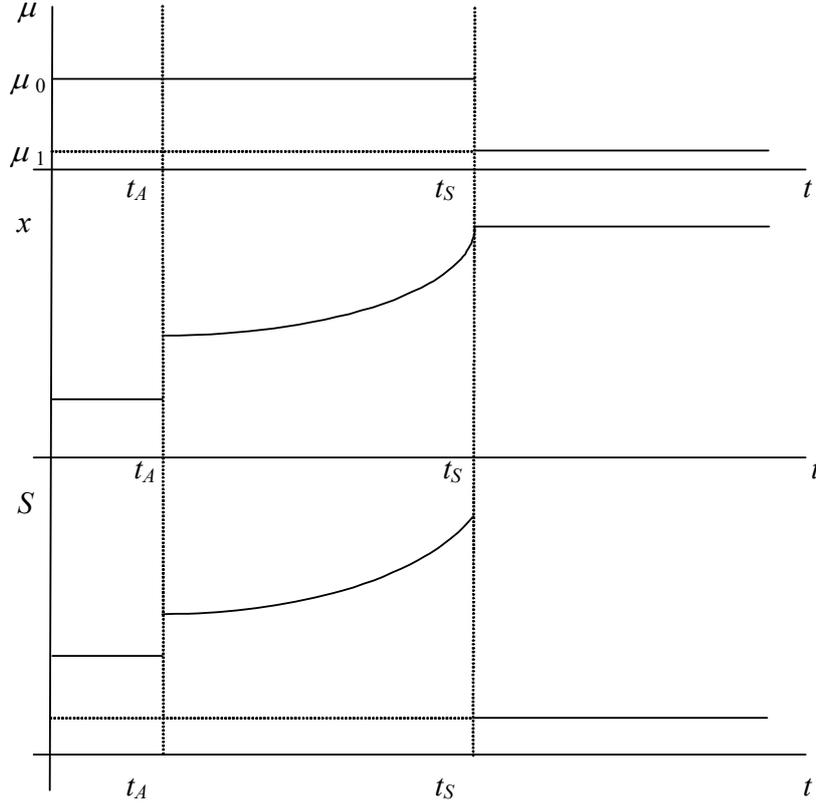
Fig. 2. The case of permanent anticipated decrease in the growth rate of base money

5. Changes in the sustainable level of public debt

Consider the behavior of the sustainable level of public debt in the case of a permanent anticipated change (either an increase or a decrease) in the growth rate of base money. Substituting (14) into (8) and assuming for expositional simplicity constant level of the primary budget deficit yields:

$$
b(t) = \begin{cases}
\dfrac{\mu_0 e^{-\alpha\mu_0} - d}{r}, & t < t_A, \\[3mm]
\displaystyle\int_t^{t_S} \mu_0 e^{-\alpha\mu_0 + \alpha(\mu_0-\mu_1)e^{-\frac{1}{\alpha}(t_S-\tau)} - r(\tau-t)}\,d\tau + \dfrac{\mu_1 e^{-\alpha\mu_1 - r(t_s-t)}}{r} - \dfrac{d}{r}, & t_A \le t < t_S, \\[3mm]
\dfrac{\mu_1 e^{-\alpha\mu_1} - d}{r}, & t \ge t_S.
\end{cases}
\tag{15}
$$

At time t_A sustainable level of public debt undergoes a discrete change of the size:

$$
\Delta b(t_A) = \int_{t_A}^{t_S} \mu_0 e^{-\alpha\mu_0 + \alpha(\mu_0-\mu_1)e^{-\frac{1}{\alpha}(t_S-\tau)} - r(\tau-t_A)}\,d\tau + \frac{1}{r}\left(\mu_1 e^{-\alpha\mu_1 - r(t_s-t_A)} - \mu_0 e^{-\alpha\mu_0}\right)
\tag{16}
$$

This change reflects the adjustment in expected present discounted value of future seigniorage revenues following the announcement of future monetary policy switch. The complexity of (16)

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makes it difficult to derive closed-form conditions for the positive value of $\Delta b(t_A)$. However, one can follow this line of reasoning (see also numerical examples in the Appendix B). The first term in the right side of (16) is always positive and decreasing in r. The second term is also decreasing in r. It is positive if $\mu_1 > \mu_0$ and r is relatively small. If $\mu_1 < \mu_0$, then the second term is negative. At the same time, the first term is an increasing function of $(\mu_0 - \mu_1)$. Putting all together we can state the following:

Proposition 1: Anticipated tight (loose) monetary policy can provide an increase in the sustainable level of public debt only if the interest rate is relatively high (small).

Proposition 2: The higher (lower) is the interest rate, the higher is an increase in the sustainable level of public debt in case when anticipated tight (loose) monetary policy produces a gain in the present value of future seigniorage revenues.

Fig. 3 and Fig. 4 illustrate Propositions (these numerical examples correspond to specification discussed in the Appendix B). An increase in the sustainable level of public debt following changes in expectations with respect to future monetary policy means that actual level of public debt, which is predetermined by past macroeconomic policy, is lower than current sustainable level. Thus future switch is monetary policy is feasible at it does not violate the sustainability constraint (8).
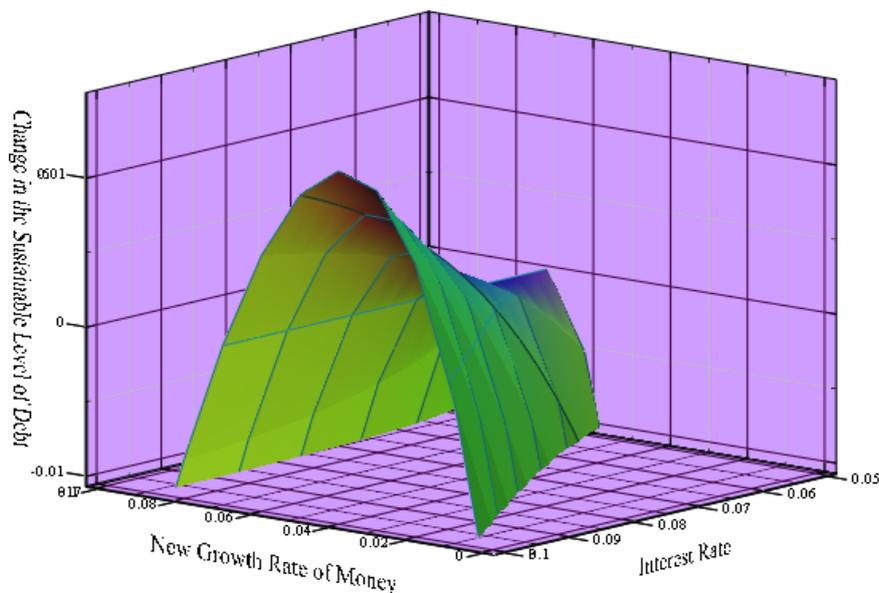

Fig. 3. Options for monetary policy when the interest rate is relatively high
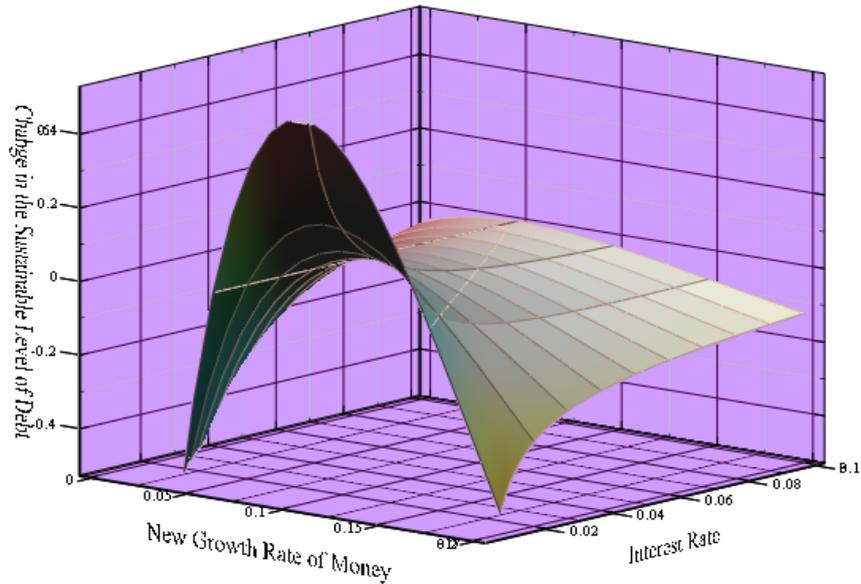
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Fig. 4. Options for monetary policy when the interest rate is relatively low

One can also consider another implication of Propositions. Assume that the government needs to increase primary budget deficit now and/or in the future. (This is what happens, for example, during and in the aftermath of economic recession.) It is not possible in absence of the adjustment of monetary policy as it violates sustainability constraint (8). Thus the question is how monetary policy can compensate an increase in the present discounted value of future budget deficits. It follows that when the interest rate is relatively high (low), monetary policy should form expectations of a decrease (increase) in the growth rate of base money in the future. In other words, permanent fiscal expansion does not necessary require accompanying loose monetary policy resulting in higher inflation now and in the future.

Another important conclusion following from (16) is that the change in the sustainable level of public debt as a function of the new growth rate of base money demonstrates its own Laffer curve property. Indeed, while the first term in the right side of (16) decreases with μ_1, the second term is a hump-shaped function of μ_1. Fig. 3 and Fig. 4 confirm this finding. In turn, it implies two major inferences. First, the possibility to use preannounced monetary policy to generate gains in the present value of future seigniorage revenue is limited from above. Second, there are two levels of μ_1 that allows two achieve the certain attainable level of $\Delta b(t_A)$.

6. Concluding remark on policy perspectives

The possibility of a transitional gain in seigniorage allows us to re-emphasize the role of the interest rate on public debt. In the setup considered by SW a high interest rate may be seen as a strengthening factor for UMA: the higher the interest rate, the faster public debt grows following

the initial cut in seigniorage revenue. However, if there is a transitional gain in seigniorage instead of a loss, the role of a high interest rate is reversed. For a given future path of budget deficits, the sustainability of public debt requires a certain *present discounted value* of future seigniorage revenues. A higher interest rate implies a higher discounting of future revenues. In this case, a short-run gain in seigniorage becomes more important than its long-run decrease.

In other words, while higher interest rates (that one would expect in the near future) will certainly worse the fiscal stance, it opens the opportunity for monetary policy to fight inflation even without otherwise necessarily fiscal adjustment.

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Appendix A. Microeconomic foundations

Consider a representative agent with an infinite time horizon who maximizes the life-time utility from consumption and real money balances.[11]

$$\max_{c,m} \int_0^\infty \left[v(c) + w(m) \right] e^{-\rho t} dt .$$

(A1)

Here c is consumption, $m = M/P$ is the real money balances, ρ is a subjective discount rate, $v(c) + w(m)$ is additively-separable instantaneous utility function with the standard properties: $v(c)$ and $w(m)$ are increasing and strictly concave. The budget constraint is given by the following equation:

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[11] The model considered here belongs to a class of models with money in the utility function (Sidrauski, 1967). In this exposition, the model is closest to the version used in Drazen (1985).

$$Pc + \dot{M} + \dot{B} = Py + RB - PT, \tag{A2}$$

where P is the price level, y is the real income (flow endowment), which we assume for simplicity to be constant, T is the lump-sum taxes, B is the nominal public debt, and R is the nominal interest rate on public debt. The representative agent spends his total disposable nominal income on consumption and saving. The latter consists of increments in the base money and in government bonds. The initial stocks of the nominal assets are given by $M(0) = M_0, B(0) = B_0$. The budget constraint (A2) can be written in terms of real variables for convenience:

$$\dot{a} = y + ra - c - (r + \pi)m - T, \tag{A3}$$

where $a = m + b$ is the real assets of a representative agent, $b = B/P$ is the real (indexed) public debt, π is the rate of inflation. Under perfect foresight hypothesis, $r = R - \pi$ is the real interest rate.

The government finances the operational budget deficit (which is defined as the government expenditure, G, minus net taxes, T, both taken for simplicity to be constant, plus debt service) by new borrowings and seigniorage, $S = \dot{M}/P = \dot{m} + m\pi$. The dynamic budget constraint of the government in terms of real variables can be written as

$$G - T + rb = S + \dot{b}. \tag{A4}$$

The budget constraint of a representative agent (A3) and the budget constraint of the government (A4) together form the fundamental macroeconomic identity:

$$y = c + G. \tag{A5}$$

The first order conditions for the intertemporal optimization problem (A1) and (A3) are

$$v'(c) = \lambda, \tag{A6}$$

$$w'(m) = \lambda(r + \pi), \tag{A7}$$

$$\dot{\lambda} = \lambda(\rho - r), \tag{A8}$$

where λ is the co-state variable.

Under assumption of constant flow of endowment y and constant government expenditure G, condition (A6) determine constant level of co-state variable and thus condition (A8) determine constant real interest rate:

$$\lambda = v'_c(y - G), \tag{A9}$$

$$r = \rho. \tag{A10}$$

Condition (A7) determines money demand function that decreases with respect to the nominal interest rate. The Cagan function (1) that we use in the analysis, $m^d = Ae^{-\alpha\pi}$, corresponds to the utility function:

$$w(m) = m(\alpha_1 - \alpha_2 \ln m), \quad \alpha_1, \alpha_2 > 0, \tag{A11}$$

where $\alpha = v'(y - G)/\alpha_2$ is constant semi-elasticity and scale parameter $A = \exp((\alpha_1 - \alpha_2 - \rho v'(y - G))/\alpha_2)$ is normalized to unity.

Finally, considering dynamic government budget constraint (A4) and imposing No-Ponzi-Game condition

$$\lim_{t \to \infty} b(t)e^{-rt} \leq 0 \tag{A12}$$

gives sustainability constraint (8).

Appendix B. Numerical examples

Here we give numerical examples for the parameterization of our model that confirm the main results. The choice of values for the parameters was made mostly for demonstrative purposes, but they are not by any means unrealistic. Absolute values, such as real money balances, deficit, public debt, and seigniorage are not important as long as it is a matter of scale. Assuming that the semi-elasticity of money demand is $\alpha = 10$, we have put the maximum of inflation tax at an inflation rate equal to 10%. In fact, the results are robust to changes in this parameter, if we also shift the scale for other relative values, such as the growth rate of money and inflation. In order to characterize regimes with low and high interest rates, we choose correspondingly 1 percent and 10 percent. The length of the time interval $(t_A - t_S)$ is 10 years.

The tables below contain values of all variables at the initial time ($t = 0$), at the moment when changes in macroeconomic policies are announced ($t = t_A$), and at the times when actual policy switches are implemented ($t = t_S$). Since seigniorage typically undergoes an additional jump at the time of policy switch, we also consider its values just before that time ($t = t_{S-}$).

Table 1. Permanent decrease in the growth rate of base money

	μ	π	x	S	b_S	
					$r = 0.01$	$r = 0.1$
$t = 0$	0.07	0.07	-0.7	0.035	2.476	0.248
$t = t_A$		0.063	-0.625	0.037	2.119	0.259
$t = t_{S-}$				0.042		
$t = t_S$	0.05	0.05	-0.5	0.03	2.033	0.203

Table 2. Permanent increase in the growth rate of base money

	μ	π	x	S	b_S	
					$r = 0.01$	$r = 0.1$
$t = 0$	0.05	0.05	-0.5	0.03	2.033	0.203
$t = t_A$		0.057	-0.574	0.028	2.4	0.199
$t = t_{S-}$				0.025		
$t = t_S$	0.07	0.07	-0.7	0.035	2.476	0.248

As Table 1 shows, an anticipated decrease in the growth rate of base money leads to an increase in the sustainable level of public debt when the interest rate equals 10 percent. But it leads to a decrease in the sustainable level of public debt when the interest rate is low at 1 percent. Thus, anticipated tight monetary policy is only feasible when the interest is relatively high.

Quite the opposite, Table 2 demonstrate that anticipated increase in the growth rate of base money is the feasible type of monetary policy when the interest rate is low at 1 percent. When the interest rate is 10 percent anticipated increase in the growth rate of base money results in a decrease in the sustainable level of public debt, which is not feasible unless the government adjust its future policy to meet sustainability constraint.