Predictable dynamics in implied volatility surfaces from OTC currency options*

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This version: March, 2009

Abstract

Recent empirical studies report predictable dynamics in the volatility surfaces implied by observed index option prices, as prescribed by general equilibrium models. Using an extensive data set from the over–the–counter options market, we document similar predictability in the factors that capture the daily variation of surfaces implied by options on 25 different foreign exchange rates. We proceed to demonstrate that simple vector autoregressive specifications for the factors can help produce accurate out–of–sample forecasts of the systematic component of the surface at short horizons. Profitable delta–hedged positions can be set up based on these forecasts; however profits disappear when transaction costs are increased and when trading rules on wide segments of the surface are sought.

JEL classification: C32; C53; G13; F37

Keywords: Implied volatility surfaces; static Factor model; Forecasting; Trading strategies.

*We are grateful to J.P. Morgan for providing the data for this study and to Kian Guan Lim, Barry O'Grady and Yueh–Neng Lin for discussions and suggestions that have improved the paper. We would also like to thank participants at the Asian FA/NFA conference in Yokohama, Japan, the 17$^{\text{th}}$ EFMA meeting in Athens, Greece, the 2$^{\text{nd}}$ Risk Management conference in N.U.S., Singapore, the 15$^{\text{th}}$ GFA meeting in Hangzhou, P. R. China, the 6$^{\text{th}}$ HFAA meeting in Patras, Greece and seminar audiences at the Athens University of Economics & Business and the University of Piraeus for comments and suggestions. Any errors are our own responsibility.

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1 Introduction

Observed option prices implicitly contain information about the volatility expectations of market participants. Using an option pricing model, these volatility expectations can be extracted, and if market participants are rational, then these implied volatilities should contain all the information that is relevant for the pricing, hedging and management of option contracts and portfolios.

Contrary to the Black–Scholes–Merton assumption of constant (or deterministically time–dependent) volatility (Black and Scholes (1973), Merton (1973)), the empirical pattern of option–implied volatilities has two features that have attracted the interest of researchers and practitioners in financial modeling. First, the volatilities implied from observed contracts systematically vary with the options' strike prices and their time to expiration, giving rise to an instantaneously non–flat *implied volatility surface* (hereafter IVS).[1] The second feature is that the IVS changes dynamically over time, as prices in the options market respond to new information that affects investors' beliefs and expectations.[2]

Three popular approaches to modeling this empirically observed profile of the IVS can be identified in the literature. The *no–arbitrage approach*, inspired by the stochastic interest rate literature, where stochastic volatility models are calibrated to today's IVS so as to preclude arbitrage, with prominent examples offered by Dupire (1993), Derman and Kani (1998) and Ledoit and Santa–Clara (1998) among others.

Secondly, there is the approach of fitting *linear parametric specifications* in the cross section of options available at a point in time, linking implied volatility to time to maturity and option "moneyness" (see Dumas, Fleming and Whaley (1998), Peña, Rubio and Serna (1999), Gonçalves and Guidolin (2006), etc.).

Finally, there has been a number of recent papers (David and Veronesi (2000), Guidolin and Timmermann (2003), Garcia, Luger and Renault (2003)) that advocate a *general equilibrium approach* to the investigation of the stylised facts regarding implied volatility surface dynamics. There, investors' uncertainty and learning (from option prices) about the processes of funda-

[1]See Canina and Figlewski (1993) and Rubinstein (1994) for evidence on the implied volatility 'smile' or 'skew', and Heynen et. al. (1994), Xu and Taylor (1994) and Campa and Chang (1995) for the 'term structure' of implied volatilities.

[2]See, for example Heston and Nandi (2000) that report significant path dependency in the volatility of the S&P 500 index returns.

mentals (the state of the economy, dividends growth, etc.) give rise to an IVS, whose dynamics are driven by latent factors. If the processes describing these latent factors are persistent, these models imply IVS predictability.

Despite the relative success of these approaches for pricing and hedging purposes, surprisingly little has been said regarding the practical problem of predicting the implied volatility surface. The no–arbitrage literature has little to say about forecasting, as it is concerned primarily with fitting the IVS at a point in time and producing plausible surface dynamics. Moreover, although several studies have identified latent (statistical) factors in the dynamics of surfaces implied from equity index options (Skiadopoulos et. al. (1999), Cont and da Fonseca (2002), Mixon (2002)), none is concerned with whether these factors can be exploited for accurate out–of–sample predictions of the IVS.

Naturally, being able to forecast the entire IVS, and thus all future option prices, would imply inefficiency of the options market. However, it might be reasonable to expect that segments of the IVS, due to non–uniform trading across the surface, adjust to new information at different speeds, making some segments more predictable than others. This would be in line with the general equilibrium models, where predictability in the IVS dynamics arises as a consequence of investors' uncertainty and learning, and it would imply the existence of predictable systematic factors that affect the shape of the IVS.

In this paper, we take an explicitly out–of–sample forecasting approach, in an attempt to demonstrate that such predictable systematic factors are present, and can be exploited in an economically significant way for risk and portfolio decision–making. Using an extensive data set of daily volatility surfaces implied from over–the–counter options on 25 different exchange rates, we first demonstrate that–in accordance with the existing literature–a few static factors completely characterise IVS variation in–sample. These static factors, although statistical in nature, are shown to have a natural interpretation in the law of motion of the IVS and exhibit significant time variation and persistence.

This leads us to examine whether modeling the time series properties of the factors can improve our ability to forecast implied volatility and hence option prices out of sample. Simple vector autoregressive (VAR) specifications are first shown to achieve remarkable fit of the factors' dynamics, and subsequently used to forecast the entire surface by forecasting the factors forward. Both statistical and economic criteria are used to assess the forecasting ability of the VAR factor models examined.

In a statistical sense, we measure the ability of the VAR factor model to accurately predict the level and the direction of change of 1 to 5–day ahead implied volatilities across each of the 25 surfaces. Results are very promising, in that forecasting accuracy is found to be very good, at least up to 3 days ahead, both in absolute terms and relatively to natural benchmarks such as a random walk for implied volatilities.

In accordance with existing literature, for short–term predictions, the VAR factor model's performance is comparable (but not statistically superior) to hard–to–beat benchmarks *on an aggregate level* across the *whole surface*; however it outperforms on many (and in many cases in most) segments of the surfaces. Thus, although modeling the latent factors driving IVS dynamics cannot lead to superior surface forecasts in a statistical sense, it can identify the most predictable segments of the surface, whose existence is dictated by general equilibrium models.

In order to establish the economic relevance of identifiable predictable IVS segments, we examine whether surface forecasts produced by the VAR factor model can support profitable trading decisions. We simulate out–of–sample, recursive, one–day strategies that construct delta–hedged straddles on the IVS segments that our VAR factor model predicts substantial 1–day–ahead deviations from the observed surface. The simulated trading strategies generate positive and statistically significant out–of–sample returns, even when low to moderate transaction costs are incurred in trades. However profit returns disappear as transaction costs are increased and as trading rules on wider segments of the surface are sought.

Although the predictability patterns we document hardly represent rejections of the informational efficiency of OTC FX options market (since profits disappear with transaction cost levels that speculators might realistically face and bid–ask spread considerations are ignored), it should be stressed that they are in line with the general equilibrium models, where predictability in the IVS dynamics arises as a consequence of investors' uncertainty and learning. Our simple VAR specifications of static latent factors—that seem to act as reduced–form analogs of such more sophisticated models, exploiting this predictability—can nevertheless improve volatility forecasting and support risk management and portfolio decisions.

A few existing papers are related to ours, with Diebold and Li (2006) and Gonçalves and Guidolin (2006) closest in spirit. Both papers first apply parametric specifications at the cross–sectional level, and then fit time series models on the coefficients estimated from the first step. Moreover, both papers are concerned with forecasting: the yield curve and the IVS of S&P

500 index options respectively.

In contrast to the aforementioned papers that estimate the factors by imposing structure on the factor loadings, we estimate common factors and their loadings by the method of asymptotic principal components (Connor and Korajzcyk (1986)). The method and its variants has a long tradition in finance (see for example the references in Wilson (1994)) and has been used for the examination of IVS dynamics (by Skiadopoulos et. al. (1999), Tompkins (2001) among others), and more recently in the context of macroeconomic forecasting (Stock and Watson (2002), Boivin and Ng (2005)).

One of the main contributions of this paper is that static factors, identified in the IVS dynamics by several authors, can be used quite successfully for forecasting purposes in an economically significant way. This is demonstrated with a use of a very extensive data set, encompassing options on both very liquid and less–traded exchange rates. Moreover, and to the best of our knowledge, no other study has examined the predictability of the IVS from the OTC FX options market.

The rest of the paper is organised as follows: Section 2 describes the data, presents the methodology for decomposing the implied volatility surface into approximate static factors that exhibit intuitive interpretation, and presents the estimation results. In Section 3 we estimate a VAR–type model that can capture the time–series dynamics of the factors identified in the previous section. Sections 4 and 5 are devoted to the assessment of the out–of–sample forecasting performance of our VAR factor model and its ability to support trading strategies respectively, while Section 6 concludes the paper.

2 The implied volatility surface

2.1 The data

The data used in this study consist of daily time–series of implied volatilities for a cross–section of OTC currency options on 25 different currencies quoted against the Euro, kindly supplied by one of the largest global market makers. The time series are from 1/1/1999 to 21/5/2007, a total of 2,184 weekdays. The currencies examined and some exchange rate statistics are reported in Table 1.

In comparison to exchange–traded currency options, the OTC market is far more liquid. According to a Bank of International Settlements survey (2007), the outstanding notional amount of OTC currency options on the

Code	Currency	Average	Min-Max
AUD	Australian \$	1.679	1.504-1.915
BRL	Brazilian Real	2.678	1.395-3.977
CAD	Canadian \$	1.496	1.256-1.804
CHF	Swiss Franc	1.544	1.444-1.656
CLP	Chilean Peso	648.5	459.6-849.3
CZK	Czech Koruny	32.17	27.48-38.68
GBP	British £	0.658	0.571-0.724
HKD	Hong Kong \$	8.564	6.463-10.68
HUF	Hungarian Forint	253.5	234.5-284.6
IDR	Indonesian Rupiahs	9916.9	6726.9-13220.4
INR	Indian Rupees	49.90	38.65-60.03
JPY	Japanese ¥	126.2	89.34-164.1
KRW	South Korean Won	1236.9	943.4-1517.8
MXN	Mexican Peso	11.41	7.576-15.31
NOK	Norwegian Kroner	8.058	7.228-8.947
NZD	New Zealand \$	1.961	1.638-2.302
PLN	Polish Zlotych	4.064	3.351-4.900
RUB	Russian Ruble	31.21	23.13-37.85
SEK	Swedish Kronor	9.064	8.070-9.937
SGD	Singapore \$	1.863	1.453-2.233
SKK	Slovakian Koruny	40.89	32.83-48.30
TRY	Turkish (New) Lira	1.341	0.370-2.139
TWD	Taiwanese (New) \$	36.32	26.48-45.47
USD	United States \$	1.100	0.829-1.366
ZAR	South African Rand	8.013	6.099-12.09

Table 1: Average, minimum and maximum middle exchange rates of 25 different currencies against the Euro from January 1999 to May 2007. Source: European Central Bank.

Euro in December 2006 was approximately 3.65 trillion US$ (2.54 trillion Euros). The corresponding amount of exchange–traded currency options was 78.64 million US$ (54.77 million Euros), far less than 1% of the notional amount outstanding in the OTC market.

As is typical in such markets, dealers do not quote option prices denominated in currency units but rather implied volatilities, which are then conventionally converted into prices using the Garman and Kohlhagen (1983) version of the Black and Scholes (1973) option pricing formula:

$$c = Se^{-r_f T}\mathcal{N}(d) - Ke^{-r_d T}\mathcal{N}\left(d - \sigma\sqrt{T}\right) \tag{1}$$

$$p = Ke^{-r_d T}\mathcal{N}\left(-d + \sigma\sqrt{T}\right) - Se^{-r_f T}\mathcal{N}(-d) \tag{2}$$

where

$$d = \frac{\ln(S/K) + \left(r_d - r_f + \sigma^2/2\right)T}{\sigma\sqrt{T}} \tag{3}$$

with r_d, r_f the risk–free interest rate in the domestic and the foreign country respectively, S the spot exchange rate, K the strike price of the option, T the time to option maturity in years, σ the exchange rate's volatility and $\mathcal{N}(.)$ the standard cumulative normal distribution.

The moneyness of the option is measured by its (Black–Scholes) delta:

$$\Delta^{BS} = \frac{\partial O}{\partial S} = \begin{cases} e^{-r_f T}\mathcal{N}(d), & \text{if the option } O \text{ is a call} \\ e^{-r_f T}\left[\mathcal{N}(d) - 1\right], & \text{if the option } O \text{ is a put} \end{cases} \tag{4}$$

The industry convention is to quote, for each maturity, implied volatilities for portfolios of options such as delta–neutral straddles and risk reversals or butterfly spreads of a certain Δ^{BS}. From these, the implied volatility for at–the–money (ATM) options and for out–of–the–money (OTM) calls and puts can be inferred.[3]

Our data–set consists of implied volatilities for the following fourteen expirations: 1 week, 1 month, 2 months, 3 months, 6 months, 9 months, 12 months, 18 months, 2–5 years, 7 years and 10 years. For each of these maturities, the implied volatility is observed for options with five different Black–Scholes deltas: OTM puts with $\Delta^{BS} = -0.10$ and $\Delta^{BS} = -0.25$, ATM calls/puts and OTM calls with $\Delta^{BS} = 0.10$ and $\Delta^{BS} = 0.25$. Hence,

[3] Carr and Wu (2007a) and Malz (1996) demonstrate this in detail, in their excellent discussions on OTC currency option quoting and trading conventions.

for each exchange rate and on each observation date, a vector of 70×1 implied volatilities is observed.

Of course, not all currencies in our sample and not all option expirations are of equal trading intensity and variation. A few necessary exclusionary criteria are applied to all surfaces, to ensure that thinly–traded segments and misrecordings do not influence our results.

First, sample days with [a] at least one of the 70 implied volatilities missing or [b] flat implied volatility profiles (i.e. no "smile" or "skew") *for all 14 maturities* are excluded as misrecordings. Secondly, maturities for which the implied volatility does not change from day to day in more than 30% of the weekdays in our sample are excluded, as thinly–traded. These are in most cases the very long–term option maturities (7 and 10 years) of the surface. Finally, to ensure that each IVS is continuous in the time domain, we discard parts of the sample that cause gaps of missing values longer than 4 weekdays. Applying the above three criteria ensures that in our reduced (both in maturities and in eligible weekdays) sample the entire surface under consideration is active. Table 2 reports the starting date, the number of weekdays and the longest option expiration (in years) remaining in our sample after the above criteria have been applied, as well as some descriptive statistics of the implied volatility surfaces.

Several different profiles of implied volatility surfaces are observed in our sample. As an indication, in Figures 1–4 the average IVS profile and the daily standard deviation of the IVS from EUR/USD and HUF/EUR options are plotted. In the EUR/USD case, the implied volatility surface exhibits a clear symmetric "smile" with an increasing term structure on average, and a fair amount of variability around this average profile (ranging from a fourth to a tenth of its typical value). In contrast, the HUF/EUR implied volatility surface exhibits a "skew", with either an increasing or a humped–shaped term structure, and a significantly asymmetric variability for short maturities. Similar patterns emerge in all currencies examined; to conserve space the corresponding figures for the remaining 25 currencies are relegated to Appendix C (available from the authors upon request).[4]

Given the origin of the data, one possible criticism is that idiosyncratic effects, specific to the market participant supplying the quotes, could influence the analysis. There are however reasons to believe that such effects (if any) are not strongly affecting our analysis. First, our focus here is on

[4]In all figures, the "moneyness" metric used is $\Delta = \left| |\Delta^{BS}| - \mathbf{1}_{\Delta^{BS}<0} \right| \times 100$, with Δ^{BS} as in (4) and $\mathbf{1}_x$ an indicator function that takes the value of one if condition x is true, and zero otherwise. It is simply a transformation of $\Delta^{BS} \in [-1, 1]$ to $\Delta \in [0, 100]$.

Currency Code	Start Date	No. of weekdays	Longest maturity	Implied Volatility (%)						
				Min.	Max.	Mean	Median	St. Dev.	Skewness	Kurtosis
AUD	08-Sep-2000	1729	5	4.122	19.264	10.049	9.955	2.256	0.410	3.166
BRL	08-Apr-2003	1064	3	5.138	52.450	18.046	17.366	4.941	0.642	3.486
CAD	03-Nov-2003	883	5	5.227	14.428	9.050	9.101	1.009	0.075	3.466
CHF	11-Jul-2000	1783	7	2.130	11.221	4.323	4.247	1.043	0.528	2.972
CLP	08-Dec-2004	638	1.5	5.152	22.882	11.746	11.333	2.848	0.704	3.261
CZK	05-Sep-2000	1744	10	2.550	20.757	6.130	6.051	1.462	0.988	6.144
GBP	05-Sep-2000	1744	5	2.953	13.961	7.099	6.938	1.701	0.744	3.573
HKD	05-Dec-2005	381	5	4.235	11.474	8.292	8.590	1.374	-0.263	1.989
HUF	05-Dec-2005	381	5	4.247	19.090	9.405	8.796	2.347	0.665	2.922
IDR	05-Apr-2005	555	1.5	4.225	20.830	11.448	11.194	2.921	0.348	2.633
INR	05-Dec-2005	381	5	5.432	14.596	8.692	8.876	1.208	-0.130	2.259
JPY	04-Sep-2000	1745	5	4.028	22.730	10.414	9.848	2.610	0.967	3.643
KRW	27-Apr-1999	2082	1.5	4.300	31.312	12.287	12.092	3.942	0.810	4.392
MXN	02-Jan-2006	361	5	6.633	19.163	11.341	11.000	2.053	0.706	3.410
NOK	04-Sep-2000	1745	1.5	3.819	12.993	6.509	6.299	0.990	0.980	4.386
NZD	05-Dec-2005	381	5	6.919	15.232	10.009	9.925	0.931	0.573	3.813
PLN	05-Dec-2005	381	5	4.596	16.428	8.473	8.400	1.523	0.413	3.130
RUB	03-Jan-2006	359	10	3.256	13.686	7.985	8.110	1.823	0.063	2.902
SEK	05-Sep-2000	1744	5	2.738	16.865	6.019	5.808	1.336	0.682	3.620
SGD	05-Dec-2005	381	5	4.049	9.433	6.957	7.138	1.110	-0.218	2.043
SKK	08-Apr-2003	1064	1.5	1.574	12.478	6.008	5.865	1.275	0.377	3.309
TRY	14-Nov-2000	1688	1.5	4.725	50.007	22.435	21.414	9.445	0.560	2.583
TWD	05-Dec-2005	381	5	4.203	11.610	7.845	7.863	1.248	0.025	2.665
USD	04-Sep-2000	1745	5	4.681	18.498	10.442	10.383	1.936	0.156	3.420
ZAR	05-Dec-2005	381	5	8.000	35.843	14.500	13.723	3.126	1.159	4.631

Table 2: For each of the twenty five different currency options in our sample, the table reports the starting date, the number of trading days in the time series, the longest option expiration of the surface (in years), and descriptive statistics of the implied volatilities. The end date in all time series is 21/5/2007.

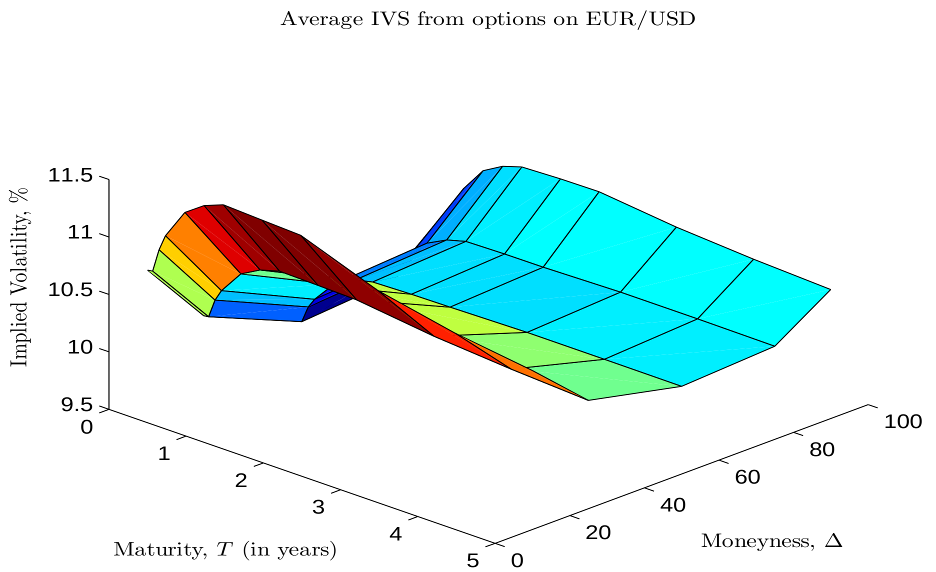

Figure 1: Average implied volatility surface from EUR/USD options, for the period 4/9/2000–21/5/2007.

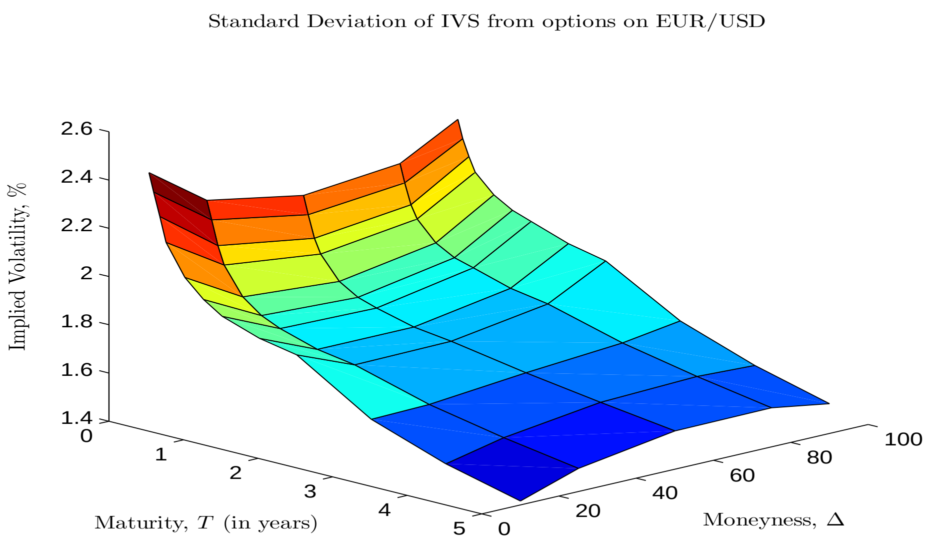

Figure 2: Daily standard deviation of EUR/USD implied volatilities as a function of moneyness and time to maturity for the period 4/9/2000–21/5/2007.

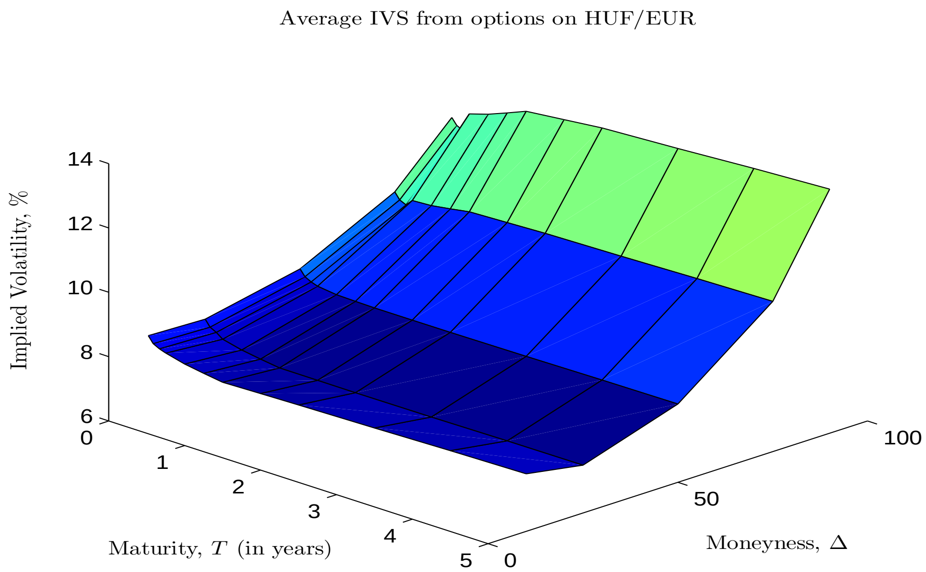

Figure 3: Average implied volatility surface from HUF/EUR options, for the period 5/12/2003–21/5/2007.

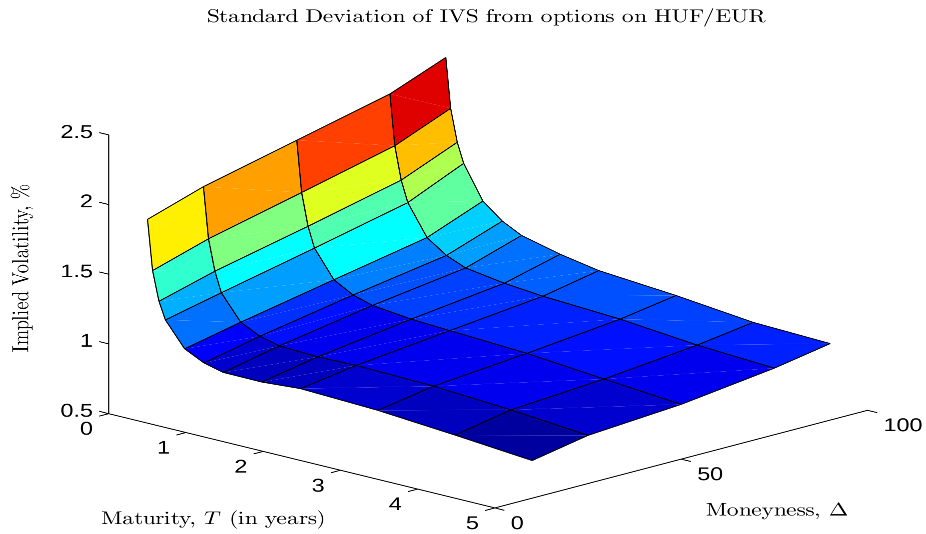

Figure 4: Daily standard deviation of HUF/EUR implied volatilities as a function of moneyness and time to maturity for the period 5/12/2005–21/5/2007.

systematic factors in the volatility surface, not on specific events or outliers of the surface. Secondly, given the liquidity of the market and the size of the market participant supplying the data, it should be fairly unlikely that our data are substantially away from typical values. Cross–checking a randomly selected sub–sample of our data set with the implied volatility quotes from another data vendor (Bloomberg) reveals that this is indeed the case.

Of course using OTC data has many advantages in comparison to exchange–traded data. Besides superior liquidity, OTC currency options are available for longer maturities than the currency options traded in exchanges. Moreover, OTC options have a constant time–to–maturity, unlike exchange–traded options whose maturity varies from day to day. In practical terms, this alleviates the need for grouping options into maturity bins (see for example Skiadopoulos et. al. (1999)) or for creating synthetic *fixed–maturity* series via interpolation (as in Alexander (2001)). This should translate to less noisy IVS's and more precision in the identification of factors affecting their dynamics. Similar OTC currency options data have been used in previous studies by Campa and Chang (1995), (1998), Carr and Wu (2007a), (2007b) and Christoffersen and Mazzotta (2005); the latter study actually concludes that OTC currency options data are of superior quality for volatility forecasting purposes.

2.2 Factor representation of the implied volatility surface

For each exchange rate, we have τ time series observations of N cross–section units of implied volatility, each unit referring to an option with a different "moneyness" and time–to–maturity.

We consider a static factor representation of the implied volatilities σ_{it} $(i = 1, \ldots, N, t = 1, \ldots \tau)$,

$$\sigma_{it} = \lambda_{i1} F_{1t} + \cdots + \lambda_{ir} F_{rt} + e_{it} = \lambda_i' F_t + e_{it} \tag{5}$$

where F_t is a vector of r common factors, λ_i is the corresponding vector of loadings for implied volatility i, and e_{it} is an idiosyncratic error. In the sense of Chamberlain and Rothschild (1983) "approximate factor model", it is assumed that factors and idiosyncratic disturbances are mutually uncorrelated, $\mathbb{E}\left(F_t e_{is}\right) = 0$ for all t, s, but weak cross–section correlation in e_{it} is allowed, as long as $\frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \left|\mathbb{E}\left(e_{it} e_{jt}\right)\right|$ is bounded. In this static representation, dynamics can be entertained by allowing both factors and idiosyncratic errors to be serially correlated.[5]

As established by Stock and Watson (2002) and Bai and Ng (2002), the method of asymptotic principal components (see Connor and Korajzcyk (1986)) can be used to consistently estimate the common factors. Starting with an arbitrary number of factor $k < \min(\tau, N)$, estimates of λ^k and F^k can be obtained by solving

$$J(k) = \min_{\Lambda^k, F^k} \frac{1}{N\tau} \sum_{i=1}^{N} \sum_{t=1}^{\tau} \left(\sigma_{it} - \lambda_i^k F_t^k\right)^2 \tag{6}$$

subject to the normalisation $\Lambda^{k\prime}\Lambda^k/N = \mathbb{I}_k$, with $\Lambda^k = \left[\lambda_1^k \ldots \lambda_N^k\right]$ the $k \times N$ matrix of loadings when k factors are allowed in the estimation.

One solution of the above optimisation is given by $\left(\widehat{\Lambda}^k, \widehat{F}^k\right)$, where $\widehat{\Lambda}^k$ is $\sqrt{N}$ times the eigenvectors corresponding to the k largest eigenvalues of the $N \times N$ matrix $[\sigma_{it}]' [\sigma_{it}] \equiv V'V$, and $\widehat{F}^k = V\widehat{\Lambda}^k/N$.[6]

The design of appropriate formal tests that can determine the number of static approximate factors in panel data is still an open research question. Bai and Ng (2002) have proposed a number of information criteria for this purpose; however these require stationarity of the panel data–which is not the case in most of the surfaces in our sample–and tend to overestimate the number of factors in small panels.

Since our focus is mainly on out–of–sample predictions of the IVS and not in–sample fitting performance, we have decided to follow a simple, common rule of setting the number of factors $\widehat{r} = k = 3$ across all surfaces. Although an ad hoc choice, several studies in the literature have offered evidence of 2 to 3 static factors driving the dynamics of surfaces implied from index options (Skiadopoulos et. al. (1999), Mixon (2002)) or options on futures contracts (Tompkins (2001)).

Estimation results are summarised in Table 3, that reports the proportion of the total variance of $V'V$ explained by three factors in–sample. As an indication of the adequacy of three factors, the Table also reports the

[5] This static factor representation is to be distinguished from the dynamic factor model of Forni, Hallin, Lippi and Reichlin (2005); however, any dynamic factor model admits a static representation like equation (5), and Bai and Ng (2007) make precise the relationship between dynamic and static factor representations.

[6] As the factors and loadings cannot be separately identified, the solution $\left(\widehat{\Lambda}^k, \widehat{F}^k\right)$ of (6) is not unique, although the minimum $J(k)$ is. As suggested by Bai and Ng (2002) however, this solution is efficient and computationally less costly when $\tau > N$, which holds for all the implied volatility surfaces in our sample.

Currency	Total Variance	% Variance Explained			G–K
Code	Explained (%)	$\widehat{F}_1$	$\widehat{F}_2$	$\widehat{F}_3$	$\overline{\lambda}$
AUD	90.82	74.44	10.81	5.57	1
BRL	91.06	69.32	12.42	9.32	2
CAD	92.00	74.88	11.85	5.27	3
CHF	91.69	73.49	10.16	8.04	4
CLP	88.17	60.54	17.72	9.91	2
CZK	86.04	64.21	13.67	8.16	4
GBP	89.29	70.39	13.26	5.64	2
HKD	94.77	83.54	7.91	3.32	3
HUF	95.69	71.10	16.09	8.50	3
IDR	91.18	73.42	10.63	7.13	3
INR	95.52	73.03	17.45	5.04	2
JPY	91.67	75.84	10.27	5.56	2
KRW	85.76	57.27	18.59	9.90	2
MXN	90.39	67.23	13.43	9.73	2
NOK	91.36	68.52	13.75	9.09	3
NZD	93.38	78.93	10.49	3.96	2
PLN	93.93	78.13	9.89	5.91	2
RUB	95.90	84.02	8.53	3.35	3
SEK	85.91	60.81	14.45	10.65	3
SGD	91.32	74.10	12.18	5.04	3
SKK	86.36	62.44	16.13	7.79	3
TRY	89.66	57.39	25.19	7.08	2
TWD	83.85	49.61	27.51	6.73	2
USD	92.87	75.18	13.38	4.31	2
ZAR	91.99	78.35	7.59	6.05	2
Average	91.00	70.20	14.08	6.72	2.5

Table 3: For each of the 25 different currency options in our sample, the table presents the proportion of the total variance of implied volatility surface explained by the variation of a given factor. Under G–K, the mean eigevalue rule of thumb $\overline{\lambda}$ of Guttman and Kaiser is reported.

Guttman–Kaiser criterion (also known as the mean eigenvalue rule of thumb) which sets k equal to the number of eigenvalues of $V'V$ that are larger that the average of all eigenvalues.

On average, three factors can explain 91% of the variance in the daily volatility surface implied by currency options. The proportion of variance explained ranges from a minimum of 83.85% for options on TWD/EUR to a maximum of 95.90% in the RUB/EUR case.

The first factor accounts for 70.20% of the IVS variation across currencies on average; it can range from 49.61% (TWD/EUR) to 84.02% (RUB/EUR). In all but three of the currency options examined it can explain more than 60% of the IVS. The second and third factors contribute, on average, an additional 14.08% and 6.72% respectively.

To get a better feeling of the results, the estimated three–factor loadings are plotted against moneyness and time–to–maturity in Figure 5 for two representative cases: options on the EUR/USD (upper panels) and HUF/EUR (lower panels) exchange rates.

The character of the factors seems intuitive. Factor 1 in Panels (a) and (d) represent shocks that affects all maturities and deltas of the surface in the same direction (same sign). The effect is strongest at the short horizons and it dampens over time. It can be interpreted as a *level effect*, and it is consistent with a mean–reverting model of stochastic volatility. Factor 1 affects OTM and ATM volatility differently. This is consistent with the notion that a change in volatility alters the steepness of the "smile" and correspondingly the skewness of the implied risk neutral density.

Factor 2 affects short–term and long–term impled volatility with different signs (it appears to change sign around the 6–month and the 3–month option maturity for EUR/USD and HUF/EUR respectively). Thus, this factor separates between different ends of the volatility term structure, i.e. it is a *term–structure effect*. The effect is almost uniform across the moneyness dimension.

Finally, the third factor appears to change sign ATM. It separates the effect between OTM puts and calls and it is present in all maturities. However its effect is more pronounced for short–dated options. Changes along this factor alter the steepness of the implied volatility smile; it can be interpreted as a *jump–fear effect*. Similar factors emerge in all currency options examined and have also been reported in investigations of surfaces from index options (e.g. Mixon (2002)) and futures options (e.g. Tompkins (2001)).

Examination of the factors suggests that they fluctuate significantly over time and are persistent. Descriptive statistics of the factors from all currency

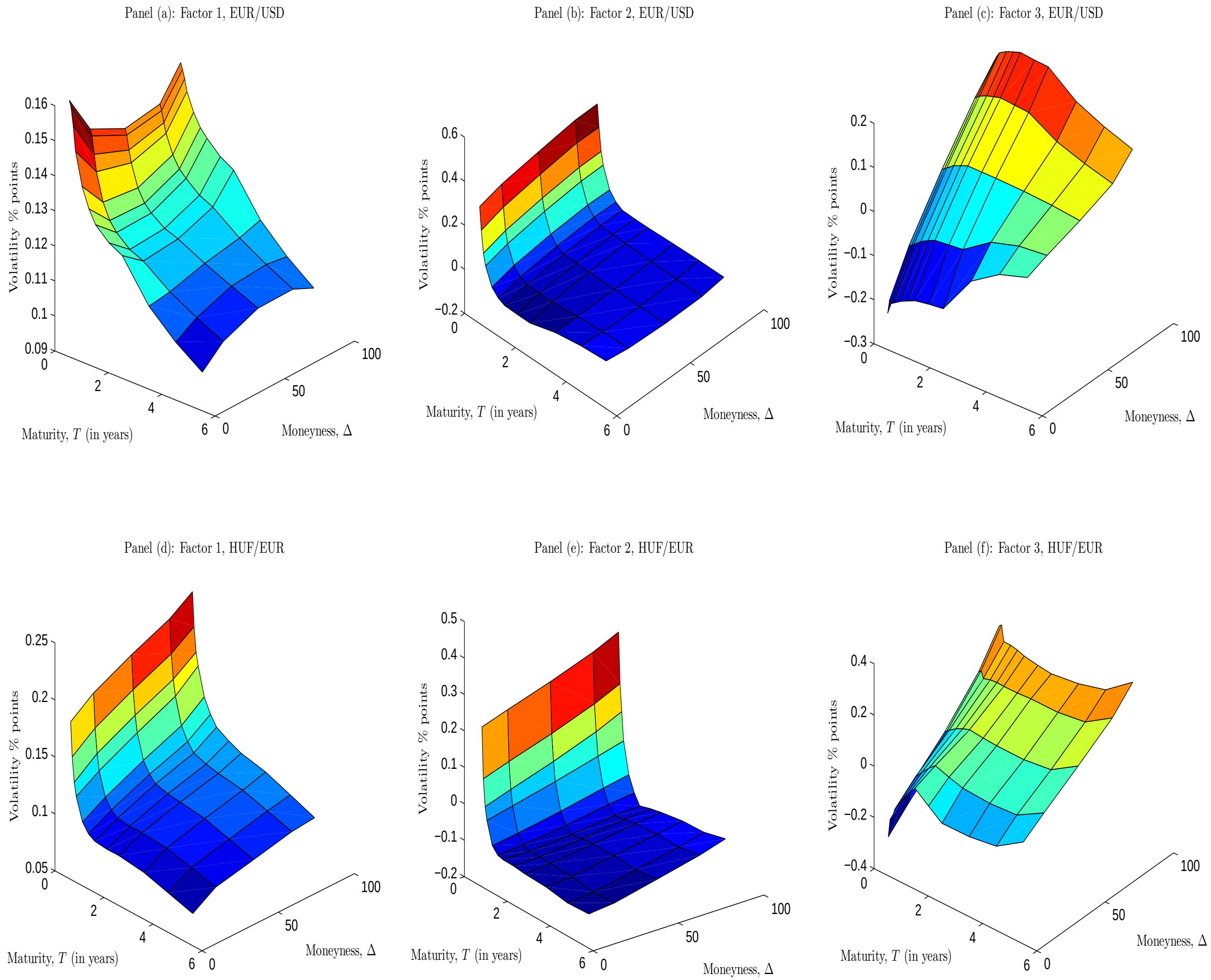

Figure 5: Loadings of the three static factors that are identified in the daily time series of the volatility surface implied by options on the EUR/USD (Panels (a)–(c)) and the HUF/EUR (Panels (d)–(f)) exchange rates respectively.

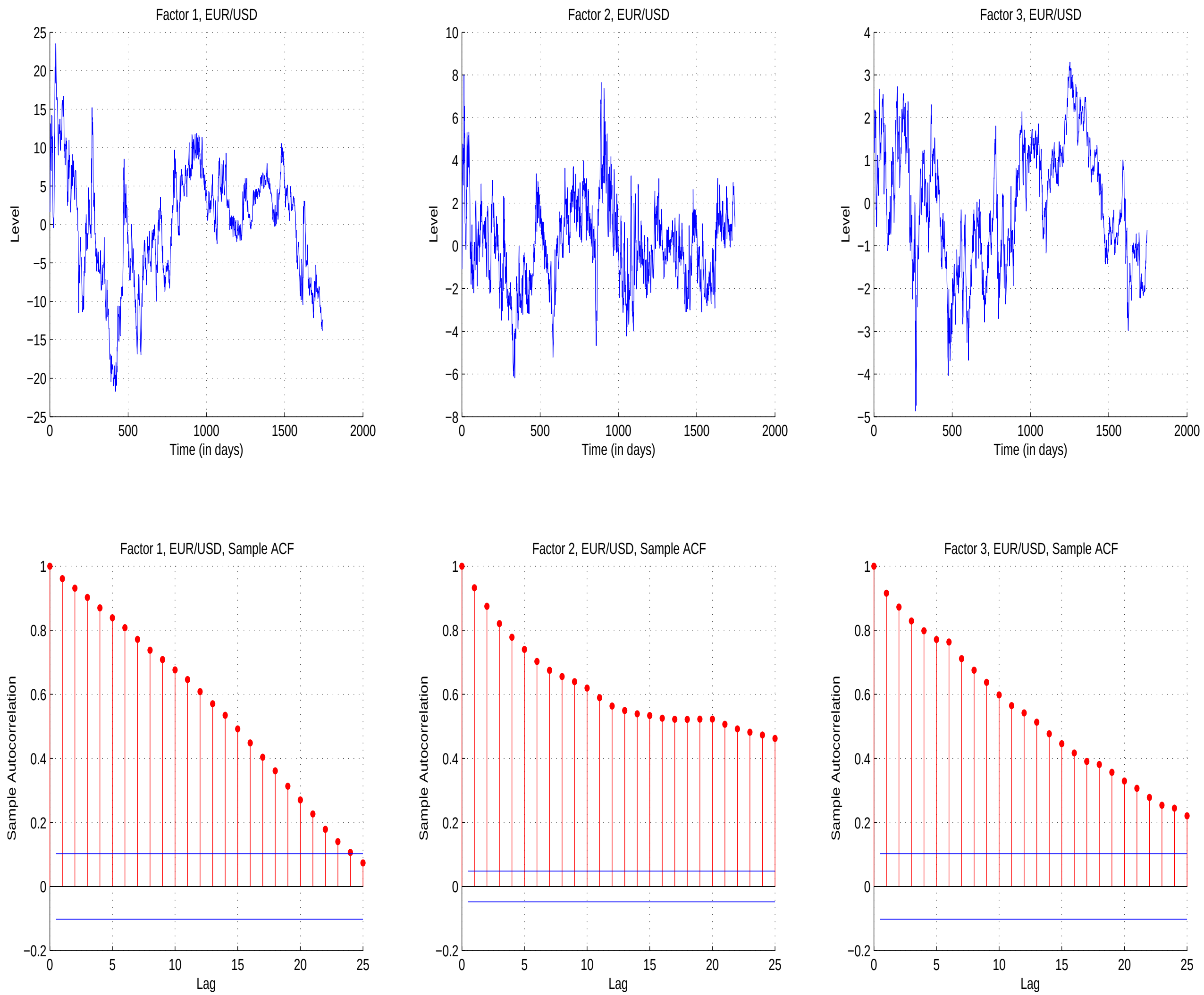

Figure 6: Factors identified in the daily time series of the volatility surface implied by options on the EUR/USD exchange rate from 4/9/2000–21/5/2007, and their autocorrelation (ACF) for up to 25 lags. The blue and red horizontal lines in the ACF graphs correspond to the $\alpha = 5\%$ significance level.

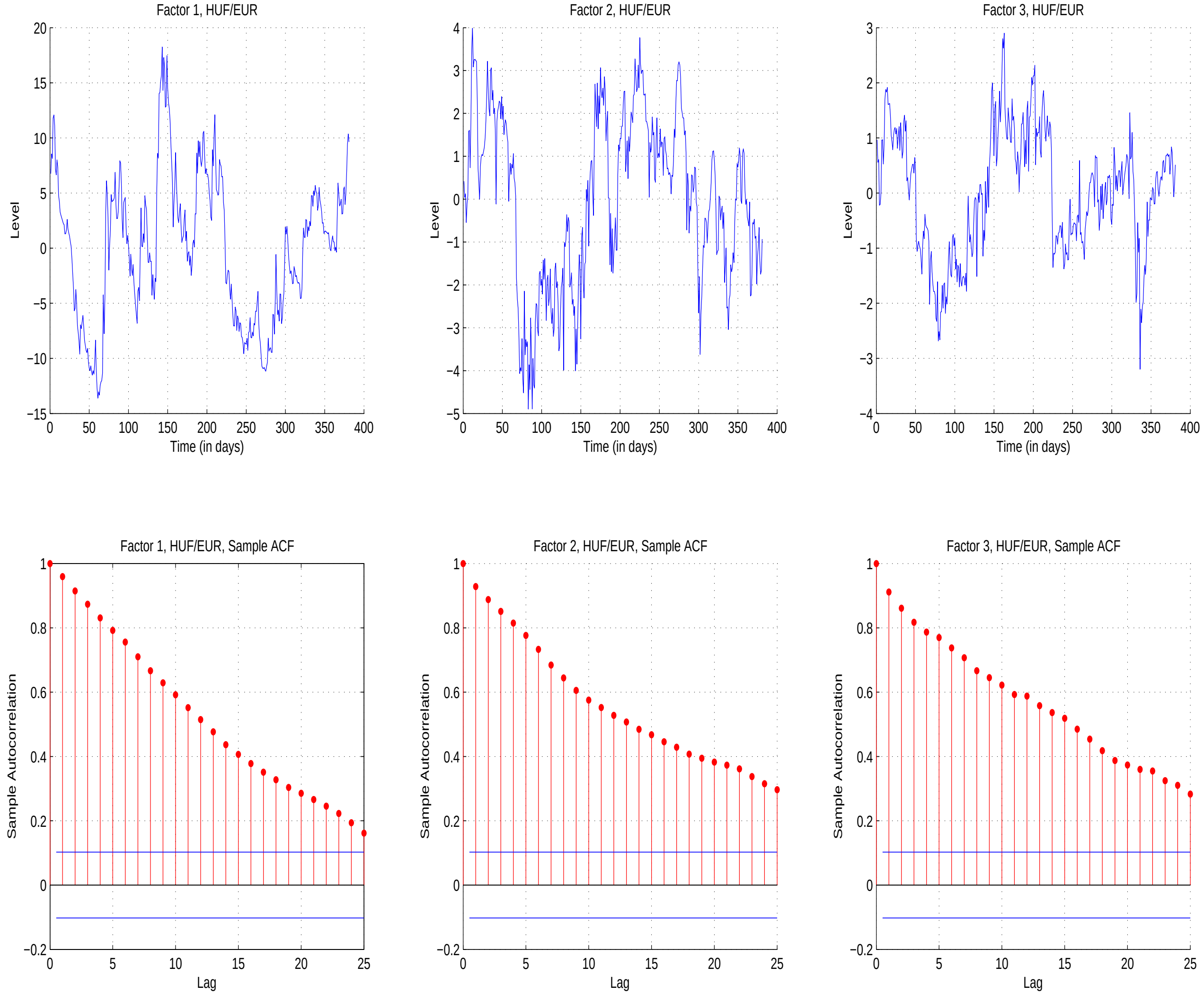

Figure 7: Factor identified in the daily time series of the volatility surface implied by options on the HUF/EUR exchange rate from 5/12/2005–21/5/2007, and their autocorrelation (ACF) for up to 25 lags. The blue and red horizontal lines in the ACF graphs correspond to the $\alpha = 5\%$ significance level.

options in our sample are relegated in Appendix A. However, an indication of the factors and their persistence is offered by Figures 6 and 7 (from EUR/USD and HUF/EUR options again) that plot the factors over time and their sample autocorrelations. In the next section, we turn our attention to the in–sample modeling of the time–series dynamics of the factors, with a view towards forecasting future implied volatility out–of–sample.

3 Modeling the time–variation of implied volatility surfaces in sample

In this section we model the time variation in the IVS as captured by the dynamics of factors identified in the cross–sectional analysis that preceded.

As Bai and Ng (2004) point out, the principal components estimators of F_t and λ_i reported in the previous section are consistent, as long as e_{it} in (5) is $I(0)$ (and regardless of whether all or some of the factors are $I(0)$). They actually go on to establish how consistent estimation of the factors can be accomplished through a "differencing and recumulating" procedure that can accommodate $I(0)$ and $I(1)$ errors. Thus, for our purposes, it suffices to establish stationarity of the idiosyncratic errors from (5) for the factors to be consistent.

Several tests for common and individual unit roots in the idiosyncratic errors are performed; to conserve space, results are reported in Table B.1 in Appendix B. The uniform conclusion that can be drawn is that the idiosyncratic disturbances from the factor representation of the IVS in our sample are stationary, and hence the identified factors are consistently estimated.

In order to model the dynamics of the factors, we consider a vector autoregressive (VAR) model for the time series of $\widehat{F}_t = \left(\widehat{F}_{1,t}, \widehat{F}_{2,t}, \widehat{F}_{3,t}\right)'$ of the form:

$$\widehat{F}_t = c + \sum_{j=1}^{d} \Phi_j \widehat{F}_{t-j} + v_t \tag{7}$$

where $v_t \sim \mathcal{N}_{\text{i.i.d.}}(0, \Omega)$.[7] If the state variables that control the dynamics

[7] In results unreported here, several alternative specifications for the factors and their first differences have been examined. These include error–correction models for the factors, VAR models of factor first–differences, as well as simple univariate autoregressive specifications of the factors and of the factor first–differences. The VAR factor model in (7) that we employ in the remainder of the paper, performs best across all surfaces, both in–sample and in terms of out–of–sample short–term forecasting. More details, as well as

underlying the fundamentals in general equilibrium option pricing models are persistent, then a VAR specification of the factors might be a reasonable parsimonious way of capturing the predictability in the IVS that these models imply.

It should be stressed that the unrestricted VAR model that we propose and estimate for the factors does not impose any no–arbitrage restrictions on the resulting, factor–based, implied volatilities. Although such restrictions could potentially improve the fitting and forecasting performance of the model, imposing them would require specification of a structural model of implied volatilities, which is beyond the scope of the present paper.[8]

Equation (7) is estimated for all surfaces by OLS, equation by equation, with d selected by the Bayesian Information Criterion of Schwarz (1978), starting with a maximum value of $d = 15$.

The results, summarised in Table 4, suggest that in all currencies a fairly parsimonious VAR specification can achieve an extremely good in–sample fit. The adjusted R^2's range from 54.26% (NZD/EUR Factor 3) to 98.67% (GBP/EUR Factor 1), with an average of 87.81%. Across currencies, the average adjusted R^2's are 93.16%, 86.61% and 83.67% for factor 1, 2 and 3 respectively. The Ljung–Box $LB\,(d)$ lack–of–fit statistic in Table 4 suggests that in the majority of cases the in–sample fit is fairly good, with uncorrelated residuals. More specifically, in 4/5 of the cases the fit seems adequate at the 5% level; in 11 out of the 75 equations fitted is the null rejected at the 1%.

Table 5, that reports Granger–causality tests as implied by the VAR estimation, provides evidence of significant (at the 1%) off–diagonal elements of Φ_j in (7). For example, the tickmarks in JPY suggest that both the "level" and the "jump–fear" factor is Granger–caused by the "term–structure" of the IVS. In only one surface (INR) are the off–diagonal terms in Φ_j redundant when compared with simple univariate autoregressive specifications of the factors. This suggests that lagged realisations of one factor (e.g. the "*jump–fear*") might influence the current realisation of another factor (e.g. the "*level*"), and is consistent with empirical observations regarding the IVS,

indications regarding the relative performance of all examined specifications can be found in Chalamandaris and Tsekrekos (2009) that attempt an extensive comparison of short and long–term forecasting methods.

[8]Moreover, since our focus here is on identifying economically significant short–term predictability of segments of the IVS, the time horizons for which such predictability is detected can be thought of as indications of the time period required for the whole surface to reach its no–arbitrage equilibrium state, as new information is incorporated to the IVS through trading across the surface at different intensities.

Factor	Code	d	$R^2_{adj.}$	$LB\,(d)$	Code	d	$R^2_{adj.}$	$LB\,(d)$	Code	d	$R^2_{adj.}$	$LB\,(d)$
1	AUD	2	0.9579	0.0209*	IDR	2	0.9443	0.0380*	RUB	4	0.8138	0.0460*
2		2	0.8883	0.4911		2	0.9348	0.0087*		4	0.6188	0.0076*
3		2	0.9167	0.6498		2	0.9635	0.7876		4	0.8762	0.7248
1	BRL	2	0.9603	0.8232	INR	2	0.9285	0.8156	SEK	3	0.9695	0.0660
2		2	0.8926	0.0008*		2	0.8470	0.6199		3	0.9369	0.1557
3		2	0.7827	0.0319*		2	0.8415	0.0530		3	0.9127	0.0000*
1	CAD	3	0.9215	0.7634	JPY	2	0.9774	0.0749	SGD	2	0.8398	0.8513
2		3	0.8051	0.9997		2	0.9600	0.8978		2	0.8538	0.4445
3		3	0.7253	0.9683		2	0.9103	0.5446		2	0.8803	0.6633
1	CHF	3	0.9438	0.9590	KRW	3	0.9782	0.7274	SKK	2	0.9514	0.0374*
2		3	0.8997	0.2282		3	0.9607	0.9871		2	0.9081	0.0643
3		3	0.9403	0.0975		3	0.9118	0.1470		3	0.6863	0.0018*
1	CLP	6	0.9061	0.9969	MXN	2	0.9620	0.7380	TRY	3	0.9753	0.9926
2		6	0.8788	0.4361		2	0.7747	0.5424		3	0.9033	0.5372
3		6	0.9573	0.9970		2	0.7057	0.0095*		3	0.6863	0.0018*
1	CZK	3	0.9574	0.9996	NOK	6	0.9699	0.6234	TWD	2	0.7255	0.0280*
2		3	0.9090	0.9958		6	0.8245	1.0000		2	0.7513	0.9841
3		3	0.9323	0.0081*		6	0.8717	0.8506		2	0.7975	0.5843
1	GBP	2	0.9867	0.5498	NZD	2	0.9343	0.3477	USD	2	0.9754	0.0000*
2		2	0.9440	0.0001*		2	0.8100	0.1670		2	0.8703	0.9509
3		2	0.9325	0.0001*		2	0.5426	0.9853		2	0.9574	0.6501
1	HKD	2	0.9081	0.6536	PLN	2	0.9329	0.9111	ZAR	2	0.9423	0.8177
2		2	0.8751	0.6940		2	0.8978	0.8261		2	0.9274	0.3140
3		2	0.6605	0.5487		2	0.7269	0.9332		2	0.7435	0.3302
1	HUF	2	0.9280	0.8787								
2		2	0.8728	0.3112								
3		2	0.8416	0.6479								

Table 4: For the volatility surface implied by the 25 different currency options in our sample, the table reports the results from the estimation of the VAR model (equation (7)) on the identified factors. The lag length, d, is selected by the Bayesian Information Criterion of Schwarz (1978), starting with a maximum value of $d = 15$. Under $LB\,(d)$, p–values for the Ljung–Box statistic (H_0 : absence of autocorrelation up to lag d in the residuals) are reported. The length of the time series for each currency is reported in Table 2.
An * denotes that the null is rejected at the $\alpha = 5\%$ significance level.

	$\widehat{F}_{1,t}$		$\widehat{F}_{2,t}$		$\widehat{F}_{3,t}$	
Currency	is Granger–caused at $\alpha = 1\%$ by					
Code	$\widehat{F}_{2,t}$	$\widehat{F}_{3,t}$	$\widehat{F}_{1,t}$	$\widehat{F}_{3,t}$	$\widehat{F}_{1,t}$	$\widehat{F}_{2,t}$
AUD					✓	
BRL		✓	✓	✓	✓	✓
CAD		✓		✓		✓
CHF			✓	✓		
CLP	✓					
CZK	✓			✓	✓	✓
GBP					✓	
HKD					✓	
HUF			✓			
IDR	✓				✓	✓
INR						
JPY	✓					✓
KRW			✓			
MXN	✓	✓	✓			
NOK	✓	✓	✓			
NZD				✓		
PLN				✓	✓	
RUB						
SEK	✓			✓	✓	✓
SGD					✓	
SKK	✓	✓	✓	✓	✓	
TRY	✓	✓		✓		✓
TWD		✓			✓	✓
USD	✓					
ZAR					✓	

Table 5: For the volatility surface implied by the 25 different currency options in our sample, the table reports the results of Granger–causality tests from the estimation of the VAR factor model (equation (7)). A ✓ denotes evidence of Granger–causality at the $\alpha = 1\%$ significance level.

such as that when the level increases, the steepness of the smile decreases, etc.

4 Out–of–sample forecasting performance

A simple factor decomposition of the implied volatility surface like equation (5), and a parsimonious model of its factor dynamics like equation (7) should not only fit well in–sample, but also produce accurate out–of–sample forecasts.

From equation (5), it is clear that predictability of σ_{it} requires that either the factors, F_t, and/or the idiosyncratic errors, e_{it} are predictable. In this section we investigate whether the VAR factor model estimated in the previous section can produce accurate out–of–sample forecasts of the whole implied volatility surface h–periods ahead, i.e. $\widehat{\sigma}_{it+h}$. If these predictions are accurate, then modeling the systematic component of the IVS, as captured by approximate static factor representations, can lead to better risk management and portfolio formation decisions as we show in the next section.

We set up this forecasting exercise as follows: For each IVS, starting from the dates in Table 2, the first 100 daily surfaces are employed to extract the factors $\widehat{F}_t$ and loadings $\widehat{\Lambda} = \left[\widehat{\lambda}_1 \dots \widehat{\lambda}_N\right]$, as in equation (6). Again, three factors, $k = 3$, are used in all surfaces.

Estimates of the VAR specification in (7) on the time series of factors $\widehat{F}_t$ are used to produce *direct* forecasts of $\widehat{F}_{t+h}$ (see Boivin and Ng (2005)): Let $\widehat{\beta}^h_{\widehat{d}}$ be the coefficients (including constant terms) of a projection of $\widehat{F}_t$ on $\widehat{F}_{t-h}$ and $\widehat{d}$ of its lags. Obviously, for one–step ahead forecasts, $h = 1$, the coefficients $\widehat{\beta}^1_{\widehat{d}}$ include $\widehat{c}$ and $\widehat{\Phi}_j$, with $j = 1, \dots, \widehat{d}$. Analogously, h–step ahead forecasts of the factors are produced by

$$\widehat{F}_{t+h} = \widehat{\beta}^h_{\widehat{d}} \widehat{F}_t, \tag{8}$$

and since the entire IVS on day $t + h$ depends on $\widehat{F}_{t+h}$, forecasts of the IVS can be produced by

$$\widehat{\sigma}_{it+h} = \widehat{\lambda}'_i \widehat{F}_{t+h} = \widehat{\lambda}'_i \widehat{\beta}^h_{\widehat{d}} \widehat{F}_t \tag{9}$$

These are then compared with σ_{it+h}, the actual IVS at time $t + h$. After the comparison, the realised period is included in the estimation sample, and the previous steps are repeated. In essence, this is the methodology of Stock and Watson (2002) that has been successful in the context of macroeconomic forecasting.

We concentrate on short–horizon forecasts, $h = 1, 2, \ldots, 5$ days ahead, where predictable dynamics have been difficult to establish in related studies, and use a "random walk" (RW) model of factors as a benchmark. This corresponds to a special case of equation (7), with $c = \mathbf{0}$, $d = 1$, $\Phi_1 = \mathbb{I}_3$ a 3×3 identity matrix, $\Phi_j = \mathbf{0}$ for $j = 2, \ldots, d$ and Ω a diagonal matrix.[9] It essentially implies that given the levels of factor estimates today, $\widehat{F}_t$, the best estimate of the IVS h–days ahead is $\widehat{\sigma}_{it+h} = \widehat{\lambda}_i' \widehat{F}_t$. If however factors exhibit short–term predictability, the IVS predictions based on the VAR model factor forecasts, $\widehat{F}_{t+h}$, would exhibit lower errors relative to the ones produced by the RW factor model.

To assess out–of–sample forecasting performance, the following three measures are computed each day t, for each model (VAR and RW):
[a] Mean squared error of implied volatility forecasts,

$$MSE_t = \frac{1}{N} \sum_{i=1}^{N} \left(\widehat{\sigma}_{it+h} - \sigma_{it+h}\right)^2$$

i.e. the average squared deviations of observed implied volatilities from the model's predicted implied volatilities,
[b] Coefficient of determination

$$R_t^2 = R^2_{\sigma_{it+h} \rightarrow \widehat{\sigma}_{it+h}}$$

i.e. the R^2 from a univariate regression of the actual implied volatilities σ_{it+h} of the surface on the model–predicted implied volatilities $\widehat{\sigma}_{it+h}$, and
[c] Mean correct prediction of the direction of change in implied volatilities,

$$MCP_t = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{sgn\{\sigma_{it+h} - \sigma_{it}\} = sgn\{\widehat{\sigma}_{it+h} - \sigma_{it}\}}$$

the percentage of the surface implied volatilities for which the model correctly predicted the sign of change h–days ahead.

The average values of the performance measures [a]–[c] across all out–of–sample days, i.e.

$$\overline{MSE} = \frac{1}{\tau - 100} \sum_{t=101}^{\tau} MSE_t \quad (10)$$

$$\overline{R^2} = \frac{1}{\tau - 100} \sum_{t=101}^{\tau} R_t^2 \quad (11)$$

[9] Alternatively stated, this corresponds to setting $\widehat{d} = 1$ and $\widehat{\beta}^h_{\widehat{d}} = \mathbb{I}_3$ in equation (8) for all h.

$$\overline{MCP} = \frac{1}{\tau - 100} \sum_{t=101}^{\tau} MCP_t \qquad (12)$$

are summarised in Table 6, for one and two–day ahead predictions.

The VAR factor model performs notably better than the (special case) random walk: average mean squared errors are most of the times two or three–and in some occasions even six–times less those of the RW model. Concentrating on the most liquid currencies (USD, JPY, GBP, CAD, CHF) for $h = 1$, the $\overline{MSE}$ of VAR is 0.0218, 0.0245, 0.0249, 0.0207, 0.0460, a marked improvement over the performance of the random walk model (0.0356, 0.0431, 0.0409, 0.609, 0.0594). In all cases our approach outperforms, in terms of average MSEs, the random walk benchmark.

A similar conclusion is reached when the second measure, the $\overline{R^2}$ from univariate regressions of implied volatilities on the model–predicted ones, is examined. There is a distinct improvement in explanatory power, ranging from just 0.23% (KRW, $h = 2$) to an impressive 52.17% (NZD, $h = 1$). Across all currencies, the average $\overline{R^2}$ improvement is 17.44% and 14.28% for one and two–days ahead respectively. In terms of the $\overline{MCP}$ measure, the VAR factor model performs better, by 2.5%–4% in most cases, than the 50% benchmark in correctly predicting the direction of change in the factors driving the IVS dynamics. In only the CLP/EUR case does our approach perform worse than flipping a coin. Taking into account that the MCP_t's (like all other performance measures examined) are averaged across (in excess of) 1500 days for some currencies (AUD, CHF, JPY, GBP, USD, NOK, etc.), this constitutes a distinct improvement in terms of correctly–predicted days.

We also employ the equal predictive ability test of Diebold and Mariano (1995), in order to formally assess the statistical significance of the superior out–of–sample performance of the VAR model over the RW. The difference in squared forecast errors of the two models is used as a performance indicator for the test. This is conducted on all surface segments $i = 1, \ldots, N$ separately.

In order to avoid cluttering the reader with tables, columns 7 and 13 in Table 6 report p–values for the Diebold and Mariano (1995) test for only one segment of the surface, namely the 1–year ATM implied volatility.[10] The tabulated p–values indicate that with the exception of KRW and TRY, we can reject the null hypothesis of equal forecasting ability between VAR and RW,

[10] Again, all unreported results of the test are available in Appendix C, which is available from the authors upon request.

Currency	One–day ahead forecasts						Two–day ahead forecasts					
	$\overline{MSE}$		$\overline{R^2}$		$\overline{MCP}$	Acc.	$\overline{MSE}$		$\overline{R^2}$		$\overline{MCP}$	Acc.
Code	VAR	RW	VAR	RW	VAR	VAR	VAR	RW	VAR	RW	VAR	VAR
AUD	.0216	.0388	.9852	.9571	52.42	.0492	.0260	.0384	.9784	.9576	53.40	.0587
BRL	.0442	.0645	.9391	.8845	52.44	.0000	.0528	.0665	.9125	.8760	52.67	.0059
CAD	.0207	.0609	.9324	.6812	52.82	.0000	.0250	.0556	.9004	.7046	52.92	.0000
CHF	.0460	.0594	.9390	.9107	52.15	.0043	.0510	.0605	.9255	.9056	52.94	.0177
CLP	.0345	.0684	.9006	.7592	48.23	.0000	.0383	.0637	.8753	.7733	49.17	.0004
CZK	.0529	.0618	.9207	.9037	50.73	.0000	.0591	.0652	.9028	.8932	51.54	.0212
GBP	.0249	.0409	.9793	.9516	51.89	.0314	.0292	.0404	.9725	.9525	52.42	.0471
HKD	.0240	.0896	.9731	.7633	53.28	.0000	.0289	.0809	.9600	.7884	53.77	.0000
HUF	.0309	.1190	.9065	.4486	53.65	.0000	.0380	.1081	.8575	.4698	57.44	.0000
IDR	.0414	.0798	.9547	.8603	52.95	.0000	.0502	.0777	.9345	.8623	54.11	.0247
INR	.0270	.1072	.9455	.5744	51.76	.0000	.0326	.0961	.9189	.6114	50.76	.0000
JPY	.0245	.0431	.9839	.9549	52.18	.0322	.0298	.0430	.9766	.9552	52.52	.0153
KRW	.0435	.0477	.9774	.9739	51.60	.4832	.0499	.0526	.9708	.9685	52.63	.3728
MXN	.0302	.1039	.9489	.6491	53.32	.0000	.0346	.0937	.9363	.6813	53.83	.0004
NOK	.0288	.0463	.9555	.8993	54.00	.0443	.0352	.0476	.9325	.8933	55.11	.0482
NZD	.0175	.1069	.9484	.4267	54.93	.0000	.0237	.0951	.8984	.4543	57.03	.0000
PLN	.0233	.1001	.9711	.6677	52.17	.0008	.0275	.0895	.9589	.7042	54.72	.0003
RUB	.0225	.1010	.9022	.5447	51.95	.0000	.0249	.0890	.8898	.5757	53.76	.0007
SEK	.0412	.0564	.9576	.9226	51.88	.0741	.0472	.0579	.9444	.9188	52.14	.0832
SGD	.0238	.0918	.9668	.7252	54.40	.0000	.0288	.0826	.9495	.7525	55.36	.0001
SKK	.0424	.0753	.9192	.7887	52.98	.0207	.0495	.0739	.8896	.7932	53.73	.0255
TRY	.0606	.0681	.9735	.9675	51.82	.3826	.0702	.0758	.9642	.9595	51.62	.4305
TWD	.0295	.0908	.9092	.6485	52.89	.0000	.0301	.0815	.9032	.6738	52.62	.0000
USD	.0218	.0356	.9792	.9512	52.18	.0098	.0264	.0362	.9695	.9485	52.54	.0141
ZAR	.0371	.1207	.8910	.4866	54.01	.0000	.0505	.1118	.8204	.4997	55.03	.0000

Table 6: For the volatility surface implied by the 25 different currency options in our sample, the table reports out–of–sample average prediction errors for one and two–day ahead factor forecasts across models. VAR corresponds to equation (7), with d selected by the BIC criterion on estimation of the model on the first 100 observations of the IVS (starting with a maximum value of 15). RW stands for the random walk model that sets h–day's ahead IVS equal to today's level and $\overline{MSE}$, $\overline{R^2}$ and $\overline{MCP}$ are as in (10)–(12). Under Acc., p–values of the Diebold and Mariano (1995) test of equal predictive ability (against the RW model) are reported for the 1–year ATM segment of the surface.

both in one and in two–day ahead forecasts, for the 1–year ATM segment of the surface. Similarly, the unreported results reveal a statistically–significant superior predictive ability of the VAR factor model for *many* (if not most), but *not all segments* of the surface. This provides statistical evidence that the VAR factor model can better predict segments of the IVS from currency options in the short–run.

Similar in nature, although less encouraging results are reported in Table 7, that summarises the out–of–sample performance of the models when three and five–day ahead forecasts of the factors are attempted. The VAR factor model continues to outperform the RW benchmark for 3–day ahead forecasts, although its absolute performance declines. For example, in all cases the VAR factor model $\overline{MSE}$'s are lower, but only in 16 out of the 25 cases is this superiority statistically significant at conventional levels for the reported IVS segment (the 1–year, ATM implied volatility, and only in CAD, JPY and CHF among the most liquid currencies). It is not until 5–day ahead forecasts are attempted that our approach is not better than a simple random walk in $\overline{MSE}$'s. However, although the accuracy of the VAR forecasts deteriorates with the forecasting horizon, the model continues to better predict the direction of the change in the surface even 5–days ahead as the $\overline{MCP}$ in Tables 6 and 7 demonstrate.

Taken together, the results of the out–of–sample forecasting exercise suggest that the factors driving the dynamics of the volatility surfaces implied by OTC currency options exhibit moderate predictability in the short–run, that can be captured by a reduced–form vector autoregressive specification. However, whether the superior ability of the VAR factor model to better predict the *systematic component* of the IVS can actually lead to better forecasts of *all* implied volatilities σ_i of the surface critically depends on the magnitude of the idiosyncratic errors e_i.

It appears that in the short–run, *some segments* of the surface are more accurately predicted than others by the VAR factor model, since new information is incorporated into the IVS at different speeds, through different trading activity across the surface. As the next section demonstrates, being able to better predict the systematic surface component captured by the identified factors (and thus the "least idiosyncratic" IVS segments) can actually lead to better risk management and portfolio decisions.

Currency	Three–day ahead forecasts							Five–day ahead forecasts						
	$\overline{MSE}$		$\overline{R^2}$		$\overline{MCP}$	Acc.		$\overline{MSE}$		$\overline{R^2}$		$\overline{MCP}$	Acc.	
Code	VAR	RW	VAR	RW	VAR	VAR		VAR	RW	VAR	RW	VAR	VAR	
AUD	.0299	.0379	.9715	.9580	53.79	.1123		.0357	.0356	.9594	.9606	54.37	.4875	
BRL	.0601	.0678	.8846	.8687	52.82	.0319		.0730	.0698	.8237	.8530	52.79	.3296	
CAD	.0275	.0494	.8775	.7372	53.78	.0006		.0318	.0304	.8352	.8570	53.49	.3987	
CHF	.0551	.0608	.9132	.9025	52.85	.0819		.0636	.0618	.8844	.8941	53.79	.3074	
CLP	.0419	.0583	.8464	.7905	49.39	.0497		.0500	.0449	.7783	.8360	46.46	.2270	
CZK	.0648	.0684	.8840	.8823	52.49	.2713		.0745	.0738	.8464	.8617	53.80	.4036	
GBP	.0331	.0396	.9656	.9537	52.54	.1108		.0396	.0376	.9518	.9565	52.77	.3372	
HKD	.0330	.0703	.9472	.8224	53.94	.0000		.0403	.0394	.9210	.9244	51.84	.4283	
HUF	.0432	.0943	.8117	.5131	58.65	.0000		.0530	.0540	.7016	.7336	57.33	.1074	
IDR	.0603	.0768	.9065	.8597	55.00	.0964		.0795	.0759	.8386	.8466	56.35	.3719	
INR	.0374	.0834	.8913	.6608	51.63	.0007		.0450	.0446	.8382	.8534	47.62	.4472	
JPY	.0346	.0428	.9688	.9552	52.63	.0566		.0426	.0415	.9533	.9564	52.39	.4874	
KRW	.0549	.0563	.9648	.9638	53.08	.4014		.0633	.0626	.9531	.9551	53.06	.4924	
MXN	.0386	.0812	.9232	.7262	55.45	.0018		.0457	.0451	.8996	.8864	56.09	.4671	
NOK	.0407	.0485	.9075	.8874	55.32	.2981		.0494	.0490	.8573	.8776	55.89	.4572	
NZD	.0289	.0812	.8358	.4994	57.68	.0066		.0370	.0361	.6893	.7972	56.76	.4926	
PLN	.0308	.0766	.9473	.7555	56.02	.0034		.0378	.0381	.9170	.9222	58.19	.3036	
RUB	.0268	.0756	.8788	.6155	54.53	.0078		.0294	.0289	.8594	.8689	58.18	.2844	
SEK	.0529	.0591	.9299	.9153	51.91	.3518		.0613	.0596	.9044	.9125	52.26	.4193	
SGD	.0324	.0717	.9346	.7901	55.28	.0075		.0373	.0375	.9064	.9164	56.46	.4013	
SKK	.0560	.0715	.8559	.8020	54.78	.1486		.0657	.0648	.7915	.8285	55.27	.4721	
TRY	.0769	.0801	.9571	.9553	51.57	.4582		.0905	.0893	.9403	.9448	52.31	.3813	
TWD	.0332	.0706	.8790	.7088	50.94	.0164		.0358	.0379	.8516	.8567	44.13	.4712	
USD	.0308	.0370	.9588	.9447	52.98	.1769		.0376	.0373	.9398	.9396	53.41	.4190	
ZAR	.0631	.1006	.7467	.5294	55.31	.0318		.0909	.0691	.5870	.6959	58.12	.3201	

Table 7: For the volatility surface implied by the 25 different currency options in our sample, the table reports out–of–sample average prediction errors for three and five–day ahead factor forecasts across models. VAR corresponds to equation (7), with d selected by the BIC criterion on estimation of the model on the first 100 observations of the IVS (starting with a maximum value of 15). RW stands for the random walk model that sets h–day's ahead IVS equal to today's level and $\overline{MSE}$, $\overline{R^2}$ and $\overline{MCP}$ are as in (10)–(12). Under Acc., p–values of the Diebold and Mariano (1995) test of equal predictive ability (against the RW model) are reported for the 1–year ATM segment of the surface.

5 Economic significance of factor forecasts

The results of Section 4 suggest that the latent factors driving the dynamics of volatility surfaces implied by OTC currency options are predictable in a statistical sense. In this section we ask whether any profitable trading strategies could be devised based on these forecasts of the systematic IVS component captured by the factors.

To accomplish this, we examine whether certain trading rules, based on forecasts $\widehat{F}_{t+h}$ by the VAR model, may generate significant profits for a hypothetical speculator.

The trading strategies we consider are based on out–of–sample forecasts of *implied volatility*, as produced by out–of–sample forecasts of the *factors* by the VAR model (described in the previous section). For a given exchange rate and on a given day, if the next–day, model–based implied volatility of contract i, $\widehat{\sigma}_{it+1} = \widehat{\lambda}_i' \widehat{F}_{t+1}$, is predicted to increase (decrease) relative to today's observed level σ_{it}, contract i is considered for purchasing (sale). More specifically, each day, depending on the sign of $\widehat{\sigma}_{it+1} - \sigma_{it}$, a net investment of 1,000 euros is made in long and short positions of straddles, struck at a volatility of σ_{it}, for the time–to–maturity that corresponds to contract i, and held for one trading day. The exercise price of each straddle executed is chosen so to make the position delta–neutral, thus any computed profits truly reflect profits in "trading volatility".

The following trading rules are considered: First, following Harvey and Whaley (1992), we consider a trading rule (henceforth trading rule A) that always trades in the closest–to–maturity, ATM segment of the IVS, i.e. one straddle is only executed. Second, we consider a strategy (trading rule B), where again only one straddle is executed, at contract $\arg\max_i |\widehat{\sigma}_{it+1} - \sigma_{it}|$, i.e. the contract for which the model predicts the largest implied volatility absolute deviation. The last strategy considered (trading rule C) buys a straddle in contract $\arg\max_i (\widehat{\sigma}_{it+1} - \sigma_{it})$ and sells a straddle in contract $\arg\min_i (\widehat{\sigma}_{it+1} - \sigma_{it})$, i.e. the contracts for which the "buying" and "selling" signals are the largest.

Two benchmarks are considered. One is the "underlying buy and hold" strategy, where each day the 1,000 euros is simply invested in the underlying exchange rate. The second one (trading rule R) is a random option strategy: every day a delta–neutral straddle at σ_{it}, $i = 1, \ldots, N$ has a chance of $1/N$ of being executed, and if selected, the position (long or short) is decided by a flip of a coin.

Tables 8 and 9 present summary statistics, in the absence of transactions

Currency Code	Rule A: Mean Moneyness S/K	Rule A: Mean Maturity (in years)	Rule A: Mean Profit (%)	Rule A: Daily % standard Deviation	Rule A: t-ratio	Rule B: Mean Moneyness S/K	Rule B: Mean Maturity (in years)	Rule B: Mean Profit (%)	Rule B: Daily % standard Deviation	Rule B: t-ratio
	Trading Rule A					Trading Rule B				
AUD	.999	.019	.396	.043	9.275	.964	1.047	.497	.034	14.586
BRL	.999	.019	.350	.070	4.967	.955	.845	1.229	.077	15.978
CAD	.999	.019	.335	.041	8.267	.966	.931	.540	.031	17.546
CHF	.999	.019	.779	.064	12.159	.900	3.317	1.218	.060	2.348
CLP	.999	.019	.719	.047	15.450	.977	.531	.795	.038	2.762
CZK	.999	.019	.645	.052	12.523	.888	3.763	.909	.046	19.833
GBP	.999	.019	.686	.047	14.605	.951	1.521	.941	.035	27.048
HKD	.999	.019	.209	.038	5.555	.955	1.032	.683	.031	22.369
HUF	.999	.019	.692	.049	14.272	.955	.965	.666	.036	18.329
IDR	.999	.019	.395	.046	8.521	.963	.791	.295	.034	8.787
INR	.999	.019	.535	.068	7.845	.945	1.248	.573	.054	1.663
JPY	.999	.019	.250	.048	5.154	.950	1.416	.629	.039	15.934
KRW	.999	.019	.506	.044	11.424	.983	.444	.525	.039	13.349
MXN	.999	.019	.289	.043	6.686	.911	1.787	.767	.034	22.374
NOK	.999	.019	.678	.051	13.343	.992	.235	.903	.042	21.290
NZD	.999	.019	.416	.041	1.167	.996	.083	.635	.035	18.090
PLN	.999	.019	.221	.044	4.984	.970	.716	.654	.034	19.334
RUB	.999	.019	.380	.036	1.562	.923	1.742	.640	.026	24.779
SEK	.999	.019	.936	.058	16.234	.976	.753	1.011	.047	21.588
SGD	.999	.019	.577	.044	13.272	.935	1.540	.707	.029	24.653
SKK	.999	.019	.432	.068	6.353	.990	.275	.918	.054	17.098
TRY	.999	.019	.393	.072	5.482	.965	.521	1.018	.073	13.944
TWD	.999	.019	.508	.044	11.415	.945	1.279	.763	.034	22.251
USD	.999	.019	.608	.046	13.195	.985	.416	.578	.039	14.822
ZAR	.999	.019	.317	.058	5.472	.968	.613	.649	.047	13.730

Table 8: From the starting dates in Table 2 and using the first 100 daily observations of the IVS, factors and loadings are estimated using (6). The VAR factor model in (7) is estimated on the factor dynamics (with d, selected by the BIC, starting with a maximum value of $d = 15$) and used to produce one–day ahead forecasts of the factors, $\widehat{F}_{t+1}$, and of the IVS, $\widehat{\sigma}_{it+1} = \widehat{\lambda}_i' \widehat{F}_{t+1}$. Each day, estimation of factors, loadings and VAR coefficients is repeated. The sign and the magnitude of the model–predicted IVS deviations, $\widehat{\sigma}_{it+1} - \sigma_{it}$ is used to identify contracts i that should be purchased or sold. A 1,000 euros are invested daily in each trading rule. Trading rule A refers to a delta–neutral straddle executed only at the ATM, closest–to–maturity contract, while Trading rule B executes a delta–neutral straddle at the implied volatility with the highest predicted absolute deviation. No transaction costs are imposed.

	Trading Rule C					Trading Rule R			Buy & Hold Underlying		
Currency Code	Mean Moneyness S/K	Mean Maturity (in years)	Mean Profit (%)	Daily % standard Deviation	t-ratio	Mean Profit (%)	Daily % standard Deviation	t-ratio	Mean Profit (%)	Daily % standard Deviation	t-ratio
AUD	.954	1.328	.319	.011	29.928	-.047	.019	-2.557	-.010	.006	-1.716
BRL	.951	.900	.973	.047	2.626	-.128	.035	-3.637	.034	.008	4.095
CAD	.955	1.236	.369	.010	36.530	.029	.015	1.925	-.010	.005	-2.201
CHF	.904	3.161	.694	.045	15.520	-.038	.030	-1.270	.018	.003	5.852
CLP	.977	.514	.268	.015	18.365	.037	.022	1.713	.018	.005	3.462
CZK	.893	3.579	.575	.029	19.868	.013	.034	.393	.005	.004	1.046
GBP	.948	1.610	.451	.015	3.163	-.010	.020	-.513	-.011	.005	-2.035
HKD	.942	1.324	.296	.012	23.904	-.011	.020	-.537	-.034	.003	-11.135
HUF	.933	1.446	.418	.018	23.176	.013	.026	.504	.000	.006	.027
IDR	.966	.713	.234	.017	13.954	.025	.030	.838	.070	.007	1.122
INR	.940	1.349	.173	.011	15.221	-.314	.047	-6.624	.020	.005	4.005
JPY	.937	1.790	.371	.014	26.097	-.016	.021	-.742	.032	.006	5.307
KRW	.979	.545	.372	.022	17.226	-.072	.027	-2.669	.002	.007	.287
MXN	.924	1.524	.304	.014	22.287	-.035	.022	-1.613	.014	.005	2.978
NOK	.985	.443	.537	.017	32.288	-.075	.024	-3.075	.004	.004	.789
NZD	.975	.545	.360	.010	35.563	-.050	.018	-2.788	-.021	.006	-3.701
PLN	.953	1.105	.391	.015	26.584	.003	.021	.149	-.005	.005	-1.046
RUB	.930	1.577	.273	.013	21.724	-.086	.017	-5.033	.010	.003	3.518
SEK	.967	1.037	.598	.024	25.068	-.179	.031	-5.727	-.010	.004	-2.463
SGD	.938	1.459	.315	.013	24.839	-.033	.018	-1.874	-.018	.003	-6.477
SKK	.986	.412	.685	.032	21.533	-.003	.037	-.076	.006	.004	1.524
TRY	.963	.559	.690	.040	17.196	-.044	.050	-.879	.026	.013	1.942
TWD	.946	1.238	.359	.020	17.690	-.086	.027	-3.238	-.014	.004	-3.520
USD	.964	1.025	.300	.011	27.226	-.012	.018	-.648	.024	.006	4.221
ZAR	.939	1.149	.318	.015	21.554	-.111	.024	-4.635	-.082	.009	-9.013

Table 9: From the starting dates in Table 2 and using the first 100 daily observations of the IVS, factors and loadings are estimated using (6). The VAR factor model in (7) is estimated on the factor dynamics (with d, selected by the BIC, starting with a maximum value of $d = 15$) and used to produce one–day ahead forecasts of the factors, $\widehat{F}_{t+1}$, and of the IVS, $\widehat{\sigma}_{it+1} = \lambda_i' \widehat{F}_{t+1}$. Each day, estimation of factors, loadings and VAR coefficients is repeated. The sign and the magnitude of the model–predicted IVS deviations, $\widehat{\sigma}_{it+1} - \sigma_{it}$ is used to identify contracts i that should be purchased or sold based on the model predictions. A 1,000 euros are invested daily in each trading rule. Trading rule C executes two delta–neutral straddles at the implied volatilities with the highest and the lowest predicted deviations, while Trading rule R executes random straddles on the IVS. No transaction costs are imposed.

costs, from profits derived from trading rules A–C and the two benchmarks considered. All trading rules yield statistically significant positive profits, ranging from 0.173% (INR, rule C) to 1.229% (BRL, rule B), and all outperform in comparison to the two benchmarks considered.

Turning our attention to the comparison of trading rules, trading rules A, B, C, R and B&H yield mean profits of 0.49%, 0.75%, 0.426%, 0.049% and 0.003% on average, across all surfaces. It is apparent that in almost all cases rule B (that invests only in the largest absolute IVS predicted deviation) produces higher mean profits than rules A and C.[11]

Interestingly, rules B and C that are not restricted in terms of contracts that can be selected, tend to take positions in maturities that are normally not available to investors in exchange–traded currency options. A final remark worth making is that trying to exploit more segments of the IVS forecasts, as rule C attempts in comparison to rules A and B, does not necessarily mean higher profits: in the vast majority of surfaces, restricting to one volatility trade leads to higher profits.

Tables 10 and 11 report results that take transaction costs into account. Rates of return for trading rules are recomputed, this time imposing transaction costs of 20 volatility basis points per traded contract.[12]

As expected, profits after transaction costs are lower on average, but the ranking of rules is the same: across all currencies, rules A, B, C, R and B&H yield on average 0.453%, 0.559%, -0.021%, -0.235% and -0.010% respectively. Trading rule C that attempts to exploit more segments of the surface (its highest positive and negative predicted deviations) appears to suffer the most in terms of mean profits from transaction costs. In only one third of the surfaces examined can rule C now yield statistically positive mean returns; however mean profits are negative for the most liquid currency surfaces.

Trading rules A and B continue to yield significant positive mean profits in most cases. Rule B can lead to mean profits ranging from virtually zero (0.045% in IDR, with $t = 1.347$) to 1.03% (BRL). For the most liquid currencies, the average mean profit is 0.528%.[13] As the mean maturity

[11]With the exception of HUF, IDR and USD where rule A yields marginally higher profits.

[12]For rule B&H that trades the underlying exchange rate, transaction costs of 0.10 euros are charged.

[13]This corresponds to approximately one fourth of the mean profits that Goncalves and Guidolin (2006, Table 7) report, when a similar trading strategy on the S&P 500 IVS dynamics is followed.

Currency Code	Trading Rule A: Mean Moneyness S/K	Trading Rule A: Mean Maturity (in years)	Trading Rule A: Mean Profit (%)	Trading Rule A: Daily % standard Deviation	Trading Rule A: t-ratio	Trading Rule B: Mean Moneyness S/K	Trading Rule B: Mean Maturity (in years)	Trading Rule B: Mean Profit (%)	Trading Rule B: Daily % standard Deviation	Trading Rule B: t-ratio
AUD	.999	.019	.354	.043	8.293	.964	1.047	.318	.034	9.278
BRL	.999	.019	.309	.070	4.378	.955	.845	1.030	.077	13.405
CAD	.999	.019	.291	.041	7.185	.966	.931	.343	.031	11.099
CHF	.999	.019	.733	.064	11.449	.900	3.317	.817	.060	13.621
CLP	.999	.019	.684	.046	14.700	.977	.531	.638	.038	16.608
CZK	.999	.019	.601	.052	11.662	.888	3.763	.495	.046	1.726
GBP	.999	.019	.650	.047	13.844	.951	1.521	.728	.035	2.758
HKD	.999	.019	.187	.038	4.976	.955	1.032	.563	.031	18.440
HUF	.999	.019	.662	.049	13.643	.955	.965	.522	.036	14.363
IDR	.999	.019	.354	.046	7.629	.963	.791	.045	.034	1.347
INR	.999	.019	.509	.068	7.461	.945	1.248	.418	.054	7.757
JPY	.999	.019	.190	.048	3.914	.950	1.416	.306	.040	7.696
KRW	.999	.019	.468	.044	1.583	.983	.444	.382	.039	9.698
MXN	.999	.019	.262	.043	6.059	.911	1.787	.548	.034	15.930
NOK	.999	.019	.637	.051	12.537	.992	.235	.799	.042	18.827
NZD	.999	.019	.387	.041	9.463	.996	.083	.583	.035	16.615
PLN	.999	.019	.191	.044	4.310	.970	.716	.537	.034	15.939
RUB	.999	.019	.335	.036	9.331	.923	1.742	.310	.026	11.846
SEK	.999	.019	.901	.058	15.632	.976	.753	.880	.047	18.751
SGD	.999	.019	.543	.043	12.494	.935	1.540	.454	.029	15.598
SKK	.999	.019	.393	.068	5.778	.990	.275	.807	.054	15.045
TRY	.999	.019	.353	.072	4.927	.965	.521	.849	.073	11.639
TWD	.999	.019	.484	.044	1.890	.945	1.279	.610	.034	17.749
USD	.999	.019	.557	.046	12.093	.985	.416	.447	.039	11.419
ZAR	.999	.019	.289	.058	4.994	.968	.613	.548	.047	11.601

Table 10: From the starting dates in Table 2 and using the first 100 daily observations of the IVS, factors and loadings are estimated using (6). The VAR factor model in (7) is estimated on the factor dynamics (with d, selected by the BIC, starting with a maximum value of $d = 15$) and used to produce one–day ahead forecasts of the factors, $\widehat{F}_{t+1}$, and of the IVS, $\widehat{\sigma}_{it+1} = \widehat{\lambda}_i'\widehat{F}_{t+1}$. Each day, estimation of factors, loadings and VAR coefficients is repeated. The sign and the magnitude of the model–predicted IVS deviations, $\widehat{\sigma}_{it+1} - \sigma_{it}$ is used to identify contracts i that should be purchased or sold based on the model predictions. A 1,000 euros are invested daily in each trading rule. Trading rules as in Table 8. Transaction costs of 20 volatility basis points per trade are imposed.

Currency Code	Trading Rule C: Mean Moneyness S/K	Trading Rule C: Mean Maturity (in years)	Trading Rule C: Mean Profit (%)	Trading Rule C: Daily % standard Deviation	Trading Rule C: t-ratio	Trading Rule R: Mean Profit (%)	Trading Rule R: Daily % standard Deviation	Trading Rule R: t-ratio	Buy & Hold Underlying: Mean Profit (%)	Buy & Hold Underlying: Daily % standard Deviation	Buy & Hold Underlying: t-ratio
AUD	.954	1.328	-.145	.011	-12.805	-.230	.019	-12.348	-.022	.006	-3.894
BRL	.951	.900	.539	.047	11.410	-.379	.038	-1.072	.021	.008	2.573
CAD	.955	1.236	-.140	.011	-12.928	-.288	.016	-18.000	-.023	.005	-4.874
CHF	.904	3.161	-.139	.045	-3.104	-.301	.024	-12.351	.005	.003	1.737
CLP	.977	.514	-.057	.015	-3.928	-.204	.024	-8.372	.005	.005	1.036
CZK	.893	3.579	-.239	.029	-8.103	-.317	.032	-9.821	-.008	.004	-1.815
GBP	.948	1.610	-.023	.015	-1.488	-.261	.021	-12.220	-.023	.005	-4.446
HKD	.942	1.324	-.011	.012	-.921	-.170	.021	-8.275	-.046	.003	-15.244
HUF	.933	1.446	.000	.018	.024	-.168	.021	-8.062	-.012	.006	-2.219
IDR	.966	.713	-.250	.017	-14.712	-.209	.032	-6.547	.058	.007	8.323
INR	.940	1.349	-.178	.011	-15.676	-.376	.028	-13.330	.007	.005	1.467
JPY	.937	1.790	-.442	.015	-29.008	-.369	.020	-18.540	.019	.006	3.214
KRW	.979	.545	.047	.022	2.178	-.110	.027	-4.040	-.011	.007	-1.536
MXN	.924	1.524	-.112	.014	-8.178	-.148	.021	-7.132	.001	.005	.287
NOK	.985	.443	.228	.017	13.547	-.092	.029	-3.175	-.009	.004	-2.025
NZD	.975	.545	.127	.010	12.389	-.226	.016	-13.847	-.034	.006	-5.881
PLN	.953	1.105	.042	.014	2.904	-.176	.018	-9.556	-.017	.005	-3.706
RUB	.930	1.577	-.405	.013	-3.166	-.428	.014	-31.211	-.002	.003	-.733
SEK	.967	1.037	.254	.024	1.597	-.248	.027	-9.043	-.023	.004	-5.501
SGD	.938	1.459	-.205	.013	-15.298	-.150	.017	-8.647	-.030	.003	-11.017
SKK	.986	.412	.393	.032	12.372	-.261	.032	-8.170	-.007	.004	-1.759
TRY	.963	.559	.325	.040	8.114	-.064	.054	-1.189	.013	.013	.990
TWD	.946	1.238	.042	.020	2.044	-.049	.029	-1.717	-.027	.004	-6.656
USD	.964	1.025	-.173	.012	-14.647	-.375	.018	-2.711	.012	.006	2.047
ZAR	.939	1.149	-.010	.015	-.666	-.270	.036	-7.568	-.094	.009	-1.388

Table 11: From the starting dates in Table 2 and using the first 100 daily observations of the IVS, factors and loadings are estimated using (6). The VAR factor model in (7) is estimated on the factor dynamics (with d, selected by the BIC, starting with a maximum value of $d = 15$) and used to produce one–day ahead forecasts of the factors, $\widehat{F}_{t+1}$, and of the IVS, $\widehat{\sigma}_{it+1} = \widehat{\lambda}_i' \widehat{F}_{t+1}$. Each day, estimation of factors, loadings and VAR coefficients is repeated. The sign and the magnitude of the model–predicted IVS deviations, $\widehat{\sigma}_{it+1} - \sigma_{it}$ is used to identify contracts i that should be purchased or sold based on the model predictions. A 1,000 euros are invested daily in each trading rule. Trading rules as in Table 9. Transaction costs of 20 volatility basis points per trade are imposed.

of options chosen by this strategy demonstrates, restricting trading to one position chosen by the VAR factor model can yield statistically significant trading profits, even if one concentrates on the short–end of the surface (as rule A does) that is the most volatile. However, profits disappear when trading rules on wide segments of the surface are sought and as higher transaction costs are incurred.

Before leaving this section, it should be stressed that the trading profits reported hardly represent rejections of the informational efficiency of OTC FX options market, since we have abstracted from bid–ask spread considerations and other measurement error/microstructure effects that might offer alternative explanation for these profits. The results of the trading strategies however demonstrate that the systematic component of the IVS, as captured by (latent static) factors identified before us by several authors, do exhibit short–term predictability that is economically significant: simple parsimonious econometric specifications like the VAR considered here can capture this predictability and provide investors with forecasts useful for portfolio decisions and risk assessment.

6 Concluding remarks

When plotted against time–to–maturity and "moneyness", the volatility of contracts in the options market describe non–flat surfaces that exhibit significant time–variation.

General equilibrium option pricing models have proposed economic justifications for the existence and the empirical characteristics of the implied volatility surface. When persistent latent variables drive the pricing fundamentals, time–varying surfaces can be derived in equilibrium, and based on information related to the latent factors the IVS can be forecasted.

In this paper, we attempt to examine whether this predictability can be exploited in an economically significant way, by building on the vast literature that has identified latent static factors in the dynamics of implied volatility surfaces.

Using an extensive data set from the over–the–counter options market, we first demonstrate that–in accordance with the existing literature–a few static statistical factors, with an intuitively clear interpretation, can completely characterise IVS variation in–sample. These factors exhibit significant time variation and persistence, and can be successfully modeled through parsimonious vector autoregressive specifications.

The VAR factor model is shown to generate accurate out–of–sample forecasts of the surface, at least up to three days ahead, both in absolute terms and relatively to natural benchmarks such as a random walk for implied volatilities. Although, superiority in short–term predictions across the whole surface is difficult to establish, we show that the VAR factor model is successful in identifying the predictable segments of the surface, whose existence is prescribed by general equilibrium models.

We demonstrate that this ability can be economically exploited for portfolio decision–making by performing volatility–based trading based on one–day ahead predictions of the IVS. In the absence of transaction costs, statistically significant profits are generated via a number of alternative trading strategies that use the IVS forecasts of the VAR factor model; however, profits decrease or even disappear completely when transaction costs are increased and when trading rules on wider segments of the surface are sought. Although bid–ask spreads and other market frictions might explain the remaining profits of the VAR factor model after transaction costs, we feel that our findings establish that the IVS predictability suggested by equilibrium models is present in the OTC FX options market, and can significantly improve our portfolio decisions and the management of risk exposures.

A Appendix: Descriptive statistics of identified factors

Table A.1 that follows, reports descriptive statistics of the factors that are identified in the implied volatility surfaces from the 25 different currency options in our sample. By construction the factors have a mean of zero.

Table A.1: Descriptive statistics of the factors identified in the volatility surfaces implied by options on 25 different exchange rates. $\widehat{\rho}(l)$ denotes the sample autocorrelation coefficient of lag l.

Code		Min.	Max.	St. Dev.	Skew.	Kurt.	$\widehat{\rho}(1)$	$\widehat{\rho}(12)$	$\widehat{\rho}(25)$
AUD	$\widehat{F}_1$	-15.02	20.20	5.25	0.42	3.92	.977	.742	.592
	$\widehat{F}_2$	-5.07	7.93	2.05	0.53	3.57	.941	.578	.414
	$\widehat{F}_3$	-4.20	5.30	1.29	0.45	3.28	.956	.712	.608
BRL	$\widehat{F}_1$	-35.64	76.58	17.93	0.82	3.70	.979	.786	.575
	$\widehat{F}_2$	-51.82	15.38	7.77	-1.37	7.68	.941	.620	.324
	$\widehat{F}_3$	-21.84	24.20	3.50	2.39	19.01	.873	.446	.057
CAD	$\widehat{F}_1$	-8.03	20.33	3.85	0.09	3.64	.956	.752	.580
	$\widehat{F}_2$	-6.66	5.00	1.50	-0.33	3.68	.892	.450	.300
	$\widehat{F}_3$	-4.22	2.32	0.80	-0.67	4.99	.845	.268	.254
CHF	$\widehat{F}_1$	-8.60	16.46	3.19	0.57	3.66	.970	.695	.496
	$\widehat{F}_2$	-4.23	7.71	1.60	0.24	4.05	.944	.691	.505
	$\widehat{F}_3$	-5.85	3.54	1.46	-0.74	4.16	.966	.823	.679
CLP	$\widehat{F}_1$	-18.14	15.24	6.37	-0.25	2.51	.939	.635	.399
	$\widehat{F}_2$	-8.05	9.09	2.59	0.80	5.94	.912	.509	.075
	$\widehat{F}_3$	-3.41	2.77	1.51	-0.34	2.36	.976	.769	.533
CZK	$\widehat{F}_1$	-14.92	32.53	6.34	1.90	8.72	.978	.791	.630
	$\widehat{F}_2$	-8.77	6.38	2.62	-0.43	3.27	.950	.736	.569
	$\widehat{F}_3$	-7.43	5.56	2.03	-0.82	3.63	.962	.802	.688

Code		Min.	Max.	St. Dev.	Skew.	Kurt.	$\widehat{\rho}(1)$	$\widehat{\rho}(12)$	$\widehat{\rho}(25)$
GBP	$\widehat{F}_1$	-20.98	18.40	7.02	-0.62	3.34	.979	.921	.869
	$\widehat{F}_2$	-5.04	5.01	2.01	0.06	2.47	.970	.838	.722
	$\widehat{F}_3$	-3.25	3.46	1.15	-0.15	3.05	.964	.793	.672
HKD	$\widehat{F}_1$	-8.73	6.81	3.26	-0.15	2.91	.951	.578	.240
	$\widehat{F}_2$	-2.91	3.70	1.43	-0.11	2.47	.933	.533	.232
	$\widehat{F}_3$	-1.70	2.24	0.53	0.73	4.87	.810	.112	-.025
HUF	$\widehat{F}_1$	-13.60	18.25	6.59	0.13	2.55	.959	.514	.162
	$\widehat{F}_2$	-4.89	3.99	1.98	-0.31	2.26	.928	.528	.297
	$\widehat{F}_3$	-3.20	2.90	1.11	-0.13	2.53	.912	.588	.283
IDR	$\widehat{F}_1$	-10.75	31.87	6.70	1.90	8.71	.969	.533	.327
	$\widehat{F}_2$	-5.82	14.66	2.97	1.31	8.65	.962	.408	.195
	$\widehat{F}_3$	-9.01	2.97	1.95	-1.28	5.26	.970	.607	.411
INR	$\widehat{F}_1$	-7.73	7.67	3.63	-0.20	2.15	.961	.608	.074
	$\widehat{F}_2$	-5.04	11.52	2.80	0.48	2.66	.915	.542	.221
	$\widehat{F}_3$	-2.67	2.41	0.82	0.73	3.79	.906	.528	.125
JPY	$\widehat{F}_1$	-20.61	23.36	8.63	0.24	2.96	.978	.830	.708
	$\widehat{F}_2$	-6.63	12.00	3.41	1.10	3.99	.978	.774	.632
	$\widehat{F}_3$	-6.59	5.63	1.96	-0.18	3.23	.950	.685	.478
KRW	$\widehat{F}_1$	-27.49	50.35	11.39	1.27	6.58	.979	.888	.794
	$\widehat{F}_2$	-16.47	13.28	4.75	-0.24	3.55	.979	.854	.717
	$\widehat{F}_3$	-13.10	7.20	2.68	-0.44	4.56	.953	.653	.484
MXN	$\widehat{F}_1$	-14.71	23.51	7.34	1.02	4.50	.979	.762	.396
	$\widehat{F}_2$	-9.92	3.83	2.05	-1.13	5.09	.872	.429	-.030
	$\widehat{F}_3$	-2.81	5.39	1.20	0.26	3.74	.837	.338	.170
NOK	$\widehat{F}_1$	-11.17	14.36	4.83	0.24	2.60	.974	.820	.679
	$\widehat{F}_2$	-2.80	5.90	1.19	0.65	4.76	.904	.476	.339
	$\widehat{F}_3$	-4.93	2.39	0.93	-0.05	3.27	.923	.690	.574

Code		Min.	Max.	St. Dev.	Skew.	Kurt.	$\widehat{\rho}(1)$	$\widehat{\rho}(12)$	$\widehat{\rho}(25)$
NZD	$\widehat{F}_1$	-9.94	11.77	4.68	0.36	2.55	.963	.525	.341
	$\widehat{F}_2$	-5.46	3.26	1.50	-0.77	4.23	.894	.312	.091
	$\widehat{F}_3$	-1.54	2.78	0.63	0.45	4.07	.731	-.196	.158
PLN	$\widehat{F}_1$	-9.45	13.11	4.34	0.13	3.05	.963	.585	.229
	$\widehat{F}_2$	-6.03	4.51	2.17	-0.58	2.85	.947	.590	.334
	$\widehat{F}_3$	-1.81	1.69	0.61	-0.20	3.16	.845	.280	.164
RUB	$\widehat{F}_1$	-5.73	8.31	2.30	-0.07	3.58	.850	.554	.346
	$\widehat{F}_2$	-4.67	2.26	1.06	-0.40	4.00	.686	.420	.160
	$\widehat{F}_3$	-2.08	2.10	0.91	0.01	2.48	.926	.720	.470
SEK	$\widehat{F}_1$	-13.27	23.13	5.66	0.18	3.45	.974	.849	.709
	$\widehat{F}_2$	-4.31	13.52	2.34	1.48	7.67	.968	.775	.557
	$\widehat{F}_3$	-7.68	4.52	1.77	0.03	3.25	.947	.816	.691
SGD	$\widehat{F}_1$	-5.90	4.73	1.96	-0.25	3.22	.915	.387	-.008
	$\widehat{F}_2$	-5.19	3.57	1.43	-0.55	4.25	.923	.495	.203
	$\widehat{F}_3$	-1.66	1.18	0.64	-0.50	2.66	.931	.631	.351
SKK	$\widehat{F}_1$	-13.83	18.50	5.11	0.47	3.52	.974	.820	.659
	$\widehat{F}_2$	-5.54	4.35	1.60	-0.25	2.90	.924	.602	.427
	$\widehat{F}_3$	-3.06	4.71	1.25	0.99	4.40	.946	.666	.453
TRY	$\widehat{F}_1$	-111.74	92.94	34.59	0.36	2.85	.984	.873	.770
	$\widehat{F}_2$	-43.76	20.11	8.49	-1.31	6.11	.946	.721	.532
	$\widehat{F}_3$	-26.85	14.45	6.00	-0.94	5.57	.805	.512	.348
TWD	$\widehat{F}_1$	-9.04	5.67	2.30	0.05	3.01	.827	.529	.169
	$\widehat{F}_2$	-3.05	2.18	0.82	-0.68	3.51	.853	.305	.036
	$\widehat{F}_3$	-1.86	1.48	0.63	-0.43	2.79	.880	.399	.049
USD	$\widehat{F}_1$	-21.72	23.53	7.49	-0.38	3.06	.967	.819	.724
	$\widehat{F}_2$	-6.18	8.01	2.00	0.18	3.50	.932	.563	.462
	$\widehat{F}_3$	-4.87	3.30	1.46	-0.11	2.37	.924	.797	.674

Code		Min.	Max.	St. Dev.	Skew.	Kurt.	$\widehat{\rho}(1)$	$\widehat{\rho}(12)$	$\widehat{\rho}(25)$
ZAR	$\widehat{F}_1$	-20.38	48.27	12.49	0.95	4.14	.968	.578	.154
	$\widehat{F}_2$	-8.36	18.64	4.06	1.61	7.71	.961	.644	.339
	$\widehat{F}_3$	-4.41	4.47	1.34	0.09	3.73	.856	.362	.221

B Appendix: Idiosyncratic errors unit root tests

The idiosyncratic errors from the factor representation in equation (5) have been examined for common and individual unit roots through a number of panel tests. Results are summarised in Table B.1.

The modified t–statistic of Levin, Lin and Chu (2002), the standardised t–statistic of Im, Pesaran and Shin (2003), and the Fisher–augmented Dickey Fuller χ^2 statistic are reported in the table. In all tests, individual intercepts are allowed in the test equations, and the number of lags used is decided by the Bayesian information criterion of Schwarz (1978). The Parzen kernel is used in the Levin, Lin and Chu (2002) common unit root test.

Currency Code	Common UR: LLC t^*	Individual URs: ADF-F χ^2	Individual URs: IPS W	Panel size: Cross sect.	Panel size: N.Obs.
AUD	-33.69	1510.2	-27.41	60	103,090
BRL	-29.09	1093.4	-22.19	50	52,810
CAD	-21.66	765.2	-14.13	60	52,313
CHF	-36.66	1690.8	-29.30	65	115,168
CLP	-24.01	817.6	-18.40	40	25,267
CZK	-34.95	1820.0	-29.73	70	120,987
GBP	-35.40	1622.4	-29.20	60	103,914
HKD	-22.46	784.9	-14.30	60	22,501
HUF	-29.08	1063.5	-20.40	60	22,449
IDR	-18.54	563.1	-12.90	40	22,002
INR	-22.45	897.9	-16.77	60	22,476
JPY	-32.43	1396.9	-25.66	60	103,917
KRW	-28.39	1010.7	-22.47	40	82,663
MXN	-27.02	995.1	-18.60	60	21,298
NOK	-30.08	1279.9	-26.76	40	69,101
NZD	-22.40	789.4	-14.51	60	22,529
PLN	-25.58	853.8	-16.00	60	22,427
RUB	-25.65	985.8	-16.81	70	24,741
SEK	-41.81	2036.1	-34.98	60	103,820
SGD	-21.49	744.3	-13.45	60	22,451
SKK	-29.75	1101.7	-23.43	40	41,894
TRY	-28.83	996.2	-21.84	40	66,877
TWD	-18.55	699.6	-12.26	60	22,566
USD	-21.03	860.3	-15.31	60	103,548
ZAR	-30.04	1180.6	-22.35	60	22,518

Table B.1: The table reports panel unit root tests for the idiosyncratic errors e_{it} from the factor representation (equation (5)) of the volatility surfaces implied by the 25 different currency options in our sample. LLC t^* denotes the modified t–statistic of Levin, Lin and Chu (2002), ADF-F χ^2 is the Fisher–augmented Dickey Fuller chi–square statistic, while IPS W stand for the properly–standardised t–statistic of Im, Pesaran and Shin (2003) that is asymptotically normal.
An * denotes that the null hypothesis of (a common or individual) unit root cannot be reject at $\alpha = 1\%$.

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