The Predictability of Returns with Regime Shifts in Consumption and Dividend Growth*

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this version: April 20, 2010

Abstract

The predictability of the market return and dividend growth is addressed in an equilibrium model with two regimes. In linear predictive regressions over $1930-2006$, the market return is more predictable by the price-dividend ratio if the probability of being in the first regime exceeds 50%; and dividend growth is more predictable by the price-dividend ratio if the probability of being in the second regime exceeds 50%. The model-implied state variables perform significantly better at predicting the equity, size, and value premia, and the variance of market return than linear regressions with the market price-dividend ratio and interest rate as predictive variables.

Keywords: Return Predictability, Consumption Growth Predictability, Dividend Growth Predictability, Regime Shifts, Cross-Section of Returns.

JEL classification: G12, E44

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*We thank John Cochrane, Rick Green, Lars Hansen, John Heaton, Burton Hollifield, Christian Julliard, Oliver Linton, Bryan Routledge, Duane Seppi, Pietro Veronesi and seminar participants at Carnegie Mellon University and University of Chicago for helpful comments. We remain responsible for errors and omissions. Ghosh acknowledges financial support from Carnegie Mellon University. Constantinides acknowledges financial support from the Center for Research in Security Prices of the University of Chicago Booth School of Business.

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1 Introduction

Stock return predictability has for long been the subject of both theoretical and empirical research in financial economics. Attempts to predict the aggregate stock market return have a long history in finance going back to as early as 1920 when Dow (1920) explored the role of dividend ratios in predicting the market return. Over the last three decades, the academic literature has explored numerous financial variables as potential predictors of the market return and equity premium. The price-dividend ratio has received extensive scrutiny as a predictive variable because, as a mathematical identity, all variation in the price-dividend ratio must be accounted for by changing expectations on future returns and/or future dividend growth (Campbell and Shiller (1988)). Welch and Goyal (2008) review this literature and undertake a comprehensive study of the in-sample and out-of-sample performance of these variables in predicting the equity premium. They conclude that "by and large, these models have predicted poorly both in-sample (IS) and out-of-sample (OOS) for 30 years now; these models seem unstable, as diagnosed by their out-of-sample predictions and other statistics; and these models would not have helped an investor with access only to available information to profitably time the market." These conclusions are controversial.

Campbell and Thompson (2008) show that, when restrictions are imposed on the theoretically expected sign of the regression coefficient and the fitted value of the equity premium, the out-of-sample R^2 improves but is still small. However, they argue that even a small value of $R^2$is economically meaningful for mean-variance investors. Cochrane (2008) provides a defense of return predictability by arguing that return and dividend growth predictability are intimately related and that the absence of dividend growth predictability gives stronger evidence against the null that returns are not forecastable than does the presence of return forecastability in the historical data.

In this paper, we shed light on this debate by arguing that there exist (at least) two economic regimes. The market return is more predictable by the price-dividend ratio in the first regime than in the second regime; and the market dividend growth is more predictable by the price-dividend ratio in the second regime than in the first one. We identify the regimes in the context of the dynamic equilibrium asset pricing model with two regimes, proposed in Constantinides and Ghosh (2009*b*). The probability that the economy is in the first regime is obtained as a non-linear function of the market price-dividend ratio and interest rate, with parameters estimated from the Euler equations of the market return, the interest rate, and the cross-section of size and book-to-market equity-sorted portfolio returns plus unconditional moments of the consumption and dividend processes. Furthermore, this non-linearity cannot be captured by a quadratic function of the log price-dividend ratio and interest rate.

Over the period $1930 - 2006$, in all years when the probability of being in the first regime exceeds 50%, in-sample linear predictive regressions of the realized one-year market real return and realized real dividend growth on the lagged log price-dividend ratio have (adjusted) $\overline{R}^2$ 9.1% for the market return and negative $\overline{R}^2$ for dividend growth. By contrast, in the second regime, the $\overline{R}^2$ for the market return is negative and for dividend growth is 20.2%. We also find that the equity premium and the returns on portfolios of "Small", "Large", "Growth", and "Value" stocks are predictable by the market-wide price-dividend ratio in the first regime (when dividend growth is not predictable) with statistically significant coefficients and $\overline{R}^2$ varying from 4.1% for the "Small" portfolio to 8.5% for the "Large" portfolio. The price-dividend ratio performs poorly at predicting returns in the second regime with the $\overline{R}^2$ varying from -4.6% for the "Value" portfolio to -1.0% for the equity premium.

In the model, a state variable x_t that drives the conditional means of the aggregate consumption and dividend growth rates reverts to its unconditional mean with a process that differs across two regimes. Based on his information set, the consumer observes x_t and also calculates the posterior probability, p_t, that the economy is in the first regime. The conditional means of the aggregate consumption and dividend growth rates are affine functions of the two state variables (x_t, p_t). The market-wide log price-dividend ratio and interest rate are approximately affine functions of (x_t, p_t) and their product, thereby rendering the (potentially latent) state variables and the expected return of each asset class known nonlinear functions of the price-dividend ratio and interest rate. The model parameters are estimated from the Euler equations of the market return, the interest rate, and the cross-section of size and book-to-market equity-sorted portfolio returns plus unconditional moments of the consumption and dividend processes.

We show that the model has superior forecasting performance for the equity premium and its variance relative to a linear forecasting model with the market-wide price-dividend ratio and risk free rate as predictive variables.

While most of the predictability literature focuses on predicting the aggregate US stock market return and equity premium, the literature on the time series forecastability of the cross-section of size and book-to-market-equity sorted portfolio returns is scant. Forecastability of the cross-section of returns is important for at least two reasons. First, the historical size premium (9.7%) and value premium (7.4%) are of the same order of magnitude as the equity premium (8.3%), based on arithmetic annual returns. Therefore, the predictability of these premia is important in active portfolio management. Second, it is also important in providing an alternative channel to examine the empirical plausibility of a given set of state variables that purport to explain the cross-section of returns. We show that the model has superior forecasting performance for the size and value premia relative to the linear forecasting model.

Our paper is related to equilibrium models by Bansal and Shaliastovich (2009), Bansal and Yaron (2004), Drechsler (2009), Hansen, Heaton and Li (2008), Lettau and Ludvigson (2001), and Menzly, Santos, and Veronesi (2004) with implications on forecasting the market return and dividend growth.

Our paper is also related to Brandt and Kang (2004), Koijen and Van Binsbergen

(2009), Pastor and Stambaugh (2009), and Rytchkov (2007), who focus on return predictability using filtering techniques. While these are reduced form models, we rely on an equilibrium model and avoid using filtering techniques by arguing that, under the model assumptions, the (potentially latent) state variables and the expected return of each asset class are known nonlinear functions of observable financial variables like the price-dividend ratio and interest rate.

Finally, our work is related to Lettau and Van Nieuwerburgh (2008), Pastor and Stambaugh (2001), and Paye and Timmermann (2006) who find evidence of structural breaks and argue that allowing for these breaks has important implications for return predictability.

The paper is organized as follows. In Section 2, we define the regime shifts model. We express the price-dividend ratio, risk free rate, and expected equity premium as functions of the state variables (x_t, p_t). The annual data over the period $1930-2006$ are discussed in Section 3. In Section 4, we estimate the model parameters by GMM from the set of the Euler equations for the market return, the interest rate, and portfolios of "Small", "Large", "Growth" and "Value" stocks, and the unconditional moments of the consumption and dividend growth. Using the point estimates of the model parameters, we invert the expressions for the price-dividend ratio and interest rate as functions of the state variables and express the state variables as functions of the price-dividend ratio and risk free rate.

Armed with the time series of the state variables, we address the questions raised in this paper. Section 5 presents empirical evidence that the predictability of returns and dividend growth differ significantly in the two-regimes. In Section 6, we present evidence on the in-sample and out-of-sample predictability of the equity, size, and value premia. In Section 7, we present evidence on the predictability of the variance of the market return. Section 8 concludes.

2 The Model and Implications for Predictability

We consider the regime shift model proposed in Constantinides and Ghosh (2009*b*). Here we provide a brief discussion of the model and its implications for the predictability of the equity premium, size premium, value premium, consumption growth, and dividend growth (see Constantinides and Ghosh (2009*b*) for further details).

2.1 Model

The model stipulates that the state variable, x_t, that simultaneously drives the conditional means of the aggregate consumption and dividend growth rates reverts to its unconditional mean with a process that differs across two regimes:

$$x_{t+1} = \rho_{s_{t+1}} x_t + \varphi_e \sigma_{s_{t+1}} e_{t+1}, \tag{1}$$
$$\Delta c_{t+1} = \mu + x_t + \sigma_{s_{t+1}} \eta_{t+1}, \tag{2}$$
$$\Delta d_{t+1} = \mu_d + \phi x_t + \varphi_d \sigma_{s_{t+1}} u_{t+1}, \tag{3}$$

where c_{t+1} is the logarithm of the aggregate consumption level; d_{t+1} is the logarithm of the aggregate stock market dividends; and $s_t = 0, 1$ is a second state variable that denotes the economic regime. The persistence parameter, ρ_{s_t}, of the state variable x_t and the level of its volatility, σ_{s_t}, are generally different in the two regimes. The shocks e_{t+1}, η_{t+1}, and u_{t+1} are assumed to be distributed with mean 0 and variance 1 and independent of the past.

Given his information set, $\mathcal{F}(t)$, the representative consumer observes x_t and calculates his subjective probability, p_t, at time t of being in regime $s_t = 0$:

$$p_t \equiv Prob\left(s_t = 0 | \mathcal{F}(t)\right) \tag{4}$$

We do not take a stand on the content of the information set, $\mathcal{F}(t)$. In one extreme case, it may be limited to the history of consumption, dividends, and past realizations of x. In the other extreme case, it may include all publicly available information. Furthermore, we do not take a stand on the optimality of the filter that the consumer applies to form his belief, p_t. The econometrician does not directly observe the state variables, p_t and x_t, and, hence, they are latent.

We assume that s_t follows a Markov process with the following transition probability matrix:

$$\Pi = \begin{pmatrix} \pi_0 & 1 - \pi_1 \\ 1 - \pi_0 & \pi_1 \end{pmatrix}, \tag{5}$$

where $0 < \pi_i < 1$ for $i = 0, 1$. Thus, the consumer's probability of being in regime $s_{t+1} = 0$ at time $t + 1$, given his information set, $\mathcal{F}(t)$, is

$$Prob\left(s_{t+1} = 0 | \mathcal{F}(t)\right) = \pi_0 p_t + (1 - \pi_1)(1 - p_t) \equiv f(p_t) \tag{6}$$

Note that $0 < f(p_t) < 1$ for all p_t, $0 \leq p_t \leq 1$.

Once the consumer updates his information set at time $t + 1$, his probability of being in regime $s_{t+1} = 0$ at time $t + 1$ is $p_{t+1} \equiv Prob\left(s_{t+1} = 0 | \mathcal{F}(t + 1)\right)$. We assume that the consumer's expectations are unbiased in that

$$p_{t+1} = f(p_t) + \varepsilon_{t+1}, \tag{7}$$

where $E\left[\varepsilon_{t+1} | \mathcal{F}(t)\right] = 0$.

We make the following assumptions regarding the shocks η_{t+1}, u_{t+1}, e_{t+1}, and ε_{t+1}:

$$E\left[y_{t+1} | \mathcal{F}(t), s_{t+1} = 0\right] = E\left[y_{t+1} | s_{t+1} = 0\right] \equiv y(0), \text{ a constant} \tag{8}$$

where $y = \eta$, u, e, and ε;

$$E\left[y_{t+1}w_{t+1}|F(t), s_{t+1} = 0\right] = E\left[y_{t+1}w_{t+1}|F(t)\right] \equiv \sigma_{y,w}, \text{ a constant} \tag{9}$$

where y, $w = \eta$, u, e, and ε, $y \neq w$; and

$$E\left[y_{t+1}^2|F(t), s_{t+1} = 0\right] = E\left[y_{t+1}^2\right] = 1 \tag{10}$$

where $y = \eta$, u, and e.

Equation (8) recognizes that the means of the residuals η_{t+1}, u_{t+1}, e_{t+1}, and ε_{t+1}, conditional on the regime at time $t+1$, may differ from their unconditional value of zero. Equation (9) recognizes that the residuals η_{t+1}, u_{t+1}, e_{t+1}, and ε_{t+1} may be correlated. Finally, equation (10) limits the number of parameters to be estimated by setting the second moments of the residuals u_{t+1}, e_{t+1}, and ε_{t+1}, conditional on the regime at time $t+1$, equal to their unconditional value of one.

We assume that the consumer has the version of Kreps and Porteus (1978) preferences adopted by Epstein and Zin (1989) and Weil (1989). These preferences allow for a separation between the coefficient of risk aversion and the elasticity of intertemporal substitution. The utility function is defined recursively as

$$V_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\left(E\left[V_{t+1}^{1-\gamma}|F(t)\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}} \tag{11}$$

where δ denotes the subjective discount factor, $\gamma > 0$ is the coefficient of risk aversion, $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$, and $\psi > 0$ is the elasticity of intertemporal substitution. Note that the sign of θ depends on the relative magnitudes of γ and ψ. The standard time-separable power utility is obtained as a special case when $\theta = 1$, i.e. $\gamma = \frac{1}{\psi}$.

For this specification of preferences, Epstein and Zin (1989) and Weil (1989) show that, for any asset j, the first-order conditions of the consumer's utility maximization yield the following Euler equations,

$$E\left[\exp(m_{t+1} + r_{j,t+1})|F(t)\right] = 1, \tag{12}$$

$$m_{t+1} = \theta \log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)r_{c,t+1}, \tag{13}$$

where m_{t+1} is the natural logarithm of the intertemporal marginal rate of substitution, $r_{j,t+1}$ is the continuously compounded return on asset j, and $r_{c,t+1}$ is the unobservable continuously compounded return on an asset that delivers aggregate consumption as its dividend each period.

We rely on log-linear approximations for the log return on the consumption claim, $r_{c,t+1}$, and that on the market portfolio (the return on the aggregate dividend claim),

$r_{m,t+1}$, as in Campbell and Shiller (1988),

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1}, \tag{14}$$
$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + \Delta d_{t+1}, \tag{15}$$

where z_t is the log price-consumption ratio and $z_{m,t}$ the log price-dividend ratio. In equation (14), $\kappa_1 = \frac{e^{\overline{z}}}{1+e^{\overline{z}}}$ and $\kappa_0 = log(1 + e^{\overline{z}}) - \kappa_1 \overline{z}$ are log-linearization constants, where $\overline{z}$ denotes the long run mean of the log price-consumption ratio. Similarly, in equation (15), $\kappa_{1,m} = \frac{e^{\overline{z}_m}}{1+e^{\overline{z}_m}}$ and $\kappa_{0,m} = log(1 + e^{\overline{z}_m}) - \kappa_1 \overline{z}_m$, where $\overline{z}_m$ denotes the long run mean of the log price-dividend ratio.

Note that the current model specification involves two state variables, x_t and p_t. We conjecture and verify the following approximate expressions for the log price-consumption ratio and log price-dividend ratio at date t, respectively, (see Appendices $A.1$ and $A.2$ in Constantinides and Ghosh (2009b) for derivations, expressions, and intuition for the parameters $A_0(0)$, $A_1(0)$, $A_0(1)$, $A_1(1)$, $A_{0,m}(0)$, $A_{1,m}(0)$, $A_{0,m}(1)$, and $A_{1,m}(1)$):

$$z_t = p_t \left[A_0(0) + A_1(0)x_t\right] + (1 - p_t)\left[A_0(1) + A_1(1)x_t\right], \tag{16}$$
$$z_{m,t} = p_t \left[A_{0,m}(0) + A_{1,m}(0)x_t\right] + (1 - p_t)\left[A_{0,m}(1) + A_{1,m}(1)x_t\right]. \tag{17}$$

The continuously compounded risk free rate, $r_{f,t}$, between periods t and $t + 1$, is a function of the two latent state variables and their product (see Appendix $A.3$ in Constantinides and Ghosh (2009b) for derivation and expressions for the parameters $A_{0,f}$, $A_{1,f}$, $A_{2,f}$, $A_{3,f}$),

$$r_{f,t} = A_{0,f} + A_{1,f} x_t + A_{2,f} p_t + A_{3,f} p_t x_t. \tag{18}$$

2.2 Predictive Implications for Returns and Growth Rates

Equations (15), (17), and (3) imply that the expected market return is given by:

$$E\left[r_{m,t+1}|\mathcal{F}(t)\right] = B_0 + B_1 x_t + B_2 p_t + B_3 p_t x_t. \tag{19}$$

Hence, from Equations (19) and (18), the expected equity premium is given by:

$$E\left[(r_{m,t+1} - r_{f,t})\,|\mathcal{F}(t)\right] = E_0 + E_1 x_t + E_2 p_t + E_3 p_t x_t, \tag{20}$$
$$E_i = B_i - A_{i,f}, \quad i = 0, 1, ..., 3.$$

The model generates time-varying expected returns and equity premium. The coefficients $\{B_i, E_i\}_{i=0}^{3}$ are known functions of the underlying time-series and preference parameters. Under the assumption that the dividend growth processes of the "Small", "Large", "Growth" and "Value" portfolios are similar to that for the market, the expected returns on these portfolios can also be shown to be affine functions of the state variables, x and p, and their product.

The regime shifts model also has implications for the predictability of the aggregate consumption and dividend growth rates. The time series specification of the model implies that the expected consumption growth rate is given by

$$\begin{aligned} E\left(\Delta c_{t+1}|\mathcal{F}(t)\right) &= \mu + x_t + E\left[\sigma_{s_{t+1}}\eta_{t+1}|\mathcal{F}(t)\right] \\ &= \mu + x_t + (\sigma_0 - \sigma_1)\,\eta(0) f(p_t), \end{aligned} \tag{21}$$

and the expected dividend growth rate is given by

$$\begin{aligned} E\left(\Delta d_{t+1}|\mathcal{F}(t)\right) &= \mu_d + \phi x_t + \varphi_d E\left[\sigma_{s_{t+1}}u_{t+1}|\mathcal{F}(t)\right] \\ &= \mu_d + \phi x_t + \varphi_d\,(\sigma_0 - \sigma_1)\,u(0) f(p_t), \end{aligned} \tag{22}$$

both linear functions of the state variables, x_t and p_t.

3 Data

We consider the predictive performance of the model at the annual frequency, using annual data over the entire available sample period 1930-2006. The asset menu consists of the equity premium, and portfolios of "Value", "Growth", "Small" capitalization, and "Large" capitalization stocks. Our market proxy is the Centre for Research in Security Prices (CRSP) value-weighted index of all stocks on the NYSE, AMEX, and NASDAQ. The proxy for the annual real risk free rate is the inflation-adjusted rolled-over return of one-month Treasury Bills from Ibbotson Associates. The equity premium is the difference in average returns on the market and the risk free rate. The construction of the size and book-to-market portfolios is as in Fama and French (1993). In particular, for the size sort, all NYSE, AMEX, and NASDAQ stocks are allocated across 10 portfolios according to their market capitalization at the end of June of each year. Value-weighted returns on these portfolios are then computed over the following twelve months. NYSE breakpoints are used in the sort. "Small" and "Large" denote the bottom and top market capitalization deciles, respectively. The size premium is the difference in average returns between the "Small" and "Large" portfolios. Similarly, value-weighted returns are computed for portfolios formed on the basis of BE/ME at the end of June of each year using NYSE breakpoints. The BE used in June of year t is the book equity for the last fiscal year end in $t-1$ and ME is the price times shares outstanding at the end of December of $t-1$. "Growth" and "Value" denote the bottom and top BE/ME deciles, respectively. The value premium is the difference in average returns between the "Value" and "Growth" portfolios. Annual returns for the "Small", "Large", "Growth", and "Value" portfolios are computed by compounding monthly returns within each year. The premia are computed as the difference in the average annual returns.

Also used in the empirical analysis are the price-dividend ratios and dividend growth rates of the above mentioned portfolios. Data on these are obtained from the CRSP files. The quarterly dividend payments within a year are added to obtain the annual aggregate dividend, i.e. we do not reinvest dividends either in T-Bills or in the aggregate stock market. All nominal quantities are converted to real, using an ARMA(1,1) forecast of the annual inflation.

4 Parameter Estimation

The parameters are estimated from the Euler equations of the market return, the interest rate, and the "Small", "Large", "Growth", and "Value" portfolio returns plus unconditional moments of the consumption and dividend processes, using the GMM approach. The laggged log price-dividend ratio of the market and the lagged risk free rate are used as instruments. The Euler equations for the six assets along with the chosen instruments give 18 moment restrictions. To this set of pricing restrictions, we add 5 moment restrictions implied by the time-series specification of the model. These moments correspond to the unconditional means and variances of aggregate consumption and dividend growth rates and the covariance between consumption and dividend growth rates. Thus, we have a total of 23 moment conditions. The total number of parameters to be estimated is 21: the 3 preference parameters (γ, ψ, δ); the 16 time-series parameters (μ, μ_d, ϕ, φ_d, ρ_0, ρ_1, σ_0, σ_1,π_0, π_1, φ_e, $e(0)$, $\eta(0)$, $u(0)$, $\varepsilon(0)$, $\sigma_{\varepsilon,e}$); and 2 combinations of all the parameters that appear in the Euler equations.

Note that the pricing kernel is a function of the aggregate consumption growth rate and the two latent (from the point of view of the econometrician) state variables, x_t and p_t. Our estimation methodology involves inversion of two non-linear equations (17) and (18) to express the latent state variables, x_t and p_t, as functions of the observables, $z_{m,t}$ and $r_{f,t}$. This procedure yields quadratic equations for x_t and p_t, with coefficients that depend on $z_{m,t}$ and $r_{f,t}$, and the time-series and preference parameters. Solving the equations gives two pairs of solutions for x_t and p_t. We report results obtained using the bigger root of the quadratic equations as this choice minimizes the value of the GMM criterion function.

The estimation results are reported in Table 1. The first row reports the point estimates of the parameters along with the associated standard errors in parentheses. The persistence parameter of the state variable, x, in the two regimes takes values 0.20 and 0.98, respectively. This suggests that in the first regime, consumption and dividend dynamics are driven by a high frequency component that has a half-life less than 1 year. In the second regime, x has a half-life of just over 34 years. The volatility of x takes values 3.5% and 0.5%, respectively, in the two regimes. These findings suggest the presence of two regimes, one in which consumption and dividend growth rates are more persistent and less volatile and the other during which the growth rates are much less persistent and have higher volatility. The point estimates of the transition probabilities, π_0 and π_1, suggest that the duration of the regimes are 10 and 5 years, respectively.

The point estimates of the subjective discount factor (0.976) and the risk aversion coefficient (12) are economically sensible. The point estimate of the IES is 0.9 and is smaller than one. However, the standard error is 0.25 and we cannot reject values of the IES slightly greater than one.

The table also reports the model-implied and the historical values of the equity premium, risk free rate, size premium, and value premium. The historically observed average level of the risk free rate is 0.8% with standard error of 0.5%. The model generates an average risk free rate of 0.2%. The model generates an equity premium of 11.9%, which is within the one standard error interval of the 8.3% value in the data. The model also generates a size premium of 7.3%, that is within the one standard error interval of the 9.7% value in the data. The model performs less well at explaining the value premium. It also generate higher returns for Value stocks relative to Growth stocks, but the magnitude of the difference is much smaller than that observed in the data. In particular, the value premium is 7.4% in the data while the model-implied value is only 1.5%. However, note that the model-implied value of 1.5% is within the 95% confidence interval of the historical value of 7.4%.

Note that the GMM estimation procedure examines the ability of the model to simultaneously explain the pricing restrictions given by the Euler equations and the restrictions on the unconditional moments of aggregate consumption and dividend growth rates implied by the time-series specification of the model. Therefore, the estimates of the time-series and preference parameters in Table 1 are also consistent with the time-series specification of the model. The unconditional means of consumption and dividend growth rates are 1.6% and 1.5%, respectively, in the data. The model-implied values of these moments are 2.0% and 3.4%, respectively. The unconditional variances of consumption and dividend growth rates are 0.06% and 1.2%, respectively, in the data while the model implies values of 0.10% and 1.0%, respectively. Finally, consumption and dividend growth have a correlation of 0.50 in the historical sample while the corresponding value in the model is 0.45.

Note that the market-wide log price-dividend ratio and risk free rate are approximately affine functions of x_t , p_t, and, $x_t p_t$ (equations (17) and (18), respectively). The coefficients $\{A_{i,m}(j)\}_{i,j=0}^{1}$ and $\{A_{i,f}\}_{i=0}^{3}$ are known functions of the underlying time-series and preference parameters. Therefore, using the point estimates of the parameters and the time series of the price-dividend ratio and risk free rate, we extract the time series of the state variables x_t and p_t and use them in the forecasting regressions for returns and growth rates. This gives the following expressions for the state variables x_t and p_t in terms of the market-wide log price-dividend ratio and risk free rate:

$$p_t = \frac{-b_t \pm \sqrt{b_t^2 - 4ac_t}}{2a}, \tag{23}$$

where

$$\begin{aligned} a &= 3.24 \times 10^{-16} \\ b_t &= -107.0r_{f,t} - \left(3.55 \times 10^{-15}\right) z_{m,t} + 5.40 \\ c_t &= 7.21r_{f,t} - 1.11z_{m,t} + 1.04, \end{aligned}$$

and

$$x_t = \frac{r_{f,t} - 0.037}{1.11 + \left(3.55 \times 10^{-15}\right) p_t}. \tag{24}$$

We choose the bigger root of p_t (and the corresponding value of x_t) as this choice minimizes the GMM criterion function. In years when the bigger root of p_t exceeds one, we set $p_t = 0.99$ and in years when the bigger root of p_t is negative, we set $p_t = 0.01$. Figure 1 plots the state variable p_t as a function of the price-dividend ratio and risk free rate.

Equations (23) and (24) imply that the expected equity premium in equation (20) and the expected return of each asset class are highly nonlinear functions of the price-dividend ratio and risk free rate. Moreover, the nonlinearity cannot be captured by including as additional predictor variables (in addition to the price-dividend ratio and risk free rate), the square of the price-dividend ratio, the square of the risk free rate, or interaction terms of the price-dividend ratio and risk free rate in linear forecasting regressions. Figure 2 plots the expected equity premium as a function of the price-dividend ratio and risk free rate.

5 Economic Interpretation of the Two Regimes

The point estimates of the model parameters in Table 1 imply that the first regime has expected duration 10 years and the consumption and dividend growth dynamics are driven by a high-frequency state variable that has half-life shorter than one year; while the second regime has expected duration 5 years and the consumption and dividend growth dynamics are driven by a low-frequency state variable that has half-life just over 34 years. These properties suggest that the regimes capture features of the economy other than the business cycle. In Figure 3 we plot the time-series of the probability that the economy is in the first regime over the period $1930 - 2006$. The shaded areas mark years with at least one quarter in recession, as defined by the NBER. The vertical dashed lines mark major stock market crashes, as defined in Mishkin and White (2002). The figure illustrates a tenuous pattern of a drop in the probability of being in the first regime after stock market crashes.

In Table 2, we present the mean, variance, and annual autocorrelation of the dividend, consumption, and GDP growth, the risk free rate, the market-wide price-dividend ratio, the market return, and the equity, size, and value premia.[1] In Panel A, we present these summary statistics for the 57 years over the period $1931-2006$ in which the posterior probability that the economy is in the first regime equals or exceeds 50%. In Panel B, we present these summary statistics for the 19 years over the period $1931-2006$ in which the posterior probability that the economy is in the first regime is below 50%. Given the small size of these subsamples, the standard errors are large and differences in the point estimates across the two regimes are often statistically insignificant. However, the differences across several financial variables make a compelling case that the two regimes are different. The average risk free rate is 2.8% (s.e. 0.6%) in the first regime while it is negative 5.4% (s.e. 1.1%) in the second regime. The average market return is 9.4% (s.e. 2.2%) in the first regime and -0.2% (s.e. 4.6%) in the second one. The variance is higher in the second regime than the first one. The market exhibits reversal in the first regime and momentum in the second one. The mean of the market-wide price-dividend ratio is similar across regimes but its variance is higher in the first regime. The equity premium is remarkably similar in the two regimes. Most of the size premium occurs in the first regime while the value premium is similar in the two regimes. Finally, the mean, variance and autocorrelation of the consumption and GDP growth rates are similar across regimes, reinforcing the implication from Figure 3 that the regimes capture aspects of the economy other than the business cycle.

In Table 3, we present the results of linear regressions of the dividend growth rate and returns with the lagged log price-dividend ratio as predictive variable in the two regimes. The first regime captures periods of dividend growth unpredictability and return predictability; the second regime captures periods of dividend growth predictability and return unpredictability. In Panel A, the aggregate dividend growth rate is not predictable by the price-dividend ratio, having a statistically insignificant coefficient and negative $\overline{R}^2$.[2] However, returns are strongly predictable by the price-dividend ratio. The equity premium and market return have statistically significant slope coefficients and $\overline{R}^2$ 5.0% and 9.1%, respectively. The price-dividend ratio also has superior predictive ability for the cross-section of size and book-to-market-equity

[1] Let t_j, t_k, ... denote consecutive (but not necessarily adjacent) years in which the posterior probability that the economy is in the first regime equals or exceeds 50%. The first order autocorrelation of dividend growth in Panel A is calculated as the correlation of Δd_{t_j} with $\Delta d_{t_{j+1}}$ and not as the correlation of with Δd_{t_j} with Δd_{t_k}. The other autocorrelations reported in the table are calculated accordingly.

[2] Throughout the paper, we use $\overline{R}^2$ to denote the adjusted R^2.

sorted portfolio returns with $\overline{R}^2$ 4.1%, 8.5%, 6.6%, and 5.6%, for the "Small", "Large", "Growth", and "Value" portfolios, respectively.

The second regime captures periods of dividend growth predictability and return unpredictability. The price-dividend ratio strongly forecasts the dividend growth rate. The slope coefficient in the predictive regression is significantly positive and the $\overline{R}^2$ rises from -1.4% in Panel A to 20.2% in Panel B. The price-dividend ratio performs poorly in predicting returns. The regressions have statistically insignificant slope coefficients for the equity premium, market return, and the cross-section of returns; the $\overline{R}^2$ is negative for all returns.

Taken as a whole, the results in Tables 2 and 3 suggest that the economy exhibits different characteristics across regimes. The differences in predictability across regimes shed light on why the empirical evidence on predictability which does not explicitly account for regime shifts is not robust in subperiods and its interpretation is controversial; and why recognition of structural breaks has important implications for return predictability (Lettau and Van Nieuwerburgh (2008), Pastor and Stambaugh (2001), and Paye and Timmermann (2006)).

6 Forecasting the Equity, Size, and Value Premia

We examine the ability of the regime shifts model to forecast the equity, size, and value premia with regressions on the model state variables, x and p, and their product. We compare the results with corresponding linear regressions on the market-wide price-dividend ratio and risk free rate. In Section 6.1, we estimate the model parameters over the period $1930 - 2006$, extract the time series of the state variables, and perform in-sample forecasting regressions over the period $1930 - 2006$. In Section 6.2, we estimate the model parameters over the subperiod $1930 - 1975$, extract the time series of the state variables, and perform in-sample forecasting regressions over the non-overlapping subperiod $1976 - 2006$. In Section 6.3, we estimate the model parameters over the subperiod $1976 - 2006$, extract the time series of the state variables, and perform out-of-sample predictive regressions over the subperiod $1976 - 2006$. In all cases, the model-implied regressions outperform the regressions based on the price-dividend ratio and risk free rate.

6.1 In-Sample Forecasting: 1930-2006

The expected equity premium implied by the model is an affine function of the two state variables and their product (equation (20)). We estimate the model parameters over the period $1930-2006$ and extract the time series of the state variables. We perform an in-sample forecasting regression of the realized equity premium on the state variables and their product. The regression coefficients are marginally significant and the $\overline{R}^2$

is 6.6% (Table 4, Panel A). The two state variables, x and p, are highly non-linear functions of the aggregate log price-dividend ratio and risk free rate (see equations (17) and (18)). Therefore, the expected equity premium is a highly nonlinear function of the price-dividend ratio and risk free rate. We investigate whether this nonlinearity is important by performing linear forecasting regressions of the realized equity premium on the aggregate log price-dividend ratio (Row 2) and the log price-dividend ratio and risk free rate (Row 3). The regression coefficients are marginally significant and the $\overline{R}^2$ is 3.7% and 3.8%, respectively. This indicates that linear forecasting regressions do not capture the highly nonlinear dependence of the expected equity premium on the log price-dividend ratio and risk free rate.

We further test the implications of the model for the expected equity premium by substituting in the model-implied expression for the expected equity premium

$$\mu_t \equiv E\left[(r_{m,t+1} - r_{f,t})\,|\mathcal{F}(t)\right] = E_0 + E_1 x_t + E_2 p_t + E_3 p_t x_t, \tag{25}$$

the coefficients $E_0 = 0.0$, $E_1 = -6.0$, $E_2 = 0.0$, and $E_3 = 23.4$ computed from the point estimates of the model parameters in Table 1. We obtain the time series of μ_t from the time series of the state variables x_t and p_t and regress the realized equity premium on μ_t over the full sample:

$$r_{m,t+1} - r_{f,t} = \beta_0 + \beta_1 \mu_t + \epsilon_{p,t+1}. \tag{26}$$

The slope coefficient is statistically significant and the $\overline{R}^2$ is 4.8% and still higher than those obtained from linear forecasting regressions on the log price-dividend ratio and risk free rate.

The superior predictive performance of the model is also revealed in Figure 4, Panel A that plots the realized equity premium (black solid line) along with its predicted value from the forecasting regression implied by the regime shift model (green dotted line) and a linear forecasting regression using the log market-wide price-dividend ratio as a predictor variable (red dashed line). Note that the time series of the equity premium predicted by the model lines up more closely with the actual realized time series compared to the time series predicted by the price-dividend ratio. In particular, the price-dividend ratio, unlike the state variables of the regime shift model, fails to account for the sharp movements in the equity premium in the historical data including the Great Depression of the early 30s followed by a very quick recovery and the huge run-up in asset prices in the mid-90s. To further illustrate these observations, Figure 4, Panel B plots the cumulative squared demeaned equity premium minus the cumulative squared regression residual from the alternative forecasting regression specifications : the predictive regression implied by the model (black solid line), and a linear predictive regression with the log price-dividend ratio as a predictor variable (red dashed line). An increase in a line indicates better performance of the named model relative to the equity premium mean while a decrease in a line indicates better performance of the equity premium mean. The figure reveals the superior predictive performance of the regime shifts model relative to the other predictor variables that is particularly pronounced during the Great depression, World War II, and the run-up in the 90s.

The historical size premium (9.7%) and value premium (7.4%) are of the same order of magnitude as the equity premium (8.3%), based on arithmetic annual returns. The predictability of these premia is important in active portfolio management. It is also important in providing an alternative channel to examine the empirical plausibility of a given set of state variables that purport to explain the cross-section of returns. The results of predictive regressions for the full sample period $1930 - 2006$ are presented in Table 4.

Panel B displays results for the size premium. The first row displays results of a regression with x, p, and their product as predictive variables. The regression coefficients of x and the product xp are statistically significant. The $\overline{R}^2$ of the regression is 8.6%. The second row displays results of a linear regression with the market-wide log price-dividend ratio as predictive variable. The coefficient on the price-dividend ratio is statistically insignificant. Moreover, the $\overline{R}^2$ is -0.2% - an order of magnitude smaller than that obtained from the model-implied regression in Row 1. Row 3 displays results from a linear regression with the risk free rate as an additional predictive variable. Neither slope coefficient is statistically significant. The $\overline{R}^2$ is only 0.5%, still an order of magnitude smaller than that obtained from the regression in Row 1.

In Panel C, the results on predicting the value premium are similar to those in Panel B. The predictive regression with x, p, and their product as predictive variables has $\overline{R}^2$ 4.5%. The linear regression with the lagged market-wide log price-dividend ratio as regressor has $\overline{R}^2$ -0.9%. The inclusion of the risk free rate further lowers the $\overline{R}^2$ to -2.3%.

Figure 4, Panel A (C) that plots the realized size (value) premium (black solid line) along with its predicted value from the forecasting regression implied by the regime shift model (green dotted line) and a linear forecasting regression using the log market-wide price-dividend ratio as a predictor variable (red dashed line). Panel B (D) plots the cumulative squared demeaned size (value) premium minus the cumulative squared regression residual from the alternative forecasting regression specifications : the predictive regression implied by the model (black solid line), and a linear predictive regression with the log price-dividend ratio as a predictor variable (green dashed line) An increase in a line indicates better performance of the named model relative to the portfolio mean return while a decrease in a line indicates better performance of the mean return. The figure reveals the substantially superior predictive performance of the regime shifts model relative to the mean return that is particularly pronounced during the Great depression and the run-up in the 90s.

Note that the results of the predictive regression of the realized equity premium on the aggregate log price-dividend ratio and risk free rate do not support the implication of the single-regime Bansal and Yaron (2004) model that the equity premium is an affine function of the aggregate log price-dividend ratio and interest rate.[3]

6.2 In-Sample Forecasting: 1976-2006

We reexamine the ability of the regime shifts model to forecast the equity, size, and value premia over the subperiod $1976 - 2006$ for two reasons. First, it facilitates comparison with the extant literature that documents poor in-sample (and out-of-sample) performance of predictive models over this subperiod. Second, it allows us to estimate the model parameters over the first subperiod $1930 - 1975$ and examine the forecasting performance of the model over the non-overlapping second subperiod $1976 - 2006$. The forecasting performance of the model is even stronger over the subperiod compared to linear forecasting regressions with the price-dividend ratio and risk free rate as predictive variables. This demonstrates that the superior forecating performance of the model over the full sample period $1930 - 2006$ is not due to the potential look-ahead bias introduced by estimating the model parameters over the same period over which we forecast the premia. The results are reported in Table 5.

In Panel A, we report results for the equity premium. The first row displays results of a regression with x, p, and their product as predictive variables. The regression coefficient of p is statistically significant. The $\overline{R}^2$ of the regression is 3.3%. The second row displays results of a linear regression with the market-wide log price-dividend ratio as predictive variable. The coefficient of the price-dividend ratio is statistically insignificant. Moreover, the $\overline{R}^2$ is -1.1%. The inclusion of the risk free rate lowers the $\overline{R}^2$ further to -4.7%. The poor forecasting performance of the price-dividend ratio and risk free rate over the last thirty years is consistent with the findings reported in Welch and Goyal (2008).

Panel B displays results for the size premium. The first row shows that the regression with x, p, and their product as predictive variables yields a statistically significant coefficient of p and $\overline{R}^2$ 43.5%. The second row shows that the coefficient of the price-dividend ratio is statistically insignificant and the $\overline{R}^2$ is -1.6%. Row 3 displays results from a linear regression with the risk free rate as an additional predictive variable. The coefficient of the risk free rate is statistically significant and the $\overline{R}^2$ rises to 11.4% but is still much smaller than that obtained from the model-implied regression in Row 1.

In Panel C, we report results on forecasting the value premium. The forecasting

[3] This implication of the Bansal and Yaron (2004) model follows from two observations. First, the aggregate log price-dividend ratio and interest rate are affine functions of the two state variables - the conditional mean of consumption growth rate and the conditional variance of its innovation. Therefore, the two state variables are affine functions of the the aggregate log price-dividend ratio and interest rate. Second, the equity premium is an affine function of the conditional variance of the innovation of the consumption growth rate. Hence, the model predicts that the equity premium is an affine function of the price-dividend ratio and interest rate.

regression with x, p, and their product as predictive variables has statistically significant coefficients of x, p, and xp, and $\overline{R}^2$ 17.7%. The linear regression with the lagged market-wide log price-dividend ratio as regressor has $\overline{R}^2$ -3.1%. The inclusion of the risk free rate further lowers the $\overline{R}^2$ to -4.1%.

Figure 6, Panel A that plots the realized equity premium (black solid line) along with its predicted value from the forecasting regression implied by the regime shift model (green dotted line) and a linear forecasting regression using the log market-wide price-dividend ratio as a predictor variable (red dashed line). Figure 6, Panel B plots the cumulative squared demeaned equity premium minus the cumulative squared regression residual from the alternative forecasting regression specifications : the predictive regression implied by the model (black solid line), and a linear predictive regression with the log price-dividend ratio as a predictor variable (red dashed line). Figure 7 reports analogous plots for the size and value premia. Note that the time series of the premia predicted by the model line up more closely with the actual realized time series compared to the time series predicted by the price-dividend ratio.

6.3 Out-of-Sample Prediction: 1976-2006

Whereas many models that forecast the equity premium and/or market return in-sample in certain subperiods spectacularly fail to predict out-of-sample, we demonstrate that our model retains its predictive power out-of-sample. We examine the out-of-sample peformance of our model forecasts in two ways. First, we use the central insight of the model of changing economic regimes that makes the equity premium predictable by the market-wide price-dividend ratio in the first regime but not in the second regime. We estimate the model parameters over the period $1930 - 1975$ and extract the time series of the state variable, p_t. At each year t, starting from 1976, we perform the following regression using data for all prior years:

$$r_{m,t+1} - r_{f,t} = \alpha_0 + \alpha_1 I_{\{p_t > 0.5\}} z_{m,t} + \upsilon_{t+1}. \tag{27}$$

We use the coefficient estimates to predict the equity premium for period $t+1$. Equation (27) implies that for those time periods in which the probability of being in the first regime, p_t, is bigger than 0.5, the price-dividend ratio is used to predict the equity premium, whereas in the time periods when $p_t < 0.5$ the forecast of the equity premium is obtained from its historical average. The out-of-sample performance of these forecasts is evaluated using an out-of-sample R^2 statistic as in Campbell and Thompson (2008) and Welch and Goyal (2008):

$$R^2_{OS} = 1 - \frac{MSE_A}{MSE_N}, \tag{28}$$

where MSE_A denotes the mean-squared prediction error from the predictive regression (27) and MSE_N denotes the mean-squared prediction error of the historical average return. If the out-of-sample R^2 is positive, then the predictive regression has lower mean-squared prediction error than the historical average return. We perform a similar predictive regression for the market return.

The results are reported in Table 6. Panels A and B report results for the equity premium and market return, respectively. Row 1 of Panel A shows that the predictive regression (27) for the equity premium gives an out-of-sample R^2 of 2.3%. We compare the predictive performance of the model-implied regression (27) to a specification that ignores the presence of regimes and performs a linear predictive regression of the realized equity premium on the lagged log price-dividend ratio. Row 2 of Panel A shows that the linear regression model gives a large negative out-of-sample R^2. Row 3 shows that addition of the risk free rate to the linear regression model does not improve its out-of-sample predictive performance and still gives a large negative out-of-sample R^2. The poor out-of-sample predictive performance of the price-dividend ratio and risk free rate over the last thirty years has also been reported in Welch and Goyal (2008).

The results in Panel B for the market return provide even stronger evidence in favour of the two-regime model. The model-implied predictive regression (27) for the market return gives a large out-of-sample R^2 of 15.3%. On the contrary, a linear predictive model with the price-dividend ratio as the predictor variable gives a large negative out-of-sample R^2, and the inclusion of the risk free rate as an additional predictor variable does not help improve the out-of-sample performance of the linear predictive model.

Figure 8, Panel A that plots the realized equity premium (black solid line) along with its predicted value from the forecasting regression implied by the regime shift model (green dotted line) and a linear forecasting regression using the log market-wide price-dividend ratio as a predictor variable (red dashed line). Figure 8, Panel B plots the cumulative squared demeaned equity premium minus the cumulative squared regression residual from the alternative forecasting regression specifications : the predictive regression implied by the model (black solid line), and a linear predictive regression with the log price-dividend ratio as a predictor variable (red dashed line). Figure 9 reports analogous plots for the market return.

Our second approach to examining the out-of-sample peformance of our model forecasts relies on the observation that the two state variables, x and p, and their product should predict the equity premium and market return. At each year t, starting from 1976, we forecast the equity premium and market return in the year $t+1$ as follows. First, we estimate the model parameters over the period $1930 - 1975$ and extract the time series of the state variables. This approach is conservative because we do not use all the information in the history from 1930 to time t in estimating the model parameters. Second, we estimate the coefficients of x, p, and xp from a regression over the period 1930 to time t.

The results are reported in Table 7. The first row of Panel A reports the out-of-sample results for the model-implied predictive regression for the equity premium. The *out-of-sample* R^2 is 5.1%. Row 2 reports results from a linear regression of the equity premium on the lagged aggregate log price-dividend ratio. In this case, the *out-of-sample* R^2 is only 2.1% - a third of that obtained from the model-implied regression in Row 1. Row 3 reports results from a linear regression of the equity premium on the lagged aggregate log price-dividend ratio and the log risk free rate. The *out-of-sample* R^2 further falls to -2.1%. The superior predictive ability of the model for the equity premium is illustrated in Figure 10. The first row of Panel B shows that the model-implied predictive regression for the market return has an *out-of-sample* R^2 is 4.0%. Row 2 shows that the price-dividend ratio performs well at predicting the market return out-of-sample once restrictions are imposed on the sign of its coefficient and on the sign of the predicted return, although the *out-of-sample* R^2 is still lower than that obtained from the model-implied regression. Row 3 shows that inclusion of the risk free rate worsens the predictive performance of the model.

Finally, the two-regime model performs very well at predicting the value premium out-of-sample. The *out-of-sample* R^2 for the model-implied predictive regression is 16.1%. The price-dividend ratio and risk free rate have poor predictive performance for the value premium giving *out-of-sample* R^2 -11.4% and -11.8%, respectively. The superior predictive ability of the model for the value premium is illustrated in Figure 11.

7 Forecasting the Variance of Market Return

We estimate the conditional variance of the annual market return as the sum of squares of the twelve monthly log returns. In Table 9, we report the results of predictive regressions of this conditional variance over $1930-2006$ on x, p, and their product (Row 1), the lagged aggregate log price- dividend ratio (Row 2), and the lagged aggregate log price-dividend ratio and interest rate (Row 3). In Row 1, the regression coefficient on xp is statistically significant and the $\overline{R}^2$ of the regression is economically very large at 55.6%. In Rows 2 and 3, the regression coefficients on the log price-dividend ratio are statistically significant but the values of $\overline{R}^2$ are smaller than that in the regression of Row 1.

The superior performance of the model in predicting the conditional variance of the annual market return is illustrated in Figure 6 that plots the realized variance (black solid line) along with its predicted value from the forecasting regression implied by the regime shift model (green dotted line) and a linear forecasting regression using the log market-wide price-dividend ratio as a predictor variable (red dashed line). Note that the time series of the variance predicted by the model lines up more closely with the actual realized time series compared to the time series predicted by the price-dividend ratio. In particular, the price-dividend ratio, unlike the state variables of the regime shift model, fails to account for the sharp movements in the variance in the historical data including the Great Depression of the early 30s, the Oil shock in the mid 70s, and the 1987 crash.

8 Concluding Remarks

We address the predictability of returns and of consumption and dividend growth in an equilibrium model with two regimes. The novel state variable is the probability that the economy is in the first regime. The economy exhibits different characteristics across regimes. The first regime captures periods of dividend growth unpredictability and return predictability; while the second regime captures periods of dividend growth predictability and return unpredictability. The differences in predictability across regimes shed light on the controversial interpretation of the extant empirical evidence on predictability and, in particular, the lack of robustness across subperiods. We show that the model-implied state variables perform significantly better at predicting the equity, size, and value premia and the variance of the market return over $1930 - 2006$ and $1976 - 2006$ than linear regressions with predictive variables the market log price-dividend ratio and log risk free rate.

The economy exhibits other differences across regimes as well. The average market return is substantially higher in the first regime than in the second one. The variance of the market return is higher in the second regime than in the first one. The market exhibits reversal in the first regime and momentum in the second one, both at the annual frequency. The size and value premia are higher in the second regime than in the first one. The value premium exhibits reversal in the first regime and momentum in the second one.

The first regime has expected duration 10 years and the consumption and dividend growth dynamics are driven by a high-frequency state variable that has half-life shorter than one year; while the second regime has expected duration 5 years and the consumption and dividend growth dynamics are driven by a low-frequency state variable that has half-life just over 34 years. These properties suggest that the regimes capture features of the economy other than the business cycle. High on our agenda is an understanding of the economic forces that differentiate the regimes.

A related goal is the investigation on the number of regimes that are needed to adequately describe the economy since there is no a priori reason that there should be only two economic regimes. The challenge is the judicious increase of the number of regimes in a model that retains computational and empirical tractability.

High also on our agenda is a unified theoretical framework that explains both the historically observed levels of returns of different classes of assets as well as their time series predictability. The current paper focuses on equities but the methodology is general and applicable to bonds, derivatives, and other asset classes.

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Table 1: Estimation of Parameters

	Identity weighting matrix															
	μ	μ_d	ϕ	φ_d	ρ_1	ρ_0	φ_e	σ_1	σ_0	π_1	π_0	$\eta(0)$	$e(0)$	$u(0)$	$\varepsilon(0)$	$\sigma_{\varepsilon e}$
Est	0.02 (0.05)	0.014 (0.08)	6.0 (0.02)	2.0 (0.01)	0.98 (0.41)	0.2 (0.89)	0.4 (0.39)	0.005 (0.14)	0.035 (0.03)	0.8 (0.30)	0.9 (0.48)	0 (1.41)	0 (3.80)	-1 (0.01)	1 (0.49)	0 (1.78
	δ	γ	ψ													
	0.976 (0.37)	12 (8.86)	0.9 (0.25)													

	$Model$	$Data$		$Model$	$Data$
R_f	0.002	0.008 (0.005)	$E(\Delta c)$	0.020	0.016 (0.005)
$R_m - R_f$	0.119	0.083 (0.026)	$Var(\Delta c)$	0.001	0.001 (0.0003)
$R_s - R_b$	0.073	0.097 (0.048)	$E(\Delta d)$	0.034	0.015 (0.014)
$R_v - R_g$	0.015	0.074 (0.031)	$Var(\Delta d)$	0.010	0.012 (0.004)
			$Cor(\Delta c, \Delta d)$	0.45	0.50 (0.29)

The table reports GMM estimates of the model using real annual data over the period 1930-2006. Results are reported for the identity weighting matrix. The asset menu consists of the market portfolio, the risk free rate, and portfolios of "Small" capitalization, "Large" capitalization, "Growth", and "Value" stocks. The lagged log price-dividend ratio of the market and the continuously compounded risk free rate are used as instruments giving 18 pricing restrictions. To this set of pricing restrictions, we add the 5 moment restrictions implied by the time-series specification of the model. This gives a total of 23 moment conditions. The total number of parameters to be estimated is 21, including 18 time-series and 3 preference parameters. The bigger root of the quadratic is used in the extraction of the latent state variables. The table reports the parameter estimates along with the associated standard errors in parentheses. The table also reports the model-implied and the historical values (along with standard errors in parentheses) of the equity premium, risk free rate, size premium, and value premium.

Table 2: Summary Statistics in the two Regimes

	Panel A: Regime 1		
	$E(.)$	$Var(.)$	$AC(1)$
Dividend Growth	0.019 (0.013)	0.013 (0.007)	0.16 (0.21)
Consumption Growth	0.015 (0.003)	0.0006 (0.0067)	0.30 (0.20)
GDP Growth	0.037 (0.005)	0.001 (0.001)	0.34 (0.25)
Risk Free Rate	0.028 (0.006)	0.001 (0.001)	0.59 (0.44)
Price Dividend Ratio	3.33 (0.08)	0.15 (0.05)	0.93 (0.36)
Market	0.094 (0.022)	0.027 (0.008)	−0.18 (0.13)
Equity Premium	0.066 (0.023)	0.030 (0.011)	−0.14 (0.13)
Size Premium	0.032 (0.032)	0.043 (0.014)	0.30 (0.12)
Value Premium	0.043 (0.026)	0.042 (0.012)	−0.07 (0.12)
	Panel B: Regime 2		
	$E(.)$	$Var(.)$	$AC(1)$
Dividend Growth	−0.002 (0.024)	0.009 (0.005)	0.39 (0.18)
Consumption Growth	0.020 (0.008)	0.0009 (0.0004)	0.38 (0.26)
GDP Growth	0.035 (0.018)	0.003 (0.002)	0.53 (0.29)
Risk Free Rate	−0.054 (0.011)	0.002 (0.001)	0.47 (0.91)
Price Dividend Ratio	3.04 (0.08)	0.08 (0.04)	0.83 (0.40)
Market	−0.002 (0.046)	0.060 (0.027)	0.14 (0.19)
Equity Premium	0.052 (0.048)	0.059 (0.027)	0.19 (0.18)
Size Premium	0.106 (0.083)	0.089 (0.037)	0.62 (0.37)
Value Premium	0.056 (0.071)	0.053 (0.025)	0.50 (0.30)

The table reports the mean, variance, and first order autocorrelation of the aggregate dividend, consumption, and GDP growth rates, the risk free rate, the market-wide price-dividend ratio, the market return, and the equity, size, and value premia in the two regimes.

Table 3: Forecastability in the two Regimes

	Panel A: Regime 1		
	const.	$log\,(P/D)$	*Adj*-R^2
Dividend Growth	−0.04 (0.13)	0.02 (0.04)	−0.014
Market	0.56 (0.18)	−0.14 (0.05)	0.091
Equity Premium	0.45 (0.20)	−0.12 (0.06)	0.050
Small	0.68 (0.30)	−0.17 (0.09)	0.041
Large	0.53 (0.18)	−0.13 (0.05)	0.085
Growth	0.54 (0.21)	−0.14 (0.06)	0.066
Value	0.72 (0.29)	−0.18 (0.09)	0.056

	Panel B: Regime 2		
	const.	$log\,(P/D)$	*Adj*-R^2
Dividend Growth	−0.45 (0.19)	0.15 (0.06)	0.202
Market	0.28 (0.55)	−0.09 (0.18)	−0.043
Equity Premium	0.54 (0.54)	−0.16 (0.17)	−0.010
Small	0.67 (1.05)	−0.18 (0.34)	−0.040
Large	0.24 (0.50)	−0.08 (0.16)	−0.043
Growth	0.32 (0.54)	−0.11 (0.17)	−0.033
Value	0.41 (0.84)	−0.12 (0.27)	−0.046

The table reports the predictive performance of the market-wide log price-dividend ratio in each of the two regimes for the aggregate dividend growth, market return, equity premium, and portfolios of "Small", "Large", "Growth", and "Value" stocks.

Table 4: Forecasting Equity, Size, Value Premia 1930-2006

Panel A: Equity Premium						
const.	x	p	xp	$log\,(P/D)$	r_f	*Adj-*R^2
0.01 (0.10)	−1.47 (1.12)	0.06 (0.11)	3.31 (2.33)			0.066
0.43 (0.19)				−0.11 (0.06)		0.037
0.41 (0.19)				−0.10 (0.06)	−0.45 (0.43)	0.038
Panel B: Size Premium						
const.	x	p	xp	$log\,(P/D)$	r_f	*Adj-*R^2
−0.02 (0.12)	−2.17 (1.32)	0.05 (0.13)	3.76 (1.62)			0.086
0.27 (0.23)				−0.07 (0.07)		−0.002
0.23 (0.23)				−0.05 (0.07)	−0.66 (0.53)	0.005
Panel C: Value Premium						
const.	x	p	xp	$log\,(P/D)$	r_f	*Adj-*R^2
0.003 (0.11)	−1.21 (1.21)	0.04 (0.11)	3.07 (1.48)			0.045
0.17 (0.21)				−0.04 (0.06)		−0.009
0.17 (0.21)				−0.04 (0.06)	−0.05 (0.49)	−0.023

The table reports results from forecasting regressions for the equity, size, and value premia over the period 1930-2006. Panels A, B, and C report results for the equity, size, and value premia, respectively. The first row of each panel reports the regression coefficients along with the associated standard errors, and the adjusted-R^2 from the forecasting regression of the realized premium on x, p, and xp. The second row reports the regression coefficients along with the associated standard errors and the adjusted-R^2 from the forecasting regression of the realized premium on the lagged aggregate log price-dividend ratio. The third row reports the regression coefficients along with the associated standard errors and the adjusted-R^2 from the forecasting regression of the realized premium on the lagged aggregate log price-dividend ratio and the log risk free rate.

Table 5: Forecasting Equity, Size, Value Premia 1976-2006

Panel A: Equity Premium						
const.	x	p	xp	$log\,(P/D)$	r_f	*Adj*-R^2
0.15 (0.07)	−1.98 (2.78)	−0.17 (0.09)	−1.08 (3.77)			0.033
0.24 (0.22)				−0.05 (0.06)		−0.011
0.23 (0.23)				−0.05 (0.06)	0.14 (1.03)	−0.047
Panel B: Size Premium						
const.	x	p	xp	$log\,(P/D)$	r_f	*Adj*-R^2
−0.22 (0.08)	4.61 (3.09)	0.41 (0.10)	−2.66 (4.19)			0.435
−0.19 (0.32)				0.07 (0.09)		−0.016
0.02 (0.31)				0.03 (0.09)	−3.17 (1.38)	0.114
Panel C: Value Premium						
const.	x	p	xp	$log\,(P/D)$	r_f	*Adj*-R^2
−0.16 (0.09)	7.92 (3.63)	0.31 (0.12)	−7.81 (4.92)			0.177
0.17 (0.31)				−0.03 (0.09)		−0.031
0.26 (0.33)				−0.04 (0.09)	−1.23 (1.45)	−0.041

The table reports results from forecasting regressions for the equity, size, and value premia over the subperiod 1976-2006. Panels A, B, and C report results for the equity, size, and value premia, respectively. The first row of each panel reports the regression coefficients along with the associated standard errors, and the adjusted-R^2 from the forecasting regression of the realized premium on x, p, and xp. The second row reports the regression coefficients along with the associated standard errors and the adjusted-R^2 from the forecasting regression of the realized premium on the lagged aggregate log price-dividend ratio. The third row reports the regression coefficients along with the associated standard errors and the adjusted-R^2 from the forecasting regression of the realized premium on the lagged aggregate log price-dividend ratio and the log risk free rate.

Table 6: Out-of-Sample Prediction, 1976-2006

Panel A: Equity Premium	
	R^2
Two-Regime Model	0.023
$log\,(P/D)$	−0.256
$log\,(P/D)$, r_f	−0.282
Panel B: Market Return	
	R^2
Two-Regime Model	0.153
$log\,(P/D)$	−0.218
$log\,(P/D)$, r_f	−0.206

The table reports results from out-of-sample predictive regressions for the equity premium and market return over the period 1976-2006. Panels A and B report results for the equity premium and market return, respectively. The first row of Panel A (B) reports the out–of-sampleR^2 from the model-implied predictive regression of the realized equity premium (market return). The second row of Panel A (B) reports the out–of-sampleR^2 from the predictive regression of the realized equity premium (market return) on the lagged aggregate log price-dividend ratio. The third row of Panel A (B) reports the out–of-sampleR^2 from the predictive regression of the realized equity premium (market return) on the lagged aggregate log price-dividend ratio and the log risk free rate.

Table 7: Out-of-Sample Prediction, 1976-2006

Panel A: Equity Premium	
	R^2
$Two\text{-}Regime\ Model$	0.061
$log\,(P/D)$	0.021
$log\,(P/D),\ r_f$	−0.018
Panel B: Market Return	
	R^2
$Two\text{-}Regime\ Model$	0.040
$log\,(P/D)$	0.030
$log\,(P/D),\ r_f$	−0.006

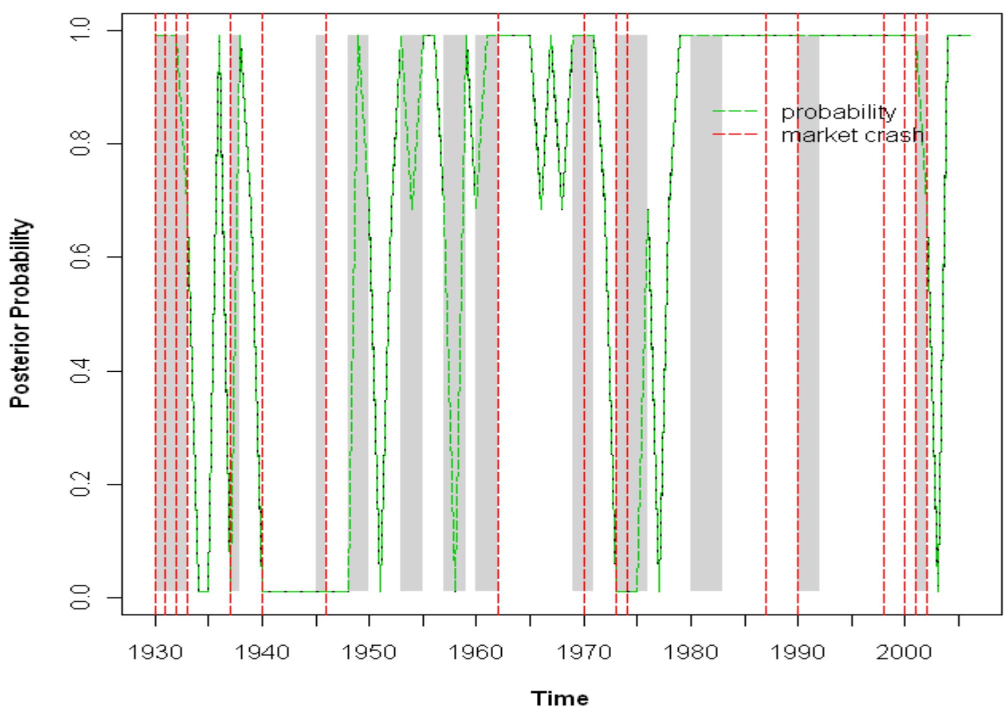
The table reports results from out-of-sample predictive regressions for the equity, size, and value premia over the period 1976-2006. Panels A, B, and C report results for the equity, size, and value premia, respectively. The first row of each panel reports the regression coefficients along with the associated standard errors, and the adjusted-R^2 from the forecasting regression of the realized premium on x, p, and xp. The second row reports the regression coefficients along with the associated standard errors and the adjusted-R^2 from the forecasting regression of the realized premium on the lagged aggregate log price-dividend ratio. The third row reports the regression coefficients along with the associated standard errors and the adjusted-R^2 from the forecasting regression of the realized premium on the lagged aggregate log price-dividend ratio and the log risk free rate.

Table 7: Forecasting Market Return Variance, 1930-2006

$const.$	x	p	xp	$log\,(P/D)$	r_f	$Adj\text{-}R^2$
0.045 (0.018)	0.153 (0.204)	0.001 (0.019)	1.394 (0.251)			0.556
0.142 (0.050)				−0.033 (0.015)		0.045
0.171 (0.043)				−0.043 (0.013)	0.544 (0.099)	0.317

The table reports results from the predictive regression for the market return variance over the period 1930-2006. The sum of squares of the monthly returns is used as a proxy for the conditional variance. The first row of reports the regression coefficients along with the associated standard errors, and the adjusted-R^2 from the forecasting regression on the two state variables, x and p, and their product. The second row reports the regression coefficients along with the associated standard errors and the adjusted-R^2 from the forecasting regression on the lagged aggregate log price-dividend ratio. The third row reports the regression coefficients along with the associated standard errors and the adjusted-R^2 from the forecasting regression on the lagged aggregate log price-dividend ratio and the log risk free rate.

Figure 3: Posterior Probability of First Regime



The figure plots the probability of being in the first regime over $1930 - 2006$ along with the NBER recessions (shaded columns) and the major stock market crashes (vertical dashed lines) identified in Mishkin and White (2002). The aggregate log price-dividend ratio and the risk free rate are affine functions only of the two state variables and their product. Hence, we can invert the system to extract the two state variables as known functions of the observable aggregate log price-dividend ratio and risk free rate. This procedure of inverting two non-linear equations yields quadratic equations for x_t and p_t, with coefficients that depend on $z_{m,t}$ and $r_{f,t}$, and the time-series and preference parameters. Solving the equations gives two pairs of solutions for x_t and p_t as functions of the observables, $z_{m,t}$ and $r_{f,t}$. We choose the bigger root as this minimizes the GMM criterion function.

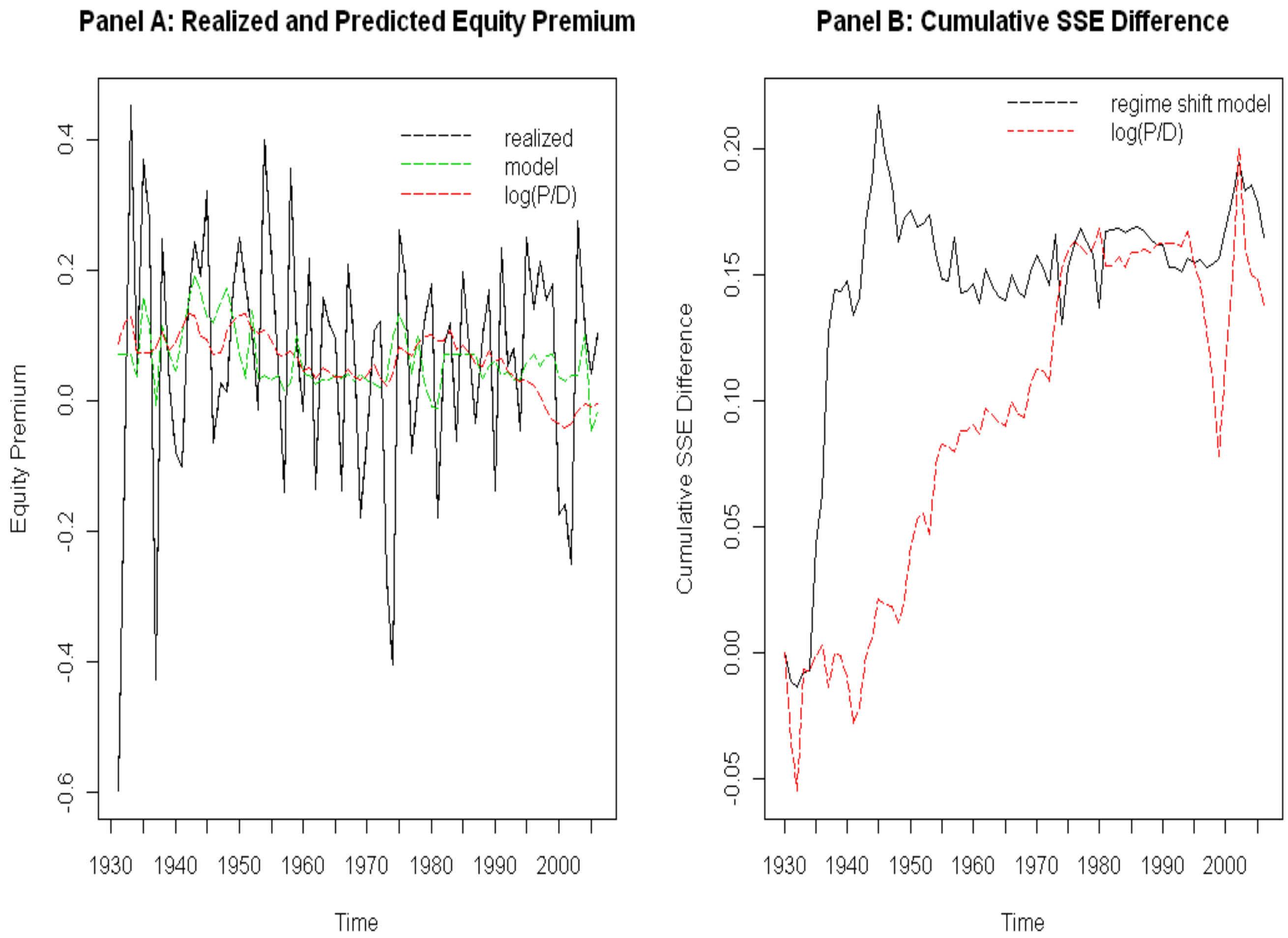


Figure 4: Panel A plots the realized equity premium along with its model forecasted value over 1931-2006. The model implies that the expected equity premium is an affine function of the two state variables, x and p, and their product, xp. Panel B shows the annual in-sample performance of the model-implied predictive regression (RSM) as well as a linear forecasting regression that uses the lagged log price-dividend ratio as a predictor variable (LM) for the equity premium. The figure plots the "Cumulative SSE Difference" defined as the cumulative squared demeaned equity premium minus the cumulative squared RSM residual. It also plots the cumulative squared demeaned equity premium minus the

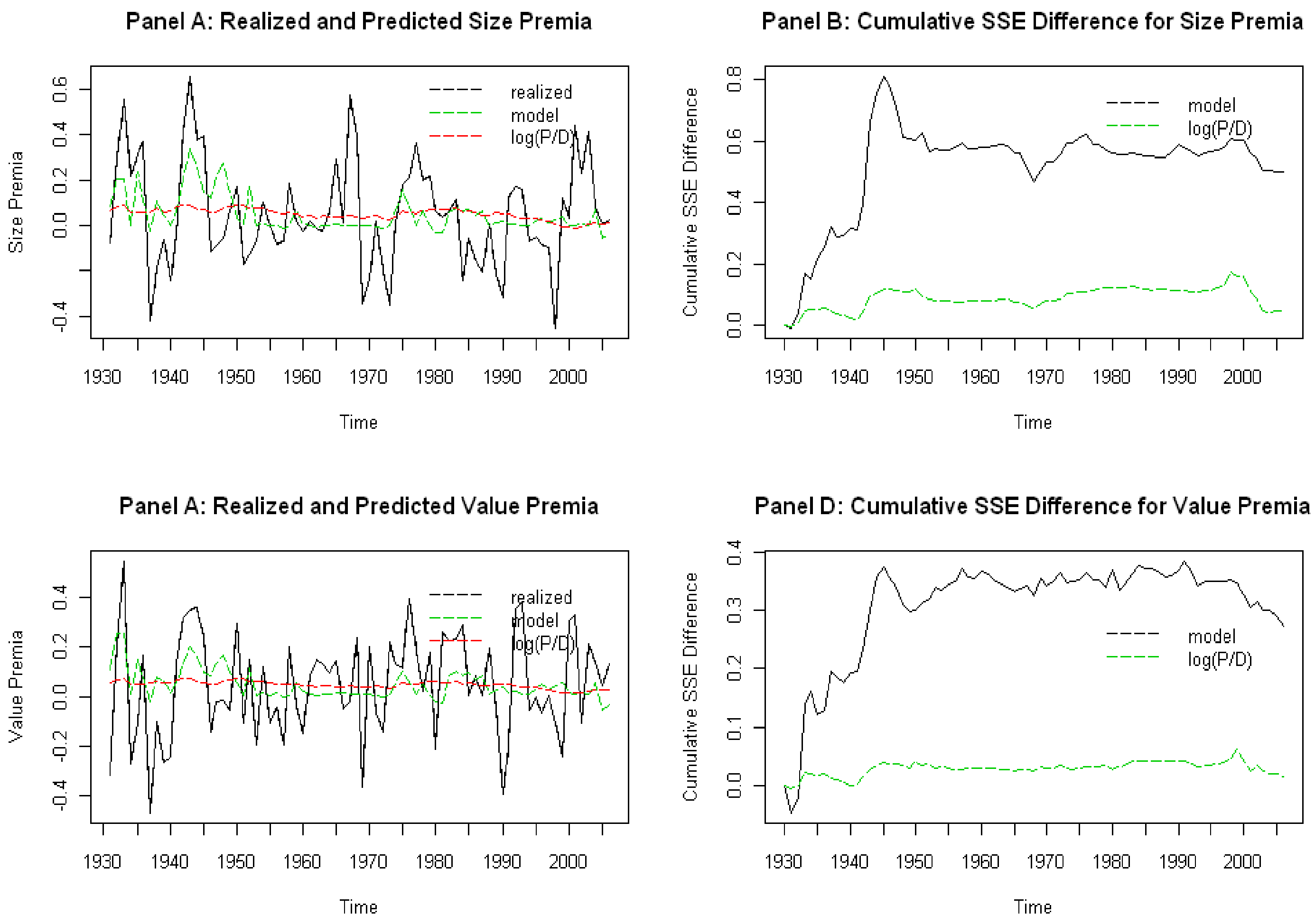


Figure 5 shows the annual in-sample performance of the model-implied predictive regression for the size premium and value premium over 1931-2006. The figure plots the "Cumulative SSE Difference" is the cumulative squared demeaned portfolio return minus the cumulative squared model-implied regression residual. In other words, it is the cumulative squared prediction error of the historical mean return minus the cumulative squared prediction error of the regime shift model.

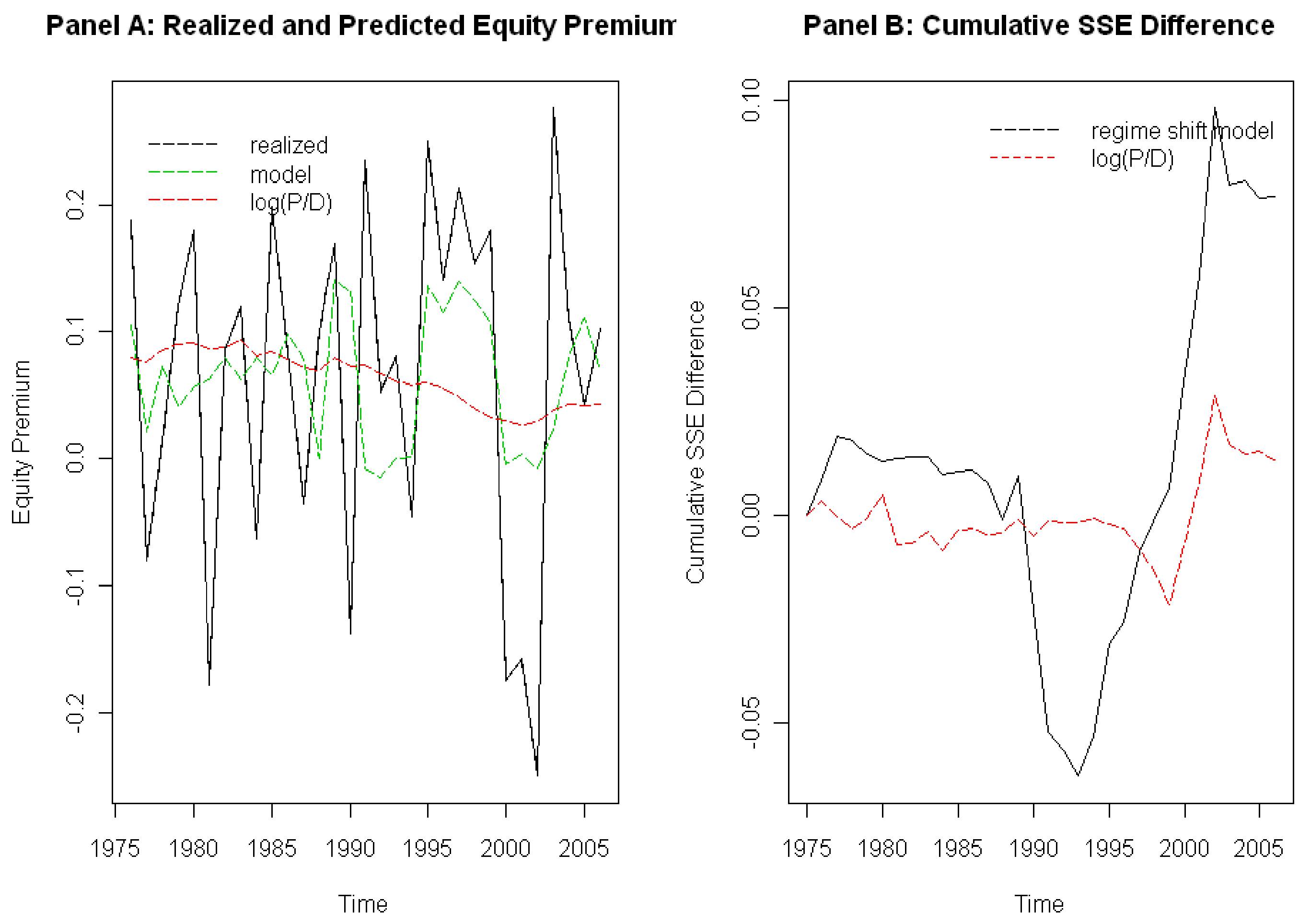


Figure 6: Panel A plots the realized equity premium along with its model forecasted value over 1976-2006. The model implies that the expected equity premium is an affine function of the two state variables, x and p, and their product, xp. Panel B shows the annual in-sample performance of the model-implied predictive regression (RSM) as well as a linear forecasting regression that uses the lagged log price-dividend ratio as a predictor variable (LM) for the equity premium. The figure plots the "Cumulative SSE Difference" defined as the cumulative squared demeaned equity premium minus the cumulative squared RSM residual. It also plots the cumulative squared demeaned equity premium minus the

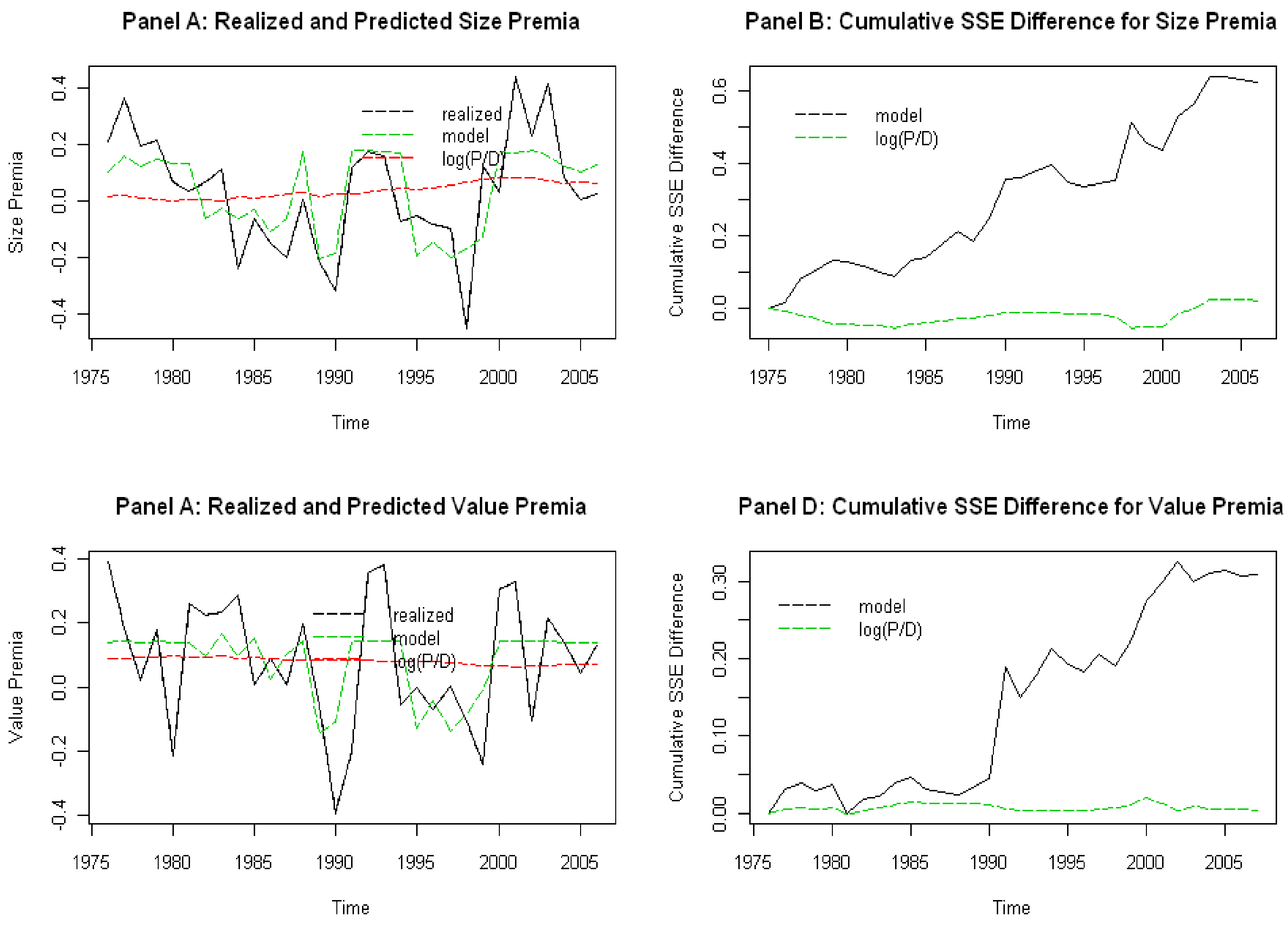


Figure 7 shows the annual in-sample performance of the model-implied predictive regression for the size premium and value premium over the subperiod 1976-2006. The figure plots the "Cumulative SSE Difference" is the cumulative squared demeaned portfolio return minus the cumulative squared model-implied regression residual. In other words, it is the cumulative squared prediction error of the historical mean return minus the cumulative squared prediction error of the regime shift model.

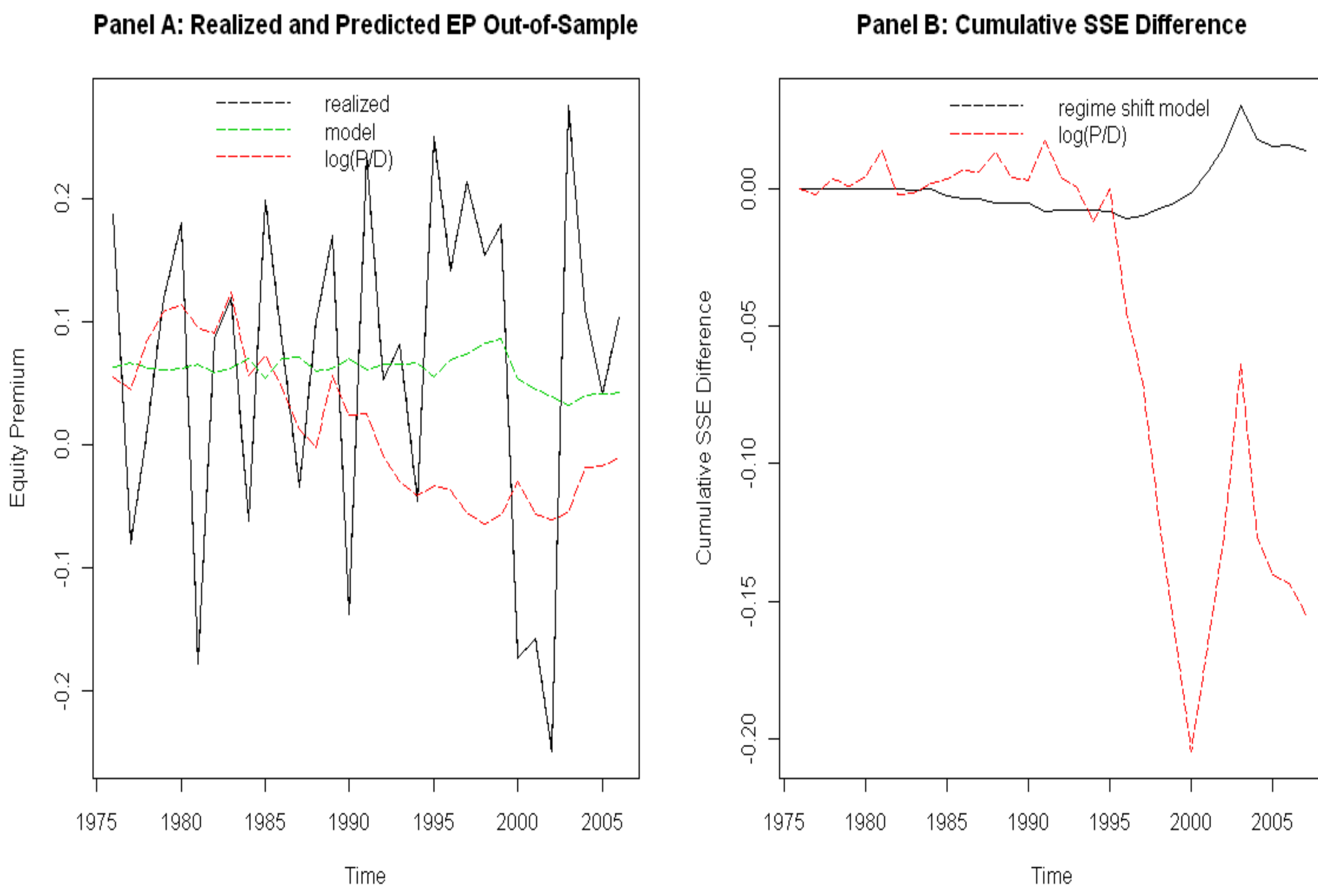


Figure 8: Panel A plots the realized equity premium along with its out-of-sample predicted value over 1976-2006. Panel B shows the annual outof-sample performance of the model-implied predictive regression (RSM) as well as a linear forecasting regression that uses the lagged log price-dividend ratio as a predictor variable (LM) for the equity premium. The figure plots the "Cumulative SSE Difference" defined as the cumulative squared demeaned equity premium minus the cumulative squared RSM residual. It also plots the cumulative squared demeaned equity premium minus the cumulative squared LM residual.

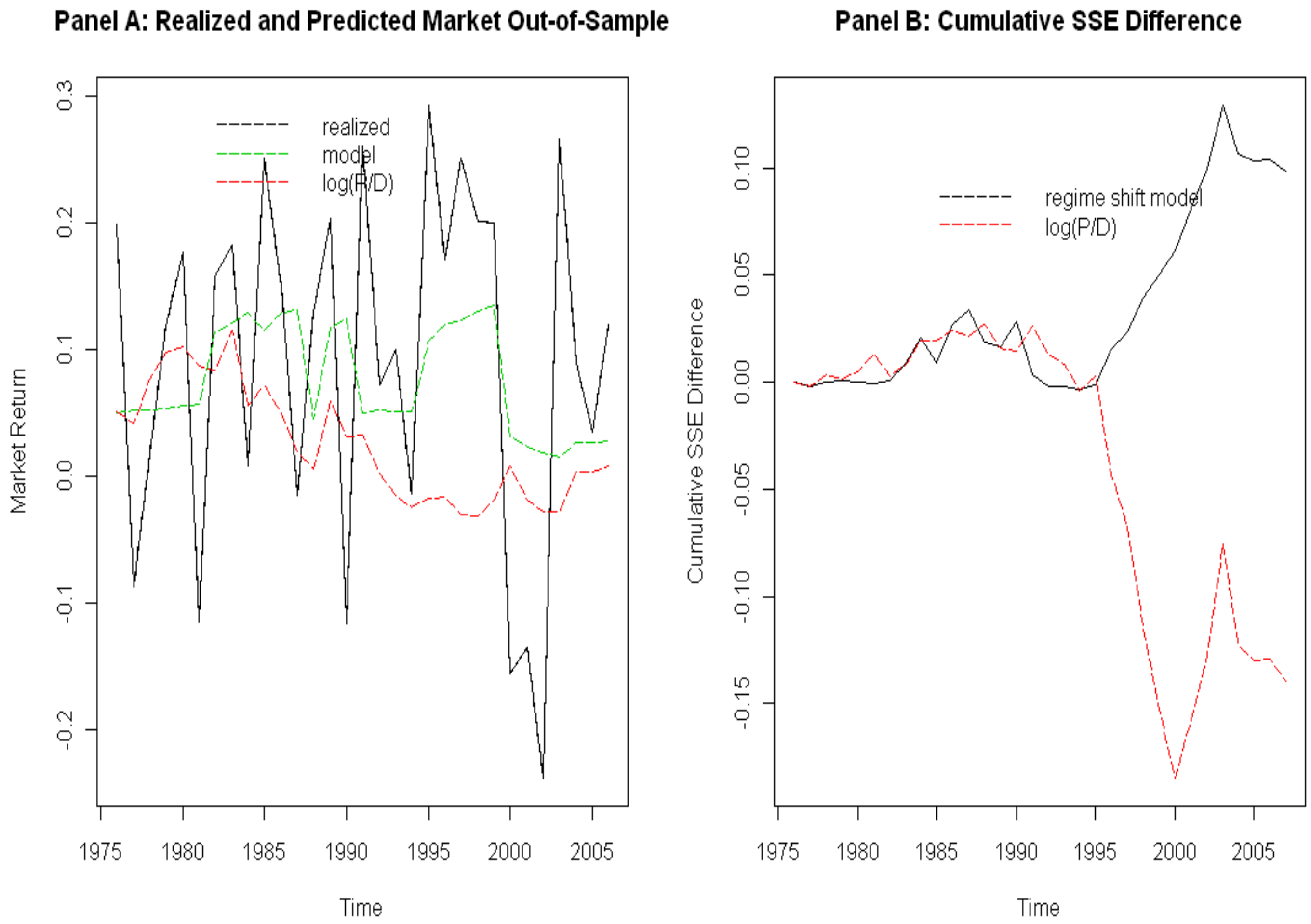


Figure 9: Panel A plots the realized market return along with its out-of-sample predicted value over 1976-2006. Panel B shows the annual out-of-sample performance of the model-implied predictive regression (RSM) as well as a linear forecasting regression that uses the lagged log price-dividend ratio as a predictor variable (LM) for the equity premium. The figure plots the "Cumulative SSE Difference" defined as the cumulative squared demeaned market return minus the cumulative squared RSM residual. It also plots the cumulative squared demeaned market return minus the cumulative squared LM residual.

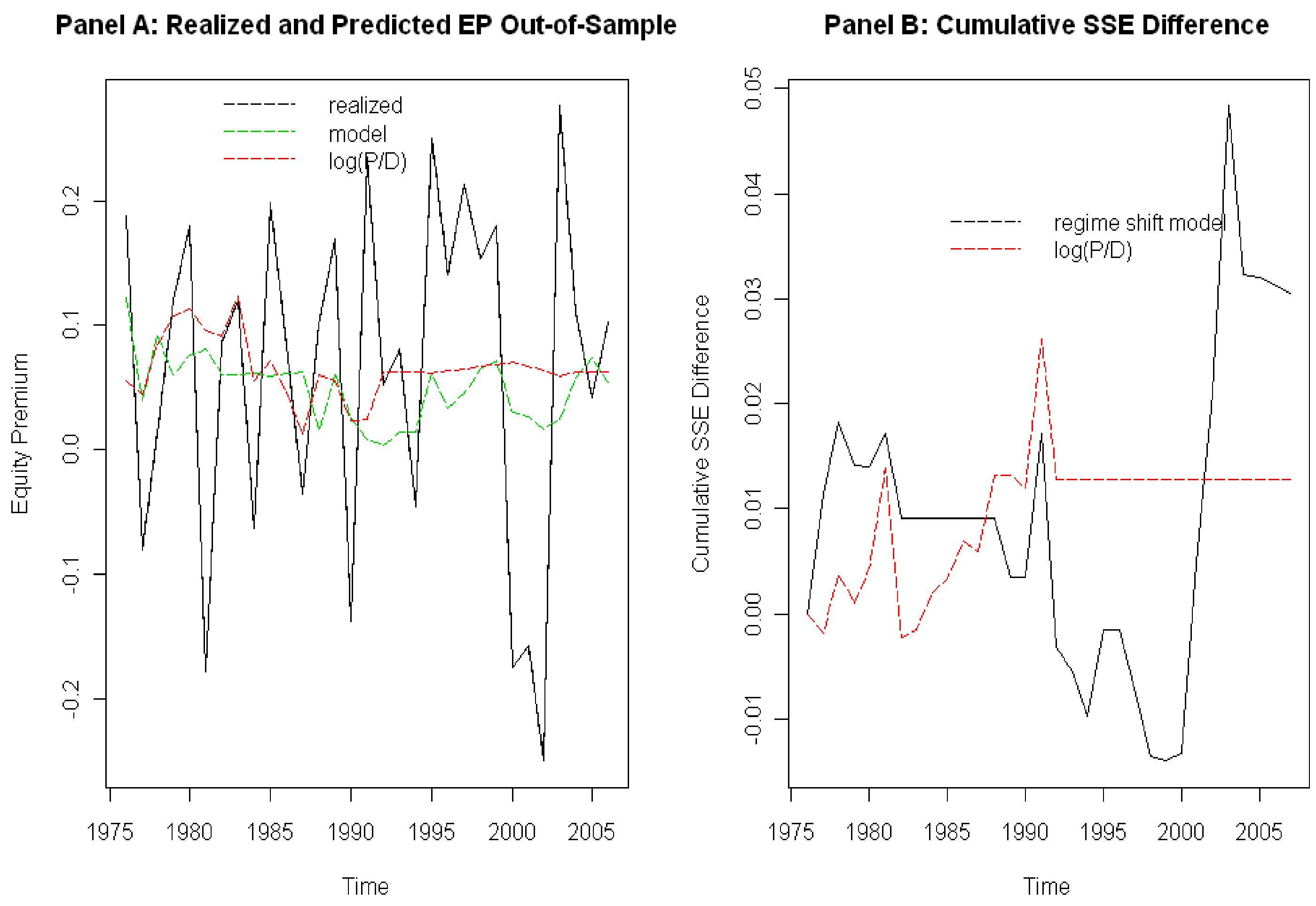


Figure 10: Panel A plots the realized and out-of-sample predictions for the equity premia over the subperiod 1976-2006. Panel B plots the "Cumulative SSE Difference", i.e. the cumulative squared demeaned portfolio return minus the cumulative squared model-implied regression residual. In other words, it is the cumulative squared prediction error of the historical mean return minus the cumulative squared prediction error of the regime shift model.

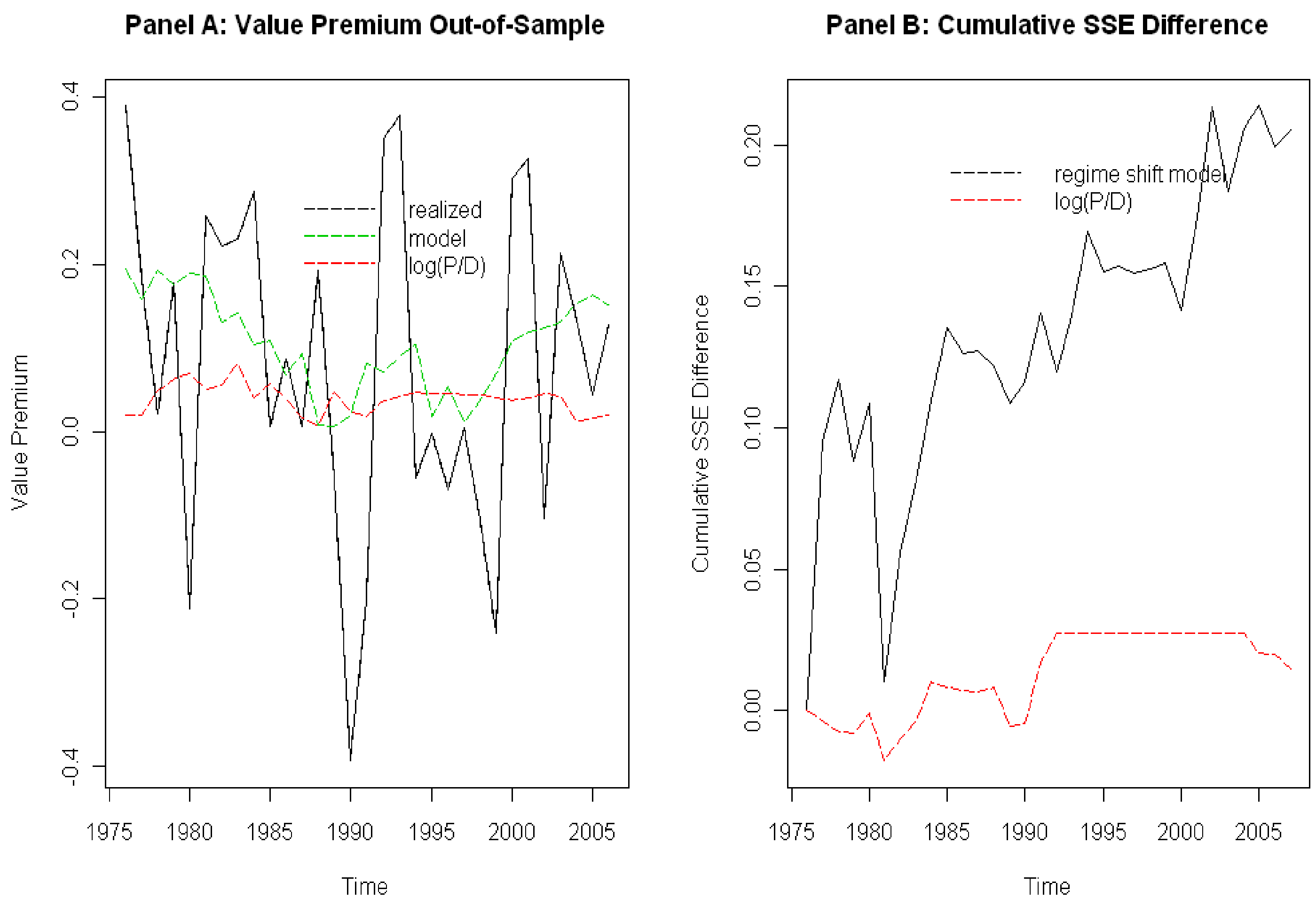
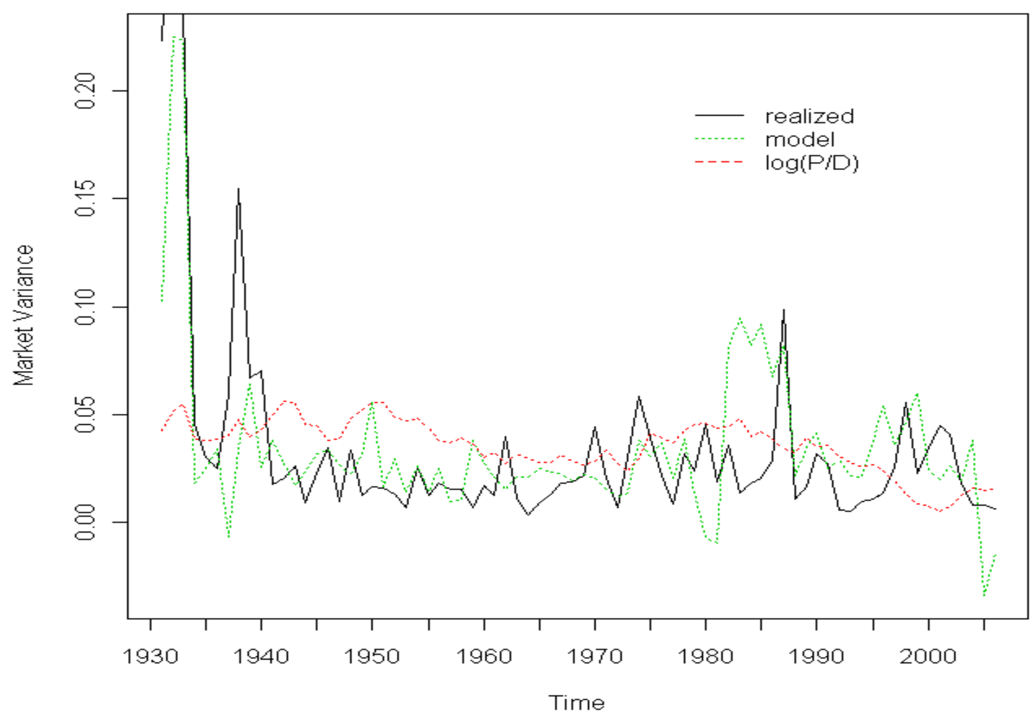


Figure 11: Panel A plots the realized and out-of-sample predictions for the value premium over the subperiod 1976-2006. Panel B plots the "Cumulative SSE Difference", i.e. the cumulative squared demeaned portfolio return minus the cumulative squared model-implied regression residual. In other words, it is the cumulative squared prediction error of the historical mean return minus the cumulative squared prediction error of the regime shift model.

Figure 6: Realized and Predicted Market Variance



The figure plots the realized market variance along with its model-predicted value. The black solid line plots the annual realized variance that is calculated as the sum of squares of the monthly log market returns. The green dotted line plots the model-predicted variance and is obtained as the fitted value from a regression of the realized variance on the two state variables and their product. The red dashed line shows the predictive power of the log price-dividend ratio for the future variance by plotting the fitted value from a regression of the realized variance on the lagged log price-dividend ratio.