Optimal Monetary Policy and Equilibrium Determinacy with Liquidity Constrained Households and Sticky Wages

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Abstract

We study Ramsey policies and optimal monetary policy rules in a model with sticky wages and prices, where financial market participation is limited, i.e. where a fraction of consumer are liquidity constrained. The interaction between liquidity coinstrained agents and wage stickiness results in a welfare-based loss function which depends on real wage gap beside depending on the output gap, wage inflation and price inflation as in previous studies. Optimal simple rules are characterized by an inflation coefficient larger than one, no matter the degree of financial market partecipation. We argue that once wage stickiness, an uncontroversial empirical fact, is considered, the degree of financial market partcipation just marginally affects the design of optimal monetary policy rules.

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1 Introduction

[TO BE DONE]

2 The Model

In what follows we follow Bilbiie (2008). However, a major difference is in the modelling of the labor market. Whereas he has a perfectly competitive market, we asssume a monopolistically competitive market characterized by nominal wage stickiness, which we describe below.

2.1 Households

The period utility function is common across households and it has the following separable form

$$U_t = \Psi_t u \left[C_t(i) \right] - v \left[L_t(i) \right] \tag{1}$$

where $C_t(i)$ is agent *i's* consumption, $L_t(i)$ are labor hours and Ψ_t is a preference shock which has the following law of motion $\log \frac{\Psi_t}{\Psi} = \rho_\Psi \log \frac{\Psi_{t-1}}{\Psi} + \varepsilon_{\Psi,t}$.[1]

We assume a continuum of differentiated labor inputs indexed by $j \in [0,1]$. As in Schmitt-Grohé and Uribe (2005), agent i supplies all labor inputs. Wage-setting decisions are taken by labor type-specific unions indexed by $j \in [0,1]$. Given the wage W_t^j fixed by union j, agents stand ready to supply as many hours on labor market j, L_t^j, as required by firms, that is

$$L_t^j = \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} L_t^d \tag{2}$$

where $\theta_w > 1$ is the elasticity of substitution between labor inputs. Here L_t^d is aggregate labor demand and W_t is an index of the wages prevailing in the economy at time t. Formal definitions of labor demand and of the wage index can be found in the section devoted to firms. Agents are distributed uniformly across unions, hence aggregate demand of labor type j is spreaded uniformly between all households.[2] It follows that the individual quantity of hours worked, $L_t(i)$, is common across households and we will denote it with L_t. This must satisfy the time resource constraint $L_t = \int_0^1 L_t^j dj$. Combining the latter with (2) we obtain

$$L_t = L_t^d \int_0^1 \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} dj \tag{3}$$

The labor market structure rules out differences in labor income between households without the need to resort to contingent markets for hours. The common labor income is given by $L_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} dj$.[3]

[1] The function u is incresing and concave while the function v is increasing and convex.

[2] Thus a share λ of the associates of the unions are non ricardian consumers, while the remaining share is composed by non ricardian agents.

[3] Erceg *et al* (2000), assume, as in most of the literature on sticky wages, that each agent is the monopolistic

2.1.1 Ricardian Households

Ricardian households face the following intertemporal budget constraint

$$E_t\Lambda_{t,t+1}X_{t+1}+\Omega_{S,t+1}V_t\leq X_t+L_t^d\int_0^1 W_t^j\left(\frac{W_t^j}{W_t}\right)^{-\theta_w}dj+\Omega_{S,t}\left(V_t+P_tD_t\right)-P_tC_{S,t}. \tag{4}$$

We distinguish shares from the other assets explicitly since their distribution plays a crucial role in the rest ofthe analysis. In each time period t, ricardian agents can purchase any desired state-contingent nominal payment X_{t+1} in period *t+1* at the dollar cost $E_t\Lambda_{t,t+1}X_{t+1}$. The variable $\Lambda_{t,t+1}$ denotes the stochastic discount factor between period $t+1$ and t. The expression $L_t^d\int_0^1 W_t^j\left(\frac{W_t^j}{W_t}\right)^{-\theta_w}dj$ represents labor income. V_t is average market value at time t shares in intermediate good firms, D$_t$ are real dividend payoffs of these shares and $\Omega_{s,t}$ are share holdings. FOCS with respect to C$_{S,t}$, $\Omega_{S,t}$ and X$_{t+1}$ are respectively

$$\Psi_t u_c\left(C_{S,t}\right)=\upsilon_t P_t, \tag{5}$$

$$V_t=E_t\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left(V_{t+1}+P_{t+1}D_{t+1}\right), \tag{6}$$

$$\Lambda_{t,t+1}=\beta\frac{\upsilon_{t+1}}{\upsilon_t}, \tag{7}$$

where υ_t is the lagrange multiplier on the flow budget constraint. The riskless nominal interest rate is a solution to

$$E_t\Lambda_{t,t+1}=\left(1+R_t\right)^{-1}. \tag{8}$$

Combining the previous conditions deliver the FOC for utility maximization of ricardian agents

$$\frac{1}{(1+R_t)}=E_t\left\{\beta\frac{\Psi_{t+1}u_c\left(C_{S,t+1}\right)}{\Psi_t u_c\left(C_{S,t}\right)}\frac{P_t}{P_{t+1}}\right\}. \tag{9}$$

2.1.2 Non Ricardian Households.

Non ricardian agents do not hold physical capital, do not enjoy firms' profits in the form of dividend income and are not able to trade in the financial markets. The nominal budget constraint of a typical non ricardian household is given by

$$P_tC_{S,t}=L_t^d\int_0^1 W_t^j\left(\frac{W_t^j}{W_t}\right)^{-\theta_w}dj. \tag{10}$$

Agents belonging to this class are forced to consume disposable income in each period and delegate wage decisions to unions. For these reasons there are no first order conditions with respect to consumption and labor supply.

supplier of a single labor input. In this case, assuming that agents are spreaded uniformly across unions allows to rule out differences in income between households providing the same labor input (no matter whether they are ricardian or not), but it does not allow to rule out difference in labor income between non ricardian agents that provide different labor inputs. This would amount to have an economy populated by an infinity of different individuals, since non ricardian agents cannot share the risk associated to labor income fluctuations. Although this framework would be of interest, it would imply a tractability problem.

2.2 Wage Setting

Nominal wage rigidities are modeled according to the Calvo (1983) mechanism. In each period a union faces a constant probability $1-\xi_w$ of being able to reoptimize the nominal wage. We extend the analysis in GVL (2007) and assume that the nominal wage newly reset at t, $\widetilde{W}_t$, is chosen to maximize a weighted average of agents' lifetime utilities. The weights attached to the utilities of ricardian and non ricardian agents are $(1-\lambda)$ and λ, respectively. The union problem is

$$\max_{\widetilde{W}_t} E_t \sum_{j=0}^{\infty} (\xi_w \beta)^j \left\{ \Psi_{t+j} \left[(1-\lambda) u \left(C_{S,t+j} \right) + \lambda u \left(C_{S,t+j} \right) \right] - v \left(L_{t+j} \right) \right\} \tag{11}$$

subject to (3), (4) and (10).[4] The FOC with respect to $\widetilde{W}_t$ is

$$E_t \sum_{j=0}^{\infty} (\beta \lambda_w)^{t+j} \Phi_{t,t+j} \left\{ \left[\lambda \frac{1}{MRS_{H,t+j}} + (1-\lambda) \frac{1}{MRS_{S,t+j}} \right] \frac{\widetilde{W}_t}{P_{t+s}} - (1+\mu^w) \right\} = 0 \tag{12}$$

where $\Phi_{t,t+j} = v_L \left(L_{t+j} \right) L^d_{t+j} W^{\theta_w}_{t+j}$ and $\mu^w = (\theta_w - 1)^{-1}$ is the, constant, net wage mark-up in the case of wage flexibility. The variables $MRS_{H,t}$ and $MRS_{S,t}$ denote the marginal rates of substitution between labor and consumption of non ricardian and ricardian agents respectively.

2.3 Firms

In each period t, a final good Y_t is produced by a perfectly competitive firm combining a continuum of intermediate inputs $Y_t(z)$ according to the following standard *CES* production function:

$$Y_t = \left(\int_0^1 Y_t(z)^{\frac{\theta_p - 1}{\theta_p}} dz \right)^{\frac{\theta_p}{\theta_p - 1}} \quad \text{with} \quad \theta_p > 1 \tag{13}$$

The producer of the final good takes prices as given and chooses the quantities of intermediate goods by maximizing its profits. This leads to the demand of intermediate good z and to the price of the final good which are respectively

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\theta_p} Y_t \quad ; \quad P_t = \left[\int_0^1 P_t(z)^{1-\theta_p} dz \right]^{\frac{1}{1-\theta_p}}$$

Intermediate inputs are produced by a continuum of monopolistic firms indexed by $z \in [0,1]$ using as inputs capital services, $K_{t-1}(z)$, and labor services, $L_t(z)$. The production technology is given by:

$$Y_t(z) = A_t L_t(z), \tag{14}$$

where A_t represents TFP. The labor input is defined as $L_t(z) = \left(\int_0^1 \left(L_t^j(z) \right)^{\frac{\theta_w - 1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w - 1}}$. Firm's z demand for labor type j and the aggregate wage index are respectively

[4] Many reasons have been provided to justify the presence of non ricardian consumers. A few of them are miopia, fear of saving and transaction costs on financial markets. None of these is, however, in contrast with rule of thumb consumers delegating wage decision to a forward looking agency, in this case a trade union.

$$L_t^j(z) = \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} L_t(z) \quad ; \quad W_t = \left(\int_0^1 \left(W_t^j\right)^{1-\theta_w} dj\right)^{1/(1-\theta_w)}.$$

The nominal marginal cost, common across producers, is given by

$$MC_t = \frac{W_t}{A_t}, \tag{15}$$

while firm z's real profits are given by $D_t(z) = \left[\frac{P_t(z)}{P_t} - \frac{MC_t}{P_t}\right] Y_t(z)$.

Price Setting Intermediate producers set prices according to the same mechanism assumed for wage setting. Firms in each period have a fixed chance $1-\xi_p$ to reoptimize their price. A price setter takes into account that the choice of its time t nominal price, $\widetilde{P}_t$, might affect not only current but also future profits. The FOC for price setting is:

$$E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \upsilon_{t+s} P_{t+s}^{\theta_p} Y_{t+s} \left[\widetilde{P}_t - (1+\mu^p) MC_{t+s}\right] = 0, \tag{16}$$

which can be given the usual interpretation.[5] Notice that $\mu^p = (\theta_p - 1)^{-1}$ represents the net markup over the price which would prevail in the absence of nominal rigidities.

2.4 Aggregation

Aggregate consumption and aggregate profits are defined as:

$$\begin{aligned} C_t &= \lambda C_{H,t} + (1-\lambda) C_{S,t}, && (17) \\ \Omega_t &= (1-\lambda)\,\Omega_{S,t} && (18) \end{aligned}$$

Since each agents holds the same quantity of shares and the sum of shares must equal 1, it has to be the case that

$$\Omega_{S,t} = \Omega_S = \frac{1}{1-\lambda}. \tag{19}$$

2.4.1 Market Clearing

The clearing of good and labor markets requires

$$\begin{cases} Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\theta_p} Y_t^d & \forall z \qquad Y_t^d = Y_t; \\ L_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} L_t^d & \forall j \quad L_t = \int_0^1 L_t^j dj \end{cases}, \tag{20}$$

where $Y_t^d = C_t$ represents aggregate demand, $L_t^j = \int_0^1 L_t^j(z)\,dz$ is the demand of labor input j and $L_t^d = \int_0^1 L_t(z)\,dz$ denotes firms' aggregate demand of the composite labor input.

[5] Recall that ν_t is the value of an additional dollar for a ricardian household. It is the lagrange multiplier on ricardian househols nominal flow budget constraint.

3 The model approximation

In this section we log-linearize the model around the efficient steady state, i.e. the steady state obtained by solving the Social Planner problem. Indeed, in order to compare our results on the optimal monetary policy with the ones get by our closed related papers, i.e. Anderson et al (2000) and Bilbiie (2008), we need to derive the central bank loss function by taking a second order approximation of the households utility functions around the efficient steady state. Moreover, since we are going to study optimal monetary policy, we want the monetary authority to target the welfare relevant output gap, i.e. the gap between the actual and the efficient equilibrium output. The latter corresponds to the equilibrium output of the Social Planner first best allocation. Appendix A1 shows how to derive the efficient equilibrium output.

3.1 The Efficient Steady State

Note that because of the presence of price and wage markups the steady state of our economy is not efficient. Since we will approximate the dynamics of our economy around an efficient steady state we assume that the Government taxes/subsidies firms through an employment subsidy/tax at a constant rate τ,and then give/take the money back to firms in a lump-sum way T. This means that the firm's profit function becomes:

$$D_t(i) = \frac{P_t(i)}{P_t} Y_t(i) - \frac{(1-\tau) W_t(i)}{P_t} N_t(i) - T_t, \tag{21}$$

where to balance the government budget we assume that $\tau \frac{W_t(i)}{P_t} N_t(i) = T_t$ .At the efficient steady state profits must be zero, which implies that $C_S = C_N = C$ and thus that agents have a common marginal rate of substituion between labor and consumption MRS. In this case the steady state labor market equilibrium would imply that:

$$w = \frac{1}{(1+\mu_p)(1-\tau)} MPL = (1+\mu^w) MRS. \tag{22}$$

Labor market equilibrium in the efficient steady implies that

$$MPL = MRS = 1, \tag{23}$$

indeed, the aggregate production function in the steady state implies production function $MPL = A = \frac{Y}{N} = 1$. Then, (22) requires τ to be such that:

$$\frac{1}{(1+\mu_p)(1-\tau)(1+\mu^w)} = 1. \tag{24}$$

Solving for τ we get $\tau = 1 - \frac{1}{(1+\mu_p)(1+\mu^w)}$. As argued above the implied value of τ leads to zero steady state profits; to see this notice that

$$D = Y - \frac{(1-\tau) W}{P} N - T = Y - \frac{Y}{(1+\mu_p)^2 (1+\mu^w)} - \left(1 - \frac{1}{(1+\mu_p)^2 (1+\mu^w)}\right) Y = 0. \tag{25}$$

3.2 The Log-linearized Model

In the remainder we adopt the following functional forms for the utility of consumption and labor hours:

$$u(C_t) = \psi_{t+1} \frac{C_t^{1-\sigma}}{1-\sigma} \quad ; \quad v(L_t) = \chi \frac{L_t^{1+\phi}}{1+\phi} \tag{26}$$

with σ representing the inverse intertemporal elasticity of substitution in consumption and ϕ measuring the elasticity of the marginal disutility of labor and ψ_t representing an AR(1) preference shock. In what follows lower case letters denote log-deviation from the efficient steady state.

The Euler equation for Ricardian households can be log-linearized as

$$c_{S,t} = E_t c_{s,t+1} - \frac{1}{\sigma} E_t (r_t - \pi_{t+1}) - \frac{1}{\sigma} E_t \Delta\psi_{t+1}, \tag{27}$$

where $r_t = \log \frac{1+R_t}{\frac{1}{\beta}}$, $\log \frac{P_{t+1}}{P_t} = \pi_{t+1}$ and $\Delta\psi_{t+1} = \log \frac{\psi_{t+1}}{\psi_t}$. Consumption of non ricardian agents reads as

$$c_{H,t} = c_t + \omega_t, \tag{28}$$

while the assumption that consumption level are equal at the steady state implies that aggregate consumption is

$$c_t = (1-\lambda) c_{S,t} + \lambda c_{H,t}. \tag{29}$$

Log-linearization of the aggregate resource constraint around the steady state yields

$$y_t = c_t. \tag{30}$$

A log-linear approximation to the aggregate production function is given by

$$y_t = l_t + a_t. \tag{31}$$

The New Keynesian Phillips Curve (NKPC) is obtained through log-linearization of condition (16) and reads as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_p mc_t, \tag{32}$$

where $\kappa_p = \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}$. The log-linear version of the first order condition for wage setting is

$$E_t \sum_{s=0}^{\infty} (\beta\xi_w)^{t+s} \left[\hat{\omega}_{t+s} - mrs_{t+s}^A\right] = 0.$$

The wage inflation curve is

$$\pi_t^w = \beta E_t \pi_{t+1}^w - \kappa_w \mu_t^w, \tag{33}$$

where $\kappa_w = \frac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w}$ and $\mu_t^w = \omega_t - (\sigma c_t + \phi l_t - \psi_t)$ is the log-deviations of the wage mark-up that unions impose over the average marginal rate of substitution.[6] Notice that since unions

[6] As pointed out by Schmitt-Grohe and Uribe (2005), the coefficient κ_w is different form that in Erceg et al (2000), which is the standard reference for the analysis of nominal wage stickiness. The reason is that we have assumed that agents provide all labor inputs. In the more standard case in which each individual is the monopolistic supplier of a given labor input, κ_w would be equal to $\frac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w(1+\phi\theta_w)}$ hence lower than in the case we consider.

maximize a weighted average of agents' utilities, the wage inflation curve takes a standard form. Equation (33) leads to a second order expectational difference equation for the log-deviation of the time t real wage:

$$\omega_t = \Gamma \left[\omega_{t-1} + \beta \left(E_t \omega_{t+1} + E_t \pi_{t+1}\right) - \pi_t\right] + \Gamma \kappa_w \left(\phi l_t + \sigma c_t - \psi_t\right), \tag{34}$$

where $\Gamma = \frac{\xi_w}{\left(1+\beta \xi_w^2\right)}$. The parameter Γ determines both the degree of forward and backward lookingness.

3.3 Log-deviations from the Efficient Equilibrium

Given our assumptions concerning steady state taxation the natural and the efficient level of output coincide. In what follows we will approximate the model around the efficient equilibrium. Log-deviations of efficient output (from the efficient steady state) are given by (see Appendix A1 for a detailed derivation):

$$y_t^{Eff} = \frac{1+\phi}{\sigma+\phi} a_t + \frac{1}{(\sigma+\phi)} \psi_t, \tag{35}$$

i.e. the efficient level of output is a function of exogenous productivity and preference shocks, and therefore as standard it is independent of policy. Also notice that the efficient real wage level is

$$\omega_t^{Eff} = a_t, \tag{36}$$

which again is known once the process for a_t is posited.
Next we move to obtain deviation from the efficient equilibrium. Recall that the NKPC reads as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_p mc_t, \tag{37}$$

where mc_t represent the log deviation of the real marginal cost. The latter can be equivalently written as

$$mc_t = \omega_t - y_t + l_t = \omega_t - a_t, \tag{38}$$

where we substitute the log-linear version of the economy production function, i.e., $y_t = a_t + n_t$. Thus, real marginal costs can be rewritten in terms of the real wage gap,

$$mc_t = \tilde{\omega}_t, \tag{39}$$

where we define $\tilde{\omega}_t = \omega_t - \omega_t^{Eff}$ as the gap between the current and the efficient equilibrium real wage. In this case the NKPC (37) can be rewrittes as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_p \tilde{\omega}_t. \tag{40}$$

Next consider the wage inflation curve

$$\pi_t^w = \beta E_t \pi_{t+1}^w - \kappa_w \mu_t^w. \tag{41}$$

Recall that μ_t^w represents the log-deviation of the wage markup from the (null) efficient level. Imposing market clearing and using the production function

$$\mu_t^w = \omega_t - (\sigma+\phi) y_t + \phi a_t + \psi_t, \tag{42}$$

thus

$$\tilde{\mu}_t^w = \tilde{\omega}_t - (\sigma + \phi) x_t, \tag{43}$$

where $x_t = y_t - y_t^{Eff}$ denotes the gap between actual output and the efficient output. We can then write

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_w (\sigma + \phi) x_t - \kappa_w \tilde{\omega}_t. \tag{44}$$

The IS curve can be written as (see Appendix A2 for a detailed derivation)

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} E_t \left(r_t - \pi_{t+1}^p - r_t^{Eff} \right) - \frac{\lambda}{1-\lambda} \left(E_t \pi_{t+1}^w - E_t \pi_{t+1}^p \right) \tag{45}$$

where r_t^{Eff} is the efficient rate of interest defined as:

$$r_t^{Eff} = \sigma \left(\Delta y_{t+1}^{Eff} + \frac{\lambda}{1-\lambda} E_t \Delta a_{t+1} - \frac{1}{\sigma} \Delta \psi_{t+1} \right). \tag{46}$$

Using the wage inflation curve and the price inflation curve

$$E_t \pi_{t+1}^w - E_t \pi_{t+1}^p = \frac{1}{\beta} \left[\left(\pi_t^w - E_t \pi_t^p \right) + (\kappa_w - \kappa_p) \tilde{\omega}_t - \kappa_w (\sigma + \phi) x_t \right]$$

substituting into the IS curve leads to

$$\left(1 - \frac{\lambda \kappa_w (\sigma + \phi)}{(1-\lambda) \beta} \right) x_t = E_t x_{t+1} - \frac{1}{\sigma} E_t \left(r_t - \pi_{t+1}^p - r_t^{Eff} \right) - \frac{\lambda}{(1-\lambda) \beta} \left[\left(\pi_t^w - \pi_t^p \right) + (\kappa_w - \kappa_p) \tilde{\omega}_t \right]$$

Further notice that by definition $\Delta \omega_t = \omega_t - \omega_{t-1} = \pi_t^w - \pi_t^p$ which implies

$$\tilde{\omega}_t - \omega_{t-1} + a_t = \pi_t^w - \pi_t^p$$

thus

$$\left(1 - \frac{\lambda \kappa_w (\sigma + \phi)}{(1-\lambda) \beta} \right) x_t = E_t x_{t+1} - \frac{1}{\sigma} E_t \left(r_t - \pi_{t+1}^p - r_t^{Eff} \right) - \frac{\lambda}{(1-\lambda) \beta} \left[a_t - \omega_{t-1} + (1 + \kappa_w - \kappa_p) \tilde{\omega}_t \right]$$

or

$$x_t = E_t x_{t+1} - \frac{(\delta^{sw})^{-1}}{\sigma} E_t \left(r_t - \pi_{t+1}^p - r_t^{Eff} \right) - \frac{\lambda}{(1-\lambda) \beta} \left[a_t - \omega_{t-1} + (1 + \kappa_w - \kappa_p) \tilde{\omega}_t \right]$$

where $\delta^{sw} = \frac{(1-\lambda)\beta - \lambda \kappa_w (\sigma + \phi)}{(1-\lambda)\beta}$. Notice that if $\lambda > \frac{1}{1 + \frac{\kappa_w}{\beta}(\sigma + \phi)} = (\lambda^*)^{sw}$ then the interest rate elasticity of aggregate demand turns positive.

3.4 A special case: sticky prices and flexible wages

When wages are flexible a log-linear approximation to the wage setting rule delivers

$$\omega_t = \sigma c_t + \phi l_t - \psi_t. \tag{47}$$

Further, and independently of the nature of price and wage setting, it holds that

$$mc_t = \omega_t - (y_t - l_t). \tag{48}$$

As standard we can rewrite real marginal costs in terms of output gap, as follows:tput gap

$$mc_t = (\phi + \sigma)\, x_t, \tag{49}$$

in this case the NKPC ca be writtens

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_p (\phi + \sigma)\, x_t, \tag{50}$$

which is the standard NKPC considered also in Bilbiie (2008). Recall that with liquidity constrained households the IS curve reads as

$$y_t = E_t y_{t+1} + \frac{\lambda}{1-\lambda} E_t \Delta a_{t+1} - \frac{\lambda}{1-\lambda} E_t \Delta \omega_{t+1} - \frac{1}{\sigma} E_t \left(r_t - \pi^p_{t+1}\right) - \frac{1}{\sigma} \Delta \psi_{t+1}, \tag{51}$$

then, substituting for $\Delta\omega_{t+1}$, using the equation of the economy production function and rearranging, we get,

$$y_t = E_t y_{t+1} - \frac{\left(\delta^{fw}\right)^{-1}}{\sigma} E_t \left(r_t - \pi^p_{t+1}\right) + \left(\delta^{fw}\right)^{-1} \frac{\lambda}{1-\lambda} (1+\phi) E_t \Delta a_{t+1} - \frac{\left(\delta^{fw}\right)^{-1}}{\sigma} \Delta \psi_{t+1}, \tag{52}$$

where $\delta^{fw} = 1 - \frac{\lambda}{1-\lambda}(\sigma + \phi)$. Notice that when $\lambda > \frac{1}{1+(\sigma+\phi)} = (\lambda^*)^{fw}$ the interest rate elasticity of aggregate demand turns positive. Taking differences with respect to the efficient equilibrium, the IS can be rewritten as follows,

$$x_t = E_t x_{t+1} - \frac{\delta^{-1}}{\sigma} E_t \left(r_t - \pi^p_{t+1} - r^{eff}_t\right), \tag{53}$$

where, in the case of flexible wages the efficient rate of interest is defined as:

$$r^{Eff}_t = \left(\frac{\sigma\delta}{(\sigma+\phi)} - 1\right) \Delta\psi_{t+1} - \sigma(1+\phi)\left[\frac{\delta}{\sigma+\phi} + \frac{\lambda}{1-\lambda}\right](1-\rho_a)\, a_t. \tag{54}$$

Notice that $(\lambda^*)^{sw} > (\lambda^*)^{fw}$ whenever $\frac{1}{1+\frac{\kappa_w}{\beta}(\sigma+\phi)} > \frac{1}{1+(\sigma+\phi)}$ i.e when $\kappa_w < \beta$ or

$$(1-\beta\xi_w)(1-\xi_w) < \beta\xi_w$$

$$(1-\beta\xi_w)(1-\xi_w)$$

$$\beta\xi_w^2 - \xi_w(2\beta+1) + 1 < 0$$

$$\xi_w = \frac{2\beta + 1 \pm \sqrt{4\beta^2 + 1}}{2\beta}$$

or $\frac{2\beta+1-\sqrt{(2\beta)^2+1}}{2\beta} \leq \xi_w \leq 1$. Assuming β=0.99 then the consition translates into

$$0.38 \leq \xi_w \leq 1$$

which implies that $(\lambda^*)^{sw} > (\lambda^*)^{fw}$ when wages have an average duration longer than 1.61 quarters. Assuming an average duration of 3 quarters as suggested by empirical evidence together with $\sigma = 2$ and $\phi = 1$ delivers

$$(\lambda^*)^{sw} = 0.79$$

Thus we need about 80 percent of liquidity constrained agents for the interest rate elasticty of aggregate demand to turn positive.

4 Determinacy

In order to close the model we need to specify an equation for the nominal interest rate. We consider the following Taylor-type interest rate rule:

$$r_t = \phi_r r_{t-1} + (1 - \phi_r)\left(\phi_\pi \pi_{t+i} + \phi_y y_{t+i}\right). \tag{55}$$

When $i = -1$, (55) reduces to a backward looking rule, when $i = 0$ it corresponds to a contemporaneous rule and when $i = 1$ it becomes a forward looking rule. The rule is simple in the sense that it depends on observable variables.

[TO BE COMPLETED]

5 Optimal Monetary Policy

In the Appendix A2 we derive a second order approximation to the average welfare losses experienced by households in economies characterized by sticky wages and prices, where a fraction of consumers are liquidity constrained, resulting from fluctuations around an efficient steady state. Those welfare losses are given by:

$$L = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\frac{(\sigma-1)\lambda}{1-\lambda}\tilde{\omega}_t^2 + (\sigma+\phi)\,x_t^2 + \frac{\theta_w}{\kappa_w}\left(\pi_t^w\right)^2 + \frac{\theta_p}{\kappa_p}\left(\pi_t^p\right)^2\right). \tag{56}$$

Notice that the welfare loss above nests that derived by Anderson et al (2000) and that in Bilbiie (2008). Indeed, for $\lambda = 0$, the term in $\tilde{\omega}_t^2$ in equation (56) vanishes and the welfare function collapses to the one found by Anderson et al (2000).[7] In the case of flexible wages, and with a walrasian labor market, i.e. in the absense of trade-unions, instead, we get the welfare function

[7] **A MENO DI UN COEFFICIENTE MI SEMBRA. DOVREMMO CITARE IL PAPER DI SGU QUI IN NOTA**

found in Bilbiie (2008). Assuming that wages are flexible and that, also in the case of flexible wages, unions set households' wage, we get the following welfare funcion:[8]

$$L = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left((\sigma+\phi)\left(1+\frac{(\sigma-1)\lambda}{1-\lambda}\right)x_t+\frac{\theta_p}{\kappa_p}\left(\pi_{p,t}\right)^2\right), \tag{57}$$

which collapses to the standard text-book welfare-loss for $\lambda = 0$. In what follows we will use (57) as a benchmark to compare the results obtained under sticky wages.

Notice also that in the case in which the intertemporal elasticity of substitution in consumption, σ, equals 1, the real wage gap does not affect society's welfare loss.

[TO BE COMPLETED]

6 Full Commitment

We now want to characterize the optimal monetary policy in the economy in which both prices and wages are sticky and a fraction of consumers are liquidity constrained. In particular we will study the optimal monetary policy in response to a positive productivity shock, a_t, and to a preference shock ψ_t. As in Galì (2008) we restrict ourselves to the case of full commitment. This means that the Central Bank maximizes the welfare function (56) subject to the following constraints:

$$\begin{cases} \pi_t^p = \beta E_t \pi_{t+1}^p + \kappa_p \tilde{\omega}_t + u_t \\ \pi_t^w = \beta E_t \pi_{t+1}^w - \kappa_w \tilde{\omega}_t + \kappa_w (\sigma+\phi) x_t \\ \tilde{\omega}_t = \tilde{\omega}_{t-1} + \pi_t^w - \pi_t^p + \Delta\omega_t^{Eff} \\ \tilde{\omega}_{t-1} \text{ given.} \end{cases} . \tag{58}$$

Defining ξ_{1t}, ξ_{2t} and ξ_{3t} the lagrange multipliers (LMs) on the constraints above, FOCs are:

$$x_t \;:\; -(\sigma+\phi)x_t + \xi_{2t}\kappa_w(\sigma+\phi) = 0, \tag{59}$$

$$\pi_t^p \;:\; -\frac{\theta_p}{\kappa_p}\pi_t^p - \xi_{1t} + \xi_{1t-1} - \xi_{3t} = 0, \tag{60}$$

$$\pi_t^w \;:\; -\frac{\theta_w}{\kappa_w}\pi_t^w - \xi_{2t} + \xi_{2t-1} + \xi_{3t} = 0, \tag{61}$$

$$\tilde{\omega}_t \;:\; -\frac{(\sigma-1)\lambda}{1-\lambda}\tilde{\omega}_t + \xi_{1t}\kappa_p - \xi_{2t}\kappa_w - \xi_{3t} + \beta E_t\xi_{3t+1}, \tag{62}$$

fully optimal policy requires setting $\xi_{1t-1} = \xi_{2t-1} = 0$, timeless policy instead sets $\xi_{1t-1} = \bar{\xi}_1$ and $\xi_{2t-1} = \bar{\xi}_2$ i.e.sets lagged LMs at their steady state value. In the remainder we characterize numerically the fully-optimal policy. Before turning to the optimal policy with the baseline calibration, notice that, as discussed above when $\sigma = 1$ the objective function is independent of the share of non ricardian agents, thus the planner can implement the same equilibrium path for x, ω and π as in the full participation economy. However the equilibrium path of the interest rate will depend on

[8] For a detailed derivation of the welfare function under flexible wages see Appendix A2.

the share of non ricardian consumers. In particular, in response to a positive productivity shock, the planner engineers a stronger decrease in the interest rate as the share of liquidity constrained agents gets larger. This can be show by looking at the derivative of the nominal interest rate in the IS curve with respect to λ, which is:

$$\frac{\partial r_t}{\partial \lambda} = \frac{1}{(\lambda - 1)^2} E_t \left(\Delta\omega_{t+1} + \Delta a_{t+1} \right) > 0. \tag{63}$$

[TO BE COMPLETED]

6.0.1 Optimal Responses to Shocks

In what follows we will show that optimal impulse response functions (OIRFs) of the main economic variables following a technology shock and a preference shock and a cost-push shock. The model is calibrated as follows.

Preferences. Time is measured in quarters. The discount factor β is set to 0.99, so that the annual interest rate is equal to 4 percent. The parameter on consumption in the utility function σ is set equal to 2 and the parameters on labour disutility, ϕ, is set equal to 1.

Production. Following Basu and Fernald (1997), the value added mark-up of prices over marginal cost is set to 0.2. This means that the intermediate goods price elasticity, θ_p is set equal to 6. The Calvo (1983) probability that firms do not reset prices, ξ_p, is set equal to 2/3.

Labour markets. The the elasticity of substitution between labor inputs, θ_w is set equal to 6. The Calvo probability that unions do not reset wages, ξ_w, is set equal to 3/4.

Exogenous shocks. The process for the aggregate productivity shock, a_t, follows an AR(1) and is calibrated so that its standard deviations is set to 1% and its persistence to 0.9. Similarly, the process for the aggregate productivity shock, ψ_t, follows an AR(1) and is calibrated so that its standard deviations is set to 1% and its persistence to 0.9. Finally, also the process for the cost-push shock u_t follows an AR(1) and is calibrated so that its standard deviations is set to 1% and its persistence to 0.9.

Figure 1 shows impulse response functions to a one percent positive productivity shock for output, hours, Ricardian and non-Ricardian households' consumption, price inflation, wage inflation, real wage and real interest rate.

Due to the increase of productivity output, wage inflation, real wage and consumption of both type of households increase. Differently, price inflation, hours and real interest rate decrease. The monetary policy in this environment faces two distortions, sticky prices and and sticky wages. The first distortion calls for zero inflation policy, i.e., to close the gap with the flexible price allocations, while the second distortion calls for an active monetary policy. As the monetary authority is endowed with a single instrument, it must trade-offs between the two competing distortions. As a result optimal policy deviates from full price stability, indeed the optimal IRF does not show an inflation rate always equal to zero, as in the case of flexible wages. Specifically, the monetary authority wants to take full advantage of the productivity increase, therefore it reduces inflation to support higher demand. Notice, that inflation shows a significant overshoot after a few periods. This captures the value of commitment as the monetary policy tries to influence future expectation to obtain faster convergence toward the steady state. At the same time the monetary authority

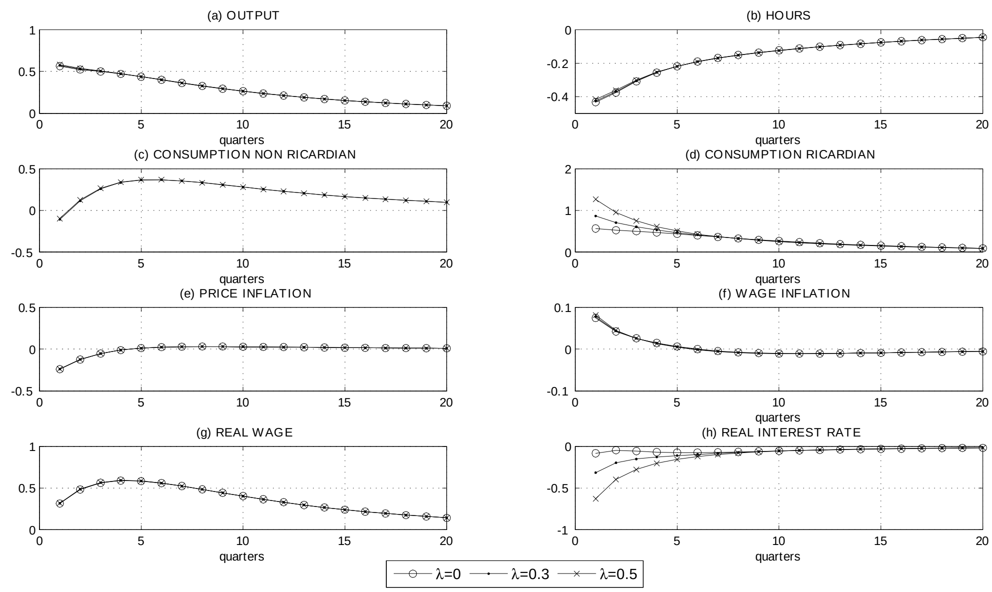

Figure 1: Response to a technology shock in the case of full commitment. Alternative values of the share of liquidity constrained agents.

allows wage inflation and real wages to increase, so that consumption of liquidity constrained household increases. Similarly, by allowing the real interest rate to decrease the central bank is able to increase ricardian household consumption, via the standard consumption smoothing mechanism. Notice that, also in the case of sticky wages, the planner engineers a stronger decrease in the real interest rate as the share λ of liquidity constrained agents gets larger. In fact, has shown in figure 1, the dynamics of real wages does not change that much as λ increases, so that also the dynamics of consumption of liquidity constrained households remains almost unaffected. This meas that, the monetary authority, in order to obtain the same reduction of aggregate consumption has to strongly reduce consumption of Ricardian household. A policy which implies a stronger reduction of the real interest rate.

Figure 2 shows impulse response functions to a one percent positive preference shock for output, hours, Ricardian and non-Ricardian households' consumption, price inflation, wage inflation, real wage and real interest rate.

In response to a preference shock optimal monetary policy implies an increase in output, hours and consumption. The real interest rate increases so that the monetary authority is able to reach both price and wage inflation stability. Overall, the deviations of the price level from the full price stability case are rather small. This is so since the shock does not affect directly labour productivity and therefore it does not generate an endogenous trade-off.

Figure 3 shows impulse response functions to a one percent positive cost-push shock for output, hours, Ricardian and non-Ricardian households' consumption, price inflation, wage inflation, real

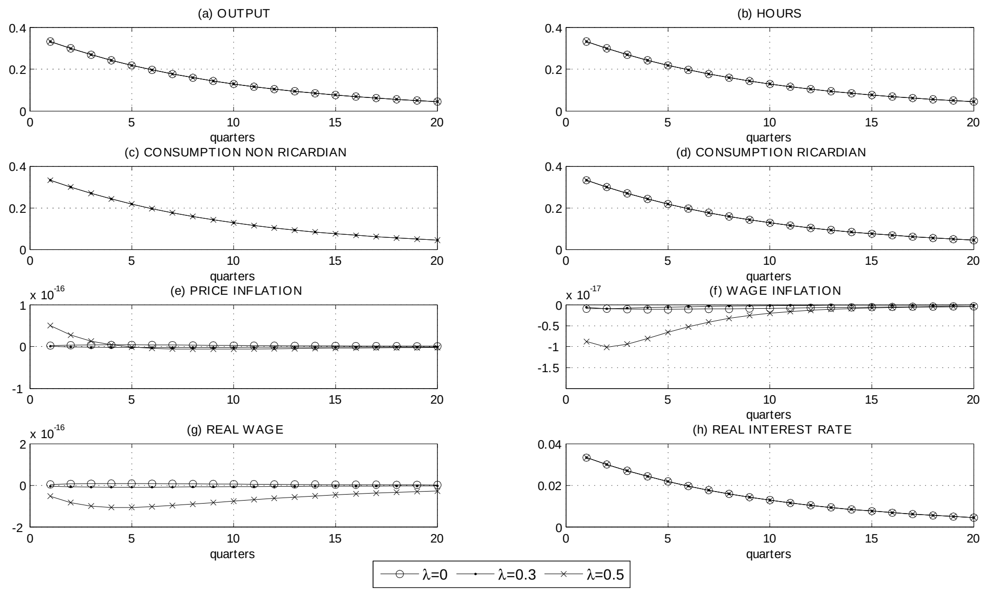

Figure 2: Response to a preference shock in the case of full commitment. Alternative values of the share of liquidity constrained agents.

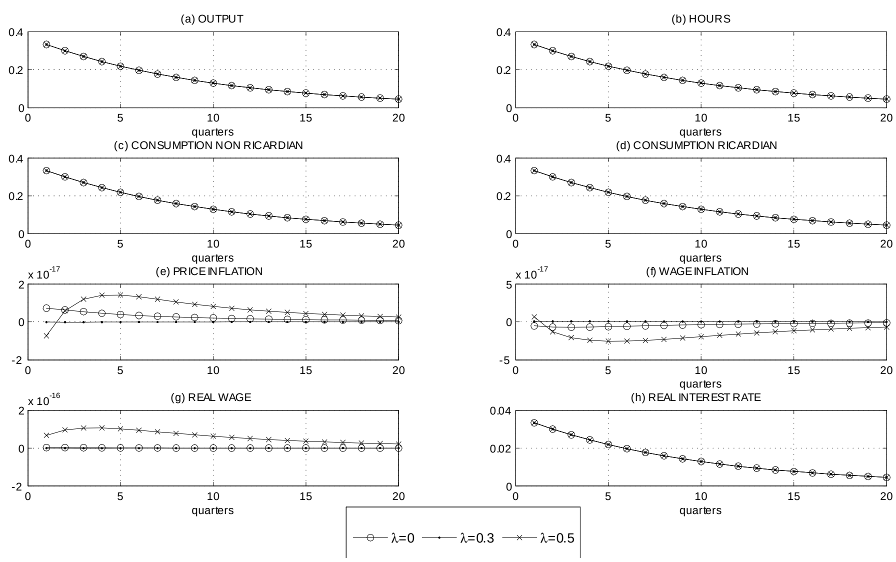

Figure 3: Response to a preference shock in the case of full commitment. Alternative values of the share of liquidity constrained agents.

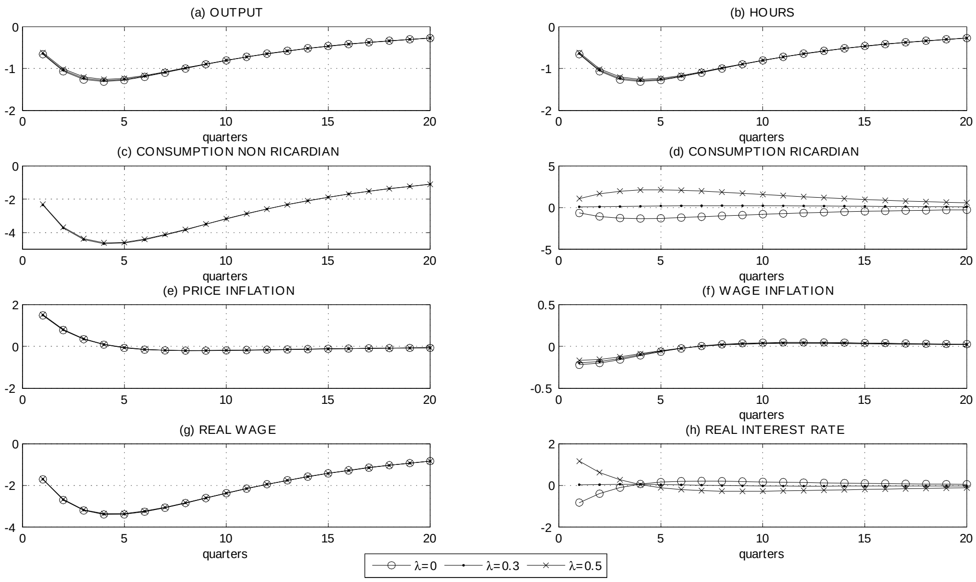

Figure 4: Response to a cost push sock under commitment. Alternative values for the share of liquidity constrained agents.

wage and real interest rate.

In response to a cost-push shock optimal monetary policy implies
[TO BE COMPLETED].

6.1 Simple Rules

In the reminder we assume that the monetary policy authority can credibly commit to a simple instrumental rule of the form of the Taylor rule (55). For the moment we set $\phi_r = 0$, so that the planner chooses the values of the coefficients ϕ_π and ϕ_y that minimize the expected, as of time zero, discounted sum of future welfare losses.

We restrict our search to policy coefficients in the interval [-5,5]. However, we will mention when our results are affected by the size of the interval.

Table 1 reports the optimal ϕ_y and ϕ_π under the variuos specification of the instrumental rule considered, together with the conditional welfare loss associated to each of them in the case of a technology shock. We assume that the system is initially at the steady state.

Consider the case of a contemporaneous rule, i.e. $i = 0$. In that case the output response is muted no matter the share of non ricardian agents, this is in line with the results in SGU (2005). The optimal inflation coefficient is always below the upper limit of the specified interval. Remarkably,

Table 1 Policy Rule	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$
Sticky Wages	ϕ_π; ϕ_y; loss	ϕ_π; ϕ_y; loss	ϕ_π; ϕ_y; loss
$i = -1$			
$i = 0$	$3.2, 0, 0.1$	$3.7, 0, 0.091$	$4.5, 0, 0.085$
$i = 1$			
Flex Wages	ϕ_π; ϕ_y; loss	ϕ_π; ϕ_y; loss	ϕ_π; ϕ_y; loss
$i = -1$			
$i = 0$	$5, -0.3, 0$	$-5, 0, 0$	$-5, -1.8, 0$
$i = 1$			

Table 1: Technology Shock. Optimal Simple Rules

the optimal inflation coefficient response gets larger as the share of non ricardian agents increases. This mimics results obtained under commitment, where the planner engineered a deeper reduction in the rate of interest as the share of liquidity constrained agent increased.

Notice that wage stickiness plays a relevant role in the design of optimal policy. Indeed, as shown in table 1, when wages are flexible the optimal policy response to inflation hits the lower bound of the specified interval, more precisely it gets negative. The optimal rule is also characterized by negative values of coefficient ϕ_y.

Table 2 shows the optimal ϕ_y and ϕ_π under the variuos specification of the instrumental rule (55) considered, together with the welfare loss associated to each of them in the case of a cost-push shock. Notice that in the case of wage stickiness the optimal inflation coefficient is always larger than one, i.e it satisfies the taylor Principle no matter tha share of non ricardian agents. This is not the case under flexible wages. Differently form the case of a technology shock, the cost push shock genertaes apolicy trade-off even under flexible wages. For this reason the optinal inflation coefficient stays within the bundaries of the interval over whcih we search. However as in the previous case it becomes negative once λ gets above a certrain threshold.

6.2 Optimal Rules Vs Commitment

[TO BE COMPLETED]

7 Conclusions

[TO BE COMPLETED]

Table 1 Policy Rule	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$
Sticky Wages	$\phi_\pi;\ \phi_y;$ loss	$\phi_\pi;\ \phi_y;$ loss	$\phi_\pi;\ \phi_y;$ loss
$i = -1$			
$i = 0$	$2.2, 0, 3.73$	$1.1, 0, 2.192$	$1.6, 0.2, 2.65$
$i = 1$			
Flex Wages	$\phi_\pi;\ \phi_y;$ loss	$\phi_\pi;\ \phi_y;$ loss	$\phi_\pi;\ \phi_y;$ loss
$i = -1$			
$i = 0$	$4.2, -1.3, 0.08$	$-2.4, 0, 0.11$	$-4.8, 2.4, 0.13$
$i = 1$			

Table 2: Cost Push Shock. Optimal Simple Rules

8 Technical Appendix

8.1 Derivation of the efficient equilibrium output

In order to derive the efficient equilibrium output, we need to first calculate the solution of the Social Planner problem (SPP) and to derive first best allocation. The equilibrium output which solve the (SPP) corresponds to efficient equilibrium output.

$$\begin{aligned} &\max_{\{C_{H,t},C_{S,t},N_t\}} \lambda\frac{\Psi_t C_{H,t}^{1-\sigma}}{1-\sigma} + (1-\lambda)\frac{\Psi_t C_{S,t}^{1-\sigma}}{1-\sigma} - \lambda\frac{L_{H,t}^{1+\phi}}{1+\phi} - (1-\lambda)\frac{L_{H,t}^{1+\phi}}{1+\phi} \\ &s.t \\ C_t &= Y_t = A_t N_t = \lambda C_{H,t} + (1-\lambda) C_{S,t} = A_t\left(\lambda L_{H,t} + (1-\lambda) L_{S,t}\right) \end{aligned}$$

note that

$$\begin{aligned} U_{CH} &= \lambda\Psi C_H^{-\sigma} = \lambda C_H^{-\sigma} \\ U_{CH\varepsilon} &= \lambda C_H^{-\sigma} \end{aligned}$$

since we assume that $\Psi = 1$, then the Lagrangean $\mathcal{L}$ is

$$\begin{aligned} \max_{\{C_{H,t},C_{S,t},N_t\}} \mathcal{L} &= \lambda\frac{\Psi_t C_{H,t}^{1-\sigma}}{1-\sigma} + (1-\lambda)\frac{\Psi_t C_{S,t}^{1-\sigma}}{1-\sigma} - \lambda\frac{L_{H,t}^{1+\phi}}{1+\phi} - (1-\lambda)\frac{L_{H,t}^{1+\phi}}{1+\phi} - \\ &\quad -\mu_t\left[\lambda C_{H,t} + (1-\lambda) C_{S,t} - A_t\left(\lambda L_{H,t} + (1-\lambda) L_{S,t}\right)\right] \end{aligned}$$

first order conditions imply

$$\begin{aligned} \frac{\partial\mathcal{L}}{\partial C_{H,t}} &= 0 : \lambda\Psi_t C_{H,t}^{-\sigma} = \mu_t\lambda \\ \frac{\partial\mathcal{L}}{\partial C_{S,t}} &= 0 : (1-\lambda)\Psi_t C_{S,t}^{-\sigma} = \mu_t(1-\lambda) \end{aligned}$$

which imply that

$$C_{H,t}^{-\sigma} = C_{S,t}^{-\sigma} = C_t^{-\sigma}$$

the marginal utility of consumption the two consumer are identical, then given that the consumer have identical preferences in consumption, it implies that $C_{H,t} = C_{S,t} = C_t$

with respect to the labor supply

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial L_{H,t}} &= 0 : \lambda L_{H,t}^{\phi} = \mu_t A_t \lambda \\ \frac{\partial \mathcal{L}}{\partial L_{S,t}} &= 0 : (1-\lambda) L_{S,t}^{\phi} = \mu_t A_t (1-\lambda) \end{aligned}$$

again it implies

$$L_{H,t}^{\phi} = L_{S,t}^{\phi} = L_t^{\phi}$$

the marginal utility of labor of the two consumer are identical, then given that the consumer have identical preferences in labor, it implies that $N_{H,t} = N_{S,t} = N_t$

Now i can combine the two generic equations

$$\begin{aligned} \Psi_t C_t^{-\sigma} &= \mu_t \\ L_t^{\phi} &= \mu_t A_t \end{aligned}$$

then

$$\Psi_t^{-1} C_t^{\sigma} L_t^{\phi} = A_t$$

which is the standard equation with one representative household

Log-linearizing and considering that $c_t = y_t$ and $y_t = a_t + l_t$, we get the efficient equilibrium output

$$y_t^{Eff} = \frac{1+\phi}{\sigma+\phi} a_t + \frac{1}{\sigma+\phi} \psi_t.$$

8.2 Derivation of the Welfare-based Loss Function

In order to derive a second-order approximation of the household' utility function, we assume that the steady state of our economy is efficient. Under this assumption, we have that in the steady state:

$$\frac{V_{N,H}}{U_{C,H}} = \frac{V_{N,S}}{U_{C,S}} = \frac{W}{P} = \frac{Y}{N} = 1 \tag{64}$$

where $N_H = N_S = N = Y$ and $C_H = C_S = C = Y$. The last equality in (64) holds since the economy production function is: $Y_t = N_t A_t$, where $A = 1$ in steady state. As shown in section once we get the efficient steady firms profits are zero in the steady state and the two households budget constraint is identical, so that $C_S = C_N = C$.

In order to derive a second order approximation of the households utility function, as in Bilbiie (2008) we assume that the Central Bank maximizes a convex combination of the utilities of two types of households, weighted by the mass of agents of each type, i.e.:

$$W_t = \lambda \left[U(C_{H,t}) - V(N_{H,t}) \right] + (1-\lambda) \left[U(C_{S,t}) - V(N_{S,t}) \right] \tag{65}$$

we know that in our model $N_{H,t} = N_{S,t} = N_t$ for each t, this means that (65) can be rewritten as

$$W_t = \lambda U(C_{H,t}) + (1-\lambda) U(C_{S,t}) - V(N_t) \tag{66}$$

A second order approximation of $\lambda U(C_{H,t})$ delivers

$$\begin{aligned}\lambda U(C_{H,t}) \ &\simeq\ \lambda\left[U(C_H) + U_{C_H}(C_{H,t}-C_H) + U_\psi(\psi_t-\psi)\right] + \\ &\quad +\frac{\lambda}{2}\left[U_{C_HC_H}(C_{H,t}-C_H)^2 + 2U_{C_H\psi}(C_{H,t}-C_H)(\psi_t-\psi) + U_{\psi\psi}(\psi_t-\psi)^2\right]\end{aligned}$$

or

$$\begin{aligned}\lambda U(C_{H,t}) \ &\simeq\ \lambda\left[U(C_H) + U_{C_H}C_H\left(c_{h,t}+\frac{1}{2}c_{h,t}^2\right) + U_\psi\psi\left(\psi_t+\frac{1}{2}\psi_t^2\right)\right] + \\ &\quad +\frac{\lambda}{2}\left[U_{C_HC_H}C_H^2c_{h,t}^2 + 2U_{C_H\psi}C_h\psi c_{h,t}\psi_t + U_{\psi\psi}\psi^2\psi_t^2\right]\end{aligned}$$

or

$$\begin{aligned}\lambda U(C_{H,t}) - \lambda U(C_H) \ &\simeq\ \lambda U_{C_H}C_H\left(c_{h,t}+\frac{1}{2}c_{h,t}^2\right) + \frac{\lambda}{2}U_{C_HC_H}C_H^2c_{h,t}^2 + \\ &\quad +\lambda U_{C_H\psi}C_h\psi c_{h,t}\psi_t + \underbrace{\frac{\lambda}{2}U_{\psi\psi}\psi\psi_t^2 + \lambda U_\psi\psi^2\left(\psi_t+\frac{1}{2}\psi_t^2\right)}_{tip}\end{aligned}$$

$$\lambda U(C_{H,t}) - \lambda U(C_H) \simeq \lambda U_{C_H}C_H\left(c_{h,t}+\frac{1}{2}c_{h,t}^2\right) + \frac{\lambda}{2}U_{C_HC_H}C_H^2c_{h,t}^2 + \lambda U_{C_H\psi}C_H\psi c_{h,t}\psi_t + tip$$

$$\lambda U(C_{H,t}) - \lambda U(C_H) \simeq \lambda U_{C_H}C_H\left(c_{h,t}+\frac{1}{2}c_{h,t}^2\right) + \frac{\lambda}{2}U_{C_HC_H}C_H^2c_{h,t}^2 + \lambda U_{C_H\psi}C_H\psi c_{h,t}\psi_t + tip$$

notice that

$$\frac{\lambda}{2}U_{C_HC_H}C_H^2c_{h,t}^2 + \lambda U_{C_H\psi}C_H\psi c_{h,t}\psi_t = \lambda U_{C_H}C_H\left(\frac{1}{2}\frac{U_{C_HC_H}}{U_{C_H}}C_Hc_{h,t}^2 + \frac{U_{C_H\psi}\psi}{U_{C_H}}c_{h,t}\psi_t\right)$$

since $\frac{U_{C_HC_H}}{U_{C_H}}C_H = -\sigma$

$$\frac{\lambda}{2}U_{C_HC_H}C_H^2c_{h,t}^2 + \lambda U_{C_H\psi}C_H\psi c_{h,t}\psi_t = \lambda U_{C_H}C_H\left(-\frac{1}{2}\sigma c_{h,t}^2 + \frac{U_{C_H\psi}\psi}{U_{C_H}}c_{h,t}\psi_t\right)$$

thus

$$\lambda U(C_{H,t}) - \lambda U(C_h) \simeq \lambda U_{C_H}C_H\left(c_{h,t}+\frac{1}{2}(1-\sigma)c_{h,t}^2 + \frac{U_{C_H\psi}\psi}{U_{C_H}}c_{h,t}\psi_t\right) + tip$$

Given the assumed functional form

$$\frac{U_{C_H\psi}\psi}{U_{C_H}} = 1$$

$$\lambda U\left(C_{H,t}\right)-\lambda U\left(C_{H}\right)\simeq\lambda U_{C_{H}}C_{H}\left(c_{h,t}+\frac{1}{2}\left(1-\sigma\right)c_{h,t}^{2}+c_{h,t}\psi_{t}\right)+tip$$

Similarly a second order approximation to the utility of ricardian agents

$$\left(1-\lambda\right)U\left(C_{s,t}\right)-\lambda U\left(C_{s}\right)\simeq\left(1-\lambda\right)U_{C_{s}}C_{s}\left(c_{s,t}+\frac{1}{2}\left(1-\sigma\right)c_{s,t}^{2}+c_{s,t}\psi_{t}\right)+tip$$

Also a second order approximation to $V\left(N_{t}\right)$ yields:

$$V\left(N_{t}\right)-V\left(N\right)\simeq V_{N}N\left(\hat{n}_{t}+\frac{1+\phi}{2}\hat{n}_{t}^{2}\right)\tag{67}$$

Summing all the terms

$$\begin{aligned}W_{t}-W &= \lambda U_{C_{H}}C_{H}\left(c_{h,t}+\frac{1}{2}\left(1-\sigma\right)c_{h,t}^{2}+c_{h,t}\psi_{t}\right)+\\ &\quad+\left(1-\lambda\right)U_{C_{s}}C_{s}\left(c_{s,t}+\frac{1}{2}\left(1-\sigma\right)c_{s,t}^{2}+c_{s,t}\psi_{t}\right)-V_{N}N\left(\hat{n}_{t}+\frac{1+\phi}{2}\hat{n}_{t}^{2}\right)\end{aligned}$$

or

$$\begin{aligned}W_{t}-W &= \lambda U_{C_{H}}C_{H}\left(c_{h,t}+\frac{1}{2}\left(1-\sigma\right)c_{h,t}^{2}+c_{h,t}\psi_{t}\right)+\\ &\quad+\left(1-\lambda\right)U_{C_{s}}C_{s}\left(c_{s,t}+\frac{1}{2}\left(1-\sigma\right)c_{s,t}^{2}+c_{s,t}\psi_{t}\right)-V_{N}N\left(\hat{n}_{t}+\frac{1+\phi}{2}\hat{n}_{t}^{2}\right)\end{aligned}$$

Given our assumptions, steady state consumption levels are identical as well as hours worked, in this case

$$\begin{aligned}W_{t}-W &= \lambda U_{C}C\left(c_{h,t}+\frac{1}{2}\left(1-\sigma\right)c_{h,t}^{2}+c_{h,t}\psi_{t}\right)+\\ &\quad+\left(1-\lambda\right)U_{C}C\left(c_{s,t}+\frac{1}{2}\left(1-\sigma\right)c_{s,t}^{2}+c_{s,t}\psi_{t}\right)-V_{N}N\left(\hat{n}_{t}+\frac{1+\phi}{2}\hat{n}_{t}^{2}\right)\end{aligned}$$

or

$$\begin{aligned}W_{t}-W &= \lambda U_{C}C\left(c_{h,t}+\frac{1}{2}\left(1-\sigma\right)c_{h,t}^{2}\right)+U_{C}Cc_{t}\psi_{t}\\ &\quad+\left(1-\lambda\right)U_{C}C\left(c_{s,t}+\frac{1}{2}\left(1-\sigma\right)c_{s,t}^{2}\right)-V_{N}N\left(\hat{n}_{t}+\frac{1+\phi}{2}\hat{n}_{t}^{2}\right)+tip\end{aligned}$$

From the economy production function we know that

$$\hat{n}_{t}=y_{t}+d_{w,t}+d_{p,t}-a_{t}$$

where $d_{w,t}=\log\int_{0}^{1}\left(\frac{W_{t}^{j}}{W_{t}}\right)^{-\theta_{w}}dj$ is the log of the wage dispersion and $d_{p,t}=\log\int_{0}^{1}\left(\frac{P_{t}^{i}}{P_{t}}\right)^{-\theta_{p}}di$ is the log of the price dispersion. Both terms are of second order and therefore thay cannot be neglected in a second order approximation. Notice that

$$\hat{n}_{t}^{2}=\left(\hat{y}_{t}+d_{w,t}+d_{p,t}-a_{t}\right)^{2}=y_{t}^{2}+a_{t}^{2}-2y_{t}a_{t}$$

thus

$$\begin{aligned} W_t - W &= \lambda U_C C \left(c_{h,t} + \frac{1}{2}(1-\sigma) c_{h,t}^2 \right) + U_C C c_t \psi_t \\ &+ (1-\lambda) U_C C \left(c_{s,t} + \frac{1}{2}(1-\sigma) c_{s,t}^2 \right) - \\ &- V_N N \left(y_t + d_{w,t} + d_{p,t} - a_t + \frac{1+\phi}{2}\left(y_t^2 + a_t^2 - 2 y_t a_t \right) \right) + tip \end{aligned}$$

or

$$\begin{aligned} W_t - W &= \lambda U_C C \left(c_{h,t} + \frac{1}{2}(1-\sigma) c_{h,t}^2 \right) + U_C C c_t \psi_t \\ &+ (1-\lambda) U_C C \left(c_{s,t} + \frac{1}{2}(1-\sigma) c_{s,t}^2 \right) - \\ &- V_N N \left(y_t + d_{w,t} + d_{p,t} - a_t + \frac{1+\phi}{2} y_t^2 - (1+\phi) y_t a_t \right) + tip \end{aligned}$$

or

$$\begin{aligned} W_t - W &= \lambda U_C C \left(c_{h,t} + \frac{1}{2}(1-\sigma) c_{h,t}^2 \right) + U_C C c_{h,t} \psi_t \\ &+ (1-\lambda) U_C C \left(c_{s,t} + \frac{1}{2}(1-\sigma) c_{s,t}^2 \right) - \\ &- V_N N \left(y_t + d_{w,t} + d_{p,t} - a_t + \frac{1+\phi}{2} y_t^2 - (1+\phi) y_t a_t \right) + tip \end{aligned}$$

or, since $U_C C = V_N N$

$$\begin{aligned} \frac{W_t - W}{U_C C} &= c_t + \frac{\lambda}{2}(1-\sigma) c_{h,t}^2 + \frac{(1-\lambda)}{2}(1-\sigma) c_{s,t}^2 + U_C C c_t \psi_t + \\ &- \left(y_t + d_{w,t} + d_{p,t} - a_t + \frac{1+\phi}{2} y_t^2 - (1+\phi) y_t a_t \right) + tip \end{aligned}$$

then using equilibrium condition $c_t = y_t$

$$\begin{aligned} \frac{W_t - W}{U_C C} &= y_t + \frac{(1-\sigma)}{2}\left[\lambda c_{h,t}^2 + (1-\lambda) c_{s,t}^2 \right] + c_t \psi_t + \\ &- \left(y_t + d_{w,t} + d_{p,t} - a_t + \frac{1+\phi}{2} y_t^2 - (1+\phi) y_t a_t \right) + tip \end{aligned}$$

Next notice that

$$\hat{c}_{H,t} = w_t + n_t$$

then

$$\begin{aligned} c_{H,t}^2 &= w_t^2 + n_t^2 + 2 w_t n_t \\ &= w_t^2 + y_t^2 + a_t^2 - 2 y_t a_t + 2 w_t y_t - 2 w_t a_t \\ &= (y_t - a_t)^2 + w_t^2 + 2 w_t y_t - 2 w_t a_t \end{aligned}$$

and

$$\hat{c}_{S,t} = \frac{1}{1-\lambda}\hat{c}_t - \frac{\lambda}{1-\lambda}\hat{c}_{H,t}$$

thus

$$\begin{aligned}
\hat{c}_{S,t}^2 &= \frac{1}{(1-\lambda)^2}\hat{c}_t^2 + \left(\frac{\lambda}{1-\lambda}\right)^2 \hat{c}_{H,t}^2 - 2\left(\frac{1}{1-\lambda}\right)\left(\frac{\lambda}{1-\lambda}\right)\hat{c}_t\hat{c}_{H,t} \\
&= \frac{1}{(1-\lambda)^2}\hat{c}_t^2 + \left(\frac{\lambda}{1-\lambda}\right)^2\left(\hat{w}_t^2 + \hat{n}_t^2 + 2\hat{w}_t\hat{n}_t\right) - \frac{2\lambda}{(1-\lambda)^2}\hat{c}_t\left(\hat{w}_t + n_t\right) \\
&= \frac{1}{(1-\lambda)^2}\hat{y}_t^2 + \left(\frac{\lambda}{1-\lambda}\right)^2\left(\hat{w}_t^2 + \hat{y}_t^2 + a_t^2 - 2\hat{y}_t a_t + 2\hat{w}_t\hat{y}_t - 2\hat{w}_t a_t\right) \\
&\quad - \frac{2\lambda}{(1-\lambda)^2}\left(\hat{y}_t\hat{w}_t + \hat{y}_t^2 - y_t a_t\right)
\end{aligned}$$

then

$$\begin{aligned}
&\left(\lambda\hat{c}_{H,t}^2 + (1-\lambda)\hat{c}_{S,t}^2\right) \\
= \;& \lambda\left(y_t^2 + a_t^2 - 2y_t a_t + \hat{w}_t^2 + 2\hat{w}_t\hat{y}_t - 2w_t a_t\right) + \\
& \frac{1}{(1-\lambda)}\hat{y}_t^2 + \frac{\lambda^2}{(1-\lambda)}\left(\hat{w}_t^2 + \hat{y}_t^2 + a_t^2 - 2\hat{y}_t a_t + 2\hat{w}_t\hat{y}_t - 2\hat{w}_t a_t\right) - \frac{2\lambda}{(1-\lambda)}\left(\hat{y}_t\hat{w}_t + \hat{y}_t^2 - y_t a_t\right)
\end{aligned}$$

collecting terms

$$\begin{aligned}
&\left(\lambda\hat{c}_{H,t}^2 + (1-\lambda)\hat{c}_{S,t}^2\right) \\
=\;& \underbrace{\left(\lambda + \frac{\lambda^2}{(1-\lambda)}\right)}_{\frac{\lambda}{1-\lambda}} w_t^2 + \underbrace{\left(\lambda + \frac{1}{1-\lambda} + \frac{\lambda^2}{(1-\lambda)} - \frac{2\lambda}{(1-\lambda)}\right)}_{1} y_t^2 + \\
& + \underbrace{\left(\lambda + \frac{\lambda^2}{(1-\lambda)}\right)}_{\frac{\lambda}{1-\lambda}} a_t^2 - 2\underbrace{\left(\lambda + \frac{\lambda^2}{(1-\lambda)} - \frac{\lambda}{(1-\lambda)}\right)}_{0} y_t a_t + \\
& + 2\underbrace{\left(\lambda + \frac{\lambda^2}{(1-\lambda)} - \frac{\lambda}{(1-\lambda)}\right)}_{0} w_t y_t - 2\underbrace{\left(\lambda + \frac{\lambda^2}{(1-\lambda)}\right)}_{\frac{\lambda}{1-\lambda}} w_t a_t
\end{aligned}$$

simplifying

$$\left(\lambda\hat{c}_{H,t}^2 + (1-\lambda)\hat{c}_{S,t}^2\right) = \left(\frac{\lambda}{(1-\lambda)}\right) w_t^2 + y_t^2 + \frac{\lambda}{(1-\lambda)} a_t^2 - 2\left(\frac{\lambda}{(1-\lambda)}\right) w_t a_t$$

Using this results and considering that a_t is independent of policy the welfare function can be rewritten as

$$\begin{aligned}
\frac{W_t - W}{U_C C} &= \frac{1}{2}\left[\frac{(1-\sigma)\lambda}{(1-\lambda)} w_t^2 - (\sigma+\phi) y_t^2 - 2\frac{(1-\sigma)\lambda}{(1-\lambda)} w_t a_t + 2y_t\psi_t + 2(1+\phi) y_t a_t\right] \\
&\quad - (d_{w,t} + d_{p,t}) + tip
\end{aligned}$$

Next we have to rewrite some terms. Recall that

$$(\sigma+\phi)\, y_t^{Eff} = (1+\phi)\, a_t + \psi_t$$

thus

$$(\sigma+\phi)\, y_t y_t^{Eff} = (1+\phi)\, y_t a_t + y_t \psi_t$$

Also

$$\begin{aligned}(\sigma+\phi)\left(y_t - y_t^{eff}\right)^2 &= (\sigma+\phi)\left(y_t^2 + \left(y_t^{eff}\right)^2 - 2y_t y_t^{eff}\right) \\ &= (\sigma+\phi)\left(y_t^2 + \left(y_t^{eff}\right)^2\right) - 2\,(\sigma+\phi)\, y_t y_t^{eff}\end{aligned}$$

substituting for the previous result

$$(\sigma+\phi)\left(y_t - y_t^{eff}\right)^2 = (\sigma+\phi)\left(y_t^2 + \left(y_t^{eff}\right)^2\right) - 2\,(1+\phi)\, y_t a_t - 2y_t\psi_t$$

which implies that

$$2\,(1+\phi)\, y_t a_t + 2y_t\psi_t = (\sigma+\phi)\left(y_t^2 + \left(y_t^{eff}\right)^2\right) - (\sigma+\phi)\left(y_t - y_t^{eff}\right)^2$$

In this case

$$\frac{W_t - W}{U_C C} = \frac{1}{2}\left[\frac{(1-\sigma)\,\lambda}{(1-\lambda)}\left(w_t^2 - 2w_t a_t\right) - (\sigma+\phi)\, x_t^2\right] - (d_{w,t} + d_{p,t}) + tip$$

where $x_t = \left(y_t - y_t^{Eff}\right)$ and given that y_t^{Eff} is independent of policy. Also notice that

$$w_t^{eff} = a_t$$

which is a term independent of policy. Multiplying w_t^{Eff} by w_t we get:

$$w_t w_t^{eff} = w_t a_t$$

Next

$$\left(w_t - w_t^{eff}\right)^2 = w_t^2 + \left(w_t^{eff}\right)^2 - 2w_t w_t^{eff}$$

combining

$$\left(w_t - w_t^{eff}\right)^2 = w_t^2 - 2w_t a_t + \left(w_t^{eff}\right)^2$$

which implies

$$w_t^2 - 2w_t a_t = \left(w_t - w_t^{eff}\right)^2 - \left(w_t^{eff}\right)^2 = \tilde{\omega}_t^2 - \left(w_t^{eff}\right)^2$$

Substituting the latter into the welfare loss funstion and considering that w_t^{eff} is a term independent of policy, we get

$$\frac{W_t - W}{U_C C} = \frac{1}{2}\left[\frac{(1-\sigma)\lambda}{(1-\lambda)}\tilde{\omega}_t^2 - (\sigma+\phi)\,x_t^2\right] - (d_{w,t} + d_{p,t}) + tip$$

Using Woodford Lemma 1 and Lemma 2, we can finally write the present discounted value of the Central Bank loss function as

$$L = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\frac{(\sigma-1)\lambda}{(1-\lambda)}\tilde{\omega}_t^2 + (\sigma+\phi)\,x_t^2 + \frac{\theta_w}{\kappa_w}\left(\pi_t^w\right)^2 + \frac{\theta_p}{\kappa_p}\left(\pi_t^p\right)^2\right) + tip$$

Notice that if $\sigma<1$ deviation of the real wage from its efficient level leads to a lower society's loss. How can this be the case? Recal that in a standard model changes in the real wage have no wealth effect on labor supply, since they are exactly off set by changes in profits for given hours. This is not the case here. However I don't whether this is the reason.

Derivation of the welfare function under flexible wages:

Remember that in the case in which wages are fully flexible, the labor supply is:

$$\begin{aligned} w_t &= \sigma c_t + \phi n_t - \psi_t \\ &= \sigma y_t + \phi y_t - \phi a_t - \phi d_{p,t} - \psi_t \\ &= (\sigma+\phi)\,y_t - \phi a_t - \psi_t - \phi d_{p,t} \end{aligned}$$

hence, subtracting the efficient equilibrium to the LHS and the RHS of the previous equation

$$\omega_t = (\sigma+\phi)\,x_t - \phi d_{p,t}$$

where we use the fact that $d_{p,t} - d_{p,t}^{Eff} = d_{p,t}$ (given that $d_{p,t}^{Eff} = 0$). Moreover, we know $a_t = a_t^{Eff}$ and that $\psi_t = \psi_t^{Eff}$ and terms multiplied by $-\phi d_{p,t}$ are terms higher than second order. Then

$$\omega_t^2 = (\sigma+\phi)\,x_t^2$$

this meas that the welfare-loss can be rerwritten as follows:

$$L = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left((\sigma+\phi)\,x_t^2 + \frac{(\sigma+\phi)(\sigma-1)\lambda}{1-\lambda}x_t^2 + \frac{\theta_p}{\kappa_p}\left(\pi_{p,t}\right)^2\right)$$

Notwistanding wage flexibility there is and additional term with respect to a fully ricardian framework, given by $\frac{(\sigma+\phi)(\sigma-1)\lambda}{1-\lambda}x_t^2$. Once again this is due to the presence of rot agents and similarly it disappears when $\sigma = 1$. Also, when $\sigma< 1$, the identified additional term leads to a reduction in society's welfare loss.

The presence of a union implies that workers supply labor on demand. To see this and neglecting exogenous shocks, the production function implies

$$\hat{n}_t = \hat{y}_t$$

where recall that $n_t = n_{h,t} = n_{s,t}$. Thus an increase in output leads to an increase in hours worked and viceversa for any given level of the real wage. Suppose now that labor markets were competitive. In this case liquidity constrained agents would be characterized by the following FOC for labor supply

$$\phi \widehat{n}_{h,t} + \sigma \widehat{c}_{h,t} = \widehat{w}_t \tag{68}$$

In can be shown that under competitive labor markets the dynamic of the aggregate wage would be identical to that obtained in the case in which unions set wages, i.e. $w_t = \sigma c_t + \phi n_t$. Substituting $c_{h,t} = w_t + n_{h,t}$ and the aggregate wage into (68) leads to

$$n_{h,t} = (1 - \sigma) y_t$$

which shows that in the case in which $\sigma > 1$ hours of liquidity constrained agents would move in the opposite direction with respect to output. Given the union forces them, no matter the value of σ, to increase hours worked as y_t icreases they suffer a loss which shows up in the welfare loss function. When σ<1 instead hours of liquidity constrained agents would move in the same direction of output, the gap with respect to the decentralized equilibrium gets narrower and the loss reduces. When $\sigma = 1$, instead, liquidity constrained agents would maintain theri labor supply constant, the loss in this case is measured uniquely by the distortions due to output changes as in the standard framework.

8.3 Derivation of the IS curve

We know that, by log-lienarizing the Ricardian Euler equation

$$c_{s,t} = E_t c_{s,t+} - \frac{1}{\sigma} E_t (r_t - \pi_{t+1}) - \frac{1}{\sigma} \Delta \psi_{t+1}$$

while from the consumption function of rule-of-tumb consumer we get:

$$c_{H,t} = l_t + \omega_t$$

while aggregate consumption is

$$c_t = (1 - \lambda) c_{s,t} + \lambda c_{H,t}$$

then solving the latter equation for $c_{s,t}$

$$c_{s,t} = \frac{1}{1-\lambda} c_t - \frac{\lambda}{1-\lambda} c_{H,t}$$

substituting in the euler equation we get

$$\frac{1}{1-\lambda} c_t - \frac{\lambda}{1-\lambda} c_{H,t} = E_t \left(\frac{1}{1-\lambda} c_{t+1} - \frac{\lambda}{1-\lambda} c_{H,t+1} \right) - \frac{1}{\sigma} E_t (r_t - \pi_{t+1}) - \frac{1}{\sigma} \Delta \psi_{t+1}$$

or

$$c_t = E_t (c_{t+1} - \lambda \Delta c_{H,t+1}) - \frac{(1-\lambda)}{\sigma} E_t (r_t - \pi_{t+1}) - \frac{(1-\lambda)}{\sigma} \Delta \psi_{t+1}$$

or substituting for $c_t = y_t$ and for $E_t \Delta c_{H,t+1} = E_t \left(\Delta l_{t+1} + \Delta \omega_{t+1}\right)$

$$y_t = E_t y_{t+1} - \lambda E_t \Delta l_{t+1} - \lambda E_t \Delta \omega_{t+1} - \frac{(1-\lambda)}{\sigma} E_t \left(r_t - \pi_{t+1}\right) - \frac{(1-\lambda)}{\sigma} \Delta \psi_{t+1}$$

given the aggregate production function

$$y_t = l_t + a_t$$

then we substitute l_t for $l_t = y_t - a_t$, then we get

$$y_t = E_t y_{t+1} - \lambda E_t \Delta y_{t+1} + \lambda E_t \Delta a_{t+1} - \lambda E_t \Delta \omega_{t+1} - \frac{(1-\lambda)}{\sigma} E_t \left(r_t - \pi_{t+1}\right) - \frac{(1-\lambda)}{\sigma} \Delta \psi_{t+1}$$

or solving for y_t

$$y_t = E_t y_{t+1} + \frac{\lambda}{1-\lambda} E_t \Delta a_{t+1} - \frac{\lambda}{1-\lambda} E_t \Delta \omega_{t+1} - \frac{1}{\sigma} E_t \left(r_t - \pi^p_{t+1}\right) - \frac{1}{\sigma} \Delta \psi_{t+1} \tag{69}$$

rewriting equation (69) in terms of output gap from the efficient equilibrium output we get:

$$y_t - y_t^{Eff} = E_t \left(y_{t+1} - y_{t+1}^{Eff}\right) + \Delta y_{t+1}^{Eff} + \frac{\lambda}{1-\lambda} E_t \Delta a_{t+1} - \frac{\lambda}{1-\lambda} E_t \Delta \omega_{t+1} - \frac{1}{\sigma} E_t \left(r_t - \pi^p_{t+1}\right) - \frac{1}{\sigma} \Delta \psi_{t+1} \tag{70}$$

we define $x_t = y_t - y_t^{Eff}$, then

$$x_t = E_t x_{t+1} + \Delta y_{t+1}^{Eff} + \frac{\lambda}{1-\lambda} E_t \Delta a_{t+1} - \frac{\lambda}{1-\lambda} E_t \Delta \omega_{t+1} - \frac{1}{\sigma} E_t \left(r_t - \pi^p_{t+1}\right) - \frac{1}{\sigma} \Delta \psi_{t+1} \tag{71}$$

remember that $E_t \Delta \omega_{t+1} = E_t \pi^w_{t+1} - E_t \pi^p_{t+1}$, then equation (71) becomes

$$x_t = E_t x_{t+1} + \Delta y_{t+1}^{Eff} + \frac{\lambda}{1-\lambda} E_t \Delta a_{t+1} - \frac{\lambda}{1-\lambda} E_t \pi^w_{t+1} + \frac{\lambda}{1-\lambda} E_t \pi^p_{t+1} - \frac{1}{\sigma} E_t \left(r_t - \pi^p_{t+1}\right) - \frac{1}{\sigma} \Delta \psi_{t+1} \tag{72}$$

We now want to rewrite the IS curve, in terms of the efficient rate of interest. We know that under the efficient equilibrium $x_t = E_t x_{t+1} = 0$, and $E_t \pi^w_{t+1} = E_t \pi^p_{t+1} = 0$, then

$$r_t^{eff} = \sigma \left(\Delta y_{t+1}^{Eff} + \frac{\lambda}{1-\lambda} E_t \Delta a_{t+1} - \frac{1}{\sigma} \Delta \psi_{t+1}\right) \tag{73}$$

given that: $E_t \Delta y_{t+1}^{Eff} = \frac{1+\phi}{\phi+\sigma} E_t \Delta a_{t+1} + \frac{1}{\phi+\sigma} E_t \Delta \psi_{t+1}$, then we can rewrite the efficient rate of interest as follows:

$$r_t^{eff} = \sigma \left[\left(\frac{1+\phi}{\phi+\sigma} + \frac{\lambda}{1-\lambda}\right) E_t \Delta a_{t+1} + \left(\frac{1}{\phi+\sigma} - \frac{1}{\sigma}\right) E_t \Delta \psi_{t+1}\right]$$

then, we can finally write the IS as

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} E_t \left(r_t - \pi^p_{t+1} - r_t^{Eff}\right) - \frac{\lambda}{1-\lambda} \left(E_t \pi^w_{t+1} - E_t \pi^p_{t+1}\right)$$