The Market Price of Risk of the Volatility Term Structure

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Abstract

In this paper I examine the market price of risk of the volatility term structure. To this end, the S&P 500 VIX volatility term structure is used as a proxy for the aggregate volatility risk. Principal component analysis shows that time variation in the term structure of volatilities can be explained by three factors, which capture changes in the level, slope and curvature. The market price of risk of each factor is estimated in the cross-section of asset returns. I find a significant negative market price of risk for the level and slope, and a positive price of risk for the curvature of the volatility term structure. It is shown that the slope of the volatility term structure predicts changes in market excess returns over intermediate quarterly horizons, while the level and curvature are more related to short-term variations in market premia. A model with market returns and the three volatility factors has similar performance to that of the Fama-French model in pricing size and book-to-market sorted portfolios.

Key words: stochastic volatility, volatility term structure, cross-section of returns

JEL classification: G10, G12

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1. Introduction

Market volatility changes stochastically over time. This is by now a well understood phenomenon that has been confirmed by many studies using market returns, option prices or both (e.g., Andersen et al. 2002, Pan, 2002, Broadie et al., 2007). While there has been a substantial progress in modeling time-varying volatility there is still some lack of understanding with respect to some of the asset pricing implications caused by stochastic volatility. Time variation in volatility affects the investment opportunity set and if volatility is systematic, intertemporal asset pricing models suggest that the expected returns of stocks should be determined by their covariation with both market returns and innovations in the state variables that drive volatility dynamics.

The cross section of volatility risk has been previously examined by Ang et al. (2006b) and Adrian and Rosenberg (2008). Ang et al. (2006b), use as a proxy for market volatility the implied volatility VIX.[1] They find a significant negative volatility risk premium in the cross section of asset returns. However, their factor model with market return and innovations in volatility reduces pricing errors marginally compared to the CAPM. Adrian and Rosenberg (2008) postulate a two component GARCH model for market returns and volatility in the spirit of Engle and Lee (1999). They find that both the short and long-run volatility components have significant negative prices of risk. They show that a model with market return and innovations in the two volatility components has lower pricing errors than the Fama and French (1993) three-factor model on portfolios sorted by size

[1] Ang et al. (2006b) use data from the old VIX, which is now termed as VXO. VXO is the Black-Scholes implied volatility of a synthetic ATM option on the S&P 100 with constant 30 calendar days to expiry.

and book-to market characteristics. They argue that the short-run component is related to market skewness risk, while the long-run component captures business cycle risk.

In this paper I also test if volatility is priced in the cross section of asset returns. However, in contrast to Adrian and Rosenberg (2008) I do not impose any particular model on volatility dynamics and unlike Ang et al. (2006b) I don't use only a single maturity implied volatility index. I adopt a market based approach using data from the S&P 500 VIX volatility term structure as a proxy for the aggregate volatility risk. Different from Black-Scholes implied volatilities, the VIX term structure is derived in a model-free manner using option prices from S&P 500, without reference to any particular form of volatility dynamics (see Carr and Wu, 2004).[2] Moreover, in contrast to GARCH models, the VIX volatility term structure is a forward looking measure of expected volatilities at different horizons and may reflect more accurately market conditions.

Principal component analysis suggests that the evolution of innovations in the volatility term structure over the 1992 to 2007 sample period can be captured by three factors. Similar to the interest rate literature, the factors capture changes in the level, slope and curvature of the volatility term structure. The market price of risk of each factor is estimated using size and book-to-market sorted portfolios. I find a significant negative market price of risk for the level and slope, and a positive price of risk for the curvature of the volatility term structure. A model

[2] The volatilities from the VIX term structure are variance swaps for different maturities. This is a forward contract where the buyer (seller) receives the difference between the realized volatility of the returns of a stated underling and a fixed volatility rate, termed variance swap rate, if the difference is positive (negative).

with the market return and the three volatility factors has similar performance to that of the Fama-French model in pricing size and book-to- market sorted portfolios.

To understand the economic underpinnings of the empirical success of the model I test whether the volatility factors capture changes in the investment opportunity set. I find that the slope of the volatility term structure captures changes in market excess returns over intermediate quarterly horizons. In particular, an increase (decrease) in the slope predicts low (high) future returns. The explanatory power of the volatility slope compares favorably to other forecasting variables such the price-dividend ratio and the consumption-wealth ratio. The other two volatility factors seem to be related to short-term variations in market premia. The results are consistent with a risk-based explanation of the Fama-French three-factor model.

The remainder of the paper is structured as follows. In the next Section I outline the asset pricing framework used in the paper. Section 3 describes the construction of the volatility term structure and discusses the methods for extracting innovations. Section 4 applies principal component analysis to historical volatility term structure data and interprets the factors that explain the time variation. In Section 5 the market price of risk of the volatility factors is estimated in the cross section using portfolios sorted by size and book-to market characteristics. Section 6 examines the risk-return relationship using the volatility factors. Section 7 conducts robustness tests by taking into account bid/offer spreads in option prices, different methods for extracting volatility innovations and different sets of test portfolios. The last Section concludes, it presents the implications of the study, and it suggests directions for future research.

2. ICAPM Formulation

I assume that investment opportunities vary over time and asset returns are driven by an Intertemporal Capital Asset Pricing Model (ICAPM) in the spirit of Merton (1973). According to ICAPM, both the excess market return and innovations in state variables that forecast changes in the future investment opportunity set should show up as pricing factors in the cross section of asset returns (see Cochrane, 2001, pp. 444). The intuition is that investors will bid up the prices of assets that do well when future investment opportunities are expected to deteriorate, and consequently lower the expected returns. These assets command a smaller risk premium because they increase the investor's ability to hedge against unfavorable changes in investment opportunities. On the other hand, investors will command a higher premium for holding assets that do badly when future investment opportunities worsen. I assume the following model for the unconditional expected excess returns of the risky assets:

$$E(R_i) = \gamma Cov(R_i, R_M) + \sum_{j=1}^{N} f_j Cov(R_i, S_j) \tag{1}$$

where $E(R_i)$ is the excess return of the risky asset i, R_M the excess return of the market and S_j is the innovation in state variable j. Let J denote the indirect utility function of the investor and W the level of wealth. The coefficient of relative risk aversion is $\gamma = \frac{-J_{WW}W}{J_W}$ while $f_j = \frac{-J_{WS_j}}{J_W}$ captures percentage changes in marginal utility due to innovations in the state variable j.

According to model (1) the risk premia of assets are determined by their covariation with the excess market return as well as covariation (scaled by γ and f_j) with innovations in state variables that capture changes in the investment opportunity set. In the next section, the factors that explain the time variation in the volatility term structure innovations are extracted from principal component analysis. I find that three factors explain approximately 96% of the total variation. The extracted factors play the role of state variable innovations for determining risk premia in equation (1).

3. Term Structure of Volatilities

The development of the VIX term structure is based on the notion of model-free implied volatility. This is the risk-neutral expectation of future integrated volatility over a fixed horizon and is constructed from out-of-the money calls and puts. Most importantly, in contrast to Black Scholes implied volatilities, does not require reference to any particular model (see, for example, Carr and Madan, 1998; Demeterfi et al., 1999; Britten-Jones and Neuberger, 2000; Jiang and Tian, 2005; Carr and Wu, 2008).

Following Carr and Wu (2008), the time t risk-neutral expectation of an asset's annualised integrated volatility over a period [t, T], denoted by $IV_{t,T}$, can be approximated by a continuum of T maturity out-of-the money puts and calls written on the asset:

$$E_t^Q\left[IV_{t,T}\right]=\frac{2}{T-t}\int_0^\infty \frac{Q_t(K,T)}{P_t(T)K^2}dK \tag{2}$$

where P_t (T) is the value of a bond at time t that pays one dollar at time T. and O_t (K,T) denotes the price of a call at time t when K is greater than the spot price, and the price of a put at time t when K is smaller than the spot price.

The Chicago Board of Exchange (CBOE) has made publicly available historical data on the VIX term structure to allow market participants to calculate the forward value of VIX, which is the underlying asset of VIX futures and VIX options (see Carr and Wu, 2004).[3] The CBOE uses a discretised version of (2) to calculate the term structure of the risk-neutral expectation of the S&P 500 integrated volatility, which is given by:

$$E_t^Q\left[IV_{t,T}\right]=\frac{2}{(T-t)_{Business}}\sum_i \frac{\Delta K_i}{K_i^2}e^{r(T-t)_{calendar}}Q(K_i,T)+\frac{1}{(T-t)_{Business}}\left[\frac{F}{K_0}-1\right]^2 \tag{3}$$

where F is the forward price derived from index options prices, r is the risk-free rate to expiration and K_0 is the first strike below the forward index level. In the formulae above the CBOE uses both calendar and business days.[4] Calendar days, $(T-t)_{calendar}$, reflect the actual financing costs of holding the replicating option portfolio. The business day measure, $(T-t)_{business}$, is calendar days excluding weekends and holidays.

[3] The data are available from: http://www.cboe.com/micro/vix/vixtermstructure.aspx.

[4] The CBOE is also subdividing each day into "business minutes" or "calendar minutes" to expiration, which are then annualized.

There are some differences between the VIX term structure and the VIX index, which is also available from CBOE. The VIX index is the model-free risk-neutral expectation of future integrated volatility with a constant 30-day maturity and is calculated using formulae (3) only with calendar days. In contrast, the VIX term structure is applied to a single strip of options having the same expiration date and does not reflect constant-maturity volatility. To correct for volatility changes due to the time decay in option prices I construct constant-maturity volatilities (V) for one month, two months, three months, six months and twelve months (T_i for i = t+30, t+60, t+90, t+180, and t+360 calendar days). I use the same interpolation scheme employed by the CBOE for the construction of the VIX index:[5]

$$E_t^Q\left[IV_{t,T_i}\right] = \left\{T_1 E_t^Q\left[IV_{t,T_1}\right]\left[\frac{T_2 - T_i}{T_2 - T_1}\right] + T_2 E_t^Q\left[IV_{t,T_2}\right]\left[\frac{T_i - T_1}{T_2 - T}\right] \times \frac{365}{T_i}\right\} \quad (4)$$

where T_2 and T_1 are the calendar days for the two nearest-term expiration months.

I use as a proxy for innovations in the term structure of volatilities the first differences of each constant-maturity volatility, $E_{t+1}^Q(IV_{t+1,T_i+1}) - E_t^Q(IV_{t,T_i})$[6]. However, there are two drawbacks from using VIX data to derive innovations in the term structure of volatilities. First, if S&P 500 returns are driven by a jump diffusion stochastic volatility process the integrated volatilities will reflect both diffusion and jump risk (see Pan, 2002). Second, integrated volatilities are

[5] Similar to the CBOE's calculations for the VIX index, volatilities that have less than eight days left to expiration are not included in the interpolation to avoid microstructure effects. In this case, the second and third nearest volatilities are used.

[6] I use first differences because, as it shown in the next section, volatility levels are highly autocorrelated. As a robustness test, in section xxx the innovations are also extracted from a vector autoregressive model applied to volatility levels.

calculated under the risk-neutral probability measure. If S&P 500 returns are driven by a diffusion stochastic volatility process integrated volatilities will also include volatility risk premia (see Carr and Wu, 2008). In the case of a jump diffusion stochastic volatility process for asset returns, integrated volatility could reflect both jump and volatility risk premia (see Carr and Wu, 2008). Since innovations are measured by first differences, the results will not be affected only if premia are constant over time.

These caveats from using VIX data can be properly addressed by assuming a particular model for asset returns, volatility dynamics and risk premia. However, even under this approach the extracted innovations will depend on the particular model parameterization. For example, in a recent study Leippold et al. (2009) show that a two-factor stochastic model provides a better fit to the dynamics of the volatility term structure compared to a one-factor model. However, in the next section I find that the variation in the term structure is captured by three factors. Similar to Ang et al. (2006b), instead of estimating a particular model I use unadulterated time series data of the VIX term structure.[7]

4. Principal Component Analysis

I use VIX term structure data over the period January 2, 1992 to December 31, 2007. The data were downloaded from the CBOE's web site. I use the volatility term structure derived from mid option prices. As mentioned in the previous section, the volatilities of the VIX term structure do not reflect constant-maturity.

[7] As a robustness test, in Section 5 the innovations are also extracted from a vector autoregressive model applied to the volatility levels.

To correct for volatility changes due to the time decay in option prices, I use the interpolation scheme in (3) and construct daily volatilities with fixed maturities at one, two, three, six and twelve months. Figure 1 depicts the evolution of the volatilities for the period under consideration. For comparison purposes I also include the VIX index. The correlation between VIX and the one-month volatility is 99% and the correlation between their first differences is 90%. All volatilities series start at relatively low levels and display spikes in 1997 and 1998 due to Asian, Russian and hedge fund crises, respectively. Spikes are also observed between 2001 and 2003.

[Figure 1]

Table 1 (Panel A) reports the summary statistics of the volatility levels. The mean term structure is humped shaped. The mean volatilities increase for maturities up to six months and then decrease moderately. The mean term structure is also plotted in Figure 1. The standard deviation of all series decreases monotonically with maturity. All series show positive skewness and kurtosis and are highly persistent (all autocorrelation are more than 0.97). Panel B shows the summary statistics for the daily first differences of the volatility series. The mean daily change is effectively zero for all maturities. All series show excess kurtosis and the first-order autocorrelation drops substantially (the maximum autocorrelation being -0.35 for the case of the 12-month volatility).

[Table 1]

Since volatility levels are highly persistent, I apply principal component analysis to the daily first differences (see also Zhu and Avellaneda, 1997) of the five volatility series. Table 2 shows that the first principal component (PC1) explains 70% of the variability in the data. The second (PC2) and third (PC3) component, explain 17% and 7%, respectively. All together, the first three principal components explain around 96% of the variability in the term structure of volatilities.

[Table 2]

Figure 2 plots the corresponding eigenvectors of the first three principal components. The first eigenvector is negative for all maturities. This suggests that the first factor is related to changes in the level of volatility. The second eigenvector is negative for maturities up to three months and positive otherwise. Hence, the second factor seems to capture changes in the slope of the term structure. The third eigenvector is positive for short and long maturities and negative for intermediate maturities and hence the third factor is related to changes in the curvature of the term structure.

[Figure 2]

Table 3 shows the correlations between the principal components and the daily changes in the volatility series. I also include daily changes in the difference between the 12-month and 1-month volatilities as a proxy for changes in the slope of the term structure. The first component is mostly correlated with short-term volatilities (1-month and 2-month with correlation -95%). Note that the correlation is negative and hence an increase in the first component should be interpreted as a decrease in the volatility level. To facilitate the interpretation of the results, in the subsequent analysis the first component is multiplied by minus one. The second component is highly correlated with changes in the difference between the 12-month and 1-month volatilities (the correlation being 77%). Therefore, an increase in the second component is associated with an increase in the slope of the term structure. The third component is positively correlated with the 1-month and 12-month volatilities and negatively correlated with all other volatilities. An increase in the third factor can be interpreted as a change in the curvature of the term structure.

[Table 3]

5. Cross-Sectional Estimation

In this section I estimate the market price of risk of the three volatility factors using cross-sectional regressions. For comparison purposes I also estimate the prices of risk according to the CAPM , the Fama-French three-factor model and the Fama-French three factor model augmented by the momentum factor. For the cross sectional estimation I use the two-stage method of Fama-Macbeth (1973) using daily data on the 25 size and book-to-market sorted portfolios. The returns on the market portfolio, the Fama-French factors and the 25 portfolios are from Professor Ken French's website and also cover the period January 3, 1992 to December 31, 2007.

In the first stage I estimate for each portfolio *i,* the factors loadings with respect to market returns and volatility factors using the full sample as in Lettau and Ludvigson (2001) and Petkova (2006):

$$R_t^i = a_i + \beta_{iM} R_t^M + \beta_{i1} PC1_t + \beta_{i2} PC2_t + \beta_{i3} PC3_t + \varepsilon_t^i \qquad (5)$$

In the second stage the market price of risk of each factor is estimated using the following cross-sectional regression:

$$\overline{R}_i = \beta_{iM}\lambda_M + \beta_{i1}\lambda_{PC1} + \beta_{i2}\lambda_{PC2} + \beta_{i3}\lambda_{PC3} + e_i \tag{6}$$

Similar to Adrian and Rosenberg (2008) I assume correct pricing of the risk-free asset and in the second-stage cross-sectional regression the intercept term is set to zero. Because factors loadings from the first regression are estimated with errors, the standard errors of Shanken (1992) are used to correct for the errors-in-variables problem in the cross-sectional regression. I have also tried to allow for time variation in factor loadings and market prices of risk using a rolling window approach. However, for rolling windows of 30 or 60 days the factor loadings of the three volatility factors were very unstable. Similar to Adrian and Rosenberg (2008) the market prices of risk were also estimated using monthly cross-sectional regressions and factor loading estimates from the full sample. The market prices of risk were similar to the ones obtained from the full sample approach. To preserve space I report the estimates from the first method only.

Table 4 reports the correlations between the first three principal components, the excess return on the market (R_M) and the Fama-French value and size factors (a zero cost portfolio that is long small-cap stocks and short in large-cap (*SMB*), and a zero cost portfolio that is short in low book-to-market stocks and long in high book-to-market stocks (*HML*)). As expected, the first component is highly negatively correlated with market returns (leverage effect). The second component is positively correlated with market returns, and hence a steepening in the term structure is associated with contemporaneous positive market returns.

The third component has a small negative correlation with the market return. The first component (PC1) has a large positive correlation (0.38) with the value factor.

[Table 4]

Table 5 reports the estimates of the factor loadings from the first-pass regression. I also include the *p* values from a system of seemingly unrelated regressions to test the joint significance of the loadings.

[Table 5]

There is substantial variability with respect to factor loadings across the various size and book-to-market portfolios. Small firms have positive loadings on the first component while large firms have negative loadings. Recall that the first component is correlated with changes in short-term volatility. Therefore, the price of small firms tends to increase when there are positive shocks to the volatility level, while the price of large firms tends to decrease. Similar to the findings of Adrian and Rosenberg (2008) low book-to-market stocks (growth stocks) tend to have positive betas and hence positive returns when there are positive shocks to the volatility level while high-book-to-market stocks (value stocks) tend to move the opposite direction.

Small firms have negative betas with respect to the slope factor and large firms positive. Across the value and growth dimension factor loadings related to changes in the slope of volatility term structure tend to be negative. All types of stocks have positive exposure to changes in the curvature factor.

Table 6 reports the estimates of the market price of risk of the three volatility factors using the 25 size- and book-to-market–sorted portfolios. To better assess the empirical results I also include the estimates from the CAPM, the Fama-French model and Carhart's (1997) momentum factor (UMD). Column (10) reports the estimates of the market price of risk of the three volatility factors.

[Table 6]

Consistent with previous results, the factor related to innovations in the volatility level carries a negative premium (-0.37% per day) in the cross section. The second principal component also carries a negative premium (-0.40% per day), while the third has a positive price of risk (0.18% per day). All principal components are significant pricing factors at the 1% significance level. However, the prices of risk that I find differ substantially from previous estimates. Ang et al. (2006b) use data for the 1986 to 2000 sample period and find that innovations in the volatility level have a negative price of risk of approximately -1% per annum. In the 1963 to 2005 sample period Adrian and Rosenberg (2008) find that the short-term volatility component has a price of risk of -2.52% per annum and the long term component a price of risk of -24.24% per annum. The annualized estimates that I obtain are -94% for PC1, -101% for PC2 and 46% for PC3, respectively. This difference may be attributed to the different sample periods or the different methods for extracting volatility risk. Note that in the literature there is still no consensus with respect to the exact market price of volatility risk. For example, Carr and Wu (2004) examine the returns from buying variance swaps on S&P 500 and find that the market price of volatility risk is around −39.7% per month.

The root-mean-square pricing error (RMSPE) and the cross sectional R^2 both show that the pricing performance of the proposed four factor model is similar to that of other competing models.[8] The RMSPE is 0.0117 and the R^2 is 20.30% (column 10) while the RMSPE and R^2 of the Fama-French model (column 2) and the CAPM (column 1) is 0.012, 14.27% and 0.0184, -98.21%, respectively. However, when the momentum factor is included in the Fama-French model (column 3) the RMSPE drops to 0.0108 and the R^2 increases to 24.95%. Note that from the Fama-French factors only the value factor has a significant price of risk in the cross section of asset returns.

Columns (4, 5 and 6) report the market price of risk when each principal component enters the cross sectional regression separately. The market price of risk of the first (column 4) and third component (column 6) remains significant but the second factor becomes insignificant (column 5). Columns (7), (8) and (9) show the prices of risk when the principal components enter in pairs. The price of risk of the second factor is significant when it is estimated along with the market price of the first component, but becomes insignificant when it is estimated along with the market price of risk of the third factor. When the three principal components are augmented by the Fama-French and momentum factors (column 11) the market price of risk of the second and third component becomes

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[8] The root-mean-square pricing error (RMSPE) is calculated as $\sqrt{\sum_{i=1}^{25} a_i^2 / N}$ and $a_i = (\bar{R}_i - \beta_i \lambda)$. The cross sectional R^2 is similar to that employed by Jagannathan and Wang (1996) and Lettau and Ludvigson (2001) and it is given by $R^2 = \frac{Var_c(\bar{R}) - Var_c(a)}{Var_c(\bar{R})}$ where $\bar{R}$ is the vector of average returns of the 25 portfolios and α is the vector of pricing errors.

insignificant. It seems that the value and momentum factors contain some additional information beyond that reflected in the volatility term structure.

To further facilitate the interpretation of the empirical results, the pricing performance of the models is also depicted in Figure 4 that plots the average realized returns of the 25 portfolios against the predicted returns derived from four different models and the estimates from Table 6. Each two-digit number represents a separate portfolio. The first digit refers to the size quintile of the portfolio (1 denotes the smallest and 5 the largest), while the second digit refers to the book-to-market quintile (1 denotes the lowest and 5 the highest). The realized average returns are computed as the time-series averages of the 25 portfolio returns using the full sample. Under perfect fit realized and predicted returns should lie on a 45-degree line. It is evident that the CAPM model is unable to explain the returns of size and book-to-market sorted portfolios. When the CAPM is augmented by the first principal component the performance improves only marginally. I have also used the VIX index as in Ang et al. (2006b) to extract innovations in volatility and the pricing performance was found to be approximately the same with that from using the first principal component. However, when all three principal components are included in the model the performance improves substantially and is very similar to that of the Fama-French model.

[Figure 4]

To examine further the empirical results, Figure 5 depicts the percentage contribution of each portfolio to the total squared pricing errors for the Fama-

French and the three factor volatility model, respectively. In both models the largest contribution to the total pricing error is coming mainly from portfolios with small size stocks (11, 12, 13, 14, and 15). The three factor volatility model performs better in pricing large value stocks (45, 55) while the Fama French model has smaller pricing errors in large growth stocks (43, 53).

[Figure 5]

6. Risk Return Tradeoff and Hedging Components

In this section I examine the economic underpinnings of the empirical success of the volatility model and I test whether the volatility factors capture changes in the investment opportunity set. The empirical results on the relationship between risk and market return have been ambiguous. For example, French et al. (1987), Glosten et al. (1993) and Brandt and Kang (2004), among others, find a positive by insignificant relationship. Ghysels et al. (2004) and Lundblad (2005) find a positive and significant relationship, while some studies even find a negative relation (e.g., Campbell, 1987, Harvey 2001, Lettau and Ludvigson, 2003). Scruggs (1998) and Guo and Whitelaw (2005) argue that the relationship is positive after controlling for the hedge component in the model specification.

Here I estimate the relationship between risk and return, but also take into account the hedging components that may arise from the remaining two volatility factors. The relationship between the future excess market returns and the three volatility factors is estimated with the following regression:

$$R_{M,t+1} = \beta_0 + \beta_1 V_t^{(1)} + \beta_2 (V_t^{(12)} - V_t^{(1)}) + \beta_3 (V_t^{(12)} + V_t^{(1)} - 2V_t^{(3)}) + u_t \qquad (7)$$

where $V^{(1)}, V^{(3)}$ and $V^{(12)}$ are the 1-month, 3-month and 12-month volatilities, respectively. The term $V^{(12)} - V^{(1)}$ is used as a proxy for the slope of the term structure, while the term $V^{(12)} + V^{(1)} - 2V^{(3)}$ is used as a proxy for the curvature (see also Ang et al. 2006a for a similar approach in yield curve modeling). Table 7, reports the estimates from regression (7). The coefficient of the short-term volatility (β_1) is positive but it is not statistically significant. The signs of the other two coefficients imply that future excess returns depend negatively on the slope of the volatility term structure (β_2) and positively on the curvature (β_3). However, both coefficients are not statistically significant.[9] Note that that the *t*-statistics of the level and curvature coefficients (1.34 and 1.54, respectively) are much higher than the *t*-statistic of the slope coefficient (-0.80). Though the significance of the results is not extremely strong, it seems that the level and curvature are more related to daily future changes of market excess returns.

[Table 6]

To better appreciate the forecasting power of the volatility factors I run predictive regressions on a monthly and quarterly frequency, since many studies (e.g., Cochrane, 2008)) have shown that market returns are more predictable at low frequencies. I use multi-period return regressions of the form:

$$\frac{1}{h}\sum_{j=1}^{h} R_{M,t+j} = \beta_0 + \beta_1 V_t^{(1)} + \beta_2 (V_t^{(12)} - V_t^{(1)}) + \beta_3 (V_t^{(12)} + V_t^{(1)} - 2V_t^{(3)}) + u_{t,t+h} \quad (8)$$

where now t, denotes end-of-month data.

[9] When regression (7) is estimated by imposing a zero intercept the signs of the coefficients remain the same, but the coefficient of the volatility level becomes significant.

Table 8 reports the estimates from regression (8) for monthly horizons (*h*=1). Because the volatilities are highly cross-correlated to account for collinearity problems I also run regressions in which the coefficients of the different volatility factors (level, slope and curvature) are estimated separately (regressions 1, 2 and 3). At the monthly horizon (*h*=1) the different volatility factors do not seem to have any predictive power. The largest R^2 (0.7%) is given by regression 2, where the dependent variable is only the slope factor. Regression 4 reports the coefficients when all three volatility factors enter as dependent variables. The coefficients are not statistically significant and the R^2 becomes negative. In Table 9 I repeat the same estimation procedure using quarterly excess returns (*h*=3). In regression 1 the coefficient of the volatility level is not statistical significant but the R^2 increases to 1.93%. The forecasting ability of the slope factor is evident from looking at the results from regression 2. The coefficient is statistically significant with a large *t*-statistic of -2.23 while the R^2 increases substantially and reaches 3.9%. The magnitude of the slope coefficient implies that a 1% increase in the slope of the volatility term structure predicts an average -0.26% decrease in the market excess returns over the next three months. The R^2 may not look very impressive but nevertheless the predictive power of the slope factor compares favorably with other traditional forecasting variables. For approximately the same sample period (January 1, 1990 to December 31, 2007), Bollerslev et al. (2008) finds that at a quarterly horizon the price-dividend ratio and the consumption-wealth ratio have $R^{2,}$s of 4.19% and 4.13%, respectively.

[Table 7]

In Figure 6 I plot the slope coefficients and the R^2's for different return horizons when the dependent variable is only the slope factor. The slope coefficient increases monotonically with the return horizon. The adjusted R^2 starts at relatively low levels at the one-month return horizon and reaches its maximum value at quarterly horizons. Then it gradually drops off and becomes negligible at the annual horizon. Interestingly enough, the pattern of the slope coefficient and R^2's as functions of return horizon look very similar to the results by Bollerslev et al. (2008) (Figure 2, pp. 39) when they regress future excess returns against the volatility risk premium. It seems that the volatility risk premium and the volatility slope share some common information. Unfortunately, data on the volatility term structure are limited to the post '90 sample period and this does not allow the examination of longer sample periods and longer return horizons. However, the results here suggest that there are strong indications that the slope of the volatility term structure carries important information with respect to time variation in market risk premium.

[Figure 6]

The empirical results from regressions (7) and (8) together with the market prices of risk estimated in Section 4 are consistent with an ICAPM interpretation. The estimated coefficients imply that a positive shock in the slope of the term structure signals a deterioration of the investment opportunity set. Given the negative market price of the slope, an asset that covaries positively with innovations in the slope is considered less risky and thus commands a smaller risk premium. A similar interpretation can be given to the curvature factor. Though the statistical results on the curvature factor are not extremely strong, regression 7 suggests that this factor seems to be related to short-term (e.g., daily) variations in market premia.

Note that the economic interpretation of the empirical success of the volatility model is somewhat different from that given by Adrian and Rosenberg (2008). Adrian and Rosenberg argue that the short volatility component captures market skewness risk and the long volatility component captures business cycle risk. Their conclusions are based on the empirical finding that the risk premium of the short-run component correlates highly with the risk premium of market skewness and the risk premium of the long-run component correlates highly with the risk premium of industrial production growth. However, they don't provide any strong evidence on the predictive ability of the volatility components and their results are closer to an APT interpretation.

7. Robustness Tests

In this section I examine whether the estimates of the market price of risk of the three volatility factors are robust to bid/offer spreads in option prices, different methods for extracting volatility innovations and different sets of test portfolios.

The CBOE has also made publicly available the VIX term structure derived from bid and offer option prices, respectively. To test whether the results are affected by market frictions I estimate the market price of risk of the volatility factors using the bid/offer volatility term structure. I also derive innovations in the volatility term structure by running a first order vector autoregressive (VAR) process with the five volatility series as state variables. From the VAR system I use the five residual series as a proxy for innovations in the term structure and then apply principal component analysis. I find that the first three components capture around 96% of the variation in the residual series. Finally, to address the concerns by Lewellen et al. (2006) and examine whether the results are affected by the choice of the test portfolios, I estimate the market price of risk of the volatility factors using as test assets the 25 portfolios augmented by the 30 industry portfolios. The 30 industry portfolios are downloaded from Professor Ken French's website. All the results are assembled in Table 10.

The estimates of market price of risk when the volatility term structure is constructed using the bid and offer option prices are reported in columns (2) and (3), respectively. In both cases the R^2's, the RMSPE and the estimates of the price of risk are very similar the ones obtained using the mid option prices in Table 6. When the volatility innovations are extracted from the VAR system (column 3) the price of risk of the curvature factor becomes insignificant. The curvature factor also becomes insignificant when the test portfolio is the 25 portfolios augmented by the 30 industry portfolios (column 4). However, in both cases the market price of risk of the level and slope remains highly significant. A drawback of the models is that the R^2 decreases substantially when the test assets are augmented by the 30 industry portfolios. However, Lewellen et al. (2006) report similar results. The show that the performance of many popular asset pricing models deteriorates substantially when they are tested against the 25 size and book-to-market sorted and 30 industry portfolios.

[Table 10]

8. Conclusions

In this paper I examine the market price of risk of the volatility term structure. Principal components analysis suggests that that time variation in the term structure of volatilities can be explained by three factors, which capture changes in the level, slope and curvature. The market price of each factor is estimated in the cross section of asset returns using size and book-to-market sorted portfolios. I find a significant negative market price of risk for the level and slope, and a positive price of risk for the curvature of the volatility term structure. The proposed asset pricing model with market return and the three volatility factors has similar performance to that of the Fama-French model in pricing size and book-to-market sorted portfolios. The volatility factors are priced in the cross section because they capture changes in the investment opportunity set in the spirit of ICAPM. The slope of volatility term structure predicts changes in market excess returns over intermediate quarterly horizons, while the level and curvature seem to be related to short-term variations in market premia. Future research should try to develop equilibrium models in order to pin down the exact economic mechanism behind the cross sectional price of risk and return predictability pattern of the volatility factors, and especially that of the slope factor. This perhaps can be done by developing multifactor stochastic volatility models in the context of Bansal and Yaron (2004) and Bollerslev et al. (2008).

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Panel A: Levels					
Maturity	Mean	Std. Dev.	Skewness	Kurtosis	1st Auto
1	0.04	0.03	2.05	8.82	0.97
2	0.04	0.03	1.73	6.88	0.98
3	0.04	0.03	1.66	6.65	0.98
6	0.04	0.02	1.43	5.45	0.99
12	0.04	0.02	1.30	6.23	0.97
Panel B: First Differences					
Maturity	Mean	Std. Dev.	Skewness	Kurtosis	1st Auto
1	4.16E-06	0.01	0.62	41.00	-0.12
2	5.75E-06	0.00	0.49	48.10	-0.05
3	5.69E-06	0.00	-0.64	29.95	-0.09
6	8.36E-06	0.00	1.03	100.59	-0.15
12	8.61E-06	0.00	0.37	745.25	-0.36

Table 1: Descriptive statistics for volatilities with fixed maturities at one, two, three, six and twelve months. Panel A reports the summary statistics for the volatility levels and Panel B for the first differences. The data are daily and cover the period from January 2, 1992 to December 31, 2007.

Principal Component	PC1	PC2	PC3	PC4	PC5
% of Total Variance	70.8	17.4	7.58	2.26	2

Table 2: Principal component analysis of the time series of daily differences of volatilities with fixed maturities at one, two, three, six and twelve months. The time period is from January 3, 1992 to December 31, 2007.

	PC1	PC2	PC3	Δ (1M)	Δ (2M)	Δ (3M)	Δ (6M)	Δ (12M)	Δ(12M-1M)
PC1	1.00								
PC2	0.00	1.00							
PC3	0.00	0.00	1.00						
Δ (1M)	**-0.94**	-0.18	**0.26**	1.00					
Δ (2M)	**-0.95**	-0.15	-0.07	0.88	1.00				
Δ (3M)	-0.83	-0.10	-0.53	0.67	0.80	1.00			
Δ (6M)	-0.80	0.38	-0.07	0.66	0.70	0.63	1.00		
Δ (12M)	-0.46	0.88	**0.05**	0.29	0.30	0.27	0.65	1.00	
Δ(12M-1M)	0.59	**0.77**	-0.22	-0.76	-0.63	-0.46	-0.18	0.41	1.00

Table 3: Correlation between the three principal components (PC1, PC2, PC3) and daily differences in volatilities with fixed maturities at one (Δ (1M)), two (Δ (2M)), three (Δ (3M)), six (Δ (6M)) and twelve (Δ (12M)) months.12M-1M is difference between the twelve and one month volatility and is used as a proxy for changes in the slope of the term structure. The time period is from January 3, 1992 to December 31, 2007.

	R_M	R_{SMB}	R_{HML}	*PC1*	*PC2*	*PC3*
R_M	1.00					
SMB	-0.08	1.00				
HML	-0.60	-0.20	1.00			
PC1	**-0.73**	0.13	**0.38**	1.00		
PC2	0.11	-0.05	-0.06	0.00	1.00	
PC3	-0.06	0.04	0.05	0.00	0.00	1.00

Table 4: Correlation between the three principal components (PC1, PC2, PC3) the excess market return (R_M) and the Fama-French Factors (*SMB*, *HML*).The time period is from January 3, 1992 to December 31, 2007.

$$R_{i,t} = a_i + \beta_{i,M} R_{M,t} + \beta_{i,PC1} PC1_t + \beta_{i,PC2} PC2_t + \beta_{i,PC3} PC3_t + \varepsilon_{i,t}$$

	Low	2.00	3.00	4.00	High	Low	2.00	3.00	4.00	High
	α					t_a				
Small	-0.03	0.01	0.02	0.04	0.04	-2.15	1.01	2.38	3.68	4.03
2.00	-0.02	0.00	0.02	0.02	0.02	-1.78	0.14	1.99	2.31	2.04
3.00	-0.02	0.01	0.02	0.01	0.03	-1.70	0.91	2.29	1.62	3.00
4.00	-0.01	0.01	0.02	0.02	0.02	-0.87	1.95	2.07	2.68	1.67
Large	-0.01	0.01	0.01	0.01	0.01	-1.32	1.90	1.21	1.10	1.27
p-Value that all 25 Intercepts Are Equal =0.00%										
	β_M					$t_{\beta M}$				
Small	1.01	0.85	0.67	0.59	0.58	25.53	27.24	25.56	24.40	25.14
2.00	1.23	0.95	0.85	0.79	0.80	46.64	44.92	38.43	35.54	33.50
3.00	1.28	0.94	0.78	0.75	0.78	47.30	59.80	37.56	36.07	32.50
4.00	1.32	0.85	0.77	0.73	0.70	45.72	50.22	38.95	27.28	25.86
Large	1.07	0.86	0.78	0.72	0.76	91.99	43.55	31.82	25.31	24.12
p-Value that all 25 Loadings Are Equal =0.00%										
	β_{PC1}					t_{PC1}				
Small	0.10	0.09	0.05	0.03	0.02	3.03	3.09	2.39	1.41	0.96
2.00	0.11	0.07	0.06	0.03	0.01	3.79	3.35	2.87	1.56	0.43
3.00	0.12	0.04	-0.03	-0.02	-0.00	3.52	1.87	-1.39	-0.89	-0.04
4.00	0.14	-0.02	-0.04	-0.01	-0.01	5.37	-1.25	-2.12	-0.31	-0.44
Large	-0.01	-0.08	-0.11	-0.04	-0.07	-0.68	-4.69	-6.20	-2.23	-2.78
p-Value that all 25 Loadings Are Equal =0.00%										
	β_{PC2}					t_{PC2}				
Small	-0.06	-0.06	-0.05	-0.04	-0.04	-2.60	-3.06	-2.80	-2.22	-2.20
2.00	-0.07	-0.04	-0.06	-0.04	-0.04	-2.61	-2.16	-3.14	-2.73	-1.93
3.00	-0.04	-0.04	0.01	-0.02	-0.03	-1.38	-3.04	0.68	-1.22	-1.53
4.00	-0.06	0.02	0.00	-0.03	-0.01	-2.87	1.44	-0.06	-1.66	-0.49
Large	0.01	0.02	0.06	0.03	0.05	1.22	1.31	3.68	1.92	2.46
p-Value that all 25 Loadings Are Equal =0.00%										
	β_{PC3}					t_{PC3}				
Small	0.05	0.10	0.06	0.06	0.06	1.00	2.55	1.84	1.96	1.95
2.00	0.07	0.10	0.10	0.12	0.05	1.62	3.18	2.64	3.02	1.18
3.00	0.03	0.06	0.08	0.05	0.05	0.72	2.22	2.09	1.44	0.94
4.00	0.08	0.02	0.06	0.07	0.03	2.34	0.77	1.79	1.38	0.50
Large	-0.06	-0.01	0.02	0.02	0.02	-2.20	-0.44	0.43	0.49	0.34
p-Value that all 25 Loadings Are Equal =0.05%										

R^2				
0.58	0.58	0.57	0.55	0.54
0.72	0.70	0.67	0.64	0.61
0.76	0.78	0.74	0.69	0.65
0.82	0.82	0.77	0.66	0.60
0.92	0.83	0.76	0.65	0.57

Table 5: This table reports factors loadings on the excess market return, R_M, and the first three principal components (PC1, PC2, PC3) of innovations in the term structure of

volatilities for 25 portfolios sorted by size and book-to-market. The corresponding *t*-statistics are corrected for autocorrelation and heteroskedasticity using the Newey–West estimator with five lags. The sample period is from January 3, 1992 to December 31, 2007. The *p* values are from a system of seemingly unrelated regressions and test the joint significance of the corresponding loadings. The R^2s from each time-series regression are reported in percentage form.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Excess Market Return (R_M)	Coef.	0.0396	0.0253	0.0297	0.0424	0.0376	0.0267	0.0343	0.0279	0.0269	**0.0283**	0.0303
	t-stat	[2.4196]	[1.6190]	[1.8990]	[2.6027]	[2.3512]	[1.6912]	[2.0314]	[1.7552]	[1.6891]	**[1.7742]**	[1.9351]
Level Factor (*PC1*)	Coef.				-0.1535			-0.4460	-0.2166		**-0.3765**	-0.2207
	t-stat				[-2.2489]			[-2.7197]	[-2.2624]		**[-2.9614[**	[-2.0689]
Slope Factor (*PC2*)	Coef.					-0.0972		-0.6342		0.1481	**-0.4027**	-0.2095
	t-stat					[-0.7066]		[-2.4054]		[0.7301]	**[-2.3262]**	[-1.5652]
Curvature Factor (*PC3*)	Coef.						0.2435		0.2949	0.2975	**0.1858**	0.1214
	t-stat						[2.5894]		[2.8993]	[2.9480]	**[2.0921]**	[1.6760]
Size Factor (*SMB*)	Coef.		0.0033	0.0009								0.0006
	t-stat		[0.3730]	[0.1001]								[0.0710]
Value Factor (*HML*)	Coef.		0.0209	0.0254								0.0233
	t-stat		[2.2702]	[2.7433]								[2.5050]
Momentum Factor (*UMD*)	Coef.			0.1476								0.1336
	t-stat			[3.2688]								[2.3282]
RMSPE		0.0184	0.0120	0.0108	0.0170	0.0183	0.0159	0.0128	0.0128	0.0155	**0.0117**	0.0100
%R^2		-98.21	14.27	24.95	-68.48	-94.4	-47.53	2.62	2.06	-41.84	**20.30**	39.89

RMSPE = *root-mean-squared pricing error*

Table 6: This table reports the Fama–MacBeth cross-sectional regressions using the excess returns on 25 portfolios sorted by book-to-market and size. In the first stage, factor loadings are obtained from regressing portfolio returns to pricing factors using the full sample over the time period January 3, 1992 to December 31, 2007. The factor loadings are reported in Table 5. In the second stage, the average excess returns of the portfolios are regressed on the loadings to obtain the estimates of the market price of risk for each factor. The *t*-statistics reported in brackets are adjusted for errors-in-variables as in Shanken (1992).

	constant	$V_t^{(1)}$	$V_t^{(12)} - V_t^{(1)}$	$V_t^{(12)} + V_t^{(1)} - 2V_t^{(3)}$	*Adj. R^2 (%)*
	-0.01 [-0.24]	1.19 [1.34]	-1.38 [-0.80]	3.23 [1.54]	0.20

Table 7: This table reports the estimates from the regression $R_{M,t+1} = \beta V_t^{(1)} + \beta_1 (V_t^{(12)} - V_t^{(1)}) + \beta_2 (V_t^{(12)} + V_t^{(1)} - 2V_t^{(3)})$, where $V^{(1)}, V^{(3)}$ and $V^{(12)}$ are the 1-month, 3-month and 12-month volatilities, respectively. The term $V^{(12)} - V^{(1)}$ is used as a proxy for the slope of the term structure and $V^{(12)} + V^{(1)} - 2V^{(3)}$ is used as a proxy for the curvature. The sample period is from January 2, 1992 to December 31, 2007. The corresponding *t*-statistics are corrected for autocorrelation and heteroskedasticity using the Newey–West estimator with nine lags.

	constant	$V_t^{(1)}$	$V_t^{(12)} - V_t^{(1)}$	$V_t^{(12)} + V_t^{(1)} - 2V_t^{(3)}$	*Adj. R^2 (%)*
1	0.14 [0.38]	12.19 [1.38]			0.3
2	0.62 [2.12]		-23.55 [-1.77]		0.7
3	0.60 [2.14]			-12.17 [-0.53]	-0.4
4	0.51 [1.1]	2.54 [0.2]	-19.85 [-1.31]	-5.07 [-0.22]	-0.3

Table 8: This table reports the estimates from the regression $R_{M,t+1} = \beta V_t^{(1)} + \beta_1 (V_t^{(12)} - V_t^{(1)}) + \beta_2 (V_t^{(12)} + V_t^{(1)} - 2V_t^{(3)})$, where $V^{(1)}, V^{(3)}$ and $V^{(12)}$ are the 1-month, 3-month and 12-month volatilities, respectively. The term $V^{(12)} - V^{(1)}$ is used as a proxy for the slope of the term structure and $V^{(12)} + V^{(1)} - 2V^{(3)}$ is used as a proxy for the curvature. The sample period is from January 2, 1992 to December 31, 2007. The regressions are based on overlapping monthly observations. The corresponding *t*-statistics are corrected for autocorrelation and heteroskedasticity using the Newey–West estimator with four lags.

	constant	$V_t^{(1)}$	$V_t^{(12)} - V_t^{(1)}$	$V_t^{(12)} + V_t^{(1)} - 2V_t^{(3)}$	*Adj. R^2 (%)*
1	0.17 [0.49]	12.04 [1.48]			1.93
2	0.63 [2.42]		-26.08 [-2.23]		3.90
3	0.61 [2.43]			-13.42 [-1.26]	-0.02
4	0.65 [1.54]	-0.58 [-0.05]	-26.00 [-1.31]	-6.18 [-0.66]	2.97

Table 9: This table reports the estimates from the regression $\frac{1}{3}\sum_{h=1}^{3} R_{M,t+h} = \beta_0 + \beta_1 V_t^{(1)} + \beta_2 (V_t^{(12)} - V_t^{(1)}) + \beta_3 (V_t^{(12)} + V_t^{(1)} - 2V_t^{(3)})$, where $V^{(1)}, V^{(3)}$ and $V^{(12)}$ are the 1-month, 3-month and 12-month volatilities, respectively. The term $V^{(12)} - V^{(1)}$ is used as a proxy for the slope of the term structure and $V^{(12)} + V^{(1)} - 2V^{(3)}$ is used as a proxy for the curvature. The sample period is from January 2, 1992 to December 31, 2007. The regressions are based on overlapping monthly observations. The corresponding *t*-statistics are corrected for autocorrelation and heteroskedasticity using the Newey–West estimator with four lags.

		bid	offer	VAR	FF25 + 30 ind.
		(1)	(2)	(3)	(4)
Excess Market Return (R_M)	Coef.	0.0288	0.0280	0.0319	0.0386
	t-stat	[1.7940]	[1.7619]	[1.9201]	[2.4257]
Level Factor (*PC1*)	Coef.	-0.3940	-0.3738	-0.3600	-0.1552
	t-stat	[-2.9274]	[-2.9542]	[-2.7368]	[-2.1167]
Slope Factor (*PC2*)	Coef.	-0.4316	-0.3735	-0.5200	-0.2142
	t-stat	[-2.3681]	[-2.2531]	[-2.2712]	[-2.1531]
Curvature Factor (*PC3*)	Coef.	0.1843	0.1902	0.1000	-0.0714
	t-stat	[2.1537]	[2.0217]	[1.2696]	[-1.2881]
RMSPE		0.0115	0.0118	0.0123	0.0165
% R^2		20.61	17.84	11.43	-13.86

RMSPE = *root-mean-squared pricing error*

Table 10: This table reports the Fama–MacBeth cross-sectional regressions using the excess returns on 25 portfolios sorted by book-to-market and size (columns 1, 2, 3) and the 25 portfolios plus the 30 industry portfolios (column 4). In the first stage, factor loadings are obtained from regressing portfolio returns to pricing factors using the full sample over the time period January 3, 1992 to December 31, 2007. In the second stage, the average excess returns of the portfolios are regressed on the loadings to obtain the estimates of the market price of risk for each factor. The *t*-statistics reported in brackets are adjusted for errors-in-variables as in Shanken (1992).

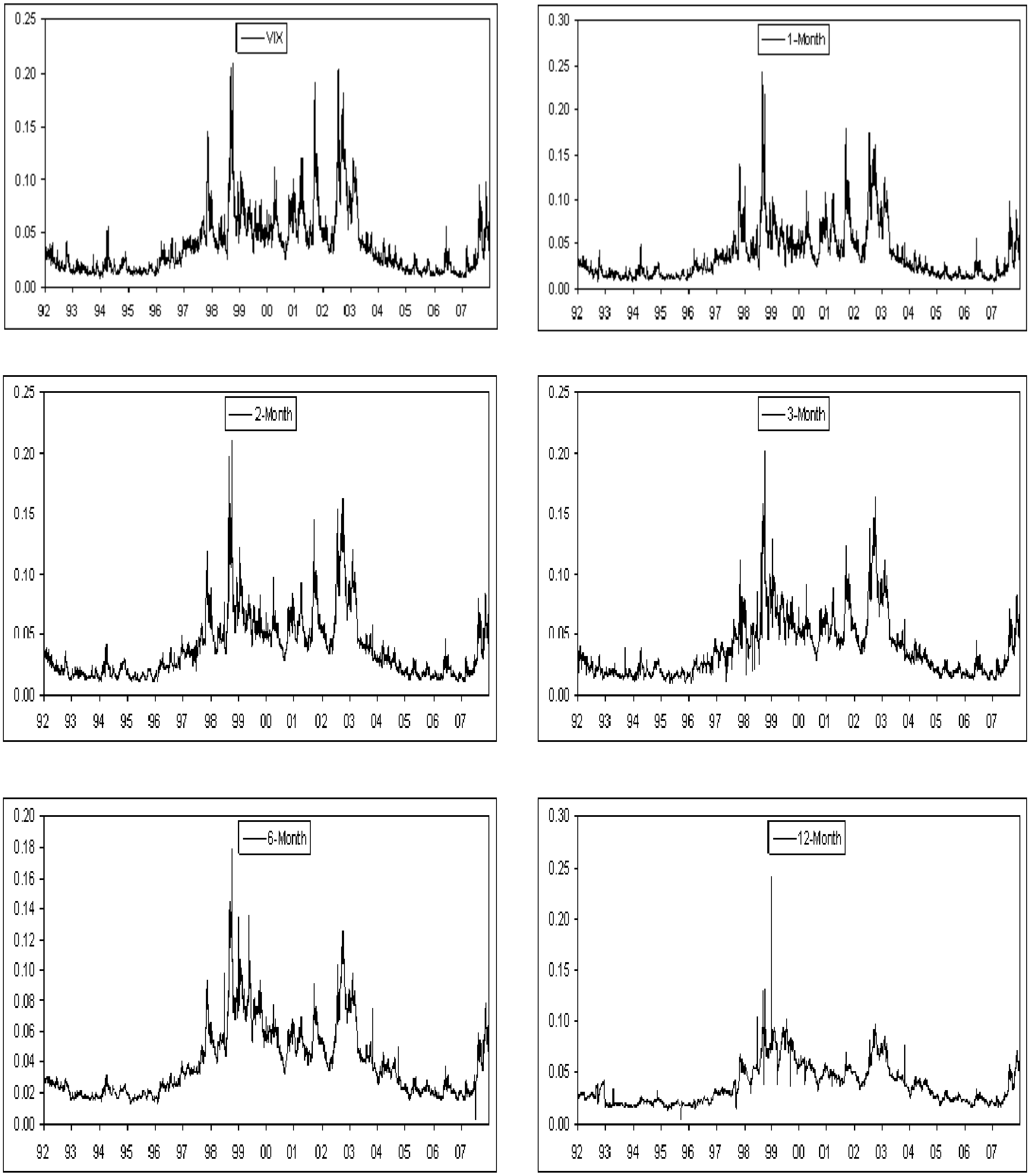

Figure 1: Time series of daily volatilities with fixed maturities at one, two, three, six and twelve months, for the time period January 2, 1992 to December 31, 2007.

Figure 2: Average term structure of volatilities for the time period January 2, 1992 to December 31, 2007. The maturities are one, two, three, six and twelve months, respectively.

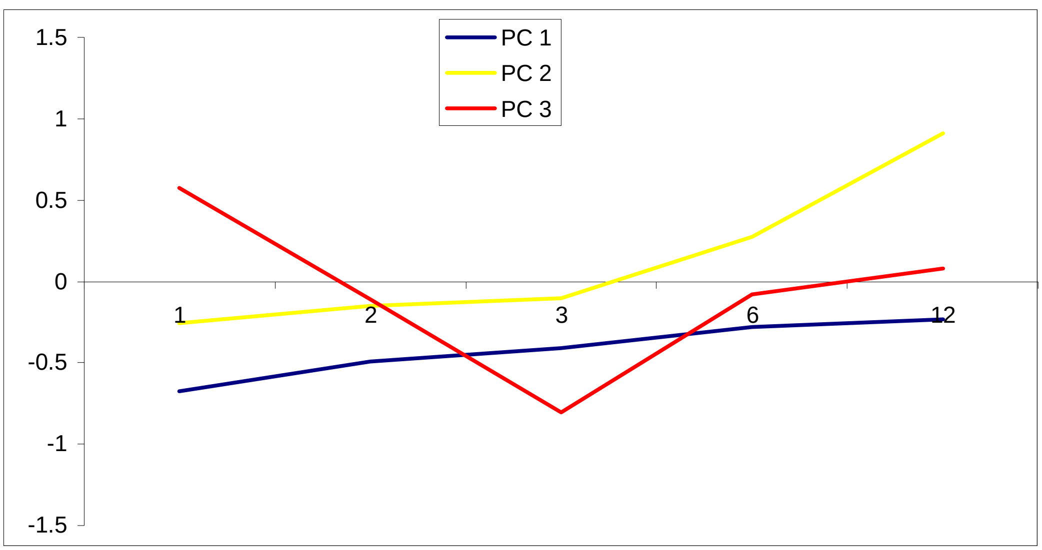

Figure 3: Eigenvectors of the first three principal components. Principal component analysis is applied to the time series of daily differences of volatilities with fixed maturities at one, two, three, six and twelve months. The time period is from January 3, 1992 to December 31, 2007.

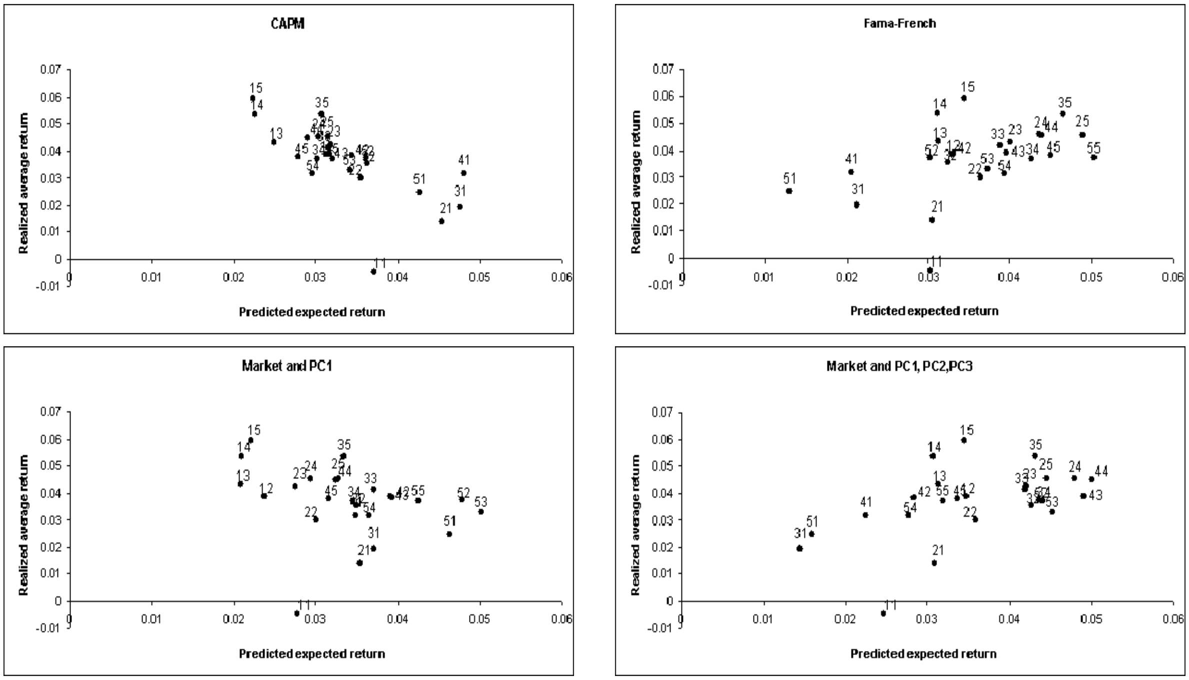

Figure 4: This figure shows the average realized excess return (%) on the horizontal axis for the size and book-to-market sorted portfolios against the predicted returns (%) from the models reported in columns (1), (2), (4) and (10) of Table xxx. The first digit refers to the size quintile of the portfolio (1 denotes the smallest and 5 the largest), while the second digit refers to the book-to-market quintile (1 denotes the lowest and 5 the highest).The sample period is from January 3, 1992 to December 31, 2007.

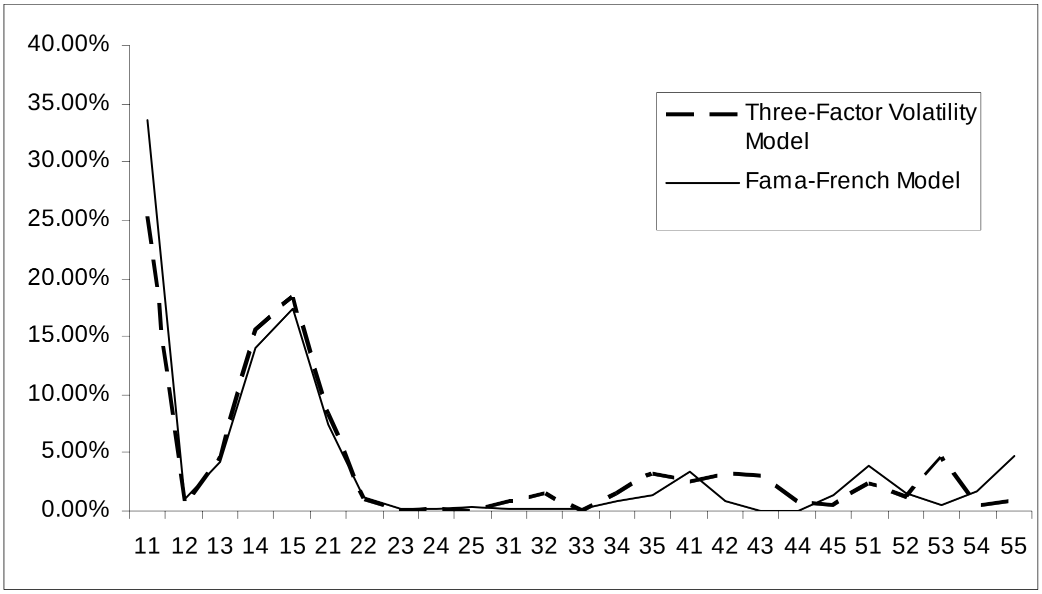

Figure 5: This figure shows the percentage contribution to the total pricing error of each one of the 25 size and book-to-market sorted portfolios for the Fama-French model (solid line) and the three-factor volatility model (dashed line) . The pricing errors are based on the parameter estimates from Table 6.In the horizontal axis the first digit refers to the size quintile of the portfolio (1 denotes the smallest and 5 the largest), while the second digit refers to the book-to-market quintile (1 denotes the lowest and 5 the highest).

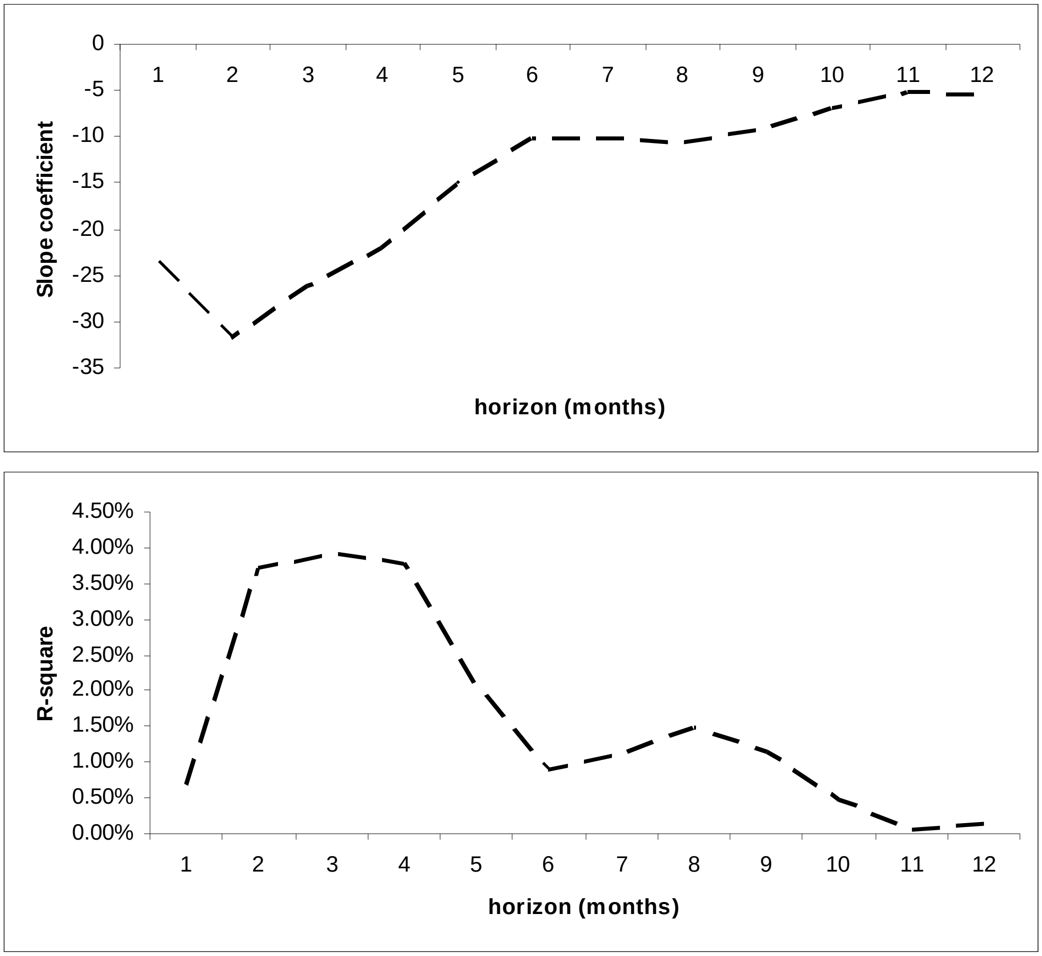

Figure 6: This figure shows the estimated slope coefficients (top figure) and the adjusted R^2's (bottom figure) from the regression $\frac{1}{h}\sum_{j=1}^{h} R_{M,t+j} = \beta_0 + \beta_1 (V_t^{(12)} - V_t^{(1)}) + u_{t,t+h}$ for different return horizons (h). The term $V^{(12)} - V^{(1)}$ is used as a proxy for the slope of the term structure, where $V^{(1)}$ and $V^{(12)}$ are the 1-month and 12-month volatilities, respectively. The sample period is from January 2, 1992 to December 31, 2007. The regressions are based on overlapping monthly observations.