

Nonlinearities in the Real Exchange Rate and Monetary Policy: Interest Rate Rules Reconsidered.

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Abstract

Empirical research during the last ten years has found significant evidence in favor of a nonlinear-threshold type behavior of the real exchange rate. Interest rate rules including the exchange rate appear to have either an insignificant effect on or generate small coefficients for the real exchange rate. However, the empirical studies do not take into account the nonlinear behavior of the exchange rate. The inclusion of nonlinearities in the real exchange rate could imply nonlinear behavior in the interest rate rule, whenever the exchange rate is included. We use a two-country sticky price model with transaction costs to show that linear and nonlinear Taylor-type rules where the exchange rate is included lead to lower variation in output and inflation.

Keywords: Taylor rules, real exchange rate, transaction costs, nonlinearities, optimal monetary policy

JEL Classification: E52, F41, F42.

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1 Introduction

Recent work on monetary economics has focused on the modeling of monetary policy in models of imperfect competition and nominal rigidities. The virtue of such models is that they provide a better insight into the evaluation of alternative monetary policies. Nominal rigidities and market power allow for real effects. Alternative policies concern specifications about the way the Central Bank conducts monetary policy. Moreover, recent work has focused on the evaluation of monetary policy for open economies (Gali and Monacelli, 2005; Benigno, 2004; Monacelli, 2003; Svensson, 2000; Ball, 1998).

Research on monetary policy, however, has focused more on models with closed economies, or on interest rate rules where the target variables are the inflation rate and the output gap (Taylor type rules). This approach relies on the fact that the real or nominal exchange rate need not be included in the rule. One reason is that the exchange rate effect exists indirectly. The exchange rate affects the other two target variables, anyway, through its pass-through effect (Taylor, 1999; Ball, 1998). Another reason is that data do not support its existence in the rule (Clarida et al., 1998)¹. On the other hand, others tend to argue for the importance of including the exchange rate in a feedback rule² (Svensson, 2000).

Exchange rate behavior has been the focus of much research since the early '90s. Rogoff (1996) originally inaugurated a new kind of approach regarding the short run and the long run dynamics of the exchange rate. The 'PPP-Puzzle' put into question the standard linear time series techniques as a way of estimating the horizons needed in order for the exchange rate to mean revert³. Simple *AR* models appeared unable to capture the behavior observed. Additionally, standard linear time series tests could not reject the null of no stationarity, implying that the real exchange rate is a random walk and, thus, invalidating long-run purchasing power parity⁴. Half life estimates appeared to be incorrect as well⁵.

The existence of transaction costs in the international trade of goods affects the trade volume and hence the behavior of the exchange rate. When transaction costs are high, international trade is less profitable. Consequently, deviations of the real exchange from PPP will be corrected very slowly. On the other hand when transaction costs are low, international trade is profitable and the real exchange rate will inherit a mean reverting property. Therefore, the existence of such costs

¹Clarida et al. found either very small or statistically insignificant coefficients for the exchange rate in a forward looking interest rate rule.

²Svensson also argues that apart from the exchange rate, foreign fundamental variables appear to be important in the feedback rule.

³The 'PPP-Puzzle' states the following: how can one reconcile long-run mean reversion with short-run high volatility.

⁴For a detailed analysis on exchange rate behavior see Coakley, Flood, Fuertes and Taylor (2005), MacDonald and Taylor (1994) and the references therein.

⁵Half life is the number of periods it takes a shock to dissipate by a half.

in international trade imply a threshold (nonlinear) behavior for the real exchange rate⁶.

In this paper, it is shown that transaction costs in goods markets affect the trade volume. Moreover, focusing on a linear rational expectations version of the model first, we show that transaction costs shocks make the real exchange rate behave as a random walk. As a second step, we consider the model in the presence of threshold effects. The home country can be either a net exporter, or a net importer depending on where the real exchange rate stands. Hence, the real exchange rate exhibits a behavior that looks like a threshold type one. That result is obtained by modifying consumer's preferences appropriately. From the consumer's maximization problem, we determine the area where international trade is not profitable. So if the real exchange rate lies within that area international trade volume will be low. In this case Home country inflation dynamics will be determined by Home goods inflation dynamics and the terms of trade effect will be low. We argue that in those cases the Central Bank need not react to exchange rate movements since there is a high degree of home bias in consumption. On the other hand, when the real exchange rate is at a level where trade volume is high, then additional factors determine Home country's CPI dynamics (significant terms of trade effect). We argue that in those cases the Central Bank is necessary to react to exchange rate movements in order to control domestic inflation and its consequences in output and employment. In the model it is shown that the Central Bank is able to affect the dynamics of the real exchange rate and make it revert to its initial level (target) faster, once monetary policy is adjusted appropriately. Therefore, the purpose of this paper is twofold. First, we want to show that it is important to include the exchange rate in an interest rate rule. Using linear interest rate rules, we show that interest rate rules where the exchange rate is included perform better. Additionally, calibration shows that interest rate rules targeting the real exchange rate too, generate half lives that are close to what the data suggest. Second, we want show that rules where the interest rate reacts to the exchange rate movements in a nonlinear fashion when the latter's deviation from PPP is high or when transaction costs are low⁷, perform better than the linear specifications. Nonlinear rules capture frictions in international goods markets. When those frictions are not taken into account, then it is likely that the model is not correctly specified, and, thus, inference may not be secure. The reason is that linear specifications do

⁶From a theoretical point of view, modeling a behavior like that described in the 'PPP-Puzzle' is nontrivial. Nominal rigidities may not be enough to generate persistence in the real exchange rate. Persistence could be generated by the degree of correlation in monetary shocks as in Chari et al. (2000) and Benigno (2004). However, this finding could be weak in case of a low degree of autocorrelation. Additionally, this approach tries to explain the persistence in real exchange rate relying on assumptions concerning exogenous variables, without endogenizing it. Gali and Monacelli (2004), in an attempt to model volatility in the real exchange rate, find that the former is determined by the degree of correlation between productivity and world output. A high positive (negative) correlation between domestic productivity and world output will tend to decrease (increase) the volatility of the nominal and real exchange rates.

⁷In other words we argue that the interest rate should react to exchange rate movement only when international trade is profitable.

not capture the threshold behavior of the exchange rate caused by those frictions.

The paper is organized as follows. In section 2 we use data for major economies against the US to provide support for the nonlinearities in the interest rate when the real exchange rate is introduced. In section 3 we develop a DSGE two country model with transaction costs to show that the existence of such costs generates a wedge between the marginal utilities of consumption between the two countries and, thus, entail a threshold type behavior for the real exchange rate. In section 4 we present the log linearized version of the model. In section 5 we introduce monetary policy by presenting alternative interest rate rules the Central Bank may adopt. In section 6 we present the calibration and simulation results. In section 7 we present the conclusions.

2 Empirical evidence and motivation

In this section we use real time series data to estimate a structural VAR model similar to that estimated in Rudebusch (2002). Rudebusch, however, does not impose any restrictions. The use of unrestricted VAR models in examining monetary policy has been criticized. However, they can constitute a simple benchmark for the dynamics of a structural model⁸. Rudebusch estimates a near VAR model of three equations, one for output, one for the inflation rate and one for the interest rate. This model is extended by considering the real exchange rate as well. We extend this model further by introducing restrictions that are in line with the theoretical DSGE model presented in the following sections. In other words we use a structural VAR. The reason is that, assuming first that the Central Bank follows point targeting, we want to explore the importance of the exchange rate having contemporaneous effects on the interest rate. In other words, the Central Bank is allowed to react to exchange rate movements as well, apart from those in the inflation rate and the output gap. Moreover, we allow the exchange rate to affect the inflation rate both contemporaneously and with a lag. In the four variable VAR two lags of the inflation rate, output and the nominal interest rate are introduced. The output gap⁹ is proxied by the hp filter. We allowed for the latter due to the fact that interest rate movements are likely to affect in the short-run the spot rate, and, thus, due to nominal rigidities, the real one. The near VAR representation is defined as

⁸For a more detailed analysis on the weaknesses and the criticism on unrestricted VARs in monetary policy analysis see Rudebusch (1998) and the references therein.

⁹The output gap was proxied using the hp filter. The latter's accuracy in capturing the actual output gap has been criticized. One reason is that the natural rate of output is proxied by a deterministic trend. However, the former may be a function of technology, monetary and demand shocks, and thus, more volatile. For a more detailed survey on the criticism on the output gap measures see Gali (2002), Gali and Gertler (1999), Sbordone (1999), Gertler, Gali and Lopez-Salido (2000) and the references therein.

$$A_0 X_t = A_1 X_{t-1} + A_2 X_{t-2} + I \Omega_t$$

where A_i , $i = 0, 1, 2, 3, 4$ are 4×4 matrices and $X_t = (i_t, q_t, y_t, \pi_t)'$, where i_t is the nominal interest rate, q_t the real exchange rate, y_t the output gap and π_t the inflation rate. I is a 4×4 identity matrix and Ω_t is a 4×1 matrix of *i.i.d.* errors.

2.1 Data

We assume a two country model. US is assumed to be the foreign country and the Eurozone to be the home country.

Monthly data were gathered from the IMF International Financial Statistics for the CPI of each country, the end of period spot exchange rate of the Euro against the US dollar respectively. The Treasury Bill rate for the US and the interbank overnight rate were used as proxies for the nominal interest rate. The dataset spans from 1999:1 to 2009:1.

2.2 Impulse response analysis

The goal in this paper is to show the importance of the real exchange in the interest rate rule in terms of inflation and output gap variation. Specifically, we want to show that the Central Bank is able to achieve better control of inflation and, if possible, the output gap. Therefore, in this first step, we assume that the ECB does point targeting and not zone targeting. This simplifies the analysis, since zone targeting implies that the standard VAR analysis is no longer valid due to time varying parameters.

The impulse response functions for the inflation rate and the output gap were computed under a one standard deviation exchange rate and policy shock. The results are shown in figures 1 and 2 below. We call the interest rate rule where the real exchange rate is also targeted, apart from the inflation and output gap, Rule 1.

Figure 1: Real exchange rate shock

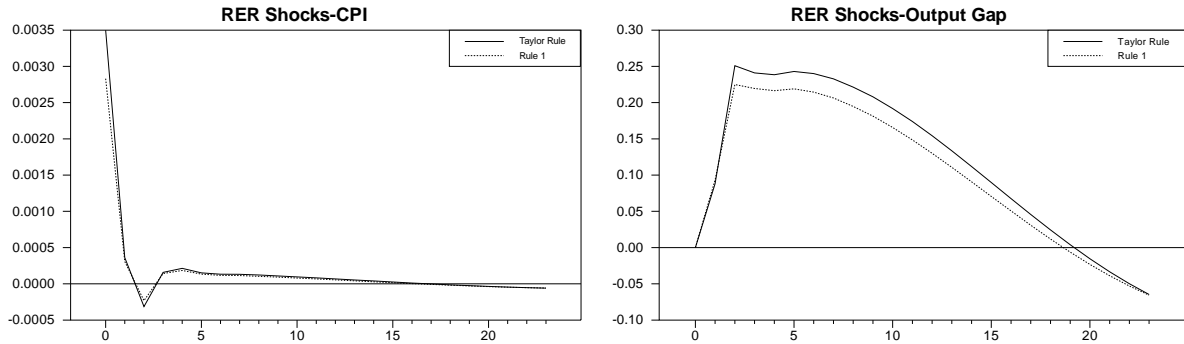
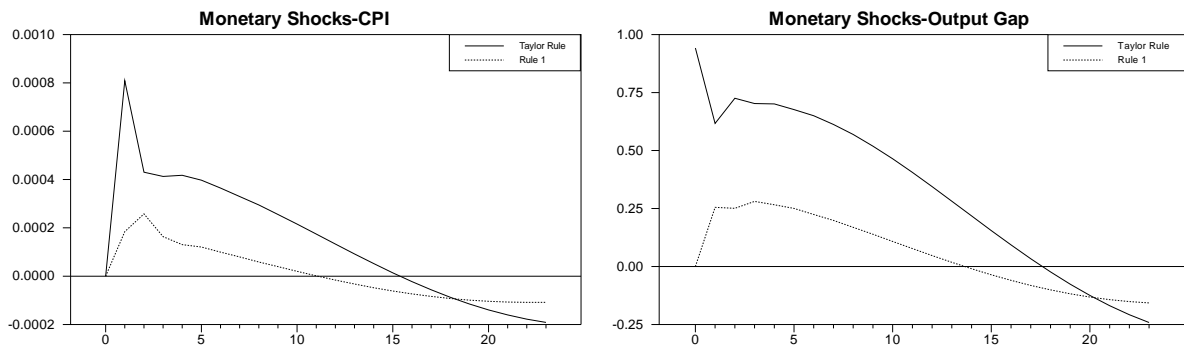


Figure 2: Monetary shock



The impulse responses show that the ECB achieves a better control of the inflation rate whenever the real exchange rate is introduced into the interest rate rule. Following a shock to the real exchange rate (Figure 1) the response of inflation is almost the same, but the initial jump is lower under Rule 1. Following a policy shock, the performance is much better. The inflation control is better as the former seems to revert faster to the initial state and the area within which it varies is smaller. Similar benefits are observed for the output gap. Therefore, data suggest that it is better for the ECB to target the real exchange rate as well.

2.3 Linearity tests

As a next step, we performed system and equation specific linearity tests. The reason for that, is that most Central Banks do zone targeting. That is, they do not have point targets, but they seem to have either tolerance intervals or allow

their targeted variables to vary only within a certain admissible region¹⁰. In other words, the Central Bank reacts to the target variable if and only if it lies outside a certain region¹¹.

One way to test for nonlinearities is to directly expand the VAR by inserting nonlinear combinations of the variables. The testing procedure followed is the same as that described in Van Dijk (1999)¹². The linear part is the same as that described by the VAR representation above. Assuming that the real exchange rate is the transition variable, the auxiliary model^{13,14} is specified as

$$X_t = \sum_{i=1}^4 \Gamma_i X_{t-i} q_{t-1}^{i-1} + I\Omega_t$$

where q_t is the real exchange rate¹⁵.

The system linearity test¹⁶ tests the null of $\Gamma_2 = \Gamma_3 = \Gamma_4 = 0$, that is, all elements of the three matrices are equal to zero. However, for the sake of power and, hence, better inference we also performed equation specific linearity tests¹⁷. The system linearity test rejects the null of linearity. The equation specific linearity tests are shown at Table 1 below.

¹⁰Orphanides and Wieland (2000) focusing on inflation zone targeting provide examples of major Central Banks which perform zone and not point targeting. They argue that uncertainty in the developments into the economy do not allow Central Banks to do point targeting since this may introduce additional volatility into the fundamentals of the economy in the case of large and unexpected shocks.

¹¹The public announcement of the admissible region (threshold) is very important in order for monetary policy to be credible.

¹²For a more detailed description of the testing procedure for both univariate and multivariate models see Terasvirta (1994), Granger and Terasvirta (1993) and Van Dijk (1999) and the references therein.

¹³The auxiliary regressions correspond to a smooth transition autoregressive model with a zero threshold, that is a two regime model. The adjustment can be either symmetric or asymmetric. The testing procedure, though, is robust to both modes of adjustment.

¹⁴Nonlinearity in monetary policy can be introduced in many different ways. Surico (2003, 2006) finds evidence in favor of asymmetric preferences of the Central Bank, that is an asymmetric loss function. Aksoy et al. (2006) and Davig and Leeper introduce nonlinearity by considering threshold (regime switching) interest rate rules. Orphanides and Wieland (2000), however, consider both ways of modelling monetary policy nonlinearities, that is both zone linear-quadratic loss function and a zone-linear Phillips curve. In our case we have both an asymmetric loss function, derived from a second order approximation of the agents utility function, and a nonlinear interest rate rule.

¹⁵The real exchange rate is lagged one period in the auxiliary regression. This is the delay parameter which, in our case, implies that it takes the real exchange rate one period (e.g. quarter) to switch from one regime to the other.

¹⁶System linearity tests were performed as F -tests. They could have also been performed as LM -tests. However, Terasvirta (1994) shows that the F -version of the test is better sized, especially when the sample is small and the number of restrictions large.

¹⁷System linearity tests are expected to have greater power as it is enough that in only one of the four equations the null of linearity is rejected, so that to reject linearity in the system. Consequently, relying only on the system linearity tests, one cannot derive secure inference about which equations in the system have statistically significant nonlinear terms.

Table 1: Linearity tests

	Eurozone
i_t	0.000
q_t	0.017
y_t	0.077
π_t	0.083

Notes: P – values from equation specific tests reported.

Results in table 1 show that linearity is rejected at all significance levels in the interest rate equation implying that the interest rate is nonlinear in the real exchange rate. The real exchange rate is nonlinear, as expected, at 5% significance level. However, the inflation rate and the output gap are linear at the 5% significance level. The rejection of linearity in the interest rate implies that the inclusion of the exchange rate in the interest rate equation in a linear fashion may lead to model misspecification, and, hence, to misleading policy conclusions. The ECB appears to be reacting in a nonlinear fashion to exchange rate movements. Consequently, in the theoretical model considered in the subsequent sections, a nonlinear interest rate rule in the real exchange rate is also considered.

3 Structure of the Model

A stochastic model is specified as in Benigno (2004), Obstfeld and Rogoff (1998, 1999). Prices adjust in a sticky way as in Calvo (1983). Each country exports and imports goods. There are shipping costs (iceberg type) in transporting goods from one country to the other. Transaction costs are modeled as in Dumas (1992), Sercu, Van Hulle and Uppal (1995) and Coeurdacier (2006)¹⁸. As a result this implies that trade will not always take place. Only when the price of the exported good is such that makes trade profitable, will each country be involved in international trade. When the real exchange rate lies inside certain bands then trade is not profitable and, hence, each economy will consume domestically produced goods more.

Monetary policy is conducted by the Central Bank which uses the short term nominal interest rate as its instrument. In the present model, the Central Bank must take into account the degree to which home country is involved in international trade. The threshold behavior of the real exchange rate, implies a threshold behavior for the instrument, once the former is introduced into the rule. Consequently, the interest rate rule will be regime dependent.

¹⁸Coeurdacier introduces transaction costs in the price aggregator assuming that the price of the imported good will be $(1 + \tau)p_j$. We follow the same approach.

3.1 Households

In this section, we specify the structure of the baseline, two country stochastic general equilibrium model. Each country is populated by a continuum of infinitely lived and identical households in the interval $[0, 1]$. Foreign variables are denoted with an asterisk. Home agent's consumption at date t is denoted by C_t , real money holdings by $\frac{M_t}{P_t}$ and labor supply by L_t . Home agent maximizes her separable utility function which is given as follows:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{(C_s + x)^{1-\sigma}}{1-\sigma} + \chi \log \frac{M_s}{P_s} - \frac{(L_s)^{1+\gamma}}{1+\gamma} \right] \quad (1)$$

where σ is the degree of relative risk aversion and $x \geq 0$ ¹⁹. C_t is a composite consumption index described as

$$C_t = \left[\delta^{\frac{1}{\rho}} C_{H,t}^{\frac{\rho-1}{\rho}} + (1-\delta)^{\frac{1}{\rho}} C_{F,t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \quad \rho > 1 \quad (2)$$

$$C_t^* = \left[(\delta^*)^{\frac{1}{\rho}} (C_{F,t}^*)^{\frac{\rho-1}{\rho}} + (1-\delta^*)^{\frac{1}{\rho}} (C_{H,t}^*)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

where ρ captures the intratemporal elasticity of substitution between home and foreign goods. $\delta \geq \frac{1}{2}$ is a parameter of home bias in preferences. C_H is the home consumption index. C_F is the foreign consumption index. Consumption indices in the home and the foreign country are defined as

$$C_{H,t} = \left[\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad C_{F,t} = \left[\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \quad (3)$$

$$C_{H,t}^* = \left[\int_0^1 c_t^*(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad C_{F,t}^* = \left[\int_0^1 c_t^*(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

¹⁹Whether x is equal or strictly greater than zero is very important in determining the existence of threshold effects in the model, or not. For $x = 0$ consumers in both countries will consume both home and foreign goods for finite prices and transaction costs. For $x > 0$, the consumer's maximization problem has corner solutions. As a result, corner solutions define, as is shown later, a no international trade area.

Money deflator is given by the aggregate consumption price index for the home and foreign country respectively, which is specified as

$$\begin{aligned}
P_t &= \left[\delta (P_{H,t})^{1-\rho} + (1-\delta) [(1+\tau_t)P_{F,t}]^{1-\rho} \right]^{\frac{1}{1-\rho}} \\
P_t^* &= \left[\delta^* (P_{F,t}^*)^{1-\rho} + (1-\delta^*) [(1+\tau_t)P_{H,t}^*]^{1-\rho} \right]^{\frac{1}{1-\rho}}
\end{aligned} \tag{4}$$

where P_H and P_F are price indices for home and foreign goods, expressed in the domestic currency and τ_t captures the time varying transaction cost assumed to follow a stationary $AR(1)$, $\tau_t = \rho_\tau \tau_{t-1} + \nu_t$, $\nu_t \sim N(0, \sigma^2)$. The price indices for the Home and Foreign country are defined as

$$\begin{aligned}
P_{H,t} &= \left[\int_0^1 p_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, & P_{F,t} &= \left[\int_0^1 p_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}} \\
P_{H,t}^* &= \left[\int_0^1 p_t^*(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, & P_{F,t}^* &= \left[\int_0^1 p_t^*(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}
\end{aligned} \tag{5}$$

In each period t the economy experiences one of the finitely many events $s^t \in \Omega$ (Ω being the set of the finitely many states). Let h^t denote the history of realized states until period t included. The probability of particular state to occur is defined as $\pi(s^{t+1}|h^t)$. The initial realization s_0 is given.

Capital markets are complete. The consumers of both countries purchase state contingent bonds denominated in the domestic currency, $B(s^{t+1})$ for domestic agents and $B^*(s^{t+1})$ for foreign agents at price $Q(S^{t+1}|h^t)$. That is $B(s^{t+1})$ denotes the home agent's holdings of a one period nominal bond paying one unit of the home currency if state s^{t+1} occurs and 0 otherwise.

The home agent maximizes her utility subject to the period budget constraint

$$P_t C_t + M_t^h + \sum_{s^{t+1} \in \Omega} Q(s^{t+1}|h^t) B_{t+1} = B_t(s^t) + W_t L_t + M_{t-1} + S_t \tag{6}$$

where W_t is the nominal wage, S_t are nominal transfers the individual receives from the government and i_t is the nominal interest rate.

Government's balanced budget requires the following

$$\int_0^1 (M_t - M_{t-1}) di = \int_0^1 S_t di \quad (7)$$

3.2 First order conditions

Maximizing the utility function (1) subject to the budget constraint (6) yields the following first order conditions

$$Q(s^{t+1}|h^t) = \frac{\beta\pi(s^{t+1}|h^t)P_t}{P_{t+1}(s_{t+1})} \left(\frac{C_t + x}{(C_{t+1}(s^{t+1}) + x)} \right)^\sigma \quad (8)$$

$$L_t = (C_t + x)^{-\frac{\sigma}{\gamma}} w_t^{\frac{1}{\gamma}} \quad (9)$$

$$\frac{M_t}{P_t} = \chi(C_t + x)^\sigma \left(\frac{1 + i_{t+1}}{i_{t+1}} \right) \quad (10)$$

where the first equation is the usual Euler equation, the second determines the labor supply schedule and the third the demand for real money balances.

Individual demands for each good z produced in the home and in the foreign country respectively are expressed as

$$c_{h,t}(z) = \left(\frac{p_t^h(z)}{P_{H,t}} \right)^{-\theta} \left(\frac{P_{H,t}}{P_t} \right)^{-\rho} \delta C_t \quad (11)$$

$$c_{f,t}(z) = \left(\frac{p_t^f(z)}{P_{F,t}} \right)^{-\theta} \left(\frac{(1 + \tau_t)P_{F,t}}{P_t} \right)^{-\rho} (1 - \delta) C_t \quad (12)$$

3.3 Real exchange rate and transaction costs

The law of one price does not hold continuously in the model. If transaction costs in international goods markets were ignored, then international goods trade would eliminate any deviation from the law implying, thus, a mean reverting behavior of the real exchange rate. In this model, however, transaction costs put restriction in the international trade of goods. Only when the total costs of shipping the good are such that profitable opportunities arise, will the international trade volume be high. Otherwise, each country's residents will consume more domestically produced goods. In other words, the presence of transaction costs generates an area where international trade volume is low.

The assumption of complete markets and identical preferences implies that the real exchange rate will be given by the ratio of the marginal utilities of consumption of the foreign and home residents. Foreign agent's budget constraint will be given by

$$P_t^* C_t^* + M_t^* + \sum_{s_{t+1}} \frac{Q(s^{t+1}|s^t) B_{t+1}^*(s^{t+1})}{\varepsilon_t} = \frac{B^*(s^t)}{\varepsilon_t} + W_t^* L_t^* + M_{t-1}^* + S_t^* \quad (13)$$

Therefore, the Euler equation from the foreign agent's maximization problem is

$$Q(s^{t+1}|h^t) = \frac{\beta \pi(s^{t+1}|s^t) P_t^* \varepsilon_t}{P_{t+1}^*(s^{t+1}) \varepsilon_{t+1}(s^{t+1})} \left(\frac{C_t^* + x}{C_{t+1}^*(s^{t+1}) + x} \right)^\sigma \quad (14)$$

and combining (8) and (14) , we receive the following expression

$$\left(\frac{C_t^f + x}{C_t^h + x} \right)^{-\sigma} = \varpi q_t \quad (15)$$

where $\varpi \equiv \left(\frac{C_0^f + x}{C_0^h + x} \right)^{-\sigma} \frac{P_0}{\varepsilon_0 P_0^*}$ depends on initial conditions and $q_t = \frac{\varepsilon_t P_t^*}{P_t}$ is the real exchange rate.

For $x = 0$,

$$\frac{\lambda_2^* P_{F,t}^*}{\lambda_2(1 + \tau_t) P_{F,t}} \left(\frac{(1 - \delta) C_{F,t}^* C_t}{\delta^* C_{F,t} C_t^*} \right)^{1/\rho} = \frac{\partial U^* / \partial C^*}{\partial U / \partial C} = \frac{\lambda_1^*(1 + \tau_t) P_{H,t}^*}{\lambda_1 P_{H,t}} \left(\frac{\delta C_{H,t}^* C_t}{(1 - \delta^*) C_{H,t} C_t^*} \right)^{1/\rho}$$

For $x > 0$, the ratio of foreign to home marginal utility of consumption is found to be bounded below and above by thresholds whose value is determined by the price levels and the levels of consumption of home and foreign goods, the transaction costs:

$$\frac{\lambda_2^* P_{F,t}^*}{\lambda_2(1 + \tau_t) P_{F,t}} \left(\frac{(1 - \delta) C_{F,t}^* C_t}{\delta^* C_{F,t} C_t^*} \right)^{1/\rho} \leq \frac{\partial U^* / \partial C^*}{\partial U / \partial C} \leq \frac{\lambda_1^*(1 + \tau_t) P_{H,t}^*}{\lambda_1 P_{H,t}} \left(\frac{\delta C_{H,t}^* C_t}{(1 - \delta^*) C_{H,t} C_t^*} \right)^{1/\rho} \quad (16)$$

When the ratio equals the upper threshold then home country increases its exports, whereas once it touches the lower threshold the home country increases its imports of foreign goods. Therefore, the behavior of the real exchange rate when international trade takes place is summarized as

$$q_t = \begin{cases} \frac{1}{\varpi} \left[\frac{\lambda_1^*(1 + \tau_t) P_{H,t}^*}{\lambda_1 P_{H,t}} \left(\frac{\delta C_{H,t}^* C_t}{(1 - \delta^*) C_{H,t} C_t^*} \right)^{1/\rho} \right] & \text{Home country} \rightarrow \text{Net exporter} \\ \frac{1}{\varpi} \left[\frac{\lambda_2^* P_{F,t}^*}{\lambda_2(1 + \tau_t) P_{F,t}} \left(\frac{(1 - \delta) C_{F,t}^* C_t}{\delta^* C_{F,t} C_t^*} \right)^{1/\rho} \right] & \text{Home country} \rightarrow \text{Net importer} \end{cases} \quad (17)$$

When the real exchange rate lies within the above thresholds, then the volume of trade will be low due to high transaction costs. Since transaction costs are high in the middle regime, traders are not interested in trading internationally due to the absence of profitable opportunities. Additionally, the Central Bank is not interested in intervening in the face of real exchange rate movements. Therefore, the exchange rate will behave either as a random walk, or as an autoregressive process with a high degree of persistence. The larger the deviations from the law of one price, the higher the profits from international trade, and the larger the volume of trade. Consequently, the speed of mean reversion of the real exchange rate will be higher, the farther away it is from the thresholds. The speed of mean reversion will be decreased as it moves closer to the bands.

3.4 Price setting

There are two types of firms, the backward looking and the forward looking. As a result, inflation will depend on both its lagged and forward values. Prices are sticky with a price setting behavior *à la* Calvo (1983). At each date, each firm changes its price with a probability $1 - \omega$, regardless of the time since it last adjusted its price. The probability of not changing the price, thus, is ω . The probability of not changing the price in the subsequent s periods is ω^s . Consequently, the price decision at time t determines profits for the next s periods. The price level for home goods at date t will be defined as

$$P_{H,t} = [\omega P_{H,t-1}^{1-\theta} + (1 - \omega) \tilde{p}_t(h)^{1-\theta}]^{\frac{1}{1-\theta}} \quad (18)$$

The price $\tilde{p}_t(h)$ that will be set at date t is specified as

$$\tilde{p}_t(h) = \zeta p_t^B(h) + (1 - \zeta) p_t^F(h) \quad (19)$$

where $\zeta \in (0, 1)$ is the fraction of backward looking firms, $p_t^B(h)$ and $p_t^F(h)$ is the price set by the backward and the forward looking firms, respectively.

Dividing (18) by $P_{H,t-1}$:

$$\Pi_{H,t}^{1-\theta} = \omega + (1 - \omega) \left(\frac{\tilde{p}_t(h)}{P_{H,t-1}} \right)^{1-\theta} \quad (20)$$

where $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$.

Similarly, for the foreign goods consumed in the home economy:

$$\Pi_{F,t}^{1-\theta} = \omega + (1 - \omega) \left(\frac{\tilde{p}_t(f)}{P_{F,t-1}} \right)^{1-\theta} \quad (21)$$

The aggregate price level dynamics are specified, thus, as

$$\Pi_t^{1-\rho} = \delta \left[\left(\frac{P_{H,t-1}}{P_{t-1}} \right) \Pi_{H,t} \right]^{1-\rho} + (1 - \delta) \left[(1 + \tau_t) \left(\frac{P_{F,t-1}}{P_{t-1}} \right) \Pi_{F,t} \right]^{1-\rho} \quad (22)$$

A continuum of firms is assumed for the home economy indexed by $z \in [0, 1]$. Each firm produces a differentiated good, with a technology

$$Y_t(z) = A_t L_t(z) \quad (23)$$

where A_t is a country specific productivity shock at date t which is assumed to follow a log stationary process $\alpha_t = \rho_{\alpha_t} \alpha_{t-1} + v_t$, where v_t is an *i.i.d.* process.

Each firm chooses a price for the home market and a price for the foreign market.

Backward looking firms.

Backward looking firms set their prices according to the following rule

$$p_t^B(h) = P_{H,t-1} + \pi_{H,t-1} \quad \text{and} \quad p_t^{B*}(h) = P_{H,t-1}^* + \pi_{H,t-1}^* \quad (24)$$

Forward looking firms.

Forward looking firms set their prices by maximizing their expected discounted profits. Their maximization problem comprises of two decisions. The one concerns the price for the domestic market and the other the price charged in the foreign market, when it exports. Hence their maximization problem is described as

$$\max E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} \left\{ \tilde{p}_t(h) y_{t+s}^h(h) + \varepsilon_t \tilde{p}_t(h)^* y_{t+s}^f(h) - W_{t+s}^h L_{t+s}^h \right\} \quad (25)$$

where $y_t^i(h)$, $i = h, f$ is the demand for the home good for home and foreign agents specified as

$$y_t^h(p_t(h)) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} \left(\frac{P_{H,t}}{P_t} \right)^{-\rho} \delta C_t, \quad (26)$$

$$y_t^f(p_t^*(h)) = \left(\frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} \left(\frac{(1 + \tau_t) P_{H,t}^*}{P_t^*} \right)^{-\rho} (1 - \delta) C_t^* \quad (27)$$

The firm maximizes its objective function (25) subject to (26) in order to find the optimal price for the Home good in the Home economy. It maximizes subject to (27), in order to find the optimal price for the Home good in the Foreign economy. The firm chooses a price for the Home good in the Home economy that satisfies the first order condition

$$E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}(p_t(h)) \left\{ p_t(h) - \frac{\theta}{\theta-1} MC_{t+s} \right\} = 0$$

where $MC_{t+s} = \frac{W_{t+s}}{A_{t+s}}$ denotes the nominal marginal cost and $\frac{\theta}{\theta-1}$ captures the optimal markup.

The optimal price, thus, for the Home good in the Home country is specified as

$$p_t^F(h) = \frac{\theta}{\theta-1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y_{t+s}^h(p_t(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}^h(p_t(h))} \quad (28)$$

Respectively, the optimal price for the Home good in the Foreign country is specified as

$$p_t^{F*}(h) = \frac{\theta}{\theta-1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y_{t+s}^f(p_t(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}^f(p_t(h)) \varepsilon_{t+s}} \quad (29)$$

4 Log linearized model

In this section we proceed to the loglinearization of the model for $x = 0$. The model is a standard linear rational expectations model without any threshold effects. We will come back to the model where $x > 0$ in section 7.

A log linearized version of the relationships found in the previous section serves in providing us with a way to deal with the problem of no closed form solution. Additionally, this is a way to end up in a state space form which can be estimated using real time series data.

4.1 Supply side

We use a first order Taylor approximation around the steady state of zero inflation rate. Log linearized variables are denoted with a hat.

After loglinearizing the first order condition (9), the price level equations (21) and (22), the production function (23) the demand schedules faced by each firm (26) and (27) and optimal price setting rules (28) and (29), we receive the two relations describing the domestically consumed home goods inflation rate and the respective of the home goods consumed in the Foreign country as in Benigno (2004)

$$\begin{aligned} \pi_{H,t} = & b_{\pi_{H,-1}}\pi_{H,t-1} + b_{\pi_{H,-1}^*}\pi_{H,t-1}^* + \beta E_t\pi_{H,t+1} + b_{\pi_H^*}\pi_{H,t}^* + b_C\hat{C}_t + \dots \\ & \dots + b_T\hat{T}_t + b_{T^*}\hat{T}_t^* + b_q\hat{q}_t + b_a a_t + \varepsilon_{H,t} \end{aligned} \quad (30)$$

$$\begin{aligned} \pi_{H,t}^* = & b_{\pi_{H,-1}}\pi_{H,t-1} + b_{\pi_{H,-1}^*}\pi_{H,t-1}^* + \beta E_t\pi_{H,t+1}^* + b_{\pi_H^*}\pi_{H,t} + b_C^*\hat{C}_t + \dots \\ & \dots + b_T^*\hat{T}_t + b_{T^*}\hat{T}_t^* + b_q^*\hat{q}_t + b_a^* a_t + \varepsilon_{H,t}^* \end{aligned} \quad (31)$$

where $\varepsilon_{H,t}$ and $\varepsilon_{H,t}^*$ are *i.i.d.* cost push shocks. $T_t = \frac{(1+\tau_t)P_{F,t}}{P_{H,t}}$ and $T_t^* = \frac{(1+\tau_t^*)P_{H,t}^*}{P_{F,t}^*}$ captures the terms of trade for the Home and Foreign country respectively.

The log linearized aggregate price level relation (22) is specified as

$$\pi_t = \pi_{H,t} + (1 - \delta)(\pi_{F,t} - \pi_{H,t} + (\rho_\tau - 1)\hat{\tau}_t) \quad (32)$$

which can be further simplified as²⁰

$$\pi_t = \pi_{H,t} + (1 - \delta)\Delta\hat{T}_t$$

²⁰To end up to that expression, we used equation $\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} + \hat{\tau}_t$ for the terms of trade which is reported later in the text.

4.2 Demand side

In this section we proceed to the loglinearization of the Euler equation

$$C_t = \kappa(i_t - E_t\pi_{t+1}) + E_t C_{t+1} \quad (33)$$

where $\kappa = -\frac{1}{\sigma}$.

Goods market clearing assumes the following two conditions

$$Y = C_H + C_H^* + G_t \quad \text{and} \quad Y^* = C_F + C_F^* + G_t^*$$

where G_t and G_t^* denote government purchases.

Using the demand schedules as in (26) and (27), and then loglinearizing using the goods market equilibrium conditions, we end up to the following expressions for consumption in the Home country

$$\hat{C}_t = \frac{1}{-1 + \delta + \delta^*} \left(\delta^* \hat{Y}_t - (1 - \delta) \hat{Y}_t^* + (1 - \delta) \left((2 - \delta^*) \hat{T}_t^* - (1 - (1 - \delta) \delta^*) \hat{T}_t \right) + (1 - \delta) g_t^* - \delta^* g_t \right) \quad (34)$$

Therefore, combining equations (33) and (34), we derive the aggregate demand equation:

$$\begin{aligned} \hat{Y}_t = E_t \hat{Y}_{t+1} + \eta(i_t - E_t \pi_{t+1}) - \frac{(1-\delta)}{\delta^*} E_t \Delta \hat{Y}_{t+1}^* + \frac{\rho(1-\delta)(1-(1-\delta)\delta^*)}{\delta^*} E_t \Delta \hat{T}_{t+1} \\ \dots - \rho(1-\delta)(-2 + \delta^*) E_t \Delta \hat{T}_{t+1}^* + (1 - \rho_g) g_t \end{aligned} \quad (35)$$

where $\eta = \frac{(-1 + \delta + \delta^*) \kappa}{\delta^*}$.

4.3 Real exchange rate behavior

As already mentioned, for $x > 0$, the real exchange rate exhibits regime switching behavior depending on whether trade takes place or not. The larger the deviation from the bands (or absolute PPP in case of a two regime model), the higher the volume of trade, and, thus, the faster the real exchange rate reverts back to the thresholds. When no trade occurs, the real exchange rate depends highly on its lagged values. Additionally, under the assumption of identical preferences, frictionless financial markets and the even stronger assumption of the same degree

of price stickiness across the two countries, the real exchange rate behaves as a random walk within the thresholds. Such a behavior is consistent with empirical literature on exchange rate, where smooth transition autoregressive models seem to solve the so-called 'PPP-Puzzle'. In particular, Taylor, Peel and Sarno (2001), Sarno and Taylor (2001), Taylor and Peel (2000), Lothian and Taylor (1997), Mac Donald and Taylor (1994) and Sarno, Taylor and Chowdhury (2004) using either quarterly or monthly data for major currencies found significant evidence in favor of threshold (or nonlinear) behavior of the real exchange rate. However, for $x = 0$, the real exchange rate is linear. The differenced real exchange rate is specified as

$$\Delta \hat{q}_t = \Delta \varepsilon_t + \pi_t^* - \pi_t \quad (36)$$

where ε_t is the nominal exchange rate and π_t^* is the foreign inflation rate.

4.4 Terms of trade

The terms of trade determine the competitive advantage of each of the two countries. For the home country the terms of trade variable is defined as $T_t = \frac{(1+\tau_t)P_{F,t}}{P_{H,t}}$, whereas for the foreign country are defined as $T_t^* = \frac{(1+\tau_t)P_{H,t}^*}{P_{F,t}^*}$. We can write the following two expressions for the terms of trade.

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} + (\rho_\tau - 1)\hat{\tau}_t, \quad \hat{T}_t^* = \hat{T}_{t-1}^* + \pi_{H,t}^* - \pi_{F,t}^* + (\rho_{\tau^*} - 1)\hat{\tau}_t^*$$

4.5 Flexible price equilibrium

At the flexible price equilibrium firms adjust their prices at each period. Each firm will set its marginal cost equal to the optimal marginal cost (i.e. $-\log\left(\frac{\theta}{\theta-1}\right)$) which is constant over time and equal across firms. Since firms adjust their prices every period, monetary policy will not have any real effects into the economy. The real marginal cost is specified by the following equations

$$mc_t = -\log\left(\frac{\theta}{\theta-1}\right) = -\mu$$

$$mC_t = w_t - \alpha_t - \nu$$

where w_t is the real wage, α_t (log) productivity and ν a subsidy to labor. Solving for the case with flexible prices, we receive the following set of equations describing the equilibrium processes for output, consumption, labor, real interest rate and real exchange rate, given by:

$$\bar{y} = \psi_\zeta \zeta + \psi_a \alpha_t + \psi_{a^*} \alpha_t^* + \psi_\tau \tau_t + \psi_{\tau^*} \tau_t^* + \psi_g g_t + \psi_{g^*} g_t^* \quad (37)$$

$$\bar{c}_t = \psi_\zeta \zeta + \left(\frac{\gamma \delta^* + \sigma}{\delta(\gamma + \sigma) - \gamma(1 - \delta^*)} \right) \psi_\alpha \alpha_t - \left(\frac{\gamma}{\sigma} \psi_{\alpha^*} \right) \alpha_t^* - \left(\frac{\gamma}{\sigma} \psi_\tau \right) \tau_t - \left(\frac{\gamma}{\sigma} \psi_{\tau^*} \right) \tau_t^* - \left(\frac{\gamma}{\sigma} \psi_g \right) g_t - \left(\frac{\gamma}{\sigma} \psi_{g^*} \right) g_t^* \quad (38)$$

$$\bar{l}_t = \psi_\zeta \zeta + \left(\frac{\gamma(\delta^*(1 - \sigma) - (1 - \delta)) - \sigma(1 - \delta)\psi_\alpha}{\delta(\gamma + \sigma) - \gamma(1 - \delta^*)} \right) \alpha_t - \psi_{a^*} \alpha_t^* + \psi_\tau \tau_t + \psi_{\tau^*} \tau_t^* + \psi_g g_t + \psi_{g^*} g_t^* \quad (39)$$

$$\begin{aligned} \bar{r}_t = & \left(\frac{\gamma \delta^* + \sigma}{\kappa \delta(\gamma + \sigma) - \gamma(1 - \delta^*)} \right) \alpha_t - \left(\frac{\gamma(1 - \rho_{a^*})\psi_{a^*}}{\kappa \sigma} \right) \alpha_t^* - \left(\frac{\gamma(1 - \rho_\tau)\psi_\tau}{\kappa \sigma} \right) \tau_t + \left(\frac{\gamma(1 - \rho_{\tau^*})\psi_{\tau^*}}{\kappa \sigma} \right) \tau_t^* - \dots \\ & \dots - \left(\frac{\gamma(1 - \rho_g)\psi_g}{\kappa \sigma} \right) g_t - \left(\frac{\gamma(1 - \rho_{g^*})\psi_{g^*}}{\kappa \sigma} \right) g_t^* \end{aligned} \quad (40)$$

$$\bar{q}_t = \left(\frac{\sigma(\gamma + 1)}{\sigma + \gamma \delta^* - \gamma(1 - \delta)} \right) (\alpha_t - \alpha_t^*) - \left(\frac{\sigma(2(\delta^* - \delta) + (\delta^{*2} - \delta^2))}{\sigma + \gamma \delta^* - \gamma(1 - \delta)} \right) \tau_t + \left(\frac{\sigma \gamma}{\sigma + \gamma \delta^* - \gamma(1 - \delta)} \right) (g_t^* - g_t) \quad (41)$$

where $\zeta = \mu - \nu$, $\psi_\zeta = \frac{\gamma(1 - \delta - \delta^*) - \sigma}{(\gamma + \sigma)(\sigma + \gamma \delta^* - \gamma(1 - \delta))}$, $\psi_a = \frac{(\gamma + 1)(\delta(\gamma + \sigma) - \gamma(1 - \delta^*))}{(\gamma + \sigma)(\sigma + \gamma \delta^* - \gamma(1 - \delta))}$, $\psi_{a^*} = \frac{(1 - \delta)(\gamma + 1)\sigma}{(\gamma + \sigma)(\sigma + \gamma \delta^* - \gamma(1 - \delta))}$,
 $\psi_\tau = \frac{\rho\sigma(1 - \delta)(\gamma(1 - \delta^*(1 - \delta)) + \sigma\delta)}{(\gamma + \sigma)(\sigma + \gamma \delta^* - \gamma(1 - \delta))}$, $\psi_g = \frac{\sigma(\gamma \delta^* + \sigma)}{(\gamma + \sigma)(\sigma + \gamma \delta^* - \gamma(1 - \delta))}$, $\psi_{g^*} = \frac{-\gamma\sigma(1 - \delta)}{(\gamma + \sigma)(\sigma + \gamma \delta^* - \gamma(1 - \delta))}$,
 $\psi_{\tau^*} = \frac{-\rho\sigma(1 - \delta)(\sigma + \gamma \delta^*(2 - \delta^*))}{(\gamma + \sigma)(\sigma + \gamma \delta^* - \gamma(1 - \delta))}$

From the interest rate equation (40) it is evident that the interest rate response to different shocks, and especially to shocks that affect it independently of the regime the real exchange rate lies (e.g. domestic productivity, Home demand shock and transaction costs), changes depending on the volume of international trade. In particular, the interest rate response to domestic productivity shocks is smaller when the volume of trade is low. The same result holds for the interest rate response to changes in transaction costs. Given $\kappa\delta(\gamma + \sigma) > \gamma(1 - \delta^*)$, the interest

rate response to domestic productivity shocks is positive, whereas it is negative with respect to foreign productivity shocks. From (41) one observes that the real exchange rate persistence and volatility depends on the degree of autocorrelation of demand shocks, productivity shocks and the cross correlations between productivity differentials and demand shocks, the Home productivity shock and the Home demand shock, the Home productivity and the Foreign demand shock, the foreign productivity and the Home demand shock, and lastly, on the correlation between the Foreign productivity shock and the Foreign demand shock. A unit serial correlation between the two demand shocks and the two productivity shocks implies absolute PPP (zero persistence and low volatility). Positive cross correlations between the productivity differential and the demand shock differential implies lower volatility for the real exchange rate. The same holds for the rest of cross correlations among the two kinds of shocks. The volatility of the real exchange rate can be specified as

$$\begin{aligned} var(\bar{q}_t) = & \sigma^2[\psi_\alpha^2(\sigma_{a^*}^2 + \sigma_a^2) + (\psi_g + \psi_{g^*})^2(\sigma_g^2 + \sigma_{g^*}^2) - 2\psi_\alpha^2\varrho_{a^*,a}\sigma_{a^*}\sigma_a - 2(\psi_g + \psi_{g^*})^2\varrho_{g^*,g}\sigma_{g^*}\sigma_g - \\ & 2\psi_\alpha(\psi_g + \psi_{g^*})(\varrho\sigma_{g^*-g}\sigma_{a^*-a} + \varrho_{g^*,a}\sigma_{g^*}\sigma_a + \varrho_{g,a}\sigma_g\sigma_a + \varrho_{g^*,a^*}\sigma_{g^*}\sigma_{a^*} + \varrho_{g,a^*}\sigma_g\sigma_{a^*}) \end{aligned}$$

where $\varrho_{g^*,g} = \frac{cov(g_t, g_t^*)}{\sigma_g\sigma_{g^*}}$, $\varrho_{a^*,a} = \frac{cov(a_t, a_t^*)}{\sigma_{a_t}\sigma_{a_t^*}}$, $\varrho_{g^*,a} = \frac{cov(a_t, g_t^*)}{\sigma_a\sigma_{g^*}}$, $\varrho_{g,a} = \frac{cov(a_t, g_t)}{\sigma_a\sigma_g}$, $\varrho_{g^*,a^*} = \frac{cov(a_t^*, g_t^*)}{\sigma_{a^*}\sigma_{g^*}}$, $\varrho_{g,a^*} = \frac{cov(a_t^*, g_t)}{\sigma_{a^*}\sigma_g}$ and $\varrho = \frac{cov((a_t^* - a_t), (g_t^* - g_t))}{\sigma_{a^*-a}\sigma_{g^*-g}}$.

Finally, the natural levels of output, consumption, labor, real interest rate and real exchange rate vary not only according to the exogenous processes of the transaction costs, technology and demand shocks, but also according to the degree of home bias.

5 Monetary Policy

Monetary policy is conducted through nominal interest rate rules by the Central Bank. The rules considered in this paper are various and serve the main goal. The latter is whether a nonlinear (or a threshold type) interest rate rule is the optimal policy rule, when the real exchange rate is introduced in it. That question relies on the real exchange rate literature that supports the view of threshold type, or more generally, nonlinear behavior of the real exchange rate. Therefore, this raises the question of whether non linearity inherent in the real exchange rate is the source of nonlinearities in the interest rate rule.

Open economy monetary policy literature has rejected the importance of the exchange rate in the interest rate feedback rules, either because it is argued that

its effect is already there, indirectly through its pass through on prices and then in inflation (Ball, 1999; Taylor, 1999), or because data do not support its significance (Clarida, Gali and Gertler, 1998). However, a weakness of that literature is that it does not take into account the potential nonlinear behavior of the real exchange rate, or alternatively, the existence of transaction costs either in the goods, or in financial markets. As already shown the existence of transaction costs determines the trade volume internationally, and, thus, the way the real exchange rate behaves. The threshold behavior of the real exchange rate (and the nominal exchange rate due to nominal rigidities) implies a threshold type feedback rule whenever the latter is introduced.

After the introduction of the real exchange rate in the interest rate feedback rules, more generalized ones could be considered. The latter allow for foreign fundamentals in the rule²¹. Svensson (2000) considers variants of Taylor type feedback rules. Simulation results exhibit a non-negligible weight of the exchange rate on the interest rate rule. Moreover, foreign fundamentals appear to be an important component in the rule. However, focusing only on the coefficients of the additional variables in the rule is not a sufficient condition for choosing the optimal policy. The focus must, rather, be on the extent to which the overall variation in output or inflation, or both (depending on the objectives of the Central Banker) is altered across the different policy rules.

5.1 Welfare

The Central Bank sets the interest rate in such a way to minimize a measure of social loss derived by a second order Taylor expansion of the consumer's utility function as in Rotemberg and Woodford (1998). It is summarized as²²

$$W = -\lambda_1(y_t - y_t^n)^2 - \lambda_2\pi_{H,t}^2 - \lambda_3(\pi_{H,t}^*)^2 + \lambda_4q_t - \lambda_5q_t^2 + \lambda_6(y_t y_t^* + y_t q_t + y_t^* q_t) + t.i.p. + O(\|\xi\|^3) \quad (42)$$

5.2 Policy rules

In this section we focus on different policy rules. We characterize optimal the rule that leads to the lowest variation in output and inflation. Each rule leads to a different system of equations and, thus, different conditions that are necessary for determinacy. The rules considered will be of a standard Taylor form to more generalized ones.

²¹Taking into account the fundamental equations found in our model, it is evident that optimal control policies allow for foreign fundamentals to be one of the determinants of the nominal interest rate.

²²The derivation of the loss function is given in detail in the Appendix.

5.2.1 A Nonlinear interest rate rule

In section 3.3 we showed that the volume of international trade depends on where the real exchange rate lies. That is, if the real exchange rate is equal to the upper bound in (18) then the home country is a net exporter. When the real exchange rate is equal to the lower bound then the Home country is a net importer.

Monetary policy must take into account the position of the real exchange rate when setting the nominal rates. In particular, when the real exchange rate is equal to the lower bound (home country is a net importer) then the domestic inflation is determined not only by the home goods inflation rate, but also by that of foreign goods. From the inflation rate expressions for the home goods at home and abroad and from the respective equations for the foreign country, it is shown that in that case the volatility of the home inflation rate (CPI) is determined by the volatility of the home and foreign PPI, output, the real exchange rate and exogenous shocks. When trade is profitable, the real exchange rate is more volatile. Consequently, when international trade takes place, domestic inflation will be more volatile as well. In other words, when trade volume is high domestic inflation will have additional sources of variation apart from those that already exist when trade volume is low. This implies that the interest rate rule must be such that the threshold behavior of the exchange rate and whatever it implies for the domestic inflation rate, are taken into account. That is, the Central Bank should adopt an interest rate rule with a threshold type nonlinearity, summarized as

$$i_t = \begin{cases} c + \phi'_y y_t + \phi'_\pi \pi_t + \phi'_q q_t, & q_{t-d} = q_U \\ c + \phi_y y_t + \phi_\pi \pi_{H,t} & q_L < q_{t-d} < q_U \\ c + \phi''_y y_t + \phi''_\pi \pi_t + \phi''_q q_t, & q_{t-d} = q_L \end{cases} \quad (43)$$

where q_U stands for the upper bound and q_L stands for the lower bound given in (18).

When the real exchange rate lies within the bounds (middle regime) the interest rate rule takes the form of a standard Taylor rule. When international trade takes place the Central Bank reacts to changes in the real exchange rate so that to eliminate its effect on the overall volatility in domestic inflation. Since in the middle regime the volatility of the exchange rate is both lower and does not affect that of the domestic inflation rate, the Central Bank need not react to changes in it. More interestingly, the Central Bank must target the CPI given in (32) when trade volume is high. Otherwise, the Central Bank reacts to the domestic PPI given in (30), since in that case home agents exhibit a high degree of home bias in their preferences ($\delta \rightarrow 1$).

5.2.2 Inflation targeting and nominal income rules

The natural rate of output, specified in (37), is determined by Home productivity and demand shocks. Additionally, when international trade takes place, the natural rate of output depends on the Foreign country productivity and demand shocks and the transaction cost, as well. That is, the natural rate of output is determined by exogenous processes. Moreover, the central bank or the private sector cannot form expectations or even observe the current and the future behavior of those processes. This causes the problem of a good and reliable measure for the output gap. Using detrended series to approximate the output gap might be risky, since the natural rate of output, and hence the output gap, is likely to be more volatile²³. Consequently, the problem of measurement error arises, which may lead to the wrong policy responses and, hence, to instability. Gali (2000), McCallum and Nelson (1999) and Rotemberg and Woodford (1999) argue that using detrended output in the interest rate rule may cause inefficiencies, especially when shocks to fundamentals call for large changes in output. The output gap induced by such a policy will, in turn, lead to unnecessary fluctuations.

Along with interest rate rules targeting the contemporaneous values of the variables, lagged data rules are also considered. This serves in exploring whether our findings are robust to various forms of an interest rate rule. Additionally, looking at lagged data rules allow us to test whether the interest rate rules considered are robust to various kinds of criticism regarding contemporaneous rules. McCallum (1999) and Bullard and Mitra (2002) question the validity of contemporaneous rules. They argue that it is difficult for a Central Bank to have knowledge of the exact level of output and inflation the time it takes its interest rate decisions²⁴.

Lagged data rules in our case are summarized as

$$\dot{i}_t = \phi_\pi \pi_{t-1} + \phi_x x_{t-1} + \varepsilon_{3t} \quad (44)$$

$$\dot{i}_t = \phi_\pi \pi_{t-1} + \phi_x x_{t-1} + \phi_q q_{t-1} + \varepsilon_{4t} \quad (45)$$

tions in inflation (Gali, 2002)²⁵. Therefore, rules where no weight is placed on the output stabilization are considered as well. Orphanides (1999) refers to the advantages of such rules, as they decrease the risks related to large and persistent measurement errors in the output gap.

²³The volatility of the natural rate of output has the same sources as the real exchange rate plus the transaction cost and its correlation with the other shocks.

²⁴Bullard and Mitra consider forward looking interest rate rules as well. Obviously, the conditions for determinacy differ with those under lagged data rules.

²⁵Gali stresses the practical difficulties in implementing such rules associated with the measurement of variables like total factor productivity.

6 Calibration and Simulation

In this section we proceed to the calibration and simulation of the model in order to evaluate alternative monetary policy rules. As already noted, alternative monetary policy rules will be compared according to the value of the welfare loss they generate²⁶. As a first step we will compare the different linear rules, the standard Taylor rule and the one where the real exchange rate enters in a linear fashion. As a next step we will proceed to the simulation of the model in order to compare linear and nonlinear interest rate rules. The model is simulated because standard techniques no longer hold since it becomes nonlinear. The procedure followed is similar to that of Aksoy et al. (2006) and Levin et al. (1999). Since we are interested in determining what the optimal interest rate rule should look like when shocks (or large variations) to the exchange rate take place, we restrict our analysis to the evaluation of alternative rules under shocks to the exchange rate. In the calibration and simulation exercise the rules considered receive the following forms²⁷

$$i_t = \phi_\pi \pi_t + \phi_x x_t + \varepsilon_{1t} \quad \textit{Taylor Rule}$$

$$i_t = \phi_\pi \pi_t + \phi_x x_t + \phi_q q_t + \varepsilon_{2t} \quad \textit{Rule 1}$$

$$i_t = \begin{cases} c + \phi'_x x_t + \phi_\pi \pi_t + \phi'_q q_t, & |q_{t-1}| \geq 1sd \\ c + \phi_x x_t + \phi_{\pi,H} \pi_{H,t} & 1sd < q_{t-1} < 1sd \end{cases} \quad \textit{Rule 2}$$

6.1 Calibration results

In this section we calibrate the model to investigate how the variables of the model respond to shocks. Firstly, we want to show the importance of including the exchange rate into an interest rate rule, in general. Therefore, the model is

²⁶As usual, optimal monetary policy is defined as one that minimizes the welfare loss as measured by (42).

²⁷The nonlinear interest rate rule was approximated by a third order Taylor approximation of a logistic transition function of the form

$$[1 + \exp(-\lambda(q_t - d))]^{-1}$$

Throughout the paper we assume that the Central Bank targets the real exchange at its PPP level. Specifying, thus, the transition function in that way, we impose the zero threshold to be the target of the Central Bank.

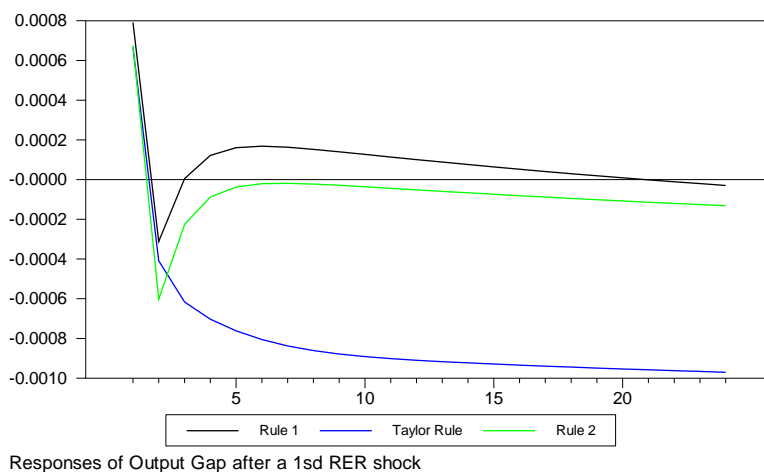
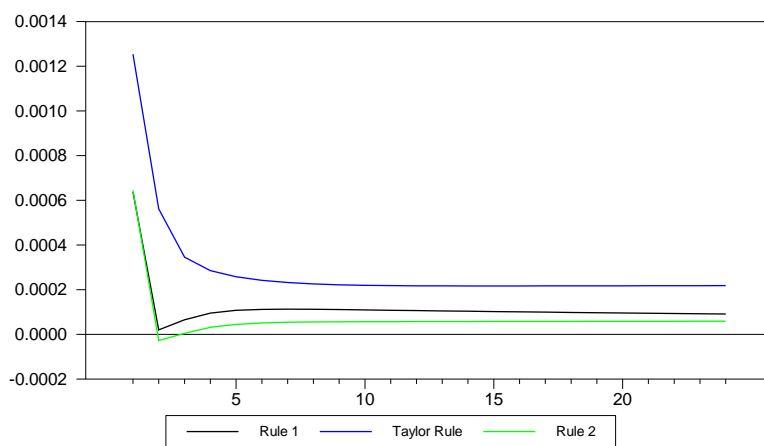
calibrated assuming that international trade takes place or that transaction costs are zero. Two kinds of shocks are considered, an exchange rate and an interest rate shock. In table 5 below we provide the values of the calibrated parameters.

Table 5: Parameter values

β	δ	ρ	θ	σ	$\zeta = \zeta^*$	$\omega = \omega^*$	χ	ϕ_π	ϕ_x	ϕ_q
0.99	0.6	1.5	1.5	1.5	0.2	0.75	1.5	1.5	0.5	0.55

Figure 3 shows the impulse response of the CPI inflation rate, the output gap and the real exchange rate under the three rules in response to a one standard deviation shock to the real exchange rate.

Figure 3: Impulse responses



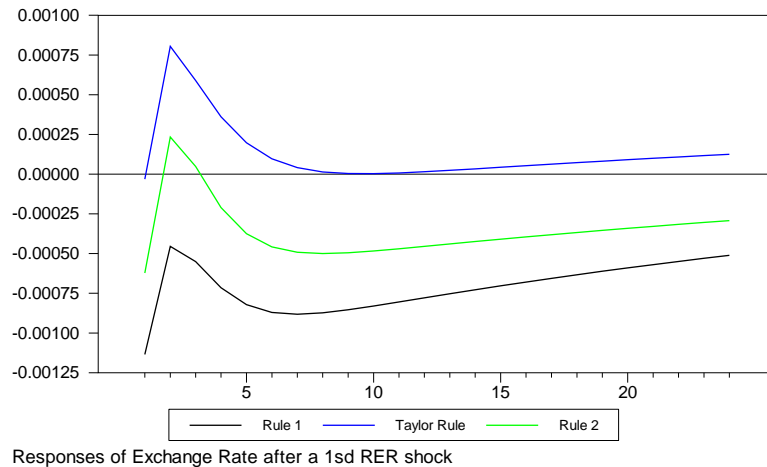
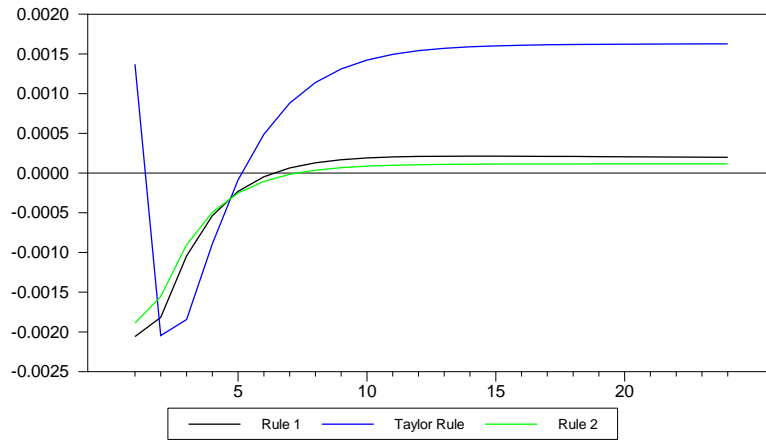


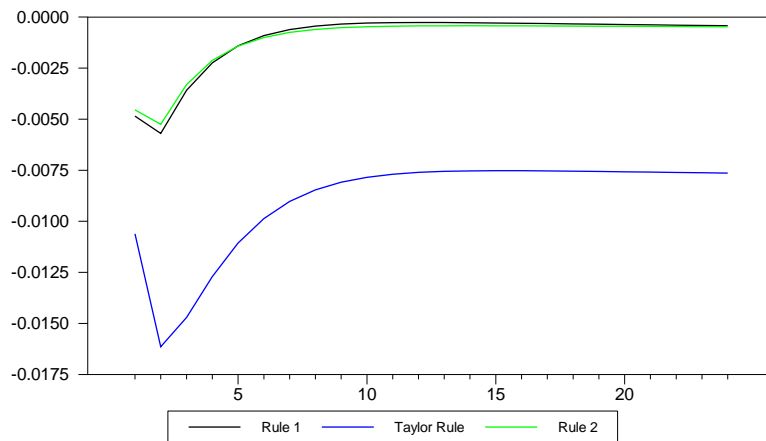
Figure 3 provides evidence in favor of Rule 2, as far as inflation stabilization is concerned. On the other hand, when the Central Bank uses a standard Taylor rule, the inflation rate and the output gap are more persistent. CPI inflation increases less under Rule 1 and Rule 2 following the real exchange rate shock. The output gap seems to be fluctuating closer to its long run level under Rule 1 and Rule 2. The area it fluctuates, is smaller than under the Taylor rule. From the impulse response analysis, it seems the the Central Bank has a better control of the inflation rate and the output gap when it includes the real exchange rate into the rule.

The model was also calibrated for a one standard deviation shock to the nominal interest rate. Rule 2 performs better again. After the shock to the interest rate, the CPI inflation experiences a fall under Rule 1 and Rule 2 compared to the standard Taylor rule, where it increases initially and then it is more highly persistent. Again, the area within which the CPI inflation fluctuates is smaller under Rule 1 and Rule 2. As far as the output gap is concerned, it experiences a smaller fall, following the shock, and it seems to revert faster. On the other hand, under the Taylor Rule, it exhibits a higher initial fall and persists more. Finally, the real exchange rate fluctuates less under Rule 2 and has lower persistence. The impulse responses following an interest rate shock are presented in Figure 4 below.

Figure 4: Impulse responses



Responses of CPI inflation after a 1sd monetary shock



Responses of Output Gap after a 1sd monetary shock

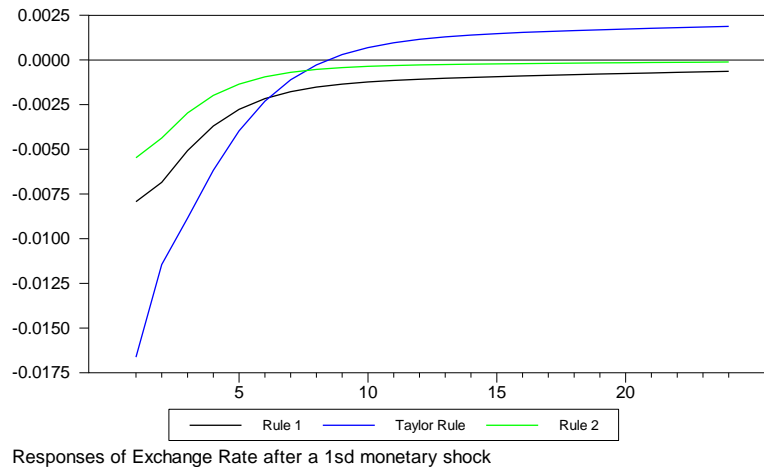


Figure 4 shows that the real exchange rate exhibits higher persistence under the Taylor rule. Additionally, looking the responses of the real exchange rate after the monetary shock, it seems that Rule 1 and Rule 2 perform equally well. When it comes to CPI inflation and the output gap Rule 1 and Rule 2 seem to stabilize the economy quite quickly, compared to the Taylor rule which introduced high persistence. Therefore, it is clear from the graphs that even after policy shocks the exchange rate is important into the rule in terms of inflation control.

6.1.1 The importance of expectations

Agents expectations about the real exchange rate and inflation rate are crucial in the overall performance of the interest rate rule. Under the assumption of symmetric information and credibility, the inclusion of the real exchange rate into the interest rate rule informs agents that the Central Bank will react with a lag (Rule 2) to exchange rate shocks²⁸. Given that the real exchange rate affects domestic inflation contemporaneously, this has an immediate impact on the path of the inflation rate.

As far as Rule 2 is concerned, agents know that when there is a shock to the real exchange rate at date t , the Central Bank will adjust its interest rate the next date. After incorporating this into their information set, they expect lower future inflation. This allows for a better control of inflation. Davig and Leeper (2006) and Leeper and Zha (2003), analyzing the benefits of regime switching interest rate rules, refer to the distance between the impulse response functions

²⁸Looking at Rule 2 again, one can see that the real exchange rate is the transition variable. Additionally, the transition variable is lagged one period. This means that whenever there is a shock to the real exchange rate at date t of degree greater than one standard deviation, agents know that the Central Bank will react to this the day after.

form the Taylor rule and Rule 2 as the expectations formation effect. Following exchange rate shocks of one or greater standard deviations, the agents expect the interest rates to be higher and hence future inflation to be lower. The persistence of exchange rate shocks is also important. The higher the persistence, the more agents expect that the economy is in regime one (exchange rate shocks of degree greater or equal to one standard deviation), and the lower they expect inflation to be in the future.

6.2 Simulation results

In this section we focus on interest rate rules where the Central Bank reacts to the current values of the targeted variables. When the Central Bank follows Rule 2 the model becomes nonlinear²⁹. In order to make the results comparable to those for nonlinear interest rate rules we evaluate the above rules by simulating the model. 2000 observations of artificial data were generated³⁰.

Welfare Losses			
	Taylor Rule	Rule 1	Rule 2
Loss	-0.000405	-0.000168	-0.000043

Notes: Unconditional losses reported.

The results in the table show that the nonlinear rule performs better than the linear rules. Additionally, Rule 1 seems to be superior to the Taylor rule, leading to lower welfare losses. The former performs also better for negative shocks. Consequently, the results from contemporaneous rules provide evidence in favor of a nonlinear rule for different shock sizes, and specifically of the importance of including the exchange rate into the interest rate rule using a specification that captures the frictions in international goods markets. The Central bank must intervene and adjust its interest rate in a nonlinear fashion after shocks to the exchange rate.

7 The model in the presence of thresholds (in progress)

In this section we consider the model when $x > 0$. As already shown, in this case the model allows for corner solutions. The latter impose thresholds in the behavior of almost all the variables of the model like domestic inflation, CPI inflation, the real exchange rate and output. Given the fact that the parameters of the model change depending on where the real exchange rate stands, the model receives a nonlinear state space form and the standard solution methods for linear rational expectations models no longer apply.

²⁹The solution technique followed is similar to that in Barthelemy, Clerc and Marx (2008).

³⁰1000 simulations were also carried out. The conclusions, however, do not change significantly.

8 Conclusions

We used a two country sticky price model with transaction costs in international goods markets to derive the optimal monetary policy based on a welfare criterion. Based on calibrating the model, it was shown that the real exchange rate is important into an interest rate rule as this leads to significant welfare gains. At first, focusing on a linear setting, calibration showed that when the exchange rate is in the interest rate rule, the former reverts faster to its initial level after either an interest rate shock or an exchange rate shock. Similarly, when lagged data rules are used, the Taylor rule is inferior to Rule 1 as it leads to higher variation in inflation, the output gap and the real exchange rate.

Analyzing data from major economies we found evidence in favor of a nonlinear behavior of the interest rate, once the real exchange rate is introduced. Therefore, we examined the performance of an alternative interest rate rule which is nonlinear with the real exchange rate being the threshold variable. Conducting 500 simulations of 200 artificial observations, we found that for small negative deviations of the real exchange rate from the PPP level, and for any size of positive deviations, the nonlinear interest rate rule performs better than the linear rules leading to lower welfare losses. The same simulation exercise was also carried out using lagged data interest rate rules. The results were similar. The nonlinear interest rate rule performed better for all positive deviations and small (-1sd) negative deviations from the PPP level. Therefore, after the correct determination of the thresholds of the real exchange rate deviations from its target, the nonlinear interest rate rule is superior, leading to lower welfare losses.

Consequently, the paper suggests that it is optimal for the Central Bank to include the real exchange rate into an interest rate rule both in terms of welfare and in terms of controlling domestic CPI. However, the way the exchange rate is introduced into the interest rate rule is crucial, in order to derive secure policy implications.

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