Interest Rate Volatility and Risk Management: Evidence from CBOE Treasury Options

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This paper investigates the US Treasury market volatility and develops new ways of dealing with the underlying interest rate volatility risk. Our innovative approach is based on a class of model-free interest rate volatility (VXI) indices we derive from options traded on the CBOE. The empirical analysis indicates substantial interest rate volatility risk for short and medium-term instruments which declines to the levels of the equity market only as the tenor increases to 30 years. This risk appears do be priced in the market since we find a significant negative interest rate volatility premium. Moreover, for the first time we present evidence that interest rate and equity volatility risk premia have a significant time-varying relationship. We also demonstrate that US Treasury market volatility is appealing from an investment diversification perspective since the VXI indices are negatively correlated with the levels of interest rates and of equity market implied volatility indices, respectively. The VXI indices are affected by macroeconomic and monetary news but are only partially spanned by information contained in the yield curve. Motivated by our results on the magnitude and the nature of interest rate volatility risk and by the phenomenal recent growth of the equity volatility derivative market, we propose the use of our VXI indices as benchmarks for monitoring, securitizing, managing and trading interest rate volatility risk. As a first step in this direction, we describe a framework of one-factor equilibrium models for pricing VXI futures and options on the basis of empirically favored mean-reverting jump-diffusions.

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1. Introduction

The volatility of interest rates is of prime importance to monetary authorities, financial institutions, policy makers and journalists since interest rates have such a central position in most economic theories, models and systems. Bond and foreign exchange market participants are also particularly concerned about the future evolution and variability of interest rates since volatility is a protagonist in the pricing, hedging and risk management of financial instruments involving interest rates (see the reviews by Chapman and Pearson, 2001; Dai and Singleton 2003; Ederington and Lee, 2007).[1] Not surprisingly, an impressive body of research over the past years investigates issues related to interest rate volatility. Although various hypotheses have been evaluated and some stylized facts have been uncovered, there is no strong consensus yet in the empirical literature on how interest volatility should be measured and modeled. More importantly, although we now understand well that interest rate volatility exhibits large swings (eg., see Ait-Sahalia, 1996; Andersen and Lund, 1997; Amin and Morton, 1994, and Amin and Ng, 1997), few advances have been made on how this particular risk should be monitored and dealt with (existing risk management practices are reviewed by Ho, 2007).

Expanding on an idea originally mentioned in Brenner and Galai (1989), we take a fresh look at interest rate volatility by employing ideas and tools from the extensive recent research on volatility indices in equity markets. This allows us to make a number of extensions to the literature. Specifically, using a well established model-free methodology which was first used for the VIX equity market volatility index, we build a set of new metrics for interest rate volatility. In our particular application, these metrics are employed as proxies of expected volatility for Treasury market instruments on the basis of information contained in interest rate options traded on the CBOE. We argue that the so-called VXI implied interest rate volatility indices we build are advantageous when compared to alternative proxies. We add to the empirical evidence on interest rate volatility by studying the behaviour of four VXI indices with maturities of 13 weeks, 5 years, 10 years and 30 years, respectively. The results indicate that implied volatility over a twelve year period is substantial in magnitude and variation and is subject to violent jumps. For example, in the case of the 5-year instrument, volatility is almost double in comparison to that of the VIX equity volatility index (39.34% *vs*. 20.41%). Over the recent credit crisis, levels of implied interest volatility of short and medium-term rates have increased sharply more than fourfold compared to

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[1] As pointed out in a recent Economist article (*Interest-rate risk. Surf's up. Banks' next big problem appears on the horizon*, Feb 25th 2010), the corporate effect of interest rates 'varies': "*'Asset-sensitive' firms, whose assets are of shorter duration than their liabilities and therefore reprice faster, tend to do well when rates rise. 'Liability-sensitive' banks are more exposed to rising funding costs and see their margins squeezed*". As discussed in a study by the research firm CreditSights cited by the article: "*Among big banks the picture is mixed. A 0.5 percentage-point rise in rates would cost Citigroup $771m in annual net interest income...Wells Fargo would gain by a similar amount.*"

the recent past. As expected, a negative premium is attached to the interest rate volatility risk which is much higher in magnitude than that reported for the equity market. An important new result is that our estimates of interest rate volatility risk premia have a time varying-correlation with equity market volatility risk premia. Another useful contribution is related to the finding that our VXI indices, as is the case with the VIX index, offer valuable diversification opportunities to bond and equity investors. Specifically, our measures of interest rate volatility have a strong negative correlation with interest rate levels (up to -85.8) and equity market implied volatility index levels (up to -24.7%).

In line with previous research, we show for the first time that macroeconomic and monetary announcements affect significantly implied interest rate volatility by decreasing (increasing) it the day before (after). Another new result is that this effect varies across the term structure and becomes more prominent at the longer maturities. In agreement with several other studies we find that interest rate implied volatility is not fully spanned by the information which is contained in the underlying yield curve. Finally, motivated by our results and the rapid development of the equity volatility derivative market, we propose our VXI indices are vehicles for developing options and futures which can be used for managing and trading interest rate volatility risk. On the basis of a horserace amongst popular continuous time models for representing the VXI index empirical behavior, we develop a single-factor pricing framework using autonomous mean-reverting jump-diffusions.

2. Methodology: Interest Rate Implied Volatility Indices (VXI)

Measuring interest rate volatility is a daunting task due to the fact that, as with the volatility of equities, it is empirically unobservable. Two main approaches are typically employed by academics and practitioners. The first resorts to historical time series of interest rates in order to derive estimates of "historical volatility" using unconditional moment estimators, exponential moving averages, ARCH models, stochastic volatility models, etc. (for a comprehensive treatment of these models see Mills and Markellos, 2008; a review of the historical interest rate volatility literature is Ederington and Lee, 2007). The second approach aims at calculating the "implied volatility" that equates actual prices of interest rate options with those given by some theoretical pricing model. Since this estimate reflects market data, it incorporates investor expectations, behaviors and risk attitudes about the future evolution of volatility. Although there is controversy in the empirical literature about which approach is superior, most researchers seem to agree that implied volatility is better than historical volatility in terms of forecasting power (see, for example, Poon and Granger, 2003). A third nonparametric approach employs intraday price data to derive so-called realized volatility measures (see Andersen and Benzoni, 2008). Although this last approach is known now to be theoretically and empirically appealing, it is still not widely applied due to the significant data requirements it has.

Turing now to the interest rate in particular, most of the previous empirical research on volatility is based on estimates derived by inverting observed option prices on the basis of the Heath, Jarrow and Morton (1992) model (see, among others, Amin and Morton, 1994; Amin and Ng, 1997). Two notable exceptions are the studies by De Jong, Driessen and Pelsser (2001) and Christiansen and Hansen (2002) which estimate implied volatility via the LIBOR market model. Unfortunately, the implied volatility estimation approach used by the above studies comes with two important disadvantages. First, the accuracy of the estimates depends critically on the validity of the option pricing model assumed. Second, at any particular moment, there are as many implied volatility estimates as strike prices of the options. In order to overcome such problems, Britten-Jones and Neuberger (2000) and Jiang and Tian (2005) propose a model-free methodology that calculates implied volatility using the entire set of the option prices at a certain point of time. In both studies, the authors provide evidence that the model-free implied volatility is better than both historical volatility and model-driven implied volatility.

In recent years there has been a great deal of research also on the construction and the properties of equity implied volatility indices (Fleming et al., 1995, Moraux et al., 1999, Whaley, 1993, 2000, 2009, Simon, 2003, Wagner and Szimayer, 2004, Giot, 2005, Carr and Wu, 2006). The first volatility index (VIX) was introduced in 1993 by the CBOE. Soon after the introduction of the index, CBOE was subject to strong criticism regarding the methodology used for the calculation of VIX. Originally the VIX was calculated as an average of the Black and Scholes (1973) at-the-money (ATM) option implied volatility, according to the methodology proposed by Whaley (1993). As a response, on September 22, 2003, the CBOE changed the Black-Scholes based methodology of VIX calculation. The new VIX methodology is independent of any model and allows VIX to be robustly replicated by a portfolio of options (see CBOE, White Paper, 2009 and Carr and Wu, 2006, for a detailed description of "new" VIX methodology and for the comparison of the two methodologies).[2] Specifically, the new VIX implied volatility index is constructed as the weighted sum of out-of-the-money (OTM) call and put option closing prices at two nearby maturities across all available strikes. The implied volatility index captures the implied volatility of a synthetically created ATM option with a constant maturity of 30 days. Several other equity implied volatility indices have also been developed. These include the VXN, the VXD and the RVX in the CBOE, which are the equivalent to VIX implied volatility indices for the NASDAQ, Dow Jones Industrial Average and Russell 2000 Index, respectively. Similarly, we have the DAX-30 volatility index (VDAX-NEW) in Germany, the CAC-40 volatility index (VCAC) in France and the Dow Jones EURO STOXX

[2] CBOE still quotes the "old" VIX, which is calculated with the old methodology, under the ticker "VXO". All volatility indices, apart from VXO, quoted in CBOE are calculated with the new model free methodology.

50 volatility index (VSTOXX) in the Eurex. Given the great success of equity implied volatility indices and the rapidly expanding market for volatility futures and options, CBOE recently decided to launch three more implied volatility indices in different asset classes than equity: the Crude Oil Volatility Index (OVX), the EuroCurrency Volatility Index (FVX) and the Gold Volatility Index (GVX).

Although the model-free methodology and index construction has been widely applied for equities, to the best of our knowledge no relevant research has been done yet in the interest rate literature. This will be one of the objectives of our paper. In order to construct the interest rate implied volatility index (VXI) we closely follow the methodology of VIX. The price of the interest rate implied volatility index at time *t* is then calculated by:

$$\sigma = \sqrt{\frac{2}{T-t}\sum_{i}\left(\frac{\Delta K_{\iota}}{K_i^2}e^{rT}Q\left(K_{\iota},T\right)\right)-\frac{1}{T}\left[\frac{F_t}{K_0}-1\right]^2} \tag{1}$$

Where T is the time to maturity of interest rate options involved in the calculation, F_t is the forward interest rate level at time t derived from the interest rate option prices, K_0 is the first strike below F_t, K_i is the strike price of i^{th} OTM option (call if $K_i>K_0$, put if $K_i< K_0$; both put and call if $K_i=K_0$, ΔK_i is the half interval between the strike prices K_{i+1} and K_{i-1}), r is the time t risk-free interest rate to expiration and $Q(K_i)$ is the average of the quoted bid-ask spread (mid-quote) for each option with strike K_i.

In short, the calculation of each daily VXI price is as follows. First, two option series with the nearest expirations are selected. Both option series must have at least one week to expiration. Otherwise, we roll to the next option series with the nearest expiration. The methodology uses all the OTM options with non-zero bid prices, except the put (call) options with a higher (lower) strike price than the strike price of two consecutive puts (calls) with zero bid prices. Second, for each option selected, we calculate the mid-quote $Q(K,T)$. Third, the forward level F_t of the underlying interest rate is determined for each expiration under consideration. F_t is derived via the put-call parity relation:

$$F_t = e^{r_t(T-t)}\left(C_t - P_t\right) + K$$

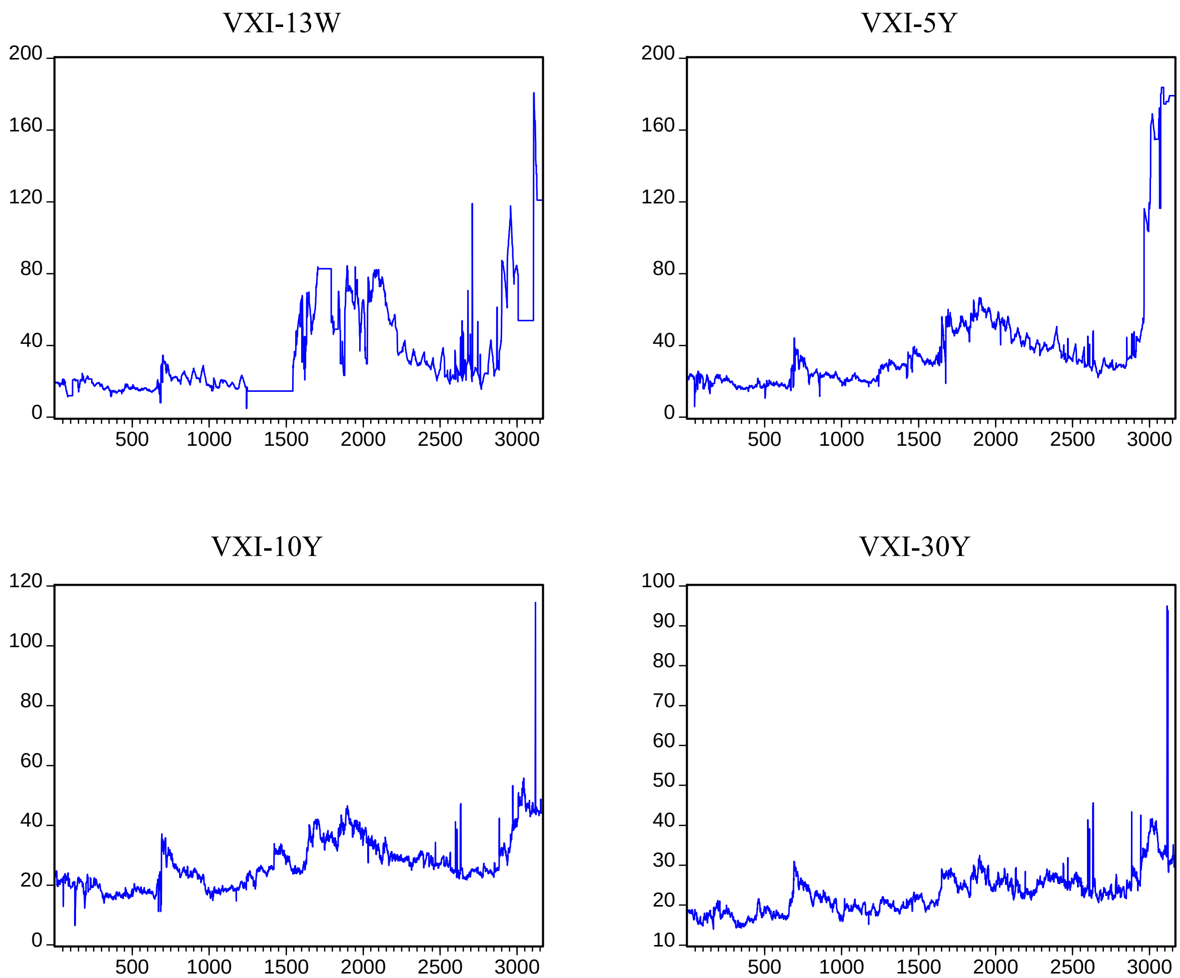
where C_t and P_t is the call and put price respectively. Fourth, we apply the formula (1) on each of the two series with mid-quotes that we have generated, and we come up with two index values, one for each expiration. Fifth, we interpolate between the two index values to obtain an index value with 30 days expiration. We use the same interpolation formula that CBOE uses for the case of VIX. Finally, VXI is the square root of that value which is then typically multiplied by 100. The VXI represents the risk-neutral expectation of the annualized volatility of the underlying interest rate over the next 30 calendar days. As in the case of equity implied volatility indices, where each implied volatility index corresponds to the implied volatility of a stock index, we can construct as many interest rate implied volatility indices, as the different maturities of interest rates. The same methodology can be applied to all interest rate markets, where there is an active options market, e.g., options on Eurodollar futures traded at CME, treasury option traded at CBOE or at CBOT, etc.

3. Empirical Results

The data employed in the empirical analysis correspond to daily market prices for interest rate call and put options traded on the Chicago Board Options Exchange (CBOE) over the period 1/4/96 to 8/29/08, a total of 3,159 trading days. These are cash-settled European style options written on the spot yield of U.S. Treasury securities. Currently there are 4 different contracts available written on: the annualized discount rate of the most recently auctioned 13-week Treasury bill, and, on the yield-to-maturity of the most recently auctioned 5 year Treasury note, the 10 year Treasury note and the 30 year Treasury bond, respectively. The ticker symbols of the underlying instruments are IRX, FVX, TNX and TYX, respectively (see the website of CBOE for more details). Most studies of interest rate implied volatility use data from swaptions (see, among others, Trolle and Schwartz, 2009) and option contracts written on Eurodollar futures (see, for example, Amin and Morton, 1994, and, Amin and Ng,1997). To the best of our knowledge, only Christiansen and Hansen (2002) uses interest rate options data from CBOE to analyze the IRX rate. However, since they calculate the implied volatilities through the LIBOR market model, their estimates are subject to model misspecification. Our dataset offers four main advantages. First, it gives us the opportunity to provide empirical evidence on a relatively unexplored market. Second, the interest rate options we analyse are much simpler than options on Eurodollar futures, since the former are written directly on interest rates. In this manner we deal directly with the quantity of interest and avoid any irrelevant effects. Third, since we are dealing with new volatility metrics and derivatives, it makes sense to base our study around the CBOE which is leader in the volatility securitisation and monitoring industry. Finally, since all the data are provided by the CBOE, we preserve homogeneity and minimize the errors that may result from asynchronous trading and variations in data quality.

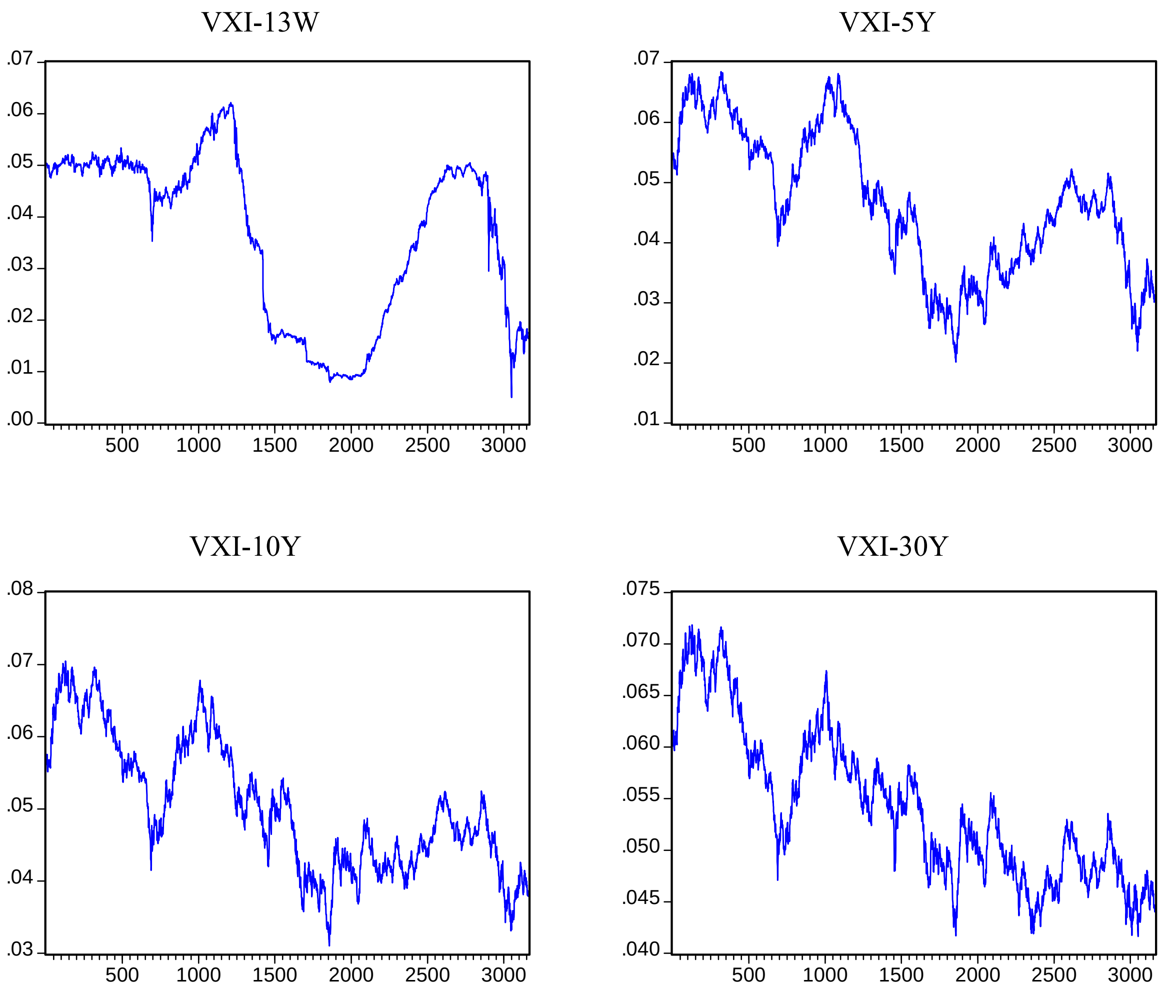
Using the model-free methodology described previously we derive the implied volatility from the option contracts on IRX, FVX, TNX and TYX and coin the corresponding indices as VXI-13W, VXI-5Y, VXI-10Y and VXI-30Y, respectively. We estimate also simple logarithmic returns for these indices and denote them as: ΔVXI-13W, ΔVXI-5Y, ΔVXI-10Y and ΔVXI-30Y (these will be refereed to simply as returns or changes in the remainder of the paper). The implementation of the model-free methodology assumes a very liquid market. However, due to the relatively low liquidity of interest rate options during some days some of the conditions imposed by the method are not met for the construction of the indices. For the days where the value of the implied volatility index cannot be computed, around 5% of the sample, we use the value of the previous trading day.

Figure 1. VXI Indices over the period 1/4/96 to 8/29/08



Time series plots of the indices and underlying yields are presented in Figures 1 and 2. Descriptive statistics of index levels and returns are given in Table 1. We also include some results for the VIX, S&P500 and the Merrill Option Volatility Expectations (MOVE©) indices in order to facilitate comparative inferences. The MOVE© is calculated by Merrill Lynch using implied yield normalised volatility from constant one-month at-the-money OTC Treasury securities with maturities of 2 years, 5 years, 10 years and 30 years, respectively. Yields of all maturities are equally weighted with 20% except for the 10-year which has a weight of 40%. No detailed information was found on the exact method that is used to normalise the volatility and to derive it from the options data.

Figure 2 Treasury rates over the period 1/4/96 to 8/29/08



In line with previous studies (see, among others, Ait-Sahalia, 1996; Andersen and Lund, 1997; Amin and Morton, 1994, and Amin and Ng, 1997), the plots indicate that interest rate volatility is substantial and varies significantly across time. The averages and standard deviations are very different cross-sectionally between the indices analyzed. The two indices with the shortest maturity of the underlying have the highest level of implied volatility ($\mu_{VXI\text{-}13W}$ = 35.24, $\mu_{VXI\text{-}5Y}$ = 39.34) and variability ($CV_{VXI\text{-}13W}$ = 72.64%, $CV_{VXI\text{-}5Y}$ = 83.69%), much higher than those for the VIX (μ_{VIX} = 20.41, CV_{VIX}= 32.83%). This other two indices have a similar average level and variability compared to the VIX. A sharp shift took place recently in volatility as the credit crisis unfolded. Specifically, the VXI-13W and VXI-5Y increased more than fourfold since June 2008 from an average level of 33.56 and 37.02 to 135.71 and 177.86, respectively. The extreme positive and negative returns confirm what can be seen visually in the plots as violent and abrupt changes. All VXI indices are non-normally distributed with a positive skewness and excess kurtosis. Again these characteristics are far more prominent for the two short term indices studied. Finally we see that the behavior of the VXI indices to the MOVE is quite different. The differences in magnitude are expected since MOVE is based on some normalization scheme. MOVE changes are far more smooth and closer a Gaussian distribution.

Table 1. Descriptive statistics of daily VXI, MOVE and VIX indices over the period 1/4/96 to 8/29/08

	VXI-13W	**VXI-5Y**	**VXI-10Y**	**VXI-30Y**	**MOVE**	**VIX**
Observations	3,159	3,159	3,159	3,159	3,159	3,159
Mean	35.2442	39.3429	27.0728	22.9346	99.3071	20.4110
Median	23.5991	29.7287	25.3819	22.4687	99.6800	19.7800
Max	180.6814	183.6313	114.4094	94.9674	195.0000	45.7400
Min	4.9360	5.9831	6.4852	13.9961	51.2000	9.8900
St. Deviation	25.6010	32.9273	8.3109	5.3406	22.8868	6.7018
CV	0.7264	0.8369	0.3070	0.2329	0.2305	0.3283
Skewness	1.7776	3.0927	1.0918	2.2918	0.3165	0.7741
Kurtosis	6.8120	12.5641	6.7058	23.7647	2.9542	3.4807
Jarque-Bera	3.5764E+03	1.7076E+04	2.4353E+03	5.9518E+04	5.3011E+01	3.4592E+02
$\rho(1)$	0.9792	0.9891	0.9624	0.8897	0.9834	0.9832
	ΔVXI-13W	**ΔVXI-5Y**	**ΔVXI-10Y**	**ΔVXI-30Y**	**ΔMOVE**	**ΔVIX**
Mean	0.0064	0.0028	0.0019	0.0018	0.0009	0.0016
Median	0.0000	0.0000	0.0000	0.0011	-0.0009	0.0000
Max	4.9648	1.8604	1.5804	1.9344	0.2875	0.6422
Min	-0.6462	-0.7241	-0.6800	-0.6755	-0.1653	-0.2591
St. Deviation	0.1464	0.0729	0.0623	0.0665	0.0413	0.0550
Skewness	17.4234	9.8764	7.8762	16.3295	0.9183	1.1050
Kurtosis	487.2888	212.8932	193.5014	467.8288	8.2960	11.1054
Jarque-Bera	3.1021E+07	5.8483E+06	4.8079E+06	2.8571E+07	5.3011E+01	9.2873E+03
$\rho(1)$	-0.1249	-0.1698	-0.2293	-0.2745	0.0312	-0.0482

Jarque-Bera is a test of normality. $\rho(1)$ is the coefficient of an AR(1) model with a constant.

In line with studies such as Dai and Singleton (2003), Litterman, Scheinkman, and Weiss (1991) and Chapman and Pearson (2001), our results across the 4 maturities studied suggest that the implied volatility term structure is hump-shaped with a peak at the 5 year period. In addition to the level of volatility, we find that the variability of the indices also has a similar hump-shaped pattern. As in Amin and Morton (1994) and Ball and Torous (1999), the preliminary analysis for all series demonstrates that interest rate volatilities are highly persistent but mean reverting. A first indication for this is given by the time series plots and the fact that autocorrelation coefficients of levels at lag 1 are just below unity. These results are confirmed by various univariate and panel econometric tests for stationarity and unit roots (results available upon request). An exception is the VXI-5Y index which has a sharp shift over the recent period and has a first order autocorrelation very close to unity. However, if we exclude from the sample the last months since September 2008, which correspond the credit crisis period, this series also appears to be stationary. In contrast to Ball and Torous (1999), we find that interest rate volatility displays similar mean-reversion to that of the equity market, as captured by the size of the first-order autocorrelation coefficient. A possible exception is the VXI-30Y which as a considerably smaller coefficient.

It is interesting to examine if the substantial volatility risk is priced by investors. In other words, if a volatility risk premium (VRP) is demanded as a compensation for assuming interest rate volatility risk. According to Bollerslev, Tauchen and Zhou (2009), and Carr and Wu (2009), the VRP over the period from t to $t+\tau$ can be defined as the difference between the realized volatility under the statistic measure P and the expectation of the future volatility under risk neutral measure Q, i.e.:[3]

$$VRP_{t,t+\tau} = E_t^P[V_{t,t+\tau}] - E_t^Q[V_{t,t+\tau}] \tag{2}$$

A number of theoretical and practical implications of the VRP have been discussed in the literature. Chernov (2007) emphasizes the role of the VRP for portfolio managers and policymakers in allowing them to form better forecasts of future volatility. Accordingly, Bollerslev, Tauchen and Zhou (2009) show that the VRP can explain a significant portion of the variation in the post-1990 aggregate stock market returns. Almeida and Vicente (2009) argue that the VRP is crucial in reconciling option market implied volatilities with spot market historical volatilities. Joslin (2007) discuss the importance of the market price of volatility risk for matching the option price dynamics. Bakshi and Kappadia (2003) point out that a negative VRP implies that option prices are higher than the prices when volatility risk is not priced. A

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[3] Bollerslev, Tauchen and Zhou (2009) use the VIX index as a proxy for the risk-neutral expectation of the future volatility along with realized volatility derived from S&P500 intraday returns. Car and Wu (2009) employ futures prices and swap rates for the calculation of the realized and future volatility, respectively. An earlier approach by Bakshi and Kapadia (2003) measures the VRP from the delta-hedged profits of options portfolios. Specifically, these authors assume a long call option position which hedged by a short position in the underlying stock, such that the net investment outcome the risk-free return. The sign and magnitude of the average delta-hedged returns determine the volatility risk premium if volatility risk is priced.

variety of option pricing models explicitly incorporate a volatility risk premium and account for stylized facts such as stochastic and the sensitivity to the level of volatility.

Empirical studies in equity (Bakshi and Kappadia, 2003; Bakshi and Madan, 2006; Bollerslev, Tauchen and Zhou, 2009; Carr and Wu, 2009; Todorov, 2010), currency (Guo and Neely, 2004) and fixed income markets (Joslin, 2007; Fornari, 2008; Almeida and Vicente, 2009) show that the volatility risk premium is negative, time-varying and dependent on the level of volatility. Estimates of the VRP range between -2% to -3% and -4% to -5% for developed equity and fixed income markets, respectively. The negative sign of the premium is explained by considering that investors that are long in volatility are willing to pay a premium in order to insure themselves against upward movements in volatility.

Figure 3. Interest rate volatility risk premia over the period 1/4/96 to 8/29/08

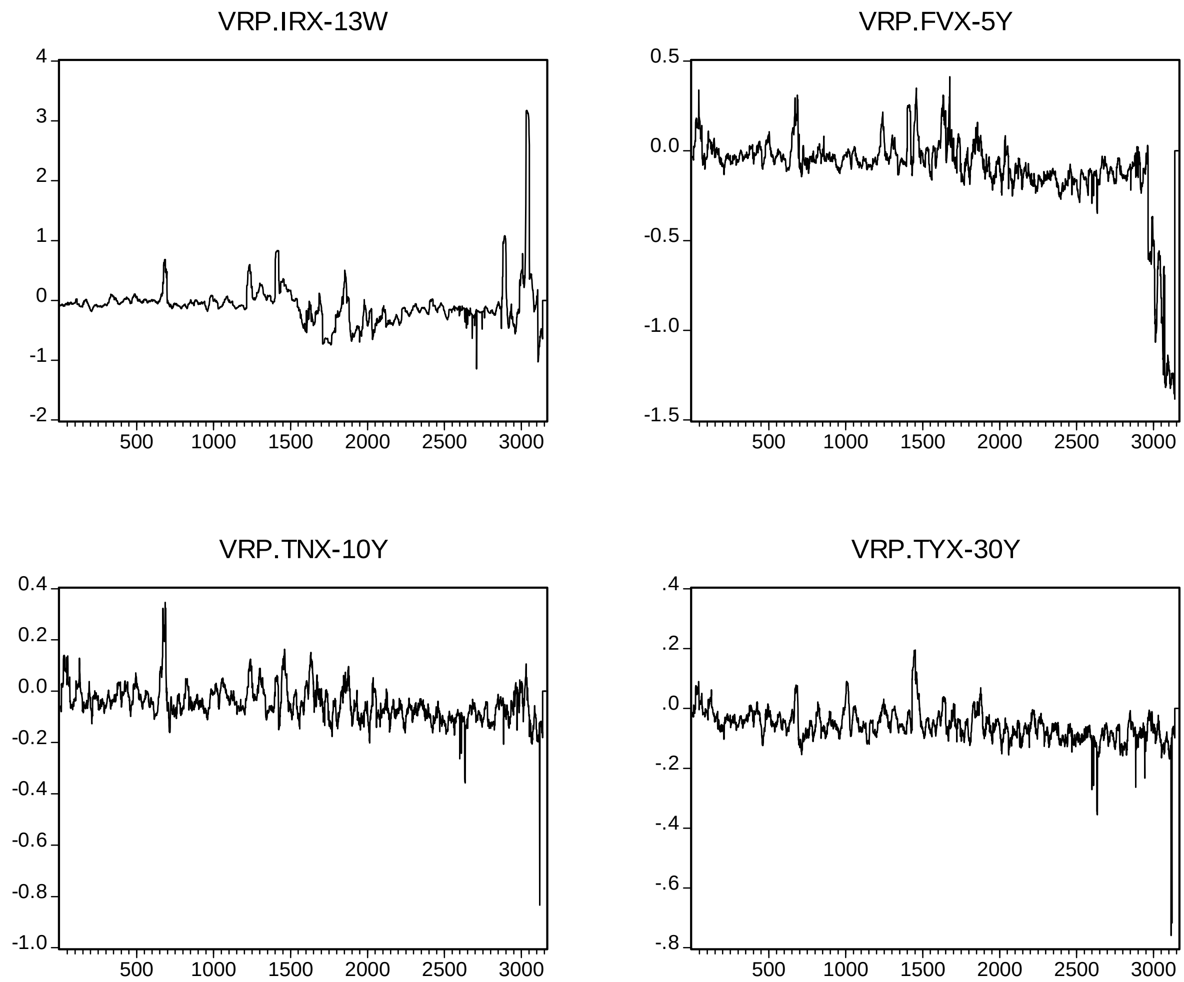


Table 2. Descriptive statistics of interest rate volatility risk premia over the period 1/4/96 to 8/29/08

	VRP.IRX-13W	**VRP.FVX-5Y**	**VRP.TNX-10Y**	**VRP.TYX-30Y**	**VRP.SP500**
Mean	-0.0755	-0.1029	-0.0477	-0.0571	-0.0194
Median	-0.0900	-0.0631	-0.0552	-0.0601	-0.0245
Max	3.1727	0.4114	0.3468	0.1944	0.2386
Min	-1.1455	-1.3833	-0.8336	-0.7588	-0.2045
St. Deviation	0.3679	0.2300	0.0661	0.0522	0.0543
Skewness	4.6106	-3.3782	0.2354	-0.8980	0.9781
Kurtosis	39.6867	16.7485	11.9236	24.3514	6.0989
Jarque-Bera	1.9053E+05	3.1292E+04	1.0503E+04	5.9352E+04	1.7846E+03
$\rho(1)$	0.9823	0.9866	0.9232	0.8668	0.9564
	***Δ*VRP.IRX-13W**	***Δ*VRP.FVX-5Y**	***Δ*VRP.TNX-10Y**	***Δ*VRP.TYX-30Y**	***Δ*VRP. SP500**
Mean	0.0113	-0.0714	0.0445	-0.0681	0.3869
Median	-0.0006	-0.0013	-0.0074	0.0002	-0.0254
Max	69.9752	22.7400	197.4842	77.6308	1973.2390
Min	-77.8863	-117.1709	-114.1927	-83.1226	-1951.7070
St. Deviation	3.0478	3.1409	5.4068	2.7911	52.5937
Skewness	1.8901	-27.9670	21.8505	-5.4509	1.6669
Kurtosis	393.3872	962.1677	930.5794	510.7815	1252.4380
Jarque-Bera	1.9916E+07	1.2100E+08	1.1300E+08	3.3707E+07	2.0400E+08

Jarque-Bera is a test of normality. ρ(1) is the coefficient of an AR(1) model with a constant.

Since volatility risk management and option pricing is a prime objective in the present paper, we examine the empirical behavior of the interest rate volatility risk premium using our VXI data. Specifically, we estimate the VRP at time *t* as following:

$$VRP_t = RV_{t,t+30} - VXI_t \tag{3}$$

Where VXI_t is the risk neutral interest rate expected volatility over the next month and $RV_{t,t+30}$ is the realized volatility for the same period. Following Carr and Wu (2009), the realized volatility is calculated from the following formula:

$$RV_{t,t+30} = \frac{365}{30}\sum_{i=1}^{30}\left(\frac{r_{t+i} - r_{t+i-1}}{r_{t+i-1}}\right)^2 \tag{4}$$

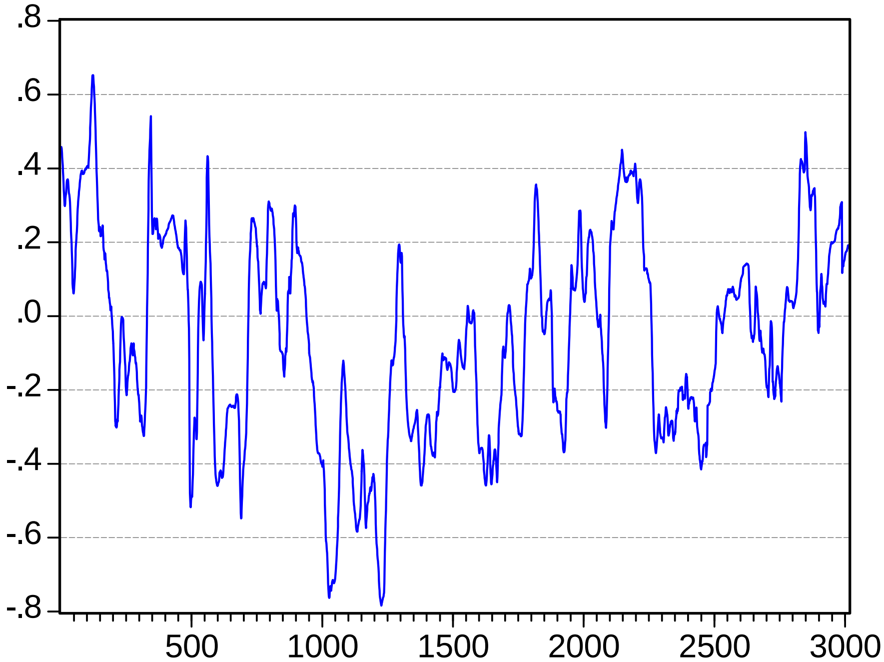
where r_t denotes the time *t* treasury rate. In approximating the second term, we use the volatility estimates based on the sum of squared interest rate daily returns over the period of one month. The estimated risk premia are depicted in Figure 3 while Table 2 gives some summary statistics. In order to facilitate comparisons we also include descriptive statistics on the SP500 *VRP*. The latter was estimated using the same methodology as for the interest rate *VRP*. The VIX index used as a proxy of the model-free S&P500 implied volatility, while realized volatility estimated over the next month using the sum of squared S&P500 index daily returns. The results indicate that interest rate *VRP* are time varying and subject to violent upward and downward shifts. The median interest rate volatility premium ranges between -5.52% for the 10 year maturity down to -9% for the 13-week instrument. These estimates are higher than the -4% premium obtained for US interest rates by Fornari (2008) using a different methodology and dataset. The interest rate *VRP* are also clearly much higher than the -2.45% premium estimate obtained for the equity market using the VIX and S&P500 returns. The interest rate *VRP* we find have high variability, especially for the two shortest tenors examined. The distributions of premia and premia changes are highly nonnormal with many violent positive and negative jumps. The plots suggest that premia are increasing over the recent past. Specifically, since June 2008 median premia have increased in magnitude by a factor of 1.92, 2.36, 1.3 and 1.15 when compared to the previous period for the case of the IRX, FVX, TNX and TYX, respectively. It is interesting to note that the VIX volatility risk premium decreased in magnitude by a factor of 0.47 since June 2008. Comparable results are obtained if averages are used rather than the outlier-robust median measures of central tendency.

Table 3. Correlation analysis of volatility risk premia over the period 1/4/96 to 8/29/08

	VRP.IRX-13W	VRP.FVX-5Y	VRP.TNX-10Y	VRP.TYX-30Y	VRP.SP500	IRX	FVX	TNX
VRP.IRX-13W	100.0	-4.4	29.7	15.2	6.9	15.1	6.9	5.3
VRP.FVX-5Y	-4.4	100.0	51.2	46.4	-5.9	24.5	32.1	35.1
VRP.TNX-10Y	29.7	51.2	100.0	78.4	-2.8	14.9	18.0	22.3
VRP.TYX-30Y	15.2	46.4	78.4	100.0	-7.7	6.0	17.6	25.4
VRP.SP500	6.9	-5.9	-2.8	-7.7	100.0	-0.5	-7.9	-8.7
IRX	15.1	24.5	14.9	6.0	-0.5	100.0	87.7	75.4
FVX	6.9	32.1	18.0	17.6	-7.9	87.7	100.0	96.9
TNX	5.3	35.1	22.3	25.4	-8.7	75.4	96.9	100.0
TYX	4.0	37.6	28.0	34.5	-10.1	55.7	85.1	95.0

The two-tailed 5% and 1% critical values for the absolute value of the correlation coefficient are 3.49% and 4.59%, respectively.

Figure 4. Correlation between VRP.TYX-30Y and VRP.SP500 risk premia over the period 1/4/96 to 8/29/08 using a rolling window of 125 trading days (6 months)



The two-tailed 5% and 1% critical values for the absolute value of the correlation coefficient are 17.5% and 22.87%, respectively.

A correlation analysis of the *VRP* with respect to the underlying interest rates is shown at Table 3. The inspection of the table reveals several interesting results. First, interest rate *VRP* are interrelated between them, especially at the longer maturities considered (eg., the *VRP* at the two longer maturities have a correlation of 78.4%). Second, there is a positive "level effect" in that VXI-derived *VRP* are correlated to the levels of the underlying interest rates. In other words, risk premia are higher at higher levels of interest rates. Third, there appears to be a weak correlation between equity and interest rate volatility risk premia. This correlation is positive only for the shortest-term maturity considered. The relationship between these two premia should be examined in the context of the voluminous literature on the association between bond and equity markets (see Baele *et al.*, 2010, *inter alia*). Volatility risk premia, as proxies of investor risk aversion and attitudes, should be equal between these two markets according to most asset pricing frameworks. However, the "flight to quality" and "flight from quality" phenomena predict an inverse relationship between the risk premia in the two markets. Specifically, when the stock market crashes (rallies) then risk aversion towards equities (bonds) increases (decreases) and investors move to bonds (stocks). Empirical evidence with respect to the direction of the relationship between stocks and bonds has been conflicting. However, recent studies suggest that this relationship is time-varying and depends on a variety of macroeconomic and microeconomic variables (see Baele *et al.*, 2010). In light of this evidence we undertook a rolling correlation analysis using a window of 125 trading days which is equivalent to a calendar period of 6 moths (for a similar approach see, for example, Connolly *et al.*, 2005). The results for the VRP.TYX-30Y, depicted in Figure 4, suggest that the relationship is indeed time-varying with correlation assuming negative values over most of the sample period under study.

Table 4. Correlation coefficients (%) of VXI levels and changes with other variables (1/4/96 to 8/29/08)

	VXI-13W	**VXI-5Y**	**VXI-10Y**	**VXI-30Y**	**MOVE**	**IRX-13W**	**FVX-5Y**	**TNX-10Y**	**TYX-30Y**	***Δ*SP500**	**VIX**
VXI-13W	100.0	67.4	73.3	65.1	35.3	-66.1	-66.5	-60.8	-51.0	-2.2	-9.8
VXI-5Y	67.4	100.0	80.3	75.0	44.4	-54.7	-61.3	-58.7	-52.3	-1.0	-20.7
VXI-10Y	73.3	80.3	100.0	86.2	49.0	-79.2	-85.8	-80.3	-68.9	-2.2	-3.7
VXI-30Y	65.1	75.0	86.2	100.0	29.7	-53.9	-71.8	-74.8	-74.4	-1.5	-23.5
MOVE	35.3	44.4	49.0	29.7	100.0	-45.0	-27.2	-11.2	7.7	-1.8	21.6
IRX-13W	-66.1	-54.7	-79.2	-53.9	-45.0	100.0	87.7	75.4	55.7	2.5	-16.9
FVX-5Y	-66.5	-61.3	-85.8	-71.8	-27.2	87.7	100.0	96.9	85.1	2.5	-2.3
TNX-10Y	-60.8	-58.7	-80.3	-74.8	-11.2	75.4	96.9	100.0	95.0	2.2	6.3
TYX-30Y	-51.0	-52.3	-68.9	-74.4	7.7	55.7	85.1	95.0	100.0	2.0	19.1
***Δ*SP500**	-2.2	-0.9	-2.0	-1.2	-1.9	2.2	2.2	1.8	1.6	100.0	-10.9
VIX	-9.8	-20.7	-3.7	-23.5	21.6	-16.9	-2.3	6.3	19.1	-10.2	100.0

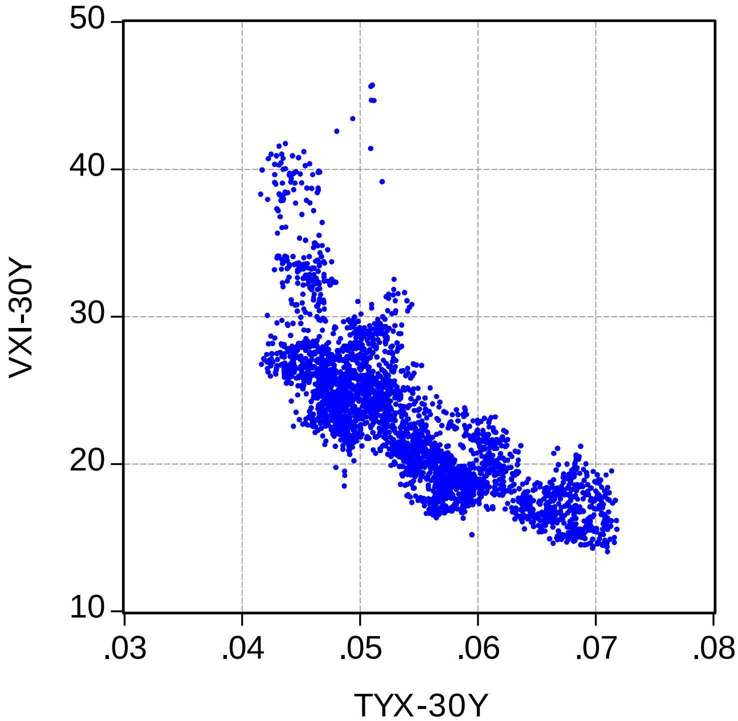
	***Δ*VXI-13W**	***Δ*VXI-5Y**	***Δ*VXI-10Y**	***Δ*VXI-30Y**	***Δ*MOVE**	***Δ*IRX-13W**	***Δ*FVX-5Y**	***Δ*TNX-10Y**	***Δ*TYX-30Y**	***Δ*SP100**	***Δ*VIX**
***Δ*VXI-13W**	100.0	4.4	5.7	4.0	0.2	2.6	3.7	2.5	1.0	-1.1	0.0
***Δ*VXI-5Y**	4.4	100.0	38.8	19.1	4.7	1.3	0.3	0.6	-1.0	0.1	-1.8
***Δ*VXI-10Y**	5.7	38.8	100.0	57.1	11.9	-2.5	3.5	4.0	2.7	0.7	-2.3
***Δ*VXI-30Y**	4.0	19.1	57.1	100.0	8.3	-1.2	1.5	0.5	-0.4	1.6	-3.2
***Δ*MOVE**	0.2	4.7	11.9	8.3	100.0	-4.2	10.2	13.5	15.4	0.2	-0.5
***Δ*IRX-13W**	2.6	1.3	-2.5	-1.2	-4.2	100.0	30.0	24.8	19.6	-0.1	2.0
***Δ*FVX-5Y**	3.7	0.3	3.5	1.5	10.2	30.0	100.0	94.6	82.5	0.3	-2.0
***Δ*TNX-10Y**	2.5	0.6	4.0	0.5	13.5	24.8	94.6	100.0	93.1	0.4	-2.0
***Δ*TYX-30Y**	1.0	-1.0	2.7	-0.4	15.4	19.6	82.5	93.1	100.0	0.3	-1.3
***Δ*SP500**	-1.1	0.1	0.7	1.6	0.2	-0.1	0.3	0.4	0.3	100.0	-71.6
***Δ*VIX**	0.0	-1.8	-2.3	-3.2	-0.5	2.0	-2.0	-2.0	-1.3	-71.6	100.0

The two-tailed 5% and 1% critical values for the absolute value of the correlation coefficient are 3.49% and 4.58%, respectively.

In order to examine the contemporaneous relationship between the VXI indices and other series, we undertake a correlation analysis. The results, shown in Table 4, demonstrate clearly that a strong positive relationship (all correlations over 65%) exists between the VXI indices at the four maturities studied. The MOVE index is positively related to VXI levels with correlation coefficients ranging between 29.7% and

49%. The negative correlation of -10.9% between the VIX and S&P500 returns confirms what is widely known in the financial industry with respect to the hedging benefits of implied equity volatility. Although we find that the VXI indices have no linear correlation with SP500 returns, they are negatively correlated with VIX levels, especially for the 5 year and 30 year maturity studied (-20.7% and -23.5, respectively). A strikingly significant result is the strong negative relationship between levels of VXI and interest rates with the correlation coefficients ranging between -51% (VXI-13W with TYX-30Y) and -85.8% (VXI-10Y with FVX-5Y).

Figure 5. VXI against underlying TYX-30Y Treasury rates over the period 1/4/96 to 8/29/08



This negative relationship, also apparent in the scatter plot given in Figure 5 for the TYX-30Y, is consistent with findings throughout the interest rate literature on the so-called "level effect" according to which interest rate volatility is sensitive to the level of interest rates. However, there is controversy with respect to the size and sign of the level effect. Earlier studies characterize this relationship as strongly positive, whereby high volatility is associated with high interest rate levels (see, for example Chan, *et al.* 1992). Later studies, which account for properties of the series such as autocorrelation and heteroskedasticty, find a much weaker positive relationship (see, for example, Brenner *et al*, 1996; Andersen and Lund, 1997; Ball and Torous, 1999). More recently, Trolle and Schwartz (2009) use interest rate implied volatility estimates from swaptions and caps, and report both positive and negative relationships between interest rate implied volatility and interest rate levels, depending on the model used to back-out implied volatilities. Our model-free estimates of volatility shed new light on this empirical puzzle.

It must be noted also that the level effect, positive or negative, has not been justified theoretically or at least intuitively in the interest rate literature. A possible explanation of the negative level-effect could be based by inverting the leverage-effect arguments that have been used in equities. Specifically, the leverage effect hypothesis proposed by Black (1976) and Christie (1982), postulates that negative returns will usually reduce the stock price and market value of the firm, which in turn means an increase in financial leverage, i.e., a higher debt to equity ratio, and this will ultimately lead to an increase in risk and equity volatility. However, from a debt market perspective, higher levels of interest rates mean that the market value of debt (or the price of bonds) decreases which it turn implies less financial leverage and, ultimately, smaller interest rate risk and volatility. In practical terms, our results concerning the negative association of VXI indices with other variables have important practical implications since they suggest that interest rate volatility can act as significant hedge against variations in the underlying interest rate levels and equity market volatility.

A similar picture to that painted above, although correlation coefficients are much smaller in magnitude, is drawn if changes in volatility and interest rates are used in the analysis. Finally, we also examine the behavior of correlations across different subsamples of our data. As in Chan et al. (1992) and Chapman and Pearson (2001) we find that the sensitivity of volatility to the level of interest rates changes through time. Specifically, the average correlation of the VXI-13W with the four interest rates considered changes from -8.46% in the first half of the data (1/4/96 to 2/5/02) to -33.81% in the second half of the data (3/5/02 to 8/29/08). For VXI_5Y, VXI_10Y and VXI_30Y the average correlation changes from -66.90%, -63.64% and -53.85% to -40.44%, -56.98% and -31.41%, respectively.

In order to assess the dynamic relationships and possible spillovers between the variables and markets under consideration we employ Granger-causality analysis. Both levels and changes of the volatility indices and interest rates are considered in order to capture possible dynamic level effects. The statistically significant results, contained in Table 5, allow two main conclusions. First, various intermarket spillovers exist in the Treasury market with interest rates Granger-causing volatility and vice versa. As Amin and Ng (1997) suggest it appears that implied volatility is useful in predicting the future interest rate implied volatilities. Second, the intermarket spillover effects that can be observed between the S&P500 and the Treasury rates involve some of the volatility variables studied. Moreover, in all cases except two, the VIX and ΔVIX appear to lead Treasury market rate variations, levels and volatilities. The two exceptions concern the relationship of ΔVXI-30Y with ΔVIX, and, ΔIRX-13W with VIX, respectively. For the other significant intermarket dynamic relationships we can see that ΔVIX (VIX) Granger-causes ΔFVX-5Y, ΔTNX-10Y, FVX-5Y and TNX-10Y (ΔVIX and VXI-30Y).

Table 5. Granger causality tests for 1 lag of the null hypothesis that "Variable A" causes "Variable B"

Variable A	Variable B	F-Statistic	Variable A	Variable B	F-Statistic
ΔVXI-13W	VXI-13W	203.37**	MOVE	VXI-30Y	5.92*
VXI-13W	ΔVXI-13W	4.99*	MOVE	ΔMOVE	34.66**
VXI-13W	ΔVXI-30Y	21.92**	ΔVIX	ΔFVX-5Y	7.97**
VXI-13W	FVX-5Y	10.72**	ΔVIX	ΔTNX-10Y	6.76**
VXI-13W	VXI-10Y	74.27**	ΔVIX	FVX-5Y	5.06*
VXI-13W	VXI-30Y	144.62**	ΔVIX	TNX-10Y	5.36*
VXI-13W	VXI-5Y	7.45**	VIX	ΔVIX	19.53**
VXI-13W	TNX-10Y	9.73**	VIX	VXI-30Y	11.61**
VXI-13W	TYX-30Y	7.73**	ΔIRX-13W	ΔVXI	13.25**
ΔVXI-5Y	ΔFVX-5Y	7.92**	ΔIRX-13W	IRX-13W	33.97**
ΔVXI-5Y	ΔTNX-10Y	10.72**	ΔIRX-13W	VIX	8.17**
ΔVXI-5Y	ΔTYX-30Y	7.74**	IRX-13W	VXI-10Y	93.10**
ΔVXI-5Y	FVX-5Y	6.99**	IRX-13W	VXI-13W	17.90**
ΔVXI-5Y	VXI-10Y	5.55*	IRX-13W	VXI-30Y	74.74**
ΔVXI-5Y	VXI-5Y	14.69**	ΔFVX-5Y	ΔVXI-10Y	20.45**
ΔVXI-5Y	TNX-10Y	9.67**	ΔFVX-5Y	ΔVXI-30Y	9.60**
ΔVXI-5Y	TYX-30Y	8.04**	ΔFVX-5Y	ΔTNX-10Y	4.17*
VXI-5Y	ΔVXI-30Y	9.86**	ΔFVX-5Y	ΔTYX-30Y	9.81**
VXI-5Y	IRX-13W	5.68*	ΔFVX-5Y	IRX-13W	18.21**
VXI-5Y	VXI-10Y	90.48**	ΔFVX-5Y	VXI-10Y	14.19**
VXI-5Y	VXI-13W	21.56**	ΔFVX-5Y	VXI-30Y	6.28*
VXI-5Y	VXI-30Y	229.26**	FVX-5Y	IRX-13W	18.14**
ΔVXI-10Y	ΔVXI-13W	15.36**	FVX-5Y	VXI-10Y	188.57**
ΔVXI-10Y	VXI-10Y	443.04**	FVX-5Y	VXI-13W	15.90**
ΔVXI-10Y	VXI-30Y	199.15**	FVX-5Y	VXI-30Y	210.97**
ΔVXI-10Y	VXI-5Y	11.00**	FVX-5Y	MOVE	5.24*
VXI-10Y	ΔVXI-10Y	10.82**	ΔTNX-10Y	ΔVXI-10Y	14.30**
VXI-10Y	IRX-13W	11.98**	ΔTNX-10Y	ΔVXI-30Y	6.21*
VXI-10Y	VXI-13W	23.72**	ΔTNX-10Y	IRX-13W	4.63*
VXI-10Y	VXI-30Y	154.98**	ΔTNX-10Y	VXI-10Y	7.55**
VXI-10Y	MOVE	8.17**	ΔTNX-10Y	TNX-10Y	4.16*
ΔVXI-30Y	ΔVIX	4.98*	TNX-10Y	IRX-13W	6.92**
ΔVXI-30Y	VXI-10Y	387.4**	TNX-10Y	VXI-10Y	125.39**
ΔVXI-30Y	VXI-30Y	1130.28**	TNX-10Y	VXI-13W	11.32**
VXI-30Y	IRX-13W	5.97*	TNX-10Y	VXI-30Y	255.74**
VXI-30Y	VXI-10Y	14.30**	TNX-10Y	MOVE	5.96*
VXI-30Y	VXI-13W	15.15**	ΔTYX-30Y	ΔVXI-10Y	12.69**
VXI-30Y	MOVE	5.13*	ΔTYX-30Y	ΔVXI-30Y	6.46*
ΔMOVE	ΔVXI-10Y	31.21**	ΔTYX-30Y	VXI-10Y	4.82*
ΔMOVE	ΔVXI-30Y	5.50*	ΔTYX-30Y	TNX-10Y	6.04*
ΔMOVE	ΔVXI-5Y	50.38**	TYX-30Y	VXI-10Y	62.90**
ΔMOVE	VXI-5Y	12.4**	TYX-30Y	VXI-13W	6.37*
ΔMOVE	MOVE	7.58**	TYX-30Y	VXI-30Y	250.64**
MOVE	ΔVXI-10Y	6.89**	TYX-30Y	MOVE	6.15*
MOVE	ΔVXI-5Y	10.76**			

One (two) stars denote statistical significance at the 95% (99%) level

Our results are consistent with evidence of volatility spillover between equity and bond markets (see, for example, Fleming, Kirby and Ostdiek, 1998 for historical volatility spillover, and, Wang, 2009 for implied volatility spillover). These spillovers can be justified on the basis of commonalities in the information set that simultaneously affects expectations in both markets, i.e., changes in the macroeconomic variables (see, for example, Harvey and Huang, 1991; Ederington and Lee, 1993). Another explanation is based on cross-market hedging which dictates that hedging a position in one asset class by taking an offsetting position in another asset class with similar price movements. Portfolio managers often shift funds from stocks into bonds and vice versa due to a new information arrival that alters their expectations about stock or bond returns. So, a shock in one market will be transferred to the other market due to trading activity meaning that volatility spillover takes place (see, for example, Fleming, Kirby and Ostdiek, 1998).

Another interesting point that receives much attention in the empirical literature is if the volatility implied from interest rate derivatives contains important unspanned components. This issue is of great practical concern since it determines if bonds can be used to hedge interest rate volatility as is predicted by most 'afine' term structure models. Most of the previous studies have used data on LIBOR, swap rates and Eurodollars with mixed results (for a review see Andersen and Benzoni, 2008). For example, Collin-Dufresne and Goldstein (2002), and Li and Zhao (2006) report unspanned stochastic volatility factors which drive interest rate derivatives without affecting the term structure. Heidari and Wu (2003) demonstrate that the level, slope, and curvature term-structure factors manage to explain only around 60% of the cross-sectional variability in option-implied volatilities. This finding is puzzling since these three factors explain over 95% of the variation in the underlying interest rates (see, for example, Litterman and Scheinkman, 1991). Andersen and Benzoni (2008) also find unspanned factors in realized interest rate volatility. In an attempt to address this issue, we undertake a principal component analysis of the four interest rate series under consideration. The results, presented in Table 6a, indicate that the first two factors are able to explain over 99% of the variation in the four interest rate series. However, the four principal components are able to explain only a portion of the variation in the implied volatility indices. Specifically, as shown in Table 6b, the first two yield curve principal components are always significant regressands of the VXI indices. However, they are able to explain only a portion of the variability in the volatility ranging from 37.37% in the case of the VXI-5Y up to 73.27% in the case of the VXI-10Y. Although, these results are only preliminary, since we examine only four instruments, they provide results in support of the hypothesis that interest rate volatility are not fully explained by information contained in the yield curve.

Table 6a. PCA of interest rate levels (1/4/96 to 8/29/08)

	Principal Component			
	1	*2*	*3*	*4*
Eigenvalue	3.4928	0.4766	0.0289	0.0016
Variance Prop.	0.8732	0.1192	0.0072	0.0004
Cumulative Prop.	0.8732	0.9924	0.9996	1.0000
	Eigenvector			
Variable	*1*	*2*	*3*	*4*
VXI-13W	-0.4543	-0.7565	-0.4629	-0.0835
VXI-5Y	-0.5301	-0.1217	0.6166	0.5692
VXI-10Y	-0.5279	0.2204	0.2959	-0.7650
VXI-30Y	-0.4837	0.6035	-0.5639	0.2895

Table 6b. Regression of yield curve principal components against VXI indices (1/4/96 to 8/29/08)

Principal Component	**VXI-13W**	**VXI-5Y**	**VXI-10Y**	**VXI-30Y**
1	8.9459	10.7120	3.7381	2.1097
2	7.4610	2.9680	1.9355	-1.3265
Constant	35.2442	39.3429	27.0728	22.9346
R-squared	0.4671	0.3737	0.7327	0.5746

One (two) stars denote statistical significance at the 95% (99%) level. Heteroskedasticity and autocorrelation consistent covariances and standard errors are estimated using the Newey and West (1987) approach.

Finally, we examined the relationship between VXI and four types of news announcements over the period 1/4/96 to 8/29/08. Specifically, we studied CPI and PPI announcements (152 and 150 events, respectively), Federal Open Market Committee (FOMC) meetings (137 events) and employment announcements (148 events). The meeting dates were downloaded from the website of the Federal Reserve. Since Goodhart and Smith (1985), several papers over the years investigate the impact of such announcements on equity market returns and volatility (for a recent overview of this literature see Chen and Clements, 2007). However, empirical evidence is mixed. Three recent papers focus on implied volatility market and give more conclusive results. Kearney and Lombra (2004) show that the VIX increases along with the surprise element in employment announcements. Nikkinen and Sahlström (2004) find that the VIX rises prior to, and falls after announcements related to the CPI, PPI and FOMC meetings. Chen and Clements (2007) find that the VIX makes a significant drop only on the day of

FOMC meetings. Motivated by this research, it is instructive to see if macroeconomic and monetary news constitute a significant factor in the fixed income market. Following Nikkinen and Sahlström (2004), we adopt the following regression framework in order to examine the impact of news on interest rate volatility:

$$\Delta VXI_t = \alpha + \phi \Delta VXI_{t-1} + \sum_{i=-1}^{1} \beta_i D_{i,t}^{CPI} + \sum_{i=-1}^{1} \gamma_i D_{i,t}^{PPI} + \sum_{i=-1}^{1} \delta_i D_{i,t}^{FOMC} + \sum_{i=-1}^{1} \zeta_i D_{i,t}^{Empl} + \varepsilon_t$$

$$\log(\sigma_t^2) = \omega + \lambda_1 \left| \varepsilon_{t-1} / \sigma_{t-1} \right| + \lambda_2 \, \varepsilon_{t-1} / \sigma_{t-1} + \lambda_3 \log(\sigma_{t-1}^2)$$

Where $D_{-1,t}^{CPI}$ ($D_{+1,t}^{CPI}$) is a dummy variable which takes the value of 1 one day prior (after) to the employment report release day and zero otherwise. On the release day $D_{0,t}^{CPI}$ assumes a value of 1 and zero otherwise. The other dummies are defined accordingly. As Nikkinen and Sahlström (2004), a lagged ΔVXI_t term is used in order to capture persistence in the dependent variable. However, rather than using a GARCH(1,1) specification with normally distributed errors, as in Nikkinen and Sahlström (2004) and Chen and Clements (2007), we adopt the more richer EGARCH(1,1) with errors following a Generalized Error Distribution (GED).

The estimation results are presented in Table 7. In general, the announcements studied have a significant effect on the volatility of all the series except for the case of the VXI-13W. In most cases for the CPI, and FOCM, this effect is negative on both the day of the announcement and the day before. In most cases for the PPI and EMPL, the effect is positive for the day before the announcement and positive on the day. Implied volatility tends to increase following the announcement day for the CPI, PPI and FOCM. These results are broadly in line with those reported by previous researchers for implied equity volatility and suggest that derivative market investors consider the meetings studied as significant for fixed income pricing. For example, Chen and Clements (2007) report a 2% drop in the VIX on the day of FOCM meetings. Here we find a somewhat milder effect with the VXI-10Y and VXI-30Y falling by 1.18%and 0.92% on the FOCM meeting day. The estimation results in Table 7 offer some further insights in the dynamics of the VXI series. The GED parameter is statistically significant and equal or less than 2 in all cases sugegsitng that the errors have a fat-tailed distribution. Implied volatility is highly persistent since all the λ_3 GARCH coefficients in the conditional variance equation are well above zero. The effect of news is asymmetric for all series ($\lambda_2 \neq 0$) and in all but one case there is a leverage effect ($\lambda_2 < 0$).

Table 7. Coefficients of regressions between ΔVXI and dummies for macroeconomic announcements

	ΔVXI-13W	**ΔVXI-5Y**	**ΔVXI-10Y**	**ΔVXI-30Y**
α	1.02E-05	9.75E-09	0.0010**	0.0016**
ϕ	0.0839**	0.0090	-0.0324**	-0.0867**
β_{-1}	9.88E-06	-0.0022**	-0.0021**	-0.0010
β_0	5.49E-05	-0.0017**	-0.0059**	-0.0083**
β_{+1}	-6.24E-05	4.07E-04	0.0023*	0.0032**
γ_{-1}	1.42E-04	0.0027**	0.0051**	0.0009
γ_0	9.98E-06	0.0006	-0.0081**	-0.0053**
γ_{+1}	3.03E-05	0.0015	0.0055**	0.0013
δ_{-1}	-1.84E-04	0.0004	-0.0074**	-0.0028**
δ_0	6.28E-05	-0.0032**	-0.0044**	-0.0064**
δ_{+1}	5.85E-05	0.0080**	0.0022*	0.0108**
ζ_{-1}	-1.18E-04	0.0063**	0.0074**	0.0034**
ζ_0	-7.78E-05	-0.0121**	-0.0117**	-0.0065**
ζ_{+1}	1.83E-04	0.0001	-0.0010	-0.0019
ω	-3.6175**	-0.6282**	-2.5272**	-1.7718**
λ_1	0.2878**	0.2338**	0.3372**	0.1400**
λ_2	-0.1198**	-0.0763**	-0.1294**	0.0486*
λ_3	0.5039**	0.9295**	0.6616**	0.7637**
GED paremeter	0.9999**	0.6251**	0.7012**	0.7449**

One (two) stars denote statistical significance at the 95% (99%) level.

4. Interest Rate Volatility Risk Management

Following the large success of equity implied volatility indices, CBOE introduced volatility futures and options written on the VIX (March 2004 and February 2006, respectively). Futures on the VXD were introduced in April 2005 and European options followed soon. According to a recent CBOE Futures Exchange press release (December 3, 2009), year-to-date through November 2009, almost 29 million VIX options have changed hands, making VIX options the second most-actively traded index option at the exchange. Motivated by the success of the VIX market and the relative magnitude of interest rate volatility risk demonstrated in the present study, we believe that it is useful to discuss relevant solutions for trading and managing interest rate volatility risk.

A first step in the direction of building pricing and risk management models is to understand and approximate empirically the continuous time dynamics of the volatility processes considered. The models under consideration are nested in the following stochastic differential equation, under the real probability measure *P*:

$$dV_t = \mu(V_t,t)dt + \sigma(V_t,t)dW_t + y(V_t,t)dq_t \tag{5}$$

where, V_t is the value of *VXI* at time *t*, W_t is a standard Wiener process, and $\mu(V_t,t)$, $\sigma(V_t,t)$ and $y(V_t,t)$ are the drift, the diffusion and the jump amplitude coefficients, respectively. The jump component is driven by a Poisson process q_t with constant arrival parameter *λ*, i.e. $\Pr\{dq_t = 1\} = \lambda dt$ and $\Pr\{dq_t = 0\} = 1 - \lambda dt$. dW_t, dq_t and *y* are assumed to be mutually independent. We allow $\mu(V_t,t)$, $\sigma(V_t,t)$ and $y(V_t,t)$ to be general functions of time and the interest rate volatility. Hence, by changing the specification of the above coefficients we come up with the following seven models:

Mean Reverting Square-Root process (MRSRP) $dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dW_t$ (6)

Mean Reverting Logarithmic process (MRLP) $d\ln(V_t) = \kappa(\theta - \ln(V_t))dt + \sigma dW_t$ (7)

Constant Elasticity of Variance (CEV) $dV_t = \kappa(\theta - V_t)dt + \sigma V_t^{\gamma} dW_t$ (8)

MRSRP with Jumps (MRSRPJ) $dS_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dW_t + (y-1)V_t dq_t$ (9)

MRLP with Jumps (MRLPJ) $d\ln(V_t) = \kappa(\theta - \ln(V_t))dt + \sigma dW_t + (y-1)V_t dq_t$ (10)

The choice of models is based on four criteria: economic intuition, stationarity, mathematical tractability, and popularity among the researchers. Random walk processes make no economic sense, as they imply that volatility can drift off to arbitrarily high levels. The inclusion of jump diffusions is motivated by our empirical findings concerning abrupt upward and downward changes in the VXI indices. All jump-diffusion processes are the natural extensions to their diffusion analogues, so as to facilitate a direct comparison. The jump size distribution is assumed double exponential, which allows for the derivation of the characteristic function of the examined processes (see Duffie et al., 2000, and Psychoyios et al., 2009 for more details on the specifications of the jump-diffusions processes under consideration)[4]. Without bounded lower support on the jump size distribution it is possible that in some of the models the volatility becomes negative. We could restrict the jump sizes to be positive to avoid such problems (see for similar assumptions Broadie et al., 2007, Eraker, 2004). However, we deliberately use "unrestricted" jump-diffusion models in order to account for the empirically observed negative jumps in implied volatility. The models under consideration have been widely used to model the dynamics of the instantaneous and

[4] The derivation of the characteristic functions can be provided by the authors upon request.

implied volatility/variance for equities in continuous time setting (see among others Brenner et al, 2006, Jones (2003), Detemple and Osakwe, 2000, Chan et al., 1992, Eraker (2004) for the models (3), (4), (5) and (6), respectively).

Estimation is done in MATLAB using a Maximum Likelihood (ML) approach (see Psychoyios et al. for details on the estimation methodology). The ML results for the four indices under study are given in Table 8. The table also provides two performance measures: the likelihood ratio test and the Bayes Information Criterion (BIC). The likelihood ratio test can be used only for comparisons between nested models, i.e., between MRLP and MRLPJ, and, between CEV, MRSRP, and MRSRPJ, respectively[5]. Comparison of the non-nested models can be made using the BIC criterion. The results provide several interesting insights. First, the MRSRPJ is the best performing model. Second, the jump-diffusion processes significantly outperform their diffusion counterparts. Third, although MRSRPJ is the best performing process, its diffusion counterpart (MRSRP) is almost the worst performing model among the diffusion processes. In this case, CEV process dominates all the other models, closely followed by the MRLP process. The only exception occurs in the case of the VXI-5Y, where MRSRP performs better than CEV. A further investigation of the results, regarding the diffusion processes, reveals that the higher the dependence of the volatility of volatility parameter (σ) on the current level of interest rate implied volatility (i.e MRLP and CEV), the higher the fitting performance[6]. In general, the findings indicate that interest rate implied volatility has a proportional, mean reverting structure with jumps, i.e., they are subject to large movements that cannot be explained by standard diffusion processes. These three main conclusions are supported by all the performance criteria used and hold for all four interest rate implied volatility indices. Moreover, they are consistent with the descriptive analysis findings from the previous section, namely: the existence of jumps, the nonormality of returns and the stationarity of the interest rate implied volatility processes.

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[5] The likelihood ratio test statistic for comparing the nested models is given by: $LR = -2\times\left(\Im_R - \Im_U\right) - \chi^2(df)$, where df is the number of parameter restrictions and $\Im_R$, $\Im_U$ are the log-likelihoods of the restricted and unrestricted model, respectively. The 5% level critical values are: $\chi^2(df) = \left[3.84\,(df=1), 7.82\,(df=3), 9.49\,(df=4)\right]$. In order to facilitate the direct comparison of the logarithmic processes (i.e. MRLPJ and MRLP) with the rest of the processes, we apply the following change of variable to the log-likelihoods of the MRLP and MRLPJ: $\Im'_R = \sum_{t=1}^{T} \ln(V_{t+\tau}) + \Im_R$.

[6] Two additional specifications were also examined: the Mean Reverting Gaussian Process ($dV_t = \kappa\left(\theta - V_t\right)dt + \sigma dW_t$), and its counterpart augmented by jumps ($dS_t = \kappa\left(\theta - V_t\right)dt + \sigma dW_t + \left(y-1\right)V_t dq_t$ However, the subsequent analysis indicated that the processes where misspecified and their performance was inferior with relation to the other models. Due to space limitations neither the processes nor the results are presented.

Table 8. Estimation results of diffusion and jump diffusion processes over the period from 1/4/96 to 8/29/08 for all four interest rate implied volatility indices. Numbers in brackets denote t-statistics. The table also gives the Log-Likelihood value ($\Im$) and the Bayes Information Criterion (BIC).

	VXI-13W					VXI-5Y				
Parameter	**MRSRP**	**MRLP**	**CEV**	**MRSRPJ**	**MRLPJ**	**MRSRP**	**MRLP**	**CEV**	**MRSRPJ**	**MRLPJ**
k	4.0257	3.1177	2.7869	0.3213	2.0902	3.7186	5.3604	4.0004	1.3102	2.0902
	(4.7331)	(4.2509)	(2.4461)	(14.8527)	(0.5220)	(1.9818)	(2.5758)	(3.8513)	(17.9884)	(2.8690)
θ	37.2561	3.4032	36.2001	22.0057	3.1475	56.7050	3.6057	33.2628	21.8304	3.1475
	(8.5905)	(23.7858)	(12.9179)	(13.4567)	(12.5589)	(2.5716)	(16.7577)	(6.6148)	(10.8815)	(23.4622)
σ	9.8093	1.5744	0.9172	0.6980	0.3819	5.5099	1.0109	1.0831	1.3761	0.3819
	(78.6747)	(78.9268)	(73.8701)	(11.8527)	(12.6980)	(79.1978)	(79.1756)	(75.5051)	(156.058)	(51.7534)
γ	0.5	1.0	1.1583	0.5	1.0	0.5	1.0	0.9857	0.5	1.0
			(0.9300)					(1.2752)		
λ	-	-	-	30.0834	20.5204	-	-	-	31.9295	20.5204
				(8.8680)	(11.2348)				(4.0786)	(10.7943)
ρ	-	-	-	0.5674	0.5139	-	-	-	0.5563	0.5139
				(20.5435)	(24.2640)				(4.2476)	(12.7674)
$1/\eta 1$	-	-	-	1.8894	9.2110	-	-	-	1.7723	9.2110
				(13.4332)	(4.8311)				(2.7580)	(7.2640)
$1/\eta 2$	-	-	-	2.3677	5.5907	-	-	-	2.1335	5.5907
				(6.4003)	(12.0988)				(3.6637)	(4.8069)
BIC	16,489	15,554	15,489	4,738	7,738	13,295	13,584	13,560	6,724	8,644
$\Im$	-8,233	-7,765	-7,728	-2,341	-3,841	-6,636	-6,780	-6,764	-3,334	-4,294

	VXI-10Y					VXI-30Y				
Parameter	**MRSRP**	**MRLP**	**CEV**	**MRSRPJ**	**MRLPJ**	**MRSRP**	**MRLP**	**CEV**	**MRSRPJ**	**MRLPJ**
k	6.0155	4.9855	4.5465	1.0194	2.0902	5.6594	4.5085	2.6467	1.2543	1.8263
	(5.9737)	(5.4621)	(1.8825)	(19.9674)	(2.1909)	(5.8536)	(5.2462)	1.4347	(0.8349)	(8.2811)
θ	27.3305	3.2644	24.6630	21.9320	3.1475	23.0471	3.1165	25.2793	38.2533	3.5317
	(20.9957)	(61.1747)	(28.1220)	(16.4180)	(15.0161)	(28.3915)	(78.2940)	0.6939	(3.4991)	(85.5782)
σ	5.2769	0.9411	0.4000	1.6458	0.3819	3.3725	0.6345	0.8208	1.3144	0.3079
	(78.4612)	(78.6474)	(76.1675)	(120.391)	(44.4812)	(78.5196)	(78.6992)	2.4081	(90.5605)	(55.2048)
γ	0.5	1.0	1.1104	0.5	1.0	0.5	1.0	0.9105	0.5	1.0
			(1.7411)					(1.8489)		
λ	-	-	-	38.5869	20.5204	-	-	-	32.0856	17.9697
				(3.4737)	(5.4977)				(2.8870)	(6.1650)
ρ	-	-	-	0.5005	0.5139	-	-	-	0.3172	0.2575
				(4.6922)	(6.6267)				(4.4092)	(3.0683)
$1/\eta 1$	-	-	-	2.2077	9.2110	-	-	-	1.8201	7.8277
				(4.1794)	(7.3612)				(2.2861)	(5.0515)
$1/\eta 2$	-	-	-	1.9525	5.5907	-	-	-	2.1371	12.6329
				(2.0673)	(3.4654)				(2.2993)	(9.8237)
BIC	12,231	11,635	11,628	6,266	7,013	8,940	8,220	8,188	4,708	5,155
$\Im$	-6,103	-5,805	-5,798	-3,105	-3,478	-4,458	-4,098	-4,078	-2,326	-2,550

Apart from improving the fitting performance, the introduction of the jump component also has two more effects. First, it significantly reduces the diffusion volatility parameter (σ), suggesting that jumps account for a substantial component of volatility and help to capture additional skewness. For example, in Table 8, in all four volatility indices the diffusion volatility drops on average to one-third its prior level (see also Das, 2002 for similar results regarding interest rate levels). Second, it significantly reduces the speed of the mean reversion parameter. This is caused by the fact that many jumps, as it can be seen also in Figure 1, have a persistent effect and the process do not pull back to its long run mean. The latter may imply that models with non-linear long run mean, or regime-switching jumps diffusion models may be more appropriate to capture the characteristics of the interest rate implied volatility (see for example Bakshi, Ju, and Ou-Yang, 2006, and Siou and Lau, 2008). However, these models are beyond the scope of this research, they require too many parameters to be estimated and they impose a large amount of mathematical complexity, which makes derivatives pricing difficult. In order to check for the stability of the above general results we estimate all the all the processes again over the period from 1/4/96 to 31/12/07. We eliminate all 2008 data that corresponds to the latest credit crash, which would bias the results in favor of finding jumps. The ranking of the processes as well as the main conclusions remain the same. Due to space limitations we do not include the table with the estimated parameters; however the results are available from the authors upon request.

Finally, we have to note that the estimated parameters of the processes that used to model the dynamics of the interest rate implied volatility index cannot be used as a proxy for the parameters of the instantaneous volatility process. This is because implied and instantaneous volatility processes do not share the same structure. However, for the processes under consideration and under certain assumptions, it can be proved that under the risk-adjusted probability measure the parameters of the implied *variance* process (i.e. VXI^2) are related to the instantaneous *variance* ones (see also Wu, 2005, and Ait-Sahalia and Kimmel, 2006)[7].

Before proceeding to futures valuation, we must rewrite equation (9) under the risk neutral probability measure Q. Following Heston (1993), Grunbichler and Longstaff (1996) and Pan (2002), we assume that the volatility risk is proportional to the current level of interest rate implied volatility, i.e., ζV_t. We also assume that there is no "volatility of volatility", "jump" risk, and model risk[8]. So, the volatility process under the risk neutral measure is given by:

[7] The proofs of these statements are available from the authors upon request

[8] We cannot use no-arbitrage arguments to price interest rate volatility derivatives, since the market is not complete. Only for the case of VXI-futures, we can derive arbitrage free bounds, following the same methodology as in the case of VIX-futures of Carr and Wu (2006). However, in order to do so, Carr and Wu assume a very liquid market of plain vanilla options and exotic OTC derivatives, such as forward-start at-the-money forward call options, written on the underlying of each VXI index.

$$dV_t = \left(k(\theta - V_t) - \zeta V_t\right)dt + \sigma\sqrt{V_t}dz + \left(y-1\right)dq \tag{11}$$

or, equivalently,

$$dV_t = k^*(\theta^* - V_t) + \sigma\sqrt{V_t}dz + \left(y-1\right)dq \tag{12}$$

where $k^* = k + \zeta$ and $\theta^* = \dfrac{k\theta}{k+\zeta}$.

Denote $F_t(V,T)$ the price of a futures contract on V_t at time t with maturity T. Under the risk-adjusted equivalent martingale measure Q, $F_t(V,T)$ equals the conditional on the information up to time t expectation of V_T at time T, i.e

$$F_t = E_t^Q(V_T) \tag{13}$$

Since the MRSRPJ process does not have a known density, $E_t^Q(V_T)$ is derived by differentiating the characteristic function once with respect to s and then evaluating the derivative at $s= -i$ (see Psychoyios *et al.*, 2009 for the derivation of the characteristic function).

$$E_t^Q\left(V_T\right) = V_t e^{-k^*(T-t)} + \theta^*(1-e^{-k^*(T-t)}) + \frac{\lambda}{k^*}(1-e^{-k^*(T-t)})\frac{1}{\eta} \tag{14}$$

In order to obtain the valuation formula for a European volatility call, we follow the approach of Bakshi and Madan (2000).The price $C(V_t,\tau;K)$ of the call option with strike price K and τ time to maturity is given by:

$$C(V_t,\tau;K) = e^{-r\tau}e^{-k^*\tau}V_t\Pi_1(t,\tau) + e^{-r\tau}(1-e^{-k^*\tau})\left(\theta^* + \frac{\lambda}{k^*\eta}\right)\Pi_1(t,\tau) - e^{-r\tau}K\Pi_2(t,\tau) \tag{15}$$

The Π_1 and Π_2 probabilities are determined by

$$\Pi_j(t,\tau) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi K}\times g_j(V_t,\tau;\phi)}{i\phi}\right]d\phi \tag{16}$$

where

$g_1(V_t,\tau;\phi) = \dfrac{F(V_t,\tau;\phi)}{F_\phi(V_t,\tau;0)}$ and $g_2(V_t,\tau;\phi) = e^{r\tau}F(V_t,\tau;\phi)$. $F_\phi(V_t,\tau;0)$ is the first derivative of $F(V_t,\tau;\phi)$ with respect to φ, evaluated at φ=0.

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