# Estimating Regime-Switching Taylor Rules with Trend Inflation<sup>\*</sup>

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#### Abstract

This paper estimates regime-switching monetary policy rules featuring trend inflation over the post-WWII U.S. data. We find evidence in favor of regime shifts, time-variation of the inflation target, and a drop in the inflation gap persistence when entering the Great Moderation sample. Estimated Taylor rule parameters and regimes are robust across different monetary policy models. We propose an "internal consistency" test to discriminate among our estimated rules. Such test relies upon the reverse causality running from the monetary policy stance to the inflation gap. Our results support the stochastic autoregressive process as the most consistent model for trend inflation, above all when conditioning to the post-1985 subsample.

*Keywords:* Active and passive Taylor rules, trend inflation, inflation gap persistence, Markov-switching models.

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### 1 Introduction

Macroeconomists often employ the Taylor (1993) rule to model monetary policymakers' conduct. In its original version, the Taylor rule postulates the systematic, *stable* reaction of a short term interest rate - assumed to be directly controlled by the Fed - to deviations of inflation with respect to a *constant* inflation target and to business cycle fluctuations. Taylor (1993) shows that a calibrated version of the rule fits remarkably well the U.S. facts for the period 1987-1992.

Both parameter and inflation target stability have been recently challenged by the empirical literature. Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), Cogley and Sargent (2005a), Kim and Nelson (2006), Boivin (2006), Boivin and Giannoni (2006) and Benati and Surico (2008a) document the significant instability of the Fed's reaction to inflation over the last 50 years, with evidence in favor of a switch to a more aggressive monetary conduct occurring at the beginning of the '80s. Lubik and Schorfheide (2004), Boivin and Giannoni (2006), Lubik and Surico (2006), and Benati and Surico (2008b) show that such a policy shift, when coupled with a new-Keynesian framework, is able to deliver the Great Moderation.<sup>1</sup> As regards the inflation target, Ireland (2007), Cogley and Sbordone (2005), Cogley and Sbordone (2007), Bjørnland, Leitemo, and Maih (2007), Stock and Watson (2007), Leigh (2008) and Cogley, Primiceri, and Sargent (2008) provide evidence in favor of a slowly evolving latent monetary policy objective, i.e. time-varying "trend inflation".<sup>2</sup> Interestingly, trend inflation turns out to

<sup>&</sup>lt;sup>1</sup>The term "Great Moderation" is usually employed to indicate the generalized reduction in output and inflation volatility observed since the mid-'80s in the U.S. and other industrialized countries (Kim and Nelson (1999), McConnell and Perez-Quiros (2000), and Blanchard and Simon (2001)). It must be acknowledged that the "good monetary policy" view is not uncontented. For contributions supporting the role of more benign macroeconomic shocks as the main driver of the Great Moderation, see Sims and Zha (2006), Justiniano and Primiceri (2008), Canova, Gambetti, and Pappa (2007), Canova, Gambetti, and Pappa (2008). For some counter-arguments, see Benati and Surico (2008b) and Giannone, Reichlin, and Lenza (2008).

<sup>&</sup>lt;sup>2</sup>Ascari (2004) coined the term "trend inflation" to indicate a strictly positive level of steady state inflation around which to approximate firms' first order condition in the derivation of the new-Keynesian Phillips curve. The literature has recently considered the case of a time-varying inflation target. We will use the terms "time-varying trend

be important in modeling inflation with a new-Keynesian Phillips curve. Cogley and Sbordone (2005), Cogley and Sbordone (2007), and Bjørnland, Leitemo, and Maih (2007) show that, when accounting for trend inflation in the derivation of the supply curve, the empirical relevance of the ad-hoc price indexation ingredient collapses to zero. In other words, trend inflation turns out be important for delivering a theoretically more satisfactory model for inflation.<sup>3</sup>

Unfortunately, empirical investigations on *policy rules* conducted so far have considered *either* sources of misspecification. Indeed, *both* parameter instability *and* trend inflation are sources of potentially severe misspecification for the estimation of objects of interest such as policy parameters, policy targets, dating of the switch from a regime to another, measures of gaps such as the inflation gap - the difference between realized inflation and its contemporaneous target - and the real interest rate gap - the wedge between the real interest rate and its steady-state value, and their interrelationships. One is then left to wonder how these objects look like when parameter instability and trend inflation are *jointly* accounted for.

This paper employes Bayesian techniques to estimate regime-switching Taylor rules featuring time-varying policy targets. First, we estimate a regime-switching rule under the assumption of fixed inflation target so to set up a reference scenario. Then, we propose two alternative policy rules, one featuring a non-stochastic time-varying inflation target computed with the widely employed Hodrick-Prescott filter, and the other one characterized by a stochastic autoregressive model for trend inflation as in Ireland (2007), Cogley and Sbordone (2005), Cogley and Sbordone (2007), Bjørnland, Leit-emo, and Maih (2007), Stock and Watson (2007), Leigh (2008) and Cogley, Primiceri, and Sargent (2008). The former alternative model is studied to understand if a "quick-fix" measure of trend inflation may deliver an estimated Taylor rule displaying interesting properties when compared to the latter framework, which encompasses the fixed-inflation target proposal.

inflation" and "trend-inflation" interchangeably.

<sup>&</sup>lt;sup>3</sup>Kozicki and Tinsley (2005) empirically investigate the impact that shifts in the private sector's perceptions over the time-varying inflation target may have on the transmission going from structural shocks to inflation, output, and interest rates.

Our main results read as follows. In line with previous contributions, we find evidence in favor of monetary policy regime-switches and time-variation of the estimated inflation target in the post-WWII era.<sup>4</sup> Our three policy rules - again, the rule with a fixed inflation target, the rule with the Hodrick-Prescott inflation trend as time-varying inflation objective, and the one with a stochastic autoregressive process to model the evolving target feature quite different inflation gaps but very similar estimated parameters as well as associated policy regimes. Importantly enough, inflation gaps turn out to be informative to discriminate among policy rules on the basis of an "internal consistency" test. Taylor rules sensibly describe policymakers' conduct if a feedback relationship going from monetary policy interventions that influence the real interest rate gap - to the inflation gap is established. In particular, policy tightenings/loosenings should induce downward/upward movements of the inflation gap, possibly with some lags. Then, we should observe a negative correlation between the real interest rate gap and the inflation gap. Interestingly, we find that the only model supporting this "reverse causality" among the ones at hand is that allowing for the stochastic autoregressive representation of the time-varying inflation target. Our results support the employment of stochastic processes for trend inflation in monetary policy frameworks.

The structure of the paper reads as follows. The next Section presents the regime-switching models we estimate, and shortly discusses the algorithm we employ to estimate them. Section 3 discusses our empirical results by highlighting similarities and differences that concern the estimated models at hand. Section 4 takes the estimated rules seriously and performs the previously discussed logical consistency check. Section 5 concludes.

 $<sup>^{4}</sup>$ We abstract from considering the role of data revisions and real time data in this paper. For a paper tackling these issues, see Orphanides (2001).

## 2 Monetary policy: Model and estimation strategy

We work with an extension of the original simple policy rule proposed by Taylor (1993). Our model reads as follows:

$$i_t = [1 - \rho(S_t)] [\overline{r} + \pi_t + \alpha(S_t)(\pi_t - \pi_t^*) + \beta(S_t)y_t] + \rho(S_t)i_{t-1} + \epsilon_t^{MP}$$
(1)

$$\pi_t^* = (1 - \rho_\pi)\pi^{LR} + \rho_\pi \pi_{t-1}^* + \epsilon_t^{\pi^*}$$
(2)

$$\pi_t = \pi_t^* + z_t \tag{3}$$

$$z_t = \phi_z(S_t) z_{t-1} + \epsilon_t^z \tag{4}$$

where  $i_t$  is the short-term nominal interest rate,  $\pi_t$  is the inflation rate,  $y_t$  is the output gap - i.e. the deviation of log-real GDP with respect to its longrun trend,  $\overline{r}$  is the long-run real interest rate,  $\pi_t^*$  is the possibly time-varying inflation target,  $z_t$  is the inflation gap, and  $S_t$  is a random variable whose realization is interpreted as the state in which the economy (most likely) is at time t.

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Model (1)-(4) relaxes the original Taylor rule model along different dimensions. First, we allow - but do not necessarily require - the policy parameters to be state dependent. In particular, in the light of the instability of the U.S. monetary policy conduct documented by several recent empirical investigations - Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), Cogley and Sargent (2005a), Kim and Nelson (2006), Boivin (2006), Boivin and Giannoni (2006) and Benati and Surico (2008a) - we enhance policy parameters  $\alpha$ ,  $\beta$ , and  $\rho$  to switch across states, so to capture possible changes in the monetary policy conduct occurred in the post-WWII U.S. economic history. Moreover, several authors - Schorfheide (2005), Justiniano and Primiceri (2008), Mojon (2007), and Cogley, Primiceri, and Sargent (2008) - have provided evidence in favor of heteroskedasticity of the U.S. monetary policy shocks. To account for it, we allow the variance of the monetary policy shock  $\epsilon_t^{MP}$  to be state dependent, i.e.  $\epsilon_t^{MP} \sim \mathcal{N}(0, \sigma_{\epsilon^{MP}}^2(S_t))$ .

As said, trend inflation is possibly an important ingredient in a Taylor rule set up. We assume the time-varying inflation target to follow the auto regressive process (2) whose persistence is driven by the parameter  $\rho_{\pi}$  and whose unconditional mean reads  $\pi^{LR}$ . We interpret  $\pi_t^*$  as the short-term goal that the Fed sets period by period conditional to the economic situation and its knowledge of the monetary policy transmission mechanism. Cogley and Sargent (2005b), Primiceri (2006), Sargent, Williams, and Zha (2006)), and Carboni and Ellison (2008) show that the Fed is likely to have learnt the structure of the economy over time and, consequently, to have revised its estimates of the disinflationary costs associated to a monetary policy tightening. It may very well be that the Fed has set the short run inflation target  $\pi_t^*$  by considering the perceived inflation-output volatility trade-off. By contrast, the long-run inflation target  $\pi^{LR}$  is meant to be compatible with long-run goals such as sustainable growth and employment. Following other authors - Ireland (2007), Cogley and Sbordone (2005), Cogley and Sbordone (2007), Bjørnland, Leitemo, and Maih (2007), Stock and Watson (2007), Leigh (2008) and Cogley, Primiceri, and Sargent (2008) - we do not propose a microfounded model for the inflation target, which is then assumed to evolve exogenously.<sup>5</sup> Both the short-run inflation target and its long-run counterpart are state-independent.<sup>6</sup>

We assume that the Fed aims to close the short-run inflation gap (3) by manipulating the real interest rate gap  $r_t - \overline{r}$ , where  $r_t \equiv i_t - \pi_t$ . Likely, the

<sup>&</sup>lt;sup>5</sup>Ireland (2007) tests the role of exogenous supply shocks in shaping such a target, but finds their role to be negligible

<sup>&</sup>lt;sup>6</sup>Schorfheide (2005) estimates a small scale macroeconomic model that allows for the inflation target and the variance of the monetary policy shock to switch between two states, and finds evidence for shifts of the inflation target and monetary policy shock heteroskedasticy in the post-WWII sample. Liu, Waggoner, and Zha (2007) embeds Schorfheide (2005)'s inflation target process in a medium scale model featuring heteroskedastic macroeconomic shocks. Their preliminary results support a constant inflation target over the same period investigated by Schorfheide (2005). While we do not admit abrupt shifts in our inflation target, we allow for quarter-by-quarter variations as in Ireland (2007), Cogley and Sbordone (2005), Cogley and Sbordone (2007), Bjørnland, Leitemo, and Maih (2007), and Cogley, Primiceri, and Sargent (2008).

monetary policy impulse takes some time before influencing inflation due to the frictional adjustments in consumption and investment decisions as well as price stickiness. Then, as suggested by eq. (4), we allow  $z_t$  to be serially correlated. Importantly, the degree of inflation gap persistence  $\phi_z$  is assumed to be state-dependent. Cogley and Sbordone (2007), Cogley, Primiceri, and Sargent (2008), and Benati and Surico (2008a) show that the inflation gap persistence is likely to have changed when moving from the '70s to the Great Moderation subsample. In the long-run, inflation and output gaps close and the stochastic elements of the model assume their unconditional mean. Consequently,  $\pi_t = \pi_t^* = \pi^{LR}$  and  $i_t = i = \bar{r} + \pi^{LR}$ , which is the Fisher equation consistent with our Taylor rule.

We model shocks to the policy target and to the inflation gap process throughout the stochastic components  $\epsilon_t^{\pi^*} \sim \mathcal{N}(0, \sigma_{\epsilon^{\pi^*}}^2)$  and  $\epsilon_t^z \sim \mathcal{N}(0, \sigma_{\epsilon^z}^2)$ .<sup>7</sup> These error terms, as well as the policy shock  $\epsilon_t^{MP}$ , are assumed to be mutually uncorrelated independently distributed martingale differences.<sup>8</sup>

#### Estimated Policy Rules and Econometric Strategy

We fit three different Taylor rules to U.S. quarterly data spanning the sample 1955Q1-2007Q2.<sup>9</sup> We focus on the following models:

1. *EFT*: A rule with fixed long-run inflation target, i.e. model (1)-(4) estimated under the constraints  $\phi_z = \rho_\pi = \sigma_a = \sigma_\eta = 0$ ;

 $<sup>^{7}</sup>$ We assume the shocks to the inflation target to be homoskedastic. For contributions relaxing this assumption, see Schorfheide (2005), Stock and Watson (2007), and Cogley, Primiceri, and Sargent (2008).

<sup>&</sup>lt;sup>8</sup>Notice that our formulation embeds a stochastic intercept. As pointed out by Cochrane (2007), if we estimated the model with a deterministic intercept, we would induce serial correlation in the - reduced form, at that point - Taylor rule residual. This can be seen by rewriting the rule as follows:  $i_t = [1 - \rho(S_t)] [\overline{r} + \tilde{\alpha}(S_t)\pi_t + \beta(S_t)y_t] + \rho(S_t)i_{t-1} + \eta_t$ ,

where  $\eta_t \equiv \epsilon_t^{MP} - \alpha(S_t)\pi_t^*$  is clearly a serially correlated process in the light of eq. (2). <sup>9</sup>The data source is the Federal Reserve Bank of St. Louis' website. We retrieved the short-term policy rate (effective federal funds rate), the real GDP level  $Y_t$ , the estimate of the potential output made by the Congressional Budget Office  $Y_t^*$ , and the GDP deflator  $P_t$ . Quarterly observations of the federal funds rate were obtained by averaging monthly observations. The output gap is the percentualized log-deviation of the real GDP from its potential level, i.e.  $y_t \equiv 100 \log(Y_t/Y_t^*)$ . The inflation rate is the annualized quarterly growth rate of the GDP deflator, i.e.  $\pi_t \equiv 400 \log(P_t/P_{t-1})$ . Our dataset is available upon request.

- 2. *HPT*: The model (1)-(4), with the latent series of the inflation target  $\pi_t^*$  computed as the Hodrick-Prescott filter of the GDP deflator inflation series (smoothing weight: 1,600);
- 3. ETVT The model (1)-(4), in which the latent series of the inflation is assumed to be an autoregressive process.

To estimate our policy rules, we assume the existence of two states labeled as 0 and 1. These two states are modeled via the random variable  $S_t = \{0, 1\}$ , which follows a first order Markov chain whose transition probability matrix reads

$$P = \left[ \begin{array}{cc} 1 - p_{01} & p_{10} \\ p_{01} & 1 - p_{10} \end{array} \right]$$

where  $p_{ij} = Pr(S_t = j | S_{t-1} = i)$  is the probability of moving from state i - the relevant state at time t - 1 - to state j at time t, and  $\sum_i p_{ij} = 1$ . Defining  $\widetilde{\mathbf{S}}_t \equiv [Pr(S_t = 0), Pr(S_t = 1)]'$ , the evolution of the states is then dictated by the law of motion  $E_t \widetilde{\mathbf{S}}_{t+1} = P \widetilde{\mathbf{S}}_t$ .

We estimate the model with Bayesian techniques by implementing an efficient Markov Chain Monte Carlo (MCMC) strategy through the Gibbs sampler. MCMC algorithms for regime-switching ARMA models have been introduced by Albert and Chib (1993), McCulloch and Tsay (1993), and successively developed by Billio, Monfort, and Robert (1999). In this framework we generalize their approach to include the extra non-observable component, i.e. the time-varying inflation target. Furthermore, we use a multi-move version of the Gibbs sampler in which we sample the states  $\mathbf{S} = (S_1, \ldots, S_T)$  from their joint distribution to improve the efficiency of the algorithm.<sup>10</sup>

As shown by Albert and Chib (1993), McCulloch and Tsay (1993), and Billio, Monfort, and Robert (1999), the imposition of constraints on the parameter space enhances the identification of the estimated regimes and model parameters. We then impose two constraints. The first one regards the reactiveness of the Fed to fluctuations in the inflation gap. The monetary policy literature has widely accepted the "Taylor principle", i.e. the prescription

 $<sup>^{10}</sup>$ A detailed explanation of our estimation strategy is provided in the Appendix.

for monetary authorities to react with the policy rate more than one-toone to fluctuations in the inflation rate to move in the right direction the real interest rate and aggregate demand so to return inflation to its target (Woodford (2003)). As in the original Taylor (1993) rule, eq. (1) has a builtin one-to-one reaction to inflation in the "Fisher equation part" of the rule. Consequently, we label a rule as "active" when the real interest rate is raised at a level higher than its natural counterpart, i.e. when  $\alpha \ge 0$ . Accordingly, we assume  $\alpha(S_t = 0) > 0 > \alpha(S_t = 1)$  and interpret  $S_t = 0$  as the state identifying "active" monetary policy.<sup>11</sup>

The second constraint regards state-contingent monetary policy shock volatility  $\sigma_{\epsilon^{MP}}^2(S_t)$ . Mojon (2007) empirically verifies that changes in the magnitude of the monetary policy shocks are important for explaining the evolution of the U.S. real GDP volatility. The relationship between more conservative monetary policies and lower monetary policy shocks has also been found by Lubik and Schorfheide (2004) and Cogley, Primiceri, and Sargent (2008). Accordingly, we assume  $\sigma_{\epsilon^{MP}}^2(S_t = 0) < \sigma_{\epsilon^{MP}}^2(S_t = 1)$ .

To reiterate, we do not impose any date-break a priori, i.e. parameter instability, if present, will be endogenously suggested by our estimation strategy.

#### Trend inflation and the Taylor principle: A caveat

The fact of allowing for a positive inflation target is not innocuous for the identification of the determinacy conditions. Ascari and Ropele (2007b) show that, in a new-Keynesian model derived consistently with a strictly positive

<sup>&</sup>lt;sup>11</sup>It is straightforward to rewrite the Taylor rule (1) in the following - somewhat more conventional - version:  $i_t = [1 - \rho(S_t)] [\gamma(S_t) + \tilde{\alpha}(S_t)\pi_t + \beta(S_t)y_t] + \rho(S_t)i_{t-1} + \epsilon_t^{MP}$ , where  $\gamma(S_t) \equiv \overline{r} - \alpha(S_t)\pi_t^*$ ,  $\tilde{\alpha}(S_t) \equiv 1 + \alpha(S_t)$ . When conditioning to a single state, Woodford (2003) shows that the systematic monetary policy reaction to inflation required to pin down a unique equilibrium in a new-Keynesian framework - i.e. the "Taylor principle" - is  $\tilde{\alpha}(S_t) > 1 - (1 - \delta)\beta(S_t)/\kappa$ , where  $\delta$  if firms' discount factor and  $\kappa$  is the slope of the Phillips curve. Notably, if policymakers' reaction to business cycle fluctuations  $\beta(S_t) > 0$ , a value of  $\tilde{\alpha}(S_t)$  lower than one - a value of  $\alpha(S_t)$  lower than zero - can still be consistent with a unique equilibrium. Therefore, the set of constraints we impose to identify the states might induce an overestimation of the "passive" monetary policy phases. We performed an ex-post check based on our estimated  $\alpha(S_t)_s$  and  $\beta(S_t)_s$  and conditional to  $\delta = 0.99$  and  $\kappa = 0.1$  (a calibration widely adopted in the literature), and we verified that such overestimation does not occur, i.e. the "passive" states we obtain remain unchanged when considering Woodford (2003)'s uniqueness condition.

steady state inflation rate, the standard Taylor principle does not hold true anymore. This is so because in presence of trend inflation the aggregate price index displays an upward trend. Then, firms able to reoptimize i) set higher prices and ii) assign a larger weight on expected - relative to current - realizations of the business cycle to avoid the erosion of relative prices and real profits. This flattens the Phillips curve and induces an increase in the sacrifice ratio. Consequently, a widening of the indeterminacy territory occurs. In other words, under trend inflation a "modified Taylor principle" applies.<sup>12</sup> Obviously, this may not be consistent with the "active"/"passive" monetary policy interpretation of our two estimated states, i.e. we may label as "active" a policy that, in Ascari and Ropele (2007b)' s world, delivers multiple equilibria.

Ropele (2007) shows that under full price indexation the "standard" Taylor principle is restored. The literature has offered a variety of point estimates for the indexation parameter, ranging from zero (Cogley and Sbordone (2005), Cogley and Sbordone (2007)) to values close to one (Giannoni and Woodford (2003)). We leave the estimation of a regime-switching framework conditional on the modified Taylor principle as in Ascari and Ropele (2007b) to future research.

## **3** Empirical evidence

As already pointed out, there are several dimensions along which we want to compare our three estimated models. First, we aim to check if the data support parameter instability and trend inflation in our framework. Given the already mentioned contributions finding evidence in favor of these objects, we see this check as a sort of validation exercise for our estimations. Second, we want to understand if possibly misspecified models such as the fixed-inflation target model (EFT) and the model approximating trend inflation with the HP-filtered inflation rate (HPT) feature different posterior estimates with re-

<sup>&</sup>lt;sup>12</sup>Ascari and Ropele (2007b) show that this "modified Taylor principle" calls for a more aggressive response to inflation fluctuations by monetary policy authorities to induce equilibrium uniqueness. The presence of interest rate smoothing in the policy rule suggests a more moderate reaction to business cycle fluctuations.

spect to our most flexible model (ETVT), which shapes trend inflation with a stochastic autoregressive process. A related point regards the estimated states  $S_t$  and the timing of the switches from a state to another. Finally, we are also willing to check the "internal consistency" of our estimated Taylor rules in the light of the reasoning proposed in the Introduction, i.e. estimated Taylor rules should deliver a negative relationship between measures the estimated monetary policy stance and the inflation gap (more on this point in the next Section).

#### Inflation Targets

Figure 1 displays the U.S. inflation rate along with the three different estimated inflation targets in the post-WWII period. The simplest rule we estimate, i.e. the EFT rule, suggests an fixed policy target - median value - equal to 1.06.<sup>13</sup> Not surprisingly, the HP filtered inflation rate smoothly follows the tendential path of observed inflation over the sample. Interestingly, also the estimated target under the ETVT rule follows fairly closely the evolution of the inflation rate, and it displays a much lower degree of smoothness with respect to the HP filter trend, so suggesting a non-negligible role played by shocks to the inflation target for the inflation target is very close to the one proposed in a DSGE context by Ireland (2007), appears to belong to the new-Keynesian Phillips curve-based credible set estimated by Cogley and Sbordone (2007), and it is in line with the latent factor identified by Cogley, Primiceri, and Sargent (2008).

Obviously, our three different Taylor rules imply strikingly different inflation gaps  $\pi_t - \pi_t^{*,14}$  In particular, the ETVT inflation gap is much less volatile than the HPT inflation gap, i.e. the sample standard deviation of the former is 0.74 vs. the latter's 1.16. Moreover, the width of the swings is also quite rule-specific. For instance, the HP inflation gap suggests that the Fed may have let the inflation gap record a value equal to 4.8% in 1974Q4

<sup>&</sup>lt;sup>13</sup>This value is somewhat lower with respect to others proposed in the literature. All results concerning the EFT model are robust to the employment of a fixed target set to the inflation sample mean, i.e. 3.58%. Notice that  $\bar{r}$  and  $\pi^{LR}$  are identified also in the EFT rule due to the state dependence of the parameter  $\alpha$ .

<sup>&</sup>lt;sup>14</sup>The Figure comparing our model-specific inflation gaps is not shown in the paper but it is available upon request.

in the aftermath of the first oil shock, while the ETVT model consistent gap assigns 2.95% to the same quarter, values much lower than the one associated to the fixed-target EFT model, which associate to that quarter an inflation gap equal to 9.72%!

#### Regime Shifts

Given the evidence presented in Figure 1, one should probably expect to find differences in the estimated regimes as well as parameters across policy rules. In fact, it turns out that this is not the case. Figure 2 plots the estimated probabilities  $p(S_t = 1)$  of being in the "passive" monetary policy state. The three different models deliver a very similar picture in terms of estimated regimes. The medians of the estimated probabilities associated to the two models that allow for a time-varying inflation target indicate more clearly the estimated state of the economy. However, apart from a single episode associated to the EFT model, the remaining switches and estimated regimes are very similar across models, as also testified by Table 1. On the one hand, one may interpret this finding as good news - a misspecified Taylor rule does not necessarily lead to any evident econometric bias in terms of estimated regimes. On the other hand, this result may be driven by identification problems affecting inference over the parameters of our monetary policy rules, an issue recently re-proposed by Canova and Sala (2006), Cochrane (2007), Beyer and Farmer (2007), and Lubik and Schorfheide (2007).

With this warning in mind, we notice that our estimated regime-switches support just in first approximation the single-regime shift at the end of the '70s often imposed when studying the U.S. monetary policy. In fact, the period 1976-1979 falls under the active monetary policy regime. This might be due to some positive realizations of the real interest rate change in the mid-'70s as well as to the reduction in the monetary policy volatility possibly occurred in those years (Mojon (2007)).<sup>15</sup> By the same token, the switch to a passive monetary policy regime detected for the year 2001 may be due to deviations with respect the systematic policy stance predicted by a standard

<sup>&</sup>lt;sup>15</sup>Notably, Sims and Zha (2006) and Mojon (2007) find that the best fitting model is a model displaying no changes in the policy rule coefficients and heteroskedastic policy shocks, but they cannot reject models with unstable policy rule on the basis of marginal likelihood comparisons.

Taylor rule due to the stock market bust taking place at the beginning of the new Millennium, a finding corroborated by Benati and Surico (2008a)'s estimates.<sup>16</sup>

As regards the '80s, we find a switch towards active monetary policy in 1985Q1 (ETVT model). This break date comes somewhat later with respect to the end of the "Volcker experiment", but it is close to the usual dating of the Great Moderation (McConnell and Perez-Quiros (2000)) and the break-down in the ability of a variety of models to predict U.S. inflation and output (D'Agostino, Giannone, and Surico (2006) and Giannone, Reichlin, and Lenza (2008)).<sup>17</sup>

#### Posterior estimates

We now turn to the analysis of our posterior estimates. Table 2 reports the median values - along with the [5th, 95th] percentiles - of our policy rules. The model of highest interest for us is ETVT. Interestingly, all the parameters featuring the inflation target process assume non-zero values. The credible set of  $\rho_{\pi}$  suggests a very persistent process for the estimated inflation target, with the posterior median reading 0.97, so suggesting an almost random walk-like behavior of the inflation target, in line with Ireland (2007), Cogley and Sbordone (2005), Cogley and Sbordone (2007), Bjørnland, Leitemo, and Maih (2007), Stock and Watson (2007), and Cogley, Primiceri, and Sargent (2008). Interestingly, the estimated persistence of the inflation gap - captured by the parameter  $\phi_z(S_t)$  - drops when shifting from passive to active monetary policy, moving from a median of 0.41 down to 0.06. While being somewhat lower with respect to some estimates recently put forward by Cogley and Sbordone (2007) and Cogley, Primiceri, and Sargent (2008), these figures are similar to the inflation gap's normalized spectrum at frequency

 $<sup>^{16}</sup>$ In the aftermath of the 9/11 terrorist attack, the Fed implemented the largest repurchase aggreement ever realized, i.e. \$81.25 billion on September 14 (Cecchetti (2007)).

<sup>&</sup>lt;sup>17</sup>Boivin and Giannoni (2006) estimate the 90% confidence interval for the break date in a VAR suited to perform monetary policy analysis to range from 1977Q1 to 1986Q2. Admittedly, the "Volcker experiment" 1979-1982 period may hardly be captured by a Taylor rule tracking a short-term nominal interest rate. It may well be that such subsample is affecting the estimated duration of the passive monetary policy regime. One could tackle this issue by estimating a model with three different regimes, so to possibly account for the Volcker experiment. Favero and Monacelli (2005) decide against this option due to high instability/sensitivity to initial conditions of the estimated parameters/regimes.

zero (median values) proposed by Benati and Surico (2008a). The three previously mentioned contributions, as well as this paper, all point towards the same direction, i.e. a lower persistence of the inflation gap in the Great Moderation sample.

To further investigate this issue, we construct the distribution of the difference  $\Delta \phi_z = \phi_z(S_t = 0) - \phi_z(S_t = 1)$  by sampling 25,000 times from  $\phi_z(S_t=0)$  and  $\phi_z(S_t=1)$  and computing the difference for each pair of draws. A reduction in the inflation gap persistence when moving from the passive to the active state would be associated to a large mass of the density  $\Delta \phi_z$  located on the left of the zero vertical line. Figure 3 depicts the so constructed density. Indeed, the density is dominated by negative realizations. Even if the 95th percentile reads 0.09, so suggesting that the 90% coverage contains the zero value, the 84th percentile – which can be related to the 68% coverage - is -0.02, a value suggesting a significant fall in the inflation gap persistence. Negative realizations when moving from the "passive" to the "active" state amount to 87%, which falls to 64% when requiring a 0.2gap when sampling from the  $\Delta \phi_z$  distribution. While not offering decisive evidence in favor of a drop in the inflation gap variable when entering the last 20 years of our sample, our results tend to favor this view.<sup>18</sup> Moreover, our estimated regimes suggest a negative correlation between monetary policy aggressiveness and the inflation gap persistence, a finding in line with Benati and Surico (2008a).

Table 3 collects further statistics regarding the posterior density of the difference in the inflation gap persistence. As already noted by Cogley, Primiceri, and Sargent (2008), the assumption of uncorrelated inflation gap proposed by Stock and Watson (2007) is hardly supported by the data. When computing the joint probability of having realizations of the inflation persistence parameter close to zero in both states, we obtained a sample  $Pr(\phi_z(S_t=0) < k \cap \phi_z(S_t=1) < k)$  equal to 3.30% for k = 0.05 and to 10.38% for k = 0.10, certainly not a large support for the "white noise inflation gap" hypothesis.

<sup>&</sup>lt;sup>18</sup>Adding some structure to our framework could strengthen the evidence in favor of a fall in the inflation gap persistence. On this point, see Cogley, Primiceri, and Sargent (2008).

#### Estimated Taylor Rules: Anything Goes?

Going back to our posterior estimates, there are no striking differences across models in terms of estimated parameters.<sup>19</sup> The median of the two models admitting time-variation in the inflation target returns more conventional values of the long run inflation target. The steady-state real interest rate assumes values in line with most of the estimates in the literature, and the same holds as regards the Taylor rule parameters. In particular, we notice the "robustness" of the estimated Taylor principle parameter  $\alpha$  as opposed to the wide difference observed in the inflation gaps across scenarios.

To wrap up, our estimated inflation targets are clearly different across models. However, the consequently different inflation gaps appear to be important neither for the estimation of the policy regimes nor for that of the Taylor rule parameters. Unfortunately, the federal funds rate is very persistent, and measures of fit associated to whatever policy rule admitting interest rate smoothing are likely not to be powerful objects to distinguish between more or less interesting models.<sup>20</sup> There, should we conclude that all these Taylor rules are all alike? Are stochastic models of trend inflation superfluous in the Taylor rule framework? To answer these questions, we develop in the next Section an "internal consistency" test.

<sup>&</sup>lt;sup>19</sup>This is confirmed by looking at the posterior distributions of the differences between a given model and the ETVT model. We constructed such posteriors by i) randomly drawing a value out of the empirical posterior of a given parameter of the model ETVT, ii) randomly drawing a value out of the empirical posterior of the same parameter of an alternative model, iii) taking the difference between the two drawn values, iv) repeating the same exercise 25,000 times for the same parameter, and v) performing steps i)-iv) for all the parameters in common between the ETVT model and the alternative model. We verified that the zero value belongs to the [5th,9th] coverage of all the so constructed posterior distributions.

<sup>&</sup>lt;sup>20</sup>On top of that, the HPT model was estimated with a set of data including on more "observable", i.e. HP-filtered inflation. Consequently, a model comparison based on marginal likelihoods would not be proper.

## 4 Estimated Taylor rules: An internal consistency test

To discriminate among our estimated policy rules, we propose the following test. In a Taylor-rule world, one should expect a two-way causality between the inflation gap  $\pi_t - \pi_t^*$  and the real interest rate gap  $r_t - \bar{r}$  to be present. The first causality link runs from the inflation gap to the real interest rate gap, and it is captured by the Taylor rule itself. Such rule suggests that positive (negative) inflationary pressures should be tackled by tight (loose) monetary policies (Taylor (1993)). This is so because of the following "reverse causality" argument. An increase in the real interest rate gap negatively affects aggregate demand by inducing households to postpone their consumption and entrepreneurs to reduce investments in productive activities. In turn, the reduction in aggregate demand triggers a downward pressure on prices, which will eventually lead inflation to go back to its target level. Therefore, when scrutinizing the causality running from the (Taylor-rule consistent) real interest rate gap to the inflation gap, one should find it negative and significant.<sup>21</sup>

What if such negative correlation does not emerge? We believe there are two (not necessarily rival) explanations. One refers to the misspecification of the simple rules at hand. If our estimated rules do not offer a good representation of policymakers' behavior, then our internal consistency test focuses on the wrong object and cannot provide us with any interesting information. The other explanation regards passive monetary policy/indeterminacy. Under the passive monetary policy scenario, monetary policy does not pin down private sector's inflation expectations and inflation, then a very *weak* relationship between the monetary policy gaps - if any - is likely to emerge. The assumption of a negative correlation between the real interest rate gap and the inflation gap is testable. In this Section we precisely aim to test this assumption.

First-glance information may be obtained by focusing on scatter-plots.

<sup>&</sup>lt;sup>21</sup>Reynard (2007) puts forward this argument and proposes a graphical analysis involving the interest rate and inflation gap without performing any formal econometric test. By contrast, in this Section we take the "internal consistency" hypothesis to the data.

Figures 4-6 plots inflation gaps vs. real interest rate gaps for the three analyzed models. We look both at the full sample (empty circles) and at the longest "active" monetary policy subsample (filled circles), i.e. the Great Moderation sample. To focus on "reverse causality" - the monetary policy transmission channel - as opposed to Taylor rule channel, we lag the real interest rate gap by four periods. In this way we possibly account for delays in the transmission of monetary policy impulses and circumvent the otherwise present endogeneity.

Figure 4 refers to the Estimated Fixed Target model. Several considerations are in order. The dispersion related to the full sample is considerable. The regression line has an evident negative slope, but the "uncertainty" surrounding such slope is huge. This is not surprising in the light of the large inflation swings occurred in the '70s which do not go much hand-in-hand with the idea of a constant inflation target. The Great Moderation subsample offers a much less dispersed scatter-plot. However, the regression line displays a somewhat puzzling positive slope (not significant, though).<sup>22</sup> Seemingly, the relationships between the monetary policy gaps conditional to the fixedtarget Taylor rule does not pass our "internal consistency" test, above all when the last 20 years are considered.

The scatter plots associated to the Hodrick-Prescott Target model - Figure 5 - depicts a different situation. Evidently, there is much less dispersion with respect to the EFT case. Moreover, a much clearer relationship emerges when moving to the Great Moderation sample. As said, if monetary policy becomes more aggressive, the link between monetary policy actions and the inflation gap should become tighter, exactly what the scatter plot suggests. Then, from a qualitative perspective, the HP filter produces a time-varying inflation target that leads the HPT model to meet our requirements.

It is comforting to notice that one may tell the same story as regards the Estimated Time-Varying Target model. Figure 6 shows that the model with the stochastic inflation trend delivers a scatter plot with even much less dispersion than in the HPT case, and with a regression line also assuming a negative value when conditioning on the Great Moderation subsample.

<sup>&</sup>lt;sup>22</sup>This positive slope might be the due to endogeneity. We show later that this result is robust to the employment of a real interest rate gap lagged eight periods.

While this preliminary overview is probably enough to cast doubts on the goodness of the fixed-target model, one is left to wonder if the effort of estimating policy rules with stochastic time-varying targets brings to higher returns with respect to the quick-fix solution of the HP time-varying trend inflation. To answer this question, Table 4 collects the estimates of the above commented regression line, i.e.

$$\pi_t - \pi_t^* = c + \xi (r_{t-i} - \overline{r}) + \zeta_t \tag{5}$$

with i = 4.

As already stressed, to pass the "internal consistency" test we require a model to deliver a negative value of the slope parameter  $\xi$ . On top of it, given that in the long run all gaps should close, to pass the test a model should also be associated to a zero value for the constant c.

Table 4 provides additional information on the relationship between the model-specific monetary policy gaps. First, when the full sample 1955Q1-2007Q2 is considered, none of the models appears to be satisfactory. This may be due to bad measurement of either/both gaps, or to the peculiarity of the employed sample. Indeed, our test implicitly assumes a sufficiently aggressive monetary policy.<sup>23</sup> According to several contributions in the literature as well as our own estimates, passive monetary policy has appeared in several subsamples in the post WWII time span. We then repeat our exercise by conditioning first to 1985Q1-2007Q2, so to consider the previously estimated monetary policy shift. Finally, we estimate eq. (5) for the sample 1985Q2-2000Q4 so to control for the 2001 year for which we found evidence supporting passive monetary policy.<sup>24</sup>

<sup>&</sup>lt;sup>23</sup>Davig and Leeper (2007) show that, if rational agents consider a non-degenerate probability distribution over the possible future regimes, then a "generalized Taylor principle" arises. The Davig-Leeper "generalized Taylor principle" states that, in a linearized version of the non-linear Markov-switching new-Keynesian model at hand, one may find parameterizations that allows for departures from the (short-run) Taylor principle but still deliver a unique equilibrium as long as such departures are brief and/or quantitatively modest. Farmer, Waggoner, and Zha (2008) elaborate on this point and show that there exist parameterizations that meet Davig-Leeper generalized Taylor principle and ensure uniqueness in Davig-Leeper's linearized model but induce multiple equilibria in the non-linear, original framework.

 $<sup>^{24}</sup>$ We performed OLS estimations with the Newey-West covariance matrix estimator (3

Interestingly, the regression outcome allows us to discriminate between the two time-varying inflation target scenarios. Indeed, the HP-based inflation gap does never pass our "internal consistency" test. The robustness check run by trimming the sample in 2000Q4 corroborates our findings. By contrast, the ETVT model passes the test, with a negative relationship between the gaps of interest equal to -0.051 and significant when considering the 95% confidence interval.

As already pointed out, if the Fed reacts to expected inflation as opposed to current inflation, then some endogeneity issue may arise. E.g. Clarida, Gali, and Gertler (2000) estimate forward looking Taylor rules by allowing for a time-horizon of one year. In the attempt of escaping the endogeneity trap, we also perform an exercise with i = 8. Table 5 proposes our exercise with eight lags. With such a transmission lag, the ETVT model is supported in all the investigated samples. We also notice that the HPT framework gets more support from the data than in the benchmark scenario. By contrast, the EFT set up still fails to satisfy our requirements for a monetary policy transmission consistent Taylor rule, with the constant being significant in all the scenarios at hand and the slope assuming a positive value in the Great Moderation subsample.

#### Robustness Check

We checked the robustness of our findings by considering the following expanded model:

$$\pi_t - \pi_t^* = c + \xi(r_{t-i} - \overline{r}) + \delta(\pi_{t-1} - \pi_{t-1}^*) + \kappa y_{t-1} + \eta_t \tag{6}$$

We first checked the case with i = 4. With respect to the previous battery of estimates, we found that the EFT model assumes a negative and significant slope - i.e.  $-0.052^*$  - in the 1985Q1-2007Q2 sample. However, the constant still takes a positive and very significant value, then leading us to reject the model. By contrast, the HPT model displays a significant slope both in the Great Moderation sample -  $-0.083^{***}$  - and in the shorter post-'85 sample -  $-0.099^{***}$ . Joint to the fact that the constant never displays significance, this result supports the HP trend as a proxy for trend inflation.

lags) to account for heteroskedasticity and serial correlation of the estimated residuals.

The check regarding the ETVT model corroborates the previously shown results. We also verified the robustness of our findings with i = 8. All the results displayed in Table 5 are robust to the additional regressors as in eq. (6).

To summarize, our models with a time-varying inflation target turn out to be superior with respect to the one with a fixed target. The HP filter appears to be a candidate to proxy the evolution of trend inflation, but the most robustly supported model is the one with the stochastic autoregressive inflation trend. Importantly, our result emerges more clearly when conditioning to the Great Moderation subsample.

### 5 Conclusions

This paper estimates regime-switching Taylor rules with trend inflation for the post-WWII U.S. economy. Our main conclusions read as follows. First, we support regime switches and a time-varying inflation target as two features of the U.S. monetary policy conduct over the last 50 years. While there is not striking evidence in favor of distortions in the estimated policy regimes as well as policy parameters when considering different models for trend inflation, policy rules need to have a time-varying representation of the inflation target to pass the "internal consistency" test adopted in this paper and deliver a statistically significant and economically meaningful relationship between the real interest rate gap and the inflation gap. Importantly, this relationship clearly emerges when conditioning on the Great Moderation subsample, a finding suggesting the presence of a monetary policy break in the '80s.

Our estimations do not allow us to make a strong case against Hodrick-Prescott filtered inflation as empirical proxy of the time-varying inflation target, but point towards the use of the stochastic autoregressive representations as those employed by Ireland (2007), Cogley and Sbordone (2005), Cogley and Sbordone (2007), Bjørnland, Leitemo, and Maih (2007), Stock and Watson (2007), Leigh (2008) and Cogley, Primiceri, and Sargent (2008). As in Cogley and Sbordone (2005), Cogley and Sbordone (2007), and Cogley, Primiceri, and Sargent (2008), we find a drop in the inflation gap persistence when entering the Great Moderation sample. Possibly, this persistence drop is driven by the shift towards a more aggressive monetary policy, a correlation already pointed out by Benati and Surico (2008a).

Our conclusions strongly suggest to allow for time-varying inflation target in the representation of the U.S. policy conduct via simple rules. Our model for the inflation target is mainly statistical. It would be interesting to understand why the inflation target evolved over time. Possibly, imperfect knowledge of the economic structure and the evolution of the perceived inflation-output volatility trade-off by the Fed is one of the candidate explanation to interpret our results. Interesting efforts in this direction have already been undertaken by Cogley and Sargent (2005b), Primiceri (2006), Sargent, Williams, and Zha (2006), and Carboni and Ellison (2008). Once established that the time-varying inflation target is an important ingredient to describe the U.S. inflation rise and fall in the post-WWII sample, the switch from the positive to the normative standpoint appears to be warranted. How should monetary policy be conducted in presence of trend inflation? This question regards a still largely unexplored research territory. An interesting analysis tackling this issue has recently been proposed by Ascari and Ropele (2007a).

### 6 Technical Appendix: Estimation Algorithm

The goal of the inferential procedure is to estimate parameters and latent processes, that in the general model are switching regimes and time varying targets. Since regimes and targets are not observed, they are treated as missing data in an Markov Chain MonteCarlo (MCMC) setup where the target distribution is  $\pi(\mathbf{S}, \boldsymbol{\pi}^*, \boldsymbol{\theta} | \mathbf{Y})$ . The vector  $\mathbf{S}$  represents the regimes,  $\boldsymbol{\theta} = \left(\bar{r}, \rho_i, \alpha_i, \beta_i, \sigma_{\epsilon_i^{MP}}^2, \pi^{LR}, \rho_{\pi}, \phi_i, \sigma_{\epsilon^{\pi^*}}^2, \sigma_{\epsilon^z}^2, p_{01}, p_{10}\right), \ i = 0, 1$  is the set of parameters,  $\boldsymbol{\pi}^*$  is the time varying target, and  $\mathbf{Y}$  is the vector of observables, i.e.,  $i_t, y_t$  and  $\pi_t, t = 1, \ldots, T$ .

All the unobserved quantities can be simulated individually through the Gibbs sampler algorithm. This approach allows not to directly compute the likelihood function, which is a highly multivariate integral. The basic idea behind MCMC is to build a Markov chain transition kernel starting from some initial state ( $\boldsymbol{\theta}^{(0)}, \mathbf{S}^{(0)}, \boldsymbol{\pi}^{*(0)}$ ), with limiting invariant distribution equal to the posterior distribution of the quantities of interest. Under suitable conditions (Robert and Casella (1999), chapters 6-7), we can build such a transition kernel generating a Markov chain { $\boldsymbol{\theta}^{(n)}, \mathbf{S}^{(n)}, \boldsymbol{\pi}^{*(n)}$ } whose elements (draws) converge in distribution to the (target) posterior density  $p(\boldsymbol{\theta}, \mathbf{S}, \boldsymbol{\pi}^* | \mathbf{Y})$ . Once convergence is achieved, we obtain a sample of serially dependent simulated "observations" on the parameter vector  $\boldsymbol{\theta}$  (and on the latent processes involved), which can be used to perform Monte Carlo inference. More precisely, estimates of the latent factors are given by averaging over the realization of the chain, i.e.  $\hat{\pi}_t^* = n^{-1} \sum_{j=1}^n \pi_t^{*(j)}$  and  $\hat{Pr}(S_t = 1) = n^{-1} \sum_{j=1}^n S_t^{(j)}$  respectively.

MCMC for switching regime ARMA models have been introduced by Albert and Chib (1993), McCulloch and Tsay (1993) and successively developed by Billio, Monfort, and Robert (1999). Here we extend their framework by considering models with an extra-latent factor, namely the time varying target  $\pi^*$ , and adapting the efficient approach by Chib (1996) to update the states S. In particular, we build up an efficient algorithm based on the multi-move Gibbs sampler proposed by Chib (1996) to update the states  $S_t$ ,  $t = 1, \ldots, T$ . We simulate  $(S_1, \ldots, S_T)$  in block from their joint distribution given the data and the other parameters. As suggested by Shephard (1994) and Carter and Kohn (1994) amongst other, this approach reduces the autocorrelation between states and speeds up the convergence of the chain to its invariant distribution. It is easy to show that the full conditional distributions of the latent target  $\pi^*$  are Gaussian. We also used conjugate priors wherever possible. This choice allows to obtain standard conditional posterior distributions from which we can draw by direct simulation.

It is easy to show that the full conditional distributions for  $\rho_i, \alpha_i, \beta_i, \rho_{\pi}, \phi_i, i = 0, 1$  are truncated Gaussian,  $\bar{r}$  and  $\pi^{LR}$  are Gaussian,  $\sigma^2_{\epsilon_i^{MP}}, i = 0, 1, \sigma^2_{\epsilon^{\pi^*}},$ and  $\sigma^2_{\epsilon^z}$  are Inverse Gamma, whereas  $p_{01}$  and  $p_{10}$  are Beta. Since all these quantities can be sampled directly, each sub-move of the chain is accepted. Furthermore, to identify the two states, we impose some constraints on the parameter space such that  $\alpha_0 > 0 > \alpha_1$  and  $\sigma^2_{\epsilon_0^{MP}} < \sigma^2_{\epsilon_1^{MP}}$ .<sup>25</sup> To simulate truncated Gamma random variables we use the accept-reject method proposed by Philippe (1997).

Our algorithms works as follows:

- Initialize the chain at  $(\boldsymbol{\theta}^{(0)}, \boldsymbol{S}^{(0)}, \boldsymbol{\pi}^{*(0)})$
- At step  $n = 1, \ldots, N$ 
  - Update  $(S_1^{(n)}, \ldots, S_T^{(n)})$  in block from  $p(\mathbf{S}|\boldsymbol{\pi}^{*(n-1)}, \boldsymbol{\theta}^{(n-1)}, \mathbf{Y})$  as suggested by Chib (1996);
  - Update  $\pi_t^*$ , t = 1, ..., T one-at-a-time from their full conditional distributions  $p(\pi_t^* | \mathbf{S}^{(n)}, \pi_{t-1}^{*(n)}, \pi_{t+1}^{*(n-1)}, \boldsymbol{\theta}^{(n-1)}, \mathbf{Y});$
  - Update  $\boldsymbol{\theta}$  one-at-a-time from  $p(\boldsymbol{\theta}_i | \boldsymbol{S}^{(n)}, \boldsymbol{\pi}^{*}{}^{(n)}, \boldsymbol{\theta}_{-i^-}^{(n)}, \boldsymbol{\theta}_{-i^+}^{(n-1)}, \boldsymbol{Y})$ , where  $\boldsymbol{\theta}_{-i^-}$  are the first (i-1) elements of  $\boldsymbol{\theta}$  and  $\boldsymbol{\theta}_{-i^+}$  are the elements form the (i+1)-th to the last.

We run this algorithm for 50,000 iterations with a burn-in of 25,000, which in our experience is large enough to render the impact of initial conditions insignificant. To account for the serial correlation of the draws, we estimated the numerical standard error of the sample mean by using the approach adopted by Kim, Shephard, and Chib (1998).

 $<sup>^{25}</sup>$ For investigations concerning identification issues related to the estimation of regimeswitching models, see Fruwirth-Schnatter (2001) and Geweke and Keane (2007).

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EFT	HPT	ETVT
1958Q1 - 1958Q2	_	—
1969Q2 - 1976Q1	1969Q3 - 1976Q1	1969Q2 - 1975Q2
1979Q3 - 1985Q1	1979Q4 - 1985Q1	1979Q3 - 1984Q4
2001Q1 - 2001Q4	2001Q3 - 2002Q1	2001Q1 - 2001Q4

Table 1: SUBSAMPLES ASSOCIATED TO PASSIVE MONETARY POL-ICY: MODEL COMPARISON. Description of the different models: See Figure 1.

Param.	Prior	EFT	HPT	ETVT
$\overline{r}$	N(2.0, 0.5)	2.25 [1.27,3.13]	2.45 [1.78,3.15]	2.50 [1.80,3.20]
$\pi^{LR}$	N(2.0, 0.5)	1.06 [0.10,2.51]	2.13 [0.96,3.25]	2.39 [1.21,3.45]
$1 + \alpha(S_t^0)$	N(2.0, 0.5)	1.53 [1.07,2.36]	1.25 [1.02,1.85]	1.29 [1.02,2.04]
$1 + \alpha(S_t^1)$	N(0.5,0.5)	0.84 [0.48,0.98]	0.72 [0.30,0.97]	0.69 [0.25,0.96]
$\beta(S_t^0)$	N(0.25, 0.15)	0.73 [0.33,0.97]	0.81 [0.52,0.98]	0.80 [0.48,0.98]
$\beta(S_t^1)$	N(0.25, 0.15)	0.52 [0.10,0.92]	0.47 [0.08,0.89]	0.47 $[0.08,0.89]$
$\rho(S_t^0)$	N(0.8, 1.0)	0.94 [0.91,0.97]	0.93 [0.90,0.95]	$\underset{\left[0.93\right.}{0.90,0.95\right]}$
$\rho(S_t^1)$	N(0.8, 1.0)	0.89 [0.80,0.96]	0.87 [0.78,0.95]	0.86 [0.77,0.94]
$\sigma^2_{\epsilon^{MP}}(S^0_t)$	IG(2.5, 0.75)	0.15 [0.11,0.20]	0.18 [0.14,0.24]	$\underset{[0.13,0.24]}{0.18}$
$\sigma^2_{\epsilon^{MP}}(S^1_t)$	IG(2.5, 0.75)	2.26 [1.62,3.27]	2.45 [1.76,3.64]	2.44 [1.71,3.77]
$\rho_{\pi}$	N(0.8, 0.1)	_	_	0.97 [0.94,0.99]
$\phi_z(S^0_t)$	N(0.5, 1.0)	_	—	0.06 [0.01,0.22]
$\phi_z(S^1_t)$	N(0.5, 1.0)	_	—	0.41 [0.03,0.83]
$\sigma^2_{\epsilon^{\pi*}}$	IG(2.5, 0.75)	_	—	0.30 [0.17,0.50]
$\sigma^2_{\epsilon^z}$	IG(2.5, 0.75)	_	—	0.78 [0.57,0.98]
<i>p</i> <sub>01</sub>	Beta(5,95)	0.04 [0.02,0.06]	$\begin{array}{c} 0.03 \\ \scriptscriptstyle [0.02, 0.05] \end{array}$	$\underset{\left[0.02,0.05\right]}{0.03}$
$p_{10}$	Beta(5,95)	$\underset{\left[0.03,0.10\right]}{0.06}$	$\underset{\left[0.03,0.09\right]}{0.05}$	0.06 [0.03,0.10]

Table 2: ESTIMATED MONETARY POLICY RULES: FIXED vs. TIME-VARYING SHORT RUN INFLATION TARGET. Figures reported in the Table are medians of the estimated posterior distributions; [5th,95th] percentile in squared brackets. Beta priors for the switching probabilities defined by their shape parameters. Description of the different models: See Figure 1. Details on estimation procedure reported in the text.

$\Delta \phi_z : p \epsilon$	ercentiles	$\Delta\phi_z < -k$		$Pr(\phi_z^0 < k \cap \phi_z^1 < k)$			
84th	95th	k = 0	k = 0.05	k = 0.1	k = 0.2	k = 0.05	k = 0.10
-0.02	0.09	86.68%	80.49%	74.99%	64.05%	3.30%	10.38%

Table 3: DIFFERENCE INFLATION GAP PERSISTENCE DENSITY: PROBABILITIES. Further explanations on the statistics reported in the table are detailed in the text.

Sample	Est.param.	EFT	HPT	ETVT
1955Q1 - 2007Q2	$\widehat{c}_{(st.dev.)}$	$2.635^{***}$ (0.370)	0.007 (0.118)	0.023 (0.052)
	$\widehat{\xi}_{(st.dev.)}$	-0.090 (0.098)	-0.007 (0.047)	-0.013 (0.020)
1985Q1 - 2007Q2	$\widehat{c}_{(st.dev.)}$	$1.500^{***}$ (0.144)	$\underset{(0.082)}{0.012}$	$\underset{(0.034)}{0.010}$
	$\widehat{\xi}_{(st.dev.)}$	$\underset{(0.056)}{-0.038}$	$-0.042$ $_{(0.036)}$	$-0.026^{*}_{(0.013)}$
1985Q1 - 2000Q4	$\widehat{c}_{(st.dev.)}$	$1.153^{***}_{(0.192)}$	$\underset{(0.107)}{0.089}$	$\underset{(0.049)}{0.052}$
	$\widehat{\xi}_{(st.dev.)}$	$\underset{(0.068)}{0.079}$	$-0.079$ $_{(0.064)}$	$-0.051^{**}$ (0.021)

Table 4: INFLATION-REAL INTEREST RATE GAPS: STATISTICAL RE-LATIONSHIPS. Inflation gap regressed on a constant and the real interest rate gap (lagged four periods). Sample: 1985Q2-2007Q2. Estimation performed via OLS with Newey-West HAC VCV matrix (lag truncation=3). \*\*\*/\*\*/\* stands for 99/95/90 per cent statistical significance. Description of the different models: See Figure 1. Details on the computation of the gaps reported in the text.

Sample	Est.param.	EFT	HPT	ETVT
1955Q1 - 2007Q2	$\widehat{c}$ (st.dev.)	$\begin{array}{c} 2.783^{***} \\ \scriptscriptstyle (0.415) \end{array}$	-0.017 (0.114)	$\underset{(0.050)}{0.025}$
	$\widehat{\xi}_{(st.dev.)}$	$-0.213^{**}$ (0.102)	$-0.068^{**}$ (0.032)	$-0.039^{***}$ (0.015)
1985Q1 - 2007Q2	$\widehat{c}_{(st.dev.)}$	$1.527^{**}$ (0.160)	$\underset{(0.082)}{0.031}$	$\underset{(0.034)}{0.019}$
	$\xi$ (st.dev.)	$\underset{(0.045)}{-0.049}$	$-0.060^{*}$ (0.031)	$-0.033^{**}$ (0.013)
1985Q1 - 2000Q4	$\widehat{c}_{(st.dev.)}$	$1.104^{***}$ (0.239)	$\underset{(0.105)}{0.134}$	$\underset{(0.043)}{0.056}$
	$\widehat{\xi}$ (st.dev.)	$0.092^{*}_{(0.046)}$	$-0.102^{***}$ (0.037)	$-0.045^{***}$ (0.017)

Table 5: INFLATION-REAL INTEREST RATE GAPS: STATISTICAL RE-LATIONSHIPS. Inflation gap regressed on a constant and the real interest rate gap (lagged eight periods). Sample: 1985Q2-2007Q2. Estimation performed via OLS with Newey-West HAC VCV matrix (lag truncation=3). \*\*\*/\*\*/\* stands for 99/95/90 per cent statistical significance. Description of the different models: See Figure 1. Details on the computation of the gaps reported in the text.

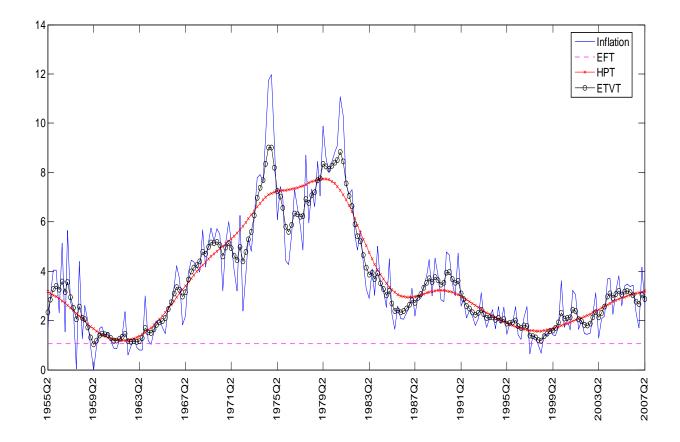


Figure 1: INFLATION RATE AND ESTIMATED TARGETS. EFT: estimated inflation target according to the fixed-inflation target model. HPT: HP-filter (weight: 1,600) of the inflation series. ETVT: time-varying inflation target estimated with a stochastic autoregressive model. EFT and ETVT refer to means of the estimated posterior densities (smoothed estimates).

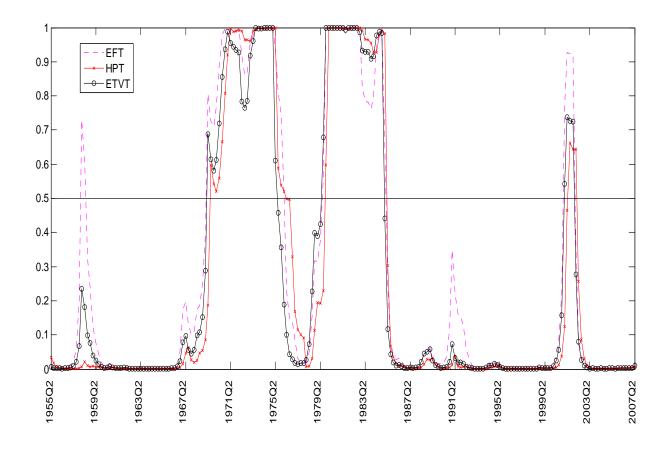


Figure 2: ESTIMATED PROBABILITY OF BEING IN THE PASSIVE MONETARY POLICY STATE: MODEL COMPARISON. Description of the different models: See Figure 1. Smoothed estimates of the probability of being in State 1.

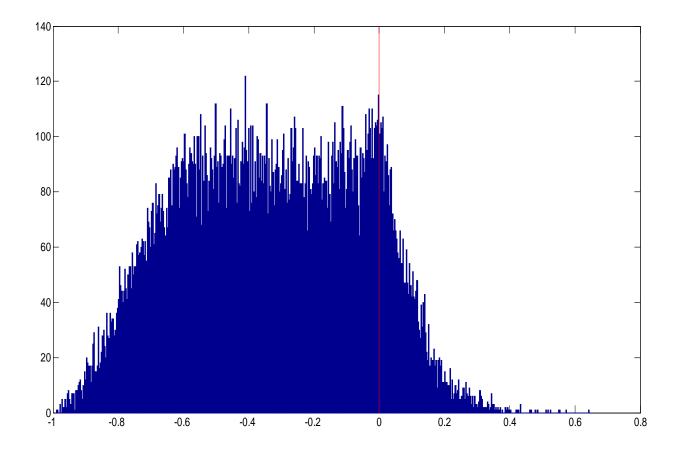


Figure 3: ESTIMATED INFLATION GAP PERSISTENCE: DIFFERENCE ACROSS REGIMES. This distribution plots 25,000 realizations of  $\Delta \phi_{z,j} = \phi_{z,j}(S_t = 0) - \phi_{z,j}(S_t = 1)$ , where *j* identifies the *jth* draw. More details on the construction of this density in the text.

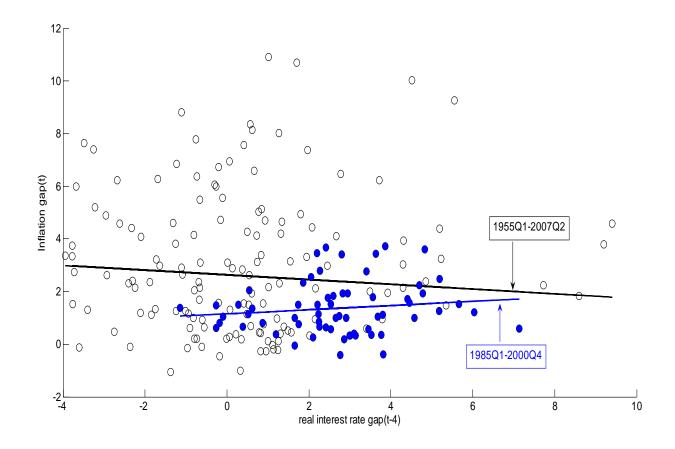


Figure 4: REAL INTEREST RATE GAP vs. INFLATION GAP: ESTI-MATED FIXED TARGET MODEL. Scatter plots involving the real interest rate gap - lagged four periods - and the inflation gap. Empty black circles, black line: Full sample analysis. Filled blue circles, blue line: Great Moderation subsample. Computation of the monetary policy gaps detailed in the text.

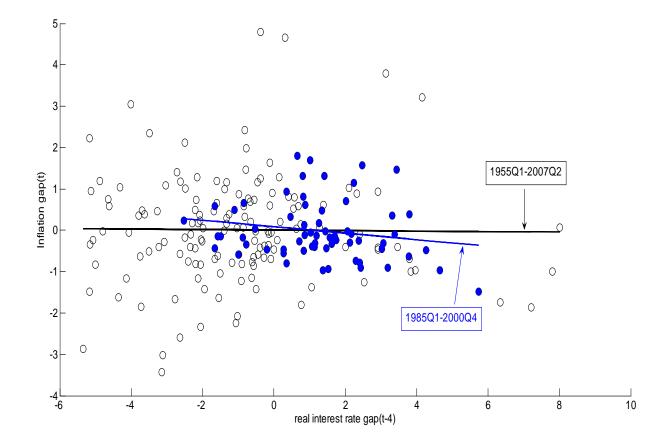


Figure 5: REAL INTEREST RATE GAP vs. INFLATION GAP: HODRICK-PRESCOTT TARGET MODEL. Scatter plots involving the real interest rate gap - lagged four periods - and the inflation gap. Empty black circles, black line: Full sample analysis. Filled blue circles, blue line: Great Moderation subsample. Computation of the monetary policy gaps detailed in the text.

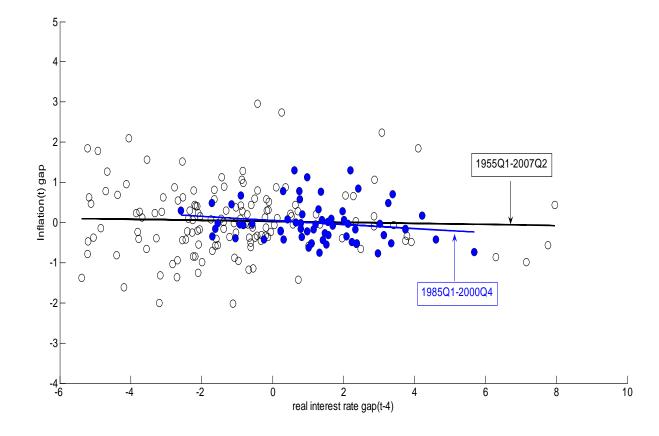


Figure 6: REAL INTEREST RATE GAP vs. INFLATION GAP: ESTI-MATED TIME-VARYING TARGET MODEL. Scatter plots involving the real interest rate gap - lagged four periods - and the inflation gap. Empty black circles, black line: Full sample analysis. Filled blue circles, blue line: Great Moderation subsample. Computation of the monetary policy gaps detailed in the text.