Multi-period putty-semi-clay production and the business cycle

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2 February 2009

Abstract

Current research has developed both "putty-putty" and "putty-clay" vintage capital models for analyzing the effects of embodied productivity shocks. Here we propose a "putty-semi-clay" model which has a C.E.S. ex post production function that nests both approaches as the two extreme cases. We find that this model exhibits the empirically relevant link between changes in capacity and movements in employment and output.

Furthermore, by introducing multi-period investment in the model we find that the model's dynamic responses differ greatly between time-to-plan, time-to-build and investment first formulations. We argue that misapplying weights may lead to a wrongful dismissal of multi-period production.

JEL classification : E22, E23
Keywords : Time-to-build, time-to-plan, putty-clay, vintage capital

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[1] I am grateful to G.C. Harcourt for his many helpful comments. I would also like to thank C. Chatzikomis, M. Baddeley, K.D. Sheedy, D. Tambakis, A. David, A. Pepelasis for helpful discussions and suggestions. Correspondence to C. Repapis, Darwin College, Silver Street, Cambridge, CB3 9EU. E-mail : cr252@cam.ac.uk. Tel : +44-1223-501330.

1 Introduction

Lately there has been a resurgence of interest in models with a time-to-build component (see Gomme, Kydland and Rupert (2001)). This is partly because empirical work at the micro level suggests that investment in most cases is a lengthy process, and therefore aggregate models that do not have production lags seem to contradict this micro evidence (see Koeva (2000)). Nevertheless, introducing multi-period construction schemes into aggregate general equilibrium models seems to have the effect of dampening the dynamic responses of these models without adding substantially to the model's ability to explain key statistics of the aggregate data (see Rouwenhorst (1991) , Cogley and Nason (1995)). One possible explanation why this is the case may be that the older generation of time-to-build models had investment applied uniformly between the different periods in production time (see Kydland and Prescott (1982), Rouwenhorst (1991)). Christiano and Todd (1996) suggested that this is a major failing since investment in new projects appears to be more heavily applied the closer the project is to completion. Further empirical research has rejected the hypothesis that investment is applied uniformly (see Jung (2006)) without, however, producing agreement in the literature on whether investment is applied more at the beginning or at the end of the project's life.

At the same time one of the key criticisms of the real business cycles models has been its reliance on exogenous disembodied productivity shocks to reproduce key moments of the aggregate data. Its reliance on the Solow residual has been called its Achille's heal (King and Rebelo, (2000), p. 34). Furthermore, it has been argued that the simple real business cycle model has problems in explaining contemporaneous correlations of key aggregate variables when the driving force in business cycle fluctuations is stochastic permanent productivity shocks instead of transitory ones (see Rotemberg and Woodford, (1996)). These criticisms have given rise to models in which fluctuations in the business cycle frequency are driven by changes in investment embodied technology instead of disembodied shocks (see Greenwood, Hercowitz, Krusell (2000)). One advantage of this is that we do not have to rely on the Solow residual as an estimate of embodied technology shocks[2] (see Greenwood, Hercowitz and Krusell (2000), Fisher (2003)). In fact, the hypothesis of embodied, productivity driven fluctu-

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[2] For example, Greenwood et al. (2000) get the stochastic structure of the investment-specific shock by relying on the time-series properties of the relative price of equipment in the postwar US data. (see Greenwood et al. (2000), p.102-108)

ations becomes more plausible as it has been suggested that investment-specific technological change may account for a sizable part of overall economy growth (see Greenwood, Hercowitz, Krusell (1997)). The best way to incorporate embodied technology shocks into the standard RBC model is by the introduction of vintages of capital. Introducing vintages permits us the further freedom of distinguishing between the *ex ante* investment decision, and the *ex post* production one, given the existing level of productivity, capital and availability of labour. Campbell (1998) suggested such a model as a vehicle for introducing embodied technology shocks and analyzing them as a source of economic fluctuations. He assumed that each firm could produce output by a variable combination of capital and labour, so a unit of capital may be worked more or less intensively depending on the need to produce output at any specific point in time. Gilchrist and Williams (2000) suggested the opposite, that once investment became capital it could combine with labour only in fixed proportions. In this sense the *ex post* production function at the machine level was Leontief, so that a unit of capital either received a specific proportion of labour input and produced output, or remained idle. To distinguish between the two types of model we call the first 'putty-putty' as substitutability between capital and labour is high both when the investment decision is made and when production is decided. The opposite extreme is suitably called 'putty-clay', as *ex post* labour and capital are non-substitutable.

The problem with both extremes is that it is highly improbable that at the firm level either holds. It is a gross simplification to assume that fixed capital is as malleable as investment, and can be combined with labour in any proportion. Equally restrictive is the scenario that capital and labour can combine in only one unalterable proportion. The introduction of a C.E.S. function for the *ex post* production decision would provide a more general framework that nests both approaches as the two extremes cases. This is because, with a C.E.S. production function we can permit a low level of substitutability between inputs, without reducing it to zero. Furthermore, vintage capital models have not generally included time-to-build elements (see e.g. Campbell (1998) and Gilchirt and Williams (2000)). This is surprising as it would be expected that combining them with the particular structure of vintage capital models might provide novel insights in the responses of key macroeconomic variables.

This paper introduces a model with time-to-build investment integrated within a vintage capital framework. Furthermore, by making the *ex post* production function C.E.S., we can accommodate both extremes and consider the case of less *ex post* capital-labour substitutability than was the case when investment was planned. As there is, to our knowledge, no other paper that has implemented in this framework an *ex post* C.E.S. production function, we will call this kind of vintage capital model a 'putty-semi-clay' model in order to compare it with the other models suggested by the literature.

The paper is organized as follows : In the next section the modeling context of the paper is presented. Section III outlines the model. Section IV looks at the particular short-run marginal cost curve for production and its relation to capital-labour substitutability. Section V discusses the stochastic results both in the case of embodied and disembodied productivity shocks. Section VI concludes.

2 A brief and selective literature review

The original formulation of vintage capital models can be traced back to the work of Salter (1960, 1965) and Johansen (1959). Salter suggested that technological growth enters the production process through investment, so that the 'stock' of capital in an economy is the outcome of vintages of capital, each embodying the technical knowledge at the time of production.[3] This analysis underlines the important distinction between investment decisions done by producers *ex ante*, and the *ex post* production decision in which producers have to utilize equipment already in existence. *Ex post* producers are restricted in their production choices by a stock of capital that may not have been built for the purpose they wish now to utilize it, and in which several production parameters are now unalterable. One such rigidity is the way capital combines with labour in production. If labour cannot be substituted for capital *ex post*, then increases in production could not be achieved by working current machines more intensively, but instead only by bringing into production machines that had been mothballed due to their low productivity, as their technology is outdated. Johansen (1959) outlines such a model in which there are *ex ante* substitution possibilities between capital and labour, but *ex post* there are none.[4] The ramifications of having a 'putty-clay' production function for capital and growth theory were investigated in a number of papers that followed Salter's and Johansen's original contributions.[5] [see Solow (1962), Solow, Tobin, von Weizsacker and Yaari (1966) Sheshinski (1967) Cass and Stiglitz (1969) and more recently Lindh (2000)]

From the early 1990's there has been a resurgence of interest in vintage capital models in a growth theory framework. These papers stressed the finding that different schemes of machine depreciation could produce complex dynamic investment behavior. The work of Benhabib and Rustichini (1991) found that in a continuous time setting if vintages are characterized by non-exponential rates

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[3] The alternative formulation of technoligical progress affecting the whole capital stock is quite unrealistic. As Solow noted "..[disembodied technological growth] conflicts with the casual observation that many if not most innovations need to be embodied in new kinds of durable equipment before they can be made effective. Improvements in technology affect output only to the extent tha they are carried into practise either by net capital formation or by the replacement of old-fashioned equipment by the latest models..." [Solow, 1960, p. 91]

[4] We may have vintage capital models in which this *ex post* rigidity does not exist. These are known as 'putty-putty' production models. [see Solow (1960)]

[5] The best survey on the literature of the period and the effects of the 'putty-clay' production function for capital theory may be found in Harcourt (1972) ch. 2.

of depreciation they may experience oscillations in investment. Several papers extend this finding and consider the possibility of oscillations in investment under different vintage specific depreciation schemes.[see Boucekkine, Germain and Licandro (1997), Boucekkine, Lindaro, Puch and del Rio (2005)].

Nevertheless, these models are not explicitly business cycle models, and do not consider the effects of embodied productivity shocks in a dynamic general equilibrium framework. Campbell (1998) builds a dynamic general equilibrium model which incorporates vintages. He assumes that the model economy produces 'plants' which are characterized by a Cobb-Douglas production function, and in which once capital is invested in, cannot be altered until the plant is scrapped. Each plant's capital content is also dependant on two shocks : 1) the level of embodied technology at the time of production and 2) an idiosyncratic shock which intends to capture how well each plant has integrated the new technology. This second condition produces a continuum of plants of different capital input, although they are of the same vintage. Finally, once capital is invested in a plant it remains dormant for a number of periods until the plant becomes operational.[6]

A number of papers have also considered the effects of embodied productivity shocks in general equilibrium models with vintage capital characteristics. Greenwood, Hercowitz and Krusell (2000) have a Cobb-Douglas production function which distinguishes investment between structures and equipment, with embodied technology affecting the latter. Benhabib and Hobijn (2001) consider shocks in embodied technology when vintages are producing complementary output.[7] Marquis and Trehan (2007) consider a model with labour differentiation where the newest vintage is paired off with the most skilled workers. However, none of these models have a putty-clay production function.

Gilchrist and Williams (2000, 2004, 2005) integrated a putty-clay effect into their model. They assumed that *ex ante* producers can create any number of machines which would, once created, be operated with a fixed labour input. This means that *ex post* a machine may either operate at full capacity, by employing a unit of labour, or remain dormant. The producer thus has two separate decisions to undertake every period. The *ex ante* decision of investing in machines, which

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[6] It should be stressed that this is equivalent to a gestation lag, not a 'time-to-build' investment scheme, since investment is committed only when production of the plant is decided. (see Campbell, 1998, p.382-84)

[7] This model relates to the literature of "jelly capital" which suggests that complimentarities may be present between the output of different vintages, so that one vintages output is used for the production of another's. (see e.g. Phelps (1962))

will become productive next period and in which he can now choose the amount of capital a unit of labour will operate with for the entirety of the machine's life, and the *ex post* decision of utilization of machines already in existence dependent on the going wage rate, the only cost of operating the machine. While the production choice is Leontief at the micro level, it permits some labour-capital substitutability at the aggregate level. This is achieved by assuming that each machine has an imbuilt idiosyncratic productivity shock.[8] This leads to a distribution of productivity levels across machines of the same vintage, so that an increase in aggregate production could be achieved by employing machines that where previously unused due to their low productivity.[9] Further work on the effects of putty-clay production was done by Ateson and Kehoe (1999) and Wei (2003) who have introduced *ex post* rigidities on factor substitution in models of energy use.[10]

One element that this generation of 'putty-clay' models has not integrated into their frameworks is the consideration that investment may take several periods to enter production, and may have to be applied at intervals, some time after the original decision to invest was taken. These 'time-to-build' investment schemes have been quite extensively used in dynamic general equilibrium models outside a vintage capital framework [see Kydland and Prescott (1982); Rouenhorst (1991); Gomme, Kydland and Rupert (2001)]. The model presented in the next section integrates time-to-build investment with a putty-semi-clay framework of production.

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[8] This is in conception close to what has been suggested by Campbell (1998), where it is argued that not all machines are equally successful in applying the latest technical advances in their production.

[9] It should be noted that production does not increase by bringing back into production whole vintages that lay dormant due to average productivity increase. Vintages are scrapped at some point.

[10] Wei (2003) applies a variation of the Gilchrist-Williams model in which he considers energy as a separate factor of production (with labour and capital) in the *ex post* production function.

3 The Model

This model is a putty-semi-clay model in which there is a time-to-build element, so that machines come into operation after several periods from the time that the investment decision is made. Furthermore, the production function for each machine has the form of a C.E.S. function from the time the capital content is decided.

Formally, a production process produces 'machines' which experience efficiency shocks when produced. These machines are ready at time t+i from investment occurring at time t, where i is an exogenously applied period of 'building' the machine. Each machine's inbuild level of working capital is dependent on two parameters : 1) the economy-wide level of θ_t which is an exogenous shock variable influencing the cost of capital at time t, the period that the machine is commissioned.[11] 2) the amount of total capital expected to be invested in the machine until their completion. Total capital investment of a machine is denoted by $k_{i,t}$ and it is the summation of all pre-decided investment installments for each machine weighted accordingly, multiplied by θ_t. Formally : $k_{i,t} \equiv \theta_t(\sum\limits_{n=1}^{i} \varphi_n S_{n,t+i-n})$ (1) where φ_n are the weights [12] and $S_{n,t+j}$ the investment applied at time t+j for a machine which is n stages from completion.

Once built, the production capability of a machine for its time of operation is : $Y_{t+j}(k_{i,t}) = z_{t+j}(v_t \left(ak^u_{i,t} + (1-a)\, l^u_{i,t,t+j}\right)^{\frac{1}{u}})$ $j \geq i$, where $l_{i,t,t+j}$ is the labour input of vintage t decided at the time t+j, v_t is an exogenous shock variable capturing the effect of a change in the embodied productivity of this vintage's machine, z_{t+j} is an exogenous disembodied productivity shock occurring at time t+j that affects all vintages in operation equally, $-\infty<u<0$ is the degree of complementarity of the factors of production, and $1<a<0$ is a constant.

The *ex post* CES production function effectively imposes some restrictions on the substitutability of capital and labour. This means that from the ex-

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[11] An alternative way to see this shock variable is as a disturbance affecting the quality of capital in a machine. This means that for a given level of saving the productivity of capital bought is affected by this shock. While this will have an effect on the interest rate, it is distinct to an exogenous shock in the interest rate (r_t), or a shock in the intertemporal wish to save. This is because a capital cost shock entails true capital productivity loss or gain, and not simply an intertemporal redistribution of consumption. In calling it a capital cost shock we keep within the usual terminology of the literature (see Gilchrist and Williams (2000) and Greenwood et al. (2000)). For a discussion on the relation between v_t and θ_t see footnote 27 below.

[12] The weights are restricted to add up to one, thus $\sum_{n=1}^{i} \varphi_n = 1$.

treme of a near Leontief *ex post* production function[13] to one where the factors of production are completely complementary (u=1), a host of different *ex post* production functions can be applied.[14] In this paper we restrict our attention to the cases where the *ex post* production function has some degree of complementarity of inputs, and therefore concentrate on the negative values of u (u<0).

3.1 The Investment Decision

The representative producer has to decide the level of investment to be committed over the number of periods that the machine is in gestation. The capital-intensity of each machine is guided by the expected discounted value of gross profits over its operating life.

Formally the profit condition is :

$$\Pi_{i,t} = E_t[-(\varphi_i S_{i,t} + \sum_{n=1}^{i-1} R_{t,t+n}\varphi_{i-n}S_{i-n,t+n})+$$

$$\sum\nolimits_{j=i}^{M} R_{t,t+j}(1-\delta_i)^{j-i}(z_{t+j}(v_t(ak_{i,t}^{u}+(1-a)\,l_{i,t,t+j}^{u})^{\frac{1}{u}})-l_{i,t,t+j}W_{t+j})]$$

where $R_{t,t+n}$ is the discount rate between periods t and t+n, W_{t+j} the wage rate at time t+j,M the total number of periods the machine will operate before it is permanently scrapped and $(1-\delta)^j$, the probability the machine will not fail. It should be noted that capital as such does not depreciate in a machine due to its usage. Instead this natural wear and tear of capital is captured by the probability that the whole machine will be prematurely scrapped.

The entrepreneur has to consider not only the chain of investments that will be necessary in order to complete the machine, but subsequently the level of labour applied at every future period in which the machine is operational. The problem can be decomposed into *ex ante* deciding capital intensity, where the future wage rates are an exogenous factor and determine future labour inputs, and an *ex post* problem at every period, in which given the level of investment and the wage rate optimum, the labour input is decided.

Furthermore, the decision on the machines' capital-intensity is undertaken at time t, and cannot be altered later on. Capital in the pipeline has an equation

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[13] which would would be the case as u→ $-\infty$, since the Leontief production function admits no substitutability between inputs.

[14] For example when u⟶ 0 it is equivalent to a Cobb-Douglas production function.

of motion, $S_{j,t} = S_{j-1,t+1}\ \forall j$ (2). Maximizing with respect to new investment to be undertaken gives us the first-order condition for an internal optimum :

$$S_{i,t} = aE_t\{(\varphi_i+\textstyle\sum_{n=1}^{i-1}\varphi_n R_{t,t+i-n})^{-1}k^u_{i,t}[\sum_{j=i}^{M} R_{t,t+j}(1-\delta)^{j-i}z_{t+j}(v_t(ak^u_{i,t}+(1-a)\,l^u_{i,t,t+j})^{\frac{1}{u}-1})]\}\ (A)$$

Similarly maximizing at every subsequent period with respect to labour we have the marginal condition for labour input. This is :

$$W_{t+j} = (1-a)l^{u-1}_{i,t,t+j}z_{t+j}(v_t(ak^u_{i,t}+(1-a)\,l^u_{i,t,t+j})^{\frac{1}{u}-1}) \quad \text{for j=i.....M} \qquad \text{(A1)}$$

which yields the familiar condition that the marginal product of labour equals the wage rate and the labour market clears.

Finally, the number of machines built at every period in time is guided by the condition that the rents received from building one more machine should be zero.[15] This effectively means that $\Pi_{i,t} = 0$, which gives the condition that expected discounted costs must always equal expected discounted returns, thus:

$$S_{i,t} = E_t\{(\varphi_i + \textstyle\sum_{n=1}^{i-1}\varphi_n R_{t,t+i-n})^{-1}\sum_{j=i}^{M} R_{t,t+j}(1-\delta_i)^{j-i}(z_{t+j}(v_t(ak^u_{i,t}+(1-a)\,l^u_{i,t,t+j})^{\frac{1}{u}})-l_{i,t,t+j}W_{t+j})\}\ (B)$$

These conditions describe the *ex ante* investment decision. The output and labour decisions are considered next.

3.2 The Production Decision

Since the aggregate the economy is a closed economy with no government sector, it is described by $C_t = Y_t - (\varphi_i S_{i,t}Q_t + \sum_{n=1}^{i-1}\varphi_{i-n}S_{i-n,t}Q_{t-n})$ (3)

Therefore, consumption is not only constrained by the investment decisions undertaken today, but also by paying for the completion of machines that are already in the production line. This would effectively mean that only a portion of

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[15] It is assumed that *ex ante* there is no cost of building one more machine. This effectively guarantees that total investment may be *ex ante* divided into more or less machines depending only on the producers optimization decision on what is expected to be the best combination of the capital-labour ratio when this vintage comes into operation.

the total investment committed today is decided today, and that any necessary re-adjustment of gross investment can only happen by changing the amount invested in new projects.

Output depends on the labour input decided at machine level for the operational vintages. Thus aggregate output is simply the summation of the outputs of all machines across all vintages and can be expressed as :

$$Y_t = z_t \sum\nolimits_{n=j}^{M} (1-\delta)^{j-i} Q_{t-j} (v_{t-j} \left(ak_{i,t-j}^{u} + (1-a)\, l_{i,t-j,t}^{u} \right)^{\frac{1}{u}}) \qquad (4)$$

The labour decided for every operating vintage is taken by the producers at time t. In order for the labour market to clear, the marginal products of labour for all machines of every vintage must equal the current wage rate, and by taking the derivative of machine output with respect to labour we have M equations characterizing the labour inputs across vintages :

$$W_t = (1-a) l_{i,t-j,t}^{u-1} z_t (v_{t-j} (ak_{i,t-j}^{u} + (1-a)\, l_{i,t-j,t}^{u})^{\frac{1}{u}-1}) \quad \text{for j=i.....M } (C)$$

Finally, aggregate labour is simply the summation of all working machines across vintages : $L_t = \sum\nolimits_{j=i}^{M} (1-\delta)^{j-i} [l_{t-j,t} Q_{t-j}]$ (5)

3.3 The Consumer and Equilibrium

The representative household has the usual characteristics found in related literature. It maximizes a utility function of the form :

$$U_t(c,l) = E_t \sum\nolimits_{j=0}^{\infty} \tfrac{1}{1-\gamma} \beta^j (C_{t+j} (1-L_{t+j})^{\zeta})^{1-\gamma}$$

where $0<\beta<1$, $1>\gamma>0$, $\zeta>0$.[16]

With the description of the utility function the maximization problem is completely defined. The household has to maximize intertemporal consumption and leisure by choosing a path for the variables $\{W_{t+j}, S_{i,t+j}, Q_{t+j}, [l_{t-s,t+j}]_{s=i}^{M}\}_{j=0}^{\infty}$, while constrained by equations (1)-(5).

In equilibrium the investment decisions are guided by the optimality conditions (A),(B). The amount of saving, however, is regulated by the household

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[16] If $\gamma = 1$ then the logged form of the utility is used, where : $u_{t+j}(c,l) = \ln(C_{t+j}) + \zeta(1 - L_{t+j})$.

willingness to save which depends on the current interest rate. Given that the household wishes to smooth consumption over time this leads to the familiar Euler equation :

$$U_{c,t} = \frac{\beta^j}{R_{t,t+j}} U_{c,t+j} \qquad \text{for j=1....M} \qquad (6)$$

where $U_{c,t}$ is the marginal utility of consumption at time t. Similarly the relative labour choice between the different vintages is regulated from the first-order conditions (C), but the aggregate amount of labour the household wishes to supply the market with depends on the going wage rate, and is regulated by the labour-leisure trade-off:

$$U_{c,t}W_t + U_{l,t} = 0 \qquad (7)$$

where $U_{l,t}$ is the marginal utility of incrementally decreasing leisure.

This completes the description of the dynamic equilibrium of this economy, which is described by equations (1)-(7) and (A)-(C). We next describe a simple neoclassical model with a time-to-build element, similar to a simplified version of the model first described by Kydland and Preskott [1982]. This will be used as a benchmark model in the simulation section.

3.4 A benchmark time-to-build model

In this model capital takes (i) periods to enter production, and once completed is indistinguishable from capital already in use. Output is determined by a Cobb-Douglas production function, formally : $Y_t = z_t k_{i,t}^a l_t^{(1-a)}$ where z_t is an aggregate shock, $k_{i,t}$ is aggregate capital, and l_t is the labour choice made at time t. The equation of capital accumulation is : $k_{i,t+1} = (1-\delta)k_{i,t} + I_{i,t-i+1}$, where $I_{i,t} = \theta_t(\sum_{n=1}^{i} \varphi_n S_{n,t+i-n})$. All other variables are as defined above with θ_t being again an exogenous shock affecting the cost of new capital similarly to the one introduced above. The unfinished projects follow (2), and with the total resource constraint : $C_t = Y_t - (\varphi_i S_{i,t} + \sum_{n=1}^{i-1} \varphi_{i-n} S_{i-n,t})$ this economy's optimization problem is completely outlined.

The first-order conditions describing equilibrium are equations : the intertemporal trade-offs (6) for j=1...i, the labour-leisure trade-off (7), the optimal allocation of labour effort given wage : $\mathrm{W}_t = (1-a)z_t k_{i,t}^{a} l_t^{-a}$ (8), the maximization of capital condition :

$$\mathrm{E}_t\{aR_{t,t+i}z_{t+i}k_{i,t+i}^{a-1}l_{t+i}^{1-a}+\sum_{j=1}^{i-1}((1-\delta)\theta_{t+1}^{-1}\varphi_{i-j}R_{t,t+j+1}-\theta_t^{-1}\varphi_{i-j}R_{t,t+j}) \\ +((1-\delta)\theta_{t+1}^{-1}\varphi_i R_{t,t+1}-\varphi_i\theta_t^{-1})\} = 0 \ (9)$$

and finally the resource constraint :

$$\mathrm{C}_t = z_t k_{i,t}^{a} l_t^{(1-a)} - \sum_{n=0}^{i-1}\varphi_{i-n}\theta_{t-n}^{-1}(k_{i,t+i-n}-(1-\delta)k_{i,t+i-n-1}) \ (10).$$

4 The Short-Run Production Function

One defining characteristic of the model is its putty-semi-clay aggregate nature. In the *ex ante* production choice the producers can vary the number of machines in production and the capital input of each machine, so there is a high degree of substitutability between capital and labour. *Ex post,* however, the number of machines is fixed, and for each machine the marginal product of labour is guided by the C.E.S. coefficient of capital-labour substitutability. This effectively means that we can increase production by using existing machines more labour-intensively, which distinguishes this model from the Gilchrist and Williams (2000) one in which there is a Leontief choice at the machine level, or any 'putty-putty' formulation in which both *ex ante* and *ex post* the degree of substitutability between capital and labour is the same. [see e.g. Campbell (1998), Greenwood, Hercowitz and Krusell (2000), Marquis and Trehan (2007)]

This production function creates interesting short-run dynamics around the steady state. Figure one plots the log variation of output and labour if we keep wages and capital fixed for various values of (u). These short-run marginal cost curves (SRMC) depict the asymmetric nature of increasing and decreasing labour from the steady state and its effect on production. Decreases in labour input proportionately lower output. The effects there differ from the simple 'putty-putty' vintage model with a Cobb-Douglas production function *ex post*, (the case where u=0) only in the slope of the SRMC curve. This is not the case for increases of labour input. There increasing the degree of complementarity between inputs means that increases of output in the short-run runs up against the lack of capital. Therefore, the extreme case of an almost Leontief production function (u$\rightarrow -\infty$) would make the SRMC curve have a kink at the point of the steady state level of capital. Assuming values less than negative infinity not only makes the SRMC function continuous and differentiable, but also considers the plausible case that some degree of substitutability is *ex post* possible at the machine level, but not to the degree that there was before investment became fixed capital.

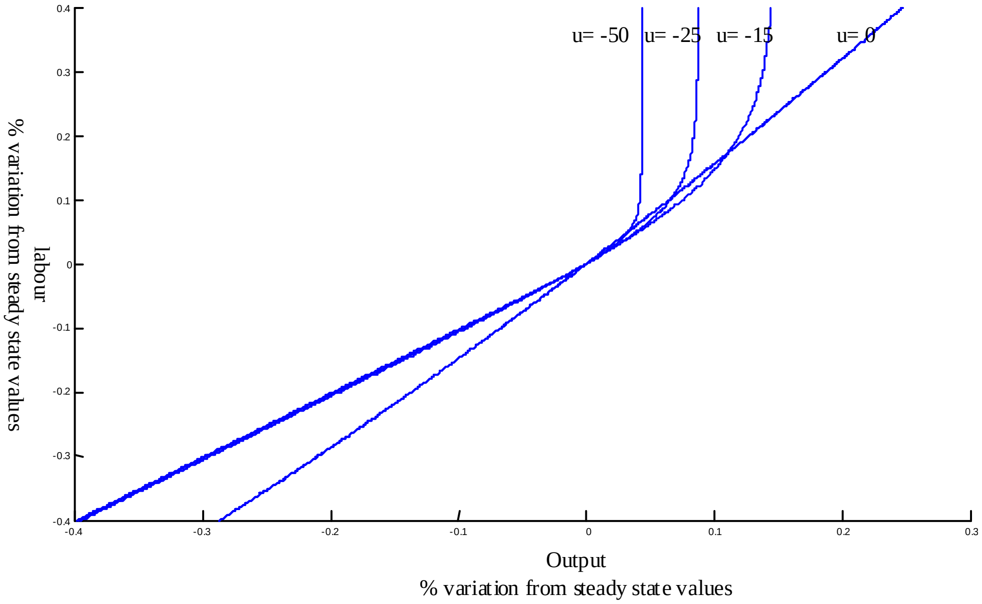

Figure 1: The short run production function for different values of u.

5 Stochastic Equilibrium

Approximating the stochastic behavior of the model is complex due to the size of the state space. For a model with M operating vintages, there are 2xM+6 economic equations describing the equilibrium, and M forward lags for variables $[\mathrm{C}_{t+s},\mathrm{W}_{t+s},\mathrm{l}_{t,t+s}]_{s=1}^{M}$ as well as M backward lags for variables $[\mathrm{Q}_{t-s},\mathrm{k}_{i,t-s},l_{t-s,t}]_{s=1}^{M}$. The linearized equations used for a first-order approximation of the stochastic equilibrium are given in Appendix A. In this paper we calibrate values of 40 working vintages (M), depreciation rate (δ) of 0.084, a discount rate (β) of 0.97 ($\mathrm{r}_t = 0.03$), a=0.36, $\gamma = 1$, and ζ=3. These values are standard for an annual version of the model, and are close to the ones used in Gilchrist and Williams (2000), Kydland and Prescott (1991) and Christiano and Eichenbaum (1992).[17]

Values of (u) which is the degree of relative complementarity between factor inputs are not readily available from the literature. However, as we are interested in the 'putty-semi-clay' characteristics of the model, we will restrict u to

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[17] The only difference from the Gilchrist and Williams (2000) numbers is the trend growth rate which here is set to zero.

negative values that are not close to zero. The case of u$\approx$ 0 is important as it approximates the case where we have a Cobb-Douglas *ex post* production function, and therefore effectively a 'putty-putty' production function, albeit with vintages. The value of u=-15 is used to show the putty-semi-clay behavior of the model.[18]

Furthermore, the period of the exogenous lag of production must de determined. In this section we set i=2, so that capital matures two years after it is planned. The two years production lag although not very common in RBC models which favour usually a one year completion time, [e.g. Kydland and Preskott(1982), Christiano and Todd (1996)][19] is broadly supported by industry specific data [see Mayer(1960), Jorgerson and Stephenson (1967), Koeva (2000), Boca et a. (2005), Montgomery (1995)].[20] A discussion has also ensued in the literature over the distribution of investment between the different phases of production.

Traditional time-to-build models have an equal distribution of investment input between the different phases of production [see Kydland and Preskott (1982), Rouwenhorst (1991)]. Time-to-plan models [see Christiano and Todd (1996), Christiano and Vigfusson (2003)] argue that during the planning phase of the cycle relatively little of the total investment has to be committed, so that the bulk of it is committed afterwords when the project is already in the pipeline. Alternatively, it can be argued that investment in new projects is usually applied early in the cycle of production, so that during the last phases of the production process very little investment is actually committed.[21] This

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[18] Values of u smaller than -15 (but still tractable) do not change substantially the behavior of the model since by that figure the effect of the *ex post* complementarity of capital and labour is strong enough to bring out the particular characteristics of the model. Alternatively, values between 0 and -10 produce responses to shocks that are of a more 'putty-putty' form as the *ex post* substitutability of inputs is greater.

[19] However, some papers have utilised a two period production lag, see Koeva (2001).

[20] The issue of what is the appropriate length to fit in with the data is unresolved. However, a number of industry specific sources suggest that the two year lag for a model which has only an aggregate capital structure and does not distinguish between different kinds of capital (e.g. durables and non-durables) or different manufacturing sectors is a good generalisation.

More specifically [Mayer (1960)], surveys U.S. companies investment projects and notes that "...half a year elapses between the decision to undertake the project and the start of construction and that more than a year elapses between the start of construction and completion" (Mayer, 1960, p.128) This finding is also corroborated by Jorgenson and Stephenson (1967).

More recently [Koeva (2000)] notes that construction time for new plants is around two years. Finally, Boca et al. (2005) find that for a panel of Italian firms there is little evidence to support a construction lag of over a year for equipment but there is evidence supporting a lag of 2-3 years for building structures.

[21] This was also suggested by Zhou (2000). He estimates from aggregate US data investment expenditure and finds that "...investment progress slows down sustantially during the final two quarters of construction.. [for construction taking six quarters]."(Zhou, 2000, p. 278) This

is close to suggesting a "learning by using" scheme where it takes some time for management and the labour force to get used to the new equipment and make it productive, while in the meantime maintaining the equipment is costly [see Benhabib and Rustichini (1991)]. The extreme case would be a pure gestation model in which all the investment is applied when the plan is made and then a period passes until investment becomes productive capital. Here this variation of the model which has a substantial part (but not all) of total investment applied at the beginning of the project will be called the 'investment first' version to distinguish it from the other two versions. In the 'time-to-build' alternative where we set the $\varphi's$, $\varphi_1 = \varphi_2 = 0.5$ so that half of the investment is applied at every period of the machines' construction, and the 'time-to-plan' alternative in which $\varphi_2 = 0.25$, $\varphi_1 = 0.75$, so that only a quarter of total investment is committed at the first year of construction, with the rest applied during the second year. For the last variation, 'investment first' the proportions are reversed with $\varphi_1 = 0.25$, $\varphi_2 = 0.75$.[22]

5.1 Shocks in Capital Costs

Investment cost shocks display interesting dynamic characteristics in vintage capital models. As shocks in embodied technology can be viewed as factor cost shocks influencing the price of investment (see Greenwood et al. (2000) Gilchrist and Williams (2000)) here θ_t affects the cost of capital embedded in a machine, whereas v_t is the cost of a new machine. To distinguish them we shall call (θ_t) a shock in capital cost, and v_t, a shock in machine embodied capital. For all models considered here, the *ex ante* substitutability of capital and labour is high, so the two shocks produce very similar dynamic behavior.[23] In this section

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can be explained since "at very late stages of investments, the major work of construction has been completed and what remains does not cost much money." (Zhou, 2000, p. 279)

[22] There are several studies that consider the weighting of project completion from industry specific data. Montgomery (1995) and Jung (2006) reject the time-to-build assumption, and assert that the distribution of completion pattern is not uniform. Jung (2006) also finds that while only a small portion of investment projects is completed in the very early stage this is significantly different from zero, rejecting a strict time-to-plan formulation. (see Jung, 2006, p. 17)

This difference of competion patterns is also noted by Boca et al. (2005) who find that 'structures' (typically larger investment projects) have different completion patterns than 'equipment' (typically smaller investment projects) (see Boca et al., 2005, p.20-22). It therefore remains an empirically open question on how best to distribute completion patterns, and for this reason all three options are analysed in the paper.

[23] The main difference between the two shocks is the labour decision per machine. The shock on the cost of capital (θ_t) alters investment per machine, and the producer changes

we consider the responses of a change in θ_t, and it is assumed that it follows an AR1 process, so that $\ln\theta_t = p_1 \ \ln\theta_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim IID(0, \sigma^2)$. A coefficient of p1=0.95 would mean that the shock has a half-life of just under 14 periods, years in this annual version of the model.

Figure 2 shows the impulse response function for output and labour for the benchmark model, for the putty-putty model as well as for the case of a one period production lag. A comparison of the first four graphs shows that the introduction of vintages, when the production choice has both *ex-ante* and *ex-post* a high degree of substitutability between capital and labour, u→ 0, does not alter the dynamic responses of the economy. Labour responds immediately to the cost shock, as increased demands are made on output for both consumption and investment purposes. After the initial increase, effort declines towards its steady-state value, displaying a stepwise pattern. This can be explained by the two periods investment takes to mature. The initial increase in investment meant that next period investment already in the pipeline demands an increased proportion of that period's total investment, reducing the number of new projects undertaken. Next period, the initial investment stream finally becomes working capital, increases output, and introduces a new cycle of higher investment. Thus, labour rises in periods when increased demands are made to complete projects already under production, sustaining the high level of output and smoothing consumption between periods. An important difference between the output and labour series is that output does not start to decrease until well into the 9-10nth year after the initial shock, in contrast to labour that peaks almost immediately, and declines soon after.

This is in marked contrast with what happens in the cases when *ex post* substitutability between capital and labour is restricted. The next two graphs show output and labour dynamics when we have a putty-semi-clay production function (u=-15), and when investment takes only one period to mature, (i=1). The labour response mirrors more closely what is happening in output. This is because the original will to increase production runs up against the scarcity of capital. Labour rises only as increased investment makes capital available, and,

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machine capital intensity by varying the number of machines. When the shock is in the cost of the machine (v_t) the producer again can decrease (increase) capital-intensity by increasing (decreasing) the number of machines.

Effectively the main difference is that because (θ_t) influences directly capital-intensity per machine, it has; due to (u) the parameter guiding the complementarity of inputs, a much smaller quantitative effect on the variables than v_t. This is why in the next section that compares simulation statistics between embodied and disembodied shocks v_t is used for the statistics presented.

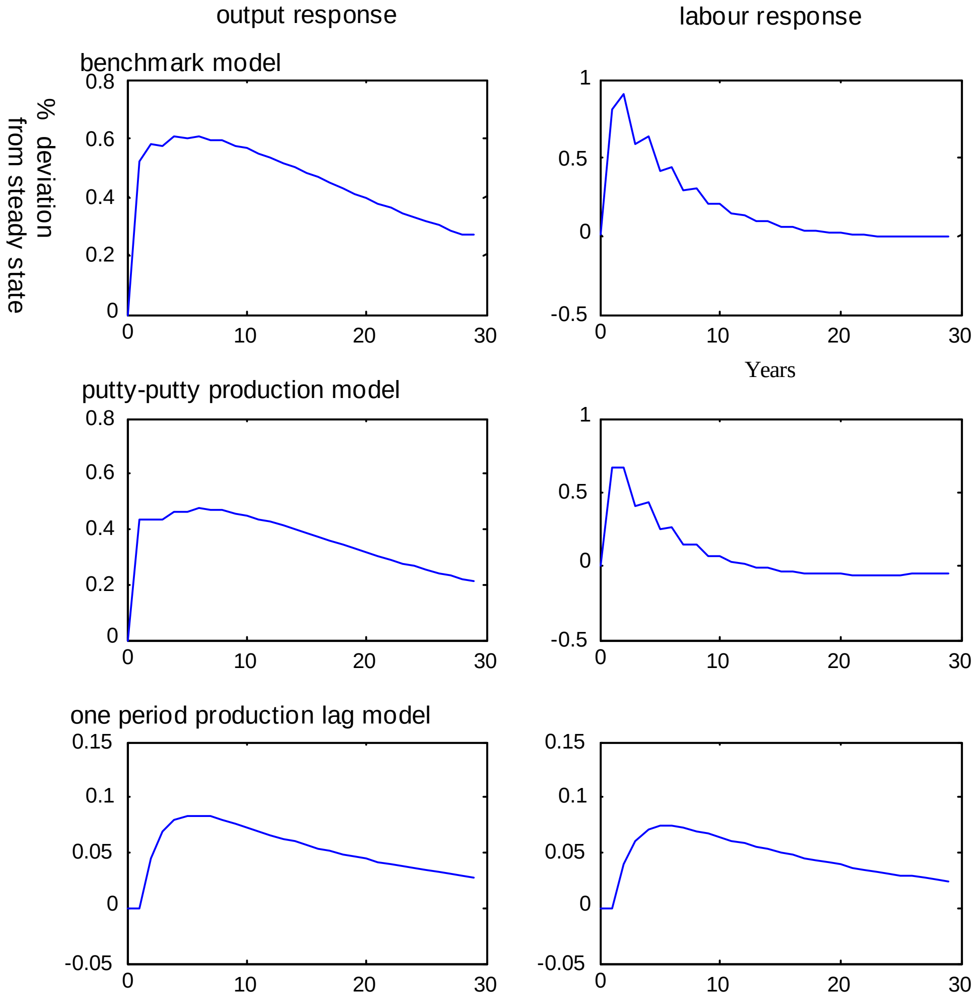

Figure 2: A persistent transitory capital cost shock

through the synergy with capital, raises output.

The introduction of lags produces an added complication for the production decision. Figure 3 compares labour and output responses for the time-to-build model, time-to-plan, and the investment-first model. A comparison of the time-to-build model with the one period production lag model demonstrates that the dynamics are not vastly altered by the introduction of multi-period construction. One important distinction is that due to the lag in production, consumption is not so smoothly adjusted and exhibits a more 'angular' pattern (see figure 4). This is because both consumption and investment are 're-adjusted' at the periods when new investment becomes productive.

In fact it is the two period investment cycle that complicates the dynamics of the time-to-build model. Whereas in the one period production model total investment decreased monotonically after the shock, the introduction of a two period construction scheme means that we have cycles of increased investment activity followed by periods of relative stagnation. Figures 5 and 6 show capital per machine and number of machines per vintage. This decomposition shows that the oscillating behavior of aggregate investment comes from a period by period variation of the number of machines rather than their capital input. In fact, investment per machine decreases steadily after the initial shock and approaches monotonically the steady-state value. Furthermore, it becomes evident that in the investment-first model there is an initial over-reaction both in machine numbers and in investment per machine. This increase in investment has the effect of raising consumption next period and therefore raising wages as leisure is valued more by the representative household. In the other two models (time-to-build and time-to-plan) the initial effect is not so strong. This is because over-extending investment may lead to plans being abandoned next period if wages dramatically rise. The smaller increase in investment leads to a lower increase in consumption next period in comparison to the investment-first model which leads to a lower increase in the wage. This not only permits the completion of the investment plans undertaken last period, but also the planning of a number of new machines. This is not the case with the investment-first model. There the increase in consumption, in conjunction with decreased output due to the rising wage rate depress the level of new machines planned to a level below that of the steady-state. This 'correction' is due entirely to the production lag as consumption rose in anticipation of future increased capital, but capital has not yet become productive and raised the MPL of machines. The effect is more pronounced when more of the investment is committed earlier in

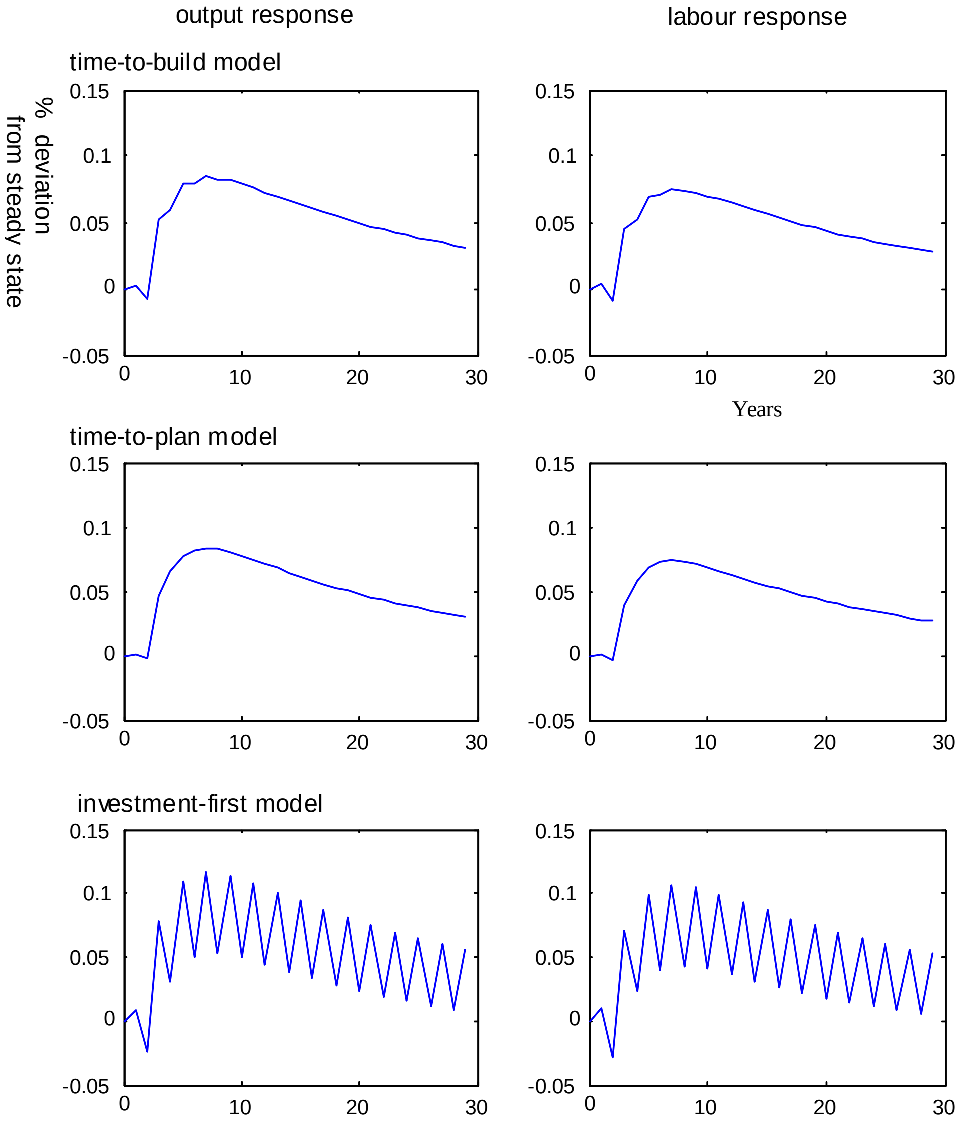

Figure 3: A peristant capital cost shock -The putty-semi-clay multiperiod models

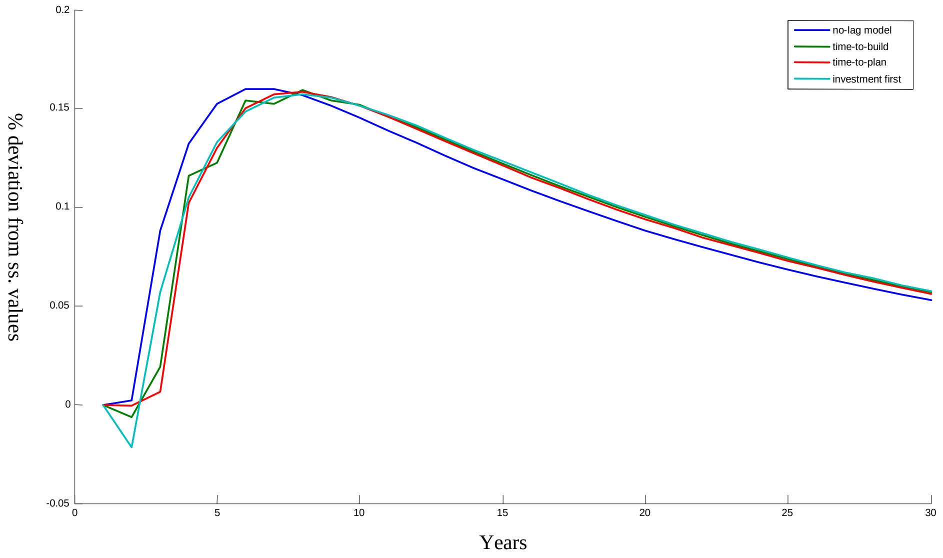

Figure 4: Percent change in consumption due to the capital cost shock.

the investment process, as investment schemes that have the bulk of investment committed during the later phases of production makes producers invest less at the planning stage. This is rational since future uncertainty means that producers react cautiously even to good news. This is because if they undertake too many obligations for next period they might have to abandon machines already in process, something that is very costly.

This minor distinction of the initial dynamic behavior of the representative household between models leads to quite divergent behavior later on. In the time-to-build and time-to-plan model investment does have 'cycles' but they are really of secondary importance, and die out quite fast. Furthermore these cycles do not seem to influence any other key variables' characteristic response, as output and labour follow similar patterns to those of the no-lag model. However, in the investment-first model the household is soon caught between alternating periods of high and low total investment, depending on the amount of machines that entered production that period. (see figure 5) This leads to a two period alternating cycle of high and then low labour activity and output.

Nevertheless, what should be noted is that if we take the average of these two period cycles we would observe that the responses for output, labour and investment are very similar to those of the time-to-build model. Thus, it can be argued that we are dealing with two distinct phenomena: a medium-period cycle propagated by the persistent capital cost shock which lasts effectively over 15-18 years and has the familiar hump shape that is associated with the putty-clay model and a high frequency repeated cycle that is a distinct feature of this model's dynamic response.

These kinds of investment echoes are not an entirely novel phenomenon. They been documented before in related literature for non-vintage models with a time-to-build element, (see Rouwenhorst (1991), Christiano and Todd (1996)), and it has been found that they are usually of importance when shocks are not autocorrelated. When shocks are correlated they are of a 'secondary character'.[24] It is the synthesis with a putty-clay production function that combines the strong high frequency response of these models with a longer period cycle driven from the persistence of the shocks.

5.2 Embodied vs. Disembodied productivity shocks

The introduction of embodied technology shocks as an important alternative propagation mechanism in business cycle fluctuations has lately received some attention. Campbell (1998) has suggested that these types of shocks can account for a significant part of economic fluctuations. Greenwood, Hercowitz and Krusell (2000) also find that while these shocks are not the main factor behind the business cycle they are a significant contributor to output fluctuation. Fisher (2003) finds that "investment-specific technology shocks account for about 50 percent of the variation in hours and about 40 percent of the variation in output" (Fisher, 2003, p.30). Further findings also suggest that investment-specific economic growth is the major source of economic growth. Greenwood, Hercowitz and Krusell (1997) argue that "approximately 60 percent of postwar productivity growth can be attributed to investment-specific technological change." (Greenwood et al., 1997, p. 359)[25] If this is the case then permanent stochastic

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[24] Rouwenhorst writes "The right panels show that increasing the autocorrelation of the shocks does not completely eliminate the cycles in the adjustment path, although they appear to be of 'second-order' importance relative to the persistence induced by the autocorrelation of the shocks" (Rouwenhorst, 1991, p.252)

[25] This result has been partly disputed. See Oulton (2007).

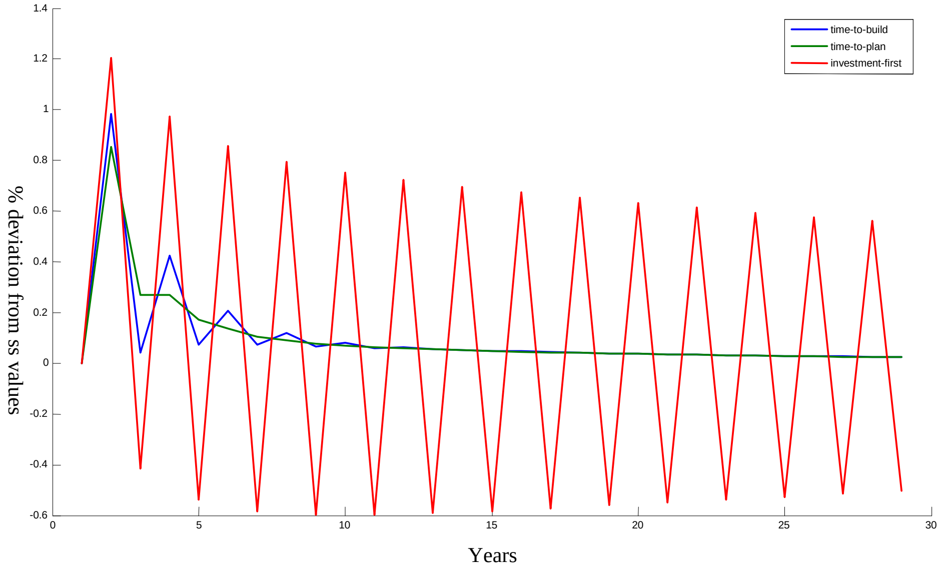

Figure 5: Variation in number of machines built due to the capital cost shock

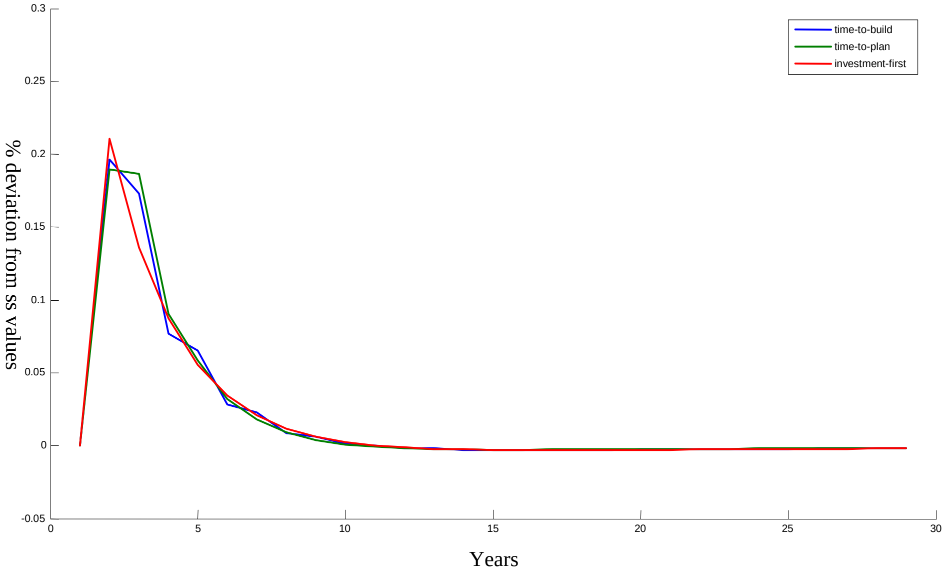

Figure 6: Machine investment change due to the capital cost shock

variations in investment-specific technology may play an important role in our understanding of the business cycle (see Fisher 2003). This section compares the putty-semi-clay model responses with the putty-putty vintage model both in the cases when shocks are embodied and disembodied. The differences in the responses of key variables when shocks are transitory and permanent are considered in turn.

5.2.1 Persistent transitory shocks

The effects of a disembodied technology shock on output and employment for the putty-putty production model as well as for the different types of putty-semi-clay models is shown in figure 7. A clear difference between the models with the putty-semi-clay production function and the putty-putty one is that in the former labour input increases only when investment has already raised the capital stock. In the latter labour input peaks immediately as it responds to a general increase in productivity and does not run against the *ex post* complementarity of capital and labour. Nevertheless the hump shaped labour responses is not mirrored in output. As the disembodied shock affects all the capital stock, output rises immediately and then monotonically decreases to its steady-state value as the shock's effect recedes in subsequent periods. The increase in labour input in the following periods does not instigate a new increase in output, but is used only to employ the new machines now in production, and as such does not provide the model with different output dynamics than its putty-putty equivalent. Even in the investment first model, which again shows the high frequency two period cycles in labour, the effect this has on output is muted by the general effect of the disembodied shock raising total factor productivity.

A comparison of the output and labour impulse response functions between the putty-semi-clay models and the putty-putty production model reveal some interesting regularities on the co-movement of labour and output. The *ex post* complementarity of capital and labour in the putty-semi-clay models produces a higher contemporaneous correlation between output and labour both in the case of embodied and disembodied shocks than in the putty-putty model In fact, a comparison of the values of Table 1 and 2 show that the putty-semi-clay models has similar responses to the putty-putty model in exhibiting strong contemporaneous co-movement between output with consumption and output per hour in the case of disembodied shocks. An important difference is the lower correlation of output and investment for the putty-semi-clay models. More divergent results are apparent in the case of embodied shocks. There the correlations of output with labour, consumption and output per unit of labour are significantly higher for the putty-semi-clay models,[26] but the correlation of output with investment is much weaker in those models.[27] The synergy of capital with labour also explains the higher autocorrelation of labour in the putty-semi-clay models. As labour shadows the changes in the capital stock, it gains some of its persistence.[28]

These findings are important if we consider stylised facts from the US. data. The strong contemporenious correlation of output with key variables is not only an important stylised fact, but also a key finding of the baseline RBC model when fluctuactions are the outcome of disembodied productivity shocks (see King and Rebelo (2000)). With embodied shocks though the simple putty-putty model is not so succesfull. More specifically, Greenwood, Hercowitz and Krusell (2000) find for US Annual data for the period 1954-1990, a stronger possitive contemporenious correlation for output with consumption and labour, than with investment (see Greenwood et al. 2000, p.105 table 3). This is also the case for the putty-semi-clay models where the correlation of output-consumption and output-labour is substantially higher than output-investment. On the downside, the output-investment correlation seems to be very low in comparison with the data, and this is a shortcomming of the putty-semi-clay models. The investment first model provides a higher output-investment correlation that seems to be

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[26] With the exception of the investment first model for the correlation of output with output per unit of labour.

[27] This is a standard result for this class of models. The limited *ex post* capital-labour substitutability means that investment rises out of decreased consumption, and only later does output boom as new capital comes into production. Thus the initial investment spike coincides with only limited increases in output. This also explains the negative correlation between investment and consumption and investment with output per hour for the putty-semi-clay models.

The investment first model provides a significantly stronger correlation between investment and output than the other putty-semi-clay models. The reason for this higher correlation is due entirely to the internal propagation mechanism that creates the two period cycles of investment and output, and raises their contemporaneous correlation. As the other putty-semi-clay models with multi period construction (time-to-build and time-to-plan) lack this mechanism they have a much lower output-investment correlation which is closer to the model with no multi-period construction.

[28] Investment-first displays again different characteristics than the other putty-semi-clay models. This is because the repeated cycles of labour lowers substantially its first-order autocorrelation, and have an effect on outputs' autocorrelation also. There the strong and repeated investment cycles explained in the last section are the reason for the strong negative autocorrelation of investment.

z_t	Putty-Putty	One Period Prod.	Time-to-build	Time-to-plan	Investment first
$\rho(y_t,y_{t-1})$	0.962 (0.0078)	0.955 (0.0087)	0.955 (0.0091)	0.956 (0.0086)	0.947 (0.0118)
$\rho(c_t,c_{t-1})$	0.981 (0.0042)	0.960 (0.0076)	0.958 (0.0086)	0.956 (0.0087)	0.962 (0.0080)
$\rho(i_t,i_{t-1})$	0.902 (0.0140)	0.612 (5.5E-06)	0.752 (0.0211)	0.686 (0.0252)	-0.683 (0.0765)
$\rho(p_t,p_{t-1})$	0.981 (0.0042)	0.947 (0.0100)	0.946 (0.0106)	0.947 (0.0101)	0.948 (0.0106)
$\rho(l_t,l_{t-1})$	0.815 (0.0170)	0.984 (0.0032)	0.971 (0.0060)	0.975 (0.0048)	0.258 (0.1633)
$\rho(y_t,i_t)$	0.927 (0.0059)	0.669 (0.0114)	0.735 (0.0123)	0.694 (0.0144)	0.370 (0.0238)
$\rho(y_t,c_t)$	0.976 (0.0042)	1 (5.83E-05)	1 (3.99E-05)	1 (4.64E-05)	0.998 (0.0008)
$\rho(y_t,l_t)$	0.548 (0.0189)	0.935 (0.0123)	0.871 (0.0258)	0.862 (0.0260)	0.740 (0.0468)
$\rho(y_t,p_t)$	0.977 (0.0041)	1 (7.83E-05)	0.999 (0.0002)	0.999 (0.0002)	0.997 (0.0008)
$\rho(p_t,l_t)$	0.359 (0.0085)	0.924 (0.0143)	0.848 (0.0297)	0.839 (0.0298)	0.686 (0.0580)
$\rho(p_t,c_t)$	1 (4.54E-07)	0.998 (0.0002)	0.998 (0.0004)	0.998 (0.0003)	0.998 (0.0004)
$\rho(p_t,i_t)$	0.828 (0.0196)	0.692 (0.0094)	0.763 (0.0097)	0.716 (0.0127)	0.325 (0.0350)
$\rho(l_t,i_t)$	0.820 (0.0211)	0.362 (0.0453)	0.313 (0.0633)	0.335 (0.0558)	0.650 (0.0467)
$\rho(l_t,c_t)$	0.357 (0.0085)	0.944 (0.0108)	0.881 (0.0240)	0.870 (0.0246)	0.709 (0.0565)
$\rho(i_t,c_t)$	0.827 (0.0198)	0.650 (0.0133)	0.720 (0.0138)	0.677 (0.0162)	0.304 (0.0340)

Table 1: The only shock here is persistant transitory disembodied technology shock. For all tables the results are averages from 500 simulations of 1000 realisations each. All variables are in logs. y, i, c, p, and l are output, total period investment (which includes new and finishing projects), consumption, output per unit of labour (defined as lny-lnl). The values in parenthesis are the standard deviations of the table values taken from the 500 simulations. Correlations between two variables are depicted as p(x,y).

v_t	Putty-Putty	One period Prod.	Time-to-build	Time-to-plan	Investment first
$\rho(y_t,y_{t-1})$	0.976 (0.0056)	0.988 (0.0024)	0.982 (0.0038)	0.988 (0.0025)	0.519 (0.1371)
$\rho(c_t,c_{t-1})$	0.974 (0.0060)	0.959 (0.0082)	0.957 (0.0086)	0.957 (0.0087)	0.959 (0.0084)
$\rho(i_t,i_{t-1})$	0.851 (0.0180)	0.531 (0.0254)	0.668 (0.0197)	0.602 (0.0233)	-0.790 (0.054)
$\rho(p_t,p_{t-1})$	0.974 (0.0059)	0.996 (0.0010)	0.996 (0.0010)	0.996 (0.0010)	0.996 (0.0008)
$\rho(l_t,l_{t-1})$	0.805 (0.0187)	0.973 (0.0036)	0.958 (0.0060)	0.976 (0.0033)	-0.052 (0.168)
$\rho(y_t,i_t)$	0.746 (0.0151)	0.158 (0.0184)	0.103 (0.0306)	0.125 (0.0270)	0.475 (0.0588)
$\rho(y_t,c_t)$	0.806 (0.0418)	0.971 (0.0056)	0.978 (0.0042)	0.976 (0.0050)	0.846 (0.0436)
$\rho(y_t,l_t)$	0.429 (0.0144)	0.853 (0.0058)	0.853 (0.0052)	0.853 (0.0056)	0.865 (0.0137)
$\rho(y_t,p_t)$	0.807 (0.0418)	0.883 (0.0201)	0.891 (0.0192)	0.888 (0.0203)	0.782 (0.0452)
$\rho(p_t,l_t)$	-0.184 (0.056)	0.509 (0.0313)	0.525 (0.0324)	0.519 (0.0330)	0.367 (0.0430)
$\rho(p_t,c_t)$	1 (8.66E-08)	0.904 (0.0144)	0.906 (0.0144)	0.905 (0.0148)	0.906 (0.0146)
$\rho(p_t,i_t)$	0.210 (0.0876)	-0.0538 (0.040)	-0.072 (0.049)	-0.065 (0.048)	-0.023 (0.018)
$\rho(l_t,i_t)$	0.921 (0.0130)	0.350 (0.0173)	0.276 (0.0277)	0.307 (0.0238)	0.732 (0.0490)
$\rho(l_t,c_t)$	-0.184 (0.056)	0.775 (0.0073)	0.795 (0.0071)	0.787 (0.0073)	0.539 (0.0534)
$\rho(i_t,c_t)$	0.209 (0.0876)	-0.083 (0.041)	-0.106 (0.050)	-0.097 (0.049)	-0.060 (0.0206)

Table 2: The only shock here is persistant transitory embodied technology shock. All variables are defined as in table 1.

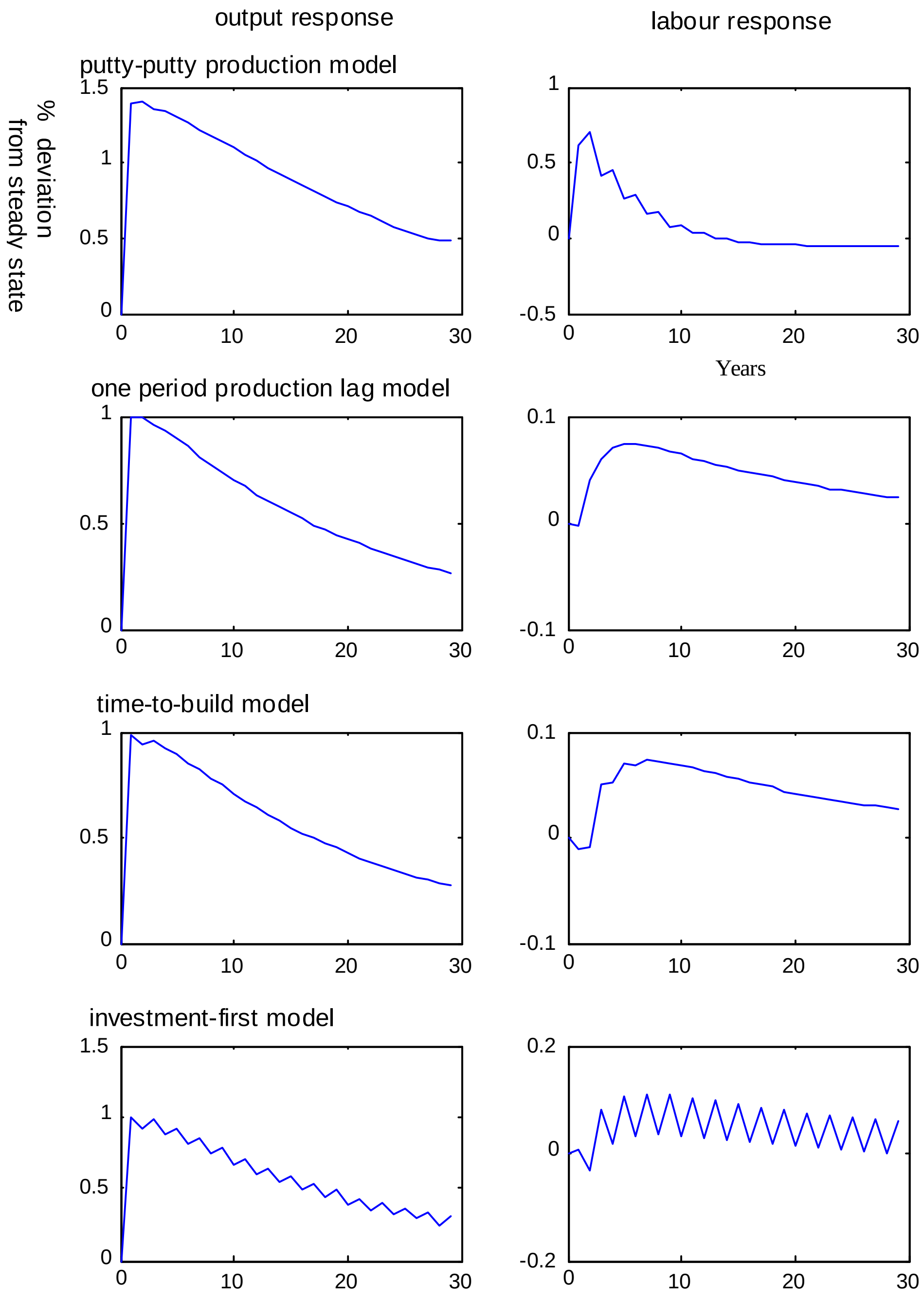

Figure 7: A persistant shock to disembodied technology.

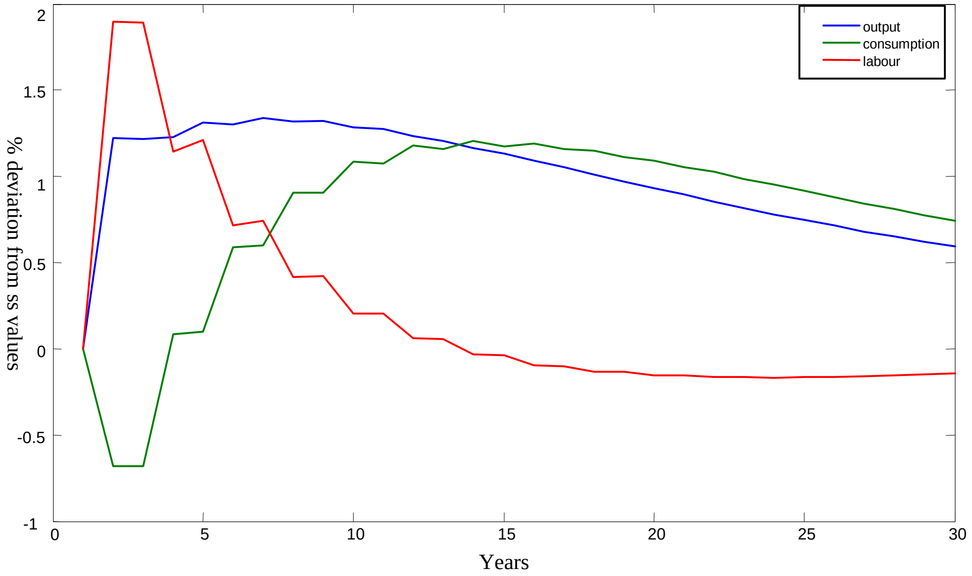

Figure 8: Embodied productivity shock- the putty-putty production function

more in accordance with the data, but this model produces negative and zero first order autocorrelations in investment and labour that contradict findings in the data.[29]

The contemporaneous correlations of the other key variables reveal some further differences between the models. The most striking is the observation that the putty-putty model finds negative contemporaneous correlation between labour and consumption, as well as labour and output per unit of labour when the shocks are embodied. This can be explained as an increased capital base raises the MPL, and subsequently through wages, consumption, while the labour input used in production falls. This can be observed in figure 8. In contrast, all the variations of the putty-semi-clay models exhibit positive, and strong contemporaneous correlation. As it can be seen from figure 9, apart from the original decrease in consumption that fuelled investment, labour rose as new

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[29] The actual data numbers are very different. They are not however readily comparible for a number of reasons. First, the data set was HP filtered whereas the simulations results in this paper are not. Secondly, there is strong evidence that the persistance coefficient for embodied shocks in annual models is lower than what has been used here, and the standard deviation of the shocks is larger. Greenwood et al. (2000) using annual data from the relative prices of new equipment find a persistance coefficient of the shock of about 0.64 and a standard deviation for the shock 0.035.

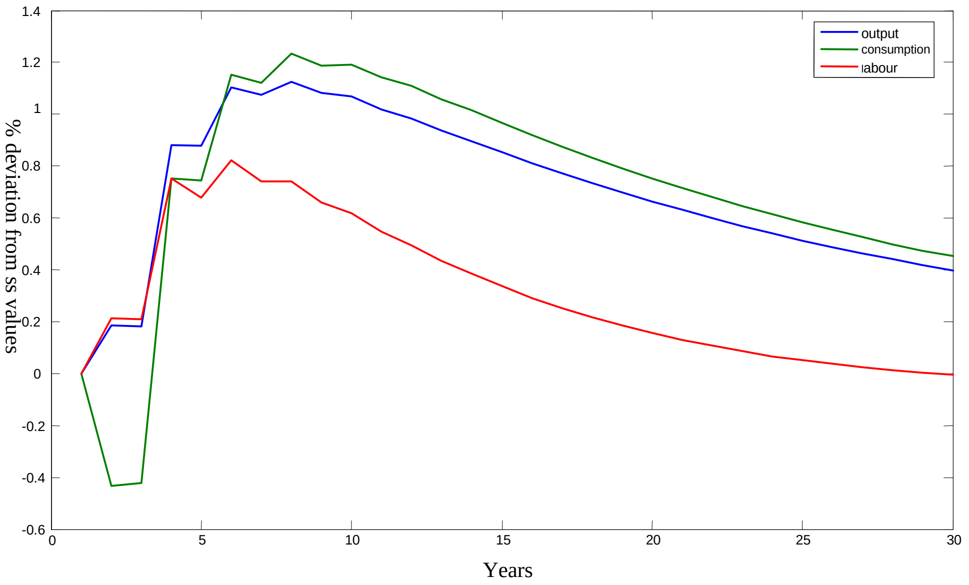

Figure 9: Embodied productivity shock the putty-semi-clay production function.

capital became available, and thus is highly correlated with output per unit of labour, and with consumption. This shortcomming of the 'putty-putty' models has been noted by Campbell (1998) who notes that "the consumer's intertemporal substitution implies that consumption covaries negatively with output and employment in the model, whereas it covaries positively with them in the U.S. economy."(Campbell, 1998, p. 402) This finding is consistent with Gilchrist and Williams (2000) who first noted this characteristic in a putty-clay production function. The results here extend this finding and notes that the *putty-clay effect*[30] also operates if we have a CES *ex post* production function at the firm level, as long as the *ex post* capital-labour substitutability is low.

5.2.2 Permanent shocks

An important criticism of the standard RBC model with disembodied growth is that the model's behavior does not correspond with empirical findings when permanent shocks to technology are considered (see Rotemberg and Woodford,

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[30] Gilchrist and Williams define the putty-clay effect as "a tight link between changes in capacity and movements in employment and output." (Gilchrist and Williams, 2000, p. 929)

1996). Permanent embodied technology may become important in reconciling the model responses with the data. Fisher (2003) finds that permanent investment specific shocks can account for a large percentage of observed business cycle variation. Furthermore, Gilchrist and Williams (2000) have shown that the putty-clay model provides different dynamic responses to that observed in the simple putty-putty model in a number of key variables. The results in this section build on and extend this finding.

Table 3 outlines the first-order autocorrelations of the growth of key variables, and their contemporaneous correlation with output. The difference in the correlation coefficients between embodied and disembodied shocks for all models shows that identifying what kind of growth can account for the key business cycle movements is crucial for applying the model to data. One key finding is that when the shocks are embodied, output growth for the one period production lag model, and the time-to-plan model is substantially more correlated than in the putty-putty model.[31] The time-to-build model shows almost no autocorrelation.[32] At the other extreme, for the investment first model the correlation is highly negative due to the period by period oscillations of output. This highlights the importance in the distribution of weights at the different periods of production. The two relative extremes, time-to-plan and investment first, show wide differences in key correlations across the table, with time-to-build usually having values between these extremes. These differences are more clearly observed when the shocks are embodied. The models provide strong contemporaneous correlations of the opposite sign for growth in output and investment. Furthermore, labour growth is strongly positively autocorrelated in the time-to-plan model and strongly negatively autoccorelated in the investment first model. A comparison of the multiperiod production models with the one period lag model shows that the later investment is committed the more model dynamics correspond to the one period production putty-semi-clay model. This may also be a first indication of which model better explains the data. An important stylized fact is that output and labour growth is strongly autocor-

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[31] Gilchrist and Williams (2000) also noted that the putty-clay model has substantially higher output growth autocorrelation (see p. 947-8).

[32] This is close to what was observed by Christiano and Todd (1996). They found that introducing time-to-plan into a simple Kydland-Prescott type model increased the autocorrelation of output growth. In fact they show that while time-to-plan has $\rho(\Delta y, \Delta y_{t-1}) = 0.4$, time-to-build and the simple RBC model have autocorrelations of around zero (see table 3 p. 27). The difference here is that while time-to-build has again very low autocorrelation, when we have no multiperiod construction, output growth autocorrelation is high. This indicates that multiperiod construction plays a decisive part in the model's dynamics, and may overshadow the characteristics assosiated with putty-semi-clay production.

related (see : Gilchrist and Williams, 2000, table 2, p. 950; Rotemberg and Woodford,1996, table 1, p. 75).[33] This would indicate that time-to-plan and one period production lag models perform not only better than time-to-build and investment first, but also to the putty-putty production model.

Finally, a number of characteristics are common in all putty-semi-clay models. These are the strong correlation in the growth of output with consumption, and output with output per unit of labour, as well as the persistance in growth of output per unit of labour. In comparsion the putty-putty model delivers very low or negative correlations. This is a marked improvement when comparing with actual data (see : Gilchrist and Williams, 2000, table 2, p. 950) as strong positive correlations are also found there. The strong correlation between output and consumption growth is due to the time it takes for investment to mature as capital and therefore raise production. This retards output growth, and it accounts for the comovement with consumption growth. The other two correlations noted can be easily explained as they capture the *par excellence* characteristic of the putty-semi-clay formulation, that the persistent increase in capacity over time is the only way to substantially increase output.

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[33] The values reported in these references are considered here only in so far as to give a general impression of the behaviour of the actual data. They are not directly comparable to the results presented here, as they report quarterly data, and the models here are calibrated in their annual version. A mored direct comparison with data is left for future research.

	Putty-	Putty	One Period	Production	Time-to	-build	Time-to	-plan	Investment	-first
	v_t	z_t	v_t	z_t	v_t	z_t	v_t	z_t	v_t	z_t
$\rho(\Delta y_{t,} \Delta y_{t-1})$	0.040	0.049	0.771	0.027	0.088	0.001	0.683	0.013	-0.962	-0.075
$\rho(\Delta c_{t,} \Delta c_{t-1})$	0.110	-0.050	0.095	0.049	0.048	-0.003	0.070	-0.025	0.080	0.041
$\rho(\Delta i_{t,} \Delta i_{t-1})$	-0.040	0.050	-0.219	-0.214	-0.013	0.023	-0.159	-0.136	-0.970	-0.966
$\rho(\Delta p_{t,} \Delta p_{t-1})$	0.112	-0.049	0.773	0.003	0.793	0	0.773	-0.002	0.889	0.002
$\rho(\Delta l_{t,} \Delta l_{t-1})$	-0.036	-0.034	0.720	0.074	-0.080	0.064	0.719	0.183	-0.979	-0.962
$\rho(\Delta y_{t,} \Delta i_t)$	0.891	0.945	-0.075	0.905	-0.347	0.842	-0.375	0.304	0.965	0.463
$\rho(\Delta y_{t,} \Delta c_t)$	-0.435	0.946	0.626	1	0.852	1	0.793	0.999	0.017	0.970
$\rho(\Delta y_{t,} \Delta l_t)$	0.845	0.809	0.879	-0.458	0.923	-0.466	0.879	-0.651	0.997	0.192
$\rho(\Delta y_{t,} \Delta p_t)$	-0.434	0.947	0.673	1	0.695	1	0.733	0.999	0.108	0.970

Table 3: Each colum presents statistics where the only shock is a permanent embodied (v) or disembodied (z) shock. All variables are in logs. y, i, c, p, and l are output, total period investment (which includes new and finishing projects), consumption, output per unit of labour (defined as lny-lnl). The deltas are defined as one period differences of the variable in logs. Correlations between the variables are depicted as p(x,y).

6 Conclusion

The introduction of vintages in stochastic dynamic general equilibrium models facilitated the analysis and comparison of embodied with disembodied productivity shocks. Modeling vintages explicitly did not dramatically alter the dynamic response of shocks when the *ex post* production function has a high degree of capital-labour substitutability ('putty-putty' models), and therefore an embodied productivity shock in a 'putty-putty' production model has very similar dynamics to an investment cost shock in a non-vintage baseline RBC model. By introducing a CES production function *ex post*, we can consider model economies in which, once investment becomes capital, there is a different degree of substitutability between capital and labour, than there was when investment was planned. This asymmetry highlights the important distinction between investment plans and existing capital, and distinguishes between the investment decision which is forward looking and the production/utilization of capital decision which depends on past investment decisions. A comparison of the dynamic response of models which integrate this characteristic (putty-semi-clay models), with their putty-putty equivalent shows that what can be called a '*putty-clay effect'* dramatically alters the behavior of the model, especially when an embodied productivity shock is concerned. The *ex post* complementarity of capital with labour means that now output can rise substantially only when new investment has increased the capital stock and its labour capacity. This creates a time lag between the original shock and the expansion of output and labour. It inevitably leads to a tighter link between increases in output per unit of labour with output, and labour. Another corollary of this delay is that now labour positively co-varies with consumption, instead of co-varying negatively to it as is the case in 'putty-putty' models. As the co-movement of these key variables has been considered as an important stylized fact of the US business cycle (see Campbell (1998)), the inability of the 'putty-putty' production model to replicate this behavior when the shocks were embodied, was an important shortcoming of the model. With putty-semi-clay production, consumption, labour and output per unit of labour positively co-vary with output.

The integration of investment lags in the 'putty-semi-clay' model showed that multiperiod investment can alter substantially the responses of the model to both embodied and disembodied shocks, and provide a very different set of statistics. An important, and empirically open question, is whether investment is applied uniformly or not during the period that it is completed. Three different variations where considered : 1) when the bulk of investment is applied close to the time that the investment decision was made, (investment first model) 2) when investment was applied uniformly, (time-to-build model) 3) when most of the investment was committed close to the time that the project was being completed, (time-to-plan model). The three models varied widely in their simulated statistics on the correlations between key macroeconomic variables. The two extremes where occupied by the time-to-build model whose behavior was generally closer to the one period investment lag model, and the investment first model, with the time-to-build model being in between. The behavior of the investment-first model was novel in that it combined a high frequency repeated cycle in labour, output and investment, with a longer period cycle which was driven by the persistence of the shock. This was very pronounced when the shock was embodied. Nevertheless, the investment first model was the weakest of the three in meeting the stylized facts of the data examined, whereas the time-to-plan model seemed to perform much better. This highlights the importance of considering the right structure of applying investment when comparing the model with aggregate data, as choosing an inappropriate weight structure may lead to a wrongful dismissal of the general class of multiperiod investment models.

7 Appendix A

All variables that have time subscripts are log deviations from the steady state values. The linearized model used in the stochastic equilibrium section is the following.

A) $\mathrm{Y}_{ss}Y_t = Y_{ss}z_t + Q_{ss}(ak_{ss}^u + (1-a)l_{ss}^u)^{\frac{1}{u}}[\sum_{j=2}^{M}(1-\delta)^{j-2}(Q_{t-j})] + Q_{ss}(ak_{ss}^u + (1-a)l_{ss}^u)^{\frac{1}{u}-1}[\sum_{j=2}^{M}(1-\delta)^{j-2}(ak_{ss}^u k_{t-j} + (1-a)l_{ss}^u l_{t,t-j})]$

B) $\mathrm{L}_{ss}L_t = Q_{ss}l_{ss}[\sum_{j=2}^{M}(1-\delta)^{j-2}(Q_{t-j} + l_{t,t-j})]$

C) $\mathrm{C}_t - R_{t,t+j} = C_{t+j}$ j=1...M

D) $\mathrm{C}_t = W_t$

E) $\mathrm{Y}_{ss}Y_t = C_{ss}C_t + Q_{ss}\phi_2 S_{2,ss}S_{2,t} + Q_{ss}\phi_2 S_{2,ss}Q_t + Q_{ss}\phi_1 S_{1,ss}S_{1,t} + Q_{ss}\phi_1 S_{1,ss}Q_{t-1}$

F) $k_{i,s} = \phi_2 S_{2,t} + \phi_1 S_{1,t+1} + \theta_t$

G) $\mathrm{W}_t = z_t + \frac{a(1-u)k_{ss}^u}{(ak_{ss}^u + (1-a)l_{ss}^u)}k_{i,t-j} + [(u-1) + \frac{(1-u)(1-a)l_{ss}^u}{(ak_{ss}^u + (1-a)l_{ss}^u)}]l_{t,t-j}$ for j=1...M

H) $(\phi_2 + \beta\phi_1)S_{2,ss}S_{2,t} + \beta\phi_1 S_{2,ss}R_{t,t+1} = \sum_{j=2}^{M}\beta^j(1-\delta)^{j-2}ak_{ss}^u(ak_{ss}^u + (1-a)l_{ss}^u)^{\frac{1}{u}-1}[R_{t,t+j} + z_{t+j}] + \sum_{j=2}^{M}\beta^j(1-\delta)^{j-2}ak_{ss}^u(ak_{ss}^u + (1-a)l_{ss}^u)^{\frac{1}{u}-2}(1-u)(1-a)l_{ss}^u l_{t,t+j} + \{\sum_{j=2}^{M}\beta^j(1-\delta)^{j-2}[auk_{ss}^u(ak_{ss}^u + (1-a)l_{ss}^u)^{\frac{1}{u}-1} + a^2k_{ss}^{2u}(1-u)(ak_{ss}^u + (1-a)l_{ss}^u)^{\frac{1}{u}-2}]\}k_{i,t}$

I) $(\phi_2 + \beta\phi_1)S_{2,ss}S_{2,t} + \beta\phi_1 S_{2,ss}R_{t,t+1} = \sum_{j=2}^{M}\beta^j(1-\delta)^{j-2}(ak_{ss}^u + (1-a)l_{ss}^u)^{\frac{1}{u}}[R_{t,t+j} + z_{t+j}] + \sum_{j=2}^{M}\beta^j(1-\delta)^{j-2}(ak_{ss}^u + (1-a)l_{ss}^u)^{\frac{1}{u}-1}[ak_{ss}^u k_{i,t} + (1-a)l_{ss}^u l_{t,t+j}] - (\sum_{j=2}^{M}\beta^j(1-\delta)^{j-2}[W_{ss}W_{t+j} + l_{ss}l_{t+j,t}])$

J) $S_{2,t} = S_{1,t+1}$

And for the benchmark model :

A) $\mathrm{c}_t = z_t + ak_{2,t} - al_t$

B) $\mathrm{c}_{ss}c_t = k_{ss}^a l_{ss}^{1-a}[z_t + (1-a)l_t + ak_{2,t}] + \varphi_2 k_{ss}[(1-\delta)k_{2,t} - k_{2,t+2}]$
$+k_{ss}(\varphi_2(1-\delta) - \varphi_1)k_{2,t+1} + \varphi_2\delta k_{ss}\theta_t + \varphi_2\delta k_{ss}\theta_{t-1}$

C) $-\varphi_2 c_t = -\beta^2 ak_{ss}^{a-1}l_{ss}^{1-a}[z_{t+2} + (1-a)l_{t+2} + (a-1)k_{2,t+2} + C_{t+2}] + \varphi_1(1-\delta)c_{t+2} - (\varphi_1\beta^2 + \varphi_2\beta)(1-\delta)\theta_{t+1} + \beta(\varphi_1 - (1-\delta)\varphi_2)c_{t+1} + (\varphi_1\beta + \varphi_2)\theta_t$

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