Nonlinear Interest Rate Reaction Functions for the UK*

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December 19, 2008

Abstract

We empirically analyze Taylor-type equations for short-term interest rates in the United Kingdom using quarterly data from 1970Q1 to 2006Q2. Starting from strong evidence against a simple linear Taylor rule, we model nonlinearities using logistic smooth transition regression (LSTR) models. The LSTR models with time varying parameters consistently track actual interest rate movements better than a linear model with constant parameters. Our preferred LSTR model uses lagged interest rates as a transition variable and suggests that in times of recessions the Bank of England puts more weight on the output gap and less so on inflation. A reverse pattern is observed in non-recession periods. Parameters of the model do not change after 1992, when an inflation target range was announced. We conclude that for the analysis of historical monetary policy, the LSTR approach is a viable alternative to linear reaction functions.

Keywords: interest rate reaction functions, smooth transition regression model, monetary policy

JEL classification: C51, C53, E43, E52, E58

*This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 'Economic Risk'. We thank participants of the UKon-ULP-Workshop on 'Empirical Economics and Econometrics' 2007, the 8th IWH-CIREQ Macroeconometric Workshop 2007, the Hejnice Seminar 2008 and the Jahrestagung des Vereins für Socialpolitik 2008, for valuable comments on earlier versions of this paper. Ralf Brüggemann: Universität Konstanz, FB Wirtschaftswissenschaften, Lehrstuhl für Statistik und Ökonometrie, Box 129, 78457 Konstanz, Germany, email: ralf.brueggemann@uni-konstanz.de; Jana Riedel: Westfälische Wilhelms-Universität Münster, Wirtschaftswissenschaftliche Fakultät, Institut für Internationale Ökonomie, Universitätsstraße 14-16, 48143 Münster, Germany, email: jana.riedel@wiwi.uni-muenster.de.

1 Introduction

Following the work of Taylor (1993), fairly simple linear interest rate reaction functions have been used to analyze and evaluate monetary policy of central banks. There is, however, an ongoing debate on how to model these decisions empirically. In line with the New Keynesian theory discussed in Clarida, Galí & Gertler (1999), a forward-looking approach to estimate a central bank's reaction function is widely used. Alternatives include backward-looking and contemporaneous Taylor rules (see e.g. Gerlach & Schnabel (2000), Gerlach-Kristen (2003), Gerdesmeier & Roffia (2004) and Surico (2003)). Typically, empirical results depend to some extent on the used estimation techniques and sample period. Another serious problem with the empirical results reported in the literature is that parameters from linear models seem to be rather unstable over time (see e.g. Judd & Rudebusch (1998) for the US economy).

A look at the history of monetary policy in the UK illustrates that the Bank of England's (BoE) policy towards inflation and interest rate setting has quite likely changed over time. Although an inflation reducing policy has been announced in 1976, a specific inflation target range was only introduced after the pound crisis that led to the breakdown of the European Monetary System (EMS) in 1992. While an average target for inflation of 2.5 percent was already officially announced in 1995, the BoE gained operational autonomy to fulfill the inflation target set by the Her Majesty's Treasury (HMT) only in May 1997. Since the beginning of 2004, the point target is set to 2 percent. Recent empirical literature (see e.g. Martin & Milas (2004), Kesriyeli, Osborn & Sensier (2004), Dolado, Pedrero & Ruge-Murcia (2004), Assenmacher-Wesche (2006), Kharel (2006), Cukierman & Muscatelli (2008), Qin & Enders (2008) and Castro (2008)) points out time varying and nonlinear evidence in the relationship between nominal interest rates set by the central bank and deviations of output and inflation from their corresponding target values. Thus, a strictly linear rule-based approach may not adequately reflect the actual interest rate setting behavior in the UK.

To analyze possible changes in monetary policy, Assenmacher-Wesche (2006) uses a Markov-switching approach to estimate central banks' reaction functions of the US, UK and Germany. She models abrupt switches, indicating different reactions dependent on existing inflationary pressure. For the UK, the only regime shift occurs in 1984. Before this, interest rate smoothing and output stabilization characterize the high inflation regime, whereas afterwards high weight is put on inflation stabilization. A drawback of the Markov-switching approach is that it does not allow for slow transitions between different states. To incorporate slowly changing behavior, some authors estimate nonlinear Taylor rules using smooth transition regressions. In these models, the speed of transition between regimes is not predetermined. Smooth changes are typically induced by a time trend as a transition variable. More abrupt switches are obtained using an economic variable as transition variable. For instance, Kesriyeli et al. (2004) conducted an analysis for the US, Germany and the United Kingdom using monthly data starting in 1984. In their backward-looking Taylor rules, the transition variable is either the first difference of lagged interest rates or a linear trend. Qin & Enders (2008) analyze univariate, linear and nonlinear Taylor rules (the latter using STR and again lagged interest rates as transition variable) for US real-time data. Their in-sample estimates and the out-of-sample forecasting exercise reveal that nonlinearities matter over different subsamples. In addition, STR models do outperform linear and univariate specifications, but are not able to beat the most simple univariate setup in terms of forecasting. Martin & Milas (2004) estimate a logistic smooth transition regression (LSTR) model with two regime switches for UK's monetary policy over a broad time span using nonlinear least squares and instrumental variable estimators. They focus on determining varying behaviour of the Bank of England (BoE) induced by inflation changes. They find stronger reactions on increasing inflation if inflation is above the target than on decreasing inflation below the target after 1992 and, more general, a smaller influence of inflation on the interest rate before 1992. Kharel (2006) estimates an LSTR model for the sample after the decision on inflation targeting in 1992Q3 using a forward-looking approach and confirms the findings of Martin & Milas (2004) detecting the bias towards deflation in BoE's monetary policy, keeping inflation below the target of 2.5 percent on average. Castro (2008) uses also an LSTR model with inflation as transition variable to confirms that the BoE keeps inflation in between a target range of 1.8 - 2.4 percent rather than following a point target. In addition, he includes a so called financial indicator variable to see if central banks consider this additional information for their interest rate settings. It only seems to be relevant for the ECB, at least for the considered sample period 1992:2007, and not for the UK and the US. However, nonlinearities are detected in the BoE's interest rate rule. In a more recent paper, Cukierman & Muscatelli (2008) models the central bank's loss function as sum of functions of the output gap and the inflation gap, respectively. In their model, similar to Dolado et al. (2004), the shape of the functions determine possibly asymmetric preferences. They use a hyperbolic tangent STR model to empirically evaluate the underlying theory using US and UK data. For the BoE 1979-2005, Cukierman & Muscatelli (2008) confirm recession avoidance preferences before the target was introduced and inflation avoidance preferences afterwards, i.e the is concave before and convex after 1992. From a theoretical point of view, Dolado et al. (2004) combine nonlinear Phillips curves and asymmetric preferences of the central bank, i.e. they demonstrate that nonlinearities occur if either the CB loss function is non-quadratic or the aggregate supply curve is convex.[1] Similarly, Surico (2004) derives and estimates a model in which nonlinerity arises due to asymmetric central bank's preferences by using a cubic loss function. Including an output gap in the objective function, his results contrast to those of Dolado et al. (2004), i.e. he does not find evidence for nonlinearities after 1979 for US data.

Thus, there is some evidence for nonlinearities in the interest rate reaction function for the UK. These nonlinearities may reflect the structural changes related to monetary policy changes in the UK that have been going on in the past decades. In this paper, we take a closer look at these nonlinearities within a smooth transition regression model (see e.g. Teräsvirta (1998), van Dijk, Terävirta & Franses (2002) and Teräsvirta (2004)). We estimate LSTR models for forward-looking interest rate reaction functions in the UK using data from 1970Q1-2006Q2. We use a forward-looking interest rate rule (in the spirit of Clarida, Galí & Gertler (1998)) to reflect that future inflation is the relevant quantity for todays central bank's interest rate decision.

Alternative logistic smooth transition regression models are specified which differ with respect to the chosen transition variable. We find that all considered nonlinear models outperform the simple linear specification in terms of model fit and the ability to track the actual interest rate. Our preferred model specification is an LSTR model where a lagged interest rate is used as a

[1]In addition, Dolado et al. (2004) provide empirical evidence from GMM estimation, that the Fed's preferences differ between the Burns-Miller and the Volcker-Greenspan period and preferences are indeed asymmetric and, in line with this, the conditional variance of inflation is positively correlated with the real interest rate in the Volcker-Greenspan period. Furthermore, convexity of the Phillips curve does not seem to be an issue in this dataset.

transition variable. From this model we indeed find evidence for changing parameters on both inflation and the output gap. In periods of recessions, the BoE seems to have put more weight on the output gap and less so on inflation. A reverse pattern is observed for non-recession periods. Another interesting observation from this model is the fact that changes in the parameters only occurred prior to 1992. After this date, which coincides with the decision for inflation targeting, the parameters of the Taylor-type relation are constant. Thus our empirical model is consistent with the fact that monetary policy goals have not greatly changed after 1992. Overall, we find for the UK, that the smooth transition regression approach of this paper is a viable alternative to the widely used linear Taylor-type rules if interest is in the analysis of historical monetary policy.

The remainder of the paper is organized as follows. Section 2 introduces the smooth transition regression modeling framework and the empirical equations for nonlinear interest rate reaction functions. The empirical analysis including a comparison to the linear model is contained in Section 3. Section 4 focuses on the economic interpretation of our results and finally Section 5 concludes.

2 The Modeling Framework

Since the seminal paper by Taylor (1993) the nominal interest rate set by central banks is often assumed to depend on the output gap and on inflation. Our starting point is the forward-looking Taylor-type reaction function (see e.g. Clarida et al. (1998)), where the nominal interest rate r_t^* depends on the deviation from an inflation target, $\mathrm{E}[\pi_{t+1}|\Omega_t]-\pi^*$, and on the output gap, $\mathrm{E}[y_t|\Omega_t]-y_t^*$. Let $\bar{r}$ denote the long-run equilibrium rate, then r_t^* can be expressed as

$$r_t^* = \bar{r} + \beta(\mathrm{E}[\pi_{t+1}|\Omega_t] - \pi^*) + \gamma(\mathrm{E}[y_t|\Omega_t] - y_t^*) \tag{2.1}$$

Following Clarida et al. (1998), we define $\alpha = \bar{r} - \beta\pi^*$ and $y_t^{\text{gap}} = y_t - y_t^*$ to write (2.1) as

$$r_t^* = \alpha + \beta\mathrm{E}[\pi_{t+1}|\Omega_t] + \gamma\mathrm{E}[y_t^{\text{gap}}|\Omega_t] \tag{2.2}$$

In empirical specifications, additional terms are needed to account for the fact that interest rate changes are smooth. Thus a typical specification assumes that the actual rate adjusts only partially according to

$$r_t = \sum_{j=1}^{J} \rho_j r_{t-j} + \left(1 - \sum_{j=1}^{J} \rho_j\right) r_t^* + e_t \tag{2.3}$$

in which e_t is an iid innovation that is assumed to represent exogenous shocks to the interest rates and $J = 1$ or $J = 2$ depending on the particular empirical implementation. For estimation, equations (2.1) and (2.3) are combined to obtain the so-called reduced form model

$$r_t = \sum_{j=1}^{J} \rho_j r_{t-j} + \alpha^* + \beta^*\pi_{t+1} + \gamma^* y_t^{\text{gap}} + e_t, \tag{2.4}$$

in which $\alpha^* = (1 - \sum_{j=1}^{J}\rho_j)\alpha$, $\beta^* = (1 - \sum_{j=1}^{J}\rho_j)\beta$ and $\gamma^* = (1 - \sum_{j=1}^{J}\rho_j)\gamma$ and inflation and output gap expectations have been replaced by realized values. In this paper, we extend this linear specification by introducing a smooth transition regression (STR) model. The STR model is discussed in detail in Teräsvirta (1998), Teräsvirta (2004) and applied to our setting allows to model smooth changes in the reaction function of the central bank. To be more specific, we start with a model where we allow all coefficients to vary over time, including those of lagged interest rates.[2] For this purpose, we introduce a nonlinear term in (2.4) such that the model is written as

$$r_t = \sum_{j=1}^{J} \rho_{0j} r_{t-j} + \alpha_0^* + \beta_0^* \pi_{t+1} + \gamma_0^* y_t^{\text{gap}} + \left[\sum_{j=1}^{J} \rho_{1j} r_{t-j} + \alpha_1^* + \beta_1^* \pi_{t+1} + \gamma_1^* y_t^{\text{gap}} \right] G(s_t, \theta, c) + \varepsilon_t. \tag{2.5}$$

α_0^*, β_0^*, γ_0^* and $\rho_{0j}, j = 1, 2$ are the parameters in the linear part of the model and α_1^*, β_1^*, γ_1^* and $\rho_{1j}, j = 1, 2$ are the parameters in the nonlinear part of the model. The error terms ε_t are assumed to be iid $(0, \sigma^2)$. The transition function

$$G(s_t, \theta, c) = \left\{ 1 + \exp\left[-\theta \prod_{k=1}^{K} (s_t - c_k) \right] \right\}^{-1}, \quad K = 1, 2 \tag{2.6}$$

is a logistic function, in which s_t denotes a particular transition variable. In our application, s_t will either be a linear deterministic trend or an economic variable. c_k denotes particular threshold values to be determined from the data. We consider models with $K = 1$ and $K = 2$. For $K = 1$ the parameters may change monotonically depending on the variable s_t. For instance, the parameter on inflation may change from β_0^* to $\beta_0^* + \beta_1^*$. We refer to this model as an LSTR1 in the following. If $K = 2$ the parameters change systematically around the point $(c_1 + c_2)/2$. This model is called the LSTR2 in the following. The choice of K is an empirical question. The parameter $\theta > 0$ governs the speed of transition between two regimes. The smaller $\theta > 0$ in equation (2.6), the smoother is the transition between regimes. The speed of transition is not predetermined in this model but estimated from the data. Note that this specification also nests the linear model for the case when the transition function is constant. In the empirical specification, tests for linearity are conducted and for this purpose, it is convenient to rewrite the model (2.5) in compact notation as

$$r_t = \phi' \mathbf{z}_t + \psi' \mathbf{z}_t G(s_t, \theta, c) + \varepsilon_t, \tag{2.7}$$

where for instance for $J = 2$ one has $\mathbf{z}_t = (1, r_{t-1}, r_{t-2}, \pi_{t+1}, y_t^{\text{gap}})'$ as the vector of regressors and the vectors $\phi = (\alpha_0^*, \rho_{01}, \rho_{02}, \beta_0^*, \gamma_0^*)'$ and $\psi = (\alpha_1^*, \rho_{11}, \rho_{12}, \beta_1^*, \gamma_1^*)'$ contain the parameters from the linear and the nonlinear part, respectively. This modeling framework provides a fairly flexible way to model possible nonlinearities in the central bank's reaction function. The choice of the transition variable s_t as well as the number of regimes is an empirical question and is therefore discussed in the empirical analysis in the Section 3.

[2] We also model nonlinearities only in the exogenous variables. However, as our specification nests this case, results are not presented here but available on request.

3 Empirical Analysis

3.1 The Data

To estimate the interest rate setting rule discussed above for the UK, we use quarterly data for a sample period of 1970Q1-2006Q2.[3] The three-month Treasury bill rate provided by the IMF-IFS database is used as the short-run nominal interest rate r_t. Another proxy would be the interbank overnight rate because of the flat yield curve between the bank rate and the interbank overnight rate. We stick to the Treasury bill rate because the other data are only available from 1978Q1 onwards and no significant differences occur. Inflation π_t is calculated as a year-to-year change $\pi_t = 100 \cdot (P_t - P_{t-4})/P_{t-4}$ of the retail price index (RPI), denoted by P_t. Since 1992 the BoE reports the retail price index without mortgage prices (RPIX). Thus, we construct the inflation series using the RPI until 1992Q3 and the RPIX afterwards. Both series are taken from the EcoWin Economics database.[4] The output gap y_t^{gap} is constructed by using HP-filtered real GDP as measure for trend output (smoothing parameter λ =1600) and substracting it from the actual GDP series. Quarterly real GDP data are taken from the OECD Main Economic Indicators.[5] Figure 1 shows the seasonally adjusted series for interest and inflation rate as well as for the output gap. While these series show some persistence, we follow the standard practice in this literature and do not consider the possibility of unit roots in the interest rate and the inflation rate as they are not plausible from an economic point of view. Besides domestic variables, we also include the US federal funds rate in some of our models to account for foreign effects.[6]

3.2 Testing for STR nonlinearities

We start the empirical analysis by estimating the linear model in equation (2.4) using ordinary least squares (OLS)[8]. Using bootstrapped Chow sample split and break point tests we find evidence for changing parameters in the model, hence a linear model with constant parameters is clearly rejected for the full sample period (1970Q1-2006Q2). Therefore, we explore the possibility that nonlinear LSTR models can capture the changes in the parameters.

To detect nonlinear pattern in the form of equation (2.5), we perform LM-type linearity test.

[3]Note, the UK was member of the Bretton Woods system and had also fixed exchange rates from 1990 to 1992 when it joint the EMS. Thus, we included the Dollar exchange rate as additional regressor in the subsequent analysis, but there is no evidence for a significant influence of the exchange rate regime on the estimation results. In line with Assenmacher-Wesche (2006) we do not consider this exchange rate contraint, not at least because of the short fixed exchange rate time spans.

[4]When switching to a point target in 2004Q1, the BoE changes its measure of inflaiton and uses the CPI. We do not include this last change in our analysis as it incorporates only the last ten data points adn might lead to erroneous results with respect to nonlinearities.

[5]Since monthly real GDP data is not available for the UK, we also estimate the monthly data model using the industrial production index. Results are available on request.

[6]We also tried to include the German call money market rate, German M3 money and the Dollar/Pound exchange spot rate[7] and, following Dolado et al. (2004), the conditional variance of inflation. These additional variables turned out not to be very important in our models. Consequently, the corresponding results are not shown.

[8]Forward-looking Taylor rules are commonly estimated using GMM estimation due to the involved expectations. However, we feel that results depend crucially on the choice of instruments and feel encouraged not to use GMM, as the linear OLS estimation fits the results of Clarida et al. (1998) TSLSE quite well.

Details on this approach are given in Teräsvirta (1998) and Teräsvirta (2006). Since the model is only identified under the alternative of nonlinearity as written in equation (2.5), a third-order Taylor approximation around $\theta = 0$ is done for $G^*(\cdot) = G(\cdot) - 1/2$ because for $\theta \to 0$, $G(\cdot) \to 1/2$. From the Taylor expansion one obtains

$$r_t = \delta_0' \mathbf{z}_t + \sum_{j=1}^{3} \delta_j' \tilde{\mathbf{z}}_t s_t^j + \varepsilon_t^*, \quad \varepsilon_t^* = \varepsilon_t + remainder. \tag{3.1}$$

$\tilde{\mathbf{z}}_t$ denotes the vector of $\mathbf{z}_t$ without the constant when s_t is an element of $\mathbf{z}_t$. Under the null hypothesis of linearity, $\delta_j = 0 \; \forall \; j$; under the alternative $\delta_j \neq 0$ for at least one j. The test procedure considers each regressor as a candidate transition variable and is implemented as an F-significance test. In case of rejecting the null for several specifications, we tend to use the variable with the strongest rejection of the null (with the lowest p-value). The test results for two different subsamples are given in Table 1. The p-values of the joint significance test are given in the first column denoted by F. To make a decision on the number of regime shifts K, we consider three other hypotheses for which the p-values of the corresponding F statistics are given in the columns labled F4, F3 and F2 in Table 1, respectively. Following Teräsvirta (2004), the three hypothesis are $H_{04}: \; \delta_3 = 0, \; H_{03}: \; \delta_2 = 0 | \delta_3 = 0$ and $H_{02}: \; \delta_1 = 0 | \delta_2 = \delta_3 = 0$ from equation (3.1), an LSTR1 model would be proposed by the strongest rejection in either H_{04} or H_{02}, whereas the smallest p-value being the one for H_{03} would imply to model nonlinearities via LSTR2 (or exponential STR) models.

Using this test strategy, we find that in a model with a trend as transition variable, the linear model with constant parameters is rejected (in line with the results from the Chow tests) and an LSTR with $K = 1$ (LSTR1) model is suggested. For the full sample, the tests suggest a linear specification if the inflation rate is used as a transition variable. In contrast, an LSTR1 model is suggested in the shorter subsample starting in 1978Q1.[9] Furthermore, the linear specifications are typically rejected when either the output gap or lagged interest rates are considered as a transition variable. Interestingly, we do not find evidence for an STR model with the US federal funds rate as a transition variable. In summary, the trend, the output gap and the lagged interest rates seem to be among the set of possible transition variables. The evidence is less clear for the one-period ahead value of the inflation rate. The choice of $K = 1$ or $K = 2$ is not very clearcut. Therefore, we estimate different models suggested by the linearity test and check their ability to describe the data. The alternative models are described in the following subsections.

3.3 LSTR Models with a Trend as a Transition Variable

In this section we discuss the results from LSTR models that use a linear deterministic trend as a transition variable. This model allows only one smooth transition between the parameters of two states. We estimate these and all following LSTR models by conditional maximum likelihood. A grid search determines initial values for the coefficients θ and c. We fix θ and c and estimate the remaining parameters as functions of both. This is done for a set of grid points with in advance specified boundaries. The one specification with the minimum sum of squared residuals is used

[9] There is a sample split in 1978Q1 to eliminate the years of the Bretton Woods breakdown, stagflation and interest rate fluctuations due to the first oil crisis. This is supported by the bootstrapped p-values of the Chow sample split test.

for conditional maximum likelihood estimation using the Newton-Raphson algorithm. Note that the proposed lag length in the linear specification does not have to be best choice for the fitted nonlinear model, but works as a good first guess. Model evaluation is done in terms of tests of residual autocorrelation at lag 2 and 4, remaining additive nonlinearity and non-constancy of parameters.[10] Furthermore, we perform residual tests for non-normality and ARCH effects. For a derivation of these tests based on third-order Taylor approximations see e.g. Teräsvirta (1998).

Estimation of the LSTR1 model for the full sample period (1970Q1-2006Q2) leads to a model that is not satisfactory. Although coefficients in the nonlinear part are significant, diagnostic tests provide evidence for remaining nonlinearity and parameter constancy has to be rejected at the 5% level. Therefore, the precise results are not shown to conserve space. Instead we report results for a sample period starting in 1978Q1.

The results for the STR models (together with the results of a linear specification) for the sample period (1978Q1-2006Q2) are given in Table 2 together with results from diagnostic tests given in Table 3. The tests do not show any evidence for remaining nonlinearity nor evidence for parameter non-constancy. Thus, the model seems to be well specified. From the parameter estimates given in Table 2 we find that inflation rate enters significantly in the linear part of the model, while it is insignificant in the nonlinear part. In comparison to the linear model, the inflation coefficient in the linear part increases and the output gap coefficient is no longer significant in the linear part but enters the nonlinear part significantly. Thus, we conclude that the nonlinear part contains substantial information. This is also obvious from looking at Figure 2 where we have plotted the transition function together with the implied linear and nonlinear part of r_t for both considered sample periods. The transition function deviates from zero right at the time of the recession in 1979 and reaches its turning point shortly before the last recession (1990Q2). The final state is not reached before 1999, i.e. just after introduction of inflation targeting and the EMS II system. Adding the federal funds rate to equation (2.5) results in a steeper transition, but the reduced form coefficients are robust to this. The linear part overstates the actual interest rate, at least since the end of the last recession; the nonlinear part is negative then and brings down the estimated interest rates.

STR models with a trend as a transition variable allow for a one-time smooth change of the coefficients only. Therefore, a more flexible modeling approach is to consider STR models, where the transition depends on different states of the economy. In other words, a more flexible model can be obtained by letting the transition variable s_t be an economic variable. In these models, changes in the parameters take place whenever the economic transition variable falls above or below a certain threshold. Such models are considered next.

3.4 LSTR Models with the Output Gap as a Transition Variable

In this section we consider LSTR models with the output gap as a transition variable. Empirical results for equation (2.5) with $s_t = y_t^{\text{gap}}$ are given in Table 4 together with results of diagnostic tests in Table 5. Following the suggestion obtained from the linearity test, we specify LSTR1 and LSTR2 models. The residuals of the LSTR2 model reported in column one of Table 4 are

[10]For the latter, hypotheses for $K = 1, 2$ or 3 are tested.

not autocorrelated but show some signs of ARCH effects. The first column of Table 5 reveals no remaining nonlinearity. Parameter constancy is rejected on the 5% but not on the 1% level in a model that excludes the federal funds rate. If the federal funds rate enters the regression, there is some evidence for remaining nonlinearity (in lagged interest rates) but parameters are constant over time. We have also estimated a model for a sample starting in 1978Q1 but find evidence for both, remaining nonlinearities and parameter non-constancy and therefore. Therefore, we do not consider this model in the following. Our preferred model is the LSTR2 for the full sample without the federal funds rate.

The nonlinear behavior of our preferred LSTR2 model is summarized in Figure 3. The left panel indicates that nonlinearity sets in whenever the economy is right before a recession period and also during the frequent fluctuations in the eighties. Including the federal funds rate results in less peaks in the transition function (results not shown).

3.5 LSTR Models with Lagged Interest Rates as Transition Variables

Linearity tests provide support for smooth transition models using lagged interest rates as transition variables and results for these models are summarized in Tables 6 and 7 and Figure 4. The first column of Table 6 presents estimated coefficients from an LSTR1 with r_{t-1} as a transition variable. Inflation enters the linear and the nonlinear part significantly, while the output gap is only significant in the nonlinear part. Note, however, that the coefficient on one-period ahead inflation is quite small in the linear part and has a negative sign in the nonlinear part. Very similar coefficient estimates are obtained when considering an LSTR2 model with r_{t-2} as a transition variable (see 2nd column of Table 6). Interestingly, the federal funds rate enters in both models significantly in the linear part. When considering the sample 1978Q1:2006Q2, the estimates of the inflation coefficient increase drastically in the linear part, which may reflect the fact that the BoE has put more weight on inflation in the later sample. In line with the estimates for the full sample, the output deviations are still important in the nonlinear part (in a model that include the federal funds rate). The results of the diagnostic tests in Table 7 suggest that (apart from some residual non-normality and ARCH effects) the LSTR1 and LSTR2 model for the full sample and the LSTR1 model with the federal funds rate for the reduced sample are reasonably well specified. In particular, there is neither evidence for remaining nonlinearity nor for non-constant parameters. In contrast, the LSTR1 model without the federal funds rate for the period 1978Q1-2006Q2 (2nd last column of Table 7) shows signs of both non-constant parameters and remaining nonlinearities.

For the LSTR2 (full sample) and the LSTR1 (reduced sample, with federal funds rate) we provide a graphical representation of the nonlinear parts and the transition functions in Figure 4. In both models the transition sets in at the beginning of the second recession in 1979 and short before the third one in 1989. The different degree of smoothness in the regime changes are due to differences in $\hat{\theta}/\hat{\sigma}_s^K$, the estimated standardized coefficient of the transition variable, with $\hat{\sigma}_s^K$ being the K's power of the standard deviation of the transition variable. Nevertheless the transition starting points are almost identical. There is mixed evidence for time varying parameter. In the LSTR2 model regime changes occur regularly and are also relevant after 1992Q3, while in the LSTR1 for the reduced sample no parameter changes occur after 1992.

3.6 Comparing Linear and Nonlinear Models

To compare the model fit of the linear and our nonlinear models, we recursively calculate the estimate for the implied target interest rate r_t^*. In analogy to the linear model the implied target rate is determined by plugging in the estimates to

$$r_t^* = \alpha_0 + \beta_0 \pi_{t+1} + \gamma_0 y_t^{\text{gap}} + (\alpha_1 + \beta_1 \pi_{t+1} + \gamma_1 y_t^{\text{gap}}) G(s_t, \theta, c) \tag{3.2}$$

Here, following Assenmacher-Wesche (2006), we do not apply interest rate smoothing to make the differences easier to visualize. We find that the nonlinear models outperform the linear ones in the sense, that the nonlinear specifications are able to track the actual rate better for most specifications. Figure 5 compares the implied target rates from the linear and nonlinear models with actual rates. The upper left panel shows the actual interest rate and the rate 'predicted' by the linear Taylor rule. For most periods the predicted rate from the linear Taylor rule is below the actual rate. In particular, the linear model does not track the actual target well in volatile times. In contrast, results for the LSTR1 model with a trend as the transition variable (upper right panel) indicate that this model tracks the actual interest rate quite well. The two remaining panels show results for the LSTR2 model with the output gap and the LSTR1 with lagged interest rates as a transition variable. Again, the nonlinear models capture actual interest rate dynamics better than the linear model.

To shed some more light on the relative gains from using the nonlinear models, we report relative mean squared errors to compare linear and nonlinear specifications. To be more precise, we compute

$$\text{RelMSE} = \frac{\sum_{t=1}^{T}(r_t - \hat{r}_{t,\text{nonlin}})^2}{\sum_{t=1}^{T}(r_t - \hat{r}_{t,\text{lin}})^2} \cdot \frac{(T - k_{\text{lin}})}{(T - k_{\text{nonlin}})}. \tag{3.3}$$

r_t denotes the actual interest rate, $\hat{r}_{t,\text{nonlin}}$ and $\hat{r}_{t,\text{lin}}$ denote the fitted values for a particular nonlinear STR model and the linear model, respectively. To make a fair comparison, we also correct for the different number of parameters to be estimated in linear and nonlinear models. The results for different nonlinear STR models and for both considered sample periods are reported in Table 8. The results are quite clearcut. Any nonlinear model specification outperforms the linear model as indicated by entries well below 1. Thus, we conclude that the model fit of our nonlinear specifications is clearly superior to the simple linear model. A possible reason is, of course, that the linear model does not capture parameter changes due to structural breaks. In the next section we turn to the economic interpretation of our results.

4 Economic Interpretation

In this section we consider the evolution over time of the key parameters in the Taylor-type equation. For this purpose we focus on the evolution of the coefficients on inflation and the output gap in the structural form of our Taylor specifications.[11] Thus, we need to calculate back the structural form parameters from the reduced form estimation results. Due to the nonlinearity of the model,

[11]Note that the values of the structural coefficients at time t depend on the realization of the transition variable at this point in time. This is inherent in the structure of the LSTR model and should be considered in the coefficient interpretation.

the structural coefficients are made up of the linear and the nonlinear part. First, we have to account for interest rate smoothing such that

$$\beta_{0t} = \frac{\beta_0^*}{(1 - \sum_j \rho_{0j} - \sum_j \rho_{1j} G(s_t, \theta, c))}, \quad \beta_{1t} = \frac{\beta_1^*}{(1 - \sum_j \rho_{0j} - \sum_j \rho_{1j} G(s_t, \theta, c))} \tag{4.1}$$

and

$$\gamma_{0t} = \frac{\gamma_0^*}{(1 - \sum_j \rho_{0j} - \sum_j \rho_{1j} G(s_t, \theta, c))}, \quad \gamma_{1t} = \frac{\gamma_1^*}{(1 - \sum_j \rho_{0j} - \sum_j \rho_{1j} G(s_t, \theta, c))}. \tag{4.2}$$

β_{0t} and γ_{0t} are the parameters from the linear part, which are now time varying due to time variation in interest rate smoothing. β_{1t} and γ_{1t} denote the parameters from the nonlinear part of the model and finally the overall coefficients are given by

$$\beta_t = \beta_{0t} + \beta_{1t} G(s_t, \theta, c) \tag{4.3}$$

and

$$\gamma_t = \gamma_{0t} + \gamma_{1t} G(s_t, \theta, c). \tag{4.4}$$

A graphical representation of these two time varying parameters obtained from the LSTR model in which a time trend is the transition variable is given in Figure 6. We show the time varying coefficient on inflation and the output gap for the LSTR1 model fitted to the sample 1978Q1:2006Q2 with and without the federal funds rate as an additional regressor. In the model with the federal funds rate included, the inflation coefficient (left panel) increases steeply around 1986. In contrast, in our preferred model specification (without the federal funds rate), the transition sets in later and the turning point of the coefficient function is exactly at the end of the last recession. The estimated inflation coefficient $\hat{\beta}_t$ increases to above unity for most specifications, thus implying an effect on the real interest rate in the later periods. The increase of the inflation coefficient over time may be interpreted as representing a more stringent policy of the BoE. Thus, our model reflects the changes in the the bank's policy towards inflation and interest rate setting over time. This is for instance reflected by events like the announcement of an inflation target in 1992 after the pound crisis or operational autonomy of the BoE to fulfill inflation target in 1997. These slow changes in preferences are captured by using a trend as transition variable. The right panel of Figure 6 plots the output gap coefficients obtained from equation (4.4). There is some evidence for the increasing importance of the output gap in the BoE's interest setting policy in the later periods. The turning point in the transition function is around the end of the recession period 1990-1992.

The left panel of Figure 7 plots the estimated coefficient on inflation obtained from the LSTR2 model for the full sample using output deviations as transition variable. We find small decreases in the inflation coefficient whenever output falls below the trend and in general higher coefficients since 1994. Since the coefficient is smaller than one there will be no influence on the real rate using this policy instrument (result not shown). The estimated coefficient of y_t^{gap} is given in the right panel of Figure 7. Regime shifts appear quite often during volatile times in the beginning of the sample until regimes become more stable at the end. The coefficient on output deviates symmetrically around the estimate from the linear model (indicated by the long-dashed line). Thus, there is no sign of asymmetric preferences. Essentially, the output coefficient changes from values around zero to large positive values. The switches also occur in the middle of recession. In case of positive output deviations from trend and increasing inflation, the BoE seems to put more weight on both, inflation and output gap.

Considering now the results based on the preferred LSTR model with lagged interest rate as a transition variable, we give the evolution of the parameters over time in Figure 8. The inflation coefficient declines sharply whenever being in a recessionary periods. In these times the weight on the output gap increases substantially. Thus, in recession periods the BoE seems to put more weight on output, while weight on inflation increases in non-recession periods. Note that the coefficients on inflation and output gap fluctuate only in the time before 1992, indicating ongoing changes in the preferences of the central bank (possibly due to volatile movements in the economy). Interestingly, after 1992/1993, the coefficient do not change anymore. Thus, in contrast to a linear model, our nonlinear model is able to capture the changing environment at the beginning our sample period and at the same time also indicates more constant parameters in the recent years. Moreover, the implied weight on inflation for the later periods is such that an effect on the real rate is obtained.

5 Conclusions

Using quarterly UK data for the period 1970Q1-2006Q2 we find strong evidence against a linear Taylor type relation. We find evidence for changing parameters using different Chow test variants and therefore test for the possibility of nonlinearities in form of smooth transition regression models. Alternative logistic smooth transition regression models are specified which differ with respect to the chosen transition variable. All considered nonlinear models outperform the simple linear specification in terms of model fit and the ability to track the actual interest rate.

First, we use a linear trend as a transition variable thereby allowing a one-time gradual change of the parameters. From this model we find evidence for the fact that the BoE's weights on inflation and output gap have indeed changed over time. Against this background we interpret the Bank of England's failure to bring down inflation rates in the 1970s and 1980s as a result of very low weights on inflation during this period.

A more flexible model that allows for more than one gradual change in the parameters uses the output gap as a transition variable. We find that parameters on inflation and the output gap have changed more frequently in the first part of the sample. This reflects the more volatile economic environment of the 1970s and 1980s and the changing UK monetary policy during that time.

Our overall preferred model specification (based on diagnostic tests) is a model with lagged interest rate used as a transition variable. From this model we again find evidence for changing parameters on both inflation and the output gap. In periods of recessions, the BoE seems to have put more weight on the output gap and less so on inflation. A reverse pattern is observed for non-recession periods. Another interesting observation from this model is the fact that changes in the parameters only occurred prior to 1992. After this date, the parameters of the Taylor-type relation are constant. This is consistent with the fact that monetary policy goals have not greatly changed after 1992.

Clearly, the estimated regression equations are based on a purely empirical approach. An extension of this study could therefore include a more detailed analysis of events that drive the nonlinearities and of how these events are related to monetary policy. This information could then be used to analyze changes in central bank preferences in more detail.

In summary, we find that estimating linear Taylor-type rules with constant parameters is not adequate for UK data. This is particular true for time spans with high interest rate and inflation volatility. Findings based on the linear model may therefore be quite misleading and may lead to inferior interest rate forecasts. For the UK case, the smooth transition regression approach followed in this paper is a viable alternative for the analysis of historical monetary policy and for forecasting interest rates.

References

Assenmacher-Wesche, K. (2006). Estimating central banks' preferences from a time-varying empirical reaction function, *European Economic Review* **50**: 1951–1974.

Castro, V. (2008). Are central banks following a linear or nonlinear (augmented) Taylor rule?, *Documentos de Trabalho Working Paper Series 19*, Universidade do Minho.

Clarida, R., Galí, J. & Gertler, M. (1998). Monetary policy rules in practice: Some international evidence, *European Economic Review* **42**: 1033–1067.

Clarida, R., Galí, J. & Gertler, M. (1999). The Science of Monetary Policy: A New Keynesian Perspective, *Journal of Economic Literature* **37**: 1661–1707.

Cukierman, A. & Muscatelli, A. (2008). Nonlinear Taylor rules and asymmetric preferences in central banking: Evidence from the United Kingdom and the United States, *Contributions to Macroeconomics* **8**(1).

Dolado, J., Pedrero, R. M.-D. & Ruge-Murcia, F. J. (2004). Nonlinear monetary policy rules: Some new evidence for the US, *Studies in Nonlinear Dynamics and Econometrics, the Berkeley Electronic Press* **8**(3).

Gerdesmeier, D. & Roffia, B. (2004). Empirical estimates of reaction functions for the Euro Area, *Schweizerische Zeitschrift für Volkswirtschaft und Statistik* **140**(1): 37–66.

Gerlach-Kristen, P. (2003). Interest rate reaction functions and the Taylor Rule in the Euro Area, *Working Paper 258*, European Central Bank.

Gerlach, S. & Schnabel, G. (2000). The taylor rule and interest rates in the EMU area, *Economic Letters* **67**: 165–171.

Judd, J. P. & Rudebusch, G. D. (1998). Taylor's rule and the FED: 1970-1997, *Federal Reserve Bank of San Francisco Economic Review* **3**: 3–16.

Kesriyeli, M., Osborn, D. R. & Sensier, M. (2004). Nonlinearity and structural change in interest rate reaction functions for the US, UK and Germany, *Working Paper 0414*, Research and Monetary Policy, Department Central Bank of the Republic of Turkey.

Kharel, R. S. (2006). Does the Bank of England target inflation precisely?, *Technical report*, Brunel University.

Krolzig, H.-M. & Toro, J. (2002). Classical and modern business cycle measurement: The European case, *Economic Working Paper E2002/20*, Centro de Estudios Andaluces.

Martin, C. & Milas, C. (2004). Modelling monetary policy: Inflation targeting in practice, *Economica* **71**(281): 209–221.

Qin, T. & Enders, W. (2008). In-sample and out-of-sample properties of linear and nonlinear Taylor rules, *Journal of Macroeconomics* **30**: 428–443.

Surico, P. (2003). Asymmetric reaction functions for the Euro Area, *Oxford Review of Economic Policy* **19**: 44–57.

Surico, P. (2004). Inflation targeting and nonlinear policy rules: The case of asymmetric preferences, *Technical report*, CESifo working paper.

Taylor, J. B. (1993). Discretion versus policy rules in practice, *Carnegie-Rochester Conference Series on Public Policy* **39**: 195–214.

Teräsvirta, T. (1998). Modeling economic relationships with smooth transition regressions, *in* A. Ullah & D. E. Giles (eds), *Handbook of Applied Economic Statistics*, Marcel Dekker Inc., NY, chapter 15, pp. 507–552.

Teräsvirta, T. (2004). Smooth transition regression modeling, *in* H. Lütkepohl & M. Krätzig (eds), *Applied Time Series Econometrics*, Cambridge University Press, chapter 6, pp. 222–242.

Teräsvirta, T. (2006). Forecasting economic variables with nonlinear models, *in* A. T. G. Elliot, C. Granger (ed.), *Handbook of Economic Forecasting*, Elsevier Publisher, chapter 8.

van Dijk, D., Terävirta, T. & Franses, P. H. (2002). Smooth transition autoregressive models - a survey of recent developments, *Econometric Reviews* **21**: 1–47.

Table 1: Linearity Test Results

	70Q1:06Q2					78Q1:06Q2				
	F	F4	F3	F2	model	F	F4	F3	F2	model
Panel A: models without foreign interest rates										
Transition Variable										
trend	0.010	0.368	0.064	0.010	LSTR1	0.000	0.614	0.087	0.000	LSTR1
inflation(t+1)	0.767	0.966	0.892	0.182	Linear	0.001	0.197	0.165	0.000	LSTR1
outgap(t)	0.031	0.187	0.003	0.979	LSTR2	0.037	0.070	0.048	0.424	LSTR2
interest rate(t-1)	0.676	0.777	0.644	0.275	Linear	0.007	0.696	0.070	0.002	LSTR1
interest rate(t-2)	0.338	0.821	0.213	0.182	Linear	-	-	-	-	-
Panel B: models including federal funds rate										
Transition Variable										
trend	0.001	0.062	0.126	0.003	LSTR1	0.003	0.684	0.013	0.003	LSTR1
inflation(t+1)	0.338	0.830	0.629	0.043	Linear	0.013	0.370	0.053	0.017	LSTR1
outgap(t)	0.025	0.071	0.178	0.083	LSTR1	0.014	0.065	0.050	0.148	LSTR2
interest rate(t-1)	0.019	0.037	0.136	0.150	LSTR1	0.017	0.282	0.110	0.016	LSTR1
interest rate(t-2)	0.001	0.047	0.012	0.051	LSTR2	-	-	-	-	-
fed funds rate(t)	0.217	0.637	0.125	0.220	Linear	0.108	0.727	0.134	0.050	Linear

The table contains the p-values of linearity tests, for which equation (2.5) indicates the model under the alternative hypothesis of nonlinearity. LSTR1 and LSTR2 denote suggested logistic smooth transition models with $K = 1$ and $K = 2$, respectively.

Table 2: Estimation results for STR models with a trend as a transition variable. Sample period: 1978Q1-2006Q2.

		Eq.(2.4)	Eq.(2.5)	Eq.(2.5), fed funds rate
		linear part		
intercept	3.8649***	0.5155**	4.1963***	4.4962***
	(0.9354)	(0.1982)	(0.8281)	(0.8002)
interest rate(t-1)	0.8666***	0.8666***	0.4533***	0.4700***
	(0.0338)	(0.0338)	(0.1026)	(0.0790)
inflation(t+1)	0.8522***	0.1137***	0.2387***	0.2744***
	(0.1477)	(0.0316)	(0.0503)	(0.0678)
outgap(t)	1.4095**	0.1880***	−0.1492	−0.2279
	(0.6777)	(0.0619)	(0.1245)	(0.1485)
fed funds rate(t)				−0.0723
				(0.0710)
		nonlinear part		
intercept			−4.2827***	−4.2763***
			(0.8835)	(0.8325)
interest rate(t-1)			0.4443***	0.3255***
			(0.1333)	(0.1097)
inflation(t+1)			−0.0217	−0.0862
			(0.1592)	(0.1222)
outgap(t)			0.5828***	0.4014**
			(0.2092)	(0.1964)
fed funds rate(t)				0.1651
				(0.1025)
$\theta/\hat{\sigma}_s^K$			0.0993	22.1026
c_1			44.3106	35.9995
AIC	−0.496	−0.496	−0.691	−0.685

The table presents the linear and nonlinear regression results using a time trend as transition variable for the latter. The first column gives the results for the structural form of the Taylor rule. Estimated standard errors are given in parentheses. *, **, *** denote significance at the 10%, 5% and 1% level, respectively.

Table 3: Diagnostic test results for STR models with a trend as a transition variable. Sample period: 1978Q1-2006Q2.

	Eq.(2.4)	Eq.(2.5)	Eq.(2.5), fed funds rate
Residual Tests			
JB	0.000	0.050	0.074
ARCH(1)	0.848	0.005	0.000
AutoC(2)	0.730	0.553	0.716
AutoC(4)	0.847	0.671	0.576
Remaining Nonlinearity: H_0 : no			
interest rate(t-1)		0.745	0.333
interest rate(t-2)		-	-
inflation(t+1)		0.206	0.740
outgap(t)		0.135	0.068
fed funds rate(t)			0.294
Parameter Constancy: H_0 : yes			
H1		0.727	0.592
H2		0.920	0.506
H3		0.686	0.152

The table presents the p-values of diagnostic tests for the corresponding models shown in Table 2. Diagnostic tests are those described in Section 6.3.3. in Teräsvirta (2004).

Table 4: Estimation results for STR models with y_t^{gap} as a transition variable. Sample periods: 1970Q1-2006Q2 and 1978Q1-2006Q2.

	70Q1:06Q2		78Q1:06Q2
	Eq.(2.5)	Eq.(2.5), fed funds rate	Eq.(2.5)
		linear part	
intercept	0.6526***	0.5463**	7.4538***
	(0.2397)	(0.2408)	(2.7878)
interest rate(t-1)	0.9702***	1.0329***	0.4868**
	(0.1055)	(0.0882)	(0.2217)
interest rate(t-2)	−0.1183	−0.2331***	-
	(0.1058)	(0.0861)	(-)
inflation(t+1)	0.0393*	0.0254	0.1278*
	(0.0234)	(0.0173)	(0.0750)
outgap(t)	0.6392***	−0.0411	−0.6311*
	(0.1061)	(0.1584)	(0.3526)
fed funds rate(t)	-	0.1158***	-
	(-)	(0.0334)	(-)
		nonlinear part	
intercept	−0.0323	6.7257	−6.9644**
	(0.5210)	(5.9897)	(2.8000)
interest rate(t-1)	0.1524	−0.5595	0.3702
	(0.1672)	(0.4406)	(0.2257)
interest rate(t-2)	−0.1113	0.4596	-
	(0.1617)	(0.4627)	(-)
inflation(t+1)	−0.0134	0.1076	0.0039
	(0.0326)	(0.2284)	(0.0870)
outgap(t)	−0.6541***	−1.1481	0.8088**
	(0.1344)	(0.8948)	(0.3583)
fed funds rate(t)	-	−0.1048	-
	(-)	(0.4778)	(-)
$\theta/\hat{\sigma}_s^K$	398.7326	2.2765	4.5825
c_1	−0.6204	2.3628	2.3171
c_2	4.5704	-	7.0851
AIC	−0.1772	−0.2308	−0.5295

The table presents the nonlinear regression results using the output gap y_t^{gap} as transition variable. Estimated standard errors are given in parentheses. *, **, *** denote significance at the 10%, 5% and 1% level, respectively.

Table 5: Diagnostic test results for STR models with y_t^{gap} as a transition variable. Sample periods: 1970Q1-2006Q2 and 1978Q1-2006Q2.

	70Q1:06Q2		78Q1:06Q2
	Eq.(2.5)	Eq.(2.5), fed funds rate	Eq.(2.5)
Residual Tests			
JB	0.000	0.000	0.000
ARCH(1)	0.001	0.000	0.663
AutoC(2)	0.837	0.477	0.735
AutoC(4)	0.936	0.579	0.789
Remaining Nonlinearity: H_0 : no			
interest rate(t-1)	0.772	0.021	0.005
interest rate(t-2)	0.364	0.006	-
inflation(t+1)	0.936	0.444	0.037
outgap(t)	0.834	0.109	0.552
federal funds rate(t)	-	0.376	-
Parameter Constancy: H_0 : yes			
H1	0.112	0.255	0.032
H2	0.011	0.331	0.045
H3	0.021	0.416	NaN

The table presents the p-values of diagnostic tests for the corresponding models shown in Table 4. Diagnostic tests are those described in Section 6.3.3. in Teräsvirta (2004).

Table 6: Estimation results for STR models with r_{t-1} or r_{t-2} as a transition variable. Sample periods: 1970Q1-2006Q2 and 1978Q1-2006Q2.

	1970Q1:2006Q2		1978Q1:2006Q2	
	Eq.(2.5), fed funds rate		Eq.(2.5)	Eq.(2.5), fed funds rate
	r_{t-1}	r_{t-2}	r_{t-1}	r_{t-1}
		linear part		
intercept	-0.0087	-0.6718	-0.2951	-0.0897
	(0.3261)	(0.6761)	(0.4606)	(0.3138)
interest rate(t-1)	1.0956^{***}	0.8311^{***}	0.8954^{***}	0.7853^{***}
	(0.1163)	(0.1247)	(0.1180)	(0.0720)
interest rate(t-2)	-0.2415^{**}	0.0085	-	-
	(0.0972)	(0.1409)	-	-
inflation(t+1)	0.0505^{**}	0.0670^{*}	0.3313^{***}	0.2796^{***}
	(0.0226)	(0.0396)	(0.0865)	(0.0801)
outgap(t)	0.0188	-0.0479	0.1024	0.0248
	(0.0768)	(0.1091)	(0.1069)	(0.1081)
fed funds rate(t)	0.1650^{***}	0.2507^{***}	-	0.1256^{**}
	(0.0508)	(0.0597)	(-)	(0.0560)
		nonlinear part		
intercept	-5.0271	1.9856^{**}	-	-
	(4.5577)	(0.9999)	(-)	(-)
interest rate(t-1)	0.4874	0.2145	0.0316	0.1011
	(0.4407)	(0.2735)	(0.0693)	(0.0800)
interest rate(t-2)	-0.1346	-0.2758	-	-
	(0.2628)	(0.2683)	-	-
inflation(t+1)	-0.2042^{***}	-0.1840^{***}	-0.2501^{**}	-0.2489^{**}
	(0.0937)	(0.0692)	(0.1041)	(0.0988)
outgap(t)	0.6563^{***}	0.6659^{***}	0.0884	0.2831^{*}
	(0.2453)	(0.2148)	(0.1493)	(0.1610)
fed funds rate(t)	0.1045	-0.0306	-	-0.0444
	(0.1082)	(0.0974)	(-)	(0.0979)
$\theta/\hat{\sigma}_s^K$	1.4039	0.7657	1.1949	1.6805
c_1	12.2128	4.3514	10.6919	10.9206
c_2	-	11.7890	-	-
AIC	-0.245	-0.304	-0.567	-0.601

*: The table presents the nonlinear regression results using the lagged interest rate $r_{t-j}, j = 1, 2$ as transition variable. Estimated standard errors are given in parentheses. *, **, *** denote significance at the 10%, 5% and 1% level, respectively.

Table 7: Diagnostic test results for STR models with r_{t-1} or r_{t-2} as a transition variable. Sample periods: 1970Q1-2006Q2 and 1978Q1-2006Q2.

	1970Q1:2006Q2		1978Q1:2006Q2	
	Eq.(2.5), fed funds rate		Eq.(2.5)	Eq.(2.5), fed funds rate
	r_{t-1}	r_{t-2}	r_{t-1}	r_{t-1}
Residual Tests				
JB	0.000	0.000	0.001	0.026
ARCH(1)	0.253	0.002	0.094	0.027
AutoC(2)	0.232	0.565	0.753	0.775
AutoC(4)	0.421	0.387	0.718	0.755
Remaining Nonlinearity: H_0 : no				
interest rate(t-1)	0.694	0.565	0.447	0.525
interest rate(t-2)	0.153	0.243	-	-
inflation(t+1)	0.330	0.806	0.024	0.207
outgap(t)	0.057	0.179	0.018	0.028
federal funds rate(t)	0.987	0.493	-	0.273
Parameter Constancy: H_0 : yes				
H1	0.089	0.154	0.009	0.072
H2	0.168	0.188	0.019	0.116
H3	0.352	0.518	0.117	0.358

The table presents the p-values of diagnostic tests for the corresponding models shown in Table 6. Diagnostic tests are those described in Section 6.3.3. in Teräsvirta (2004).

Table 8: Relative mean squared error according to (3.3)

Transition Variable	1970Q1:2006Q2		1978Q1:2006Q2	
	w/o FFR	w.FFR	w/o FFR	w.FFR
Trend	0.878	0.876	0.782	0.820
Output Gap	0.880	0.924	0.904	-
Lagged Interest Rates	0.911	0.846	0.902	0.909

Entries are mean squared error of nonlinear LSTR models relative to MSEs of linear model. The first columns gives the corresponding transition variable. Model with and without the federal funds rate (FFR) are considered.

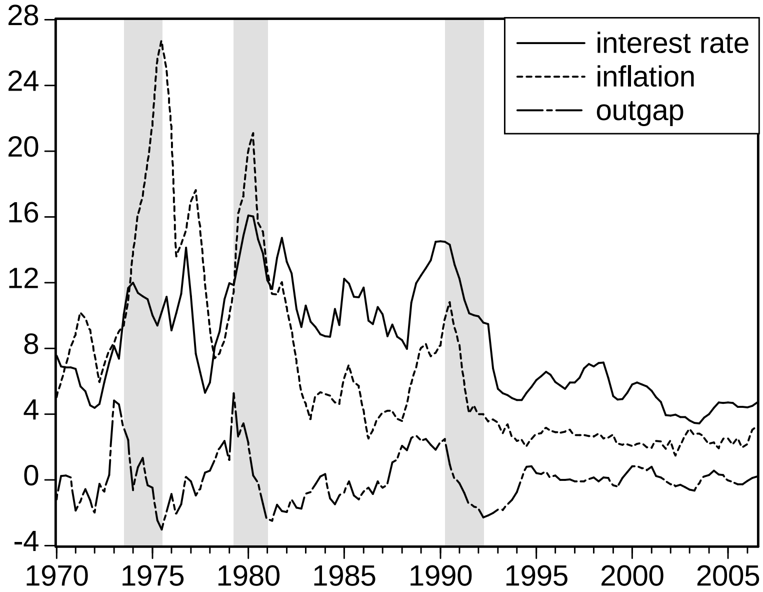

Figure 1: Plots of the Treasury bill rate (solid line), the inflation rate (short-dashed line) and the output deviations from the HP-filtered real GDP (long-dashed line) for the UK, 1970Q1:2006Q2. The shaded areas indicate times of recession following Krolzig & Toro (2002).

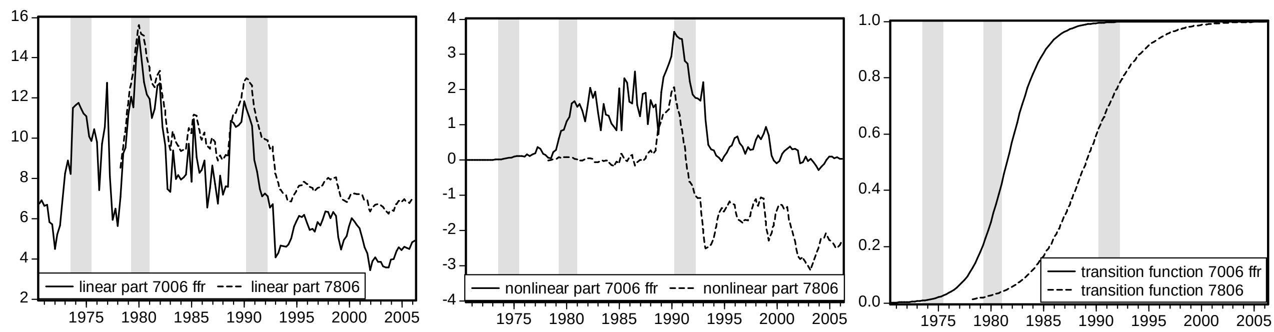

Figure 2: The left and the center panel indicate the difference in the linear and nonlinear parts of two samples (1970Q1:2006Q2 (solid line) and 1978Q1:2006Q2 (short-dashed line)) induced by the estimated transition functions in the right panel. Results based on model in equation (2.5) with a time trend as a transition variable. The federal funds rate is included as additional regressor for the full sample.

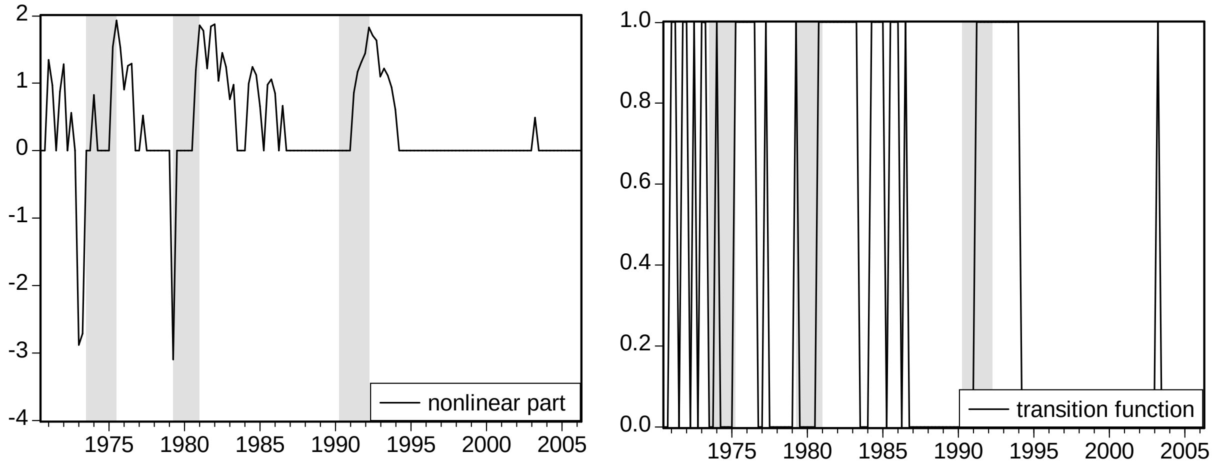

Figure 3: Nonlinear part and transition function of LSTR2 in (2.5) with y_t^{gap} as a transition variable. Sample period: 1970Q1-2006Q2.

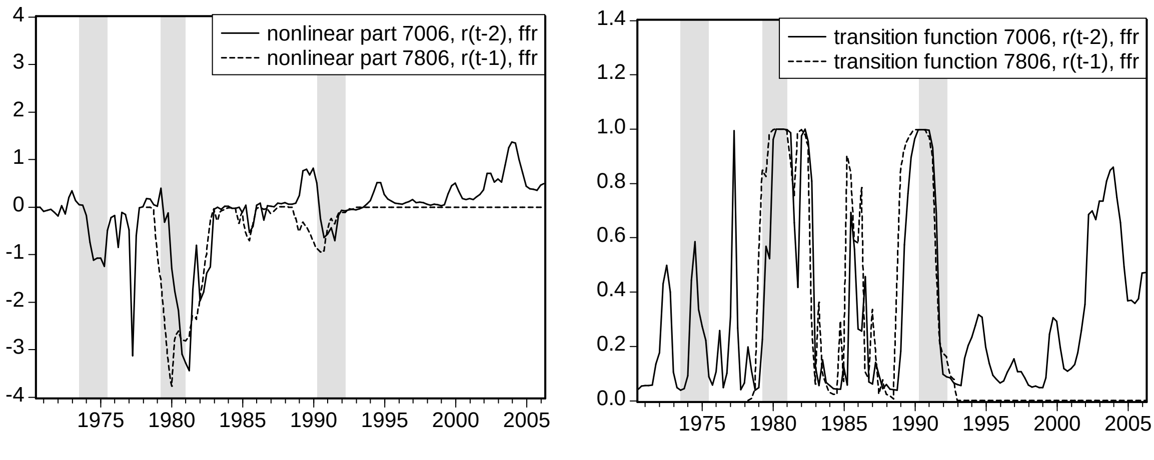

Figure 4: Nonlinear parts and transition functions of LSTR1 and LSTR2 model with $s_t = r_{t-i},\ i = 1, 2$ as a transition variable. Sample periods: 1970Q1-2006Q2 and 1978Q1-2006Q2. Both models include the US federal funds rate.

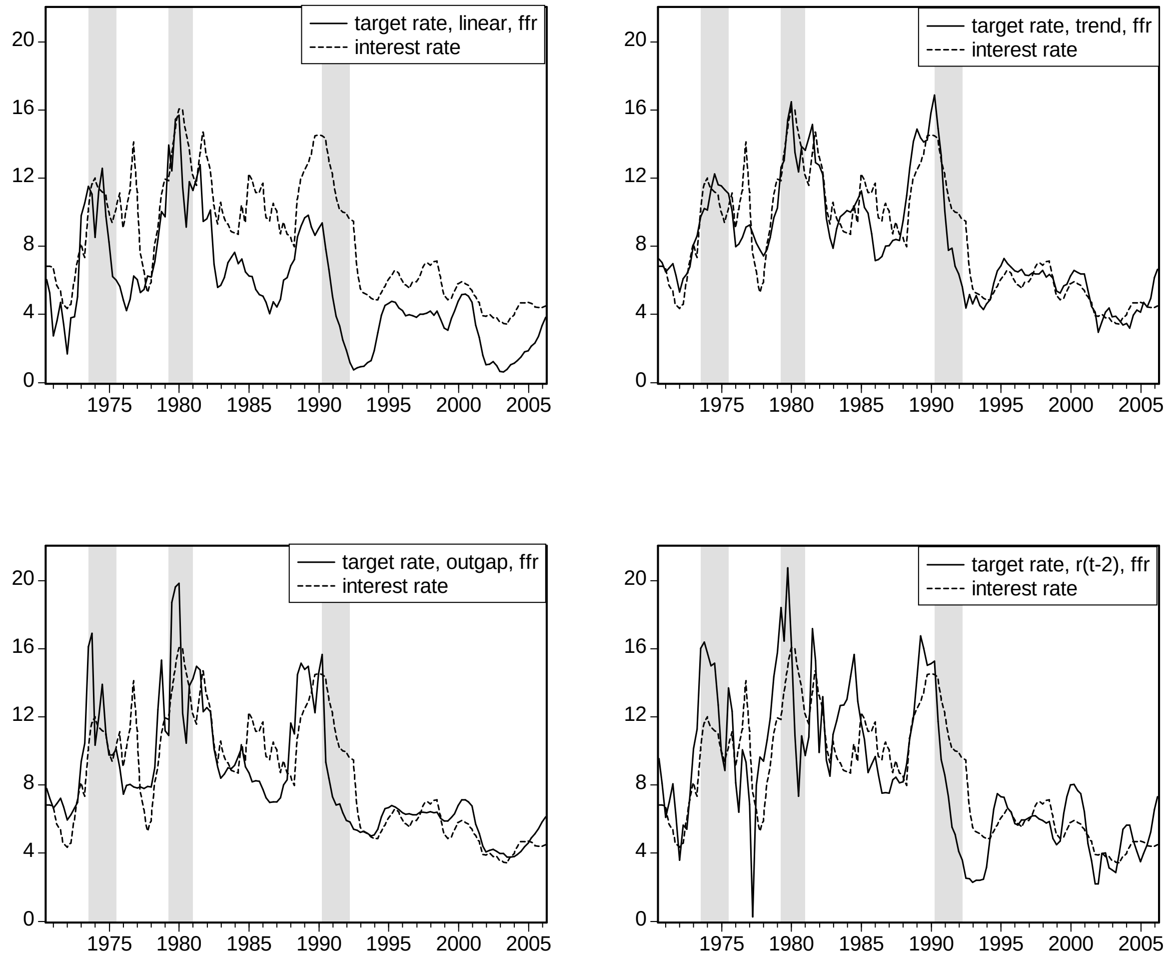

Figure 5: Treasury bill rate r_t (short-dashed line) and implied target nominal interest rate r_t^* (solid line) from linear model (upper left panel), LSTR1 with trend as a transition variable (upper right), LSTR2 with output gap as a transition variable (lower left) and LSTR1 with r_{t-2} as a transition variable (lower right panel). All models include the federal funds rate. Sample period: 1970Q1-2006Q2.

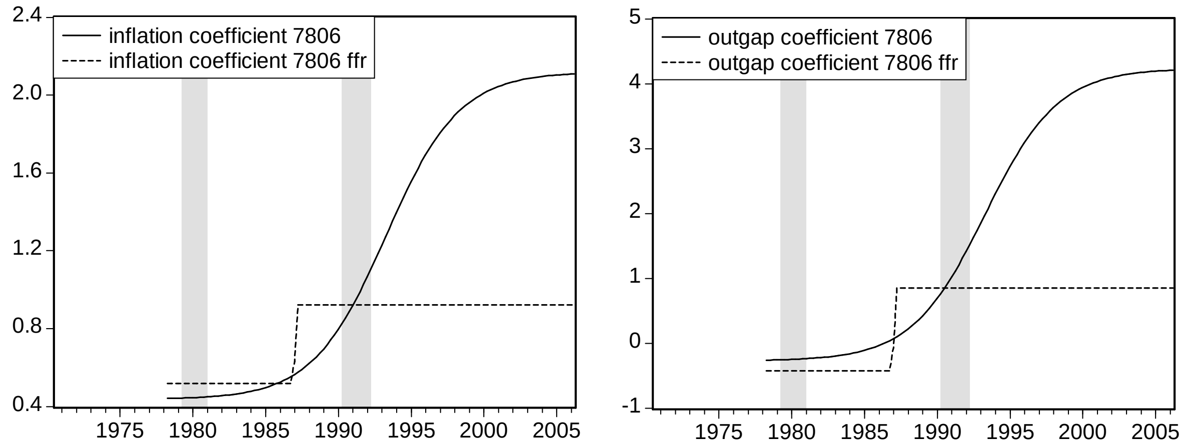

Figure 6: Time varying coefficients for inflation (β_t) and output gap (γ_t) calculated from equations (4.4) and (4.3). The models are LSTR1 with a time trend as a transition variable. The solid line indicates the coefficient from a model without, the dashed line indicates the coefficient from a model with the federal funds rate. Sample period: 1978Q1-2006Q2.

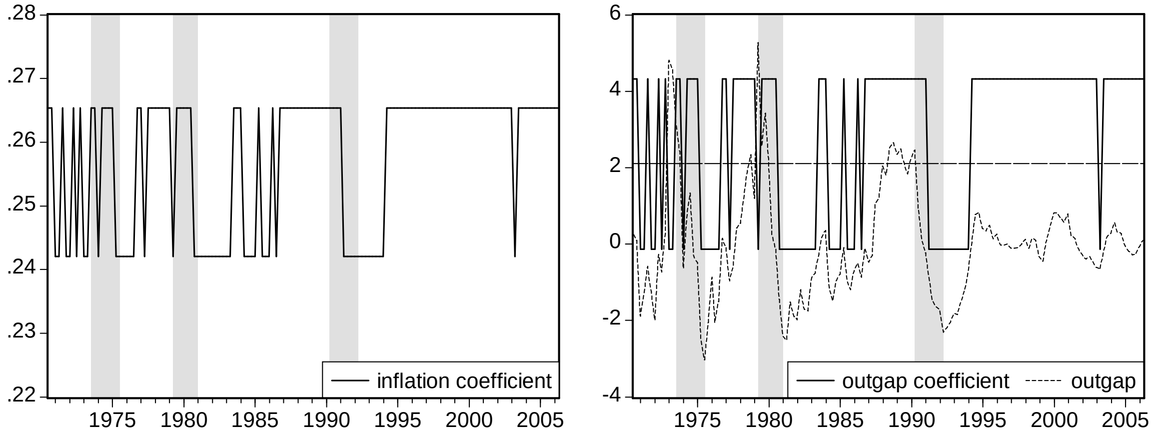

Figure 7: Time varying coefficients for inflation (β_t) and output gap (γ_t) calculated from equations (4.4) and (4.3). The model is an LSTR2 with the output gap as a transition variable. The estimated coefficient for the linear model is denoted by a long-dashed line. Sample period 1970Q1-2006Q2.

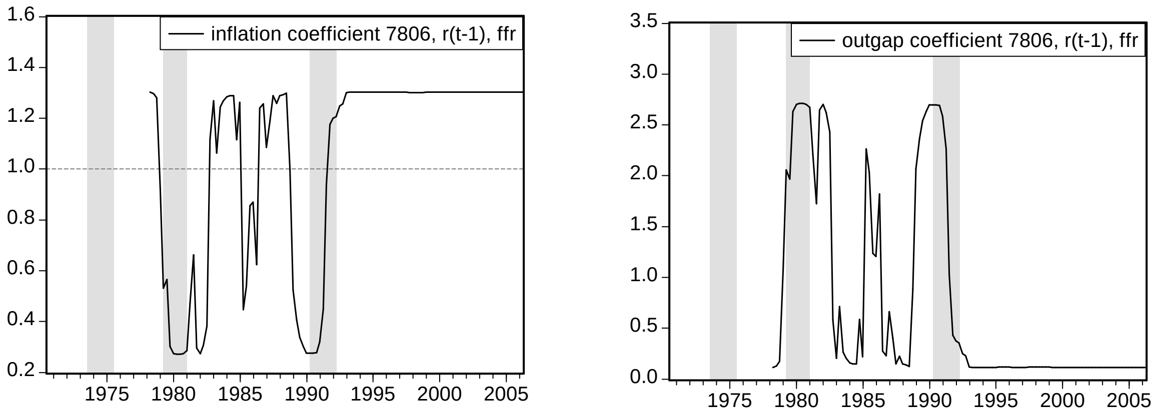

Figure 8: Time varying coefficients for inflation (β_t) and output gap (γ_t) calculated from equations (4.4) and (4.3). The model is an LSTR1 with r_{t-1} as a transition variable. Sample period: 1978Q1-2006Q2.