Debt and inflation risk in a monetary union

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Comments welcome

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Abstract

This paper investigates how member states of a monetary union are affected by government debt and the risk of inflation that might be involved when different pension schemes are in place. I use a stochastic two-country two-period overlapping-generations model, where one country uses a PAYG pension scheme and the other country uses a fully funded retirement system. There is productivity risk on stocks and inflation risk on government bonds. The paper first shows that a country using a PAYG pension system gains from unexpected inflation at the cost of the country that uses a funded system. If it is not clear to market participants how the central bank will react to these conflicting interests about inflation, inflation *risk* may rise with the level of government debt in the PAYG country. Higher inflation risk affects both countries negatively as the rate of return on bonds, and thus the interest obligations on government debt will rise. The scenarios sketched in this paper might be especially relevant in the coming decades when the population ages and the higher pension burden will put the public finances more under pressure.

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1 Introduction

In the coming decades population ageing will put public finances in the European Economic and Monetary Union (EMU) under great pressure. This especially holds for countries that have large public pension schemes financed on the basis of pay-as-you-go (PAYG), where the working population pay taxes to finance the pension benefits of the elderly. In these countries the projected increases in government expenditures are enormous (see EuropeanCommission (2006)) and the temptation for governments to use debt instead of raising taxes/lowering pension benefits will be large[1].

High levels of nominal government debt gives governments an incentive to lobby for surprise inflation at the central bank as this will reduce the fiscal burden of debt service. The question is of course whether the central bank will give in and create unexpected inflation. The European Central Bank (ECB), for example, is formally independent. The decision-making process about monetary policy, however, is not always transparent to market participants. This implies that the market does not exactly know how the ECB will react when debt levels are high and investors may perceive that the risk of inflation is higher. Higher levels of government debt will in that case go together with a higher *risk* of inflation. In a monetary union like the EMU this will necessarily affect other members that accumulated large pension funds.

This paper analyses the effects of government debt and the associated risk of inflation when countries with different pension schemes form a monetary union.

To address both the issue of government debt and inflation risk I develop a stochastic two-period overlapping generations (OLG) model with two countries that form a monetary union. One country has fully funded pensions and the other country relies on PAYG-financed defined benefit pensions. Consumers allocate their investment portfolio between stocks and government bonds. Both assets are risky, productivity risk makes the return on stocks uncertain and there is inflation risk on government bonds. In a model where the asset returns are uncertain the difference in pension schemes implies that people in the two countries have different attitudes towards risk. People who receive a safe PAYG pension benefit act like less risk averse individuals and prefer to have a more risky portfolio compared to people that do not receive a safe pension benefit, i.e., have fully funded pensions.

The major advantage of the stochastic general equilibrium model in this paper is that it is fairly simple so that explicit solutions are still feasible. Most papers that develop stochastic general equilibrium OLG models use computable models (e.g., Storesletten et al. (1999), Sánchez-Marcos and Sánchez-Martin (2006), and Krueger and Kubler (2006)). There are a few papers that develop an analytical stochastic general equilibrium OLG model, like Bohn (1998, 2001, 2003) and Beetsma and Bovenberg

[1]Given the exceptions made for Germany and France in 2003 this scenario might actually occur.

(2007). These papers do not, however, derive the optimal conditions for savings- and portfolio decisions of consumers as I do in my model.

To obtain an explicit expression for the optimal portfolio share I use the approach of Campbell and Viceira (2002). This approach is also taken by Matsen and Thogersen (2004) who develop a partial equilibrium model where the PAYG pension system is treated as a 'quasi-asset' and they derive the optimal share invested in this PAYG asset. Matsen and Thogersen (2004) assume that people only consume in the second period of life, so that the complete net labour income received in the first period of life is saved. In contrast to Matsen and Thogersen (2004) I model the savings decisions of individuals and more importantly, I develop a general equilibrium model where the effects on the rates of return are taken into account.

An important channel in this paper is that countries with high debt levels might be able to put pressure on the central bank to follow an accommodating policy in order to reduce the debt burden. This implies that we assume that the monetary authority might not be completely independent and may have an incentive to inflate away nominal debt. One could argue that this assumption is not very realistic as monetary policy has become more and more independent in the last decades. I argue, however, that in case PAYG countries finance their increased pension obligations by issuing more debt, that, given the scope of projected increases in expenditures (see EuropeanCommission (2006)), the accumulation of debt is extensive. These high debt levels substantially increase the pressure on the central bank to give in. In the European Economic and Monetary Union (EMU) it may happen, for example, that a group of large countries that rely to a large extent on PAYG-financed pensions (e.g. Italy, Germany and France) put pressure on its national delegates on the board of the ECB to accommodate.

Another argument, put forward for example by Leith and Wren-Lewis (2006), Schabert and Van Wijnbergen (2006), Annicchiarico et al. (2006), could be that the combination of high nominal debt and an active inflation targeting policy by the central bank may result in unstable debt dynamics. The reasoning is that a high level of government debt increases inflationary pressures. If the central bank raises the nominal interest rate in response to higher expected inflation, the service costs of debt rise, which further increases the real debt and thus leads to unstable debt dynamics. To avoid an explosive path for debt, the monetary authority has to follow a "passive" policy such that the rise in inflation reduces real interest rates and the debt is stabilised. These studies assume Calvo-type price rigidities and the central bank follows a Taylor-type interest rate rule. In contrast to these papers I do not model monetary policy explicitly, but I introduce a simple ad-hoc specification of the link between government debt and inflation (risk). Moreover, I assume that governments only increase their debt for a while and make sure that the debt dynamics are stable. The nominal rigidity in the model does not originate from the fact that imperfectly competitive firms can only change their prices infrequently, but from the fact that people invest in long-term nominal bonds. This assumption is justified by the fact that the introduction of market-based valuation by regulators forces pension funds to use the yields of fixed-income instruments as the basis for discounting liabilities. In response to this, pension funds hold a larger fraction of long-term bonds to limit the volatility of the regulatory funding ratios and to reduce the duration gap between assets and liabilities. Long-term bonds are still mostly denominated in nominal terms[2]. Unexpected inflation has real effects because it creates a wedge between the ex post real interest rate and the ex ante rate.

Unexpected inflation decreases the real value of government debt and the real return on government bonds[3]. This creates a gain for the government, while debt holders lose. In a closed-economy setting unexpected inflation is a zero-sum game, that is, the gain for the government is exactly high enough to compensate the people who lose from surprise inflation. In an asymmetric monetary union where one country relies on PAYG pensions and the other country has a fully funded pension scheme, however, the PAYG country gains from unexpected inflation at the cost of the funded countr. This means that there is a conflict of interest on monetary policy when countries with different pension schemes form a monetary union. The reason for this result is that residents of the funded country hold a relatively large share of the total amount of government bonds because they save more and have a more conservative investment portfolio. This implies that the PAYG country can export part of the inflationary tax on debt holders to the funded country, while it still receives the full gain of a lower debt burden and a net gain results. The PAYG country therefore has an incentive to lobby for suprise inflation at the central bank when it forms a monetary union with a funded country.

This gain of unexpected inflation for a PAYG country rises with the amount of nominal government debt. In the coming decades the ageing of the population will put the public finances more under pressure. In case PAYG countries finance their increased pension obligations by issuing more debt, the incentive of PAYG governments to lobby for surprise inflation will rise. On the other hand will unexpected inflation harm funded countries more if PAYG countries issue large amounts of nominal government debt. The conflict of interests on monetary policy between PAYG- and funded countries will therefore be reinforced if population ageing raises government debt in PAYG countries.

[2]This is especially relevant for the Netherlands, where a large part of old-age consumption is financed by pension benefits which are taken care of by pension funds. In 2005 the investment portfolio of the largest pension fund in the Netherlands, the ABP, consisted for 40 percent of nominal bonds (see ABP, 2006, Table 5.5).

[3]Expected inflation will not have any real effects because people will ask a higher nominal rate of return on government bonds, so that the real rate of return does not change.

The question is of course whether the central bank will give in and create unexpected inflation. The European Central Bank (ECB), for example, is formally independent. If, however, the decision-making process of the central bank is not completely transparent, it will not be clear to market participants how the central bank will react to these conflicting interests about the creation of inflation between PAYG-and funded countries. As a consequence, there will be more uncertainty about what the final outcome for inflation will be and inflation *risk* may rise with the level of government debt in PAYG countries. Higher inflation risk makes government bonds more risky and less attractive to hold. The rate of return on bonds will rise relative to the return on equity (i.e, the equity premium falls) to induce people to buy the existing stock of government debt. The increase of the rate of return on government bonds implies that the costs of government debt rise, which harms both groups of countries. Actually, the negative utility effects from a rise in inflation risk is larger for the PAYG country if it forms a monetary union with a country that uses a fully funded pension scheme instead of a PAYG system. This result arises because the funded country holds a relatively large part of the government bonds and people in the funded country are more risk averse because they do not receive a safe PAYG pension benefit. Therefore these people need to be compensated more in order to hold the more risky government bonds, that is, the rate of return on government bonds has rise to a larger extent when a funded pension scheme is in place.

The analysis in this paper therefore points out that in a monetary union like the EMU it may be in the interest of both funded- and PAYG countries to obey the fiscal constraints stated in the Stability and Growth Pact to prevent large increases in government debt, which may raise inflation risk. Moreover, it is important for all countries that a central bank like the ECB is independent, credible *and* transparent to prevent an increase in inflation risk when debt levels are high.

A number of other papers have also argued in favour of fiscal restrictions in case of monetary unification. Chari and Kehoe (2007), for example argue that there will be a free-rider problem in fiscal policy in case there is a time-inconsistency problem in monetary policy resulting from an inflation bias by the central bank. This free-rider problem arises because the fiscal authorities do not fully take into account the cost of higher inflation due to a higher stock of public debt, as part of these costs wil be borne by residents of other countries in the monetary union. This results in too much debt accumulation and an inflation rate that is too high. Beetsma and Bovenberg (1999) also argue that monetary unification leads to excessive debt accumulation due to the lack of commitment in monetary policy and myopic governments. In these papers the countries in the monetary union are symmetric and the interaction between monetary policy and fiscal policy is modelled as a game. The objective function of the central bank comprises the inflation rate, output (gap) and government debt. In contrast to these papers I take monetary policy as exogenous, we just note that high debt levels may lead to high inflation (risk). Moreover, the countries in our model are asymmetric and I show that this asymmetry has the effect that one country gains from unexpected inflation at the expense of the other country.

This paper is organised as follows. Section 2 presents the stochastic OLG model. In Section 3 I discuss the effects of government debt. Section 4 analyses the effects of an unexpected inflation shock where both countries try to compensate the people that lose from this shock; the elderly. As explained above the government in the PAYG country can implement a policy that is Pareto improving in its own country, while the residents in the funded country necessarily lose. This result shows that within the EMU there will be conflicting interests between different groups of countries in setting monetary policy in case debt of capital-importing (PAYG) countries reach high levels. Section 5 explores the spillover effects in case these conflicts of interest on monetary policy raise perceived inflation risk. Section 6 concludes the paper.

2 Model

Following Buiter (1981) and Persson (1985), I will use a two-period overlapping-generations model of an open economy. The world consists of two countries, country F and country P, which differ in the way the pensions are financed. Country P relies on a PAYG pension system and country F has a fully funded pension scheme. I assume a constant population size[4] and dynamic efficiency in both countries. Countries may, however, differ in population size. In this way, I allow for scale differences between the two countries. Normalizing the working population in country P, L^P, to one and define $\frac{L^F}{L^P} = \nu$, then ν tells us the relative size of the active population in country F, L^F. The countries are identical in all other respects. I use the framework of Campbell and Viceira (2002) to derive an explicit solution for the portfolio choice of consumers. All variables in the model are expressed as the amount per young individual in the country and lowercase letters refer to the logarithm of the respective variable.

[4]This assumption may come as a surprise as the main motivation of this paper, as sketched in the Introduction, is that population ageing will exert great pressure on public finances in the coming decades. To keep the results tractable, however, we leave population ageing out of the analysis. The upcoming rise of the relative number of elderly compared to the working people provides a good argument why the scenarios described in this paper might occur in the future. The interested reader is referred to Adema (2008*b*) where the rise in government debt evolves endogenously after a rise in life expectancy. The international spillover effects of pensions under population ageing without the use of government debt are analysed in Adema et al. (2008*a*).

2.1 Risk factors

There are two risk factors, there is productivity risk on stocks and inflation risk on government bonds.

Production

Production per worker is described by a standard neoclassical constant-returns-to-scale Cobb-Douglas production function:

$$F(A_t, K_t) = A_t(K_t^i)^\alpha \tag{1}$$

where A_t denotes productivity at time t, α the production elasticity, and K_t^i the amount of capital per young individual at time t in country i, $i = P, F$. Profit maximization and perfect competition among producers results in the usual equilibrium conditions:

$$W_t^i = (1-\alpha)A_t(K_t^i)^\alpha \tag{2}$$

$$R_{k,t}^i + \delta = \alpha A_t(K_t^i)^{\alpha-1} \tag{3}$$

where W_t^i is the real wage, $R_{k,t}^i$ the return to capital and δ the depreciation rate of capital. There is perfect capital mobility between the two countries, but labor is immobile. Since capital can freely move across countries, the rates of return will be equalized, i.e., $R_{k,t}^P = R_{k,t}^F = R_{k,t}$, $\forall t$. As both countries are endowed with the same production technology, we have $K_t^P = K_t^F = K_t$, and consequently $W_t^P = W_t^F = W_t$. Following Campbell and Viceira (2002) I assume that the gross return on capital $(1 + R_{k,t})$ is lognormal distributed[5]. To achieve this I assume that A_t is lognormal distributed and that there is 100% depreciation, i.e., $\delta = 1$. This implies that both W_t and $R_{k,t}$ are stochastic. People do not have to form expectations about W_t, however, as A_t is already known before W_t is paid (see Section 2.2, Figure 1). People base their saving- and portfolio decisions on the future return of capital, so they *do* have to form expectations about this variable. The variance of the log of the gross return on capital is equal to the variance of the log of productivity (see Appendix A.1 for details), i.e.,

$$\text{Var}_t[\log(1 + R_{k,t+1})] = \text{Var}_t[\log A_{t+1}] \tag{4}$$

$$\sigma_{k,t}^2 = \sigma_{a,t}^2 \tag{5}$$

[5]In case a variable X is lognormally distributed, this means that:

$$\log X \sim N\left(E(\log X), \sigma_x^2\right)$$

The expectation and variance of X are equal to:

$$E(X) = \exp[E(\log X) + \frac{1}{2}\sigma_x^2]$$

$$\sigma_X^2 = \exp[2E(\log X) + \sigma_x^2][\exp(\sigma_x^2) - 1]$$

Inflation

Government debt is denominated in nominal terms and therefore there is inflation risk on the return on government bonds. Inflation (risk) is the same in the two countries as they form a monetary union and there is no country-specific risk, like default risk, on government bonds[6]. Perfect capital mobility equalizes the rates of return on government bonds. The real return on government bonds is equal to:

$$1 + R_{b,t} = \frac{1 + R^N_{b,t}}{1 + \pi_t} \tag{6}$$

where $R^N_{b,t}$ is the nominal return on government bonds and π_t the inflation rate between $t-1$ and t. The nominal return $R^N_{b,t}$ is a predetermined variable and I assume that the inflation factor ($\frac{1}{1+\pi_t}$) is lognormally distributed. In Appendix A.2 we derive that:

$$\begin{aligned} \mathrm{Var}_t\left[\log(1 + R_{b,t+1})\right] &= \mathrm{Var}_t\left[\log\left(\frac{1}{1+\pi_{t+1}}\right)\right] && (7) \\ \sigma^2_{b,t} &= \sigma^2_{\pi,t} && (8) \end{aligned}$$

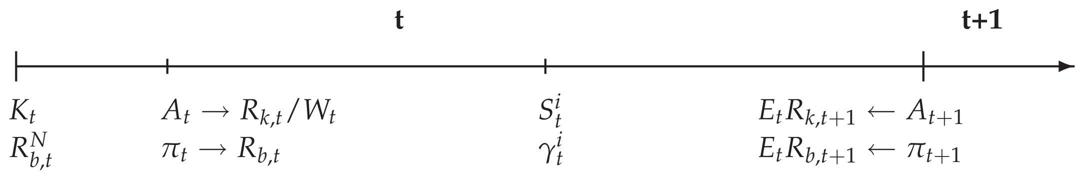
There are no risk-free indexed bonds, which implies that there is a missing market. Safe income in the second period of life, like defined-benefit pensions, would (partly) fill this gap.

2.2 Timing

The sequence of events is shown in Figure 1. At the beginning of period t, the capital stock K_t and the nominal interest rate on government bonds $R^N_{b,t}$ are inherited from the previous period, as they are determined by the savings and portfolio decisions made in period $t-1$. Then, productivity and inflation are revealed. With this knowledge firms choose factor employment and the real return on government bonds is determined. Subsequently, households make their portfolio choice γ^i_t and saving decisions S^i_t (and thereby their consumption decisions), $i = P, F$, which are also based on the expected future asset returns. Consumers only face uncertainty about the return on their savings.

[6]Harald Uhlig: Maybe better just to say that the return on bonds is stochastic instead of calling it inflation risk as the model is real for rest. **But:** in current version model risk on bonds should be the same in the two countries, so I cannot call it default risk because this can differ between governments. Inflation risk, however, should be the same in the two countries when they form a monetary union. $\rightarrow$ Maar bij de paper over H4 kan ik dit wel doen!

Figure 1: Timing of events



2.3 Pensions and government debt

Intially, the government in country P runs a balanced PAYG pension system, that is, pension benefits of the elderly (Z^P_t) are covered by lump-sum taxes of the young (T^P_t)[7]:

$$Z^P = T^P \tag{9}$$

An important assumption in this paper is that PAYG pension benefits are guaranteed in real terms, that is, PAYG pension benefits are safe. This implies that the PAYG pension scheme partly removes the market incompleteness that people cannot invest in any risk-free asset, as the system offers the possibility to transfer income from the working period to retirement at a non-stochastic rate of return. In this way I incorporate the fact that a PAYG pension scheme also serves as a risk-sharing and diversification device.[8]

Governments issue one-period debt, which yields the real interest rate $R_{b,t}$. Instead of levying taxes on the young (T^P_t), the government in the PAYG country can also use public debt for a while to finance the pension benefits of the elderly. At a later stage, additional contributions ($T^{B,P}_t$) are raised to finance the interest obligations on the debt, so as to keep debt per worker constant. The government budget constraint

[7]By omitting time subscripts, I denote the initial steady state value of the respective variable.

[8]The PAYG pension scheme has a comparable role as in the literature that focuses on the intergenerational risk-sharing properties of pension schemes. The idea of this literature is that financial markets are incomplete because there cannot be trade with unborn generations and human capital is not traded. As a result of these missing markets the young are too much exposed to wage risk and the old bear too much financial market risk. In case financial market returns are imperfectly correlated with wages, this results in suboptimal diversification. By linking PAYG pension benefits to wages, retired households obtain a claim to human capital which is not traded on financial markets. In this way PAYG pension schemes can contribute to better intergenerational risk sharing and diversification. I do not link pension benefits to wages because in this model wages are perfectly correlated with stock market returns (both are affected by productivity risk) and therefore wage-linked pension benefits will not improve diversifaction opportunities. PAYG pension benefits are modeled as safe lump-sum benefits and in this way pension benefits are imperfectly correlated with financial market returns and therefore contribute to better diversification. To allow for imperfect correlation between labour and capital income an often-used trick is to assume that there is is depreciation risk besides productivity risk. As soon as PAYG pension benefits are linked to (past or current) wages, Z^P_t becomes stochastic. I leave this for future research as this would complicate the analysis to a large extent.

in the PAYG country is given by:

$$B_{t+1}^{P} = (1 + R_{b,t})B_t^P + Z_t^P - T_t^P - T_t^{B,P} \tag{10}$$

where public debt per young individual at time t is denoted by B_t^P. In case of a balanced PAYG system, i.e., $Z_t^P = T_t^P$, debt per worker is stabilized at B^P if:

$$T_t^{B,P} = R_{b,t}B^P \tag{11}$$

Note that $R_{b,t}$ is known at time t as both $R_{b,t}^N$ and π_t are already known at the beginning of period t (see Figure 1).

In country F, pension funds invest the contributions of the young (T_t^F) and return them with interest in the next period in the form of transfers to the then old agents (Z_{t+1}^F). The funded scheme has fixed contributions and the pension fund has the same investment strategy as individuals. This implies that contributions to the pension scheme are exactly offset by an equal reduction in private savings. The funded pension system is neutral in the sense that the economy behaves exactly the same as if there were no pension scheme. Therefore, I do not distinguish between contributions to the funded pension scheme and private savings, that is, pension contributions T_t^F are included in total savings S_t^F. Moreover, opposed to the PAYG country, the pension benefits in the funded are just as risky as savings.

As pensions in the funded country are organised by pension funds and not by the government, they do not enter the government budget constraint:

$$B_{t+1}^F = (1 + R_{b,t})B_t^F - T_t^{B,F} \tag{12}$$

where public debt per young individual is denoted by B_t^F. The government in the funded country keeps its debt constant at B^F by raising a debt tax $T_t^{B,F}$ that is equal to:

$$T_t^{B,F} = R_{b,t}B^F \tag{13}$$

It is assumed that the level of government debt is the same in both countries in the initial steady state, i.e., $B^P = B^F = B$.

2.4 Households

Expected lifetime utility of a representative individual born at t is given by the following utility function:

$$E_t U(C_t^{Y,i}, C_{t+1}^{O,i}) = \log(C_t^{Y,i}) + \frac{1}{1+\rho} E_t \left[\log(C_{t+1}^{O,i})\right] \tag{14}$$

where $C_t^{Y,i}$ is consumption when young of an individual living in country i, $i = P, F$, and $C_{t+1}^{O,i}$ is consumption in the second period of life, ρ is the rate of time preference.

Only consumption in the second period of life is uncertain because the rates of return depend on the realizations of A_{t+1} and π_{t+1}, which are unknown at t when decisions about savings S_t^i are made (see Figure 1).

People can either invest in firm stocks which yield the stochastic return $R_{k,t+1}$ or in government bonds with the stochastic return $R_{b,t+1}$[9]. The part of savings that is invested in equities is denoted by γ_t^i, so that the return on the portfolio can be defined as:

$$R^i_{p,t+1} \equiv \gamma_t^i R_{k,t+1} + (1-\gamma_t^i) R_{b,t+1} \tag{15}$$

Young agents inelastically supply one unit of labour. The consolidated lifetime budget constraint is[10]:

$$W_t - T_t^i - T_t^{B,i} - C_t^{Y,i} = \frac{1}{1+R^i_{p,t+1}} \left(C^{O,i}_{t+1} - Z^i_{t+1} \right) \tag{16}$$

Maximizing lifetime utility with respect to the lifetime budget constraint gives the Euler condition:

$$1 = \frac{1}{1+\rho}\, C_t^{Y,i}\, E_t \left[\left(C^{O,i}_{t+1} \right)^{-1} (1 + R_{j,t+1}) \right] \tag{17}$$

where $j = p, k, b$. To derive an explicit solution for the portfolio choice γ_t^i, we follow the approach of Hansen and Singleton (1983) and Campbell and Viceira (2002) and assume that the joint distribution of consumption and gross returns is lognormal. Optimal portfolio choice in the funded and PAYG country, γ_t^P and γ_t^F, are given by (see Appendix B for the details):

$$\gamma_t^P = \frac{\log E_t(1+R_{k,t+1}) - \log E_t(1+R_{b,t+1})}{(1-z_t)\sigma^2_{k-b,t}} - \frac{\sigma_{k-b,bt}}{\sigma^2_{k-b,t}} \tag{18}$$

$$\gamma_t^F = \frac{\log E_t(1+R_{k,t+1}) - \log E_t(1+R_{b,t+1})}{\sigma^2_{k-b,t}} - \frac{\sigma_{k-b,bt}}{\sigma^2_{k-b,t}} \tag{19}$$

where $\sigma^2_{k-b,t}$ is the variance of the excess return of stocks over bonds, $\sigma_{k-b,bt}$ is the covariance between the excess return and the return on bonds, and z_t is the part of expected old-age consumption financed by PAYG pensions:

$$z_t = \frac{Z^P_{t+1}}{E_t(1+R^P_{p,t+1}) \exp(-\frac{1}{2}(\sigma^2_{pt})^P) S_t^P + Z^P_{t+1}} \tag{20}$$

[9]We assume that individuals or firms do not issue bonds. In a two-period OLG model individuals can only issue bonds if agents are heterogenous. Moreover, if firms can issue bonds besides stocks the decision-making process about the financing structure of firms has to be modeled. Both extensions will complicate the analysis to a large extent and therefore we leave these for future research.

[10]Debt tax is levied on the young. This is an important assumption because it affects savings decisions and in that way capital accumulation.

People in the country with the PAYG pension scheme invest more in the risky asset, i.e., $\gamma_t^P > \gamma_t^F$, as they receive a safe PAYG pension benefit when they are old (this is shown by the $(1 - z_t)$ term in equation (18)).

Optimal savings are given by[11] (see Appendix C for details):

$$
\begin{aligned}
S_t^P &= \frac{\exp\left[\frac{1}{2}\, z_t^2 (\sigma_{pt}^2)^P\right]}{1+\rho+\exp\left[\frac{1}{2}\, z_t^2 (\sigma_{pt}^2)^P\right]} \left(W_t - T_t^P - T_t^{B,P}\right) \\
&\quad - \frac{1+\rho}{1+\rho+\exp\left(\frac{1}{2}\, z_t^2 (\sigma_{pt}^2)^P\right)} \, \frac{Z_{t+1}^P}{E_t(1+R_{p,t+1}^P)\exp(-\frac{1}{2}(\sigma_{pt}^2)^P)} \qquad (21) \\
S_t^F &= \frac{1}{2+\rho}\left(W_t - T_t^{B,F}\right) \qquad (22)
\end{aligned}
$$

Notice that people in the funded country do not react to changes in uncertainty or rates of return. The reason is that, given a logarithmic utility function, the coefficient of relative risk aversion and the intertemporal substitution elasticity are equal to one. People in the PAYG country, however, do react to changes in the portfolio return $R_{p,t+1}^P$ and the variance of the portfolio $(\sigma_{pt}^2)^P$. A higher portfolio return decreases the net present value of the PAYG pension benefit Z_{t+1}^P and therefore affects savings in the PAYG country positively. The effect of portfolio return uncertainty is somewhat less straightforward, however. In general, savings are affected in two ways when there is uncertainty (see for example Sandmo, 1970, Rothschild and Stiglitz, 1971). First, higher riskiness makes it necessary to save more in order to protect oneself against very low levels of future consumption, this is the income effect. Secondly, there is a substitution effect; an increase in the degree of risk makes the consumer less inclined to expose his/her resources to the possibility of loss, so that savings fall. As shown in Adema (2008*a*) people with a logarithmic utility function, who receive a safe PAYG pension benefit become less risk averse and act like someone with a coefficient of relative risk aversion between 0 and 1 that does receive a safe pension benefit. In that case the substitution effect of uncertainty dominates the income effect, implying that savings fall when the risk on the portfolio rises.

2.5 Equilibrium international capital market

Individuals invest their savings either in the home country or abroad. The international capital market is in equilibrium when savings at time t finance the capital stock and the total amount of government debt in the next period:

$$S_t^P + \nu S_t^F = (1+\nu)K_{t+1} + (B_{t+1}^P + \nu B_{t+1}^F) \qquad (23)$$

[11]Savings in the funded country S_t^F also contain contributions to the pension fund T_t^F.

Moreover, portfolio choice has to be such that the right part of savings goes to the capital stock and government debt:

$$\gamma_t^P S_t^P + \nu \gamma_t^F S_t^F = (1+\nu) K_{t+1} \quad (24)$$

$$(1-\gamma_t^P) S_t^P + \nu (1-\gamma_t^F) S_t^F = B_{t+1}^P + \nu B_{t+1}^F \quad (25)$$

where one of the equations is abundant. This implies that there are two equilibrium conditions and K_{t+1} and $R_{b,t+1}^N$ adjust to make sure that these equilibrium conditions hold[12].

3 Government debt

With a fixed retirement age, population ageing will increase the relative number of inactive elderly compared to the number of active young people. This will put pressure on social services that redistribute from the young to the old. One example is the public provision of pensions financed on the basis of pay-as-you-go. Given the scope of projected increases in expenditures (see EuropeanCommission, 2006), countries with large PAYG pension schemes might finance their increased pension obligations by issuing more debt. This section analyses the (international) effects of the use of government debt by the PAYG country.

The government temporarily uses debt to (partly) finance the pension obligations, implying that the PAYG pension scheme is not balanced for a while ($T_t^P \neq Z_t^P$). This is modelled as follows:

$$T_t^P = \mu_t Z_t^P \quad (26)$$

where $\mu_t \leq 1$. To calculate the effect of public debt analytically, I employ the method of comparative dynamics from Judd (1982). The process for μ_t is given by:

$$\mu_t = 1 + \zeta f_t \quad (27)$$

where $f_t \leq 0$ describes the time pattern of μ_t from its steady state value ($= 1$) and ζ reflects the magnitude of this perturbation. The effects of the use of government debt can be traced by linearizing the various equations with respect to ζ around the initial steady state. The change in PAYG contributions T_t^P is then equal to:

$$\frac{\partial T_t^P}{\partial \zeta} = Z^P f_t \quad (28)$$

The government can only increase its debt for a certain number of periods, otherwise the government debt dynamics will be unstable. Assume that the government uses

[12] I checked for stability by verifying whether the two eigenvalues of the dynamic system given in equations (23) and (24) were within the unit circle. This was the case for various parameter values.

debt until period t^* and from period t^*+1 onwards the PAYG scheme is balanced again:

$$f_t = \begin{cases} \in [-1,0) & \text{for } t = [0,t^*] \\ 0 & \text{for } t = [t^*+1,\infty) \end{cases} \tag{29}$$

We assume that as soon the PAYG scheme is balanced again, the debt tax is increased in such a way that the debt per capita is constant again, but at a higher level than in the initial steady state:

$$\frac{\partial T_t^{B,P}}{\partial \zeta} = \begin{cases} \dfrac{B^P}{1+\pi}\dfrac{\partial R_{b,t}^N}{\partial \zeta} & \text{for } t = [0,t^*] \\ \dfrac{B^P}{1+\pi}\dfrac{\partial R_{b,t}^N}{\partial \zeta} + R_b\dfrac{\partial B_t^P}{\partial \zeta} & \text{for } t = [t^*+1,\infty) \end{cases} \tag{30}$$

The debt tax in the funded country only changes because the nominal interest rate on government debt changes:

$$\frac{\partial T_t^{B,F}}{\partial \zeta} = \frac{B^F}{1+\pi}\frac{\partial R_{b,t}^N}{\partial \zeta} \tag{31}$$

The debt dynamics are given by:

$$\frac{\partial B_{t+1}^P}{\partial \zeta} = (1+R_b)\frac{\partial B_t^P}{\partial \zeta} + \frac{B^P}{1+\pi}\frac{\partial R_{b,t}^N}{\partial \zeta} - \frac{\partial T_t^P}{\partial \zeta} - \frac{\partial T_t^{B,P}}{\partial \zeta} \tag{32}$$

Appendix D derives the following system of first-order difference equations:

$$\begin{aligned} \frac{\partial K_{t+1}}{\partial \zeta} &= \underbrace{\frac{\alpha W}{\Psi K}\frac{\partial K_t}{\partial \zeta}}_{5} \underbrace{- \frac{\Omega \nu}{\Psi^F}\frac{\partial T_t^{B,F}}{\partial \zeta} - \frac{1}{\Psi^P}\frac{\partial T_t^{B,P}}{\partial \zeta}}_{4} \underbrace{- \frac{1}{\Psi^P}\frac{\partial T_t^P}{\partial \zeta}}_{1} \qquad (33) \\ &\quad - \underbrace{\frac{1}{1+\nu - S^P_{\sigma_p^2}\Delta_{\sigma_p^2} - S^P_{R_p}\Delta_{R_p}}}_{\gtrless 0}\left[\underbrace{\frac{\partial B_{t+1}^P}{\partial \zeta}}_{2} \underbrace{- \Omega_{B^P}\frac{\partial B_{t+1}^P}{\partial \zeta}}_{6}\right] \\ \frac{\partial S_t^F}{\partial \zeta} &= \underbrace{\frac{\alpha W}{(2+\rho)K}\frac{\partial K_t}{\partial \zeta}}_{5} \underbrace{- \frac{1}{2+\rho}\frac{\partial T_t^{B,F}}{\partial \zeta}}_{4} \qquad (34) \\ \frac{\partial R_{b,t+1}^N}{\partial \zeta} &= -\Phi\frac{\partial K_{t+1}}{\partial \zeta} - \frac{\nu(\gamma^P-\gamma^F)\sigma_{k-b}^2(1+R_b^N)}{\nu S^F + \frac{S^P}{1-z}}\frac{\partial S_t^F}{\partial \zeta} \\ &\quad + \underbrace{\frac{\gamma^P\sigma_{k-b}^2(1+R_b^N)}{\nu S^F + \frac{S^P}{1-z}}\frac{\partial B_{t+1}^P}{\partial \zeta}}_{3} \qquad (35) \end{aligned}$$

where Ω, Ω_{B^P}, Ψ, Ψ^P and Ψ^F are defined in Appendix D.

Equations (33) and (35) show the change in the capital-labour ratio and the nominal interest rate after an increase in goverment debt in the PAYG country when the two economies have a joint capital market. To analyze the international spillover effects, I derive the same kind of equations for the case where the two economies are closed. Obviously, in country F nothing happens when it is a closed economy, as it does not use government debt to temporarily finance its pensions. By comparing the results in the closed-economy case with the effects when the two countries have integrated capital markets, I derive the pure spillover effects of the use of government debt in a common capital market[13]. I do not show the system of dynamic equations for the closed-economy case, however, as the system above can easily be used to explain the spillover effects.

The government in the PAYG country decides at $t = 0$ to temporarily use debt to (partly) finance the pension obligations and I assume that debt is only used during one period (so $t^* = 0$). This implies that the level of government debt increases at $t = 1$ and stays at this higher level afterwards. The young at $t = 0$ obtain a windfall gain as their PAYG tax is lowered, while their pension benefit stays the same. This induces higher savings. Higher savings at $t = 0$ imply a higher capital-labour ratio in period $t = 1$. The positive effect of lower pension contributions T_0^P on the capital-labour ratio is indicated by a 1 in equation (33). Public debt, however, has a direct crowding-out effect on the capital stock (effect 2). This negative effect on the capital-labour ratio is larger than the positive effect that results from the rise in savings, so that the capital-labour ratio decreases at $t = 1$. The intuition for this result is as follows. The young generation that receives the windfall gain, consumes part of its gain and saves part of it. As the gain this generation receives equals the created debt, the increase in savings at $t = 0$ is smaller than the created debt, so that public debt crowds out part of the capital stock.

To finance the higher level of government debt, the nominal interest rate on government debt will rise to induce people to invest a larger part of their savings in government bonds. This effect is indicated by a 3 in equation (35). Moreover, the fall in the capital-labour ratio implies that the part of savings invested in the capital stock γ_0^i should fall, i.e., individuals should invest even more in government bonds. Therefore the nominal return on government bonds has to rise even more. This effect is shown in the first term of equation (35); if the capital-labour ratio K_1 falls, $R_{b,1}^N$ will rise. The fact that consumers have to reallocate their investment portfolio towards government bonds implies that the return on government bonds has to rise to a larger extent than the return on stocks, i.e., the excess return on stocks will fall.

The higher interest rate on government debt increases the debt tax in both countries at $t = 1$. The debt tax in the PAYG country also increases because the level of debt is higher (see equation (30)). A higher debt tax decreases the disposable income of people and reduces savings (effect 4). This negative effect on savings is reinforced by

[13]To exclude the effects of integration, it is assumed that the initial steady state is the same in all cases.

the fact that a lower capital-labour ratio reduces wages (effect 5). Both these effects imply that the capital-labour ratio continues to decline and the nominal return on government bonds continues to rise.

Equation (33) shows that besides the direct negative crowding-out effect (effect 2), government debt also has a positive effect on capital accumulation (effect 6). This effect arises because people in the PAYG country also adjust their savings in response to changes in the portfolio return and the variance of the portfolio (see equation (21)). The fall in the capital-labour ratio implies that the return to capital and thus the return on stocks increases. Combined with the fact that the return on bonds also increases, we know that the return on the total portfolio will rise. A higher portfolio return decreases the net present value of the PAYG pension benefit and therefore affects savings in the PAYG country positively. The variance on the portfolio, on the other hand, will fall because people invest less in stocks and more in bonds, which are relatively less risky. As explained in Section 2.4, a lower variance on the portfolio increases savings in the PAYG country. This implies that these effects via $(\sigma_p^2)^P$ and R_p^P dampen the direct negative crowding-out effect of debt on the capital-labour ratio. It is unlikely however, that these indirect savings effects turn around the effect on the capital-labour ratio as they are of a second-order nature. Simulations show that it is indeed the case that the direct negative crowding-out effect of debt dominates the indirect positive effects via savings and the capital-labour ratio falls over time.

In case country P has an integrated capital market with country F, it can finance part of its government debt with savings of country F. This implies that the fall in the capital-labour ratio and the rise in the nominal interest rate of government debt will be larger when country P does not have a common capital market with country F. To illustrate the mechanics of the model, I also show some numerical simulation experiments[14]. Figures 2 and 3 show the change in the capital-labour ratio and the nominal interest rate on government bonds for the different cases.

When the change in the capital-labour ratio and the nominal return on bonds are known, the changes in all other variables can be derived. In Figures 4 and 5 we show the effects on utility in the different cases.

[14] The graphs are based on simulations with $F(A_t, K_t) = K_t^{0.3}$ so I take $E(A) = 1$, $\delta = 1$, $E(\pi) = 0$, $\nu = 1$, $\frac{T^P}{W} = 0.2$. Agents are relatively patient with a time preference rate of 1% per year, which gives $\rho = (1.01)^{30} - 1 = 0.3478$ when one period is assumed to equal 30 years. PAYG contributions are lowered by 20% at $t = 0$ during one period, i.e., $f_0 = -0.2$. The initial level of government debt equals 0.025, which is chosen in such a way that there is still an equilibrium after the debt increase. I take $\sigma_a^2 = 0.2$ and $\sigma_\pi^2 = 0.01$, which roughly corresponds to an annual standard deviation of 8.2% for stock returns and 1.8% for bond returns, assuming that the returns are serially uncorrelated. Here we follow Campbell and Viceira (2005) who show that returns on stocks are significantly less volatile when the investment horizon is long and inflation risk on nominal bonds increases with the investment horizon.

Figure 2: Change in K

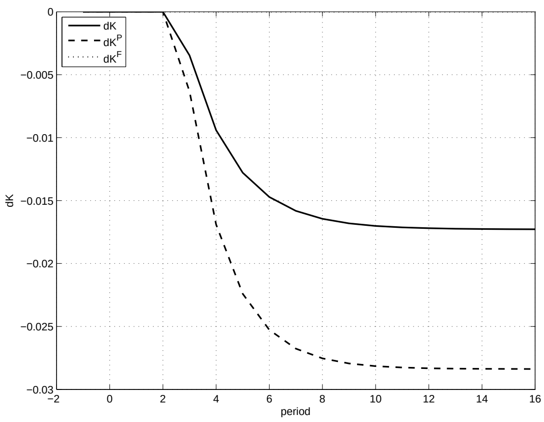
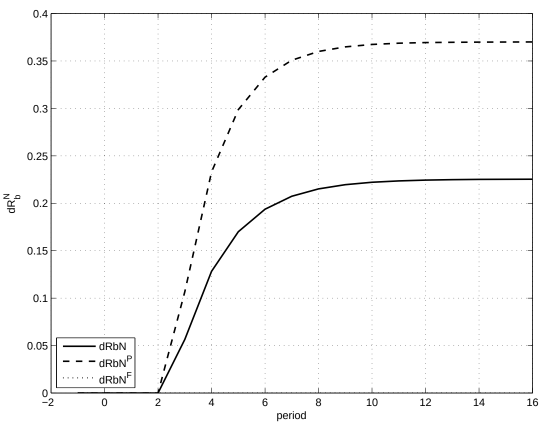


Figure 3: Change in R_b^N



Notes: The solid lines refer to the case where one country uses a funded pension scheme and the other country relies on a PAYG pension system. The dotted lines show the changes of the variables in case the funded country is closed, while the striped lines indicate the changes for the case where the PAYG country is closed.

Figure 4: Change in U^F

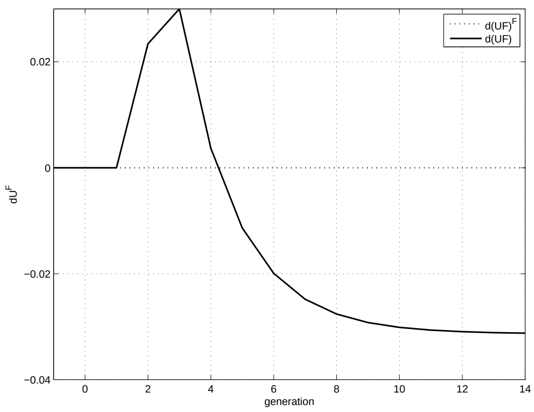
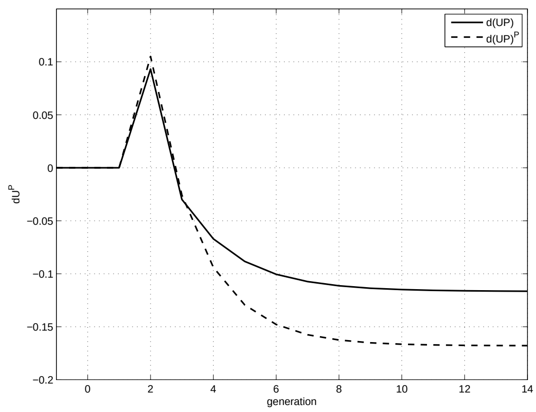


Figure 5: Change in U^P



The fact that people in the funded country partly finance the higher government debt in the PAYG country implies that they also experience a falling capital-labour ratio (see Figure 2). In the long run a lower capital-labour ratio has negative utility effects in dynamically efficient economy (see Adema et al., 2008*b*). The initial generations, however, gain from the higher return on their portfolio. This implies that the funded country experiences positive spillover effects in the short run and negative long-run spillover effects from the fact that the PAYG country increases its government debt.

Figure 5 shows the utility effects for the PAYG country in the various cases. First note that using government debt instead of levying PAYG contributions makes people born at $t = 1$ better off. Future generations lose, however, as the capital stock is crowded out. This policy can be implemented by governments that put more weight on the welfare of current generations than future generations' welfare. The long-run utility losses are much smaller in case the PAYG country shares one capital market with a funded country, as the funded country absorbs part of the extra government debt so that the crowding out of capital is less. This implies that it is easier to implement such a policy in case the PAYG country forms a monetary union with a country that uses a funded system instead of a PAYG scheme.

4 Unexpected inflation

In case government debt is very large and this debt is denominated in nominal terms, the government has an incentive to put pressure on the central bank to create surprise inflation as this will erode the real value of government debt and lower the real return on government bonds. In this section we will analyse the effects of an unexpected inflation shock. The idea behind this is that the PAYG country has an incentive to lobby for unexpected inflation after it created government debt to cope with the ageing costs. First we consider the situation where the other country also uses a PAYG scheme. This implies that there are no capital flows between the two countries and can therefore be regarded as a closed economy. In that case the gain for the government is exactly high enough to compensate the elderly that experience a loss when inflation rises unexpectedly. So in a closed economy unexpected inflation is only a matter of redistribution between the old and the young (or future generations) and can be implemented in a neutral way (so that no generation loses). In an open economy where the other country uses a funded scheme, however, unexpected inflation affects the PAYG country positively, while the funded country is affected negatively. The reason for this result is that the funded country exports capital to the PAYG country and individuals in the funded country have a more conservative portfolio. This implies that residents of the funded country own a relatively large part of the government bonds, so they are affected more negatively by the unexpected inflation shock. The government in the funded country cannot fully compensate its residents for this loss. To isolate the effects of an unexpected inflation shock, we leave out the effects of government debt in the analytical analysis below. In the simulation graphs, however, we combine the use of government debt and unexpected inflation shock.

4.1 Closed economy

First define the inflation factor:

$$\beta_t = \frac{1}{1+\pi_t} \tag{36}$$

Suppose that the time pattern of β_t is as follows:

$$\beta_t = \beta + \zeta g_t \tag{37}$$

where $g_t < 0$ in case of inflation. By taking the derivative with respect to ζ we analyse the effects of inflation.

In case of unexpected inflation there will be a difference between the expected real return on government bonds and the realised real return:

$$
\begin{aligned}
\frac{\partial R_{b,t+1}}{\partial \zeta} &= (1+R_b^N)\,\frac{\partial \beta_{t+1}}{\partial \zeta} + \beta\,\frac{\partial R_{b,t+1}^N}{\partial \zeta} && (38)\\
\frac{\partial E_t R_{b,t+1}}{\partial \zeta} &= (1+R_b^N)\,\frac{\partial E_t \beta_{t+1}}{\partial \zeta} + \beta\,\frac{R_{b,t+1}^N}{\partial \zeta} && (39)\\
\frac{\partial R_{b,t+1}}{\partial \zeta} &= \frac{\partial E_t R_{b,t+1}}{\partial \zeta} + (1+R_b^N)\left[\frac{\partial \beta_{t+1}}{\partial \zeta} - \frac{\partial E_t \beta_{t+1}}{\partial \zeta}\right] && (40)
\end{aligned}
$$

where we assumed that in steady state $\beta = E(\beta)$, so the steady state value of the inflation factor is equal to its expected value. In case people do not expect any inflation, i.e., $\frac{\partial E_t \beta_{t+1}}{\partial \zeta} = 0$ and unexpected inflation is created $\frac{\partial \beta_{t+1}}{\partial \zeta} < 0$, the real return on bonds will fall, while nothing has happened with the expected return.

We assume that consumers have static expectations; their inflation expectation is based on the current inflation rate:

$$
\frac{\partial E_t \beta_{t+1}}{\partial \zeta} = \frac{\partial \beta_t}{\partial \zeta} \tag{41}
$$

This implies that inflation can only be raised unexpectedly during one period, after that people adjust their inflation expectations. Suppose that inflation rises unexpectedly at time τ, then:

$$
\frac{\partial \beta_{\tau+1}}{\partial \zeta} = \frac{\partial \beta_\tau}{\partial \zeta} = \frac{\partial E_\tau \beta_{\tau+1}}{\partial \zeta} \tag{42}
$$

so the inflation expectations for period $\tau + 1$ are correct again.

The result of this policy is that the real interest rate on government debt is lower, which affects the financial position of the government positively, but will harm people that own government bonds. There are various options for the government how to use this interest gain. Firstly, it can give the benefit to the young people by lowering the debt tax in the same period. Second, it can repay part of its debt so that future generations gain. In both these scenarios however, the elderly alive at the time of the unexpected inflation episode will lose. The third policy option would therefore be to compensate these elderly. We will analyse this last policy option.

The compensation to the elderly is denoted by $Z_t^{B,P}$. In case the elderly receive the whole advantage of a lower real interest rate on government debt the change of $Z_t^{B,P}$ is equal to:

$$
\frac{\partial Z_\tau^{B,P}}{\partial \zeta} = -\frac{1+n}{\varepsilon}\,B^P(1+R_b^N)\,\frac{\partial \beta_\tau}{\partial \zeta} \tag{43}
$$

where we used the fact that at time τ only a fraction ε of young people at $\tau-1$ reached period τ and the population grows with n. The change in old-age consumption is:

$$\begin{aligned}\frac{\partial C_\tau^{o,P}}{\partial \zeta} &= \frac{S^P}{\varepsilon}\frac{\partial R_{p,\tau}}{\partial \zeta} + \frac{\partial Z_\tau^{B,P}}{\partial \zeta} \\ &= \frac{S^P}{\varepsilon}(1-\gamma^P)(1+R_b^N)\frac{\partial \beta_\tau}{\partial \zeta} - \frac{B^P(1+n)(1+R_b^N)}{\varepsilon}\frac{\partial \beta_\tau}{\partial \zeta} \\ &= \frac{1+R_b^N}{\varepsilon}\left[S^P(1-\gamma^P) - B^P(1+n)\right]\frac{\partial \beta_\tau}{\partial \zeta} \qquad (44)\end{aligned}$$

In the closed-economy case it has to hold that $S^P(1-\gamma^P) = (1+n)B^P$, so that $\frac{\partial C_\tau^{o,P}}{\partial \zeta} = 0$ and the elderly are exactly compensated. Young people at time τ know that inflation is higher and will ask a higher nominal rate of return on government bonds, and the real rate of return will be back at its old value. Combined with the fact that government debt does not change, this implies that an unexpected inflation shock does not have any real effects. So indeed the government can set its policy in such a way that no generation is hurt after an unexpected rise in inflation. The gain of the lower real interest rate on government debt is exactly high enough to compensate the people who lose from the unexpected inflation shock.

4.2 Open economy

In this subsection we analyse the effects of unexpected inflation in the open-economy case where one country uses a PAYG pension scheme, while the other country relies on funded pensions. First we consider the scenario discussed in the previous subsection where the whole interest gain is used to compensate the old at the time of the shock. It turns out that in an open economy where the PAYG country imports capital from the funded country, the interest gain for the PAYG country is larger than the loss of the old owners of capital, so that these people gain. In the funded country we have exactly the opposite result, the interest gain is not high enough to fully compensate the elderly, so they lose. In the second scenario we consider the effects of unexpected inflation where the current old are fully compensated and the rest of the gain/loss is transferred to future generations in the form of a lower/higher debt tax.

Scenario 1

In case a PAYG country has an integrated capital market with a funded country, the policy described in the previous subsection will have real effects. Equation (44) shows the change in old-age consumption in the PAYG country in case the whole interest gain is transferred to the old at time τ. In the open-economy case equation (25) has to hold. Because residents of the funded country save more and invest a larger part of their savings in government bonds than people living in the PAYG country, we know that $S^F(1-\gamma^F) > S^P(1-\gamma^P)$. Defining $S^F(1-\gamma^F) \equiv \alpha S^P(1-\gamma^P)$ where $\alpha > 1$, we can use equation (25) to derive[15]:

$$S^P(1-\gamma^P) = \underbrace{\frac{1+\nu}{1+\nu\alpha}}_{<1}(1+n)B \tag{45}$$

so that $S^P(1-\gamma^P) < (1+n)B$ and the term between brackets in equation (44) is smaller than zero. Combining this result with the fact that $\frac{\partial\beta_\tau}{\partial\zeta} < 0$ in case of unexpected inflation, we see that old-age consumption at time τ rises. So there is at least one generation in the PAYG country that gains from the unexpected inflation shock, while no other generation loses.

In the funded country the government also gains from the lower real interest rate on government debt at time τ. Suppose that this government follows the same strategy as the government in the PAYG country, that is, the gain of the lower real interest rate is used to compensate the elderly at the time of the inflation shock:

$$\frac{\partial Z_\tau^{B,F}}{\partial\zeta} = -\frac{1+n}{\varepsilon}B^F(1+R_b^N)\frac{\partial\beta_\tau}{\partial\zeta} \tag{46}$$

The change of old-age consumption at τ is:

$$\begin{aligned}\frac{\partial C_\tau^{o,F}}{\partial\zeta} &= \frac{S^F}{\varepsilon}\frac{\partial R_{p,T}}{\partial\zeta} + \frac{\partial Z_\tau^{B,F}}{\partial\zeta} \\ &= \frac{1+R_b^N}{\varepsilon}\Big[S^F(1-\gamma^F) - B^F(1+n)\Big]\frac{\partial\beta_\tau}{\partial\zeta} \end{aligned} \tag{47}$$

Using the same method as for the PAYG country above, we can write:

$$S^F(1-\gamma^F) = \underbrace{\frac{1+\nu}{\frac{1}{\alpha}+\nu}}_{>1}(1+n)B \tag{48}$$

which implies that $S^F(1-\gamma^F) > (1+n)B$ and the term between brackets in equation (47) is positive. This means that the effects of an unexpected inflation shock are still negative for the elderly, that is, $\frac{\partial C_\tau^{o,F}}{\partial\zeta} < 0$. Even though the government in the funded country uses the whole interest gain to compensate the pensioners, they still experience welfare losses.

So in an open economy the PAYG country gains from an unexpected inflation shock at the expense of the funded country. It takes advantage of the fact that the funded

[15] We assume that the two countries have the same level of government debt in the initial steady state, i.e., $B^F = B^P = B$.

country owns a relatively large part of the total supply of government bonds in the two economies. It is actually the case that:

$$\frac{\partial C_{\tau}^{o,P}}{\partial \zeta} = -\nu \, \frac{\partial C_{\tau}^{o,F}}{\partial \zeta} \tag{49}$$

so the effects on old-age consumption are opposite to each other. In case the two countries are of equal size, i.e., if $\nu = 1$, the gain of the elderly in the PAYG country is exactly as large as the loss of the elderly in the funded country. If $\nu > 1$, the funded country is relatively large and there will be more capital flows to the PAYG country. The larger ν, the more the PAYG country can profit from these capital flows and the larger the gain of unexpected inflation is. There is no possibility, however, that the PAYG country compensates the funded country for its loss and the union as a whole experiences a Pareto improvement. It is true that the gain in old-age consumption in the PAYG country is larger than the loss of the old in the funded country in case $\nu > 1$, but this is exactly offset by the fact that the funded country has more inhabitants.

Scenario 2

Instead of transferring the whole gain of the lower interest rate to the elderly alive at the time of the shock, the government in the PAYG country can also decide to compensate those elderly in such a way that they do not experience any welfare gains or losses. This means that these people get full compensation but do not gain as was the case when the whole advantage was transferred to them. The rest of the gain is used to pay off some of the government debt, to decrease the debt burden for future generations. The compensation is such that the consumption of the elderly at τ does not change:

$$\frac{\partial Z_{\tau}^{B,P}}{\partial \zeta} = -\frac{S^P}{\varepsilon}(1-\gamma^P)(1+R_b^N)\frac{\partial \beta_\tau}{\partial \zeta} \tag{50}$$

The fall in government debt is then:

$$\frac{\partial B_{\tau+1}^{P}}{\partial \zeta} = \frac{1+R_b^N}{(1+n)^2}\left[B^P(1+n) - S^P(1-\gamma^P)\right]\frac{\partial \beta_\tau}{\partial \zeta} \tag{51}$$

Suppose that from period $\tau + 1$ onwards the government debt is kept constant again, but then at this lower level. Future generations indeed profit from the lower debt burden because the interest obligations on the debt and thus the debt tax paid by the young decrease:

$$\frac{\partial T_{\tau+1}^{B,P}}{\partial \zeta} = (R_b - n)\,\frac{\partial B_{\tau+1}^{P}}{\partial \zeta} \tag{52}$$

Suppose that the government in the funded country also fully compensates the elderly at time τ:

$$\frac{\partial Z_{\tau}^{B,F}}{\partial \zeta} = -\frac{S^F}{\varepsilon}(1-\gamma^F)(1+R_b^N)\frac{\partial \beta_\tau}{\partial \zeta} \tag{53}$$

As the gain of the lower interest rate was smaller then the loss of the elderly, the funded country has to use government debt to be able to give full compensation. The rise in government debt is equal to:

$$\frac{\partial B^F_{\tau+1}}{\partial \zeta} = \frac{1+R^N_b}{(1+n)^2}\left[B^F(1+n) - S^F(1-\gamma^F)\right]\frac{\partial \beta_\tau}{\partial \zeta} \tag{54}$$

so that future generations have to pay a higher debt tax:

$$\frac{\partial T^{B,F}_{\tau+1}}{\partial \zeta} = (R_b - n)\,\frac{\partial B^F_{\tau+1}}{\partial \zeta} \tag{55}$$

The total amount of government debt in the two countries does not change:

$$\begin{aligned}\frac{\partial B^P_{\tau+1}}{\partial \zeta} + \nu\frac{\partial B^F_{\tau+1}}{\partial \zeta} &= \\ &\frac{1+R^N_b}{(1+n)^2}\left[B^P(1+n) - S^P(1-\gamma^P) + \nu B^F(1+n) - \nu S^F(1-\gamma^F)\right]\frac{\partial \beta_\tau}{\partial \zeta}\end{aligned} \tag{56}$$

because the term between the brackets equals zero as can be seen from equation (25). So the rise of government debt in the funded country and the fall in government debt in the PAYG country exactly cancel. This implies that the capital-labour at $\tau+1$ will not change.

In the next period people adjust their inflation expectations and demand a higher nominal rate of return:

$$\frac{\partial R^N_{b,\tau+1}}{\partial \zeta} = -\frac{1+R^N_b}{\beta}\frac{\partial E_\tau \beta_{\tau+1}}{\partial \zeta} \tag{57}$$

so that the real rate of return is back at its old level. The debt tax in the PAYG country is lower at $\tau+1$, which affects savings positively, while the debt tax is higher for inhabitants of the funded country, which decreases their savings. So it is ambiguous what happens to the capital-labour ratio at $\tau+2$. Simulations show, however, that these effects more or less cancel and that the effect on the capital-labour ratio is negligible. This implies that the effect on the debt tax determines the long-run welfare effects of this policy. Future generations in the funded country lose and people in the PAYG country gain.

The utility effects of the use of government debt and the policy described here are shown in Figures 6 and 7 by the lines with the diamonds. Inflation rises unexpectedly at time $t=7$. Comparing the utility effects of this scenario with those of the scenario described in the previous section where only government debt was used (the solid lines), shows indeed that the generations after the unexpected inflation shock gain in the PAYG country and lose in the funded country. This implies that the negative long-run spillovers of the PAYG scheme for the funded country become larger.

Unexpected inflation

Figure 6: Change in U^F

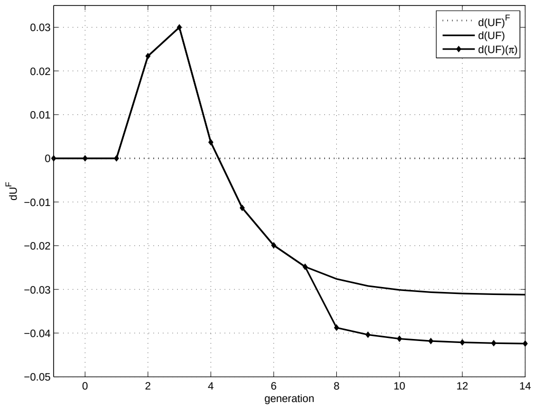
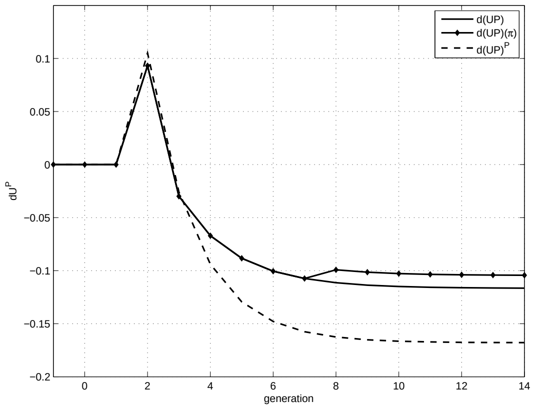


Figure 7: Change in U^P



Notes: The dotted line in Figure 6 refers to the case where both countries use a funded scheme, where no government debt is used and no surprise inflation is created. The solid lines demonstrate the utility effects in case one country uses a funded scheme while the other country has PAYG pensions. The lines with the diamonds indicate the effects where not only government debt is used but also unexpected inflation is created. The inflation rate rises to 3.58 percentage point. The elderly at the time of the unexpected inflation shock are fully compensated and the rest of the gain/loss is transferred to future generations in the form of a lower/higher debt tax. The case where both countries rely on PAYG pensions is denoted by the striped line with the bullets in Figure 7. As explained in the main text there are no capital flows between the two countries in that case and the gain of surprise inflation is exactly high enough to compensate the elderly. Therefore the line with the diamonds (unexpected inflation) coincides with the solid line (only government debt) in case both countries have a PAYG pension scheme.

The two scenarios discussed in this subsection show that the governments in both countries can decide how the total gain or loss of unexpected inflation is shared between generations. The main result is, however, that unexpected inflation is always advantageous for capital-importing PAYG countries, while in funded countries there are always some generations that lose. The gains of unexpected inflation for PAYG countries will be larger the larger the debt burden is. In the coming decades the public finances will be put more under pressure because of the ageing of the population. In case government debt is used to cope with the ageing costs, the incentive of PAYG countries to put pressure on the central bank to accommodate and create surprise inflation will rise. On the other hand will unexpected inflation harm funded countries more if PAYG countries issued large amounts of nominal government debt. The conflict of interest on monetary policy between PAYG- and funded countries will therefore be reinforced if population ageing raises government debt in PAYG countries. If the decision-making process of the central bank is not completely transparent and it is not clear how the central bank will react to these conflicting interests, inflation risk may rise when public debt levels are high. This scenario will be analysed in the next section.

5 Inflation risk

In the previous section we showed that PAYG governments have an incentive to lobby for surprise inflation when their debt burden is large. If it is not completely clear how monetary policy is determined inflation risk may rise, that is, inflation risk rises with the level of government debt[16]. In this section we will analyse this scenario. This means that inflation risk is a function of the level of government debt in the PAYG country:

$$\sigma^2_{\pi t}\left(B^P_{t+1}\right) = \lambda B^P_{t+1} \tag{58}$$

where λ shows the responsiveness of inflation risk to government debt, so when government debt changes inflation risk also changes:

$$\frac{\partial \sigma^2_{\pi t}}{\partial \zeta} = \lambda \frac{\partial B^P_{t+1}}{\partial \zeta} \tag{59}$$

Government debt in the funded country does not affect inflation risk as the funded country does not have an incentive to lobby for unexpected inflation.

Inflation risk has a direct effect on the portfolio choice of consumers and the variance of the portfolio. Following the calculations in Appendix D we get the following system of equations:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \left.\frac{\partial K_{t+1}}{\partial \zeta}\right|_{\text{debt}} + \underbrace{\frac{\Omega_{\sigma^2_b}}{(1+\nu)(1+n) - S^P_{\sigma^2_p}\Delta_{\sigma^2_p} - S^P_{R_p}\Delta_{R_p}}\frac{\partial \sigma^2_{bt}}{\partial \zeta}}_{10} \tag{60}$$

$$\frac{\partial R^N_{b,t+1}}{\partial \zeta} = \left.\frac{\partial R^N_{b,t+1}}{\partial \zeta}\right|_{\text{debt}} + \underbrace{\frac{[(1-\gamma^P)S^P + \nu S^F(1-\gamma^F)](1+R^N_b)}{\nu S^F + \frac{S^P}{1-z}}\frac{\partial \sigma^2_{bt}}{\partial \zeta}}_{9} \tag{61}$$

where:

$$\begin{aligned}\Omega_{\sigma^2_b} \equiv{}& \frac{(1-\gamma^P)S^P + \nu S^F(1-\gamma^F)}{(\nu S^F(1-z) + S^P)\sigma^2_{k-b}}\Big\{S^P_{R_p}(1-\gamma^P)E(1+R_b)(1-z)\sigma^2_{k-b} \\ & - S^P_{R_p}[E(R_k) - E(R_b)] - S^P_{\sigma^2_p}(2\gamma^P\sigma^2_{k-b} + 2\sigma_{k-b,b})\Big\} \\ & + S^P_{R_p}\frac{[E(R_k) - E(R_b)](1-\gamma^P)}{\sigma^2_{k-b}} + S^P_{\sigma^2_p}(1-\gamma^P)\left(1 + \gamma^P + \frac{2\sigma_{k-b,b}}{\sigma^2_{k-b}}\right)\end{aligned} \tag{62}$$

and:

$$\frac{\partial \sigma^2_{bt}}{\partial \zeta} = \frac{\partial \sigma^2_{\pi t}}{\partial \zeta} \tag{63}$$

[16]The increase in riskiness of government bonds in case government debt rises can also be interpreted as a rise in default risk.

For a given rise in the riskiness of government bonds people would like to switch to stocks and this effect will be stronger in the funded country. The intuition for this result is twofold. First, people in the funded country do not receive a safe PAYG pension benefit during retirement, which makes them relatively more risk averse (compare the denominators of equations (19) and (18)). Second, in the initial steady state inhabitants of the funded country own a relatively large part of the total amount of government debt in the two countries because they save more and invest a larger part of their savings in government bonds than the PAYG country. The upward pressure on γ causes the nominal rate of return on government bonds to rise to make sure that government debt is financed. This is effect 9 in equation (61) and the rise of R_b^N has to be larger in case the PAYG country has integrated capital markets with a country that uses a funded pension scheme instead of PAYG pensions.

The rise in R_b^N increases the debt tax and implies lower savings, which affects the capital-labour ratio negatively. A lower capital-labour ratio in turn, affects wages negatively so that savings continue to decrease. As can be seen from equation (60) there is an extra effect 10 on the capital-labour ratio. This is the indirect (second-order) effect of changes in $(\sigma_{pt}^2)^P$ and $E_t R_{p,t+1}^P$, which affect savings in the PAYG country.

The higher riskiness of government bonds makes bonds much less attractive to hold and the return on government bonds has to rise to a much larger extent than the return on stocks to make sure that the capital markets clears. This implies that the excess return of stocks over bonds will fall.

The fall in savings implies that γ has to fall in case both countries use a PAYG scheme. When the two countries use different pension schemes the effects on portfolio choice are not that straightforward, however. The equilibrium condition on the bond market, equation (25), shows that the part of savings invested in bonds in the two countries together ($(1-\gamma^P)+(1-\gamma^F)$) has to rise in case total savings ($S^P + S^F$) decline. Simulations show that the movement of γ^F is opposite to the movement of γ^P. People in the funded country move to stocks; they move away from the asset that becomes more risky. People in the PAYG country, on the other hand, move to government bonds. Consumers in the funded country invest more in stocks even though the excess return on stocks falls, so the direct effect of the higher risk on bonds dominates for these people. They can do this because people in the PAYG country put less weight on risk in their portfolio decision and decrease γ^P in response to the lower excess return.

In Figures 8 and 9 we compare the utility effects of a longevity shock in the different cases. The dotted line in Figure 8 shows the utility effects for the funded country when it has an integrated capital market with a country that uses a funded scheme too. In that case no government debt is created and inflation risk will not rise. The solid lines show the effect in case one country uses a funded system and the other country relies

Inflation risk

Figure 8: Change in U^F

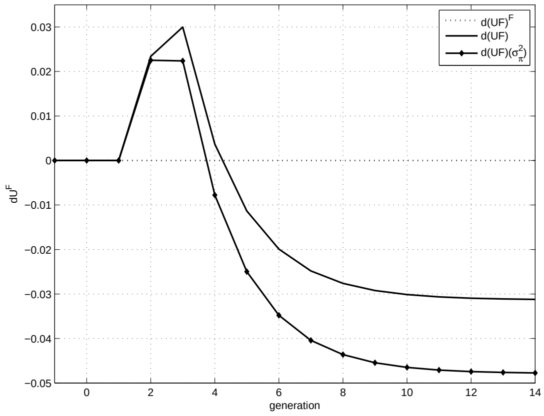
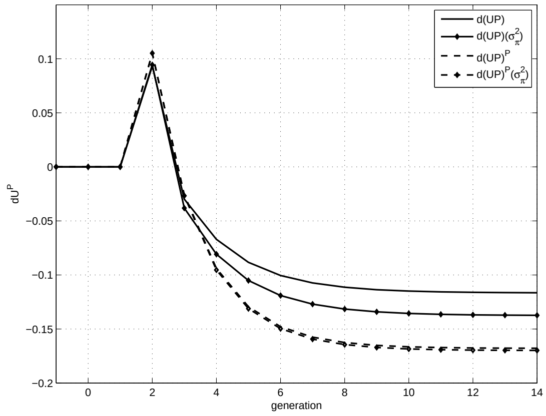


Figure 9: Change in U^P



Notes: The dotted line in Figure 8 refers to the case where both countries use a funded scheme, where no government debt is used and inflation risk does not rise. The case where both countries rely on PAYG pensions is denoted by the striped lines in Figure 9. The solid lines demonstrate the utility effects in case one country uses a funded scheme while the other country has PAYG pensions. The lines with the diamonds indicate the effects where not only government debt is used but inflation risk also rises. Inflation risk (σ_π^2) rises with 0.08 to 0.09.

on PAYG pensions. The diamonds denote the case where inflation risk also rises. The spillovers are determined by the relative change of the capital-labour ratio. A rise in inflation risk increases the interest rate on government debt, which implies a higher debt tax and lower savings. This results in a smaller capital-labour ratio and lower wages and utility is negatively affected[17]. This implies that the negative spillover effects in the long run increase for the funded country.

Figure 9 shows that the change in utility effects in the PAYG country after a rise in inflation risk is much larger in case the PAYG country has a common capital market with a funded country (the solid lines), compared to the case where it shares one capital market with a country that also uses a PAYG scheme (the striped lines). This means that higher inflation risk affects residents in the PAYG country more negatively in case they form a monetary union with a funded country. The reason for this result is that in that case a large part of government bonds is in the hands of citizens of the funded country and because these people are more risk averse, the rise of the nominal interest rate on government bonds has to be much larger compared to the case where both countries rely on PAYG pensions. This implies that the effects on the debt tax and thus the negative effects on savings and the capital-labour ratio are also much larger. This shows that the PAYG country cannot share the adverse effects of higher inflation risk with the funded country, which was the case for example with the use

[17]Higher inflation risk also has a direct negative effect on utility because people are risk averse. This direct effect is negligible compared to the negative utility effects of the higher debt tax, however.

of government debt. The positive long-run spillovers of having a common capital market with a funded country are therefore smaller for people in the PAYG country.

In contrast to unexpected inflation, a rise in inflation risk has adverse effects in both countries of the monetary union. Inflation risk rises in case PAYG countries have a large debt burden, which results in conflicting interests between PAYG- and funded countries about the creation of inflation. If the decision-making process about monetary policy is not completely transparent, it is not clear what the final outcome will be for inflation and this uncertainty raises inflation risk. The fact that both funded- and PAYG countries lose from a rise in inflation risk shows that it may be in the interest of both countries that government debt levels stay at low levels. In case government bonds are treated as more risky when debt levels are high, there is a trade-off for the PAYG country between the short-run benefits of debt and the increase in inflation risk this debt induces. This implies that not only funded countries in the EMU benefit from the rules in the Stability and Growth Pact, but that PAYG countries may benefit from these rules as well. Moreover, the independence, credibility and transparency of the ECB is important for all countries to prevent an increase in inflation risk.

6 Conclusion

The ageing of the population will confront governments in developed countries with serious challenges. As a large number of countries rely on pension schemes that are financed on the basis of Pay-As-You-Go, the rise of the number of retirees relative to the number of working people forces governments to think about the way their pension systems are organised. This paper analyses how countries that rely to a different extent on PAYG pensions affect each other when PAYG governments use debt. This will benefit the initial generations, but future generations will lose as government debt crowds out capital. The utility effects spill over to the funded country that does not use government debt. The initial generations gain from the higher rates of return, while later generations are affected negatively because wage levels fall. This implies that the use of government debt by the PAYG country increases both the positive short-run spillovers and the negative long-run spillovers for the funded country.

I also show that in a monetary union where a PAYG country has a joint capital market with a funded country, unexpected inflation will affect the PAYG country positively. In a closed economy, unexpected inflation is only a matter of redistribution from the old to the young- or future generations and can be implemented in such a way that no generation gains or loses. The reason why unexpected inflation creates a net gain in the PAYG country when it forms a monetary union with a funded country is that capital flows from the funded country to the PAYG country, so that part of the government bonds of the PAYG country is in the hands of citizens in the funded country. Therefore, part of the costs of unexpected inflation are transmitted to the funded country, while the gain of the lower debt burden does not change and a net gain results. So when a PAYG country forms a monetary union with a funded country it has an incentive to put pressure on the central bank to increase inflation unexpectedly. The gains from unexpected inflation for the PAYG country will be larger the larger the level of government debt is. In the coming decades the ageing of the population will put the public finances more under pressure and when PAYG countries finance their increased pension obligations by issuing more debt, the incentive of PAYG governments to lobby for surprise inflation will rise. It will lead to negative spillovers for the funded country when the central bank gives in.

These opposing utility effects of surprise inflation in the PAYG country and the funded country imply that there are conflicting interests between the two countries about the direction of monetary policy. Because the monetary policy strategy is not completely transparent, it is not clear how the central bank will react to these conflicting interests. As a consequence, inflation risk may rise with the level of government debt. Higher inflation risk makes government bonds less attractive to hold and the rate of return on bonds will have to increase to induce people to buy the existing stock of government bonds. This increases the debt tax and lowers savings and the capital-labour ratio, and both countries experience negative utility effects from the rise in inflation risk. Actually, the PAYG country experiences negative spillover effects from the fact that they form a monetary union with a funded country in case inflation risk increases. This result arises because the funded country owns a relatively large part of the government bonds and people in the funded country are more risk averse as they do not receive a safe PAYG pension benefit. This implies that these people need to be compensated more in order to be willing to hold the more risky government bonds, that is, the rate of return on government bonds has to increase more in case a funded pension scheme is in place. The rise in the debt tax and its negative consequences will therefore be larger in the PAYG country when it forms a monetary union with a country that uses a fully funded pension scheme instead of a PAYG pension system.

All the scenarios analysed in this paper show that in the long run funded countries are affected negatively by the fact that other countries in the monetary union rely on PAYG-financed pensions when these countries use government debt to cope with the ageing costs. The use of government debt itself has negative long-run spillover effects for funded countries. Moreover a high debt burden may lead to unexpected inflation or a higher inflation risk, which both increase the negative spillover effects even more. In the coming decades it will therefore be important for funded countries that the rules of the Stability and Growth Pact are met, so that debt levels of PAYG countries do not rise to too high levels. The use of government debt by PAYG countries to cope with the costs of ageing will benefit the initial generations in these countries that suffer most from ageing. Future generations, however, are harmed by the larger debt burden. In a monetary union part of this burden can be shifted to the funded country. The debt burden for the PAYG country can be reduced by an unexpected rise in inflation when this country successfully lobbies at the central bank. If this lobbying only raises perceived inflation risk, the negative effects of a rise in the riskiness of government bonds in response to a higher level of public debt cannot be shared with the funded country, however. It may therefore also be in the interest of PAYG countries to obey the fiscal constraints stated in the Stability and Growth Pact to prevent that government debt rises to too high levels. Moreover, it is important for all countries that a central bank like the ECB is independent, credible and transparent about its monetary policy strategy, so that it is clear how monetary policy is determined and the risk of inflation does not rise when debt levels are high.

A Derivation variances

In this appendix I derive the variances of the log gross returns.

A.1 Return on capital/stocks

Take the logarithm of optimality condition (3) and recall the assumption that $\delta = 1$:

$$\log(1 + R_{k,t}) = \log(\alpha) + \log(A_t) + (\alpha - 1)\log(K_t) \tag{64}$$

Now define $\log(1 + R_{k,t}) \equiv r_{k,t}$ and $\log X_t \equiv x_t$, where X_t can be any variable, with the exception of the returns. Now we can write:

$$r_{k,t+1} = \log(\alpha) + a_{t+1} + (\alpha - 1)k_{t+1} \tag{65}$$

The expectation of $r_{k,t+1}$ at time t is:

$$E_t r_{k,t+1} = \log(\alpha) + E_t a_{t+1} + (\alpha - 1)k_{t+1} \tag{66}$$

The variance of $r_{k,t+1}$ can be derived using equations (65) and (66). Note that there is no expectations operator E_t in front of k_{t+1} in equation (66). The capital stock is determined by the savings and portfolio decisions made in the previous period. This implies that the capital stock at time $t + 1$ is already known at the end of period t.

A.2 Return on bonds

Taking logs of equation (6) gives:

$$\log(1 + R_{b,t}) = \log(1 + R^N_{b,t}) + \log\left(\frac{1}{1 + \pi_t}\right) \tag{67}$$

which can be used to write:

$$r_{b,t+1} = r^N_{b,t+1} + \log\left(\frac{1}{1 + \pi_{t+1}}\right) \tag{68}$$

$$E_t r_{b,t+1} = r^N_{b,t+1} + E_t \log\left(\frac{1}{1 + \pi_{t+1}}\right) \tag{69}$$

These two equations can be used to the derive the variance of $r_{b,t+1}$.

B Portfolio choice

B.1 PAYG country

Using equation (17) we take logs of the portfolio-return Euler condition, i.e., $j = p$:

$$\log 1 = \log\left(\frac{1}{1+\rho}\right) + \log(C_t^{Y,P}) + E_t\left[-\log(C_{t+1}^{O,P}) + \log(1+R_{p,t+1}^P)\right] + \underbrace{\frac{1}{2}\text{Var}_t\left[-\log(C_{t+1}^{O,P}) + \log(1+R_{p,t+1}^P)\right]}_{\frac{1}{2}(\sigma_{c^o t}^2)^P + \frac{1}{2}(\sigma_{pt}^2)^P - \text{Cov}_t\left(c_{t+1}^{o,P}; r_{p,t+1}^P\right)} \tag{70}$$

where we used the Jensen's inequality condition for a lognormal random variable X (see also footnote 5):

$$\log E_t X_{t+1} = E_t \log X_{t+1} + \frac{1}{2}\text{Var}_t \log X_{t+1} \tag{71}$$

Equation (70) can be rewritten to:

$$E_t c_{t+1}^{o,P} - c_t^{y,P} = \log\left(\frac{1}{1+\rho}\right) + E_t r_{p,t+1}^P + \frac{1}{2}(\sigma_{c^o t}^2)^P + \frac{1}{2}(\sigma_{pt}^2)^P - \text{Cov}_t\left(c_{t+1}^{o,P}; r_{p,t+1}^P\right) \tag{72}$$

where

$$\begin{aligned}(\sigma_{c^o t}^2)^P &\equiv \text{Var}_t\left[\log(C_{t+1}^{O,P})\right] \\ (\sigma_{pt}^2)^P &\equiv \text{Var}_t\left[\log(1+R_{p,t+1}^P)\right]\end{aligned}$$

In the same way we can derive the log of the Euler equation of the return on bonds, we call this the benchmark-return Euler condition:

$$E_t c_{t+1}^{o,P} - c_t^{y,P} = \log\left(\frac{1}{1+\rho}\right) + E_t r_{b,t+1} + \frac{1}{2}(\sigma_{c^o t}^2)^P + \frac{1}{2}\sigma_{bt}^2 - \text{Cov}_t\left(c_{t+1}^{o,P}; r_{b,t+1}\right) \tag{73}$$

Now subtract the benchmark equation (73) from portfolio-return equation (72):

$$E_t r_{p,t+1}^P - E_t r_{b,t+1} + \frac{1}{2}((\sigma_{pt}^2)^P - \sigma_{bt}^2) = \left[\text{Cov}_t(c_{t+1}^{o,P}; r_{p,t+1}^P) - \text{Cov}_t(c_{t+1}^{o,P}; r_{b,t+1})\right] \tag{74}$$

First we derive the terms on the left-hand side of equation (74). As the return on the portfolio is a linear combination of the return on stocks and the return on bonds (see equation (15)) and the log of a linear combination is not the same as a linear combination of logs, we follow Campbell and Viceira (2002) and use a Taylor approximation of the nonlinear function relating log individual-asset returns to log portfolio returns[18].

[18] This approximation holds exactly in continuous time and is an accurate approximation over short discrete time periods. One may wonder, however, whether this approximation can still be used in a two-period OLG model with social security, where one period is around 30 years. Viceira (2001), Campbell and Viceira (2002) and in particular the detailed calculations of Barberis (2000) show that the magnitude of the horizon effects are negligible, which implies that the approximation is still satisfactory for longer holding periods.

First note that equation (15) can be written as:

$$1 + R^P_{p,t+1} = 1 + R_{b,t+1} + \gamma^P_t [(1 + R_{k,t+1}) - (1 + R_{b,t+1})]$$

Dividing by $(1 + R_{b,t+1})$ gives:

$$\frac{1 + R^P_{p,t+1}}{1 + R_{b,t+1}} = 1 + \gamma^P_t \left[\frac{1 + R_{k,t+1}}{1 + R_{b,t+1}} - 1\right] \tag{75}$$

The log of equation (75) is:

$$r^P_{p,t+1} - r_{b,t+1} = \underbrace{\log\left[1 + \gamma^P_t (\exp(r_{k,t+1} - r_{b,t+1}) - 1)\right]}_{f(r_{k,t+1} - r_{b,t+1})} \tag{76}$$

Now we take a second-order Taylor expansion of $f(\cdot)$ around $r_{k,t+1} - r_{b,t+1} = 0$, which gives:

$$r^P_{p,t+1} \approx r_{b,t+1} + \gamma^P_t (r_{k,t+1} - r_{b,t+1}) + \frac{1}{2}\gamma^P_t (1 - \gamma^P_t)\sigma^2_{k-b,t} \tag{77}$$

where $\sigma^2_{k-b,t} \equiv \mathrm{Var}_t[\log(1 + R_{k,t+1}) - \log(1 + R_{b,t+1})] = \sigma^2_{kt} + \sigma^2_{bt} - 2\sigma_{kb,t}$, is the variance of the excess return of the risky asset over the benchmark return. From equation (77) we know that:

$$E_t r^P_{p,t+1} \approx E_t r_{b,t+1} + \gamma^P_t (E_t r_{k,t+1} - E_t r_{b,t+1}) + \frac{1}{2}\gamma^P_t (1 - \gamma^P_t)\sigma^2_{k-b,t} \tag{78}$$

Using equation (77) and (78) we can derive the variance of the log gross portfolio return:

$$(\sigma^2_{pt})^P = \sigma^2_{bt} + (\gamma^P_t)^2 \sigma^2_{k-b,t} + 2\gamma^P_t \sigma_{k-b,bt} \tag{79}$$

where $\sigma_{k-b,bt}$ is the covariance of the excess return with the benchmark return, that is, $\sigma_{k-b,bt} \equiv \mathrm{Cov}_t[\log(1 + R_{k,t+1}) - \log(1 + R_{b,t+1}); \log(1 + R_{b,t+1})] = \sigma_{kb,t} - \sigma^2_{bt}$.

We derive the terms on the right-hand side of equation (74) using the lifetime budget constraint of an individual. Equation (16) can be rewritten to:

$$C^{O,P}_{t+1} = \left(1 + R^P_{p,t+1}\right)\left[W_t - T^P_t - T^{B,P}_t + \frac{Z^P_{t+1}}{1 + R^P_{p,t+1}} - C^{Y,P}_t\right] \tag{80}$$

Taking logs gives:

$$\log C^{O,P}_{t+1} = \log(1 + R^P_{p,t+1}) + \log\left[W_t - T^P_t - T^{B,P}_t + \frac{Z^P_{t+1}}{1 + R^P_{p,t+1}} - C^{Y,P}_t\right] \tag{81}$$

which is equal to:

$$c^{o,P}_{t+1} = r^P_{p,t+1} + \log\left[\exp w_t - \exp \tau^P_t - \exp \tau^{B,P}_t + \exp(z^P_{t+1} - r^P_{p,t+1}) - \exp c^{y,P}_t\right] \tag{82}$$

We approximate the term between the brackets with a first-order Taylor expansion around $r^P_{p,t+1} = E_t r^P_{p,t+1}$[19]:

$$c^{o,P}_{t+1} \approx r^P_{p,t+1} + \log \underbrace{\left[\exp w_t - \exp \tau^P_t - \exp \tau^{B,P}_t + \exp(z^P_{t+1} - E_t r^P_{p,t+1}) - \exp c^{y,P}_t\right]}_{l_t}$$
$$- \underbrace{\frac{\exp(z^P_{t+1} - E_t r^P_{p,t+1})}{l_t}}_{z_t} (r^P_{p,t+1} - E_t r^P_{p,t+1}) \tag{83}$$

The term z_t can be rewritten to:

$$z_t = \frac{Z^P_{t+1}}{E_t(1 + R^P_{p,t+1}) \exp(-\frac{1}{2}(\sigma^2_{pt})^P)\, S^P_t + Z^P_{t+1}} \approx \frac{\text{PAYG pension benefit}}{\text{expected old-age consumption}} \tag{84}$$

so z_t is the part of expected old-age consumption financed by PAYG pensions.

Now define $y_t \equiv \log l_t + z_t E_t r^P_{p,t+1}$, so that we can write:

$$c^{o,P}_{t+1} \approx r^P_{p,t+1} + y_t - z_t\, r^P_{p,t+1} \tag{85}$$
$$E_t c^{o,P}_{t+1} \approx E_t r^P_{p,t+1} + y_t - z_t\, E_t r^P_{p,t+1} \tag{86}$$

Equation (85) and (86) can be used to derive the variance of the log of old-age consumption and the covariance between the log of old-age consumption and the log portfolio return:

$$(\sigma^2_{c^o t})^P = (1 - z_t)^2 (\sigma^2_{pt})^P \tag{87}$$
$$\mathrm{Cov}_t(c^{o,P}_{t+1}; r^P_{p,t+1}) = (1 - z_t)(\sigma^2_{pt})^P \tag{88}$$

and the covariance with the log benchmark return:

$$\mathrm{Cov}_t(c^{o,P}_{t+1}; r_{b,t+1}) = (1 - z_t)\sigma^2_{bt} + (1 - z_t)\gamma^P_t \sigma_{k-b,bt} \tag{89}$$

Equation (87) shows very clearly that safe PAYG pension benefits lower the variance of old-age consumption.

Substituting equations (78), (79), (88) and (89) into equation (74) gives:

$$\gamma^P_t (E_t r_{k,t+1} - E_t r_{b,t+1}) + \frac{1}{2}\gamma^P_t \sigma^2_{k-b,t} + \gamma^P_t \sigma_{k-b,bt} = (1 - z_t)(\gamma^P_t)^2 \sigma^2_{k-b,t} + (1 - z_t)\gamma^P_t \sigma_{k-b,bt} \tag{90}$$

Dividing by γ^P_t and rearranging gives:

$$\gamma^P_t = \frac{E_t r_{k,t+1} - E_t r_{b,t+1} + \frac{1}{2}\sigma^2_{k-b,t}}{(1 - z_t)\sigma^2_{k-b,t}} + \frac{z_t}{1 - z_t}\frac{\sigma_{k-b,bt}}{\sigma^2_{k-b,t}} \tag{91}$$

[19]This is comparable to the approximation of a linear budget constraint in Campbell (1993).

Using the fact that:

$$E_t r_{k,t+1} = E_t \log(1 + R_{k,t+1}) = \log E_t(1 + R_{k,t+1}) - \frac{1}{2}\,\sigma_{kt}^2 \tag{92}$$

$$E_t r_{b,t+1} = E_t \log(1 + R_{b,t+1}) = \log E_t(1 + R_{b,t+1}) - \frac{1}{2}\,\sigma_{bt}^2 \tag{93}$$

gives the expression for γ_t^P in terms of simple returns (equation (18)).

B.2 Funded country

In the funded country contributions of the young (T_t^F) are invested and returned to them with interest in the next period when they are retired (Z_{t+1}^F), that is, $Z_{t+1}^F = (1 + R_{p,t+1}^F)T_t^F$. The funded scheme has fixed contributions, which implies that contributions to the pension scheme are exactly offset by an equal reduction in private savings. This means that the pension fund is neutral and the economy behaves in exactly the same way as if there were no pension scheme. This makes the derivation of the optimal portfolio decision a lot more simple, as we do not have to take a first-order Taylor approximation of the consumer's budget constraint to obtain the covariance between the returns and old-age consumption.

Taking logs of the portfolio-return and benchmark-return Euler condition and subtracting gives (see equation (74)):

$$E_t r_{p,t+1}^F - E_t r_{b,t+1} + \frac{1}{2}((\sigma_{pt}^2)^F - \sigma_{bt}^2) = \mathrm{Cov}_t(c_{t+1}^{o,F}; r_{p,t+1}) - \mathrm{Cov}_t(c_{t+1}^{o,F}; r_{b,t+1}) \tag{94}$$

We can use the second-order Taylor approximation of $r_{p,t+1}$ to derive:

$$E_t r_{p,t+1}^F \approx E_t r_{b,t+1} + \gamma_t^F(E_t r_{k,t+1} - E_t r_{b,t+1}) + \frac{1}{2}\,\gamma_t^F(1 - \gamma_t^F)\sigma_{k-b,t}^2 \tag{95}$$

$$(\sigma_{pt}^2)^F \approx \sigma_{bt}^2 + \left(\gamma_t^F\right)^2 \sigma_{k-b,t}^2 + 2\gamma_t^F \sigma_{k-b,bt} \tag{96}$$

The budget constraint of an individual in a funded country is:

$$C_{t+1}^{O,F} = \left(1 + R_{p,t+1}^F\right)\left[W_t - T_t^F - T_t^{B,F} + \frac{Z_{t+1}^F}{1 + R_{p,t+1}} - C_t^{Y,F}\right] \tag{97}$$

Using the fact that $Z_{t+1}^F = (1 + R_{p,t+1})\,T_t^F$, equation (97) can be written as:

$$C_{t+1}^{O,F} = \left(1 + R_{p,t+1}^F\right)\left[W_t - T_t^{B,F} - C_t^{Y,F}\right] \tag{98}$$

Dividing by $C_t^{Y,F}$ gives:

$$\frac{C_{t+1}^{O,F}}{C_t^{Y,F}} = \left(1 + R_{p,t+1}^F\right)\underbrace{\left[\frac{W_t - T_t^{B,F}}{C_t^{Y,F}} - 1\right]}_{g_t} \tag{99}$$

In case of a logarithmic utility function people consume a fixed proportion of their wealth, which implies that the term g_t is constant. Taking logs gives:

$$\log C_{t+1}^{O,F} - \log C_t^{Y,F} = \log(1 + R_{p,t+1}^F) + \log g_t \tag{100}$$

which gives:

$$\begin{aligned} c_{t+1}^{o,F} &= c_t^{y,F} + r_{p,t+1}^F + \log g_t & (101) \\ E_t c_{t+1}^{o,F} &= c_t^{y,F} + E_t r_{p,t+1}^F + \log g_t & (102) \end{aligned}$$

Equations (101) and (102) imply that the variance of the log of old-age consumption equals the variance of the log portfolio return, that is, $(\sigma_{c^o t}^2)^F = (\sigma_{pt}^2)^F$, and that:

$$\begin{aligned} \text{Cov}_t(c_{t+1}^{o,F}, r_{p,t+1}^F) &= (\sigma_{pt}^2)^F & (103) \\ \text{Cov}_t(c_{t+1}^{o,F}, r_{b,t+1}) &= \sigma_{bt}^2 + \gamma_t^F \sigma_{k-b,bt} & (104) \end{aligned}$$

Substituting equations (95), (96), (103) and (104) into equation (94) gives:

$$\gamma_t^F(E_t r_{k,t+1} - E_t r_{b,t+1}) + \frac{1}{2}\gamma_t^F \sigma_{k-b,t}^2 + \gamma_t^F \sigma_{k-b,bt} = (\gamma_t^F)^2 \sigma_{k-b,t}^2 + \gamma_t^F \sigma_{k-b,bt} \tag{105}$$

Dividing by γ_t^F and rearranging gives:

$$\gamma_t^F = \frac{E_t r_{k,t+1} - E_t r_{b,t+1} + \frac{1}{2}\sigma_{k-b,t}^2}{\sigma_{k-b,t}^2} \tag{106}$$

Or in simple returns:

$$\gamma_t^F = \frac{\log E_t(1 + R_{k,t+1}) - \log E_t(1 + R_{b,t+1})}{\sigma_{k-b,t}^2} - \frac{\sigma_{k-b,bt}}{\sigma_{k-b,t}^2} \tag{107}$$

C Savings

C.1 PAYG country

Using equation (86) and the definition of y_t we can write:

$$E_t c_{t+1}^{o,P} = E_t r_{p,t+1}^P + \log l_t \tag{108}$$

Substitute this equation into the log portfolio-return Euler condition (72):

$$\log l_t - c_t^{y,P} = \log\left(\frac{1}{1+\rho}\right) + \frac{1}{2}\left(\sigma_{c^o t}^2\right)^P + \frac{1}{2}\left(\sigma_{pt}^2\right)^P - \text{Cov}_t\left(c_{t+1}^{o,P}; r_{p,t+1}^P\right) \tag{109}$$

Using equations (87) and (88) this can be written as:

$$\log l_t - c_t^{y,P} - \frac{1}{2} z_t^2 \left(\sigma_{pt}^2\right)^P = \log\left(\frac{1}{1+\rho}\right) \tag{110}$$

This can be rewritten to:

$$\left[W_t - T_t^P - T_t^{B,P} + \frac{Z_{t+1}^P}{E_t(1+R_{p,t+1}^P)\exp(-\frac{1}{2}(\sigma_{pt}^2)^P)} - C_t^{Y,P}\right] \frac{1}{C_t^{Y,P}} \left(\frac{1}{1+\rho}\right)^{-1} = \exp\left[\frac{1}{2} z_t^2 \left(\sigma_{pt}^2\right)^P\right] \tag{111}$$

where I used the fact that $E_t r_{p,t+1}^P = E_t \log(1+R_{p,t+1}^P) = \log E_t(1+R_{p,t+1}^P) - \frac{1}{2}\left(\sigma_{pt}^2\right)^P$. Equation (111) can be used to derive the optimal solutions for consumption when young and savings:

$$C_t^{Y,P} = \frac{1+\rho}{1+\rho+\exp\left[\frac{1}{2} z_t^2 (\sigma_{pt}^2)^P\right]} \left[W_t - T_t^P - T_t^{B,P} + \frac{Z_{t+1}^P}{E_t(1+R_{p,t+1}^P)\exp(-\frac{1}{2}(\sigma_{pt}^2)^P)}\right] \tag{112}$$

$$\begin{aligned} S_t^P = {} & \frac{\exp\left[\frac{1}{2} z_t^2 (\sigma_{pt}^2)^P\right]}{1+\rho+\exp\left[\frac{1}{2} z_t^2 (\sigma_{pt}^2)^P\right]} \left(W_t - T_t^P - T_t^{B,P}\right) \\ & - \frac{1+\rho}{1+\rho+\exp\left[\frac{1}{2} z_t^2 (\sigma_{pt}^2)^P\right]} \frac{Z_{t+1}^P}{E_t(1+R_{p,t+1}^P)\exp(-\frac{1}{2}(\sigma_{pt}^2)^P)} \end{aligned} \tag{113}$$

C.2 Funded country

Substituting equation (102) into the log portfolio-return Euler condition (72) we can write:

$$\log g_t = \log\left(\frac{1}{1+\rho}\right) + \frac{1}{2}\left(\sigma_{c^o t}^2\right)^F + \frac{1}{2}\left(\sigma_{pt}^2\right)^F - \mathrm{Cov}_t\left(c_{t+1}^{o,F}; r_{p,t+1}^F\right) \tag{114}$$

Using the fact that $\left(\sigma_{c^o t}^2\right)^F = \left(\sigma_{pt}^2\right)^F$ and equation (103) we can write:

$$\log g_t = \log\left(\frac{1}{1+\rho}\right) \tag{115}$$

Using the definition of g_t in equation (99) we can derive the optimal solutions for consumption when young and savings:

$$C_t^{Y,F} = \frac{1+\rho}{2+\rho}\left(W_t - T_t^{B,F}\right) \tag{116}$$

$$S_t^F = \frac{1}{2+\rho}\left(W_t - T_t^{B,F}\right) \tag{117}$$

D Government debt

In this appendix we show the derivation of the first-order difference equations in case the government uses debt instead of taxes to finance the pension benefits. The so-called debt tax $T^{B,P}$ to keep the debt dynamics sustainable is levied on the young.

Linearise equation (23) with respect to ζ around the initial steady state:

$$\frac{\partial S_t^P}{\partial \zeta} + \nu \frac{\partial S_t^F}{\partial \zeta} = (1+\nu)\frac{\partial K_{t+1}}{\partial \zeta} + \frac{\partial B_{t+1}^P}{\partial \zeta} \tag{118}$$

Simplifying gives:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \frac{1}{1+\nu}\left[\frac{\partial S_t^P}{\partial \zeta} + \nu \frac{\partial S_t^F}{\partial \zeta} - \frac{\partial B_{t+1}^P}{\partial \zeta}\right] \tag{119}$$

From equation (22) we can derive the change of savings in the funded country:

$$\frac{\partial S_t^F}{\partial \zeta} = \frac{1}{2+\rho}\left(\frac{\partial W_t}{\partial \zeta} - \frac{\partial T_t^{B,F}}{\partial \zeta}\right) \tag{120}$$

The total derivative of savings in the PAYG country is[20]:

$$\frac{\partial S_t^P}{\partial \zeta} = \frac{\partial S_t^P}{\partial (\sigma_{pt}^2)^P}\frac{\partial (\sigma_{pt}^2)^P}{\partial \zeta} + \frac{\partial S_t^P}{\partial W_t}\frac{\partial W_t}{\partial \zeta} + \frac{\partial S_t^P}{\partial T_t^P}\frac{\partial T_t^P}{\partial \zeta} + \frac{\partial S_t^P}{\partial T_t^{B,P}}\frac{\partial T_t^{B,P}}{\partial \zeta} + \frac{\partial S_t^P}{\partial E_t R_{p,t+1}^P}\frac{\partial E_t R_{p,t+1}^P}{\partial \zeta} \tag{121}$$

Using equation (21) we can derive:

$$\begin{aligned}\frac{\partial S_t^P}{\partial (\sigma_{pt}^2)^P} = &\frac{\frac{1}{2}z^2(1+\rho)\exp(\frac{1}{2}z^2(\sigma_p^2)^P)}{\left[\exp(\frac{1}{2}z^2(\sigma_p^2)^P)+1+\rho\right]^2}\left[W - T^P - T^{B,P} + \frac{Z^P}{E(1+R_p^P)\exp(-\frac{1}{2}(\sigma_p^2)^P)}\right] \\ &- \frac{\frac{1}{2}(1+\rho)Z^P}{\left[\exp(\frac{1}{2}z^2(\sigma_p^2)^P)+1+\rho\right]E(1+R_p^P)\exp(-\frac{1}{2}(\sigma_p^2)^P)} \gtrless 0\end{aligned} \tag{122}$$

$$\frac{\partial S_t^P}{\partial W_t} = \frac{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)}{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)+1+\rho} > 0 \tag{123}$$

$$\frac{\partial S_t^P}{\partial T_t^P} = \frac{\partial S_t^P}{\partial T_t^{B,P}} = -\frac{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)}{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)+1+\rho} < 0 \tag{124}$$

$$\frac{\partial S_t^P}{\partial E_t R_{p,t+1}^P} = \frac{(1+\rho)Z^P\exp(-\frac{1}{2}(\sigma_p^2)^P)}{\left[\exp(\frac{1}{2}z^2(\sigma_p^2)^P)+1+\rho\right]\left[E(1+R_p^P)\exp(-\frac{1}{2}(\sigma_p^2)^P)\right]^2} > 0 \tag{125}$$

In order to derive the change in $(\sigma_{pt}^2)^P$ we first need to know how γ_t^P changes:

$$\frac{\partial \gamma_t^P}{\partial \zeta} = \frac{1}{(1-z)\sigma_{k-b}^2 E(1+R_k)}\frac{\partial E_t R_{k,t+1}}{\partial \zeta} - \frac{1}{(1-z)\sigma_{k-b}^2 E(1+R_b)}\frac{\partial E_t R_{b,t+1}}{\partial \zeta} \tag{126}$$

[20]We do not take into account the second-order effects on z_t, as this complicates the analysis to a large extent.

where:

$$\begin{aligned}
\frac{\partial E_t R_{k,t+1}}{\partial \zeta} &= \frac{(\alpha-1)E(1+R_k)}{K}\frac{\partial K_{t+1}}{\partial \zeta} && (127)\\
\frac{\partial E_t R_{b,t+1}}{\partial \zeta} &= \frac{E(1+R_b)}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta} && (128)
\end{aligned}$$

Then the changes in γ_t^P and $(\sigma_{pt}^2)^P$ are:

$$\begin{aligned}
\frac{\partial \gamma_t^P}{\partial \zeta} &= \frac{1}{(1-z)\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta}\right] && (129)\\
\frac{\partial (\sigma_{pt}^2)^P}{\partial \zeta} &= \frac{2\gamma^P\sigma_{k-b}^2 + 2\sigma_{k-b,b}}{(1-z)\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta}\right] && (130)
\end{aligned}$$

The changes of W_t and $E_t R_{p,t+1}^P$ are given by:

$$\begin{aligned}
\frac{\partial W_t}{\partial \zeta} &= \frac{\alpha W}{K}\frac{\partial K_t}{\partial \zeta} && (131)\\
\frac{\partial E_t R_{p,t+1}^P}{\partial \zeta} &= \frac{E(R_k)-E(R_b)}{(1-z)\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta}\right] && (132)\\
&\quad + \frac{\gamma^P(\alpha-1)E(1+R_k)}{K}\frac{\partial K_{t+1}}{\partial \zeta} + \frac{(1-\gamma^P)E(1+R_b)}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta}
\end{aligned}$$

Using all these equations we can write equations (120) and (121) as:

$$\begin{aligned}
\frac{\partial S_t^F}{\partial \zeta} &= \frac{\alpha W}{(2+\rho)K}\frac{\partial K_t}{\partial \zeta} - \frac{1}{2+\rho}\frac{\partial T_t^{B,F}}{\partial \zeta} && (133)\\
\frac{\partial S_t^P}{\partial \zeta} &= \frac{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\alpha W}{[\exp(\frac{1}{2}z^2(\sigma_p^2)^P)+1+\rho]K}\frac{K_t}{\partial \zeta} - \frac{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)}{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)+1+\rho}\left(\frac{\partial T_t^P}{\partial \zeta} + \frac{\partial T_t^{B,P}}{\partial \zeta}\right)\\
&+ S_{\sigma_p^2}^P \frac{2\gamma^P\sigma_{k-b}^2 + 2\sigma_{k-b,b}}{(1-z)\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta}\right]\\
&+ S_{R_p}^P \left\{\frac{E(R_k)-E(R_b)}{(1-z)\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta}\right]\right. && (134)\\
&+ \left.\frac{\gamma^P(\alpha-1)E(1+R_k)}{K}\frac{\partial K_{t+1}}{\partial \zeta} + \frac{(1-\gamma^P)E(1+R_b)}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta}\right\}
\end{aligned}$$

where $S_{\sigma_p^2}^P \equiv \frac{\partial S_t^P}{\partial (\sigma_{pt}^2)^P}$ and $S_{R_p}^P \equiv \frac{\partial S_t^P}{\partial E_t R_{p,t+1}^P}$, see equations (122) and (125).

Now use the other dynamic equation (24) to derive:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \frac{1}{1+\nu}\left[\gamma^P\frac{\partial S_t^P}{\partial \zeta} + S^P\frac{\partial \gamma_t^P}{\partial \zeta} + \nu\gamma^F\frac{\partial S_t^F}{\partial \zeta} + \nu S^F\frac{\gamma_t^F}{\partial \zeta}\right] \tag{135}$$

Using equation (119) we can rewrite this equation to:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \frac{1}{(1-\gamma^P)(1+\nu)}\left[\nu(\gamma^F-\gamma^P)\frac{\partial S_t^F}{\partial \zeta} + \gamma^P \frac{\partial B_{t+1}^P}{\partial \zeta} + S^P\frac{\partial \gamma_t^P}{\partial \zeta} + \nu S^F\frac{\partial \gamma_t^F}{\partial \zeta}\right] \tag{136}$$

The change of γ_t^P is given in equation (129). The change of γ_t^F is:

$$\frac{\partial \gamma_t^F}{\partial \zeta} = \frac{1}{\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta}\right] \tag{137}$$

Substituting this information into equation (136) and simplifying gives:

$$\frac{\partial K_{t+1}}{\partial \zeta} = -\frac{1}{\Phi}\frac{\partial R_{b,t+1}^N}{\partial \zeta} - \frac{\nu(\gamma^P-\gamma^F)\sigma_{k-b}^2(1+R_b^N)}{\Phi(\nu S^F+\frac{S^P}{1-z})}\frac{\partial S_t^F}{\partial \zeta} + \frac{\gamma^P\sigma_{k-b}^2(1+R_b^N)}{\Phi(\nu S^F+\frac{S^P}{1-z})}\frac{\partial B_{t+1}^P}{\partial \zeta} \tag{138}$$

where $\Phi \equiv \frac{[(1+\nu)K(1-\gamma^P)\sigma_{k-b}^2+(1-\alpha)(\nu S^F+\frac{S^P}{1-z})](1+R_b^N)}{(\nu S^F+\frac{S^P}{1-z})K} > 0$. Equation (138) can be rewritten to:

$$\frac{\partial R_{b,t+1}^N}{\partial \zeta} = -\Phi\frac{\partial K_{t+1}}{\partial \zeta} - \frac{\nu(\gamma^P-\gamma^F)\sigma_{k-b}^2(1+R_b^N)}{\nu S^F+\frac{S^P}{1-z}}\frac{\partial S_t^F}{\partial \zeta} + \frac{\gamma^P\sigma_{k-b}^2(1+R_b^N)}{\nu S^F+\frac{S^P}{1-z}}\frac{\partial B_{t+1}^P}{\partial \zeta} \tag{139}$$

Use this equation to substitute for $\frac{\partial R_{b,t+1}^N}{\partial \zeta}$ in equation (134):

$$\begin{aligned}\frac{\partial S_t^P}{\partial \zeta} = {} & \frac{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\alpha W}{[\exp(\frac{1}{2}z^2(\sigma_p^2)^P)+1+\rho]K}\frac{\partial K_t}{\partial \zeta} - \frac{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)}{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)+1+\rho}\left(\frac{\partial T_t^{B,P}}{\partial \zeta}+\frac{\partial T_t^P}{\partial \zeta}\right) \\ & + S_{\sigma_p^2}^P\left\{\Delta_{\sigma_p^2}\frac{\partial K_{t+1}}{\partial \zeta} + \frac{\nu(\gamma^P-\gamma^F)(2\gamma^P\sigma_{k-b}^2+2\sigma_{k-b,b})}{\nu S^F(1-z)+S^P}\frac{\partial S_t^F}{\partial \zeta}\right. \\ & \left. - \frac{\gamma^P(2\gamma^P\sigma_{k-b}^2+2\sigma_{k-b,b})}{\nu S^F(1-z)+S^P}\frac{\partial B_{t+1}^P}{\partial \zeta}\right\} \\ & + S_{R_p}^P\left\{\Delta_{R_p}\frac{\partial K_{t+1}}{\partial \zeta} + \left[(1-\gamma^P)E(1+R_b) - \frac{E(R_k)-E(R_b)}{(1-z)\sigma_{k-b}^2}\right]\cdot\right. \\ & \left.\left[\frac{\gamma^P\sigma_{k-b}^2(1-z)}{\nu S^F(1-z)+S^P}\frac{\partial B_{t+1}^P}{\partial \zeta} - \frac{\nu(\gamma^P-\gamma^F)\sigma_{k-b}^2(1-z)}{\nu S^F(1-z)+S^P}\frac{\partial S_t^F}{\partial \zeta}\right]\right\}\end{aligned} \tag{140}$$

where

$$\Delta_{\sigma_p^2} \equiv \frac{2\gamma^P\sigma_{k-b}^2+2\sigma_{k-b,b}}{(1-z)\sigma_{k-b}^2}\frac{\overbrace{\Phi K-(1-\alpha)(1+R_b^N)}^{>0}}{K(1+R_b^N)} > 0 \tag{141}$$

$$\begin{aligned}\Delta_{R_p} \equiv {} & \frac{[E(R_k)-E(R_b)][\Phi K-(1-\alpha)(1+R_b^N)]}{(1-z)\sigma_{k-b}^2K(1+R_b^N)} \\ & - \frac{\gamma^P(1-\alpha)E(1+R_k)}{K} - \frac{(1-\gamma^P)E(1+R_b)\Phi}{1+R_b^N} \lessapprox 0\end{aligned} \tag{142}$$

Substituting equations (133) and (140) into equation (119) and simplifying gives:

$$\begin{aligned}\frac{\partial K_{t+1}}{\partial \zeta} = \frac{\alpha W}{\Psi K}\frac{\partial K_t}{\partial \zeta} - \frac{\Omega \nu}{\Psi^F}\frac{\partial T_t^{B,F}}{\partial \zeta} - \frac{1}{\Psi^P}\frac{\partial T_t^{B,P}}{\partial \zeta} - \frac{1}{\Psi^P}\frac{\partial T_t^{P}}{\partial \zeta} \\ - \frac{1}{1+\nu - S^P_{\sigma_p^2}\Delta_{\sigma_p^2} - S^P_{R_p}}\left[\frac{\partial B^P_{t+1}}{\partial \zeta} - \Omega_{B^P}\frac{\partial B^P_{t+1}}{\partial \zeta}\right]\end{aligned} \tag{143}$$

where:

$$\begin{aligned}\Omega \equiv 1 + \frac{\gamma^P - \gamma^F}{\nu S^F(1-z) + S^P}\Big\{ & S^P_{\sigma_p^2}(2\gamma^P\sigma^2_{k-b} + 2\sigma_{k-b,b}) \\ & - S^P_{R_p}(1-\gamma^P)E(1+R_b)(1-z)\sigma^2_{k-b} + S^P_{R_p}[E(R_k) - E(R_b)]\Big\} \gtrless 0\end{aligned} \tag{144}$$

$$\begin{aligned}\Omega_{B^P} \equiv \frac{\gamma^P}{\nu S^F(1-z) + S^P}\Big\{ & S^P_{R_p}(1-\gamma^P)E(1+R_b)(1-z)\sigma^2_{k-b} - S^P_{R_p}[E(R_k) - E(R_b)] \\ & - S^P_{\sigma_p^2}(2\gamma^P\sigma^2_{k-b} + 2\sigma_{k-b,b})\Big\} \gtrless 0\end{aligned} \tag{145}$$

$$\Psi \equiv \frac{\left[\exp(\frac{1}{2}z^2(\sigma_p^2)^P) + 1 + \rho\right](2+\rho)\left[1+\nu - S^P_{\sigma_p^2}\Delta_{\sigma_p^2} - S^P_{R_p}\Delta_{R_p}\right]}{\exp\left(\frac{1}{2}z^2(\sigma_p^2)^P\right)(2+\rho) + \Omega\nu\left[\exp(\frac{1}{2}z^2(\sigma_p^2)^P)1 + \rho\right]} \gtrless 0 \tag{146}$$

$$\Psi^P \equiv \frac{\left[\exp(\frac{1}{2}z^2(\sigma_p^2)^P) + 1 + \rho\right]\left[1+\nu - S^P_{\sigma_p^2}\Delta_{\sigma_p^2} - S^P_{R_p}\Delta_{R_p}\right]}{\exp\left(\frac{1}{2}z^2(\sigma_p^2)^P\right)} \gtrless 0 \tag{147}$$

$$\Psi^F \equiv (2+\rho)\left[1+\nu - S^P_{\sigma_p^2}\Delta_{\sigma_p^2} - S^P_{R_p}\Delta_{R_p}\right] \gtrless 0 \tag{148}$$

Equations (143), (139) and (133) together give the system of equations in Section 3.

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