The Credit Channel in the Euro Area: a Dynamic Stochastic General Equilibrium Analysis

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Abstract. In this paper I estimate a New Keynesian Dynamic Stochastic General Equilibrium model à la Smets and Wouters (2003, 2004, 2007) featured with financial frictions à la Bernanke, Gertler Gilchrist (1999) for the Euro Area. The main aim is to obtain a time series for the unobserved risk premium of entrepreneurs loans, with the further aim of providing a dynamic analysis of it (IRFs analysis, variance decomposition analysis, etc.). Results confirm in general what recently found for the US by De Graeve (2008), namely that the model with financial frictions can generate a series for the premium, without using any financial macroeconomic aggregates, highly correlated with available proxies for the premium. The advantage of using a structural model to obtain the premium lies in the fact the it allows for the dynamic analysis above mentioned.

[1] The view expressed in the paper are solely the ones of the author and they do not necessarily reflect the view neither of the European Central Bank nor of the Bank of Estonia.

1 INTRODUCTION

The main goal of the paper is to provide a time series for a relevant economic variable which is unobserved. This is the external risk premium, i.e. the premium that risky entrepreneurs (because of the uncertainty of their projects) have to pay when they borrow funds from the banks, and in addition there is a problem of asymmetric information and costly state verification between the two types of agents, or in other words when they operate in a world of credit frictions. The analysis concerns the Euro Area and covers the period from 1980 to 2008.

In order to achieve that aim I based my analysis on a New Keynesian Dynamic Stochastic General Equilibrium (NK DSGE henceforth) which closely follows the structure of the model developed by Smets and Wouters (2003, 2005, 2007) but with the addition of the so called financial accelerator mechanism developed by Bernanke in the 1980s, and already included in a basic version a DSGE model (Bernanke, Gertler and Gilchrist (1999) (BGG henceforth)). The main advantages to use the model I used here are that, contrary to the last quoted theoretical contribution, several source of nominal and real rigidities are considered (which help in many ways in an estimated model[2]) and a large set of structural shocks.

It is the use of a structural model and the presence of those numerous shocks which justify the importance of this paper. In fact, many proxies for the financial premium are available, more often represented by the difference of some risky interest rates or yields (e.g. corporate bonds yields) and a measure of the risk free interest rate. Nevertheless, it is in general not possible to do a proper dynamic analysis of such series. In addition, the series available for the Euro Area are really short (they start in the first quarter of 2002). Once obtained the series from the model, which in my case goes back until the 1980s, I will compare it with the available proxies (at least for the last few years of the sample) and I will provide some dynamic analysis of the recovered premium. In particular, I will highlight how the premium responds to the different shocks (IRFs analysis) and which shocks are responsible and in which extent for the variability in the premium itself (variance decomposition analysis).[3]

In this respect, the same analysis has been done previously by De Graeve (2008) who estimated the same model as me but using US data from 1954 to 2004. He found that "the estimate - based solely on non financial macroeconomic data - picks up over the 70 percent of the dynamics of lower grade corporate bond spreads. ... [in addition there is] A gain in fitting key macroeconomic aggregates by including financial frictions in the model". I confirm those main results[4], with the specification that the high correlation with the proxies holds also for the spreads on less risky bonds. Moreover, this result is confirmed only if the smoothed series is considered until the third quarter of 2007, otherwise the correlation is always very low. Also other interesting results in terms of variance decomposition and pro-ciclicality of the premium turn out to hold.

Turning to the estimation of the model, I follow a Bayesian approach by combining the likelihood function with the prior distributions for the parameters of the model, to form the posterior density function. This posterior can then be optimized with the respect to the model parameters either directly or through Monte-Carlo Markov-Chain sampling methods (see Fernandez-Villaverde and Rubio-Ramirez (2001)). This part will be developed using the software Dynare for Matlab (see Juillard (2004)).

[2]See CEE (2005) and Smets and Wouters (2003, 2005, 2007) for a detailed discussion about their importance in an estimated model.

[3]Another interesting exercise is the shock decomposition analysis, i.e. evaluate which shocks and in what extent they are responsible for the observed trajectories of the series. I will provide that analysis in the near future.

[4]With respect to confirmation of the empirical relevance of the financial frictions in term of better fit, a model for the Euro Area with such features has been already estimated by Virginia Queijo (Queijo (2005, 2008)). Her estimation ends at the fourth quarter of 2002, so in a sense my estimation is an up-date, and her paper is silent in terms of the analysis of the fitted risk premium series described in the text.

The paper is structured as follows. In the first section I will present the model. In section 2 I will discuss about the data I used for the estimation and the estimation methodology adopted. In the subsequent section, I present the estimation results. In the fourth section I provide the dynamic analysis of the risk premium and in the end the concluding remarks.

2 THE MODEL

The model is based on two previous contributions. The main structure is taken from Smets and Wouters (2003, 2005 and 2007). That model is then extended introducing the financial accelerator mechanism as in BGG.

2.1 Households

Household i maximizes its intertemporal utility function choosing how to consume $\left(C_t^i\right)$, how to invest in order to build (today) the capital that will be used tomorrow $\left(I_t^i\right)$, the hours the want to work $\left(L_t^i\right)$, the utilization rate of capital (z_t^i) how much capital to rent to the firms $\left(K_t^i\right)$ and how many domestic bonds to buy (B_t^i)

$$\max_{\{C_t^i, I_t^i, L_t^i, z_t^i, K_t^i, B_t^i\}} E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^{\beta} U_t^i$$

$$U_t^i = \left[\frac{1}{1-\sigma_c}\left(C_t^i - hC_{t-1}\right)^{(1-\sigma_c)} - \frac{\varepsilon_t^L}{1+\sigma_L}\left(L_t^i\right)^{(1+\sigma_L)}\right]$$

where $\log \varepsilon_t^{\beta} = \rho_\beta \log \varepsilon_{t-1}^{\beta} + u_t^{\beta}$ with $(u_t^{\beta} \sim N\left(0, \sigma_\beta^2\right))$ is the discount factor shock (or preference shock) and $\log \varepsilon_t^{L} = \rho_L \log \varepsilon_{t-1}^{L} + u_t^{L}$ with $(u_t^{L} \sim N\left(0, \sigma_L^2\right))$ is the labor supply shock. The household behaviour is characterized by external habit formation, whose degree is established by parameter h. Households have a positive utility in period t only if they are able to consume something more that what was consumed last period on average. The inverse of the intertemporal elasticity of substitution in consumption (or equivalently the coefficient relative risk aversion) and the inverse of the elasticity of work effort with respect to the real wage are σ_c and σ_L respectively.

The maximization is constrained. On the one hand the household faces the budget constraint (in real terms)

$$C_t^i + I_t^i + B_t^i = R_{t-1}^n \frac{B_{t-1}^i}{\pi_t^c} + \frac{W_t^i}{P_t^c} L_t^i + Re_t^k z_t^i K_{t-1}^i - \Psi\left(z_t^i\right) K_{t-1}^i + T_t^i + Div_t^i$$

where $\mathrm{R}_t^n = (1+i_t)$, T_t^i are the net transfers, Div_t^i are dividends from the final good sector firms (owned by the households), π_t^c is the gross inflation rate $(1 + \frac{P_t^c - P_{t-1}^c}{P_{t-1}^c}$ or equivalently $\frac{P_t^c}{P_{t-1}^c}$, with P_t^c the CPI), W_t^i is the wage earned by the household, Re_t^k is the rental rate of capital, $\Psi\left(z_t^i\right)$ is the cost of capital utilization function.[5]

[5] Adjustment cost of capital utilization is represented by the following function,

$$\Psi(z_t) = Re^k \psi \left[\exp\left(\frac{z_t - 1}{\psi}\right) - 1\right],$$

where $\Psi(1) = 0$, $\Psi'(1) = Re^k$ and $\Psi'(1)/\Psi''(1) = \psi$. The degree of capital utilization is determined by condition

On the other hand, the capital accumulation equation

$$K_t=(1-\delta)\,K_{t-1}+\left[1-S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t x_t \tag{1}$$

where δ is the depreciation rate of capital, $\log x_t=\rho_x\log x_{t-1}+u_t^x$ with $(u_t^x\sim N\left(0,\sigma_x^2\right))$ and $S\left(\frac{I_t}{I_{t-1}}\right)$ is the investment adjustment costs function. It has the same properties assumed in many previous papers (see for instance CCE 2005), namely $S\,(1)=S'\,(1)=0$ and $S''\,(1)>0$.

The first order conditions[6] are

$$\frac{\partial L}{\partial C_t}=0: \qquad \beta^t\varepsilon_t^\beta\left(C_t-hC_{t-1}\right)^{-\sigma_c}-\beta^t\lambda_t=0 \tag{2}$$

$$\frac{\partial L}{\partial I_t}=0: \quad -\beta^t\lambda_t+\beta^t Q_t x_t\left[1-S\left(\frac{I_t^i}{I_{t-1}^i}\right)-S'\left(\frac{I_t^i}{I_{t-1}^i}\right)\frac{I_t^i}{I_{t-1}^i}\right]+\beta^{t+1}E_t\left\{Q_{t+1}x_{t+1}S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right\}=0 \tag{3}$$

$$\frac{\partial L}{\partial z_t}=0 \qquad Re_t^k=\Psi'\left(z_t\right)$$

$$\frac{\partial L}{\partial K_t}=0: \qquad \beta^{t+1}E_t\left\{\lambda_{t+1}\left[z_{t+1}Re_{t+1}^k-\Psi\left(z_{t+1}\right)\right]\right\}-\beta^t Q_t+\beta^{t+1}E_t\left\{Q_{t+1}\left(1-\delta\right)\right\}=0 \tag{4}$$

$$\frac{\partial L}{\partial B_t}=0: \qquad \beta^t\lambda_t-\beta^{t+1}E_t\left\{\lambda_{t+1}R_t^n\frac{1}{\pi_{t+1}^c}\right\}=0 \tag{5}$$

where λ_t is the lagrangian multiplier associated with the budget constraint and Q_t is the multiplier associated with the capital accumulation constraint. The first order condition for the labour supply is derived in the next section because households are assumed to be able to supply labour monopolistically. We report here the derivative in the case in which households offer labour in a competitive way, underlying that this is also the equation which the next section one reduces to when their non-competitive nature disappears

$$\frac{\partial L}{\partial L_t}=0: \qquad -\beta^t\varepsilon_t^\beta\varepsilon_t^L\left(L_t\right)^{\sigma_L}+\beta^t\lambda_t\frac{W_t^i}{P_t^c}=0 \tag{6}$$

From the first order condition (equation 30) it is possible to derive the consumption Euler equation

$$E_t\left\{\frac{\varepsilon_t^\beta\left(C_t-hC_{t-1}\right)^{-\sigma_c}}{\varepsilon_{t+1}^\beta\left(C_{t+1}-hC_t\right)^{-\sigma_c}}\right\}=E_t\left\{\frac{\lambda_t}{\lambda_{t+1}}\right\} \tag{7}$$

Using equation 5

$\Psi'(z_t)=Re_t^k$. This implies that

$$\begin{aligned} z_t &= \psi\ln\left(\frac{r_t^k}{r^k}\right)+1, \\ \Psi(z_t) &= \psi\left(r_t^k-r^k\right). \end{aligned}$$

The above two expressions are used to replace variable z_t by r_t^k.

[6] I removed the index i because the decentralized solution is the same of the centralized one, hence the first order conditions are the same.

$$E_t\left\{\frac{\lambda_t}{\lambda_{t+1}}\right\}=\beta E_t\left\{R_t^n\frac{1}{\pi_{t+1}^c}\right\}$$

which can be combined with equation 7 to obtain a more familiar version of the Euler equation.

Equation 33 and 28 may be re-written defining the marginal Tobin Q as $q_t=\frac{Q_t}{\lambda_t}$ (the ratio of the two lagrangian multipliers, or more loosely the value of installed capital in terms of its replacement cost). They become respectively[7]

$$1=q_t x_t\left[1-S\left(\frac{I_t^i}{I_{t-1}^i}\right)-S'\left(\frac{I_t^i}{I_{t-1}^i}\right)\frac{I_t^i}{I_{t-1}^i}\right]+\beta E_t\left\{q_{t+1}\frac{\lambda_{t+1}}{\lambda_t}x_{t+1}S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right\} \tag{8}$$

$$q_t=\beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[q_{t+1}\left(1-\delta\right)+z_{t+1}Re_{t+1}^k-\Psi\left(z_{t+1}\right)\right]\right\} \tag{9}$$

Equation 8 is nothing more then an investment Euler equation which describe the optimal path for investment in time. Equation 9 establishes the optimal way to determine the price of capital, taking into account its future return and its depreciation rate.

2.1.1 Labour Supply

Each household is a monopoly supplier of a differentiated labour service requested by the domestic firms[8]. This implies that the households can determine their own wage. After having set their wages, households inelastically supply the firms' demand for labour at the going wage rate.

The framework is similar to the one of the New Keynesian Phillips curve (see next section). In fact, there is a firm which hires labour form the households and transforms it into a homogenous input good L_t using the following production function

$$L_t=\left[\int_0^1 L_t\left(i\right)^{\frac{\theta_w-1}{\theta_w}}di\right]^{\frac{\theta_w}{\theta_w-1}}$$

where L_t^i is the household i labour supply, L_t is the aggregate labour demand and $\theta_w>1$.

The problem firms have to solve is

$$\max_{L_t^i} W_tL_t-\int_0^1 W_t^iL_t^i di$$

s.t.

$$L_t=\left[\int_0^1\left(L_t^i\right)^{\frac{1}{1+\lambda_t^w}}di\right]^{1+\lambda_t^w}$$

[7]Note that when there are not investment adjustment costs $\left(i.e.\ S\left(\frac{I_t^i}{I_{t-1}^i}\right)=0\right)$ the investment dynamics equation implies that

$$q_t=\frac{1}{x_t}$$

namely the Tobin's Q is equal to the replacement cost of capital (the relative price of capital). Furthermore, if $x_t=1$, as in the standard neoclassical growth model, $q_t=1$.

[8]The main references are Kollmann (1997), Erceg et al. (2000), CEE (2005). Most recent references are Adolfson et al. (2005) and Fernandez-Villaverde and Rubio-Ramirez (2006). The latter has very good mathematical derivations.

and he implied solution for L_t^i is

$$L_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{\theta_w} L_t$$

where

$$W_t = \left[\int_0^1 W_t(i)^{1-\theta_w} di\right]^{\frac{1}{1-\theta_w}}$$

It is also assumed that not all households can optimally set their wage each period. On the basis of the Calvo assumption, only a fraction 1-ξ_w of households can re-optimize. For those who cannot, wages evolve as follows

$$W_{t+1}(i) = (\pi_t^c)^{\tau_w} W_t(i)$$

Given this set up, households optimize their wages conditionally upon the fact that there is a certain probability that they cannot re-optimize in the future.

2.2 Firms

Firms are modeled as in Bernanke, Gertler e Gilchrist (1999). There are three types of producers: entrepreneurs and retailers. Entrepreneurs produce intermediate goods. They borrow from a financial intermediary that converts household deposits into business financing for the purchase of capital. The presence of asymmetric information between entrepreneurs and lenders creates a financial friction that makes the entrepreneurial demand for capital depends on their financial position. Retailers re described in the following section.

2.3 Retailers

Firms in this sector operate in a monopolistically competitive market. That is, products of individual firms, y_t(j), are *not perfect substitutes* and they are aggregated by the following *Dixit-Stiglitz* technology

$$Y_t = \left(\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$, it measures the elasticity of substitution. This implies that the demand for the product of an individual firms is determined by

$$y_t(j) = \left(\frac{P_t^c}{P_t(j)}\right)^{-\theta} Y_t, \tag{10}$$

where P_t^c is the aggregate price index, $P_t(j)$ is the price of firm j.

2.3.1 Entrepreneurs

The activity of entrepreneurs is at the heart of the model, therefore I will focus on their behaviour in a greater detail than the other two types of firms. They are involved into two kind of activities: the production of wholesale goods and the stipulation of financial contracts to obtain funds to finance the former activity. I will describe those two activities, starting with the problem of setting the loan contract with the financial intermediaries.

The entrepreneurs' behaviour follows that proposed by Bernanke, Gertler and Gilchrist (1999). Entrepreneurs manage firms that produce wholesale goods and borrow to finance the capital used in the production process. Entrepreneurs are risk neutral and have a finite expected horizon for planning purposes. The probability that an entrepreneur will survive until the next period is ϑ^e, so the expected lifetime horizon is $1/(1-\vartheta^e)$. This assumption ensures that entrepreneurs' net worth (the firm equity) will never be enough to fully finance the new capital acquisition. In essence, they issue debt contracts to finance their desired investment expenditures in excess of net worth. At the end of period t, entrepreneurs purchase capital, K^j_{t+1}, that will be used in the next period t+1 at the real price Q_t. Thus the cost of capital is $Q_t K^j_{t+1}$.The capital acquisition is financed partly by their net worth $\overline{NW}^j_{t+1}$ and by borrowing

$$B^j_{t+1} = \overline{NW}^j_{t+1} - Q_t K^j_{t+1} \tag{11}$$

from a financial intermediary. This intermediary obtains its funds from household deposits and faces an opportunity cost of funds equal to the economy's riskless rate of return, R^n_t. Thus, in order to acquire a loan the entrepreneurs have to engage in a financial contract before the realization of an idiosyncratic shock ω^j (with a payoff paid after the realization of the same shock). The ex-post return on capital for firm j is $\omega^j R^k_{t+1}$, where R^k_{t+1} is the ex-post aggregate return to capital (i.e. the gross return averaged across firms). The idiosyncratic shock has positive support, is independently distributed (across entrepreneurs and time) with a cumulative distribution function $F(\omega^j)$[9], with unitary mean $(E\{\omega^j\} = 1)$, and density function $f(\omega^j)$. The return of the entrepreneurial investment is observable to the outsider only through the payment of a monitoring cost $\mu\omega^j R^k_{t+1} Q_t K^j_{t+1}$, where μ is the fraction of lender's output lost in monitoring costs. Hence this cost is proportional to the expected return on capital purchased at the end of period t.

Turning to the loan contract, the entrepreneur chooses the value of firm capital, $Q_t K^j_{t+1}$, and the associated level of borrowing, B_{t+1}, prior to the realization of the idiosyncratic shock. Given $Q_t K^j_{t+1}$, B^j_{t+1} and R^k_{t+1}, the optimal contract may characterized by a gross non-default loan rate W^j_{t+1}, and a threshold value of the idiosyncratic ω^j, call it $\overline{\omega}^j$, such that for values of the idiosyncratic shock greater than or equal to $\overline{\omega}^j$, the entrepreneur is able to repay the loan at the contractual rate. In other words, entrepreneur default if

$$\omega^j < \overline{\omega}^j \equiv \frac{W^j_{t+1} B^j_{t+1}}{R^k_{t+1} Q_t K^j_{t+1}} \tag{12}$$

In this situation the lending intermediary pays the auditing cost and gets to keep what it finds. That is, the intermediary's net receipts are $(1-\mu)\omega^j R^k_{t+1} Q_t K^j_{t+1}$. A defaulting entrepreneur receive nothing. On the other hand, if $\omega^j > \overline{\omega}^j$, the entrepreneur repays the promised amount $W^j_{t+1} B^j_{t+1}$ and keeps the difference, equal to $\omega^j R^k_{t+1} Q_t K^j_{t+1} - W^j_{t+1} B^j_{t+1}$.

The values of $\overline{\omega}^j$ and W^j_{t+1} under the optimal contract are determined by the requirement that the financial intermediary receive an expected return equal to the opportunity cost of its funds. Because

[9] As in BGG I assume a log normal distribution which has a positive support.

the loan risk in this case is perfectly diversifiable the relevant opportunity cost is the riskless rate, R_{t+1}. Accordingly, noting that $F(\overline{\omega}^j)$ denotes the probability of default, the loan contract must satisfy

$$\left[1-F(\overline{\omega}^j)\right] W_{t+1}^j B_{t+1}^j + (1-\mu)\int_0^{\overline{\omega}^j} \omega^j R_{t+1}^k Q_t K_{t+1}^j dF(\omega) = R_{t+1}^n W_{t+1}^j \tag{13}$$

Combining equations 11, 12 and 13, thus eliminating W_{t+1}^j, one obtains the participation constraint of the maximization problem from which the optimal contract is determined, i.e.

$$\left\{\left[1-F(\overline{\omega}^j)\right]\overline{\omega}^j + (1-\mu)\int_0^{\overline{\omega}^j} \omega^j dF(\omega)\right\} R_{t+1}^k Q_t K_{t+1}^j = R_{t+1}^n\left(\overline{NW}_{t+1}^j - Q_t K_{t+1}^j\right) \tag{14}$$

Let's define by $\Gamma(\overline{\omega}^j)$ and 1-$\Gamma(\overline{\omega}^j)$ the fractions of net capital output received by the lender and the entrepreneur respectively. Hence I have:

$$\Gamma(\overline{\omega}^j) \equiv \int_0^{\overline{\omega}^j} \omega^j f(\omega) d\omega + \overline{\omega}^j \int_{\overline{\omega}^j}^{\infty j} f(\omega) d\omega$$

Expected monitoring costs are defined as:

$$\mu M(\overline{\omega}^j) \equiv \mu \int_0^{\overline{\omega}} \omega^j f(\omega)\, d\omega$$

Hence the net share accruing to the lender is $\Gamma(\overline{\omega}^j) - \mu M(\overline{\omega}^j)$.

The contract specifies a pair $\left\{\overline{\omega}^j \; W_{t+1}^j\right\}$ which solves the following maximization problem

$$\max_{\left\{\overline{\omega}^j \; K_{t+1}^j\right\}} \left[1-\Gamma(\overline{\omega}^j)\right] R_{t+1}^k Q_t K_{t+1}^j$$

subject to equation 14[10].

The first order conditions are given by[11]:

$$\Gamma'(\overline{\omega}^j) = \Psi_t\left[\Gamma'(\overline{\omega}^j) - \mu M'(\overline{\omega}^j)\right]$$

$$\frac{R_{t+1}^k}{R_t^n}\left\{\left[1-\Gamma(\overline{\omega}^j)\right] + \Psi_t\left[\Gamma(\overline{\omega}^j) - \mu M(\overline{\omega}^j)\right]\right\} = \Psi_t$$

where Ψ_t is the Lagrangian multiplier.

Combining those two equations with equation 14 yields a linear relationship between capital demand and net worth. Let's starting with deriving the following relation between the expected return on capital and the safe return paid on deposit

$$E_t\left\{R_{t+1}^k\right\} = \varrho(\overline{\omega})\, R_t^n$$

where

[10] Following the definitions of $\Gamma(\overline{\omega}^j)$ and of $\mu M(\overline{\omega})$, this equation may be rewritten in the following more convenient way:

$$\left[\Gamma(\overline{\omega}^j) - \mu M(\overline{\omega})\right] R_{t+1}^k Q_t K_{t+1}^j = R_{t+1}\left(\overline{NW}_{t+1}^j - Q_t K_{t+1}^j\right)$$

[11] See the BGG appendix for further details.

$$\varrho\left(\overline{\omega}\right)=\left\{\frac{\left[1-\Gamma(\overline{\omega})\right]\left[\Gamma'(\overline{\omega})-\mu M'(\overline{\omega})\right]}{\Gamma'(\overline{\omega})}+\left[\Gamma(\overline{\omega}^{j})-\mu M(\overline{\omega}^{j})\right]\right\}^{-1}\quad\text{with }\varrho\left(\overline{\omega}\right)>0 \tag{15}$$

Let's now define $\mathrm{S}_t \equiv E_t\left\{\frac{R^k_{t+1}}{R^n_t}\right\}$ as the external finance premium[12] This ratio captures the difference between the cost of finance reflecting the existence of monitoring costs, and the safe interest rate (which per se reflects the opportunity cost for the lender). By combining equation 15 with equation 14 one can write a relationship between the aggregate capital expenditure Q_tK_{t+1} and the aggregate net worth $\overline{NW}_{t+1}$, i.e.:

$$Q_tK_{t+1}=\left\{\frac{1}{1-s_t\left[\Gamma(\overline{\omega})-\mu M(\overline{\omega})\right]}\right\}\overline{NW}_{t+1} \tag{16}$$

Equation 16[13] is a key relationship in this context, for it explicitly shows the link between capital expenditure and entrepreneurs' financial conditions (summarized by aggregate net worth). One can view 16 as a demand equation, in which the demand of capital depends inversely on the price and positively on the aggregate financial conditions.

Equation 16 may be re-written in order to highlight another important relationship. In fact it becomes:

$$E_t\left\{R^k_{t+1}\right\}=s\left(\frac{\overline{NW}_{t+1}}{Q_tK_{t+1}}\right)R^n_t\quad\text{with }s'(\cdot)<0 \tag{17}$$

This formulation is very important. In fact, this synthesizes the idea underlying the financial accelerator. This idea is that the external financial premium is negatively related with the net worth of potential borrower. The intuition is that firms with higher leverage (lower capital to net worth ratio) will have a greater probability of defaulting and will therefore have to pay a higher premium. Since net worth is procyclical (because of the procyclicality of profits and asset prices), the external finance premium becomes countercyclical and amplifies business cycles through an accelerator effect on investment, production and spending.

I can now turn to the production activity. Entrepreneurs goods operate in a competitive market. They hire labour form households, paying the salary W_t, and rent the capital they need paying a return R^k_t. Firm j produces output Y^j_t on the basis of the following Cobb-Douglas production function

$$Y^j_t=A_t\left(\widetilde{K}^j_{t-1}\right)^{\alpha}\left(L^j_t\right)^{1-\alpha}-\Phi \tag{18}$$

where $\widetilde{K}_{t-1}$ is the effective utilization of the capital stock given by $\widetilde{K}_{t-1}=z_tK_{t-1}$, Φ is a fix cost to assure that profits are zero and $\log A_t=\rho_a\log A_{t-1}+u^a_t$ (with $u^a_t\sim N\left(0,\sigma^2_a\right)$) is the technology shock.

Firms minimize costs under the production function constraint. The objective function is

$$\min_{\left\{\widetilde{K}^j_{t-1},L_t\right\}}\left(\frac{W_t}{P^c_t}\right)L^j_t+Re^k_t\widetilde{K}^j_{t-1}$$

Using the f.o.c.s we have an expression for the real marginal cost

[12]Given that $\varrho\left(\overline{\omega}\right)>0$, $s_t$is necessary bigger than 1.

[13]It is easy to note that $\frac{\partial\left\{\frac{1}{1-s_t\left[\Gamma(\overline{\omega})-\mu M(\overline{\omega})\right]}\right\}}{\partial s_t}>0$. Moreover, $\left\{\frac{1}{1-s_t\left[\Gamma(\overline{\omega})-\mu M(\overline{\omega})\right]}\right\}=1$ if $\mathrm{s}_t=1$. In fact, in that case there is not risk. This means that $R^k_{t+1}=R^n_t$, $\mu=0$ and $\Gamma(\overline{\omega}^j)=1$.

$$\frac{W_t}{P_t^c} + \zeta_t A_t (1-\alpha) \left(\frac{L_t^i}{\widetilde{K}_{t-1}^j} \right)^{-\alpha} = 0$$

$$\zeta_t = MC_t = \left(\frac{W_t}{P_t^c} \right) (A_t)^{-1} (1-\alpha)^{-1} \left[\left(\frac{W_t}{P_t^c} \right)^{-1} Re_t^k \frac{1-\alpha}{\alpha} \right]^{\alpha}$$

$$MC_t = \frac{1}{A_t} \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} \left(\frac{W_t}{P_t^c} \right)^{1-\alpha} (Re_t^k)^{\alpha}$$

Now define the wholesale goods price as $P_{w,t}$, the depreciation rate of capital as δ, then the gross return of the entrepreneur's project GY_t^w is define as the sum of the total revenges $\left(\frac{P_{w,t}}{P_t^c} Y_t^w \right)$ and the market value of the undepreciated capital $(Q_t K_t - Q_t \delta_t K_t)$, i.e.:

$$GY_t^w \equiv \frac{P_{w,t}}{P_t^c} Y_t^w + (1-\delta_t) Q_t K_t \tag{19}$$

As for the demand for capital, it depends on the expected return and expected cost of capital. The marginal return is given by the gross output less the labour cost, normalized by the value of capital at time t+1[14]

$$R_{t+1}^k = \frac{GY_{t+1} - \frac{W_{t+1}}{P_{t+1}^c} L_{t+1}}{Q_t K_{t+1}} \tag{20}$$

$$R_{t+1}^k = \frac{\left[\frac{P_{w,t}}{P_{t+1}^c} \alpha \frac{Y_{t+1}}{K_{t+1}} - Q_{t+1}\delta + Q_{t+1} \right]}{Q_t} \tag{21}$$

Equation 21 can be re-written as

$$E_t \left\{ R_{t+1}^k \right\} = E_{\acute{t}} \left[\frac{Re_{t+1}^k + (1-\delta) Q_{t+1}}{Q_t} \right] \tag{22}$$

where $Re_{t+1}^k = \frac{P_{w,t+1}}{P_{t+1}^c} \alpha \frac{Y_{t+1}}{K_{t+1}}$ is the marginal productivity of capital.

Let V_t be the entrepreneurial equity (i.e. the wealth accumulated by entrepreneurs from operating firms), let W_t^e denote the entrepreneurial wage[15], and let $\overline{\omega}_t$ denote the state contingent value of $\overline{\omega}$ set in period t. Then aggregate entrepreneurial net worth at the end of period t, $\overline{NW}_{t+1}$ is given by

$$\overline{NW}_{t+1} = \vartheta^e V_t + W_t^e \tag{23}$$

with

$$V_t = R_t^k Q_{t-1} K_t - \left(R_t^n + \frac{\mu M(\overline{\omega}) R_t^k Q_{t-1} K_t}{Q_{t-1} K_t - \overline{NW}_t} \right) (Q_{t-1} K_t - \overline{NW}_t) \tag{24}$$

where $\mu M(\overline{\omega}) = \int_0^{\overline{\omega}} \omega f(\omega) d\omega$ are the expected monitoring costs and $\vartheta^e V_t$ is the equity held by entrepreneurs at t-1 who are still in business at t. Equation 24 states that the entrepreneurial equity is equal to the return on capital minus the its cost minus the cost of an eventual default.

[14]Substituting in the equation 21 the production function (eq. 18) and the expression for Q_t deriving from equation the f.o.c.s of the household it is possible to obtain the demand for capital.

[15]Since I assumed that entrepreneurs don't offer labour services, there should not be the eage in the net worth, and in fact I will not consider it in the simulation and estimation.

Entrepreneurs who fail in t consume the residual equity $((1-\vartheta^e)V_t)$. That is their consumption is

$$C_t^e = (1-\vartheta^e)V_t \tag{25}$$

As in BGG, this consumption is quite low, and not relevant. I what follows I will not consider it in the estimation of the model.

Substituting equations **??**, 24 e 18 in equation 23 yields a difference equation for $\overline{NW}_{t+1}$

$$\overline{NW}_{t+1} = \vartheta^e \left[R_t^k Q_{t-1}K_t - \left(R_t^n + \frac{\mu M(\overline{\omega})R_{t+1}^k Q_t K_{t+1}}{Q_{t-1}K_t - \overline{NW}_t}\right)(Q_{t-1}K_t - \overline{NW}_t)\right] + A_t\left(\widetilde{K}_{t-1}^j\right)^{\alpha}\left(L_t^j\right)^{1-\alpha} - \Phi$$

which is the second basic ingredient of the financial accelerator.

In order to close this part, a condition for the demand for labour has to derived. It satisfies the following condition

$$(1-\alpha)\frac{Y_t^w}{L_t} = \frac{W_t^e}{P_{w,t}} \tag{26}$$

Intermediate goods producers face another type of problem. Each period, only a fraction 1-ξ_w of them, randomly chosen, can optimally adjust their prices (see Calvo (1983)). For those that cannot re-optimize, prices are adjusted as follows

$$P_{t+1}^c = (\pi_t^c)^{\tau_\pi} P_t^c$$

where τ_π is the parameter which governs the degree of price indexation to past inflation.

Maximizing the expected discounted profits[16]

$$\max_{\{P_t(j)\}} E_0 \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\lambda_{t+i}}{\lambda_t}\left\{\left[\prod_{s=1}^{i}\left(\pi_{t+s-1}^c\right)^{\tau_\pi}\frac{P_t(j)}{P_{t+i}^c} - MC_{t+i}\right]Y_{t+i}(j)\right\}$$

subject to the constraint represented by the demand expressed by the final good producers for the intermediate goods (equation 10)

$$Y_{t+i}(j) = \left[\prod_{s=1}^{i}\left(\pi_{t+s-1}^c\right)^{\tau_\pi}\frac{P_t(j)}{P_{t+i}^c}\right]^{-\theta} Y_{t+i}$$

it is possible to derive the condition for the optimal price[17] P_t^* and consequently the NKPC.

[16] In order to maintain the paper self-contained we do not report the derivation of the New Keynesian Phillips curve. Moreover, it has been derived in many papers and books, so we refer to them. See Walsh (2003), Adolfson et al. (2005), Fernandez-Villaverde (2006) among others.

[17] We did not use P_t^{j*} because we assume that all firms are identical, and that they fix the same optimal price P_t^*. Moreover, solving this equation for P_t^* and assuming flexible prices ($\theta = 0$), it reduces to the standard monopolistic competition results that firms set their price a mark up over their nominal marginal cost

$$P_t^* = \left(1+\lambda_t^p\right)MC_t$$

2.4 Monetary policy

It is useful to compare optimal monetary policy with alternative rules. As a benchmark rule, the empirical interest-rate rule of the SW model is added:

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{\phi_m} \left[\left(\frac{\Pi_{t-1}}{\Pi}\right)^{r_\pi} \mathsf{y}_{t-1}^{r_y}\right]^{1-r_i} \left(\frac{\Pi_t}{\Pi_{t-1}}\right)^{r_{\Delta\pi}} \left(\frac{\mathsf{y}_t}{\mathsf{y}_{t-1}}\right)^{r_{\Delta y}} \varepsilon_t^{ru} \tag{27}$$

where

$$\mathsf{y}_t = \frac{y_t}{y_t^\star}$$

represents the output gap and $y_t^\star$ is the flexible-price level of output[18] and $\log \varepsilon_t^{ru} = \rho_{ru} \log \varepsilon_{t-1}^{ru} + u_t^{ru}$ with $\left(u_t^{ru} \sim N\left(0, \sigma_{ru}^2\right)\right)$ is the monetary policy shock.

2.5 Government

Fiscal policy is exogenous and it is assumed to behaves as follows

$$\log g_t = \rho_g \log g_{t-1} + u_t^g$$

where $u_t^g \sim N\left(0, \sigma_g^2\right)$. In addition there is the equilibrium condition that $G_t = T_t$.

3 DATA AND ESTIMATION METHODOLOGY

Before moving to the estimation, two steps have to be done. The first consists on log linearizing all the equations of the model. Those are reported in appendix B. Then the solution step. In order to solve the model for the rational expectations I used Dynare. The solution method implemented is the one proposed by Sims (2000) and Klein (2000). In particular the log linearized model can be written as follows

$$\Gamma_0 Y_t = \Gamma_1 Y_{t-1} + \Psi Z_t + \Pi \eta_t$$

where Γ_0, Γ_1, Ψ Π are matrices of structural coefficients

The model has 43 parameters. I set some parameters prior to estimation because the data used contain little information about them. The discount factor β is set equal to 0.99, implying an annual steady state real interest rate of 4% (or equivalently a quarterly rate of 1%). The parameter θ that measures the degree of retailers' monopoly power, is set equal to 6, implying a steady-state price markup of 20%, a common value used in the literature. The depreciation rate, δ, is assigned the commonly used values of 0.025. The parameter of the Cobb-Douglas function, α, is set equal to 0.3. As in BGG, in order to have an annualized business failure rate, F($\overline{\omega}$), of 3% (0.75% quarterly), a steady state risk spread, $\mathrm{R}^k - R^n$, equal to 200 basis points, and a ratio of capital to net worth, $\frac{K}{NW}$, of 2 (or equivalently a leverage ration of 0.5), I take the idiosyncratic productivity variable, log(ω), to be log-normally distributed with variance equal to 0.07, and I set the fraction of realized payoffs lost in bankruptcy, μ, to 0.12. The entrepreneurs' rate of survival is fixed at 0.975. The steady state share of government spending is set equal to 0.195. Table 3 summarizes the calibrated parameters. The other 30 parameters are estimated using Bayesian techniques.

[18] It is eventually worth considering the following alternative rule,

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_t}{1+i}\right)^{r_i} \frac{\Pi_t^{r_\pi}}{\Pi_{t-1}^{r_\pi - 1}} \left(\frac{w_t}{w_{t-1}}\right)^{r_w} \mathsf{y}_t^{r_y}.$$

Parameters	Definition	Values
β	discount factor	0.99
θ	intermediate goods elasticity of substitution	6
δ	capital depreciation rate	0.025
α	capital share on output	0.3
$\mathrm{F}(\overline{\omega})$	annualized business failure rate	0.03
$\mathrm{R}^k - R$	a steady state risk spread	0.02
$\frac{K}{NW}$	capital to net worth	2
μ	fraction of realized payoffs lost in bankruptcy	0.12
$\frac{G}{Y}$	steady state share of government spending	0.195
$\frac{C}{Y}$	steady state consumption ratio	0.6269
$\frac{I}{Y}$	steady state investment ratio	0.1781
σ_ω	variance of the log-normal distribution of ω	0.28
ϑ^e	Entrepreneur's rate of survival	0.975

Table 1: Calibrated Parameters

3.1 Data

I used aggregate data for the Euro Area. I took them from the Area Wide Model (AWM) database[19], the recent updated version. The sample period goes from the first quarter of 1980 to the third quarter of 2008; hence I have 115 quarterly observations. Given the number of shocks in the model, i.e to avoid the stochastic singularity problem[20], I cannot use more than seven observable variables in the estimation.

I have chosen the following: real GDP, real consumption, real gross investment, hours worked, nominal short term interest rate, real wages per head and inflation rate. As in Smets and Wouter (2003) and Queijo (2005) all real variables are in per capita terms (obtained dividing real aggregate variables divided by the labour force). Inflation rate is the quarter by quarter variation in the GDP deflator. As for the hours worked, there are not available data . Assuming that in any period only a fraction of firms, ξ_E, is able to adjust employment to its desired total labour input, they are obtained using the following formula[21]

$$\widehat{E}_t = \beta \widehat{E}_{t+1} + \frac{(1-\xi_E)(1-\beta\xi_E)}{\xi_E}\left(\widehat{l}_t - \widehat{E}_t\right)$$

where E_t is the total employment,$\widehat{E}_t$ is the percentage deviation of the employment from the mean $\left(i.e.\ \widehat{E}_t = \frac{E_t - E}{E}\right)$. The parameter ξ_E is estimated. In the end all variables are demeaned and detrended using a linear trend, except inflation, which is HP filtered (with $\lambda = 40000$), and the nominal interest rate which is detrended with the same trend as inflation.

[19]See Fagan et al. (2001).
[20]Put a note for this. Betty Ingram
[21]See Adolfson et al. (2004) for an explanation of this formula.

3.2 Methodology

The methodology used here has been adopted by many other papers.[22] It consists in solving the model for an initial set of parameters. Then, a numerical optimization procedure[23] has been used to calculate the likelihood function of the data (for given parameters) and the modes of the posterior distributions. Combining prior distributions with the likelihood of the data gives the posterior kernel which is proportional to the posterior density. Since the posterior distribution is unknown, I use Markov Chain Monte Carlo (MCMC) simulation methods to conduct inference about the parameters. Al the procedure is implemented in Dynare for Matlab (see Juillard (2004)).

More formally, the procedure starts with the estimation of the mode of the posterior distribution maximizing the posterior density p($\Im|Y$) with respect to the vector of parameters $\Im$ and given the data Y. The objective is to maximize the logarithm version of the Bayes theorem

$$\log p(\Im|Y) = \log p(Y|\Im) + \log p(\Im) - \log p(Y)$$

where $p(Y|\Im)$ is the sample density or likelihood function, p($\Im$) is the prior density of the parameters and p(Y) is the marginal likelihood. Since p(Y) does not depend on $\Im$, this is equivalent to maximize

$$\log p(\Im|Y) = \log p(Y|\Im) + \log p(\Im)$$

Markov Chain Monte Carlo (MCMC) simulation methods are used to obtain the posterior distribution. This is necessary since it is not possible to sample the parameters directly from the posterior distribution. The idea behind MCMC is to draw values of the parameters from an approximate distribution and then correct these draws to better approximate the posterior distribution. Starting from an initial arbitrary value of the parameters, the samples are drawn sequentially, such that each draw will depend on the previous value. The approximate distribution of the parameters is improved at each step of the simulation until it converges to the posterior. The posterior output can then be used to compute any posterior function of the parameters: impulse responses, moments, etc.

To perform the simulations, I used the so-called Metropolis-Hasting algorithm, which uses an acceptance/rejection rule to converge to the posterior distribution. The algorithm samples a proposal vector of parameters $\Im$ from a jumping distribution which it is assumed to be distributed as N($\Im^l, c\Sigma$) where Σ is the inverse of the Hessian computed at the joint posterior mode, and c is a scale factor set to obtain efficient algorithms. After the first round of simulations, the exercise was instead repeated setting Σ equal to the estimated covariance matrix. The purpose when choosing the scale factor is to tune the acceptance rate around 25 percent as suggested by Gelman, Carlin, Stern, and Rubin (2004). My acceptance ratio has been always less but close to 30 per cent.

3.2.1 Priors

Priors are taken from the Smets and Wouters (2003) for the common parameters. It is common to assign beta distribution to the autoregressive to the coefficients defined in the range 0-1, typically the autoregressive coefficients.

For what concerns the BGG parameters, I assign a beta distribution also to the entrepreneur's rate of survival. As for the elasticity of the external finance premium with respect to firm leverage, I assumed the same prior of Virginia (2007), but I assugned an infinite variance to the Gamma distribution.

[22] See Smets and Wouter (2003) and Queijo (2005) among others.

[23] Dynare allows for different kind of optimization procedure. Here, I used the Sims' optimizer.

Table 3.2.1 summarizes the distribution assigned with the means and the standard deviations assumed. Moreover I reported the posterior modes and the associated standard errors obtained applying the numerical optimization.

Parameters			Priors		Post. Mode	Post	Post. Mode	Post
		Distribution	Mean	St. deviation	with FA	St. dev.	without FA	St. dev.
σ_β	Std. dev. preference shock	Inv. Gamma	0.2	Inf	0.238	0.0649	0.2435	0.0718
σ_L	Std. dev. Labour supply shock	Inv. Gamma	1	Inf	1.0119	0.3205	0.9297	0.2977
σ_x	Std. dev. investment shock	Inv. Gamma	0.1	Inf	0.3194	0.0587	0.2369	0.0435
σ_a	Std. dev. techology shock	Inv. Gamma	0.4	Inf	0.8175	0.1227	0.7765	0.1191
σ_{ru}	Std. dev. Monetary policy shock	Inv. Gamma	0.1	Inf	0.1336	0.011	0.1305	0.0108
σ_g	Std. dev. Gov. spending shock	Inv. Gamma	0.3	Inf	1.4032	0.0921	1.3883	0.0907
σ_w	Std. dev. Wage mark up shock	Inv. Gamma	0.25	Inf	0.2024	0.0154	0.2026	0.0152
σ_{λ_π}	Std. dev. Price mark up shock	Inv. Gamma	0.15	Inf	0.1705	0.0141	0.1692	0.0139
$\varkappa$	elasticity of external finance wrt leverage	Inv. Gamma	0.05	Inf	0.0224	0.0068	-	-
ϕ_m	Smooth parameter in instrument rule	beta	0.8	0.05	0.9001	0.0176	0.9129	0.0164
ρ_β	Persistence param. Preference shock	beta	0.85	0.1	0.8943	0.0344	0.9061	0.035
ρ_L	Persistence param. Labour supply shock	beta	0.85	0.1	0.9899	0.0077	0.9886	0.0098
ρ_x	Persistence param. Investment shock	beta	0.85	0.1	0.6756	0.1126	0.8275	0.0951
ρ_a	Persistence param. Techology shock	beta	0.85	0.1	0.9746	0.015	0.9702	0.017
ρ_g	Persistence param. Gov. spending shock	beta	0.85	0.1	0.9137	0.0461	0.9041	0.0435
r_π	Response of interest rate to inflation	norm	1.7	0.1	1.6969	0.0978	1.6847	0.1
r_y	Response of interest rate to uotput	norm	0.125	0.05	0.1006	0.0356	0.095	0.0362
h	Habit formation	beta	0.7	0.05	0.6518	0.042	0.6628	0.0412
σ_l	inverse of the elasticity of work effort wrt real wage	norm	2	0.75	1.8034	0.5492	1.8171	0.5256
σ_c	inverse of elasticity of substitution in consumption	norm	1	0.375	1.3261	0.2347	1.4791	0.2415
τ_w	Past wage indexation	beta	0.75	0.15	0.3813	0.1309	0.4019	0.1355
τ_π	Past inflation indexation	beta	0.75	0.15	0.2202	0.0763	0.2172	0.0753
ξ_w	Calvo wage	beta	0.7	0.05	0.8415	0.0226	0.8484	0.0205
ξ_π	Calvo price	beta	0.75	0.05	0.871	0.0126	0.875	0.0133
φ	inverse elast. of the capital util. cost function	norm	4	1.5	5.6746	1.2484	4.7254	1.1457
ϕ	1+fix cost over output	norm	1.45	0.25	1.0652	0.1488	1.2195	0.1828
ψ	inverse of investment adjustment cost	norm	0.2	0.075	-0.0161	0.0443	-0.0334	0.04
r_{Δ_y}	Response of interest rate to output gorwth	norm	0.3	0.1	0.2181	0.0435	0.1991	0.0432
r_{Δ_π}	Response of interest rate to infl. First diff.	norm	0.063	0.05	0.1298	0.0298	0.1209	0.0289
ξ_E	Calvo Employment	beta	0.5	0.15	0.759	0.0227	0.7533	0.0253

Table 2: Priors

3.3 Model Comparison

There are many ways to evaluate the goodness of the fit between the two models. The main two are comparing the fitted values with the actual data and computing some test statistics. In this section we explain how a specific statistic of the Bayesian econometrics, the Bayes factor, is built and we will comment out the results in the next section. First, the models' marginal data density must be calculated. Let us label a model with financial frictions by M_f and an alternative specification of the model without financial frictions by M_f .

The marginal data density for each model will be

$$p(Y|M_i) = \int p(Y|\Im_i, M_i)p(\Im_i|M_i)d\Im_i$$

where $\Im_i$ is a vector of parameters of model i, $p(Y|\Im_i, M_i)$ is the sample density of model i and $p(\Im_i|M_i)$ is the prior density of the parameters for model i. The posterior probability for each model will be:

$$p(M_i|Y) = \frac{p(Y|M_i)p\left(M_i\right)}{\sum_i p(Y|M_i)p\left(M_i\right)}$$

Bayesian model selection is done pairwise, comparing the models in terms of the posterior odds ratio:

$$PO_{i,j} = \frac{p(M_i|Y)}{p(M_j|Y)} = \frac{p(Y|M_i)p\left(M_i\right)}{p(Y|M_j)p\left(M_j\right)}$$

where the prior odds $\frac{p(M_i|Y)}{p(M_j|Y)}$ are updated by the Bayes factor, $B_{ij}=\frac{p(Y|M_i)}{p(Y|M_j)}$. Jeffreys (1961) suggested rules of thumb to interpret the Bayes factor as follows:

$B_{ij} < 1$	support for M_j
$1 < B_{ij} < 3$	very slight evidence against M_j
$3 < B_{ij} < 10$	slight evidence against M_j
$10 < B_{ij} < 100$	strong evidence against M_j
$B_{ij} > 100$	decisive evidence against M_j

4 ESTIMATION RESULTS

4.1 Fit

In appendix C.1 I reported the fitted and the actual values of the series used in the estimations. The graphical analysis is quite intuitive, but it gives no clear understanding of which model better fits the data in this case. That is the reason why we need some statistics to properly judge the fit. As anticipated we use first the odds ratio. They are reported in table 3 and we can see that there is (slight) evidence against the model without the financial accelerator effect.

Using the same information about the Log data density, I can use the likelihood-ratio test to test the restriction imposed by the model without financial accelerator (i.e. that $\varkappa$=0) against the model with the financial accelerator.

Let L^u and L^c denote the maximum values of the log-likelihood function for the unconstrained and constrained models, respectively. The likelihood-ratio statistic $-2(L^c - L^u)$ has a χ^2 distribution with

Log data density	Model with FA	Model without FA	Odds ratio $\left(\frac{\text{FA}}{\text{NOFA}}\right)$
Laplace approximation	-280.60	-282.97	$exp^{2.37.}$
Harmonic mean	-280.28	-282.88	$exp^{1.28.}$

Table 3: Log data density

one degree of freedom under the null hypothesis that the constrained model is valid. The value of L^u is -280.28 and that of L^c is 282.88, giving a test statistic of 5.64. The 5 percent critical value for a χ^2 is 3.84. Therefore, the likelihood-ratio test easily rejects the restriction of the constrained model in favor of the model that includes a financial accelerator. Thus, the introduction of the accelerator mechanism improves the model's ability to fit the data.

4.2 Posteriors

In appendix C.2 I report the posterior distribution obtained by the M-H algorithm for both the model with and without financial frictions. In table 4.2 I reports the means values of those posterior distributions, together with their confidence intervals.

In general the posteriors means are in line with the previous estimation of the Euro Area and they are not very different between the two specifications of the model.[24] I will not describe them because already extensively described in the previous literature and I will focus on the most relevant estimated parameter $\varkappa$.

It is estimated at about 0.03. This is lower than the BGG calibrated one (0.05) but still in line with it and with the empirical evidence. In fact, the previous estimation for the Euro Area reports a value for that parameter of 0.05 in Queijo (2005) (revised at 0.04 in the 2008 version of her 2005 paper)[25]. De Graeve (2008) finds a higher value for the US (a posterior mode of 0.1). Christensen and Did (2007) estimate it at 0.042 for Canada and Lopez and Rodriguez (2008) 0.059 for Colombia.

[24] The same is true in Queijo (2005, 2008), but for instance not in De Graeve (2008).

[25] The author does not estimate that parameter, but other structural parameters whose combination gives $\varkappa$

Parameters	Posterior mean with FA	Condindence interval		Posterior mean without FA	Condindence interval	
σ_β	0.2886	0.1588	0.4070	0.3213	0.1671	0.4786
σ_L	1.3355	0.6339	2.0090	1.3645	0.5884	2.1005
σ_x	0.3329	0.2442	0.4162	0.2861	0.1990	0.3729
σ_a	0.8142	0.6133	1.0078	0.8000	0.5985	0.9976
σ_{ru}	0.1370	0.1180	0.1550	0.1366	0.1174	0.1542
σ_g	1.4222	1.2718	1.5836	1.4120	1.2622	1.5603
σ_w	0.2069	0.1795	0.2325	0.2066	0.1806	0.2340
σ_{λ_π}	0.1720	0.1466	0.1952	0.1709	0.1480	0.1938
$\varkappa$	0.0257	0.0127	0.0375	-	-	-
ϕ_m	0.8965	0.8706	0.9254	0.9029	0.8737	0.9328
ρ_β	0.8785	0.8240	0.9378	0.8675	0.8031	0.9362
ρ_L	0.9843	0.9707	0.9984	0.9839	0.9692	0.9986
ρ_x	0.6706	0.5162	0.8327	0.7390	0.5713	0.9182
ρ_a	0.9684	0.9439	0.9942	0.9668	0.9390	0.9947
ρ_g	0.9070	0.8461	0.9755	0.9023	0.8385	0.9732
r_π	1.7075	1.5525	1.8643	1.7031	1.5455	1.8623
r_y	0.1002	0.0469	0.1537	0.0858	0.0282	0.1393
h	0.6510	0.5849	0.7192	0.6579	0.5874	0.7226
σ_l	2.0062	1.1059	2.8922	1.9854	1.1100	2.8420
σ_c	1.4104	1.0392	1.8165	1.5155	1.1236	1.9131
τ_w	0.4129	0.2042	0.6233	0.4217	0.2060	0.6449
τ_π	0.2347	0.1116	0.3576	0.2381	0.1039	0.3608
ξ_w	0.8378	0.7979	0.8795	0.8421	0.7973	0.8901
ξ_π	0.8698	0.8467	0.8906	0.8759	0.8536	0.8985
φ	5.9298	3.9239	7.7855	5.3605	3.3722	7.3577
ϕ	1.1007	0.8395	1.3371	1.1781	0.8905	1.4416
ψ	-0.0162	-0.0903	0.0602	-0.0321	-0.1067	0.0434
r_{Δ_y}	0.2231	0.1565	0.2964	0.2245	0.1483	0.2989
r_{Δ_π}	0.1204	0.0716	0.1685	0.1206	0.0680	0.1744
ξ_E	0.7580	0.7200	0.7974	0.7583	0.7163	0.7980

Table 4: Posterior Means

5 PREMIUM

5.1 A series for the premium

Figure 5.1 reports the Smoothed series for the risk premium obtained by the estimation of the DSGE model. The dashed horizontal line is the (annual) steady state value for the premium we have assumed.

As we can see from the figure, three main periods can be highlighted. The first of low (below the steady state) risk premium during the 1980s. The second one in the 1990s with a decisive increase of the premium above its steady state, most probably due to the turmoil in the European monetary system. There is then a strong decline since the beginning of 2000s, due to the benefic effects of the creation of the monetary union, up to the extremely low premium of the last years, which was in the opinion of many economists the signs of an undervaluation of the real risks. It is worth noting that the model fail to reproduce the observed high increase of premium starting from the third quarter 2007, when the financial crises began. This is a feature I will deal with in the following sections.

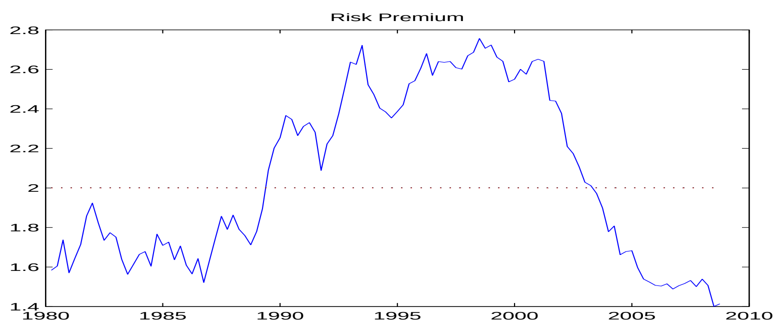

Figure 1: Series for the premium generate by the model with financial frictions (Smoothed values).

5.2 External validation

One of the main goal of the paper is to evaluate the strength of the model to produce a sensible and meaningful series for the unobserved risk premium. In order to evaluate that feature for our model we compare the smoothed series generated by the DSGE model with some available proxies for the premium of the Euro Area. They are spreads computed as the difference between some risky interest rates and the risk free interest rate represented in our case by the rate on the ten years government bonds.

Unfortunately, those series are shorter that the sample period, so we have to consider only the last part of the generated series. In particular, the spreads are available only from the first quarter of 2002. Those spreads are computed as the difference between the AAA, AA, A and BBB rated bonds and the ten years government bonds.[26]

Figures 5.2, 5.2, 5.2 and 5.2 show the series for those spreads compared with the last part of the series of the premium in figure 5.1.

AS anticipate in the previous section, it is clear that there is a high correlation between the series until approximately the third quarter of 2007 and after that date the correlation is very low. In fact, if we look at the contemporaneous correlations reported in table 5.2 we note that they are very low (never

[26]Data from the ECB.

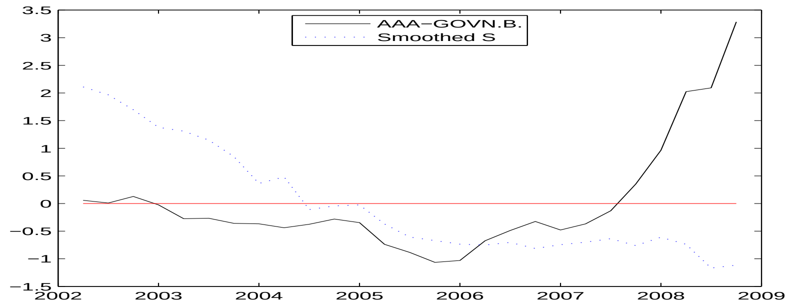

Figure 2: Comparison of the Risk premium with the proxy AAA RATED BONDS - TEN YEARS GOVERNMENT BONDS.

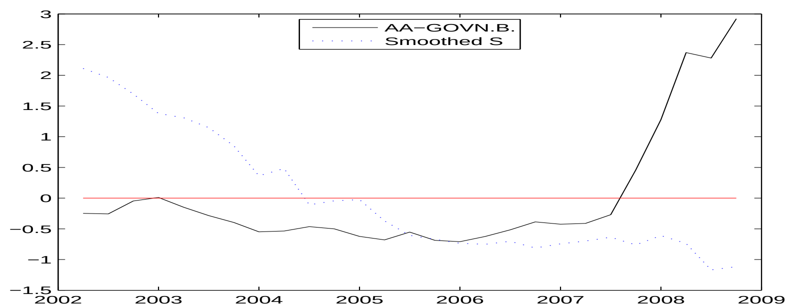

Figure 3: Comparison of the Risk premium with the proxy AA RATED BONDS - TEN YEARS GOVERNMENT BONDS.

more that 30 percent) if referred to the entire period 2002 - 2008. Nevertheless, if we compute them for the period 2002q1 - 2007q3, they significantly increase, displaying a correlation with all the proxies of 70 percent. This in part confirms the De Graeve (2008) results, in the sense that he finds very high correlation with the lower grade corporate bonds (as in my case), but it contradicts them because he found low correlation between the premium and the spread on safer bonds (AAA and AA).

	AAA-GOVN.B.	AA-GOVN.B.	A-GOVN.B.	BBB-GOVN.B.
corr. with premium 2002q1 - 2008q3	-0.204	-0.293	0.06	0.136
corr. with premium 2002q1 - 2007q3	0.720	0.710	0.744	0.696

Table 5: Contemporaneous correlation between the series generated by the model and the proxies for the premium.

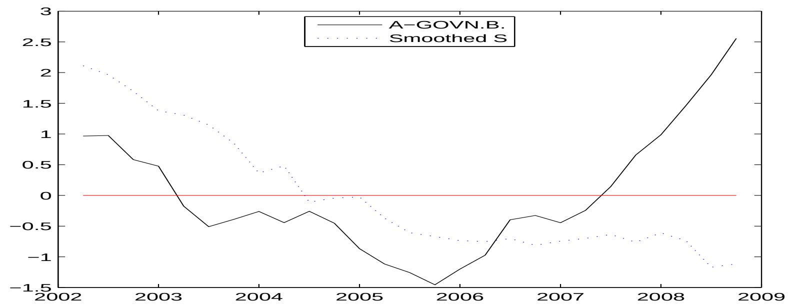

Figure 4: Comparison of the Risk premium with the proxy A RATED BONDS - TEN YEARS GOVERNMENT BONDS.

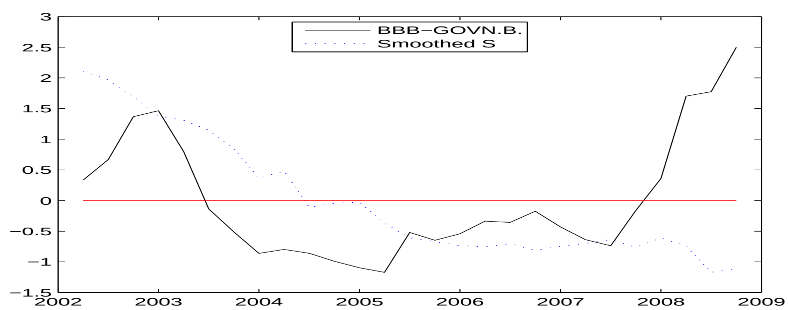

Figure 5: Comparison of the Risk premium with the proxy BBB RATED BONDS - TEN YEARS GOVERNMENT BONDS.

5.3 Variance decomposition

The variance decomposition presented in table 5.3 well shows which shock is more relevant in the Euro Area and which is the most responsible of the variance of the single variables.

	u^x	u^β	u^L	u^a	u^{ru}	u^g	u^{λ_π}	u^w
r	6.11	3.6	28.3	7.53	48.9	0.6	3.4	1.55
c	1.89	43.45	2.36	45.07	5.23	1.32	0.39	0.28
l	18.9	12.17	9.54	36.24	17	4.4	1.38	0.37
inv	41.34	3.93	7.39	36.02	10.02	0.35	0.86	0.1
q	14.48	2.02	21.38	28.8	29.56	0.39	3.02	0.34
k	34.27	5.28	7.19	42.3	9.68	0.39	0.76	0.13
nw	11.4	1.74	24.93	21.53	36.15	0.27	3.49	0.49
r^k	8.73	1.32	25.99	20.41	39.18	0.23	3.54	0.61
y	12.25	8.94	5.69	58.51	10.53	3	0.84	0.24
π	1.39	5.62	34.57	5.91	5.97	0.04	43.86	2.63
z	14.8	9.52	9.96	39.96	15.26	2.59	3.78	4.12
mc	2.98	4.64	4.44	68.69	4.69	0.51	3.94	10.11
S	12.98	2.94	24.74	16.17	38.34	0.24	3.62	0.96
r^n	12.55	24.71	14.72	9.54	33.48	0.75	3.62	0.63
w	1.38	4.6	5.67	51.85	2.19	0.05	6.88	27.37
E	21.45	13.31	13.49	20	27.13	1.81	1.89	0.92
$y^\star$	2.99	1.83	7.75	85.03	0	2.4	0	0

Table 6: Variance decomposition FA in percentage

Again, the results are in line with the literature related to the Euro Area. I will first describe them in general, focusing afterwards on the most interesting decomposition of the premium.

In general in the long run the most important shock driving the variability of the real variables is the technology shock, whose role is calmed down by the presence of the investment specific shock. The latter is the second source of variability for the most part of the real variables, with the exception of consumption, which is driven by the preference shock. The monetary policy shock plays a relevant role for the nominal variables, except for inflation which is mostly driven by the price mark up shock and by the labour supply shock. The latter seems to play a relevant role also for the price of capital and the return on capital (accounting for around 20 percent). Government, price mark up and wage mark up shocks play no role except for the last one explaining almost the 30 percent of the variability on the real wages. In the end, the flexible price output (as expected) is mostly driven by the technology shock (85 percent).

Turning to the risk premium, as in De Graeve (2008), the most responsible shocks for its variability in the long run are the supply shocks (90 percent in the US case). In my case they count for 54 percent. The remaining part is almost entirely explained by the monetary policy shock (38 percent), which represents a difference with respect to the US where that shock is less relevant at these long horizons (around 10 percent).

5.4 Impulse response functions

To illustrate the model dynamics implied by the financial accelerator, I plot the impulse response functions of key macroeconomic variables. Here I report only those related to the most interesting shocks, i.e. those which allow me to highlight some specific aspects of the model.

In figure 5.4, I report the consequences of a monetary policy shock. The mechanism of the financial accelerator is clearly represented and clearly showed by the response of investment. After the tightening of the monetary policy, investments decrease as in the normal set up. This has the usual effect to reduce the demand for capital and then its price. In the financial accelerator framework, the latter reduction leads to a decrease of the net worth which makes the entrepreneur riskier. He has than to pay a higher premium and this fact further depresses investments, generating the extra response displayed in figure 5.4.

It is worth noting that in this case, contrary to the BGG theoretical prescriptions, but also with the US empirical evidence, the accelerator effect is not transmitted to output. Rather, that variable seems to responde slightly more in the case of no financial frictions. This is due to the consumption reactions,[27] which is much stronger when the accelerator is not working, more than counteracting the effect on investments.

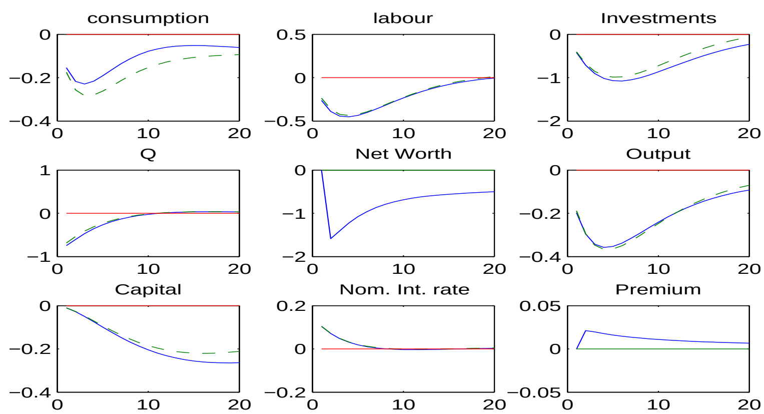

Figure 6: Variables' responses to a one standard deviation orthogonalized monetary policy shock. Percentage deviation from the steady state. Dashed line: without financial frictions. Solid line: with financial frictions.

I then analyze the investment specific shocks and the technology shocks, in figure 5.4 and 5.4 respectively, to highlight two important results holding for the Euro Area like in the US, namely the not necessarily countercyclicality of the premium and the not necessarily stronger response of investments when the accelerator is on (although the latter effect is less accentuated in the Euro Area).

In fact, in the case of the positive investment specific shock both those properties are there. Investments increase less in the case of the model without frictions[28] and although the premium is increasing[29] output still augments (i.e. the premium is pro-cyclical).

The same is true for instance in the case of the technology shock, where the premium is countercyclical, but still the response of investments is not amplified by the accelerator.

[27]This response may be explained by the fact that consumption dynamic is described solely by the Euler equation, i.e. the real interest rate is its main determinant. Hence that response is not directly related to financial fictions which operate through the investment channel. This suggests that a different modellization of the households may be necessary. This is left for future research.

[28]This is due, as in general for all the other shocks, to the presence of investment adjustment cost rather than capital adjustment costs. See De Graeve (2008) for further details.

[29]The premium increases because the investment specific shocks is a supply shock given the fact that it implies a reduction in the price of capital, despite the fact that investments increase, and this leads to a decreases of the net worth which gives less collateral to the entrepreneurs who in turn has to face a high premium.

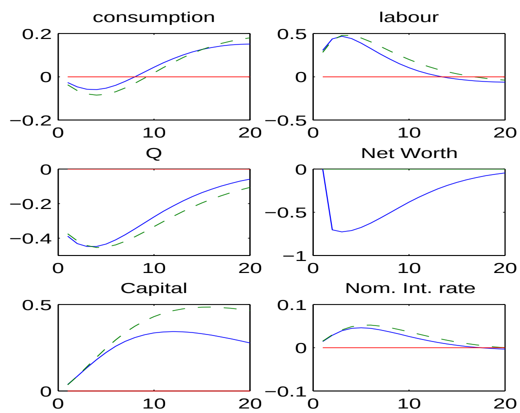

Figure 7: Variables' responses to a one standard deviation orthogonalized investment specific shock. Percentage deviation from the steady state. Dashed line: without financial frictions. Solid line: with financial frictions.

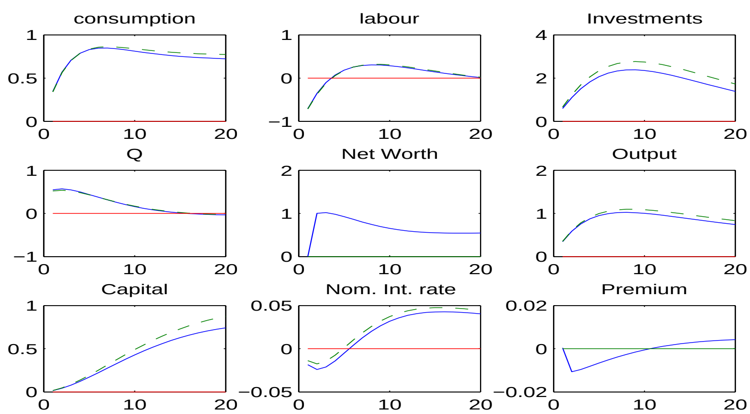

Figure 8: Variables' responses to a one standard deviation orthogonalized technology shock. Percentage deviation from the steady state. Dashed line: without financial frictions. Solid line: with financial frictions.

6 CONCLUDING REMARKS

In this paper I estimated a New Keynesian Dynamic Stochastic General Equilibrium model à la Smets and Wouters (2003, 2004, 2007) featured with financial frictions à la Bernanke, Gertler Gilchrist (1999), i.e. featured with the financial accelerator mechanism, for the Euro Area for the period 1980q1 to 2008q3.

The main aim is to estimate a time series for the unobserved risk premium the entrepreneurs have to pay on their loan given the risky nature of their projects and the asymmetric information existing between them and the banks providing the funds.

Once obtained the series and before doing the dynamic analysis, I needed to have a sort of external validation of that series. In order to achieve that aim, I compared the premium with its available proxies for the Euro Area, represented by the spreads of the risk return over the risk free interest rate (the ten year government bonds interest rate in my case). The comparison is for the period 2002q1 until 2008q3 (because of the shortness of the available proxies).

That validation shows that the model with financial frictions can generate a series for the premium, without using any financial macroeconomic aggregates, highly correlated (up to 70 percent) with the lower grade bonds (A or BBB), but only until 2007q3 (right before the financial crises begun). In fact, the model fails to reproduce the big increase in the spreads observed after that date. In addition, and this is different from what found for the US, the series is highly related (still around 70 percent) also with the upper grades bonds (AAA and AA).

As for the dynamic properties of the premium we have found that the impulse responses analysis confirms the presence of the accelerator mechanism when a monetary policy shock occurs, i.e. the amplification of the responses of the macro aggregates with respect to the model without financial frictions. Nevertheless, differently from what the model would predict in theory and from the US evidence, the mechanism produces its effects only on investments, and not on output. This because the total effect on the latter variable is more that counteracted by the response of consumption, which is much stronger in the case of no financial friction.

In the end, the impulse response analysis highlights further that the risk premium is not necessary counter-cyclical (e.g. it is not in the case of the investment specific shock), and not always the response of investments is amplified when the accelerator is on. Those two results are there because of the presence of different channel which the shocks transmit to the economy through.

A Steady state values

The real marginal cost is the inverse of the mark-up

$$MC = \left(\frac{\theta - 1}{\theta}\right)$$

The steady state value of the return on capital R^k is

$$R^k = SR^n$$

$$R^k = Re + 1 - \delta$$

where $S = 1.02$ is the steady state level of the finance premium. Remembering that $R^n = (1 + i) = (1 + r) = \frac{1}{\beta}$ (because of the zero inflation steady state), I can write Re as

$$Re + 1 - \delta = SR^n$$

$$Re = S\frac{1}{\beta} - 1 + \delta \tag{28}$$

Q=1

Profits are

$$\Pi = \lambda_d Y - ReK - WL - F$$

where λ_d is the price mark up. We know that in equilibrium $Y = ReK + WL$, and that thanks to the fix cost profits are zero in steady state. Hence.

$$\Pi = \lambda_d Y - Y - F = 0$$

Solving for F

$$F = (\lambda_d - 1) Y$$

But Y still includes F. Hence an alternative way to write it is

$$F = (\lambda_d - 1)\left[\left(\frac{K}{L}\right)^{\alpha} L - F\right]$$

Solving again for F

$$F = \frac{(\lambda_d - 1)}{\lambda_d}\left(\frac{K}{L}\right)^{\alpha} L \tag{29}$$

The the steady state value of the fixed cost in production is

$$F = \frac{\lambda_d - 1}{\lambda_d}\left(\frac{K}{L}\right)^{\alpha} L \tag{30}$$

This implies that the steady state value of Y is

$$Y = \frac{1}{\lambda_d}\left(\frac{K}{L}\right)^{\alpha} L \tag{31}$$

From the resource constraint we can derive an expression for C

$$Y = C + I + G$$

Then

$$C = Y - I - gY$$

where $g \equiv \frac{G}{Y}$
Using the production function

$$C = (1-g)\left[\left(\frac{K}{L}\right)^{\alpha} L - F\right] - I$$

Substitute out F using equation 30 and I with its steady state expression δK

$$C = (1-g)\left[\left(\frac{K}{L}\right)^{\alpha} L - \frac{\lambda_d - 1}{\lambda_d}\left(\frac{K}{L}\right)^{\alpha} L\right] - \delta K$$

$$C = (1-g)\frac{1}{\lambda_d}\left(\frac{K}{L}\right)^{\alpha} L - \delta K$$

Or equivalently

$$C = \left[(1-g)\frac{1}{\lambda_d}\left(\frac{K}{L}\right)^{\alpha} - \delta\frac{K}{L}\right] L \tag{32}$$

We need an expression for $\frac{K}{L}$
From the cost minimization problem we have

$$Re = \alpha MC\frac{Y}{K}$$

$$W = (1-a)\, MC\frac{Y}{L}$$

where W is the steady state value of the real wage.
Combining the two

$$W = (1-a)\, Re\frac{1}{\alpha}\frac{K}{Y}\frac{Y}{L}$$

Re arranging and solving for $\frac{K}{L}$

$$\frac{K}{L} = \frac{\alpha}{(1-a)}\frac{W}{Re} \tag{33}$$

We know that the marginal costs are

$$\frac{1}{\lambda_d} = \left(\frac{1}{1-a}\right)^{(1-\alpha)}\left(\frac{1}{\alpha}\right)^{\alpha}(Re)^{\alpha}(W)^{1-a}$$

Solving for W

$$W=\left[\frac{1}{\lambda_d\left(\frac{1}{1-a}\right)^{(1-\alpha)}\left(\frac{1}{\alpha}\right)^{\alpha}(Re)^{\alpha}}\right]^{\frac{1}{1-a}} \tag{34}$$

Using the expression for Z (equation 28)

$$W=\left[\frac{1}{\lambda_d\left(\frac{1}{1-a}\right)^{(1-\alpha)}\left(\frac{1}{\alpha}\right)^{\alpha}\left(S\frac{1}{\beta}-1+\delta\right)^{\alpha}}\right]^{\frac{1}{1-a}}$$

Substituting in equation 33

$$\frac{K}{L}=\left[\frac{1}{\lambda_d\left(\frac{1}{1-a}\right)^{(1-\alpha)}\left(\frac{1}{\alpha}\right)^{\alpha}\left(S\frac{1}{\beta}-1+\delta\right)^{\alpha}}\right]^{\frac{1}{1-a}}\frac{\alpha}{(1-a)\,Re}$$

$$\frac{K}{L}=\frac{\alpha}{(1-a)\left(S\frac{1}{\beta}-1+\delta\right)}\frac{1}{\lambda_d^{\frac{1}{1-a}}\left(\frac{1}{1-a}\right)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{1-a}}\left(S\frac{1}{\beta}-1+\delta\right)^{\frac{\alpha}{1-a}}}$$

$$\frac{K}{L}=\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{1}{1-a}} \tag{35}$$

Substituting the previous in the consumption equation (equation 32)

$$C=\left\{(1-g)\,\frac{1}{\lambda_d}\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{\alpha}{1-a}}-\delta\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{1}{1-a}}\right\}L \tag{36}$$

From the labour supply condition we know that

$$W=\frac{L^{\sigma_l}}{[(1-h)\,C]^{\sigma_c}}$$

Solving for L and substitute out for W (using equation 34)

$$L=\left\{\left[\frac{1}{\lambda_d\left(\frac{1}{1-a}\right)^{(1-\alpha)}\left(\frac{1}{\alpha}\right)^{\alpha}(Re)^{\alpha}}\right]^{\frac{1}{1-a}}[(1-h)\,C]^{\sigma_c}\right\}^{\frac{1}{\sigma_l}} \tag{37}$$

Using equation 36 to substitute C in equation 37

$$L=\left\{\frac{1}{\lambda_d^{\frac{1}{1-a}}\left(\frac{1}{1-a}\right)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{1-a}}(Re)^{\frac{\alpha}{1-a}}}\left[(1-h)\,L\left\{\begin{array}{c}(1-g)\,\frac{1}{\lambda_d}\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{\alpha}{1-a}}-\\-\delta\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{1}{1-a}}\end{array}\right\}\right]^{\sigma_c}\right\}^{\frac{1}{\sigma_l}}$$

Re arranging

$$L = \left[\frac{1}{\lambda_d^{\frac{1}{1-a}} \left(\frac{1}{1-a}\right) \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{1-a}} (Re)^{\frac{\alpha}{1-a}}} \right]^{\frac{1}{\sigma_l}} (1-h)^{\frac{\sigma_c}{\sigma_l}} L^{\frac{\sigma_c}{\sigma_l}} \left[\begin{array}{c} (1-g)\frac{1}{\lambda_d} \left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)} \right]^{\frac{\alpha}{1-a}} - \\ -\delta \left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)} \right]^{\frac{1}{1-a}} \end{array} \right]^{\frac{\sigma_c}{\sigma_l}}$$

The solution for L is

$$L = \left(\frac{1}{A}\right)^{\frac{\sigma_l}{\sigma_c - \sigma_l}}$$

where $A = \left[\frac{1}{\lambda_d^{\frac{1}{1-a}} \left(\frac{1}{1-a}\right) \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{1-a}} (Re)^{\frac{\alpha}{1-a}}} \right]^{\frac{1}{\sigma_l}} (1-h)^{\frac{\sigma_c}{\sigma_l}} \left[\begin{array}{c} (1-g)\frac{1}{\lambda_d} \left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)} \right]^{\frac{\alpha}{1-a}} - \\ -\delta \left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)} \right]^{\frac{1}{1-a}} \end{array} \right]^{\frac{\sigma_c}{\sigma_l}}$

I can then derive the steady state of C substituting out L in equation 36

$$C = \left\{ (1-g)\frac{1}{\lambda_d} \left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)} \right]^{\frac{\alpha}{1-a}} - \delta \left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)} \right]^{\frac{1}{1-a}} \right\} \left(\frac{1}{A}\right)^{\frac{\sigma_l}{\sigma_c - \sigma_l}}$$

The steady state value of the fix cost is (from equation 29)

$$F = \frac{(\lambda_d - 1)}{\lambda_d} \left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)} \right]^{\frac{\alpha}{1-a}} \left(\frac{1}{A}\right)^{\frac{\sigma_l}{\sigma_c - \sigma_l}}$$

The steady state value for Y is (using equation 31)

$$Y = \frac{1}{\lambda_d} \left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)} \right]^{\frac{\alpha}{1-a}} \left(\frac{1}{A}\right)^{\frac{\sigma_l}{\sigma_c - \sigma_l}}$$

Using equation 35 we can campute the steady state value of K

$$K = \left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)} \right]^{\frac{1}{1-a}} L$$

$$K = \left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)} \right]^{\frac{1}{1-a}} \left(\frac{1}{A}\right)^{\frac{\sigma_l}{\sigma_c - \sigma_l}}$$

In turns the investement steady state

$$INV = \delta \left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(S\frac{1}{\beta}-1+\delta\right)} \right]^{\frac{1}{1-a}} \left(\frac{1}{A}\right)^{\frac{\sigma_l}{\sigma_c - \sigma_l}}$$

In the end the net worth

$$NW = \theta^e \left[R^k QK - R^n QK + R^n NW \right]$$

$$NW = \frac{\theta^e K}{1 - \theta^e R^n} \left(R^k - R^n \right)$$

$$NW = \frac{\theta^e K R^n}{1 - \theta^e R^n} \left(S - 1 \right)$$

$$NW = \frac{\frac{\theta^e}{\beta} \left[\frac{1}{\lambda_d \left(\frac{1}{\alpha} \right) \left(S \frac{1}{\beta} - 1 + \delta \right)} \right]^{\frac{1}{1-a}} \left(\frac{1}{A} \right)^{\frac{\sigma_l}{\sigma_c - \sigma_l}}}{1 - \frac{\theta^e}{\beta}} \left(S - 1 \right)$$

B The Log linearized model

The resource constraint

$$\widehat{y}_t = \frac{C}{Y}\widehat{c}_t + \frac{INV}{Y}\widehat{inv}_t + \frac{G}{Y}\widehat{g}_t + \frac{K}{Y}\psi Re^k \widehat{re}_t^k + \frac{K}{Y}S\left(1 - \frac{NW}{K}\right)\left(\widehat{r}_t^k + \widehat{q}_{t-1} + \widehat{k}_t\right)$$

The Fisher equation

$$\widehat{r}_t = \widehat{r}_t^n - E_t\left\{\widehat{\pi}_{t+1}^c\right\}$$

The consumption Euler equation

$$\widehat{c}_t = \frac{h}{1+h}\widehat{c}_{t-1} + \frac{1}{1+h}E_t\left\{\widehat{c}_{t+1}\right\} - \frac{1-h}{\sigma_c(1+h)}\widehat{r}_t + \frac{1-h}{\sigma_c(1+h)}\widehat{\varepsilon}_t^\beta$$

The household labour equation

$$\begin{aligned}\widehat{w}_t \quad = \quad & \frac{\beta}{1+\beta}E_t\left\{\widehat{w}_{t+1}\right\} + \frac{1}{1+\beta}\widehat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\left\{\widehat{\pi}_{t+1}^c\right\} - \frac{1+\beta\tau_w}{1+\beta}\widehat{\pi}_t^c + \frac{\tau_w}{1+\beta}\widehat{\pi}_{t-1}^c + \\ & -\frac{1}{1+\beta}\frac{(1-\beta\xi_w)(1-\xi_w)}{\left[1+\frac{(1+\lambda_w)\sigma_l}{\lambda_w}\right]\xi_w}\left[\widehat{w}_t - \sigma_l\widehat{l}_t - \frac{\sigma_c}{1-h}\left(\widehat{c}_t - h\widehat{c}_{t-1}\right) + \widehat{\varepsilon}_t^L\right] + u_t^w\end{aligned}$$

The household investment decision equation

$$\widehat{Inv}_t = \frac{1}{1+\beta}\widehat{Inv}_{t-1} + \frac{\beta}{1+\beta}E_t\left\{\widehat{Inv}_{t+1}\right\} + \frac{\varphi}{1+\beta}\widehat{q}_t + \widehat{x}_t$$

where φ is the inverse of the elasticity of the capital utilization cost function.
The condition on the price of capital

$$\widehat{q}_t = -\left(\widehat{r}_t\right) + \frac{1-\delta}{1-\delta+R^n}E_t\left\{\widehat{q}_{t+1}\right\} + \frac{R^n}{1-\delta+R^n}E_t\left\{\widehat{re}_{t+1}^k\right\} + u_t^q$$

The equation for the ex-post aggregate return to capita

$$E_t\widehat{r}_{t+1}^k = \widehat{r}_t - \varkappa\left(\widehat{nw}_{t+1} - \widehat{q}_t - \widehat{k}_{t+1}\right)$$

The financial premium

$$\widehat{s}_t = E_t\widehat{r}_{t+1}^k - \widehat{r}_t$$

The production function

$$\widehat{y}_t = \phi\widehat{a}_t + \phi\alpha\widehat{k}_{t-1} + \phi\alpha\psi\widehat{re}_{t+1}^k + \phi(1-\alpha)\widehat{l}_t$$

where $\psi = \frac{\Psi'(1)}{\Psi''(1)}$ is the inverse inverse of investment adjustment cost and ϕ is one plus the share of fixed cost in production.

The condition for the return of capital[30]

$$(1+\psi)\widehat{re}_t^k = \widehat{l}_t + \widehat{w}_t - \widehat{k}_{t-1}$$

[30] See the labour demand equation in the next section for a derivation of the parameter ψ.

The marginal costs

$$\widehat{mc}_t = \alpha \widehat{re}_t^k + (1-\alpha)\,\widehat{w}_t - \widehat{a}_t$$

The accumulation of capital

$$\widehat{k}_t = \delta \widehat{inv}_t + (1-\delta)\,\widehat{k}_{t-1}$$

The net wealth accumulation law[31]

$$\begin{aligned}\widehat{nw}_{t+1} &= \theta^e R^k \frac{K}{NW}\widehat{r}_t^k + \theta^e \left(R^k - R^n\right)\frac{K}{NW}\left(\widehat{q}_{t-1} + \widehat{k}_t\right) + \\ &+\theta^e R^n \left(1 - \frac{K}{NW}\right)\widehat{r}_t + \theta^e R^n \widehat{nw}_t - \\ &-\theta^e \mu M(\overline{\omega}) R^k \frac{K}{NW}\left[\widehat{r}_{t+1}^k + \widehat{q}_t + \widehat{k}_{t+1}\right] + (1-\alpha)\frac{Y}{NW}\widehat{y}_t\end{aligned}$$

The New Keynesian Phillips Curve (NKPC) for the domestic inflation

$$\widehat{\pi}_t^c = \frac{\beta}{1+\beta\tau_\pi} E_t\left\{\widehat{\pi}_{t+1}^c\right\} + \frac{\tau_\pi}{1+\beta\tau_\pi}\widehat{\pi}_{t-1}^c + \frac{1}{1+\beta\tau_\pi}\frac{(1-\beta\xi_\pi)(1-xi_\pi)}{xi_\pi}\left(\widehat{mc}_t\right) + u_t^{\lambda^p}$$

Monetary policy rule

$$\begin{aligned}\widehat{r}_t^n &= \phi_m \widehat{r}_{t-1}^n + (1-\phi_m)\left[r_\pi\left(\widehat{\pi}_{t-1}\right) + r_y\left(\widehat{y}_{t-1} - \widehat{y}_{t-1}^P\right)\right] + \\ &+ r_{\Delta\pi}\left(\widehat{\pi}_t - \widehat{\pi}_{t-1}\right) + r_{\Delta y}\left[\widehat{y}_t - \widehat{y}_t^P - \left(\widehat{y}_{t-1} - \widehat{y}_{t-1}^P\right)\right] + u_t^{r^n}\end{aligned}$$

Fiscal policy[32]

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + u_t^g$$

Shocks (those shocks which are not reported here are white noise processes. They are: u_t^w, $u_t^{\lambda^p}$, $u_t^{r^n}$, u_t^q)

Preference

$$\widehat{\varepsilon}_t^\beta = \rho_\beta \widehat{\varepsilon}_{t-1}^\beta + u_t^\beta$$

[31] This equation is particularly complex. In the simulation and estimation I will use the one reported in BGG:

$$\widehat{nw}_{t+1} = \theta^e R^k \frac{K}{NW}\left(\widehat{r}_t^k - \widehat{r}_t\right) + \widehat{r}_t + \widehat{nw}_t$$

[32] Since this process is

$$\log g_t = \rho_g \log g_{t-1} + u_t^g$$

substituting the variable with its definition in terms of percentage deviation from its steady state value, i.e. $g_t \equiv G\left(1+\widehat{g}_t\right)$ we have

$$\log G\left(1+\widehat{g}_t\right) = \rho_g \log G\left(1+\widehat{g}_{t-1}\right) + u_t^g$$

which can be approximated as

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + u_t^g$$

Labour supply

$$\widehat{\varepsilon}_t^L = \rho_L \widehat{\varepsilon}_{t-1}^L + u_t^L$$

Technology

$$\widehat{a}_t = \rho_a \widehat{a}_{t-1} + u_t^a$$

Investment specific

$$\widehat{x}_t = \rho_x \widehat{x}_{t-1} + u_t^x$$

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C Table and Figures

C.1 Data and Fitted Values

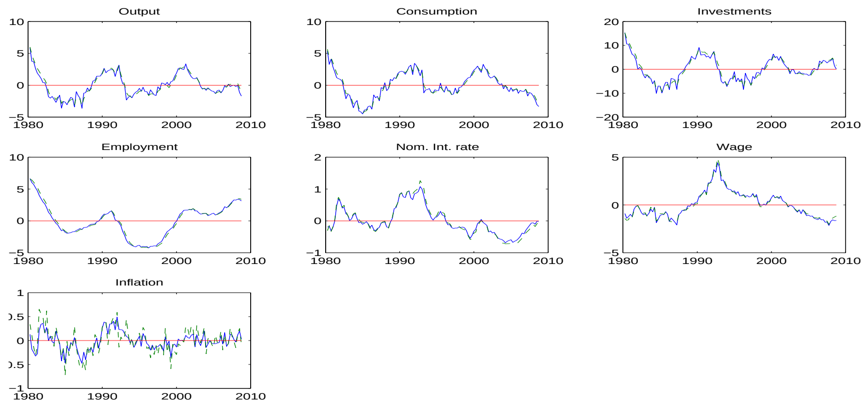

Figure 9: Data (dashed green line) and fitted values (solid blue line) from the model with financial frictions.

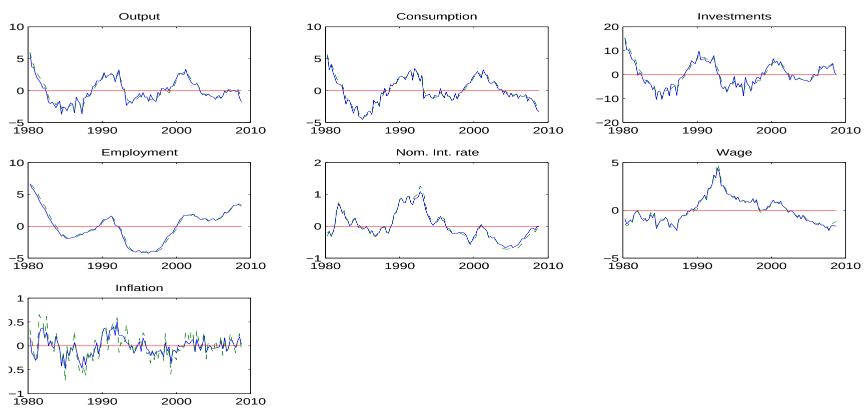

Figure 10: Data (dashed green line) and fitted values (solid blue line) from the model without financial frictions.

C.2 Priors and Posteriors distribution

C.2.1 Model with Financial Frictions

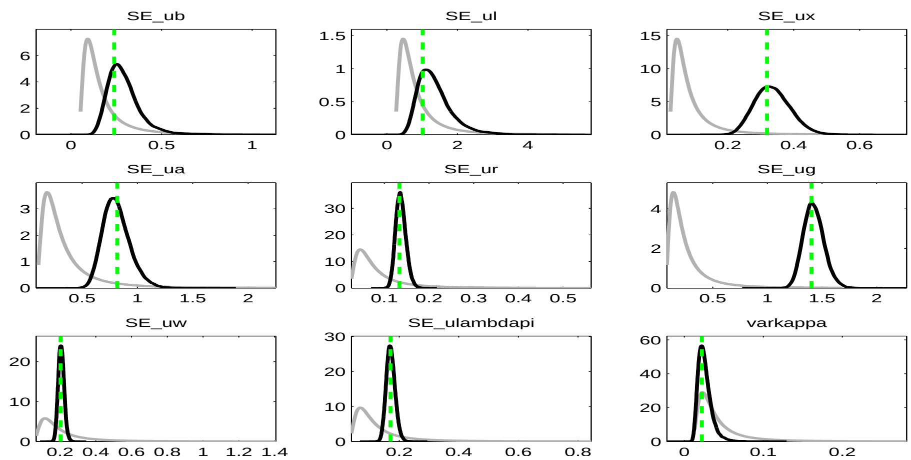

Figure 11: Posterior distributions. The green vertical line is the posterior mode computed with the likelihood maximization. The darker distribution is the posterior and the brighter one is the prior distribution.

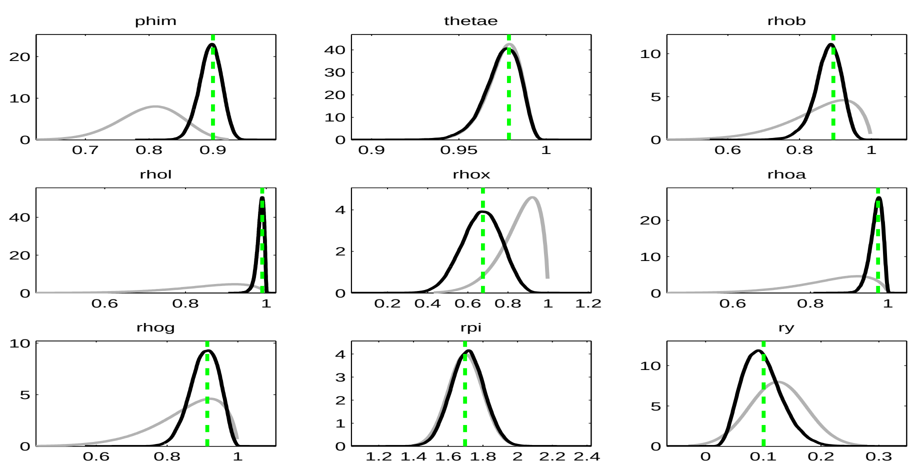

Figure 12: Posterior distributions. The green vertical line is the posterior mode computed with the likelihood maximization. The darker distribution is the posterior and the brighter one is the prior distribution.

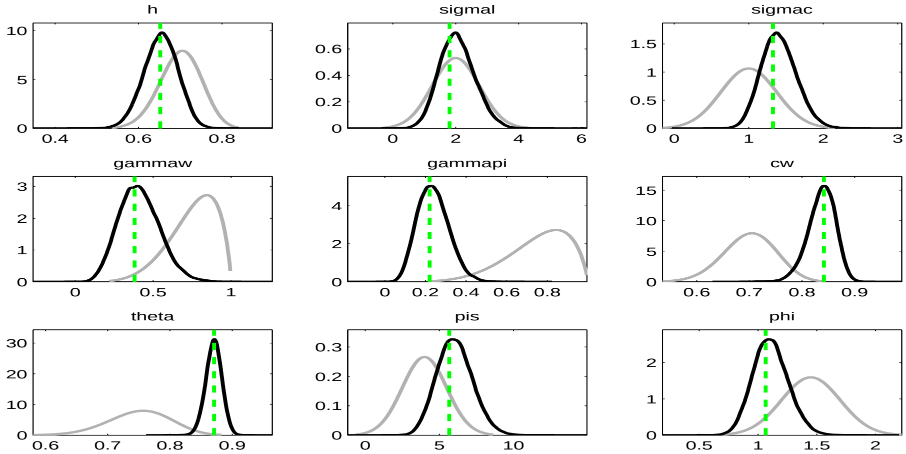

Figure 13: Posterior distributions. The green vertical line is the posterior mode computed with the likelihood maximization. The darker distribution is the posterior and the brighter one is the prior distribution.

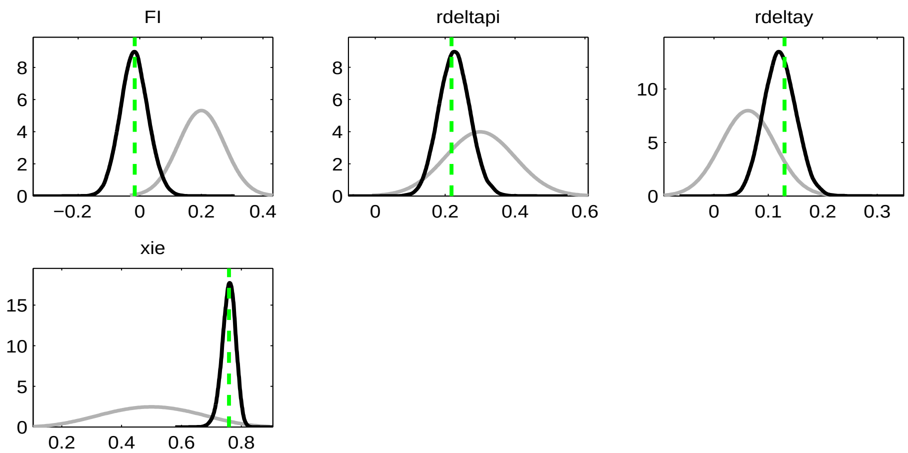

Figure 14: Posterior distributions. The green vertical line is the posterior mode computed with the likelihood maximization. The darker distribution is the posterior and the brighter one is the prior distribution.

C.2.2 Model without Financial Frictions

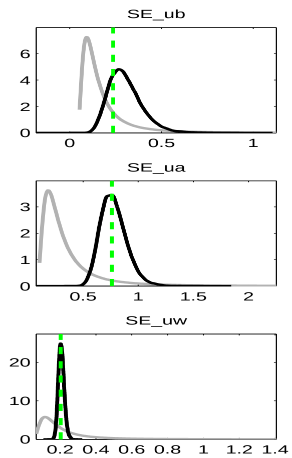

Figure 15: Posterior distributions. The green vertical line is the posterior mode computed with the likelihood maximization. The darker distribution is the posterior and the brighter one is the prior distribution.

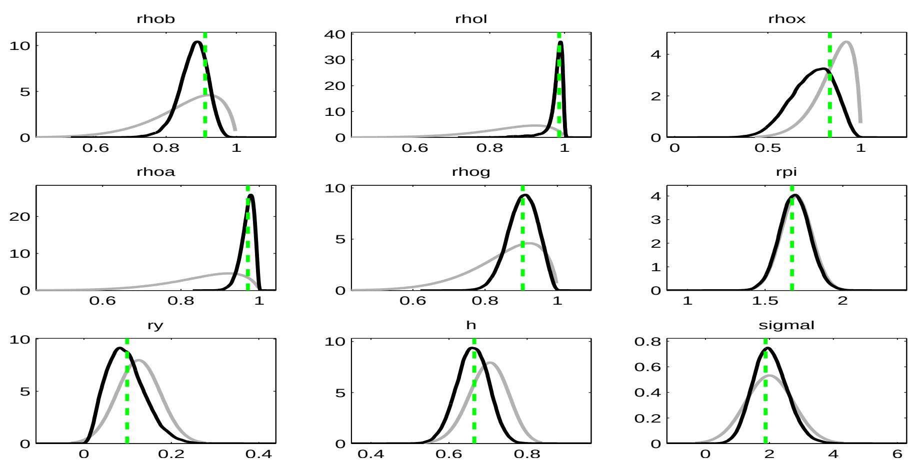

Figure 16: Posterior distributions. The green vertical line is the posterior mode computed with the likelihood maximization. The darker distribution is the posterior and the brighter one is the prior distribution.

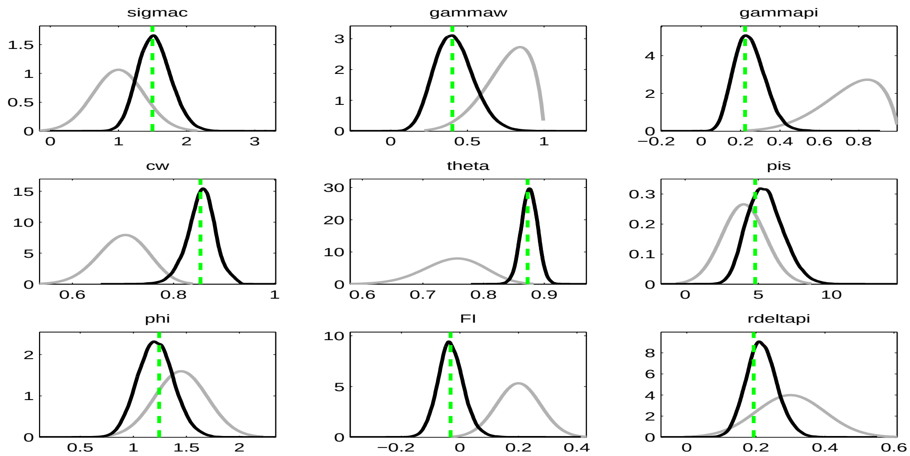

Figure 17: Posterior distributions. The green vertical line is the posterior mode computed with the likelihood maximization. The darker distribution is the posterior and the brighter one is the prior distribution.

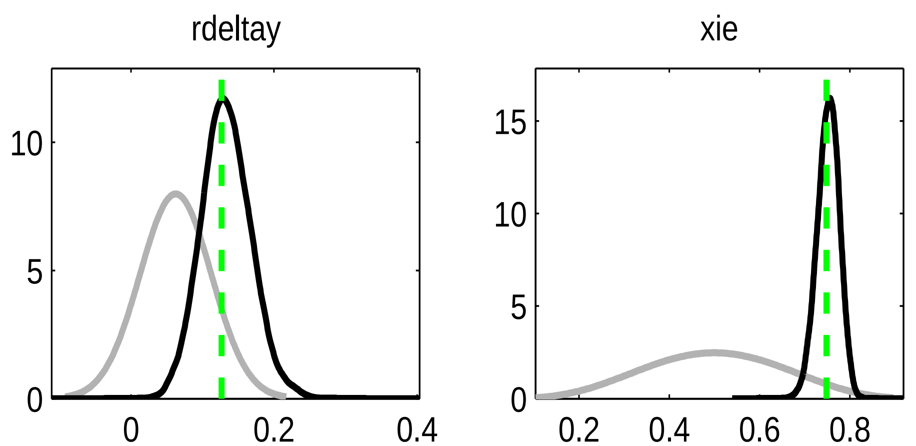

Figure 18: Posterior distributions. The green vertical line is the posterior mode computed with the likelihood maximization. The darker distribution is the posterior and the brighter one is the prior distribution.