Gibrat's Law for Countries[1]

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Abstract A re-assessment of Gibrat's Law in the context of country size is carried out in this paper. In addition, how similarly population is distributed in cities and countries is analyzed from a temporal perspective. Although evidence of Gibrat's Law is found, it is weaker than that previously established in Rose (2006). This is due to the methodology applied and is especially appreciable in very small countries. Nonetheless, we observe that the population growth process in countries is similar to that of cities. As a result, the similarities between how the population is distributed in these two geographical categories have increased over time.

Keywords Gibrat's Law · Country Size.

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1 Introduction

Gibrat's Law - also known as the Law of Proportionate Growth (Gibrat, 1931) - establishes that the growth rate of a variable is independent of its initial size. Since its formulation, it has been the subject of a large number of empirical studies as to its validity in different contexts like financial returns, firms and city sizes. More interestingly, the observation of this empirical regularity for city size has motivated theoretical developments in regional and urban economics (Gabaix, 1999; Córdoba, 2008).

Rose (2006) has gone further and analyzed whether Gibrat's Law also holds in another context relevant to population issues: country size. Among other fields, this question is interesting for economic growth (Alesina et al., 2005). The related literature started with Malthusian Theory and evolved towards modern theoretical economic growth models (Ehrlich and Lui, 1997; Jaeger and Kuhle, 2009). An implication of country population growth consistent with Gibrat's Law is that per capita income growth differences across countries would only be explained by differences in labour productivity. Country population growth also has policy implications. An example is the Chinese "One Child Policy", introduced in 1979. Therefore, it is interesting to have a clear characterization of the evolution of country population growth.

The only existing analysis regarding the fulfilment of Gibrat's Law for country sizes (Rose, 2006) was carried out using both visual (scatter plots and histograms) and econometric (β-convergence regressions and normality tests) tools. Our paper tries to further contribute to this recently established strand in the literature by implementing alternative tests. This will be done by applying the most commonly used techniques in another demographic context where Gibrat's Law is relevant: city size.

On one hand, non-parametric tests will be implemented. First, kernel regression techniques that establish a functional form-free relationship between population growth and country size for the entire distribution will be used (Ioannides and Overman, 2003; Eeckhout, 2004). Second, transition matrices (Quah, 1993) will be estimated in order to obtain information about the degree of intra-distributional mobility. On the other, Clark and Stabler (1991) suggested that testing for Gibrat's Law is equivalent to testing for the presence of a unit root. This idea has also been emphasized by Gabaix and Ioannides (2004) who expect *"that the next generation of [city] evolution empirics could draw from the sophisticated econometric literature on unit roots"*. Given the structure of the data analyzed, the panel unit root test recently proposed in Pesaran (2007) will be applied. As well as controlling for the possible dependence among countries, it has nice size and power when dealing with a cross-sectional dimension greater than the temporal one, as is the case here.

A related regularity commonly known as Zipf's Law was also explored for country size by Rose[2]. It implies that, if country sizes are ordered from the largest to the smallest, the product between rank and population size is constant. These two complementary analyses (Zipf's and Gibrat's laws) led to the conclusion that country and city size distributions were similar even if the theories explaining city size distributions do not apply to country size distribution. This finding is quite surprising since urban structure models rely on the assumption of the free mobility of workers, which is less reliable for countries because international emigrants usually face transport costs as well as cultural and legal barriers.

As another contribution in the context of country population empirics, evidence regarding Zipf's Law will be studied from a temporal perspective. Moreover, and in line with the spirit of Rose's work, a parallel analysis for the city size distributions in the United States (US) and Italy has also been carried out. Formal statistical tests of the similarity between the size distributions studied in the paper will be reported.

Although the results obtained are mixed, evidence of an independent population growth with respect to initial size is found. Gibrat's Law does not always hold for very small units when using non-parametric kernel regressions. In addition, there is little favourable evidence of Gibrat's Law from the panel unit root tests. These conclusions apply for both countries and cities. Zipf's Law estimation results also show similarities between the size distributions of cities and countries in their upper tails. However, they differ when the whole distributions are considered. Even so, it can be concluded that similarities in the population growth processes of both cities and countries have led to more similar population distributions between these two geographical categories over time.

The paper is structured as follows. Section 2 details the data sources from which the information on the population of countries and cities has been obtained. Section 3 describes the non-parametric and parametric tests for Gibrat's Law applied to country size and presents the results. The corresponding parallel analysis for US and Italian cities is also carried out. Section 4, studies the fulfilment of Zipf's Law and statistically compares country and city size distributions from a temporal perspective. Section 5 summarizes the most relevant findings and concludes.

[2] Zipf's Law for country size has also been analyzed by Di Guilmi et al. (2003) and Furceri (2008) with the difference that country size was measured in terms of GDP per capita.

2 Data sources

Country population data have been extracted from the Penn World Table (PWT, hereafter) release 6.2. As all units included in this database are considered, the only definition of "country" that we use is if it appears as such in this database. The PWT contains balanced panel data for the population of 187 countries during the period 1950-2004. Therefore, the analysis of the world population growth process and its distribution presented here will mainly refer to the second half of the 20th century.

Data from two developed countries, the US and Italy, have been used in order to carry out a parallel analysis for the size distribution of cities. This will allow us to study two different urban structures. While the US is a relatively young country whose inhabitants are characterised by high mobility, Italy has a much older urban structure and its inhabitants present greater resistance to moving. The highest number of urban units will be considered when possible. In addition, the period for which the city analysis will be carried out is almost the same as that for countries.

The US data have been extracted from the Census Bureau and refer to the units labelled as "incorporated places". They include governmental units classified under the Statal Laws as cities, towns, boroughs or villages. Alaska, Hawaii and Puerto Rico have not been considered due to data limitations. The population in each urban unit is observed every ten years. The number of observations increases from 17,113 in 1950 to 19,296 in 2000.

The geographical unit for Italy is the municipality and the data have been obtained from the Italian National Statistical Office (Instituto Nazionale di Statistica / Servizio Biblioteca e Servizi all'utenza de la Direzione Centrale per la Diffusione della Cultura e dell'informazione Statistica). The number of units is 8,100 during the whole period and the population data is also observed on a 10-year basis starting in 1951.

3 Gibrat's Law and country size

As noted before, the only empirical assessment of Gibrat's Law for countries to date was carried out by Rose (2006). It mainly relies on graphical (scatter plots and histograms) and statistical methods (traditional β-convergence regressions and normality tests). Our paper tries to contribute to this recently established strand in the literature by applying the most commonly used techniques in another demographic context where Gibrat's Law is relevant: city size. A summary of the post-1990 empirical studies about the fulfilment of Gibrat's Law in city population growth is shown in Table 1. In line with the suggestion in Gabaix and Ioannides (2004), most studies now apply unit root tests. Probably because of their flexibility, the second most commonly used tests are based on non-parametric techniques. This is especially true for kernel regressions and the estimation of transition matrices.

[Insert Table 1 here]

So, the analysis presented in this section will be based on these three methodologies. The non-parametric tests will be implemented first, followed by the application of panel unit root tests. The fundamentals of each technique will be described as well as the results obtained from country population data. In addition, the relevant parallel comparison with the results obtained from the application of these methods to city size data will be provided. The reason is that one of the main ideas in Rose's work was the similarity between how population evolved and was distributed in cities and countries.

3.1 Non-parametric tests

3.1.1 Kernel regression

Description Following Ioannides and Overman (2003) and Eeckhout (2004), the logarithmic growth rate of a given country i (g_i) can be specified in a non-parametric way as:

$$g_i = m(s_i) + \varepsilon_i \tag{1}$$

Thus, this variable is expressed as a function $m(\cdot)$ of the natural logarithm of its relative size (s_i). The latter is defined as the ratio of the country size over the contemporary world sample average. Consequently, instead of assuming a linear relationship between these two variables, as in the conventional β-convergence regressions framework, $m(\cdot)$ is estimated as a local average. This is done using a kernel function $K(\cdot)$, assumed to be symmetric, weighted and continuous. ε_i is the error term.

Population growth rates have been calculated yearly over the entire sample period. As normalized rates have been considered, Gibrat's Law would be observed if the estimated mean is a straight line close to zero and its variance is around one. Deviations from these values imply rejections of this empirical regularity.

In order to estimate the non-parametric function $m(\cdot)$ in (1), the Nadayara-Watson estimator has been applied:

$$\hat{m}(s) = \frac{n^{-1}\sum_{i=1}^{n} K_h(s-s_i)\,g_i}{n^{-1}\sum_{i=1}^{n} K_h(s-s_i)} \tag{2}$$

n is the number of observations, and K_h reflects the dependence of the kernel function on the bandwidth (h). This parameter has been fixed to 0.5. The Epanechnikov kernel has been used.

As with the estimation of the mean in (2), the variance can be obtained as:

$$\hat{\sigma}^2(s) = \frac{n^{-1}\sum_{i=1}^{n} K_h(s-s_i)\,(g_i-\hat{m}(s))^2}{n^{-1}\sum_{i=1}^{n} K_h(s-s_i)} \tag{3}$$

Results for countries Kernel estimation results for country size are plotted in the two graphs on the left of Figure 1**.** All the available information for the sample period 1950-2004 has been pooled, totalling 10,098 observations. Bootstrapped 95% confidence bands obtained using 500 random samples with replacement are reported.

[Insert Figure 1 here]

It can be concluded from the top left graph in Figure 1 that the null hypothesis of mean population growth conditional on country relative size being zero cannot be rejected at a 5% significance level for the great majority of relative sizes. However, some contrary evidence is found for some values in the lower tail of the country size distribution. These findings are related to the fact that small countries such as Djibouti, Grenade, Vanuatu or Brunei have been included in the analysis. For them, a small change in population leads to a high population growth in percentage terms, making their estimated growth rate statistically different to the sample mean. Moreover, it should be noted that this estimated conditional mean has an inverted U-shape around zero in the middle of the relative size distribution. Therefore, the fulfilment of Gibrat's Law cannot be rejected once the lower tail of the country size distribution is neglected.

A differential behaviour of the smallest countries in the sample with respect to the estimated conditional variance is also observed in the bottom left graph of Figure 1.

The null hypothesis of this variance being equal to one is rejected for some relative sizes in the lower tail. As was also the case for the mean, evidence of Gibrat's Law is found as we move towards the right of the distribution. This function reaches a peak around the lower end and returns to a value close to one in the upper part of the distribution.

Therefore, it can be concluded that the application of kernel regressions leads us to find evidence of the fulfilment of Gibrat's Law in country size with some exceptions in the lower tail of the distribution.

Comparison with city size Kernel regression estimation results for the US incorporated places and Italian cities are displayed in the middle and on the right of Figure 1, respectively. The number of observations is 89,500 for the US and 38,555 for Italy. In both cases, it can be observed in the two graphs in the upper part that the null hypothesis of zero conditional mean growth is rejected at the 5% significance level for the smallest units. In addition, the null hypothesis of the standardized conditional growth variance being equal to one is also rejected at the lower end of both size distributions.

These rejections are a consequence of the consideration of the whole size distribution since they correspond to the smallest units, represented by cities with less than 200 inhabitants. Although their urban character is debatable, Eeckhout (2004) suggested considering the whole distribution when testing for Gibrat's Law. On the contrary, other authors impose a minimum population threshold of 2,000 - 3,000 inhabitants. In contrast to what happens for cities, the definition of a country is a political issue rather than one of size.

All the findings presented above lead us to conclude that there are similarities for the growth experienced by the population of cities and countries: (i) there is evidence in favour of Gibrat's Law for most of the size distribution, (ii) rejections are especially appreciable in the smallest units, (iii) estimated conditional means and variances follow similar patterns. With respect to the latter, it should be noted that the estimated functions for countries are more similar to those of Italian cities than to those of the US incorporated places. This may be related to the fact that urban mobility in Europe is lower than in the US (Cheshire and Magrini, 2006) and, hence, more similar to that observed among countries.

3.1.2 Transition matrices

Description If population growth does not depend on the initial state, the size distribution of countries will be persistent. Therefore, evidence of Gibrat's Law will be found when changes are rarely observed in the estimated transition matrices.

As is habitual, let's begin by assumming that the world population distribution evolves according to a homogeneous first-order stationary Markov process. Its evolution is described by a transition matrix of probabilities of change between the groups in which the size distribution of countries is divided. The pattern of growth will be consistent with Gibrat's Law as the elements in the diagonal approach one. Elements different from zero outside the diagonal represent intra-distributional movements.

This methodology requires a discretization of the size distribution at each point in time into cells whose cut-off points are defined by specific values. Each country is assigned to one of a predetermined number of groups depending on its relative size. This is why the world population distribution has been divided into five states determined by the following upper bounds[3]: 0.25, 0.5, 0.75, 2 and ∞ times the contemporary sample average. In 1950, they correspond to cell shares starting from the bottom of 57, 14, 11, 9 and 9% of the total number of countries, respectively.

Let F_t be the distribution of world population at time t. As noted before, it is assumed that it evolves according to the following Law of Motion (Quah, 1993):

$$F_{t+1} = M \cdot F_t \tag{4}$$

M is the 5×5 Markov chain transition matrix that maps the distribution at one point in time to that in the following period. That is, it tracks where a given country in F_t ends up in F_{t+1} in probability terms. Each m_{ij} element in M is the estimated probability that a country in group i in period t moves to group j in period $t+1$. These transition probabilities are estimated as:

$$\hat{m}_{ij} = \frac{\sum_{t=1}^{T-1} n_{it,jt+1}}{\sum_{t=1}^{T-1} n_{it}} \tag{5}$$

where $n_{it,jt+1}$ denotes the number of countries moving from group i in year t to group j in year $t+1$ and n_{it} the number of countries in group i in year t.

Results for countries Transition matrices estimation results for countries are reported in the upper panel of Table 2. They correspond to the one-step decennial estimated transitions obtained by averaging the observed transitions for each one of the five decades in the period 1950-2000. The transition matrix on the left of the upper panel in Table 2 shows the results for the whole country sample. The main feature

[3] Changing these cut-off points does not qualitatively affect the results presented below.

observed is that the country size distribution is persistent because some values in the diagonal exceed 0.90. Specifically, most countries with a relative size less than 0.25 times the mean and 94 per cent of the biggest countries remained in the same state during the following decade. Although intermediate states are somewhat less persistent, all of their diagonal entries are greater than 0.75. Therefore, it can be concluded that there is evidence of the fulfilment of Gibrat's Law for countries when applying a transition matrix-based test.

[Insert Table 2 here]

Previous results from kernel regressions have suggested that the smallest countries seem to have a different growth pattern to those in the upper tail of the size distribution. This is why the estimated transition matrices for the latter have also been reported in the upper panel of Table 2. Specifically, the biggest 100 and 50 countries have been considered separately. Before presenting these results it should be noted that the state with the 0.25 upper bound has been dropped in order to be comparable with the city size distribution[4]. In addition, a fifth state denoted as "Rest" has been added (Lanaspa et al., 2003) because many countries either enter or leave the sample of the largest 100 or 50 in different time periods. "Rest" will include the 87 countries ranked from position 101 to 187 for the Top 100 matrix and the 50 countries ranked from position 51 to 100 for the Top 50 matrix. The transitions from the other states to this fifth one correspond to the countries that leave the sample of the 100 largest, while the transitions from the fifth state to the others reflect the countries that enter it.

Estimated probabilities in the diagonal increase for the intermediate states when considering the biggest countries. This increase is more noticeable for the biggest 100 countries and implies a higher persistence for the upper-tail distribution of country size. Therefore, and in line with our previous results, it can be concluded that the evidence favourable to Gibrat's Law increases as we move towards the upper tail of the country size distribution.

Comparison with city sizes One result commonly found in the literature about urban growth and relative size distributions is that the smallest cities tend to have higher intra-distributional mobility than the biggest ones, the latter being those that present a higher persistence. This is true for different time periods, countries and sample sizes. In light of our previous findings, this also seems to be the case for country size.

[4]This state remained empty in almost all decades for city sizes.

The transition matrices for our city size data have also been calculated in order to establish further comparisons. To do so, three different sample sizes have been considered: the 200, 100 and 50 biggest cities in each country. The results obtained for the average transition matrices are displayed in the panels in the middle and at the bottom of Table 2 for the US and Italy, respectively. It can be observed that they are similar to those for countries. That is to say, the persistence of the intermediate states increases as we move towards the upper tail of the distribution. Also in line with previous results, transition probabilities in the upper tail of the country size distribution are more similar to those in the Italian case than in the US one.

3.2 Parametric analysis: Panel unit root testing

Description The model of Clark and Stabler (1991) for city population growth with autocorrelated errors under the assumption that Gibrat's Law holds can be directly applied to countries. Within this framework, testing for Gibrat's Law in country size is equivalent to testing for the presence of a unit root in the natural logarithm of population. Specifically, if the null hypothesis that (the natural logarithm of) the country population time series (y_{it}) has a unit root is rejected, the null hypothesis that its population evolves according to Gibrat's Law is also rejected. The panel structure of the available country population data has been exploited in order to test for a unit root.

The first question to be aware of when testing for unit roots with panel data methods is the possible presence of cross-sectional dependence. This is because it has been well established in the literature that panel unit root and stationarity tests that do not explicitly allow for this feature among individuals present size distortions (Banerjee et al., 2005). The importance of this characteristic in the PWT country population data has been shown using the simple test of Pesaran (2004). It is based on the average of pair-wise correlation coefficients of the OLS residuals obtained from standard augmented Dickey-Fuller (DF) regressions for each individual i (e_{it}). Let $\hat{\rho}_{ij}$ be the sample estimate of the pair-wise correlation coefficient for countries i and j calculated over T time periods:

$$\hat{\rho}_{ij} = \hat{\rho}_{ji} = \frac{\sum_{t=1}^{T} e_{it} e_{jt}}{\left(\sum_{t=1}^{T} e_{it}\right)^{\frac{1}{2}} \left(\sum_{t=1}^{T} e_{jt}\right)^{\frac{1}{2}}} \tag{6}$$

One virtue of Pesaran's test is that it does not depend on any particular spatial weight matrix when the cross-sectional dimension (N) is large. Its null hypothesis is cross-sectional independence and is asymptotically distributed as a two-tailed standard normal distribution. The test statistic is calculated as:

$$CD = \sqrt{\frac{2T}{N(N-1)}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{ij}\right)} \longrightarrow N(0,1) \tag{7}$$

Having shown that the units in the country population panel are cross-sectionally correlated, the presence of a unit root has been tested for taking this into account. This has been done as in Pesaran (2007), who proposed augmenting DF regressions with the cross-sectional mean and some of its lags in order to proxy for a single unobserved factor. The resulting individual DF test statistics are then averaged in a similar fashion to Im et al. (2003) (CIPS test). Following Choi (2001), the p-values of the individual tests can also be combined (CZ test). Critical values are obtained with Monte Carlo simulations for a given specification of the deterministic component and depend on both the cross-sectional and temporal dimensions.

In order to avoid the size distortions of unit root tests in the presence of serially correlated errors, additional lags of the augmentation terms have been included. The latter have been chosen using the Modified Akaike information criterion proposed by Ng and Perron (2001) considering a maximum of 8. Only a constant has been included as the deterministic term. The reason is twofold. First, it is consistent with the model originally proposed by Clark and Stabler. Second, it will allow comparison with city size for which it is not possible to include a trend because the low temporal dimension available in that case imposes a degrees of freedom problem.

Results for countries The upper panel of Table 3 presents both the cross-sectional dependence and unit root tests results for country sizes. Results for the whole country sample are those on the left. The null hypothesis of no cross-sectional correlation is rejected at the 1% significance level. More interestingly, the null hypothesis of a unit root in country population is also rejected by both tests with the same level of significance. Therefore, it can be concluded that evidence against Gibrat's Law in country size is obtained from the application of panel unit root tests to the whole sample.

[Insert Table 3 here]

In order to determine whether or not this rejection is driven by the smallest countries, the same tests have been applied to the 100 and the 50 biggest countries. The results are reported in the middle and on the right of the upper panel in Table 3, respectively. The main difference is that the null hypothesis of no cross-sectional correlation cannot be rejected at the 10% significance level for the 50 biggest countries. It can be observed that the null hypothesis corresponding to the fulfilment of Gibrat's Law is also rejected in these two sub-samples.

Comparison with city sizes As was also the case for transition matrices, three different sample sizes have been considered when applying the panel unit root tests to city size: the biggest 200, 100 and 50. The results are in the panels in the middle and at the bottom of Table 3 for the US and Italian cities, respectively. The null hypothesis of no cross-sectional dependence is rejected at the 1% significance level in all the cases. Moreover, it is found that the null hypothesis of a unit root is rejected for sample sizes of 200 and 100 cities. These rejections are stronger for the CZ test and for the Italian cities. Contrary to what happened when analyzing country size, there is evidence of the fulfilment of Gibrat's Law when considering only the 50 biggest cities in both the US and Italy.

Summarizing, the use of panel unit root tests has given evidence against the fulfilment of Gibrat's Law for country sizes. In contrast to the results obtained with the non-parametric tests, rejections do not seem to be caused by considering the smallest countries. This evidence contrary to Gibrat's Law is also found in city size. However, this empirical regularity is fulfilled in the upper tail of the city size distribution of the US and Italy.

4 Zipf's Law and country size

One stylized fact in urban economics is that the city size distribution in many countries can be approximated by a Pareto distribution whose exponent is equal to one. If this is the case, it can be concluded that there is evidence of Zipf's Law (Zipf, 1949). The latter is closely related to Gibrat's Law to the extent that the two empirical regularities are considered to be the two sides of the same coin. While Gibrat's Law has to do with the population growth process, Zipf's Law refers to its resulting population distribution. Several authors have modelled this relationship theoretically. Gabaix (1999) showed how deviations from Zipf's Law are determined by deviations from Gibrat's Law using a model based on local amenity shocks. More recently, Cordoba (2008) concluded that

Zipf's Law is equivalent to Gibrat's Law under plausible conditions.

Zipf's Law has already been tested for in the context of country size by Di Guilmi et al. (2003), Rose (2006) and Furceri (2008). Because of the relationship between Gibrat and Zipf's Laws, the fulfilment of the latter in the size distributions studied in this paper has also been analyzed. Our main contributions are that a temporal perspective is adopted and that the comparison of size distributions is carried out through the use of non-parametric kernel density estimators and formal statistical tests.

4.1 Temporal evolution of the Pareto exponent

The Pareto distribution was originally used as a statistical approximation to studying income distributions. Using the notation related to country population, let us denote s as the relative size and R its corresponding rank (1 for the biggest, 2 for the second and successively). These two variables are related following a Power Law if $R(s) = As^{-a}$, which is usually specified and estimated in its logarithmic version in order to check its fulfilment and estimate the magnitude of the relevant parameters:

$$\ln R = b - a \ln s + \xi \tag{8}$$

ξ is the error term. b and a are the parameters that characterize the distribution. The second parameter is known as the Pareto exponent, and Zipf's Law holds when $a = 1$. Gabaix and Ibragimov (2007) proposed specifying equation (8) by substracting $\frac{1}{2}$ from the rank to obtain an unbiased estimation of a:

$$\ln\left(R - \frac{1}{2}\right) = b - a \ln s + \varepsilon \tag{9}$$

Equation (9) has been estimated by OLS for the PWT population data, US incorporated places and Italian cities at the different points in time when they are observed during the period 1950-2004. As in the previous section, the analysis refers to three different sample sizes: all units, the biggest 100 and 50. The estimated Pareto exponents and their corresponding 95% confidence bands are plotted in Figure 2.

[Insert Figure 2 here]

The graphs at the top of Figure 2 show the estimation results obtained when considering all the units in each geographical category. It can be observed that the estimated Pareto exponent for countries remains almost constant during the whole sample period, being clearly lower than 1. In contrast, the estimated coefficients for cities display a higher variability. Once again, this reflects that population movements at an international level are more difficult than within countries. The estimated Pareto exponents for the US incorporated places and Italian cities decrease during the period analyzed, getting closer to that for the country size distribution over time. This exponent is higher for the Italian cities, decreasing from 0.9 in 1951 to 0.7 in 2001. The corresponding exponent for the US incorporated places was around 0.64 in 1950 and 0.53 in 2000[5]. Note that this decreasing evolution might indicate a divergent behaviour of city size. It may be explained in the US by the appearance of new cities with very small relative sizes in the sample rather than by different growth rates of cities.

It can be observed in the graphs in the middle and at the bottom of Figure 2 that the estimated magnitude of the Pareto exponent increases and gets closer to one as only the bigger countries are considered. This implies that Zipf's Law holds in the upper tail of the country size distribution. The Pareto coefficient remains almost unchanged around 0.8 for the 100 biggest countries and presents an increasing trend for the 50 biggest. This upward evolution of the Pareto exponent is also found for the US incorporated places and Italian cities in the upper-tail of the size distribution. In these two latter cases, the estimated coefficients are above one and are greater for the US than for Italy.

These results are in line with Eeckhout (2004) and Soo (2005) who showed that estimated Pareto exponent is clearly dependent on the sample size as well as the geographical unit chosen. In addition, they also coincide with those in Rose (2006) who obtained favourable evidence of Zipf's Law only in the upper-tail distribution for both cities and countries.

4.2 Comparison of distributions

The aim of this last subsection is to statistically compare how similarly population is distributed among countries and cities. To do so, their relative size density functions have been estimated using an adaptative kernel density estimator. This has been done for both the initial and final time periods in order to adopt a temporal perspective. These distributions in each geographical category analyzed have been plotted at the top of Figure 3. In addition, the two-variable Kolmogorov-Smirnov (KS) test statistic of the null that the distributions are equal has also been reported.

[Insert Figure 3 here]

[5] This value almost coincides with that obtained by Eeckhout (2004). The differences might be a consequence of not working with "unincorporated places".

The smallest countries have an important weight in the distribution that has decreased in time. More interestingly, the distribution of countries has not significantly changed from 1950 to 2000. This is in line with the estimated Pareto exponents in the previous subsection and is corroborated by the KS test since the null hypothesis cannot be rejected at the 10% significance level. On the contrary, the size distributions of cities in the US and Italy have changed over time, starting from a leptokurtic distribution with a higher density around the mean at the beginning of the sample period. Similarly to the country size distribution, there is less density in the lower tails of the distributions in the final period. The graphs in the middle and bottom panels of Figure 3 compare the distributions of countries and cities at the beginning and the end of the sample, respectively. Although the KS test statistic rejects the null hypothesis that the distributions are equal in all cases, country and city size distributions are more similar at the end of the period analyzed than at the beginning. This finding also confirms those obtained from the the evolution of the Pareto coefficients and are reflected in a reduction of the KS test statistics. In fact, both countries and cities seem to be log-normally distributed around 2000. Note that this does not contradict the findings regarding the Pareto coefficient in the upper tail. As pointed out by Eeckhout (2008), a log-normal distribution of the tails does not mean that a Pareto fit does not exist.

5 Summary and concluding remarks

This paper has implemented further tests of Gibrat's Law in country sizes. Our main contribution is that the analysis has been carried out using the techniques most commonly applied in another demographic context where this empirical regularity is relevant: city size. In line with Rose (2006), we find evidence of an independent growth of country population with respect to its initial size. However, when using non-parametric kernel regressions, this hypothesis is rejected for the smallest countries. In addition, it is not possible to find favourable evidence of Gibrat's Law when using panel unit root tests. Therefore, it should be concluded that the theoretical modelling of country population in accordance with Gibrat's Law should not concern us so much until stronger evidence for it is found.

City size data of two developed countries - Italy and the US - have been used in order to establish comparisons between the population growth of countries and cities. Although population movements at an international level are more restricted than those within the same country, some similarities between these two different geographical categories have been obtained. This reinforces Rose's findings and is especially true for the Italian cities.

An analysis of population distributions has also been carried out. The estimated Pareto exponents show a common behaviour between cities and countries in the upper-tail of the distribution, where Zipf's Law holds. When the whole distribution is considered, the estimated Pareto exponent for countries remains almost unchanged over time while those for cities present a decreasing trend. This fact could indicate that a process of divergence has brought the distributions of cities closer to that of countries. In the US, this divergence would be explained not so much by differences in the growth rate of cities but by the appearance of new cities which enter with very small relative sizes. The same conclusion applies when analyzing the size distributions using non-parametric methods. That is, the distribution of countries has not significantly changed from 1950 to 2000 while city size distributions have become more similar to that of countries over time.

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Table 1: Summary of the empirical literature examining Gibrat's Law for city size, 1990-2008.

Authors	Country	Sample period	Methodology	Conclusion
Clark and Stabler (1991)	Canada	1975-1984	Unit root testing	Acceptance
Guérin-Pace (1995)	France	1836-1990	Pearson's correlation coefficient	Rejection
Eaton and Eckstein (1997)	Japan	1925-1985	Transition matrices and Lorenz Curves	Acceptance
	France	1876-1990		Acceptance
Petrakos et al. (2000)	Greece	1981-1991	Growth regressions	Rejection
Davis and Weinstein (2002)	Japan	1925-1965	Unit root testing	Acceptance
Black and Henderson (2003)	USA	1900-1990	Transition matrices and Unit root testing	Rejection
Ioannides and Overman (2003)	USA	1900-1990	Kernel regressions	Acceptance
Eeckhout (2004)	USA	1990-2000	Growth and Kernel regressions	Acceptance
Gabaix and Ioannides (2004)	USA	1900-1990	Kernel regressions	Acceptance
Resende (2004)	Brazil	1980-2000	Unit root testing	Acceptance
Anderson and Ge (2005)	China	1961-1999	Rank regressions and Transition matrices	Mixed
Henderson and Wang (2007)	World	1960-2000	Unit root testing	Rejection
Soo (2007)	Malaysia	1957-2000	Unit root testing	Rejection
Bosker et al. (2008)	West Germany	1925-1999	Transition matrices, Kernel regressions and Unit root testing	Mixed

Table 2: Average 10-year transition matrices.

Penn World Table countries (1950-2000)																	
All sample (N=187)						Top 100						Top 50					
Initial	0.09	0.09	0.11	0.14	0.57	Initial	0.05	0.09	0.04	0.36	0.47	Initial	0.04	0.10	0.05	0.31	0.50
	∞	2	0.75	0.50	0.25		∞	2	0.75	0.50	Rest		∞	2	0.75	0.50	Rest
∞	0.94	0.06				∞	0.95	0.05				∞	0.94	0.06			
2	0.06	0.92	0.02			2	0.04	0.94	0.02			2	0.02	0.90	0.08		
0.75		0.12	0.76	0.12		0.75		0.05	0.92	0.03		0.75		0.05	0.88	0.07	
0.50			0.09	0.85	0.06	0.50			0.02	0.95	0.03	0.50			0.04	0.88	0.08
0.25				0.02	0.98	Rest				0.03	0.97	Rest				0.04	0.96
US incorporated places (1950-2000)																	
Top 200						Top 100						Top 50					
Initial	0.04	0.08	0.07	0.31	0.50	Initial	0.04	0.13	0.07	0.26	0.50	Initial	0.05	0.11	0.14	0.20	0.50
	∞	2	0.75	0.50	Rest		∞	2	0.75	0.50	Rest		∞	2	0.75	0.50	Rest
∞	0.87	0.13				∞	0.90	0.10				∞	0.91	0.09			
2	0.04	0.89	0.07			2	0.04	0.89	0.07			2	0.02	0.82	0.16		
0.75		0.12	0.74	0.14		0.75		0.12	0.74	0.14		0.75		0.14	0.70	0.16	
0.50			0.10	0.70	0.20	0.50		0.01	0.09	0.68	0.22	0.50			0.12	0.70	0.18
Rest			0.01	0.10	0.89	Rest				0.10	0.90	Rest			0.02	0.06	0.92
Italian cities (1951-2001)																	
Top 200						Top 100						Top 50					
Initial	0.04	0.10	0.10	0.26	0.50	Initial	0.05	0.07	0.10	0.28	0.50	Initial	0.06	0.08	0.08	0.28	0.50
	∞	2	0.75	0.50	Rest		∞	2	0.75	0.50	Rest		∞	2	0.75	0.50	Rest
∞	0.96	0.04				∞	0.96	0.04				∞	1				
2	0.01	0.93	0.06			2		0.97	0.03			2		0.98	0.02		
0.75		0.09	0.80	0.11		0.75		0.05	0.88	0.07		0.75		0.02	0.93	0.05	
0.50			0.06	0.83	0.11	0.50			0.08	0.83	0.09	0.50			0.04	0.87	0.09
Rest				0.06	0.94	Rest				0.05	0.95	Rest				0.06	0.94

Table 3: Cross-sectional dependence and panel unit root tests. Total population (in natural logarithms).

Penn World Table countries (1950-2004, T=55)			
	All (N=187)	Top 100	Top 50
CD	4.90***	3.80***	1.36
CIPS	-2.29***	-2.27***	-2.17**
CZ	-7.55***	-5.36***	-3.01***

US incorporated places (1950-2000, T=6)			
	Top 200	Top 100	Top 50
CD	55.79***	37.93***	29.50***
CIPS	-1.99*	-2.16**	-1.76
CZ	-3.12***	-3.51***	0.36

Italian cities (1951-2001, T=6)			
	Top 200	Top 100	Top 50
CD	76.63***	74.46***	46.69***
CIPS	-2.17**	-2.56***	-1.57
CZ	-5.09***	-6.45***	0.65

Note: A constant has been included as a deterministic term. The number of lags in order to correct for autocorrelation in country population data has been selected using the MAIC criterion by Ng and Perron (2001) considering a maximum of eight. This correction has not been used for city size. ***, ** and * denote rejection of the null hypothesis at the 1, 5 and 10% significance level, respectively

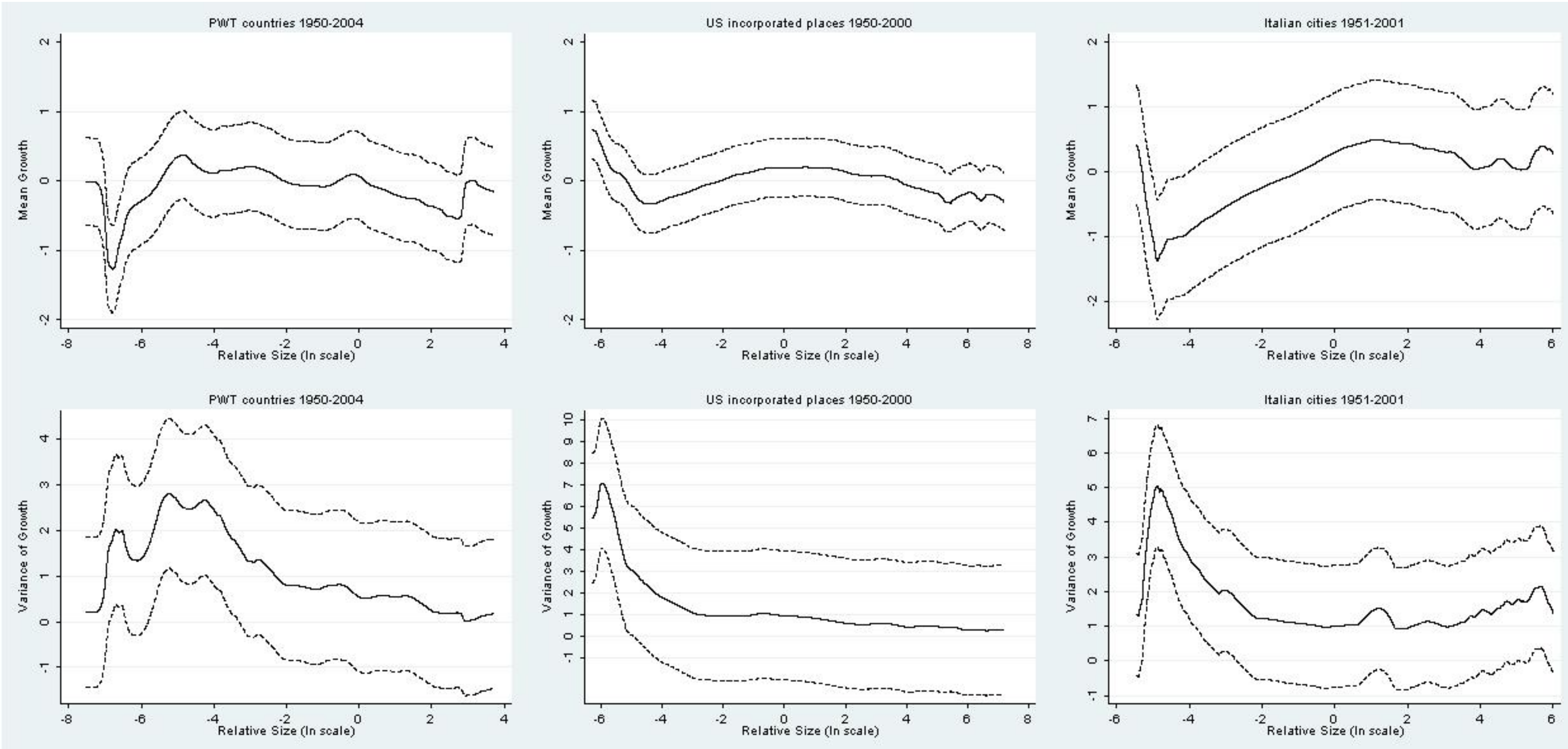

Figure 1: Estimated mean and variance of the standardized population growth in terms of relative size using kernel regression.

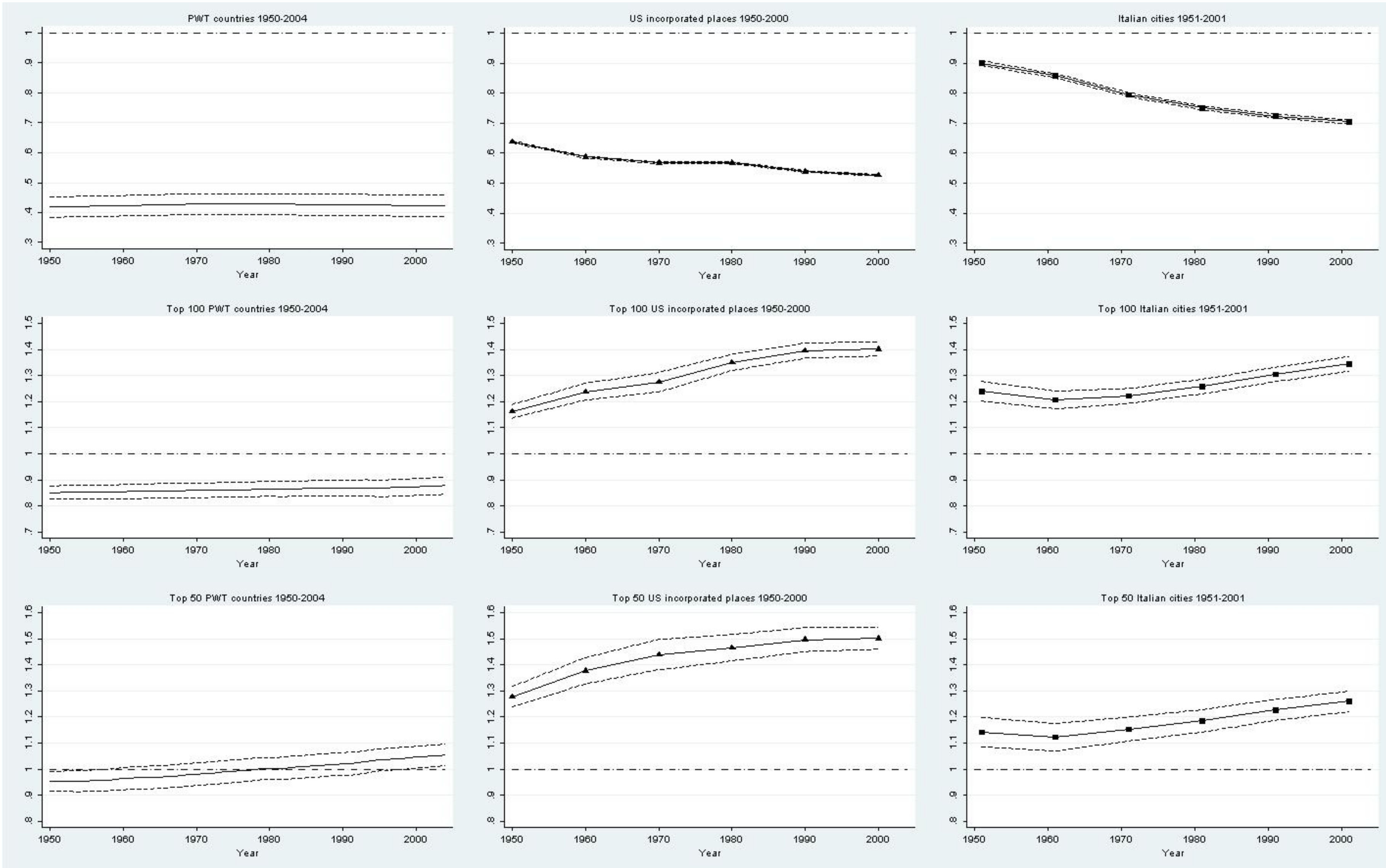

Figure 2: Evolution of the estimated Pareto exponents.

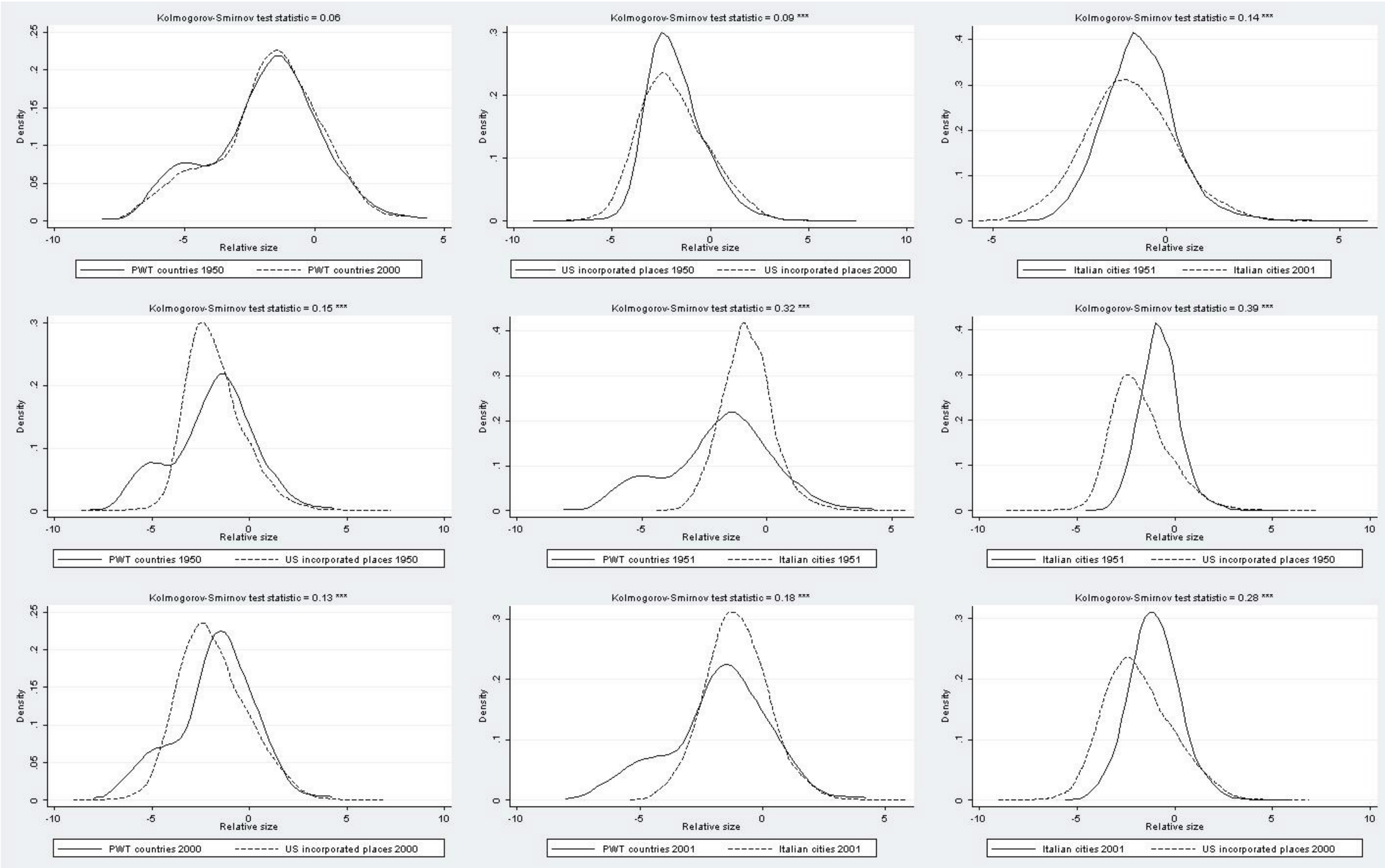

Figure 3: Kernel density estimation of relative size distributions.