

# Real Exchange Rates and Time-Varying Trade Costs

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## Abstract

In a recent paper Jacks et al. (2008) advocate a micro-founded measure for bilateral trade costs. We employ this measure and extend a nonlinear model for the real exchange rate so as to re-examine the Purchasing Power Parity hypothesis in the presence of time-varying trade costs. Using data for the dollar-sterling real exchange rate from 1830 to 2005, we provide significant evidence in favor of a positive relation between the level of trade costs and the degree of persistence of the real exchange rate.

## 1 Introduction

Trade costs can exhibit significant economic magnitudes and can play an essential role in addressing several major puzzles in international economics (Obstfeld and Rogoff, 2000; Anderson and van Wincoop, 2004). In the Purchasing Power Parity (PPP) framework, equilibrium models of real exchange rate determination demonstrate how trade costs induce nonlinear but mean reverting adjustment toward PPP and, hence, provide a possible explanation for the well-documented persistence in the real exchange rate (Dumas, 1992; O'Connell and Wei, 2002; Taylor and Taylor, 2004). For example, O'Connell and Wei (2002) extend the iceberg model of trade to allow for fixed as well as proportional costs of arbitrage. As a consequence, the tendency of the real exchange rate to return to the equilibrium rate will become apparent only for misalignments which cover the level of transactions costs and imply arbitrage opportunities. Small misalignments, close to equilibrium and within the transactions band, will be left uncorrected so that the real exchange rate will exhibit near unit root behavior.

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In a number of empirical contributions trade costs are assumed constant and the implied type of nonlinear behavior of the real exchange rate is modeled by the Exponential Smooth Transition Autoregressive (ESTAR) model (see, e.g., Michael et al., 1997; Kilian and Taylor, 2003; Taylor, Peel and Sarno, 2001). However, it can be argued that this assumption is too restrictive over long time periods.<sup>1</sup> In a recent contribution, inspired by the gravity literature, Jacks et al. (2008) present an aggregate micro-founded model which allows the construction of long span trade costs series. The authors illustrate that trade costs related to the exchange of goods across countries, far from being constant, have exhibited substantial and nonmonotonic changes from 1870 to 2000.<sup>2</sup> This finding has potentially important implications concerning the behavior of the real exchange rate. Because trade costs vary in time so does the speed of mean reversion for a given PPP deviation (see, e.g., Dumas, 1992; Sercu et al., 1995). Intuitively, when trade costs increase (decrease) the trade costs band—in which no trade takes place—widens (narrows) and the real exchange rate process becomes more (less) persistent. Hence, the persistence of the real exchange rate does not only depend on the size of the deviation but also on the level of trade costs at each particular point in time. Neglecting significant changes in trade costs leads to underestimating/overestimating the degree of persistence and the time required for the process to absorb shocks at specific periods.

The contribution of this paper is to report estimates and the properties of a smooth transition regression model of the real exchange rate which incorporates time-varying trade costs. The model is fitted to a long span of data (1830-2005) for the dollar-sterling real exchange rate and the trade costs index for the United Kingdom-United States country pair. Our choice is based on the fact that the relationship between trade frictions and the persistence of the real exchange rate should become apparent over long time periods in which large fluctuations of trade costs occur.

The rest of the paper is structured as follows. In Section 2, we present the trade costs measure of Jacks et al. (2008). Section outlines our nonlinear model of the real exchange rate. Section 4 deals with the description of the data and the empirical results. A summary and concluding comments are offered in the last section.

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<sup>1</sup>Clemens and Williamson (2001) and Mohammed and Williamson (2004) among others illustrate that tariffs and global freight rates have fluctuated substantially in the last century. These studies focus on specific impediments of trade costs and, therefore, provide indirect evidence of time-varying trade costs. A survey on recent developments in the measurement of total trade costs and their components is provided by Anderson and van Wincoop (2004).

<sup>2</sup>Consequently, the effect of trade costs cannot be approximated by deterministic trends.

## 2 Trade Costs

“Trade costs, broadly defined, include all costs incurred in getting a good to a final user other than the marginal cost of producing the good itself ” (Anderson and van Wincoop, 2004, p. 691). Obviously, trade costs break down into a vast number of components such as transportation costs (freight rates and time costs), policy barriers (tariffs and nontariff barriers), informational costs and costs associated with the use of different currencies. The fact that several of these components are unobservable and data limitations pose serious problems in obtaining accurate estimates of the magnitude of total trade costs by direct atheoretical measures. The gravity literature circumvents this obstacle on the basis of theoretical models which enable measuring the degree of trade restrictiveness by extracting information from trade flows.

In this framework, Jacks et al. (2008) present a micro-founded measure of aggregate bilateral trade costs that captures trade frictions. The key idea in the derivation of their measure is that changes in trade barriers have an effect on both international and intranational trade. By establishing a relationship between countries’ average international trade barriers and intranational trade, trade costs can be obtained directly from observable trade data without imposing a particular trade cost function (Novy, 2008).

Consider a world consisting of  $N$  countries and a continuum of differentiated goods. Anderson and van Wincoop (2003) derive the following gravity equation of international trade

$$x_{i,j} = \frac{y_i y_j}{y_w} \left( \frac{t_{i,j}}{\Pi_i P_j} \right)^{1-\sigma}, \quad (1)$$

where  $x_{i,j}$  are nominal exports from country  $i$  to  $j$ . Income levels of country  $i$ , country  $j$  and world income are denoted by  $y_i$ ,  $y_j$  and  $y_w$ , respectively. The elasticity of substitution,  $\sigma$ , is assumed to be constant and greater than unity. The cost of importing a good or, equivalently, the trade cost barrier (one plus the tariff equivalent) is  $t_{i,j} \geq 1$ . Finally, the price indices (or outward and inward multilateral resistance variables)  $\Pi_i$  and  $P_j$  for countries  $i$  and  $j$  represent the average trade restrictiveness of the countries. Novy (2008) uses Equation (1) to obtain a bidirectional gravity equation, which includes inward and outward multilateral resistance variables for both countries,

$$x_{i,j} x_{j,i} = \left( \frac{y_i y_j}{y_w} \right)^2 \left( \frac{t_{i,j} t_{j,i}}{\Pi_i P_j \Pi_j P_i} \right)^{1-\sigma}. \quad (2)$$

In turn, the author makes use of the fact that intranational trade, like international trade, depends on the magnitude of trade barriers,  $x_{i,i} = ((y_i y_i)/y_w)(t_{i,i})/(\Pi_i P_i)^{1-\sigma}$ , so as to control for multilateral

resistance. Substituting into the bidirectional gravity equation yields

$$x_{i,j}x_{j,i} = x_{i,i}x_{j,j} \left( \frac{t_{i,j}t_{j,i}}{t_{i,i}t_{j,j}} \right)^{1-\sigma}. \quad (3)$$

The geometric average of the tariff equivalent can now be obtained by

$$\tau \equiv \left( \frac{t_{i,j}t_{j,i}}{t_{i,i}t_{j,j}} \right)^{\frac{1}{2}} - 1 = \left( \frac{x_{i,i}x_{j,j}}{x_{i,j}x_{j,i}} \right)^{\frac{1}{2(\sigma-1)}} - 1. \quad (4)$$

The above equation states that a drop in trade flows between countries with respect to trade flows within countries is associated with higher trade costs. Note that the micro-founded measure evaluates bilateral trade costs against the domestic trade cost benchmark. Further, it enables the construction of long span trade costs series since its estimation only requires data for bilateral exports and intranational trade. The latter variable can be approximated by subtracting aggregate exports from a country's Gross Domestic Product (GDP) (Jacks et al., 2008).

### 3 Nonlinear Adjustment & Time-Varying Trade Costs

A nonlinear model that captures the theoretical insights of the authors above and allows us to parsimoniously encompass the influence of fixed and proportional time-varying trade costs is the Quadratic Logistic Smooth Transition Autoregressive (QLSTAR) model of Jansen and Teräsvirta (1996). The main advantage of the QLSSTAR model over the widely used ESTAR model is that it can approximate a trade costs band more closely and, at the same time, it can preserve all the other properties of the ESTAR model.<sup>3</sup>

Let us define the log real exchange rate as  $q_t = s_t - p_t + p_t^*$ , where  $s_t$  is the logarithm of the spot exchange rate (the domestic price of foreign currency),  $p_t$  is the logarithm of the domestic price level and  $p_t^*$  the logarithm of the foreign price level. A STAR model for the process  $\{q_t\}$  may

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<sup>3</sup>The appealing feature of the QLSSTAR model, as with the ESTAR model, is that it allows transitions between a continuum of regimes to occur smoothly and symmetrically rather than discretely as would occur with an explicit threshold model. This property is suggested by the analysis of Dumas (1992). It also captures the conjecture of Teräsvirta (1994) who argues that the aggregate nonlinear process may be smooth rather than discrete as long as heterogeneous agents do not act simultaneously even if they individually make dichotomous decisions. Berka (2005) provides a theoretical justification of this conjecture. We did estimate the ESTAR model with constant transactions costs and also a modified version which incorporates time-varying transactions costs in the speed of adjustment. The estimates of these models suggest a statistically significant relationship between trade restrictiveness and the persistence of the real exchange rate in line with the theoretical analysis. However, the QLSSTAR model provided a more parsimonious fit. Results for the ESTAR models are available upon request.

be written as

$$q_t - \mu = \sum_{p=1}^{\bar{p}} \phi_p (q_{t-p} - \mu) G(\cdot) + \epsilon_t, \quad (5)$$

where  $\mu$  is a constant representing the long run equilibrium,  $\epsilon_t$  is a white noise process with mean 0 and variance  $\sigma_\epsilon$ , and  $G(\cdot)$  is the transition function. For a given AR structure,  $\sum_{p=1}^{\bar{p}} \phi_p$ , the transition function,  $G(\cdot)$ , specifies the degree of persistence of the real exchange rate at each point in time. In the case of the Time-Varying Trade Costs QLSTAR (TVTC-QLSTAR) model<sup>4</sup>

$$G(q_{t-d}, \tau_{t-d}) = \left[ 1 - \left( 1 + \exp \left( - \frac{\gamma^2}{(c + c_\tau \tau_{t-d})^2} (q_{t-d} + c_1)(q_{t-d} + c_2) \right) \right)^{-1} \right] + \epsilon_t, \quad (6)$$

where  $c_1 = \mu - c - c_\tau \tau_{t-d}$  and  $c_2 = \mu + c + c_\tau \tau_{t-d}$  with  $c_1 < c_2$  are the time-varying band coefficients,  $c$  is a positive constant,  $c_\tau$  is the coefficient on trade costs  $\tau$ , and  $q_{t-d}$  denotes the transition variable with the delay parameter  $d$  being a positive integer.<sup>5</sup> Controlling for  $\gamma$ , the speed of mean reversion decreases with the absolute value of the band coefficients  $c_1$  and  $c_2$ , and increases with the past deviation from the equilibrium rate (this point is illustrated in the following section and in the Appendix). In this setting, the null hypothesis that changes in trade costs have no effect on the degree of persistence of PPP deviations is  $H_0 : c_\tau = 0$ . While the alternative hypothesis is  $H_0 : c_\tau > 0$ . We examine the impact of trade costs on the speed of mean reversion of the real exchange rate in the next section.

## 4 Empirical Results

Our data set consists of annual observations for the dollar-sterling real exchange rate and the corresponding trade costs index from 1830 to 2005. For the construction of the real exchange rate we use the International Financial Statistics database to update the nominal exchange rate and the price indexes analyzed in Lothian and Taylor (1996). International trade data are obtained by Mitchell (2008b,a) and GDP series for the United States and the United Kingdom are taken from Officer (2008) and Johnston and Williamson (2008), respectively. Figures 1 and 2 show the demeaned real exchange rate and the trade costs series, respectively. In line with Jacks et al. (2008), the latter exhibits significant fluctuations throughout the period. Specifically, until the beginning of

<sup>4</sup>Equation (6) is an extension of the quadratic logistic function proposed by Jansen and Teräsvirta (1996). A thorough analysis of the extended QLSTAR model is provided in the Appendix.

<sup>5</sup>We have scaled the trade costs index so as to have a minimum value of zero. Consequently,  $c$  reflects the lowest level of trade costs in time.

the 20th century trade costs were relatively low. Subsequently, the war and interwar periods were associated with a remarkable increase of bilateral trade costs with respect to intranational domestic costs. During this time interval the series displays two peaks, the first in 1935 following the Great Depression, and the second in 1946 at the end of the second World War and the establishment of the Bretton Woods system. A gradual decline has occurred since then.

After running a battery of linearity tests on the real exchange rate series, which indicate the presence of smooth transition nonlinearity, we examine whether trade costs are an important constituent of the nonlinear adjustment mechanism of the real exchange rate.<sup>6</sup> The results for the nonlinear models with constant and time-varying trade costs are reported in Table 1.<sup>7</sup> Overall, both models provide a parsimonious fit to the real exchange rate. However, the incorporation of time-varying trade costs leads to a radically different adjustment process. The statistical significance of the band coefficient  $\hat{c}_\tau$  of the TVTC-QLSTAR model indicates that movements in trade costs can help explain changes in the level of persistence of the real exchange rate.<sup>8</sup> An increase in trade costs widens the “band of inaction” and reduces the speed of mean reversion for a given PPP deviation. Figure 3 shows the transition function of the TVTC-QLSTAR model for three representative time periods, namely 1900, 1950 and 2000, which correspond to relatively low, large and moderate levels of trade costs, respectively. At those time periods, a PPP deviation of 0.4, which is roughly the maximum realized deviation, would suggest that the real exchange rate behaves similar to a white noise, a near unit root and an AR process with coefficient around 0.2. While according

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<sup>6</sup>Specifically, we employ the testing procedures proposed by Teräsvirta (1994), Harvey and Leybourne (2007), and Kapetanios et al. (2003). The first two are general procedures for testing linearity against smooth transition nonlinearity. The main difference between them lies in the fact that the null critical values for the test of Teräsvirta (1994) are based on the assumption of an  $I(0)$  process, whilst, the test of Harvey and Leybourne (2007) allows for both  $I(0)$  and  $I(1)$  processes. We find that the hypothesis of linearity can be rejected at the 5 and 10 percent significance levels, respectively. Finally, the test of Kapetanios et al. (2003) shows that the null hypothesis of a unit root in the real exchange rate against the alternative hypothesis of a globally stationary exponential smooth transition autoregressive process can be rejected at all conventional levels of significance. All results are available upon request in an extended version of the paper.

<sup>7</sup>The models are fitted to the demeaned real exchange rate. The lag length of the autoregressive part and the variables which enter the transition function are specified on the basis of residual diagnostics and, subsequently, the statistical significance of the coefficients of the models. In the estimation procedure we impose the restriction  $\phi_1 = 1$ . This choice is based on the fact that the AR coefficient is not statistically different from unity in the estimated ESTAR models with constant and time-varying trade costs and in the TVTC-QLSTAR model. Further, the results for the unrestricted models are qualitatively the same. For the standard QLSTAR model imposing the restriction  $\phi_1 = 1$  allows convergence of the nonlinear least squares algorithm. Note that this restriction does not necessarily imply a unit root behavior of  $\{q_t\}$  in the inner regime when QLSTAR models are applied since the maximum value of the transition function may differ from unity.

<sup>8</sup>Paya and Peel (2006) emphasize that the high degree of persistence of both the dependent and explanatory variables (such as the trade costs series) that enter the transition function may give rise to a spurious regression problem. To this end, we report the bootstrap  $p$ -value for the coefficient on trade costs. The null Data Generating Process in the simulation experiment is given by the fitted QLSTAR model.

to the QLSTAR model with constant trade costs the real exchange rate would behave as an AR process with coefficient of about 0.5.

A natural question that arises in the nonlinear framework under examination is how fast does the process adjust to such a large deviation. We employ the fitted models so as to simulate data and compute the half-life of a shock which drives the process from its equilibrium value to the maximum realized PPP deviation.<sup>9</sup> Starting with the constant trade costs model, the real exchange rate process would absorb half of the shock in four years. Turning to the TVTC-QLSTAR model, we consider three scenarios. As above, we set trade costs equal to their 1900, 1950 and 2000 levels. In the former and latter cases, the time required for the process to absorb half of the maximum PPP deviation is only two years, which is half of that corresponding to constant trade costs. Obviously, large deviations of the real exchange rate appear to mean revert much faster during the beginning of the 20th century and the recent floating period. On the contrary, the high level of trade costs around the middle of the 20th century leads to a one year increase in the half-life of the shock with respect to the constant trade costs benchmark.<sup>10</sup>

Clearly, the assumption of constant trade costs can result in severe overestimation / underestimation of persistence. Figure 4 depicts the difference between the degrees of persistence estimated by the TVTC-QLSTAR and QLSTAR models.<sup>11</sup> A broad conclusion that emerges is that the exclusion of time-varying trade costs results more frequently in underestimating rather than overestimating the degree of persistence of the process. Hence, it appears that most of the time the real exchange rate series is more persistent than what nonlinear models with constant trade costs suggest. On the other hand, overestimation occurs on rare occasions, such as 1986, which are usually associated with substantially larger differences in the speed of mean reversion. This implies much faster mean reversion of the dollar-sterling real exchange rate toward its equilibrium value.

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<sup>9</sup>Half-lives for the nonlinear models are computed by using a procedure similar to the Generalized Impulse Response Function proposed by Koop et al. (1996). Specifically, we employ the estimated coefficients and bootstrap from the residuals so as to generate one thousand artificial real exchange rate series. The initial values of these series are set equal to zero and the size of the shock equal to the maximum PPP deviation of the model under consideration. Next, we average the generated series at each horizon and compute the horizon at which the resultant series crosses the threshold of half the shock.

<sup>10</sup>The half-life of the shock corresponding to the maximum level of trade costs, which occurs in 1946, is twelve years.

<sup>11</sup>The degree of persistence in this case is measured by the value of the transition function of the corresponding model.

## 5 Conclusion

The insights of Jacks et al. (2008), motivated by the recent gravity literature, show how to compute a long span trade costs series. The authors illustrate that trade costs related to the exchange of goods across countries have exhibited substantial and non-monotonic changes from 1870 to 2000. In empirical work on the dynamic behavior of the real exchange rates trade costs have typically been assumed constant even though trade costs play a key role in theoretical models in determining real exchange rate dynamics. Essentially, arbitrage will commence, *ceteris paribus*, when it is profitable and PPP deviations are outside the transactions band. We develop and estimate a nonlinear model for the real exchange rate which incorporates time-varying trade costs and is a generalization of the ESTAR nonlinear model employed in previous analysis, where trade costs were assumed constant. Our empirical approach is supported by a battery of statistical tests and simulation methods. Our results provide strong evidence in favor of a time-varying “band of inaction”, which widens with the level of trade costs. The persistence of the real exchange rate is found to depend on both the magnitude of trade frictions and the size of the deviation from PPP. For instance, a given shock to the real exchange rate would be absorbed at significantly different speeds in 1950 and 2000 due to the existence of different trade costs levels. Although trade costs appear to have declined substantially since the second World War, their magnitude is still significant. Consequently, our empirical results are also consistent with the documented high persistence of real exchange rates in the post Bretton Woods era.

## Appendix

### The QLSTAR model

A QLSTAR model for the process  $\{q_t\}$  may be written as

$$q_t - \mu = \sum_{p=1}^{\bar{p}} \phi_p (q_{t-p} - \mu) G(\cdot) + \epsilon_t, \quad (7)$$

where  $\mu$  is a constant representing the long run equilibrium,  $\epsilon_t$  is a white noise process with mean 0 and variance  $\sigma_\epsilon$ , and  $G(\cdot)$  is the transition function. For expositional reasons, we assume that  $\sum_{p=1}^{\bar{p}} \phi_p = 1$  in Equation (7). Jansen and Teräsvirta (1996) propose the following quadratic logis-



tic function

$$G_{QL}^*(q_{t-d}) = 1 - \left(1 + \exp\left(-\gamma^2(q_{t-d} + c_1)(q_{t-d} + c_2)\right)\right)^{-1}, \quad (8)$$

where  $c_1 = \mu - c$  and  $c_2 = \mu + c$  with  $c > 0$  are the band coefficients,  $q_{t-d}$  is the transition variable and  $\gamma$  is the smoothness (or transition) parameter. The quadratic logistic transition function  $G_{QL}^*(\cdot)$  is particularly applicable because it implies symmetric adjustment for positive and negative deviations from the equilibrium. Further, the QLSTAR model specified by Equation (8) can approximate ESTAR models but also nests three regime Threshold Autoregressive (TAR) models and linear AR models. In contrast to TAR and ESTAR models, the QLSTAR allows the type of adjustment (smooth or discrete) between regimes to be specified by the data and, at the same time, can approximate narrow and wide “bands of inaction”. Hence, the model allows for both fixed and proportional costs. Overall, the model is particularly applicable when one is agnostic about the range of the “band of inaction” and the type of transition.

Suppose that regime changes occur abruptly rather than gradually (see Sercu et al., 1995), which favors the use of TAR over ESTAR models. If  $\gamma \rightarrow \infty$  and  $q_{t-d} < c_1$  or  $q_{t-d} > c_2$  the transition function value equals zero and  $q_t$  becomes white noise. Whilst, inside the “band of inaction”,  $c_1 < q_{t-d} < c_2$ ,  $G_{QL}(\cdot)$  equals one and  $q_t$  behaves as a unit root process. Note that an increase in trade costs will widen the “band of inaction” and, therefore, result in higher absolute values of the band coefficients,  $c_1$  and  $c_2$ . At the other extreme, when  $\gamma = 0$  the model becomes linear. For moderate values of  $\gamma$ , the QLSTAR model can approximate both ESTAR and TAR models. The speed of mean reversion increases with the deviation from the equilibrium  $q_{t-d} - \mu$ . If  $q_{t-d} - \mu \rightarrow \infty$  the process approaches the white noise regime (outer regime). Whilst, in the inner regime,  $q_{t-d} - \mu = 0$ , the degree of persistence is given by the maximum value of the transition function  $G_{QL}^*$

$$G_{QL}^*(\mu) = 1 - \left(1 + \exp\left(\gamma^2 c^2\right)\right)^{-1}, \quad (9)$$

which is determined by the transition parameter  $\gamma$  and the coefficient  $c$ . Consequently, changes in  $\gamma$  or  $c$  due to movements in trade costs lead to different degrees of persistence at the equilibrium. Due to the fact that there is no *a priori* reason why changes in trade costs should alter the degree of persistence in the inner regime, we modify Equation (8) as follows

$$G_{QL}(q_{t-d}) = 1 - \left(1 + \exp\left(-\frac{\gamma^2}{c^2}(q_{t-d} + c_1)(q_{t-d} + c_2)\right)\right)^{-1}. \quad (10)$$

The maximum value of  $G_{QL}(\cdot)$ , which again occurs at the equilibrium rate, is

$$G_{QL}(\mu) = 1 - (1 + \exp(\gamma^2))^{-1}, \quad (11)$$

and is independent of the value of the band coefficient. Note that dividing the smoothness parameter  $\gamma^2$  by  $c^2$  also implies that changes in the persistence of the process become more abrupt as  $c$  decreases (see Figure 3). This behavior is in line with the presence of both fixed and proportional costs which move together in time (O'Connell and Wei, 2002). The above modification enables the incorporation of time-varying trade costs in the QLSTAR model in a straightforward manner (see Equation (6)).

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Table 1: Estimated Nonlinear Models

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Panel A, QLSTAR

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$$q_t + \frac{0.014}{(0.656)} = (q_{t-1} + \frac{0.014}{(0.656)}) \left[ 1 - \left( 1 + \exp\left( \frac{-1.829^2}{(6.700)} / \frac{0.402^2}{(5.853)} (q_{t-1} - 0.387)(q_{t-1} + 0.416) \right) \right)^{-1} \right].$$

$s = 0.064$ ;  $Q_1 = 0.141$  [0.061];  $Q_5 = -0.126$  [0.219];  $\text{ARCH}_1 = 0.535$  [0.465];  
 $\text{ARCH}_5 = 0.786$  [0.561].

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Panel B, TVTC-QLSTAR

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$$q_t - \frac{0.059}{(4.064)} = (q_{t-1} - \frac{0.059}{(4.064)}) \left[ 1 - \left( 1 + \exp\left( \frac{-2.146^2}{(7.811)} / \left( \frac{0.172}{(6.929)} + \frac{0.587}{(4.488)} \tau_{t-2} \right)^2 \right. \right. \right. \\ \left. \left. \left. \times (q_{t-2} - 0.231 - \frac{0.587}{(4.488)} \tau_{t-2})(q_{t-2} + 0.1128 + \frac{0.587}{(4.488)} \tau_{t-2}) \right) \right)^{-1} \right].$$

$[0.008]$   $[0.008]$   $[0.008]$

$s = 0.063$ ;  $Q_1 = 0.020$  [0.787];  $Q_5 = -0.154$  [0.426];  $\text{ARCH}_1 = 0.667$  [0.411];  
 $\text{ARCH}_5 = 0.344$  [0.886].

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Notes: Figures in parentheses and square brackets denote absolute  $t$ -statistics and  $p$ -values, respectively. The  $p$ -value for the coefficient  $\hat{c}_\tau$  on trade costs is obtained through a simulation exercise, where the bootstrap DGP is the fitted QLSTAR model reported in Panel A. For illustration purposes, we report the summation of the long run equilibrium estimate and the constant part of the band coefficients  $\hat{\mu} \pm \hat{c}$ .  $s$  is the standard error of the regression.  $Q_1$  and  $Q_5$  denote the Ljung-Box  $Q$ -statistic for serial correlation up to order 1 and 5, respectively.  $\text{ARCH}_1$  and  $\text{ARCH}_5$  denote the LM test statistic for conditional heteroskedasticity up to order 1 and 5, respectively.

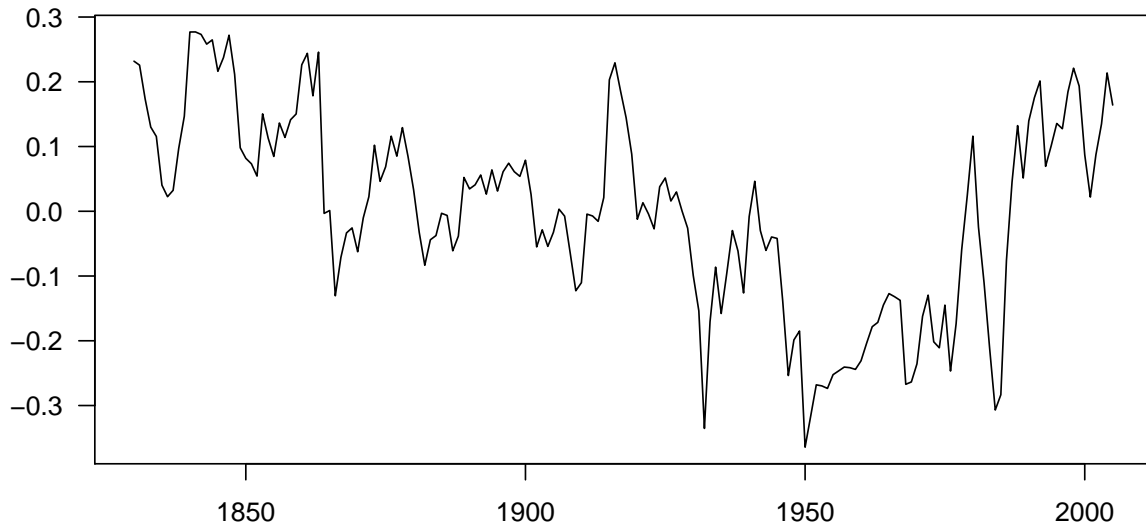


Figure 1: The dollar-sterling real exchange rate (demeaned).

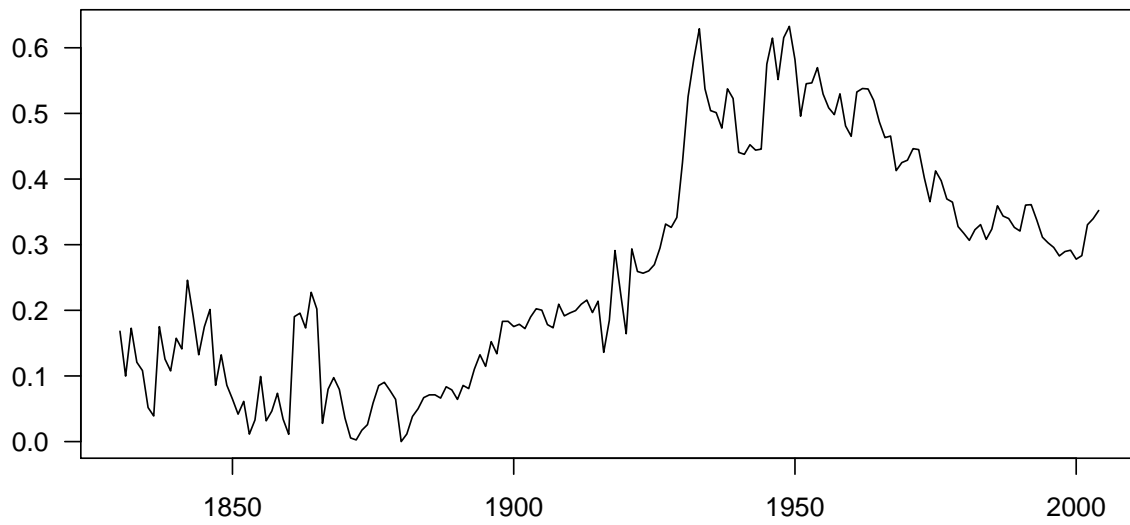


Figure 2: The United States-United Kingdom trade costs index.

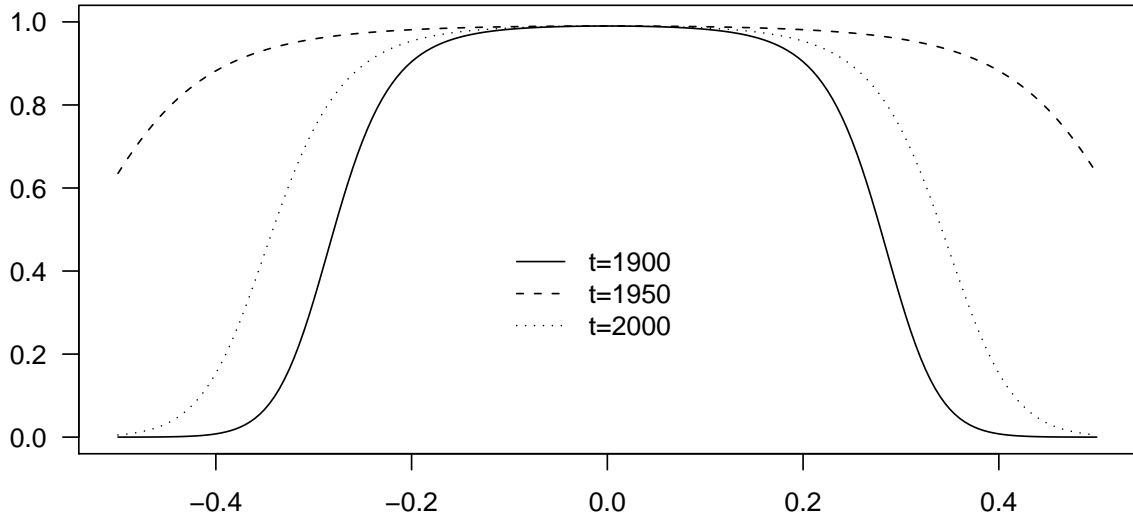


Figure 3: The quadratic logistic function for the estimated TVTC-QLSTAR model at 1900, 1950 and 2000.

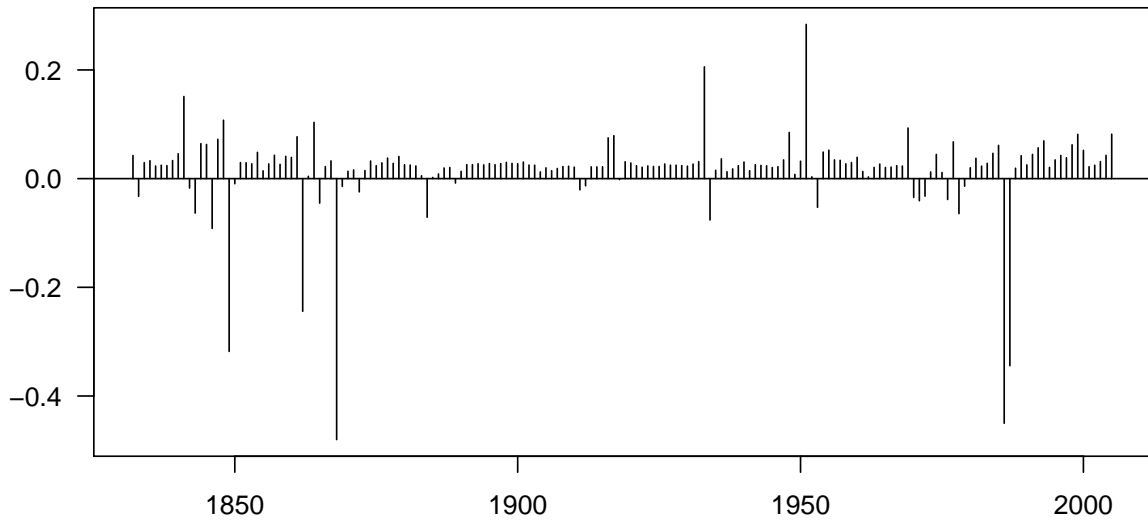


Figure 4: Difference in the degree of persistence between the TVTC-QLSTAR and QLSTAR models.