The Relationship between the Hybrid New Keynesian Phillips Curve and the NAIRU over Time

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Abstract

New Keynesian models of the Phillips Curve generally assume a short-run trade-off between inflation and a measure of excess demand due to nominal rigidities, while in the long run inflation is constant at the NAIRU. By contrast, models such as the 'Triangle Model' of inflation explicitly allow for a time-varying NAIRU. We combine both approaches and estimate state-space models of the hybrid New Keynesian Phillips curve (NKPC), allowing the NAIRU to vary over time. Moreover, households' inflation expectations are measured directly from consumer surveys by the University of Michigan and the European Commission and are not instrumented for. Our model is estimated for the US, the UK, Italy and Spain and finds considerable variation in the NAIRU over time with NAIRU estimates significantly different from HP-filter derived measures such as usually employed in dynamic stochastic general equilibrium (DSGE) models.

Keywords: Hybrid New Keynesian Phillips curve, time-varying NAIRU, state-space models

JEL-classification: C32, E31

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1 Introduction

The most commonly used model of the Phillips curve in modern macroeconomics is the hybrid New Keynesian Phillips curve (NKPC) as developed in Galí and Gertler (1999), relating the inflation rate to lagged inflation, inflation expectations and a measure of excess demand and stating a shortrun trade-off between inflation and unemployment and long-run equilibrium with constant inflation at the NAIRU. However, the NAIRU may change over time if the market characteristics underlying the equilibrium relation between inflation and unemployment change (Friedman, 1968, and Phelps, 1968). Feedback effects between labour productivity and unemployment as in Phelps' (1994) structural slumps and, accordingly, hysteresis of unemployment (e.g. Stiglitz, 1997) may also cause underlying potential unemployment to shift. With a time-varying NAIRU, the unemployment rate that will keep inflation constant changes so that knowledge of these movements is of great importance for efficient monetary policy targeting. In this paper, we investigate the relationship between the hybrid NKPC and a time-varying NAIRU for the US, the UK, Italy and Spain. We estimate state-space models of the hybrid New Keynesian Phillips curve, where the time-varying NAIRU is estimated as an unobserved component. Thus, we can analyse changes in the NAIRU within the theory-based system of the hybrid NKPC, taking account of the interdependencies between inflation, inflation expectations and the unemployment gap when determining changes in potential unemployment. We contrast our estimates of the time-varying NAIRU from the state-space model with mechanically calculated steady-state unemployment from an HP-filter, such as usually employed in dynamic stochastic general equilibrium (DSGE) models (Dees et al., 2008).

Most empirical studies of the hybrid NKPC make use of the Generalized Method of Moments (GMM), instrumenting for inflation expectations with the output gap, the interest rate and additional lags of inflation.¹ These models generally find that while backward looking behaviour with regard to inflation is statistically significant, forward looking behaviour is quantitatively more important. If excess demand is measured by the output gap, it is often found insignificant; therefore, Galí and Gertler (1999) propose to use real marginal cost instead. Proxying this with unit labour cost, most stud-

¹For examples of GMM estimates of the hybrid NKPC for the US see Galí and Gertler (1999) and Galí et al. (2001, 2003, 2005).

ies find a significant and correctly signed coefficient. However, the GMM approach may be biased due to identification problems and weak instrument bias with regard to inflation expectations that impede the recovery of unique structural coefficients (e.g. Lindé, 2005, Rudd and Whelan, 2005, and Dees et al., 2008). We avoid this problem by using direct survey measures for households' inflation expectations from the University of Michigan's Survey of Consumers for the US and the Joint Harmonized EC Consumer Survey for the European economies in our estimations.² Overall, our state-space model of the hybrid NKPC thus avoids the identification problems encountered in standard GMM models and obtains time-varying estimates of the NAIRU within the theory-based system, where the restrictions on coefficients of the model can be tested directly. We find significant changes in the NAIRU over time in all the countries under investigation, which seem to move closely with actual unemployment rates. The paper is structured as follows: A short discussion of theories of the Phillips curve is given in Section 2, while Section 3 presents the model and methodology used for the econometric estimations. Section 4 presents the results from our estimations of the state-space models of the time-varying NAIRU in a hybrid NKPC setting. Finally, Section 5 summarises and concludes.

2 Theories of the Phillips Curve

2.1 The New Keynesian Model of the Phillips Curve

Assuming Calvo (1983) pricing with sticky prices and rational firms, the New Keynesian Phillips curve (NKPC)³ is a function of expected inflation $E_t \pi_{t+1}$ and a measure of excess demand y_t , which according to the theory of profit-maximising firms is represented by the percentage deviation of firms' real

²Other empiricial studies of the New Keynesian Phillips curve that employ survey measures of inflation expectations are, e.g., Roberts (1995, 1997), Adam and Padula (2003) and Paloviita (2008).

³An extensive summary of the literature on New Keynesian theories of monetary policy is given in Clarida et al. (1999). Roberts (1995, 1997) and Mankiw and Reis (2002a), inter alia, provide empirical estimates of the sticky-prices New Keynesian Phillips curve.

marginal cost from their steady-state value (Galí and Gertler, 1999)⁴:

$$\pi_t = \lambda y_t + \beta E_t \pi_{t+1} + u_t, \tag{1}$$

where π_t denotes the inflation rate $(p_t - p_{t-1}), u$ is an *i.i.d.* disturbance term and $\lambda \equiv (1 - \theta)(1 - \beta\theta)$ is a function of the probability of price adjustment $(1 - \theta)$ and the subjective discount factor β .

With rational expectations, unexpected movements in inflation will only have short-run real effects, since inflation expectations will adjust and influence current inflation. Iterating equation (1) forward gives the following closed form of the NKPC:

$$\pi_t = \lambda \sum_{j=0}^{\infty} \beta^j E_t y_{t+j} \tag{2}$$

Inflation should thus equal future discounted expected marginal costs. More recently, New Keynesian models of the Phillips curve have incorporated a lagged inflation term to account for the strong persistence of inflation typically observed in empirical data. First introduced by Galí and Gertler (1999)⁵, it is assumed that of the firms who are able to adjust prices in any period, only a fraction adjusts to their optimal prices, while the others update last period's optimal prices with lagged inflation as a 'rule of tumb'. This results in the so-called hybrid NKPC:

$$\pi_{t} = \phi \pi_{t-1} + (1 - \phi) E_{t} (\pi_{t+1}) + \gamma y_{t} + \epsilon_{t}, \qquad (3)$$

with $0 \le \phi \le 1$ and $\epsilon_t \sim IID(0, \sigma_{\epsilon}^2)$.

The hybrid NKPC presented in equation (3) thus incorporates sticky prices as well as inflation inertia and has become the workhorse of modern

⁴Studies previous to Galí and Gertler (1999) usually employed the output gap as the measure of excess demand. However, Galí and Gertler (1999) as well as Galí et al. (2005) stress the importance of using real marginal cost (which, assuming a Cobb-Douglas production function, can be proxied by the labour share) instead of the output gap for empirical estimation of the NKPC.

⁵Fuhrer and Moore (1995) also observe the missing persistence in inflation in standard New Keynesian models of the Phillips curve with staggered contracts à la Taylor (1980) and present a model similar to the hybrid NKPC, the so-called 'relative contracting model', where agents negotiate wages relative to existing wage contracts during the time their wage contract will be in effect. This introduces persistence both in inflation and excess demand and the authors show that the dynamics of the model match actual dynamics in inflation quite closely.

macroeconomics. The lagged inflation term might also be explained by sticky information as in Mankiw and Reis (2001, 2002a,b) which could be due to rational inattention (Sims, 2003 and Reis, 2006) related, for instance, to media coverage on inflation (Carroll, 2001, 2003).

Most empirical studies of the hybrid NKPC in the literature obtain estimates of the coefficients of the model using the Generalized Method of Moments (GMM). By assuming rational expectations and i.i.d. errors, the forecast error of inflation must be uncorrelated with variables dated t and earlier, providing the following orthogonality condition:

$$E_t \left[\left(\pi_t - \lambda y_t - \beta \pi_{t+1} \right) z_t \right] = 0, \tag{4}$$

where z_t is a vector of variables dated t and earlier. Galí and Gertler (1999) as well as Galí et al. (2001, 2003, 2005) amongst many others present GMM estimates of the hybrid NKPC for the US. While they find a significant impact of inflation inertia on current inflation, the effect of forward-looking behaviour, i.e. inflation expectations, on inflation seems to be quantitatively more important. The coefficient on excess demand is usually found significant and correctly signed. However, Lindé (2005), Rudd and Whelan (2005) and Dees et al. (2008) argue that the GMM approach to the hybrid NKPC often suffers from identification problems and weak instrument bias: As is common practice in most papers in the literature, apart from the output gap and the interest rate, additional lags on inflation are used to instrument for inflation expectations. Dees et al. (2008) show that this is only appropriate if the output gap depends on past values of inflation, either directly or indirectly. If this is not the case, instruments do not fulfil the rank condition and results may be seriously biased due to the weak instruments.

Lindé (2005) proposes the use of full information maximum likelihood estimators (FIML) to avoid the possible bias in GMM single equation estimations. Nason and Smith (2005) also acknowledge the identification problems of GMM methods. They present an alternative identification method where a structural vector autoregressive (SVAR) system of the hybrid NKPC is estimated, introducing an additional error-covariance restriction between the two equations in the system: The output gap is assumed to follow a first-order autoregressive process and does not depend on current inflation, which is described by the hybrid NKPC.

Batini et al. (2005) furthermore address the problem of a possible omitted variable bias of the standard NKPC for the case of an open economy such as the UK, including proxies for material input prices, foreign competition and employment adjustment costs. They find that marginal cost is inaccurately proxied by the labour share if employment adjustment costs are not accounted for and that inflation in the UK is significantly explained by shifts in real import prices and foreign competition. Bjørnstad and Nymoen (2008) also discuss a possible omitted variable bias for NKPC estimations, namely a linear combination of unit labour costs and the real exchange rate, since the NKPC is encompassed by imperfect competition models of inflation but not vice versa.

Another empirical approach to the hybrid NKPC is developed by Sbordone (2002, 2005) who estimates the closed form of the NKPC in equation (2). Using a two-step estimation procedure, she finds, similar to Galí and Gertler (1999) and subsequent papers, that while backward looking behaviour with regard to inflation is significant, forward looking behaviour is relatively more important. Her approach has been criticised by Kurmann (2005): Kurmann analyses the fit of the inflation path derived from the closed form NKPC with respect to actual inflation and concludes that while the fit of the model seems impressive, the confidence interval around the point estimates is relatively large so that it remains uncertain whether backward looking or forward looking behaviour dominates. In that sense, his critique applies also to Galí and Gertler (1999).

There exist several other studies of the New Keynesian Phillips curve that employ direct survey measures of inflation expectations instead of instruments: Roberts (1995, 1997) uses the Michigan survey of households' inflation expectations and the Livingston survey of professional forecasters' inflation expectations for the US in his study of the NKPC. He finds that expectations are not perfectly rational and there is evidence of a role for lagged inflation in explaining current inflation. Similarly, Adam and Padula (2003) analyse the NKPC for the US with data from the Survey of Professional Forecasters (SPF) and also find that survey data of inflation expectations do not confirm the rationality hypothesis needed for the orthogonality assumption of forecast errors with respect to output, so that estimations instrumenting for expectations may be severely distorted. Furthermore, they find that lagged inflation enters the hybrid NKPC significantly. Finally, Paloviita (2008) estimates different models of the Phillips curve for European economies using survey data from Consensus Economics for inflation expectations. She reports that the hybrid NKPC model performs best and even when allowing for possible non-rationality of expectations, the lagged inflation term still enters significantly. Thus overall, there seems to be a strong case for including lagged inflation in the hybrid NKPC and using survey data to account for possible distortions due to non-rationality of expectations.

2.2 Modelling the NAIRU over Time

Gordon (1997) proposes a different model of the Phillips curve in his 'Triangle model', where inflation depends on inflation inertia in the form of lagged values of inflation, present and past measures of excess demand (D) as well as present and past supply shocks (z) (Gordon, 1997):

$$\pi_t = \alpha \left(L \right) \pi_{t-1} + \beta \left(L \right) D_t + \gamma \left(L \right) z_t + \epsilon_t, \tag{5}$$

where (L) stands for the lag operator. Excess demand D is normalised to zero and can be represented by the output gap or the unemployment gap, which is defined as the gap between the current unemployment rate and its 'natural' value $(U - U^N)$. If the sum of the α -coefficients equals exactly unity, it can be shown that there exists a 'natural' rate of unemployment consistent with constant inflation, hence a NAIRU. Long-run steady-state unemployment is thus explicitly modelled in equation (5).

The notion of changes in the NAIRU attributable to changes in the microeconomic relations governing the product and labour markets was acknowledged by Friedman and Phelps already in 1968 and later ascribed for example to 'structural slumps' (Phelps, 1994) or hysteresis of unemployment (e.g. Stiglitz, 1997). Nevertheless, most empirical approaches to the Phillips curve test the performance of an assumed fixed value for the NAIRU. Gordon (1997) resigns from this approach and instead estimates a timevarying NAIRU in equation (5), specifying it as an unobserved component following a simple random walk (Gordon, 1997, p. 20):

$$\pi_t = a\left(L\right)\pi_{t-1} + b\left(L\right)\left(U_t - U_t^N\right) + c\left(L\right)z_t + e_t \qquad \text{with } e_t \sim IID\left(0, \sigma_e^2\right) \tag{6}$$

$$U_t^N = U_{t-1}^N + v_t \qquad \text{with } v_t \sim IID\left(0, \sigma_v^2\right) \tag{7}$$

The NAIRU is allowed to vary over time according to the state-equation in (7) and exists if the sum of the *a*-coefficients equals one and the sum of the *b*-coefficients is significantly negative. Thus, by using the unobserved components approach in a state-space model of the Phillips curve, Gordon (1997) employs a specific econometric technique to estimate changes in the NAIRU over time within the system set out by the Triangle model, thereby providing testable estimates of those changes.

Gordon (1997) finds for the US in the time-period 1955(q2) - 1996(q2) that the NAIRU has varied significantly between 5.3 % and 6.5 %, contrary to the 'textbook' assumption of a constant NAIRU at 6 % for the US after 1978.⁶ Fabiani and Mestre (2004) provide estimates of the time-varying NAIRU for the Euro area using a state-space model similar to the 'Triangle model' by Gordon (1997) that is augmented by an Okun relation between the output gap and the unemployment gap. Additionally, the authors assume that both potential output and potential unemployment follow a random walk with a stochastic trend. The authors find that this specification of the NAIRU yields robust estimates across various model variants with reasonably small confidence bands. Over the estimation period, it is shown that the NAIRU for the Euro area increased continuously, in line with an increase in average unemployment rates.

In a recent paper, Harvey (2008) uses the unobserved component approach to model a hybrid NKPC, where lagged inflation π_{t-1} is substituted for a random walk μ^* :

$$\pi_t = (1 - \gamma) \mu_t^* + \gamma E_t (\pi_{t+1}) + \beta^* x_t + \epsilon_t^* \qquad \text{with } \epsilon_t^* \sim IID \left(0, \sigma_{\epsilon^*}^2\right), \quad (8)$$
$$\mu_t^* = \mu_{t-1}^* + \eta_t^* \qquad \text{with } \eta_t^* \sim IID \left(0, \sigma_{\eta^*}^2\right), \quad (9)$$

where $0 \leq \gamma \leq 1$ and x_t represents the output gap in period t. Since inflation π_t is most commonly found to be integrated of order one, but the output gap x_t is stationary by construction, the unobserved component μ_t^* captures the long-run forecast of π_t and can thus be regarded as a measure of core inflation. Harvey (2008) then shows that in steady-state, a reduced form of (8) can be derived as

$$\pi_t = \tilde{\mu}_t + \gamma \beta^* \sum_{j=0}^{\infty} \gamma^j E_t \left(x_{t+1+j} \right) + \beta^* x_t + \tilde{\epsilon}_t \qquad \text{with } \tilde{\epsilon}_t \sim IID \left(0, \sigma_{\tilde{\epsilon}}^2 \right),$$
(10)

$$\tilde{\mu}_t = \tilde{\mu}_{t-1} + \tilde{\eta}_t \qquad \text{with } \tilde{\eta}_t \sim IID\left(0, \sigma_{\tilde{\eta}}^2\right). \tag{11}$$

⁶Staiger et al. (1997) use a similar model to estimate a time-varying NAIRU for the US over the time period 1961(q1) - 1996(q4). However, they solve the model to include the NAIRU in the constant term, which is then estimated with a flexible polynomial ('spline'). The authors find estimates of the NAIRU in a 95 % confidence interval between 5 % and 8.5 %.

However, assuming that x is driven by an AR(1) process with root $|\phi| < 1$, equation (10) becomes:

$$\pi_t = \tilde{\mu}_t + \frac{\beta^*}{1 - \phi\gamma} x_t + \tilde{\epsilon}_t.$$
(12)

The model of the hybrid NKPC thus reverts back to a simple Phillips curve without expectations or dynamics and identification of γ is not possible unless the output gap follows a higher order AR(p) process with $p \geq 2$. The unobserved component $\tilde{\mu}$ captures both core inflation and inflation expectations, making a direct interpretation difficult.

3 Model and Methodology

The model used in this paper combines the hybrid NKPC as developed by Galí and Gertler (1999) and the unobserved components approaches by Gordon (1997) and Harvey (2008). The hybrid NKPC is chosen as the baseline model because it has become the most widely used model of the Phillips curve in recent years and incorporates both nominal rigidities in the form of sticky prices and inflation inertia which might be due to some form of sticky information. Nevertheless, as in the original model developed by Friedman (1968) and Phelps (1968), the assumption of a vertical long-run Phillips curve at the NAIRU, hence no long-run trade-off, is retained, but the NAIRU may vary over time if structural characteristics of the labour and commodity markets change. It thus seems to be a good starting point for the analysis of the relationship between the short-run New Keynesian Phillips curve and the NAIRU over time. As in Gordon (1997), the NAIRU is modelled directly by substituting the output gap for the unemployment gap and modelling the time-varying NAIRU as an unobserved component in a state-space representation. In order to ensure that the unobserved component measures the time-varying NAIRU and to avoid the identification problem in Harvey (2008), we include survey measures of inflation expectations directly in the model.

This gives the following model of the time-varying NAIRU in a hybrid NKPC setting, taking full account of sticky prices and inflation inertia:

$$\pi_{t} = \alpha \pi_{t-1} + (1 - \alpha) E_{t}^{survey} (\pi_{t+1}) + \gamma \left(U_{t} - U_{t}^{N} \right) + \epsilon_{t}$$
with $\epsilon_{t} \sim IID \left(0, \sigma_{\epsilon}^{2} \right)$ (13)
$$U_{t}^{N} = U_{t}^{N} + U_$$

 $U_t^N = U_{t-1}^N + v_t$

with $v_t \sim IID(0, \sigma_v^2)$, (14)

where $0 < \alpha < 1$.

In line with Gordon (1997), an unemployment rate consistent with stable inflation (a NAIRU) exists, if the coefficients on lagged and expected inflation sum to one. If in addition, the γ -coefficient on the unemployment gap is found significantly negative, we find a short-run trade-off between inflation and unemployment. By allowing the NAIRU to vary over time according to the state equation in (14), we can thus estimate changes in equilibrium unemployment within the system of the hybrid NKPC, controlling for the interdependencies between inflation, inflation expectations and unemployment. Thus, rather than assuming a fixed value of the NAIRU and testing its empirical performance, this approach provides econometrically testable estimates of structural changes in the NAIRU over time.

The state-space model of the hybrid NKPC presented in equation (13) has a number of advantages over other specifications and estimation methods found in the literature: Our model in equations (13) and (14) avoids the possible weak identification bias of GMM estimations of the hybrid NKPC described above by using independent survey measures of inflation expectations instead of IV procedures with further lags of inflation as instruments. Thus, survey measures of household's inflation expectations provide raw data that does not depend on any underlying econometric methodology.

A further advantage of the model given in equations (13) and (14) is that it allows the time-varying NAIRU to be estimated within the system set out by the hybrid NKPC. The interdependencies between inflation, inflation expectations and unemployment are used to determine steady-state unemployment over time as given by the state-variable U^N . The systems' approach thus provides estimates of the time-varying NAIRU that are grounded in macroeconomic theory rather than mechanically obtained as HP-filtered steady-state measures, such as usually applied in DSGE models (Dees et al. (2008)).

Finally, the estimates of the unobserved component of the time-varying NAIRU can be compared to mechanically calculated steady-state measures of

unemployment, such as HP-filtered trend unemployment. Furthermore, the significance of the restriction imposed on the coefficients of lagged and expected inflation in (13) $(\alpha + \beta = \alpha + (1 - \alpha) = 1)$ can be tested within the model. Overall, the state-space representation of the hybrid NKPC avoids identification problems of GMM approaches and provides a flexible and testable estimation method both for the standard short-run hybrid NKPC and the time-varying NAIRU.

4 Empirical Results

4.1 Description of the Data

The model of the hybrid NKPC presented above was estimated for the US for the time period 1961(q1) to 2007(q3), for the UK and Italy for the time period 1985(q1) to 2007(q3) and for Spain for the period 1986(q3) to 2007(q3). The shorter estimation period for the European countries was due to shorter time series of survey data of household's inflation expectations.

We used quarterly data for consumer prices, the unemployment rate and inflation expectations. Data for the consumer price index (CPI) for all items and the standardised unemployment rate were taken from the OECD Main Economic Indicators (MEI) (OECD (2008)) database. The inflation rate was then calculated as the annual growth rate of the CPI. Survey measures of households' inflation expectations in the United States were provided by the University of Michigan's Surveys of Consumers (SCA), while for the European economies in our sample we employed survey data from the Consumer Survey of the 'Joint Harmonised EU Programme of Business and Consumer Surveys' directed by the European Commission.⁷ The Michigan Survey asks directly for a quantitative estimate of expected inflation, whereas the EC Survey uses a qualitative measure of inflation expectations, asking interviewees about the direction of the expected price movement, rather than a specific point estimate. In order to derive a quantitative time series of inflation expectations, the qualitative answers were converted with the probabil-

⁷Although the surveys are conducted by country-specific institutes, the questionnaire and timing of the survey are identical across European countries and sample sizes are similar, so that the data are consistent over time and across countries. Papers using the Joint Harmonized EC Consumer Survey data include Nielsen (2003) and Döpke et al. (2008).

ity method of Carlson and Parkin (1975) as adapted for a pentachotomous survey by Batchelor and Orr (1988), scaling inflation expectations with oneperiod lagged inflation, recursive mean inflation until last period, recursively HP-filtered inflation and the fitted values obtained from an ARMA(4,4)model of inflation that were also filtered with a recursive HP-filter as in Döpke et al. (2008).

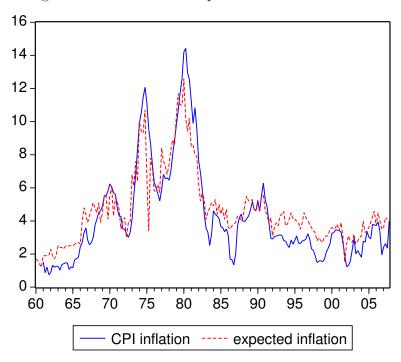


Figure 1: Inflation and Expected Inflation for the US

Source: OECD and SCA data, own calculations and graphs.

As can be seen from Figure 1, households' inflation expectations match actual inflation for the US relatively well, especially during the oil price shocks of the 70s and 80s. After a period of overshooting during the 90s, inflation expectations seem to have stabilised at around 3 - 4 % since the beginning of the new millennium in line with actual inflation.

Figure 2 presents the resulting time series of expected inflation for the UK, Italy and Spain. The graph for the UK also shows the time series of expected inflation of the Inflation Attitudes Survey by the Bank of England

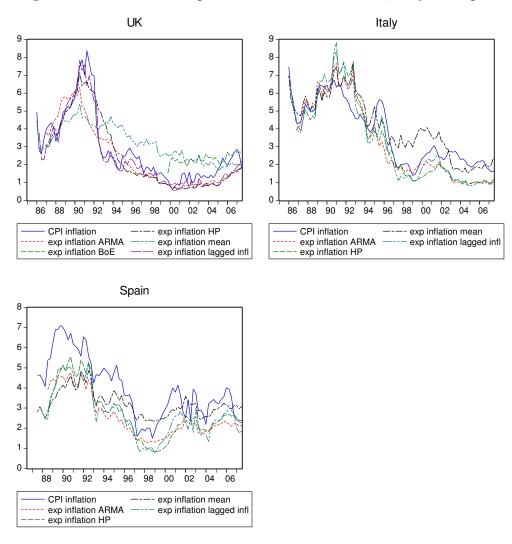


Figure 2: Inflation and Expected Inflation for the UK, Italy and Spain

Source: OECD, BoE and EC Consumer Survey data, own calculations and graphs.

for the time period 1999(q4) - 2007(q3).

The time series' of expected inflation derived with the probability method are generally quite close for the three countries analysed here: Time series' of expected inflation for the UK fit actual inflation relatively closely, only expectations scaled with recursive mean inflation overshoot from 1992 onwards, but converge towards actual inflation rates towards the end of the sample period. Furthermore, they are found very close to the series of expected inflation published by the Bank of England from 2000 onwards.

Inflation expectations in Italy match actual inflation rates closely until 1995; thereafter inflation expectations scaled with recursive mean inflation overshoot actual inflation rates until 2004. This coincides with the observation by several studies that inflation was severely overestimated during the time of the Euro introduction.⁸ The remaining time series of expected inflation for Italy are below actual inflation after 2002. In Spain, inflation expectations seem to have generally underestimated actual inflation up until the mid-90s. After a considerable drop in inflation rates, expected inflation rates approach actual rates in the second half of the sample period.

To discriminate more formally between the different series of inflation expectations derived from the probability method, we calculated the root mean squared error (RMSE) of the inflation expectations series with respect to actual inflation four quarter ahead:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} \left(\pi_{t+4} - E_t\left(\pi_{t+4}\right)\right)}{n}}$$
(15)

The RMSE thus gives a measure of forecasting accuracy of inflation expectations. Table 1 presents values of the RMSE for different scaling factors of expected inflation for the UK, Italy and Spain.

The lowest forecasting error is achieved with the HP-filtered fitted values for inflation from the ARMA-model ($infl_exp_arma$) in all three countries under investigation here, although RMSEs of expected inflation with other scaling factors are quite close in the case of Italy and Spain. We thus decided to use $infl_exp_arma$ in our model of the hybrid NKPC.

⁸See Malgarini (2008) for a summary of studies on Italian inflation expectations.

Table 1: Root Mean Squared Errors of Time Series of Inflation Expectation

Country	Scaling Factor	RMSE
UK	HP-trend from ARMA model of inflation	0.868
	Recursive HP-trend	1.300
	Recursive mean	1.545
	Last period's inflation	1.145
Italy	HP-trend from ARMA model of inflation	1.006
	Recursive HP-trend	1.310
	Recursive mean	1.181
	Last period's inflation	1.223
Spain	HP-trend from ARMA model of inflation	1.516
	Recursive HP-trend	1.543
	Recursive mean	1.549
	Last period's inflation	1.676

4.2 Testing for Unit Roots

Before we carried out any estimations, all time series in the model were tested for unit roots with the augmented Dickey-Fuller test (ADF test, Dickey and Fuller (1981)). Inflation and its expectations seem to be non-stationary in all the countries under investigation here (Table A1 in the Appendix). In the case of the US for the sample 1961(q1) - 2007(q4), this might be due to a structural break in inflation after the oil price shocks, when inflation rates in the US were stabilised substantially. Inflation rates of the European countries for the shorter sample from 1986(q1) - 2007(q3) seem to have stabilised after the turbulences of the ERM currency crisis 1991-1992. While the unemployment rate for the US was found to be stationary, the ADF tests could not reject the null of a unit root for the UK, Italy and Spain. This could be due to the significant fall in unemployment rates in the European countries from the mid-90s onwards. As mentioned by Fanelli (2008), most empirical studies on the hybrid NKPC fail to acknowledge the non-stationarity of inflation and inflation expectations. The author argues that non-stationarity may originate from the aggregation of sectoral and regional/national Phillips curves, with stationary variables at the firm level as assumed in theory. To rule out spurious results, we estimated simple OLS models of the hybrid NKPC with HP-filter derived output and unemployment gaps and tested the residuals for stationarity using special critical values from MacKinnon (1991). For all the models, residuals were stationary at the 1 % level, suggesting cointegration of the variables.⁹

4.3 State-Space Models of the Time-Varying NAIRU

The state-space model of the hybrid NKPC presented in equations (13) and (14) was estimated in two different models: In the first specification, the coefficients of lagged inflation and expected inflation were estimated freely, while in the second specification they were restricted to sum to exactly one. We then extracted estimates of the time-varying NAIRU with the Kalman filter (Kalman (1960)). This enabled us to test for the significance of the restriction $\alpha + \beta = \alpha + (1 - \alpha) = 1$ on the coefficients of lagged and expected inflation and compare the estimates of the time-varying NAIRU from the two models. In order to enable convergence, the variances of the observation equation and the state equation had to be restricted. Variances of the observation equation vary with each model, but the variance of the state equation was set uniformly to $\sigma_v^2 = 0.20$ in accordance with Gordon (1997). To provide starting values for the iterations, the estimation periods were shortened, usually by 4 quarters.

4.3.1 Fit of the Models

The estimated coefficients of the observation equation for both the restricted and the unrestricted model for the US, the UK, Italy and Spain are given in Tables A2 - A9 in the Appendix. Surprisingly, in contrast to the results of Galí and Gertler (1999), Galí et al. (2001, 2003, 2005) and Sbordone (2002, 2005), we find that the coefficient on lagged inflation is larger than that on expected inflation for all countries in our sample, with the notable exception of Spain. The reason for this finding might be the different estimation method

 $^{^{9}}$ We omit the results from the OLS models for reasons of space limitation, but they can be obtained from the author upon request.

employed here, where we use survey measures of inflation expectations instead of instruments and the different specification with the unemployment gap instead of real marginal cost. In the case of the UK, forward looking behaviour becomes quantitatively more important when the model is estimated in the restricted form.

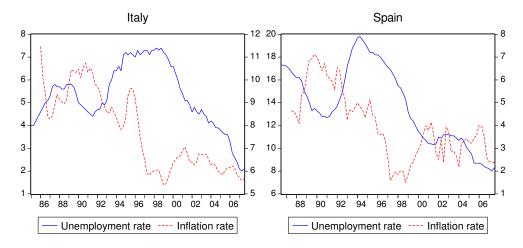


Figure 3: Unemployment and Inflation in Italy and Spain

Source: OECD data, own calculations and graphs.

The unemployment gap generally enters the hybrid NKPC with a highly significant coefficient. For the US and the UK, the γ -coefficient is negatively signed, as expected, but for Italy and Spain we find a significantly positive coefficient. This surprising result could be due to the estimation period used here, where a simultaneous drop in both inflation and unemployment occurred in the two countries in the latter half of the sample period. This was caused by monetary policies aimed at joining the EMU as well as labour market reforms and a boom that boosted employment in Italy and Spain. Nevertheless, a Phillips curve relation between inflation and unemployment is still visible at least in the first half of the sample period (Figure 3).

In order to check for misspecification, we tested the residuals of all models for normality and stationarity. The ADF test rejected the null of a unit root for the residuals at the 1 % level for all models, whereas the Anderson-Darling test for normality (Anderson and Darling (1952, 1954)) could not

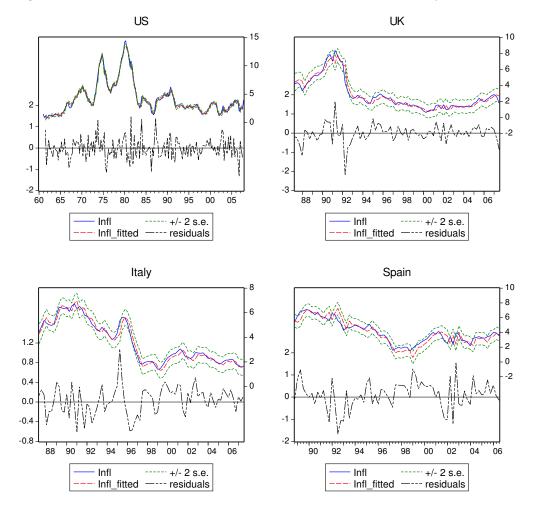


Figure 4: Fitted Values from the Unrestricted Model of the Hybrid NKPC

Source: OECD, EC Consumer Survey and SCA data, own estimations, own graphs.

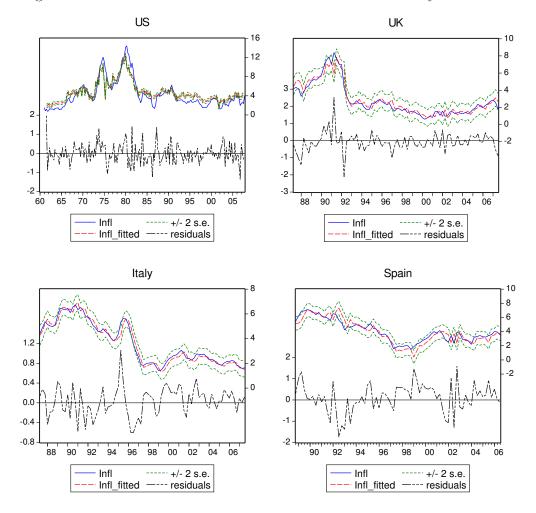


Figure 5: Fitted Values from the Restricted Model of the Hybrid NKPC

Source: OECD, EC Consumer Survey and SCA data, own estimations, own graphs.

reject the null of a normal distribution for all models except those for the UK, where two large outliers (1991/1992) distorted the outcome. Fitted values of the unrestricted and the restricted model (where coefficients on lagged and expected inflation were restricted to sum to one), as well as the residuals, are plotted in Figures 4 and 5, respectively.

In most of the countries under investigation here, fitted values from the unrestricted and the restricted model differ only marginally, and the fit of the model generally seems very close with respect to actual inflation rates. Only in the case of the US, it seems that the fit from the unrestricted model is tighter, with exceptionally low standard errors. Nevertheless, fitted values from the restricted model for the US still fit actual inflation rates very closely. As indicated by the tests for stationarity and normality, the residuals plotted in Figures 4 and 5 generally seem to follow white noise processes around mean zero.

4.3.2 Time-Varying NAIRU Estimates

From the state-space model of the hybrid NKPC as in equations (13) and (14) we derived smoothed estimates of the time-varying NAIRU with the Kalman filter. As an additional test for misspecification, we tested the NAIRU estimates from both models for a unit-root with an ADF-test (see Tables A2 - A9 in the Appendix). By construction, the NAIRU estimates should be unit roots. With the exception of the unrestricted model for the US, for all other NAIRU estimates we cannot reject the null of a unit root, usually with high p-values.

Figure 6 presents the time-varying NAIRU estimates for the US, the UK, Italy and Spain from the unrestricted model. Generally, unrestricted NAIRU estimates for the four countries under investigation here show considerable variation, usually in line with actual unemployment rates, with the notable exception of the UK, where NAIRU point estimates seem relatively stable. For the US and the UK, the unrestricted model yields rather implausible values of the NAIRU, suggesting that unemployment was significantly above the NAIRU in the US over the whole estimation period, albeit with very large confidence bands. By contrast, NAIRU estimates for Spain show a tight confidence band and are found close to actual unemployment rates, implying that unemployment was above the NAIRU only at the peak in 1994/95 and below in 2000. A similar result applies for Italy, with a NAIRU close to actual unemployment from 1994 onwards, and unemployment below

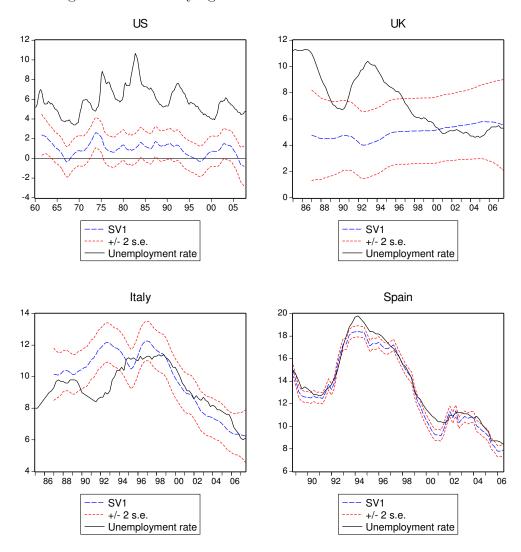


Figure 6: Time-Varying NAIRUs from the Unrestricted Model

Source: OECD, EC Consumer Survey and SCA data, own estimations, own graphs.

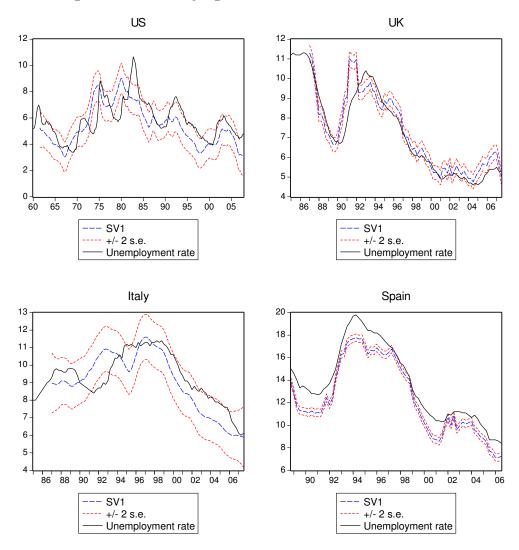


Figure 7: Time-Varying NAIRUs from the Restricted Model

Source: OECD, EC Consumer Survey and SCA data, own estimations, own graphs.

its potential rate from 1990 to 1994.

Contrasting time-varying estimates of the NAIRU from the restricted models with those from the unrestricted model in Figure 7, the improvement in significance and variation of the NAIRU is remarkable. Especially for the US and the UK we now find highly significant NAIRU estimates with low standard errors close to actual unemployment rates. It seems that although coefficients in the unrestricted model sum closely to one, the exact restriction is necessary in order to recover the NAIRU, in line with the argument of Gordon (1997). While we find for the UK that unemployment was below the NAIRU only in 1990 - 1992 and in the last years of the sample period, in the US unemployment seems to fluctuate around the NAIRU, with the NAIRU leading actual unemployment during the period of oil price shocks in the 70s and 80s. In the case of Italy, the picture seems mostly unchanged compared to the unrestricted model, with being 1992 - 1994 the only years where confidence bands of the NAIRU are above actual unemployment. For Spain, the fit of the time-varying NAIRU is again remarkable, but we now find that unemployment was above the NAIRU for most of the sample period.

4.3.3 Comparison of the Models

Finally, we compare the Kalman-filtered smoothed estimates of the timevarying NAIRU from the unrestricted and the restricted model of the hybrid NKPC to each other and to an HP-filter derived NAIRU, shown in Figure 8. Overall, we find three main results: First, as already noted above, NAIRU estimates from the restricted model of the hybrid NKPC generally seem more plausible in relation to actual unemployment and in the case of the US and the UK yield significantly different estimates. Second, all NAIRU estimates derived from the state-space models are significantly different from the mechanically derived HP-filtered NAIRUS, suggesting that estimating the NAIRU in a theoretically grounded macroeconomic model, taking account of the interaction of inflation, inflation expectations and the unemployment gap, yields significant new insights. Third, all NAIRU estimates from the statespace models of the hybrid NKPC exhibit a drop in the second half of the 90s, which in the case of Italy extends until the end of the estimation period.¹⁰ For the US, Italy and Spain, the drop in the NAIRU is even more pronounced than the fall in actual unemployment rates, suggesting that unemployment

¹⁰This result is in line with those in Gordon (1997) and Staiger et al. (1997) for the US.

remained above the NAIRU in this period.

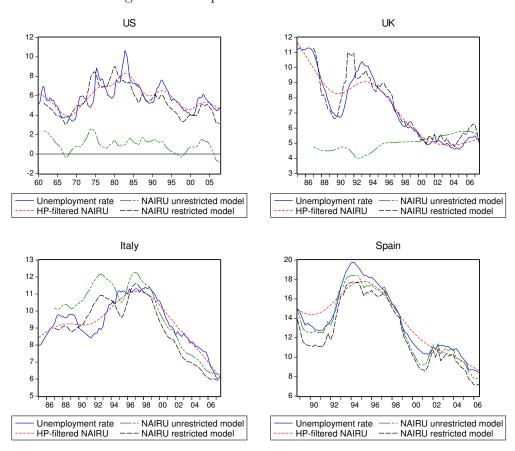


Figure 8: Comparison of NAIRU Estimates

Source: OECD, EC Consumer Survey and SCA data, own estimations, own graphs.

As noted above, in the case of the US and the UK the unrestricted model gives implausibly low values of the NAIRU, suggesting that actual unemployment was always significantly above the NAIRU. These results are not in line with those found in the literature for the US, stressing again the importance of restricting the coefficients of lagged and expected inflation to exactly one in the country models (e.g. Gordon, 1997, and Staiger et al., 1997). By contrast, time-varying estimates of the NAIRU from the restricted model imply a mean structural unemployment of 5.5 % (7.25 %) for the US (UK), close to

actual mean unemployment of 5.8 % (7.37 %). Note that the average NAIRU is still estimated to be lower than average actual unemployment. For Italy, estimates of the NAIRU from the state-space models differ mostly in the first half of the sample period, where the NAIRU implied by the restricted model is lower. Overall, the restricted NAIRU (9.23 %) has a mean closer to average actual unemployment rates in Italy (9.39 %) than the unrestricted NAIRU (10.0 %). By contrast, in the case of Spain the time-varying NAIRU from the unrestricted model (mean: 13.23 %) seems to be closer to actual unemployment (mean: 13.69 %) than the time-varying NAIRU from the restricted model (mean: 12.44 %).

In order to discriminate more formally between the unrestricted and the restricted model of the hybrid NKPC, we analysed the information criteria and conducted a Wald test on the restriction $\alpha + (1 - \alpha) = \alpha + \beta = 1$. Furthermore, since the unrestricted model encompasses the restricted one, we can run a likelihood ratio test according to the formula

$$2\left[\mathcal{L}\left(\theta\right) - \mathcal{L}\left(\theta^{*}\right)\right] \approx \chi^{2}(m),\tag{16}$$

where $\mathcal{L}(\theta)$ is the log likelihood of the unrestricted model, $\mathcal{L}(\theta^*)$ the log likelihood of the restricted model and m the number of restrictions, which here equals one.¹¹ The information criteria of the models are found in Tables A2 - A9 in the Appendix, and test values for the Wald test and the likelihood ratio test are shown in Table 2.

Generally, coefficients on lagged and expected inflation summed closely to one in all the unrestricted models of the hybrid NKPC, so that the Wald test could not reject the null hypothesis of the restriction $\alpha + (1 - \alpha) = \alpha + \beta = 1$ in all countries except for the US, thus confirming the theoretical argument of Gordon (1997) for the identification of the NAIRU. However, the information criteria and the likelihood ratio test are less conclusive: While both also favour the restricted model in the case of Italy; for the US, the UK and Spain information criteria are smaller for the unrestricted model and the likelihood ratio test rejects the null of the validity of the restriction. In the case of the UK this might be due to the non-normality of the residuals which violate a condition for a valid likelihood ratio test. Judging from the very tight fit of the model for the US, it might be the case that the unrestricted state-space model assigns too much of the variability in the data to the coefficients of the model, leading to the implausible estimate of the NAIRU. Finally, in the

¹¹See Hamilton (1994).

Country	Wald Test		Likelihood Ratio Test		
	$\chi^2(1)$	Prob.	$\chi^2(1)$	Prob.	
US	23.393	0.000	114.742	0.001	
UK	2.053	0.152	40.155	0.000	
Italy	0.287	0.592	0.328	0.567	
Spain	3.563	0.059	8.660	0.003	

Table 2: Comparing Unrestricted and Restricted Models of the HybridNKPC

Source: OECD, EC Consumer Survey data and SCA data, own calculations.

case of Spain, estimates of the NAIRU from both models are very close so that the restriction might not be necessary.

5 Conclusion

Most models of the Phillips curve assume that there is no long-run tradeoff between inflation and unemployment or output due to rational expectations of agents and that the long-run Phillips curve is hence vertical at the NAIRU. Pioneered by Friedman (1968) and Phelps (1968), this concept is by now well accepted and embodied in the most commonly used model of the Phillips curve, the hybrid New Keynesian Phillips curve (NKPC) derived by Galí and Gertler (1999). Introducing additional rigidities such as information stickiness (Mankiw and Reis, 2001, 2002a,b) yields a much slower adjustment process of expectations, more inertia in inflation and, thus, a longer-lived trade-off between inflation and unemployment.

We estimated the shifts in the NAIRU as an unobserved component in a state-space model of the hybrid NKPC, combining approaches of Gordon (1997) and Harvey (2008). Using direct survey data for inflation expectations from the University of Michigan's Surveys of Consumers and the EC Consumer Survey to avoid the problems of weak instrument bias often encountered in standard GMM approaches, the model was estimated for the US, the UK, Italy and Spain. Both the models for the US and the UK showed a significant short-run trade-off between inflation and output or unemployment, whereas in the case of Italy and Spain, we found a significantly positive coefficient. Nevertheless, in the first part of the estimation period, a Phillips curve relation between inflation and unemployment is also visible in the latter two countries. As expected, coefficients on lagged and expected inflation sum closely to one in all the countries and the restriction $\alpha + (1 - \alpha) = \alpha + \beta = 1$ could not be rejected except in the model for the US.

The Kalman-filtered smoothed estimates of the time-varying NAIRU all showed considerable variation over time, usually in line with variation in unemployment rates. Comparing estimates from an unrestricted and a restricted hybrid NKPC model, estimates from the restricted model generally gave more plausible values and the restriction could not be rejected except for the US. However, likelihood ratio tests preferred the unrestricted model for the US, the UK and Spain. Furthermore, all estimates of the time-varying NAIRU differed significantly from steady-state measures of unemployment calculated from the HP-filter, implying that a theory-based systems' approach yields important new information.

It is thus suggested for all countries investigated here that the NAIRU has shifted considerably with the business cycle and economic shocks during the estimation period, with actual unemployment rates fluctuating around it. This has important implications for monetary policy, since inflation targeting and stabilisation will be the more accurate, the better the knowledge of the NAIRU at any given point in time. Still further questions remain for future research: What is the direction of causality between changes in unemployment and changes in the NAIRU - is it unemployment that continually adjusts to changing potential unemployment or is the opposite the case? And how do changes in the NAIRU feed back into unemployment and inflation?

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6 Appendix

E	I_0 : The variable has a	unit root. E	xogenous: cons	stant
Country	Variable	t-adf stat.	Prob. value ¹	Lag length
US	π	-1.930	0.318	8
	$\Delta(\pi)$	-6.965	0.000	7
	$E_t(\pi_{t+1})$	-2.334	0.162	5
	$\Delta(E_t(\pi_{t+1})$	-5.049	0.000	5
_	u	-3.272	0.018	1
UK	π	-1.485	0.536	5
	$\Delta(\pi)$	-8.599	0.000	0
	$E_t(\pi_{t+1})_arma$	-2.025	0.276	5
	$\Delta(E_t(\pi_{t+1})_arma)$	-3.037	0.036	2
	u	-2.270	0.184	5
	Δu	-3.523	0.010	0
Italy	π	-0.795	0.815	5
	$\Delta(\pi)$	-6.103	0.000	3
	$E_t(\pi_{t+1})_arma$	-1.364	0.595	7
	$\Delta(E_t(\pi_{t+1})_arma)$	-3.285	0.019	5
	u	-0.920	0.777	5
	Δu	-2.856	0.055	$\frac{2}{5}$
Spain	π	-1.207	0.668	
	$\Delta(\pi)$	-9.215	0.000	0
	$E_t(\pi_{t+1})_arma$	-2.065	0.259	5
	$\Delta(E_t(\pi_{t+1})_arma)$	-4.621	0.000	2
		-1.477	0.540	5
	Δu	-3.594	0.008	0

Table A1: ADF tests for unit roots

¹ MacKinnon (1996) one-sided p-values.

Source: OECD, EC Consumer Survey and SCA data, own estimations.

Observation equation:	$\pi_t = \alpha \pi_t$	$-1 + \beta \pi_t^e + \gamma ($	$(u_t - u_t^N) + e$	e_t	
State equation:	$u_t^N = u_{t-1}^N + v_t$				
	Co efficient	Std. Error	z-Statistic	Prob.	
α	0.745630	0.019850	3.756.404	0.0000	
β	0.387542	0.031542	1.228.658	0.0000	
γ	-0.150547	0.029268	-5.143.814	0.0000	
	Final State	$Root\ MSE$	$z ext{-}Statistic$	Prob.	
u_t^N	-0.811555	1.136.853	-0.713861	0.4753	
No. of observations	185				
Log likelihood	-1.231.620				
No. of iterations	23				
Akaike info criterion	1.363.914				
Schwarz criterion	1.416.136				
Hannan-Quinn criterion	1.385.078				
Anderson-Darling test					
for normality of the resid	uals	0.604	Prob.	0.115	
ADF test on residuals		-7.825	Prob.	0.000	
ADF test on u_t^N		-3.353	Prob.	0.008	

Table A2: Results of the unrestricted state-space model for the US

Source: OECD and SCA data, own estimations.

Observation equation:	$\pi_t = \alpha \pi_t$	$_{-1} + (1 - \alpha)\pi$	$\frac{e}{t} + \gamma \left(u_t - u_t \right)$	$u_t^N + e_t$
State equation:	$u_t^N = u_{t-}^N$			
	Coefficient	Std. Error	z-Statistic	Prob.
α	0.729131	0.027256	2.675.116	0.0000
γ	-0.281417	0.037613	-7.482.014	0.0000
	Final State	Root MSE	z-Statistic	Prob.
u_t^N	3.118.526	0.870759	3.581.388	0.0003
No. of observations	185			
Log likelihood	-1.288.991			
No. of iterations	17			
Akaike info criterion	1.415.126			
Schwarz criterion	1.449.940			
Hannan-Quinn criterion	1.429.235			
Anderson-Darling test				
for normality of the resid	uals	0.706	Prob.	0.065
ADF test on residuals		-5.333	Prob.	0.000
ADF test on u_t^N		-1.349	Prob.	0.606

Table A3: Results of the restricted state-space model for the US

Source: OECD and SCA data, own estimations.

Observation equation:	$\pi_t = \alpha \pi_t$	$-1 + \beta \pi_t^e + \gamma \theta$	$(u_t - u_t^N) + \epsilon$	e_t	
State equation:	$u_t^N = u_{t-1}^N + v_t$				
	Co efficient	Std. Error	z-Statistic	Prob.	
α	0.806362	0.027987	2.881.187	0.0000	
β	0.264229	0.040499	6.524.410	0.0000	
γ	-0.058089	0.029610	-1.961.797	0.0498	
	Final State	Root MSE	z-Statistic	Prob.	
u_t^N	5.542.817	1.783.607	3.107.646	0.0019	
No. of observations	83				
Log likelihood	-5.954.409				
No. of iterations	48				
Akaike info criterion	1.507.086				
Schwarz criterion	1.594.514				
Hannan-Quinn criterion	1.542.210				
Anderson-Darling test					
for normality of the resid	uals	1.457	Prob.	0.001	
ADF test on residuals		-8.427	Prob.	0.000	
ADF test on u_t^N		-1.947	Prob.	0.309	

Table A4: Results of the unrestricted state-space model for the UK

Observation equation:	$\pi_t = \alpha \pi_t$	$_{-1} + (1 - \alpha)\pi$	$r_t^e + \gamma (u_t - u_t)$	$\binom{N}{t} + e_t$	
State equation:	$u_t^N = u_{t-1}^N + v_t$				
	Coefficient	Std. Error	z-Statistic	Prob.	
α	0.407533	0.081924	4.974.512	0.0000	
γ	-0.897442	0.042933	-2.090.341	0.0000	
	Final State	Root MSE	z-Statistic	Prob.	
u_t^N	5.036.696	0.488386	1.031.295	0.0000	
No. of observations	83				
Log likelihood	-7.962.151				
No. of iterations	27				
Akaike info criterion	1.966.783				
Schwarz criterion	2.025.069				
Hannan-Quinn criterion	1.990.199				
Anderson-Darling test					
for normality of the resid	uals	1.766	Prob.	0.000	
ADF test on residuals		-20.387	Prob.	0.000	
ADF test on u_t^N		-1.886	Prob.	0.337	

Table A5: Results of the restricted state-space model for the UK

Observation equation:	Observation equation: $\pi_t = \alpha \pi_{t-1} + \beta \pi_t^e + \gamma (u_t - u_t^N) + e_t$			
State equation:	$u_t^N = u_{t-1}^N$	$_{1} + v_{t}$		
	Co efficient	Std. Error	z-Statistic	Prob.
α	0.828536	0.085636	9.675.086	0.0000
eta	0.208977	0.078227	2.671.404	0.0076
γ	0.161354	0.056595	2.851.042	0.0044
	Final State	Root MSE	z-Statistic	Prob.
u_t^N	6.233.446	0.943550	6.606.376	0.0000
No. of observations	83			
Log likelihood	-2.124.911			
No. of iterations	47			
Akaike info criterion	0.584316			
Schwarz criterion	0.671744			
Hannan-Quinn criterion	0.619440			
Anderson-Darling test				
for normality of the residuals		0.618	Prob.	0.104
ADF test on residuals		-8.385	Prob.	0.000
ADF test on u_t^N		-0.115	Prob.	0.944

Table A6: Results of the unrestricted state-space model for Italy

Observation equation:	$\pi_t = \alpha \pi_t$	$-1 + (1 - \alpha)\pi$	$\frac{e}{t} + \gamma (u_t - u_t)$	$\binom{N}{t} + e_t$	
State equation:	$u_t^N = u_{t-1}^N + v_t$				
	Coefficient	Std. Error	z-Statistic	Prob.	
α	0.814319	0.071115	1.145.080	0.0000	
γ	0.151379	0.057554	2.630.211	0.0085	
	Final State	Root MSE	z-Statistic	Prob.	
u_t^N	5.897.647	0.970336	6.077.943	0.0000	
No. of observations	83				
Log likelihood	-2.141.292				
No. of iterations	30				
Akaike info criterion	0.564167				
Schwarz criterion	0.622452				
Hannan-Quinn criterion	0.587582				
Anderson-Darling test					
for normality of the resid	uals	0.544	Prob.	0.157	
ADF test on residuals		-4.471	Prob.	0.000	
ADF test on u_t^N		-0.767	Prob.	0.822	

Table A7: Results of the restricted state-space model for Italy

Observation equation:	$\pi_t = \alpha \pi_t$	$-1 + \beta \pi_t^e + \gamma \theta$	$(u_t - u_t^N) + \epsilon$	2t
State equation:	$u_t^N = u_{t-1}^N + v_t$			
	Coefficient	Std. Error	z-Statistic	Prob.
lpha	0.334502	0.060608	5.519.084	0.0000
β	0.910038	0.121401	7.496.107	0.0000
γ	0.683568	0.078028	8.760.543	0.0000
	Final State	Root MSE	z-Statistic	Prob.
u_t^N	7.889.068	0.526501	1.498.395	0.0000
No. of observations	73			
Log likelihood	-8.976.147			
No. of iterations	19			
Akaike info criterion	2.541.410			
Schwarz criterion	2.635.538			
Hannan-Quinn criterion	2.578.922			
Anderson-Darling test				
for normality of the residuals		0.655	Prob.	0.084
ADF test on residuals		-17.953	Prob.	0.000
ADF test on u_t^N		-0.609	Prob.	0.861

Table A8: Results of the unrestricted state-space model for Spain

Observation equation:	$\pi_t = \alpha \pi_t$	$_{-1} + (1 - \alpha)\pi$	$r_t^e + \gamma (u_t - u_t)$	$\binom{N}{t} + e_t$	
State equation:	$u_t^N = u_{t-1}^N + v_t$				
	Coefficient	Std. Error	z-Statistic	Prob.	
α	0.217899	0.061100	3.566.259	0.0004	
γ	0.903304	0.074232	1.216.867	0.0000	
	Final State	Root MSE	z-Statistic	Prob.	
u_t^N	7.231.519	0.481385	1.502.230	0.0000	
No. of observations	73				
Log likelihood	-9.409.141				
No. of iterations	21				
Akaike info criterion	2.632.641				
Schwarz criterion	2.695.394				
Hannan-Quinn criterion	2.657.649				
Anderson-Darling test					
for normality of the resid	uals	0.643	Prob.	0.090	
ADF test on residuals		-6.354	Prob.	0.000	
ADF test on u_t^N		-1.003	Prob.	0.748	

Table A9: Results of the restricted state-space model for Spain