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The Role of Optimal Fiscal Policy in a Currency Union*

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Abstract

By constructing a dynamic stochastic general equilibrium model, this paper verifies the role of optimal fiscal policy in a currency union with. Under the distorted setting, a single optimal monetary policy has decisive role to enhance social welfare by stabilizing inflation and the output gap in each country simultaneously when all goods are tradable. However, approximately half of goods are non-tradable in the Euro area, this implication is not applied directly. When there has non-tradable goods, optimal monetary policy alone cannot maximize social welfare because of real exchange rate anomaly or Balassa–Samuelson theorem, which holds that stabilizing inflation and the output gap in each country simultaneously is prevented. In this case, an optimal monetary and cooperative optimal fiscal policy mix is essential to maximize social welfare. Also, our game theoretical analysis shows that self-oriented setting can bring optimal allocation which corresponds to one brought about by cooperative setting.

Keywords: currency union, DSGE, real exchange rate anomaly, optimal monetary policy, monetary and fiscal policy mix

JEL Classification: E52; E62; F41

1 Introduction

A currency union that was previously a pure matter of academic speculation became a reality when the European Monetary Union (EMU) was established. The creation of the EMU led to new challenges for policy makers. This study provides a tractable framework suitable for the analysis of fiscal and monetary policy in a currency union and studies its implications for the optimal design of such policies, not only within the context of the union-wide economy but also within the context of the individual countries that comprise the currency union.

Discussions of optimal monetary policy in a currency union become vigorous in such a situation.[1] Assuming that all goods are tradable, Benigno (2004) analyzes optimal monetary policy in a simple situation such as perfect risk sharing and the economies of the two countries. He finds that the solitary central bank in a currency union can achieve welfare maximization not only in union-wide but also in each country. In contrast, Gali and Monacelli (Forthcoming) insist on a monetary and fiscal policy mix using a currency union model consisting not of two countries but infinite infinitesimal countries to maximize social welfare. Under this framework, the solitary central bank can maximize welfare at a union-wide level whereas it needs strong support from the fiscal authority to maximize welfare at the level of each country. Ferrero (2007) and Beetsma and Jensen (2005) analyze optimal monetary and fiscal policy in a two-country currency union, finding that that optimal fiscal policy is essential in a currency union to maximize social welfare.[2]

[1]Note that there are some papers discussing a role of optimal monetary and fiscal policy. Schmitt-Grohe and Uribe (2004a), (2004b) and (Forthcoming) find relationship between fiscal policy, output gap and inflation in a closed economy. Benigno and De Paoli (2006) find that it is optimal not to smooth taxes once the open economy dimension of the analysis is taken into account.

[2]Gali and Monacelli (Forthcoming), Ferrero (2007) and Beetsma and Jensen (2005) assume that all goods are tradable. Gali and Monacelli (Forthcoming) does not consider a government

It is noteworthy that we refer to papers that do not follow the dynamic stochastic general equilibrium (DSGE) model. Focusing on fiscal theory of price level, Canzoneri, Cumby and Diba (2001) discuss fiscal discipline and exchange rate system. To maintain a common currency union, the discipline of a Ricardian regime is essential. They also point out that the Stability and Growth Pact which is written into the Maastricht treaty is sufficient for a Ricardian regime. This contrasts with policy implications derived by Gali and Monacelli (Forthcoming) and Ferrero (2007). Also, a conventional paper should be cited. McKinnon (1963) insists on the necessity of moving fiscal policy control from the local or national government to the central government. The existence of non-tradable goods generates disparity among countries in a currency union. Thus, he advocates that fiscal transfer among countries under a centralized government is essential. It is clear that their policy implications on the role of fiscal policy are not consistent but vary by reviewing outstanding papers to which we refer above. To stabilizes both inflation and the output gap immediately under the assumption that all goods are tradable, Canzoneri, Cumby and Diba (2001) and Benigno (2004) imply that additional fiscal policy is not essential while Gali and Monacelli (Forthcoming) and Ferrero (2007) discuss contradictory policy implications which support the necessity of fiscal policy. Assuming existence of non-tradable goods, McKinnon (1963) advocates fiscal transfer among countries via a centralized government. There is significant difference in the policy implications of the preceding studies discuss optimal policy in the currency union. The papers clearly leave some questions. Is fiscal policy needed in a currency union from the view point of maximizing social welfare? What is the causation which creates differences in their policy implications among these papers? Can

budget constraint explicitly while Ferrero (2007) does so.

fiscal policy conducted by local government enhance social welfare, instead of income transfer by a centralized government? Can a decentralized setting be designed?

We should consider two key elements to solve the questions left by preceding papers. Although much DSGE literature do not premise form of government budget constraint, we should assume such a constraint to clarify the role of fiscal policy. Next, we should analyze the role of fiscal policy in a currency union with non-tradable goods although few studies assume the existence of non-tradable goods in recent DSGE literature.[3] Assuming existence of non-tradable goods may derive the policy implication that fiscal policy has an important role in a currency union because McKinnon (1963) advocates the necessity of fiscal policy assuming non-tradable goods. Also, assuming non-tradable goods is plausible from the view point of a well considered, actual currency union, namely, the Euro area. Following the definition that regards goods produced in the manufacturing industry, agriculture, forestry, fishery and mining as tradable goods and regards goods produced in other industries as non-tradable goods, as used by Canzoneri, Cumby and Diba (1999), non-tradable goods, in terms of both current and purchaser's prices, accounted for 50.3% of the sum of non-tradable goods and tradable goods in major Euro area countries such as Belgium, Germany, France, Greece, Italy, the Netherlands, Portugal and Spain in 1999. It is obvious that the sharing of non-tradable goods should not be ignored in analyzing monetary policy.

This study constructs a DSGE model depicting a currency union consisting

[3]Neither papers on monetary policy in a currency union nor papers on monetary policy in an open economy, such as Benigno (2004), Benigno and Benigno (2008), Gali and Monacelli (Forthcoming) and Okano (2007), consider the existence of nontradable goods, although these papers derive some important implications.

of two countries with non-tradable goods and explicit government budget constraint to obtain appropriate answers to the preceding questions. The model developed in this study has two distinctive features for analyzing an optimal policy design. Because it has two features, our model is a nested model of preceding papers such as Benigno (2004), Gali and Monacelli (Forthcoming) and Ferrero (2007). First, because of the existence of non-tradable goods, we focus on the Balassa–Samuelson theorem and real exchange rate anomaly, which explain deviations in the nominal exchange rate from purchasing power parity.[4] Needless to say, the effect of non-tradable goods on an open economy is not widely considered in the DSGE literature. Whereas the nominal exchange rate does not appear in our model because the model is a closed currency union system, the Balassa–Samuelson theorem and real exchange rate anomaly explain the disparity in the consumer price indices (CPIs) between two countries comprising a currency union. Second, we allow implementation of fiscal policy by local governments to eliminate the effects of the non-tradable goods. While the appropriateness of a centralized government is advocated by and McKinnon (1963) for welfare maximization, this study verifies that a centralized government does not need to maximize social welfare although all goods are not tradable. We prove that self-oriented local governments can bring optimal allocation which is brought about by cooperative local governments. Note that our contribution on developing the model is to mix non-tradable goods with government budget constraint.[5]

[4]Empirical Analysis has focused on nontrdable goods and Balassa–Samuelson theorem. Analyzing exchange rate volatility rather than monetary policy, Stockman and Tesar (1995), Benigno and Thoenissen (2005) and Selaive and Tuesta (2006) focus on nontradable goods where the real exchange rate is volatile and tends to move in the opposite direction to relative consumption across countries, namely, the consumption–real exchange rate anomaly. These papers based on the Balassa–Samuelson theorem point out the relationship between the anomaly and the theorem. In this paper, the another side effect reffers such anomaly.

[5]Under simple settings, monetary and fiscal interactions in an open economy have already

Summary of answers which can be obtained are as follows. We show social welfare and macroeconomic volatilities under the cases that all goods are tradable and half of goods are nontradable. In the case that all goods are tradable, which is corresponding to the setting in Ferrero (2007), the role of optimal fiscal policy is substantially small although it supports to maximize welfare. However, in the case that half of goods are tradable, the role of optimal fiscal policy is significantly large. We can clarify the causation which makes disparities in policy implications among papers. Difference in policy implications among these papers mainly depends on price index which is assumed in each paper. In addition, we can prove that self-oriented government can maximize social welfare using game theory framework. Thus, neither income transfer nor centralized government is needed to enhance social welfare. Details on this discussion are given in latter sections.

The paper is organized as follows. Section 2 constructs the model. Section 3 log-linearizes the model. Section 4 analyzes a role of optimal fiscal policy under a cooperative setting. Section 5 considers the possibility of a decentralized setting to attain the allocation brought about by the cooperative solution. Section 6 concludes the paper.

2 The Model

We construct a closed-system currency union model belonging to the class of DSGE models with nominal rigidities and imperfect competition, following Obstfeld and Rogoff (2000) and Gali and Monacelli (2005).[6] The union-wide economy consists of two equally sized countries, countries H and F. Country H

investigated by Lombardo and Sutherland (2004).

[6]Appendices clarifying not only derivation of the model but also details on other technical skills are provided at the URL: http://www.cku.ac.jp/okano/papers_e.html.

produces an array of differentiated goods indexed by the interval $h \in [0,1]$, while country F produces an array of differentiated goods indexed by $f \in [1,2]$.

2.1 Households

The preferences of the representative household in country H are given by:

$$\mathcal{U} \equiv \mathrm{E}_t \sum_{t=0}^{\infty} \delta^t U_t, \tag{1}$$

where $U_t \equiv \ln C_t - \frac{1}{1+\varphi} N_t^{1+\varphi}$ denotes the period utility in country H, E_t denotes the expectation, conditional on the information set at period t, $\delta \in (0,1)$ denotes the subjective discount factor, C_t denotes consumption in country H, $N_t \equiv N_{H,t} + N_{\mathcal{N},t}$ denote hours of work in country H, $N_{H,t}$ and $N_{\mathcal{N},t}$ denote hours of work to produce tradable goods produced in country H and nontradable goods produced in country H, respectively, γ denotes the share of tradables in the CPI and φ denotes the inverse of the labor supply elasticity. $\mathcal{U}^*$, denoting preference of the representative household in country F, is defined analogously. We note that quantities and prices peculiar to country F are denoted by asterisks while quantities and prices without asterisks are those in country H.

More precisely, private consumption is a composite index defined by:

$$C_t \equiv \left[\gamma^{\frac{1}{\eta}} C_{\mathcal{T},t}^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} C_{\mathcal{N},t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $C_{\mathcal{T},t} \equiv 2 C_{H,t}^{\frac{1}{2}} C_{F,t}^{\frac{1}{2}}$ denotes the consumption index for tradables, $C_{H,t}$, $C_{F,t}$ and $C_{\mathcal{N},t}$ denote Dixit–Stiglitz-type indices of consumption across the tradables produced in country H and produced in country F, and nontradables produced in country H, respectively, $\theta > 1$ denotes the elasticity of substitution across goods produced within a country, and $\eta > 0$ denotes the elasticity of substitution between tradable and nontradable goods. Note that C_t^* is defined analogously to

Eq.(2) whereas $C^*_{\mathcal{N},t}$, denoting the nontradables produced in country F, relaces $C_{\mathcal{N},t}$.[7]

Total consumption expenditures by households in country H are given by $P_{H,t}C_{H,t}+P_{F,t}C_{F,t}+P_{\mathcal{N},t}C_{\mathcal{N},t} = P_tC_t$, with $P_{H,t}$ and $P_{F,t}$ being Dixit–Stiglitz-type indices of the price of tradable goods produced in countries H and F, respectively, and $P_{\mathcal{N},t}$ being Dixit–Stiglitz-type indices of the price of nontradable goods produced in country H. A sequence of budget constraints in country H is given by:

$$D^n_t + W_tN_t + S_t \geq P_tC_t + \mathrm{E}_tQ_{t,t+1}D^n_{t+1}, \tag{3}$$

where $Q_{t,t+1}$ denotes the stochastic discount factor, D^n_t denotes the nominal payoff of the bond portfolio purchased by households, W_t denotes the nominal wage,and S_t denotes profits (net taxation) from ownership of the firms. The budget constraint in country F is defined analogously. Furthermore:

$$P_t \equiv \left[\gamma P^{1-\eta}_{\mathcal{T},t} + (1-\gamma)\, P^{1-\eta}_{\mathcal{N},t}\right]^{\frac{1}{1-\eta}} \tag{4}$$

denotes the CPI, $P_{\mathcal{T},t} \equiv P^{\frac{1}{2}}_{H,t}P^{\frac{1}{2}}_{F,t}$ denotes the price index of tradables, $P_{H,t}$ and $P_{F,t}$ denote the price of tradables produced in countries H and F, respectively, and $P_{\mathcal{N},t}$ denotes the price of nontradables produced in country H. P^*_t is defined analogously to Eq.(4) whereas $P^*_{\mathcal{N},t}$, denoting the price of nontradables in country F, replaces $P_{\mathcal{N},t}$. We assume that the law of one price always holds, thus $P_{H,t} = P^*_{H,t}$ and $P_{F,t} = P^*_{F,t}$, implying that the prices of tradables are equal in both countries. However, $P_{\mathcal{N},t}$ and $P^*_{\mathcal{N},t}$ are not necessarily equal in both countries because these represent the prices of different goods. These facts

[7] Following Stockman and Tesar (1995), we assume that η is not necessarily unity, while Obstfeld and Rogoff (2000) implicitly assume that η is unity. Obstfeld and Rogoff (2000) assume $C_t \equiv \frac{C^{\gamma}_{\mathcal{T},t}C^{1-\gamma}_{\mathcal{N},t}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$. This implies $\eta = 1$ in our paper.

imply that purchasing power parity (PPP) does not necessarily hold. When all goods are tradable, Eq.(4) implies $P_t = P_t^*$, namely, PPP always holds.

The optimal allocation of any given expenditure within each category of goods implies the demand functions as follows:

$$
\begin{aligned}
C_{H,t} &= \frac{1}{2}\left(\frac{P_{H,t}}{P_{\mathcal{T},t}}\right)^{-1} C_{\mathcal{T},t} \quad ; \quad C_{F,t} = \frac{1}{2}\left(\frac{P_{F,t}}{P_{\mathcal{T},t}}\right)^{-1} C_{\mathcal{T},t}, \\
C_{\mathcal{T},t} &= \gamma\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} C_t \quad ; \quad C_{\mathcal{N},t} = (1-\gamma)\left(\frac{P_{\mathcal{N},t}}{P_t}\right)^{-\eta} C_t,
\end{aligned}
\tag{5}
$$

where $P_{\mathcal{T},t} \equiv P_{H,t}^{\frac{1}{2}} P_{F,t}^{\frac{1}{2}}$ denotes the tradables price index.

The representative household maximizes Eq.(1) subject to Eq.(3). The optimality conditions are given by:

$$
\delta \mathrm{E}_t\left(\frac{C_{t+1}^{-1} P_t}{C_t^{-1} P_{t+1}}\right) = \frac{1}{R_t}, \tag{6}
$$

$$
C_t N_t^{\varphi} = \frac{W_t}{P_t}, \tag{7}
$$

where $R_t \equiv 1 + r_t$ satisfying $R_t^{-1} = Q_{t,t+1}$ denotes the gross nominal return on a riskless one-period discount bond paying off one unit of the common currency (for short, the gross nominal interest rate), and r_t denotes the net nominal interest rate. Eq.(6) is an intertemporal optimality condition, namely the Euler equation, and Eq.(7) is an intratemporal optimality condition. Optimality conditions in country F are given analogously to Eqs.(6) and (7).

Combining and iterating Eq.(6) with an initial condition, we have the following optimal risk-sharing condition:

$$
C_t = \vartheta C_t^* \mathsf{Q}_t, \tag{8}
$$

with $\mathsf{Q}_t \equiv \frac{P_t^*}{P_t}$ denoting the CPI differential between the two countries and ϑ denoting a constant depending on the initial value. When $C_{-1} = C_{-1}^* = P_{-1} =$

$P^*_{-1} = 1$, we have $\vartheta = 1$.[8]

2.2 Firms

Each producer can use a linear technology to produce a differentiated good as follows:

$$Y_{H,t}(h) = A_{H,t} N_{H,t}(h), \quad ; \quad Y_{\mathcal{N},t}(h) = A_{\mathcal{N},t} N_{\mathcal{N},t}(h), \tag{9}$$

where $Y_{H,t}(h)$ and $Y_{\mathcal{N},t}(h)$ denote the output of tradables h produced in country H, output of nontradables h produced in country H, respectively, and $A_{H,t}$ and $A_{\mathcal{N},t}$ denote stochastic productivity shifters associated with tradables produced in country H and nontradables produced in country H, respectively. Each producer in country F can use a technology similar to that in country H.

Each firm produces a single differentiated good and prices its goods to reflect the elasticity of substitution across goods produced given the CPI. This is because each firm plays an active part in the monopolistically competitive market. We assume that Calvo–Yun-style price-setting behavior applies, and, therefore, that each firm resets its price with a probability of $1-\alpha$ in each period, independently of the time elapsed since the last adjustment.

When setting a new price in period t, firms seek to maximize the expected discounted value of profits. The first order necessary conditions (FONCs) are as follows:

$$\begin{aligned} \mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k (P_{t+k} C_{t+k})^{-1} \tilde{Y}_{H,t+k} \left(\tilde{P}_{H,t} - \zeta MC^n_{H,t+k} \right) \right] &= 0, \\ \mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k (P_{t+k} C_{t+k})^{-1} \tilde{Y}_{N,t+k} \left(\tilde{P}_{\mathcal{N},t} - \zeta MC^n_{N,t+k} \right) \right] &= 0, \end{aligned} \tag{10}$$

where $MC^n_{H,t} \equiv \frac{W_t}{(1-\tau)A_{H,t}}$ and $MC^n_{\mathcal{N},t} \equiv \frac{W_t}{(1-\tau)A_{\mathcal{N},t}}$ denote the nominal marginal

[8] See Chari, Kehoe and McGrattan (2002) for details.

costs associated with tradables produced in country H and nontradables produced in country H, respectively, $\tilde{Y}_{H,t}$ and $\tilde{Y}_{\mathcal{N},t}$ denote the total demands following changes in the prices of tradables produced in country H and nontradables produced in country H, respectively, $\tilde{P}_{H,t}$ and $\tilde{P}_{\mathcal{N},t}$ denote the adjusted prices of tradables produced in country H and nontradables produced in country H, respectively, $\zeta \equiv \frac{\theta}{\theta-1}$ is a constant markup and τ denotes the tax rate.[9] Note that $(P_{t+k}C_{t+k})^{-1}$ is marginal utility of nominal income.

Let $MC_{H,t} \equiv \frac{MC^n_{H,t}}{P_{P,t}}$ and $MC_{\mathcal{N},t} \equiv \frac{MC^n_{\mathcal{N},t}}{P_{P,t}}$ define as the real marginal cost of tradables produced in country H and nontradables in country H, respectively where $P_{P,t} \equiv \frac{P_{H,t}Y_{H,t}+P_{\mathcal{N},t}Y_{\mathcal{N},t}}{Y_{H,t}+Y_{\mathcal{N},t}}$ denotes weighted average price of goods produced in country H. Combining Eq.(7): and the definition of real marginal cost, we have:

$$MC_{H,t} = \frac{C_t N_t^{\varphi} P_t}{(1-\tau)\, P_{P,t} A_{H,t}}, \quad ; \quad MC_{\mathcal{N},t} = \frac{C_t N_t^{\varphi} P_t}{(1-\tau)\, P_{P,t} A_{\mathcal{N},t}}. \tag{11}$$

Note that, we call $P_{P,t}$ as Producer Price Index (PPI) in country H, hereafter. Note that we define country wide real marginal cost as $MC_t \equiv \frac{MC_{H,t}Y_{H,t}+MC_{\mathcal{N},t}Y_{\mathcal{N},t}}{Y_{H,t}+Y_{\mathcal{N},t}}$.

2.3 Local Government

Whereas monetary frictions are omitted and the limit of a "cashless economy" is considered following Woodford (2003) throughout this paper, monetary policy has important implications for fiscal decisions, as the level of the interest rate determines the debt burden and the inflation rate affects the real value of debt. Fiscal policy can choose one-period nominal risk free debt to finance an exogenous process of public spending.[10] The flow government budget constraint

[9] Ferrero (2007) regards it as a value-added tax rate.

[10] We assume the tax rate is common in each country and constant over time for simplicity while Ferrero (2007), Schmitt-Grohe and Uribe (2004a) and Canzoneri, Cumby and Diba (2001) assume allow it can vary in each country and over time. On contrary, Gali and Monacelli

in country H is given by:

$$B_t^n = R_{t-1}B_{t-1}^n - \left[P_{P,t}\tau\left(Y_{H,t}+Y_{\mathcal{N},t}\right) - P_{G,t}\left(G_{H,t}+G_{\mathcal{N},t}\right)\right], \tag{12}$$

where $B_t^n \equiv P_t B_t$ denotes the nominal risk-free bonds issued by local government in country H, B_t denotes the real risk-free bonds issued by local government in country H and $P_{G,t} \equiv \frac{P_{H,t}G_{H,t}+P_{\mathcal{N},t}G_{\mathcal{N},t}}{G_{H,t}+G_{\mathcal{N},t}}$ denotes average price of goods purchased by government in country H. Local government in country F has a budget constraint similar to Eq.(12). For simplicity, we assume that government purchases are fully allocated to a domestically produced good and that the total amount of these is exogenous.[11] For any given level of public consumption, the government allocates expenditures across goods in order to minimize total cost.

Note that there is no borrowing or lending among all households in equilibrium because all of them are identical. Thus, all interest bearing asset holding by households are in the form of government securities. That is:

$$D_t^n = R_{t-1}\left[B_{t-1}^n + \left(B_{t-1}^*\right)^n\right]$$

at all dates and all contingencies.[12]

Starting from Eq.(12) with an appropriate transversality condition, the resulting consolidated intertemporal budget constraint can be written as:

$$\frac{C_t^{-1}}{\Pi_t}R_{t-1}B_{t-1} = \mathrm{E}_t\left\{\sum_{k=0}^{\infty}\delta^k C_{t+k}^{-1}\frac{\left[P_{P,t+k}\tau\left(Y_{H,t+k}+Y_{\mathcal{N},t+k}\right) - P_{G,t+k}\left(G_{H,t+k}+G_{\mathcal{N},t+k}\right)\right]}{P_{t+k}}\right\},$$

with $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ being the gross CPI inflation rate in countries H where we use intertemporal optimality conditions Eq.(6). These equalities can be rewritten

(Forthcoming) assume a constant (minus) tax rate similar to our paper. Constant tax rate over time depicts actual regimes not only in the Euro area but also other countries because an adjustment of tax rate is not frequent.

[11] Thus, government purchases do not distort relative price between tradables and nontradables.

[12] Thus, our households and government budget constraints boils down to them in Schmitt-Grohe and Uribe (2004a) and (2004b), Canzoneri, Cumby and Diba (2001) and Woodford (1996).

as:

$$\frac{C_t^{-1}}{\Pi_t}R_{t-1}B_{t-1} = C_t^{-1}\frac{\left[P_{P,t}\tau\left(Y_{H,t}+Y_{\mathcal{N},t}\right)-P_{G,t}\left(G_{H,t}+G_{\mathcal{N},t}\right)\right]}{P_t} + \delta \mathrm{E}_t\left(\frac{C_{t+1}^{-1}}{\Pi_{t+1}}R_t B_t\right). \quad (13)$$

Different from the setting in Ferrero (2007), $\mathsf{Q}_t = 1$, which derives $C_t = C_t^*$, is no longer applied when all goods are not tradable. When $C_t = C_t^*$ is applied, Eq.(13) become similar to one derived by Ferrero (2007). Note that because Eq.(13) derived by combining the intertemporal government budget constraint and Euler equation, the equation become new Keynessian IS curve (NKIS) by log-linearization.

2.4 Market Clearing

The market in country H for tradables clears when domestic demand equals domestic supply as follows:

$$Y_{H,t}(h) = C_{H,t}(h) + C_{H,t}^*(h) + G_{H,t}(h), \quad (14)$$

where $C_{H,t}^*(h)$ denotes country F's demand for generic tradables produced in country H. As for nontradables, equilibrium requires that:

$$Y_{\mathcal{N},t}(h) = C_{\mathcal{N},t}(h) + G_{\mathcal{N},t}(h). \quad (15)$$

As mentioned above, government purchases are fully allocated to a domestically produced good. Thus, the term $G_t(h)$ appears in both equalities. Market clearing conditions in country F are analogous to Eqs.(14) and (15).

Let $Y_{H,t}$ denote a Dixit–Stiglitz-type index of the aggregate output of tradables produced in country H. Combining this definition and Eqs.(5) and (8), Eq.(14) can be rewritten as:

$$Y_{H,t} = \frac{\gamma}{2}\left(\frac{P_{H,t}}{P_{\mathcal{T},t}}\right)^{-1} C_t\left[\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} + \left(\frac{P_{\mathcal{T},t}}{P_t^*}\right)^{-\eta}\mathsf{Q}_t^{-1}\right] + G_{H,t}. \quad (16)$$

Eq.(16) and its counterpart in country F imply that:

$$\frac{Y_{H,t}-G_{H,t}}{Y_{F,t}-G_{F,t}}=\mathsf{T}_t,$$

where $Y_{F,t}$ denotes the aggregate output of tradables produced in country F and $\mathsf{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}$ denotes the terms of trade (TOT). Thus, the difference in the output of tradables between country H and country F is equal to the TOT.

Let $Y_{\mathcal{N},t}$ denote a Dixit–Stiglitz-type index of the aggregate output of nontradables produced in country H. Combining this definition and Eq.(5), Eq.(15) can be rewritten as follows:

$$Y_{\mathcal{N},t}=(1-\gamma)\left(\frac{P_{\mathcal{N},t}}{P_t}\right)^{-\eta}C_t+G_{\mathcal{N},t}. \tag{17}$$

Eq.(17) and its counterpart in country F imply that:

$$\frac{Y_{\mathcal{N},t}-G_{\mathcal{N},t}}{Y^*_{\mathcal{N},t}-G^*_{\mathcal{N},t}}=\mathsf{N}_t^{\eta}\mathsf{Q}_t^{-(\eta-1)},$$

where $\mathsf{N}_t \equiv \frac{P^*_{\mathcal{N},t}}{P_{\mathcal{N},t}}$ denotes the nontradables price difference between countries H and F (NPD). Analogous to the difference in output of tradables, the difference in output of nontradables between the two countries is equal to the price difference of nontradables between them.

Finally, we define country-wide output and government expenditure as follows:

$$Y_t \equiv \frac{P_{H,t}}{P_{P,t}}Y_{H,t}+\frac{P_{N,t}}{P_{P,t}}Y_{\mathcal{N},t} \quad ; \quad G_t \equiv \frac{P_{H,t}}{P_{G,t}}G_{H,t}+\frac{P_{N,t}}{P_{G,t}}G_{\mathcal{N},t} \tag{18}$$

where Y_t and G_t denote output and government expenditure in country H, respectively.

3 Log-linearization of the Model

This section describes the stochastic equilibrium that arises from perturbations around the deterministic equilibrium. Lowercase letters denote percentage deviations of steady-state values for the respective uppercase letters when there is no note to the contrary; i.e., $v_t \equiv \frac{dV_t}{V}$, where V_t denotes the voluntary variable and V denotes the steady-state value of V_t. Lowercase letters accompanied by R as a superscript indicate the logarithmic difference between the two countries for the respective uppercase letters, i.e., $v_t^R \equiv v_t - v_t^*$, while lowercase letters accompanied by W as a superscript indicate the logarithmic weighted sum of the two countries for the respective uppercase letters; i.e., $v_t^W \equiv \frac{1}{2}(v_t + v_t^*)$. Finally, small letters accompanied by Δ indicate changes in the large-letter variable; i.e., $\Delta v_t \equiv v_t - v_{t-1}$.

3.1 Aggregate Demand and Output

Log-linearizing Eq.(8), we obtain the following:

$$c_t^R = \mathsf{q}_t, \tag{19}$$

where $\hat{r}_t \equiv \frac{dR_t}{R}$ denotes deviation of the nominal interest rate from its steady-state value, π_t denotes the CPI inflation rate in country H, and q_t denotes the logarithmic CPI differential between the two countries. Notice that the second equality in Eq.(19) implies that the logarithmic consumption differential between the two countries depends on the logarithmic CPI differential.

Log-linearizing and manipulating Eq.(4), we obtain:

$$\pi_t = \gamma \pi_{\mathcal{T},t} + (1-\gamma)\, \pi_{\mathcal{N},t}, \tag{20}$$

with $\pi_{\mathcal{T},t} = \frac{1}{2}\pi_{H,t} + \frac{1}{2}\pi_{F,t}$, where $\pi_{\mathcal{T},t}$ denotes the tradable goods price inflation rate, $\pi_{H,t}$ and $\pi_{F,t}$ denote the inflation rates of tradables produced in countries H and F, respectively, $\pi_{\mathcal{N},t}$ denotes the inflation rate of nontradables produced in country H.

Log-linearizing the definition of PPI, we have $p_{P,t} = \gamma p_{H,t} + (1-\gamma) p_{\mathcal{N},t}$ which implies that logarithmic PPI is weighted sum of both logarithmic price of tradables produced in country H and it of nontradables produced in country H.[13] This equality implies that:

$$\pi_{P,t} = \gamma \pi_{H,t} + (1-\gamma) \pi_{\mathcal{N},t} \tag{21}$$

where $\pi_{P,t}$ denotes the PPI inflation rate in country H. Note that $\pi_{P,t} = \pi_{H,t}$ is applied when all goods are tradable, i.e., $\gamma = 1$. Eq.(20) implies that PPI inflation rate is weighted sum of inflation rate of both tradables and nontradables produced in country H.

Log-linearizing Eq.(18), we have:

$$\begin{aligned} y_t &= \gamma y_{H,t} + (1-\gamma) y_{\mathcal{N},t} \\ g_t &= \gamma g_{H,t} + (1-\gamma) g_{\mathcal{N},t}. \end{aligned} \tag{22}$$

Log-linearizing Eqs.(16) and (17) and substituting these equalities into Eq.(22), we have:

$$y_t = (1-\sigma_G) c_t + \frac{(1-\sigma_G)\gamma}{2} \mathsf{t}_t + \frac{(1-\sigma_G)\psi}{2} \mathsf{n}_t + \sigma_G g_t, \tag{23}$$

with $\psi \equiv (1-\gamma)\gamma(\eta-1)$, where $\sigma_G \equiv \frac{G}{Y}$ denotes the steady-state ratio of the government expenditure to output, t_t denotes the logarithmic TOT, n_t denotes the logarithmic NPD, and g_t denotes the percentage deviation of government spending from the steady-state output level in country H.

Subtracting the counterpart of Eq.(23) from Eq.(23), we have:

$$y_t^R = \gamma (1-\sigma_G) \mathsf{t}_t + (1-\gamma) \varpi (1-\sigma_G) \mathsf{n}_t + \sigma_G g_t^R, \tag{24}$$

[13]Note that we have $p_{P,t} = p_{G,t}$ by log-linearizing the definition of $P_{G,t}$.

with $\varpi \equiv 1+(\eta-1)\gamma$. Because of existing nontradables, the output differential between the two countries depends not only on the TOT but also on the NPD. When $\gamma = 1$, implying that there are no nontradables, this equality is reduced to $y_t^R = (1-\sigma_G)\,\mathsf{t}_t + (1-\sigma_G)\,g_t^R$, which is familiar in many new open economy macroeconomics (NOEM) studies. This equality shows that an increase in the price of domestic nontradables decreases domestic output when we ignore the effect of η.

Log-linearizing Eq.(13), we have:

$$\begin{aligned} b_t \;&=\; \mathrm{E}_t c_{t+1} - c_t - \frac{1}{\delta}\pi_t + \mathrm{E}_t\pi_{t+1} + \frac{1}{\delta}\hat{r}_{t-1} - \hat{r}_t + \frac{1}{\delta}b_{t-1} + \left(\frac{1-\delta}{\delta}\right)\frac{\gamma}{2}\mathsf{t}_t \\ &\quad - \frac{\tau}{\sigma_B}y_t + \frac{\sigma_G}{\sigma_B}g_t, \end{aligned} \tag{25}$$

where $\sigma_B \equiv \frac{B}{Y}$ denotes the steady-state ratio of the risk-free bonds to output.

Combining Eqs.(20), (22), (25) and the counterpart of Eq.(25), we have NKISs as follows:

$$\begin{aligned} y_t^W \;&=\; \frac{\beta_W}{1-\sigma_G}\mathrm{E}_t y_{t+1}^W + \beta_W \mathrm{E}_t\pi_{t+1}^W - \beta_W\hat{r}_t + \frac{\beta_W}{\delta}\hat{r}_{t-1} - \beta_W b_t^W + \frac{\beta_W}{\delta}b_{t-1}^W - \frac{\beta_W}{\delta}\pi_t^W \\ &\quad + \sigma_G\nu_W g_t^W, \\ y_t^R \;&=\; -\beta_R\delta b_t^R + \beta_R(1-\gamma)\,\upsilon\mathsf{n}_t - \beta_R(1-\gamma)\,\mathsf{n}_{t-1} + \beta_R b_{t-1}^R + \sigma_G\nu_R g_t^R, \end{aligned} \tag{26}$$

with $\beta_W \equiv \frac{(1-\sigma_G)\sigma_B}{\sigma_B+(1-\sigma_G)\tau}$, $\beta_R \equiv \frac{\sigma_B(1-\sigma_G)}{(1-\sigma_G)\delta\tau-(1-\delta)\sigma_B}$, $\upsilon \equiv 1-(1-\delta)\,\varpi$, $\nu_W \equiv \frac{[\sigma_B(1-\rho_G)+1-\sigma_G]}{\sigma_B+(1-\sigma_G)\tau}$, $\nu_R \equiv \frac{[(1-\sigma_G)\delta-(1-\delta)\sigma_B]}{(1-\sigma_G)\delta\tau-(1-\delta)\sigma_B}$ and $\rho_G < 1$ being the coefficient associated with exogenous government expenditure processes.[14] Note that two equalities are not only demand curves but also government budget constraints. The first equality in Eq.(26) can be derived by not only summing up Eq.(25) and its counterpart in country F but also by log-linearizing union-wide government budget constraint Eq.(**??**). The model needs another one demand curve such as

[14] We assume the government expenditure and productivity shifters follow AR (1) processes. See subsection 4.2.

the second equality in Eq.(26). As mentioned by Ferrero (2007), two intertemporal government budget constraints are not necessarily satisfied although union-wide budget constraint is satisfied. Thus, relative block of NKIS, namely, the second equality in Eq.(26) should be needed. This can be explained another way. Although all goods are tradable, namely, $\gamma = 1$ is applied, $y_t^R = 0$ is not satisfied, namely, relative block of intertemporal government budget constraint affects the economy. This depicts that two intertemporal government budget constraints are not necessarily satisfied although union-wide budget constraint is satisfied.

Note that Eq.(26) accounts for the difference in policy implications between Benigno (2004) and Ferrero (2007). When all goods are tradable, i.e., $\gamma = 1$, the second equality of Eq.(26) reduces to:

$$y_t^R = -\beta_R \delta b_t^R + \beta_R b_{t-1}^R + \sigma_G \nu_R g_t^R$$

thus only government bonds and government expenditure create a disparity between output in the two countries. This equality corresponds to logarithmic relative block of government budget constraint in Ferrero (2007).[15] On contrary, Benigno (2004) assumes zero steady state value of government expenditure, that the governments do not issue bonds, and that all goods are tradable. Thus, above equality become $y_t^R = 0$, which implies there is no disparity in output between both countries, under the assumption of Benigno (2004). When changes in relative government expenditure occur, output disparity is amplified under Ferrero (2007)'s settings. To eliminate output disparity, issuing government bonds is essential because monetary policy can only cope with shifters on union-wide or average block of NKIS under his setting. However, $y_t^R = 0$ is always

[15]Note that Ferrero (2007) do not show logarithmic relative block of government budget constraint explicitly.

hold although relative shocks hit the economy under the assumption of Benigno (2004). Thus, optimal monetary policy alone can dissolve trade-offs at each country level under the assumption of Benigno (2004). It is clear that the difference in policy implications between both papers stems from settings of steady state value or the assumption of government budget constraint.

3.2 Aggregate Supply and Inflation

Log-linearizing Eq.(10) and rearranging, we can describe the dynamics of inflation in terms of marginal cost as follows:

$$\begin{aligned} \pi_{H,t} &= \delta \mathrm{E}_t \pi_{H,t+1} + \kappa (1-\gamma) p_{\mathcal{N},t} - \kappa (1-\gamma) p_{H,t} + \kappa mc_{H,t}, \\ \pi_{\mathcal{N},t} &= \delta \mathrm{E}_t \pi_{N,t+1} - \kappa \gamma p_{\mathcal{N},t} + \kappa \gamma p_{H,t} + \kappa mc_{\mathcal{N},t}, \end{aligned} \tag{27}$$

with $\kappa \equiv \frac{(1-\alpha)(1-\alpha\delta)}{\alpha}$.

Plugging Eq.(27) into Eq.(20), we have a PPI-based inflation dynamics equation, namely, a new Keynesian Philips curve (NKPC), as follows:

$$\pi_{P,t} = \delta \mathrm{E}_t \pi_{P,t+1} + \kappa mc_t, \tag{28}$$

where $mc_t = \gamma mc_{H,t} + (1-\gamma) mc_{\mathcal{N},t}$ which is derived by log-linearizing definition of marginal cost of country level. Combining the second equality of Eq.(27) and its counterpart in country F, the nontradables inflation differential is given by:

$$\pi^R_{\mathcal{N},t} = \delta \mathrm{E}_t \pi^R_{N,t+1} + \kappa\gamma \mathsf{n}_t - \kappa\gamma \mathsf{t}_t + \kappa mc^R_{\mathcal{N},t}, \tag{29}$$

with:

$$\pi^R_{\mathcal{N},t} \equiv -\Delta \mathsf{n}_t \tag{30}$$

being relative nontradables inflation, which is a type of NKPC where, at first glance, Eq.(29) evolves into this version of the real exchange rate determiner and can be called the new Keynesian real exchange rate determiner (NKRD). Our model is a closed system while a two-country economy is assumed; however, as with the Balassa–Samuelson theorem, Eq.(29) explains the CPI disparity between the two countries, although the Balassa–Samuelson theorem addresses the problem of why the nominal exchange rate deviates from purchasing power parity in the international money and finance literature. Details on Eq.(29) are provided in a later section.

By log-linearizing the aggregated Eq.(9) and combining it with Eq.(22), we have:

$$y_t = \gamma a_{H,t} + (1-\gamma)\, a_{\mathcal{N},t} + n_t. \tag{31}$$

where we also use the log-linearized definition of hours of work, $n_t = \gamma n_{H,t} + (1-\gamma)\, n_{\mathcal{N},t}$.

Combining log-linearized Eq.(11), and Eqs.(23) and (31), we have:

$$\begin{aligned} mc_{H,t} &= \frac{\lambda}{1-\sigma_G} y_t - \frac{\psi}{2}\mathsf{n}_t - (1+\varphi\gamma)\, a_{H,t} - (1-\gamma)\,\varphi a_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g_t, \\ mc_{\mathcal{N},t} &= \frac{\lambda}{1-\sigma_G} y_t - \frac{\psi}{2}\mathsf{n}_t - \varphi\gamma a_{H,t} - [1+(1-\gamma)\,\varphi]\, a_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g_t, \end{aligned} \tag{32}$$

with $\lambda \equiv 1 + (1-\sigma_G)\,\varphi$ which implies that marginal cost depends not only on domestic output but also on the NPD.

Using the log-linearized definition of the marginal cost $mc_t = \gamma mc_{H,t} + (1-\gamma)\, mc_{\mathcal{N},t}$, Eq.(32) can be rewritten as follows:

$$mc_t = \frac{\lambda}{1-\sigma_G} y_t - \frac{\psi}{2}\mathsf{n}_t - (1+\varphi)\,\gamma a_{H,t} - (1+\varphi)(1-\gamma)\, a_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g_t. \tag{33}$$

When $\gamma = 1$, Eq.(33) reduces to:

$$mc_t = \frac{\lambda}{1-\sigma_G} y_t - (1+\varphi)\, a_{H,t} - \frac{\sigma_G}{1-\sigma_G} g_t,$$

because of $\psi = 0$ when $\gamma = 1$. This equality is a familiar expression for DSGE models applied to the NOEM literature.

Combining the second equality in Eq.(32) and its counterpart in country F, the logarithmic marginal cost differential associated with nontradables is given by:

$$\begin{aligned} mc^R_{\mathcal{N},t} &= \frac{\lambda}{1-\sigma_G} y^R_t - \psi \mathsf{n}_t - \varphi\gamma a_{H,t} + \varphi\gamma a_{F,t} - \left[1 + (1-\gamma)\varphi\right] a_{\mathcal{N},t} \\ &\quad + \left[1 + (1-\gamma)\varphi\right] a^*_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g^R_t. \end{aligned} \tag{34}$$

3.3 Relative Price and Consumption

Log-linearizing Eq.(4) and rearranging yields:

$$\mathsf{q}_t = (1-\gamma)\,\mathsf{n}_t. \tag{35}$$

It is clear by paying attention to Eqs.(19) and (35) that the logarithmic consumption differential depends on both the logarithmic CPI differential and the logarithmic NPD. When $\gamma = 1$, Eq.(35) is altered as $\mathsf{q}_t = 0$, implying that the CPI is the same in both countries. In the international finance literature, this means that purchasing power parity holds.

Combining Eqs.(19) and (35) and the first equality in Eq.(20), and rearranging, we have:

$$\mathsf{n}_t = \frac{1}{1-\gamma} c^R_t.$$

This equality and Eq.(35) imply that the existence of nontradables creates disparity in consumption between country H and country F. When all goods are tradable, i.e., $\gamma = 1$, $\mathsf{q}_t = 0$ and $c^R_t = 0$. These imply that PPP holds and consumption is the same in both countries.

Benigno (2004) and Gali and Monacelli (Forthcoming) assume that all goods are tradable and the law of one price in these papers. However, the PPP is not necessarily hold in Gali and Monacelli (Forthcoming). They assumes currency union which consists of infinitesimal countries while Benigno (2004) assumes currency union consists of two countries. Settings in Gali and Monacelli (2005) makes disparity between one infinitesimal country and union wide economy because union wide CPI do not include any price index in one infinitesimal country. This stems from assumption of small open economy. Thus, $\mathsf{q}_t = 0$ is not applied in Gali and Monacelli (Forthcoming) although all goods are tradable. In latter section, we suggest that optimal fiscal policy is needed to stabilize both inflation and output gap simultaneously because of nontradables. An assumption of small open economy does not permit applying $\mathsf{q}_t = 0$. Thus, policy implications between Gali and Monacelli (Forthcoming) and this paper are very close while policy implications derived by Gali and Monacelli (Forthcoming) and Benigno (2004) are contrary.

3.4 Marginal Cost and Output Gap

Following Gali and Monacelli (2005), we define the relationship between output, its natural level and the output gap as follows:

$$y_t \equiv \bar{y}_t + \tilde{y}_t,$$

where $\tilde{y}_t$ denotes the logarithmic output gap measured from its natural level, and $\bar{y}_t$ denotes the logarithmic natural output level. Under the long-run equilibrium, $\tilde{y}_t = 0$ must hold.[16]

When the fiscal authorities design their policies to reduce the distortion gen-

[16]Following Gali and Monacelli (2005), nominal rigidities disappear in the long-run equilibrium.

erated by monopolistically competitive markets, real marginal costs under the long-run equilibrium are unity, and their logarithm is given by $mc_t = 0$. In addition, under the long-run equilibrium, PPP is applied.[17] Thus, the logarithmic NPD under the long-run equilibrium is given by $\mathsf{n}_t = 0$.

Combining these facts, Eq.(33) implies that:

$$\bar{y}_t = \bar{\beta}\gamma a_{H,t} + \bar{\beta}(1-\gamma)a_{\mathcal{N},t} + \frac{\sigma_G}{\lambda}g_t \tag{36}$$

with $\bar{\beta} \equiv \frac{(1-\sigma_G)(1+\varphi)}{\lambda}$. Eq.(36) implies that the natural level of output consists of productivity, disutility of work, and government spending while does not include any nominal variables reflecting classical dichotomy. Our setting does not assume the level of investment or the capital stock, and the natural level of output is not determined by either of these variables.

Using Eq.(36), the log-linear approximated model can be rewritten in terms of the output gap. Eq.(26) can be rewritten as:

$$\begin{aligned}
\tilde{y}_t^W &= \frac{\beta_W}{1-\sigma_G}\mathrm{E}_t\tilde{y}_{t+1}^W - \beta_W\hat{r}_t + \beta_W\mathrm{E}_t\pi_{t+1}^W + \frac{\beta_W}{\delta}\hat{r}_{t-1} - \beta_W b_t^W + \frac{\beta_W}{\delta}b_{t-1}^W - \frac{\beta_W}{\delta}\pi_t^W \\
&\quad - \frac{\gamma\bar{\beta}\beta_{\mathcal{T}}}{2}a_{H,t} - \frac{(1-\gamma)\bar{\beta}\beta_{\mathcal{N}}}{2}a_{\mathcal{N},t} - \frac{\gamma\bar{\beta}\beta_{\mathcal{T}}}{2}a_{F,t} - \frac{(1-\gamma)\bar{\beta}\beta_{\mathcal{N}}}{2}a_{\mathcal{N},t}^* + \sigma_G\varsigma_W g_t^W, \\
\tilde{y}_t^R &= -\beta_R\delta b_t^R + \beta_R(1-\gamma)\upsilon\mathsf{n}_t - \beta_R(1-\gamma)\mathsf{n}_{t-1} + \beta_R b_{t-1}^R - \bar{\beta}\gamma a_{H,t} \\
&\quad + \bar{\beta}\gamma a_{F,t} - \bar{\beta}(1-\gamma)a_{\mathcal{N},t} + \bar{\beta}(1-\gamma)a_{\mathcal{N},t}^* + \varsigma_R\sigma_G g_t^R,
\end{aligned} \tag{37}$$

with $\beta_{\mathcal{T}} \equiv 1 - \frac{\beta_W\rho_{\mathcal{T}}}{1-\sigma_G}$, $\beta_{\mathcal{N}} \equiv 1 - \frac{\beta_W\rho_{\mathcal{N}}}{1-\sigma_G}$, $\varsigma_W \equiv \nu_W + \frac{\beta_W\rho_G}{(1-\sigma_G)\lambda} - \frac{1}{\lambda}$ and $\varsigma_R \equiv \nu_R - \frac{1}{\lambda}$ where $\rho_{\mathcal{T}} < 1$, $\rho_{\mathcal{N}} < 1$ and $\rho_G < 1$ denote the coefficients associated with exogenous processes on productivity shifter of tradables and nontradables, respectively and government expenditure, respectively. Both equalities in Eq.(37) imply that the NKISs in the two countries are no longer homogeneous although risk sharing is perfect internationally. As mentioned when we derive the second

[17]Following Gali and Monacelli (2005), we assume the steady state where PPP is applied.

equality of Eq.(26), there are two major causes of disparity in the demand block between the countries: (1) A nonzero steady state value of government bonds and expenditure and (2) Nontradable goods. When the steady state value of government bonds and expenditure are zero, $\sigma_R = \sigma_G = 0$ is applied. When all goods are tradable, $1 - \gamma = 0$ is applied. Many open economy DSGE models, such as the model in Benigno (2004) and Benigno and Benigno (2008) adopt both $\sigma_R = \sigma_G = 0$ and $1 - \gamma = 0$. Thus there is little or no output gap disparity between both countries. The model in Ferrero (2007) assumes a nonzero steady state value of government bonds and expenditure because of an explicit government budget constraint. Thus, his model has a somewhat larger disparity. This paper includes both the earlier and latter cases. Our model has a larger disparity when nontradables exist.

The NKPCs in terms of the output gap are given by:

$$\pi_{P,t} = \delta \mathrm{E}_t \pi_{P,t+1} + \kappa \frac{\lambda}{1 - \sigma_G} \tilde{y}_t - \frac{\psi \kappa}{2} \mathsf{n}_t, \tag{38}$$

along with its counterpart in country F. These expressions become familiar when $\gamma = 1$. In this case, Eq.(38) can be rewritten as:

$$\pi_{P,t} = \delta \mathrm{E}_t \pi_{P,t+1} + \kappa \frac{\lambda}{1 - \sigma_G} \tilde{y}_t,$$

which corresponds one derived by Gali and Monacelli (2005), who insist that inflation–output trade-offs can be dissolved simultaneously in a small open economy, under strong parameter restrictions, by inflation targeting. Indeed, when inflation targeting, such as $\pi_{P,t} = \pi^*_{P,t} = 0$ for all t, is introduced in our currency union with special restrictions, i.e. $\gamma = 1$ and $\sigma_B = \sigma_G = 0$, these equalities imply that $\tilde{y}_t = \tilde{y}^*_t = 0$ for all t and that output gap is eliminated.

3.5 Balassa–Samuelson Theorem and Real Exchange Rate Anomaly

As mentioned in the previous subsection, we now turn to the relationship between the real exchange rate and the NKRD. Using Eq.(36), NKRD Eq.(29) can be rewritten as:

$$\begin{aligned}
\pi_{\mathcal{N},t}^{R} &= \delta \mathrm{E}_t \pi_{N,t+1}^{R} + \kappa\varphi\tilde{y}_t^{R} + \kappa \mathsf{n}_t - \kappa\varphi\gamma\left(1-\bar{\beta}\right) a_{H,t} + \kappa\varphi\gamma\left(1-\bar{\beta}\right) a_{F,t} \\
&\quad - \kappa\left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right] a_{\mathcal{N},t} + \kappa\left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right] a_{\mathcal{N},t}^{*} \\
&\quad - \frac{\kappa\sigma_G}{1-\sigma_G}\left(1-\frac{\varphi}{\lambda}\right) g_t^{R} \qquad (39)
\end{aligned}$$

Using Eq.(35), Eq.(39) can be rewritten as follows:

$$\begin{aligned}
\mathsf{q}_t &= \left(1-\gamma\right)\left\{\frac{1}{\kappa}\pi_{\mathcal{N},t}^{R} - \frac{\delta}{\kappa}\mathrm{E}_t\pi_{N,t+1}^{R} - \varphi\tilde{y}_t^{R} + \varphi\gamma\left(1-\bar{\beta}\right) a_{H,t} + \left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right] a_{\mathcal{N},t}\right. \\
&\quad \left. -\varphi\gamma\left(1-\bar{\beta}\right) a_{F,t} + \kappa\left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right] a_{\mathcal{N},t}^{*} - \frac{\varphi\sigma_G}{1+\varphi} g_t^{R}\right\},
\end{aligned}$$

which implies that the CPI disparity is dissolved between the two countries, namely, $\mathsf{q}_t = 0$ holds when the currency union has no nontradables; i.e., as $\gamma = 1$. A problem with the CPI disparity is resolved, as each country has the same CPI. This implies that purchasing power parity holds in an ordinary open-economy model. Eq.(39) depicts the Balassa–Samuelson theorem in the international money and finance literature. However it cannnot be easily understood because Eq.(39) is different from the familiar equation of the Balassa–Samuelson theorem because it is a dynamic equation, as in the New Keynesian literature, which has correctly assumed nominal rigidities. To understand this characteristic easily, we inspect Eq.(39) without nominal rigidity. Under a such condition, Eq.(39) can be rewritten as:

$$\mathsf{q}_t = \left(1-\gamma\right)\left\{\varphi\gamma\left(1-\bar{\beta}\right) a_{H,t} + \left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right] a_{\mathcal{N},t} - \varphi\gamma\left(1-\bar{\beta}\right) a_{F,t}\right.$$

$$-\left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right]a^{*}_{\mathcal{N},t}-\frac{\varphi\sigma_G}{1+\varphi}g^{R}_{t}\Bigg\}, \tag{40}$$

because $\alpha = 0$ and $\tilde{y}_t = \tilde{y}^*_t = 0$ hold. And note that when government expenditure is zero on the etady state, Eq.(40) can be rewritten as :

$$\mathsf{q}_t = (1-\gamma)\left(a_{\mathcal{N},t} - a^{*}_{\mathcal{N},t} - \frac{\varphi\sigma_G}{1+\varphi}g^{R}_{t}\right), \tag{41}$$

because $\sigma_G = 0$ implies $\bar{\beta} = 1$. In Eq.(41), increasing the relative productivity of tradables produced in country H, i.e., decreasing the productivity of nontradables produced in country H, causes a decrease in the CPI disparity q_t. As the Balassa–Samuelson theorem explains, a rise in the productivity of the tradables sector in the home country causes a decrease (appreciation) in the real exchange rate through an increase in nontradables prices in the home country, which stems from an increase in wages in both the tradables and the nontradables sector because of perfect labor mobility between each sector.

$\sigma_G = 0$ is not consistent to our setting because we assume positive government expenditure in the steady state. Eq.(40) implies that an increase in the productivity of tradables produced in country H causes an increase (depreciation) in the real exchange rate. Because of an increase in the real marginal cost in nontradables sector, CPI increases. However, CPI includes PPI partially. Thus, the real marginal cost in both sector decreases. This decreases CPI via a decrease in PPI. Finally, real exchange rate is increased (depreciated). Benigno and Thoenissen (2005) and Canzoneri, Cumby and Diba (1999) report real exchange rate anomaly. They find that direction of real exchange rate cannnot be explained by Balassa–Samuelson theorem. It can be said that Eq.(40) well reflects their findings under well microfounded settings. Note that Eqs.(40) and (41) imply that real exchange rate anomaly depends on steady state share of government expenditure to output. When the share is zero, namely, government expenditure is zero at the steady state, changes in the real exchange rate following Balassa–Samuelson theorem. However, when the share is positive, namely, government expenditure is positive at the steady state, effect of Balassa–Samuelson theorem is defeated by another side eeffect which is pointed out by Benigno and Thoenissen (2005) and Canzoneri, Cumby and Diba (1999).

4 Optimal Cooperative Solution

In this section, we analyze the macroeconomic implications of an alternative policy regime for the Euro area: an optimal monetary policy without a fiscal policy regime and an optimal monetary and fiscal policy regime under a cooperative setting. Furthermore, we assume that each policy authority is responsible for minimizing social losses. Under an optimal monetary policy regime without fiscal policy, the central bank is the only policy authority while the central bank and local governments in the two countries are both authorities under an optimal monetary and fiscal policy regime. Policy authorities seek to minimize the social loss function subject to our structural model.[18] The period loss function is derived by a second-order Taylor expansion as approximated by the definition of period utility in Eq.(1), which is given by:

$$U_t^W = -L_t^W + \text{t.i.p.} + o\left(\|\xi\|^3\right),$$

with:

$$L_t^W = \frac{1}{1-\sigma_G}\left[\frac{\theta}{4\kappa}\pi_{P,t}^2 + \frac{\theta}{4\kappa}\left(\pi_{P,t}^*\right)^2 + \frac{1+\varphi}{2}\left(\tilde{y}_t^W\right)^2 + \frac{1+\varphi}{8}\left(\tilde{y}_t^R\right)^2\right], \tag{42}$$

where $U_t^W \equiv \frac{1}{2}\left(U_t + U_t^*\right)$ denotes union-wide utility function, L_t^W denotes the period loss function in country H, t.i.p. denotes the terms of the independent

[18]Our structural model consists of Eqs.(30),(37), (38), (39) and a counterpart of Eq.(38) in country F.

policy, and $o\left(\|\xi\|^3\right)$ denotes terms that are higher than third order.[19]

We now consider the difference in welfare loss between our paper and the DSGE literature for an open economy. Using Eq.(36) we can obtain

$$\tilde{y}_t^R = \gamma\left(1-\sigma_G\right)\mathsf{t}_t + \left(1-\sigma_G\right)\left(1-\gamma\right)\varpi\mathsf{n}_t + \sigma_G g_t^R - \bar{y}_t^R \tag{43}$$

In the DSGE literature for an open economy, the welfare loss includes not only inflation and the output gap but also the TOT, namely t_t, while Eq.(42) includes only inflation and the output gap as if it is one for the closed economy. Paying attention to Eq.(43), however, it is clear that the third term on the RHS of Eq.(42) includes the NPD n_t. When all goods are tradable, namely, $\gamma = 1$, Eq.(43) becomes $\tilde{y}_t^R = \left(1-\sigma_G\right)\mathsf{t}_t + \sigma_G g_t^R - \bar{y}_t^R$. Both this equality and Eq.(42) imply that the welfare loss does not include the square of the NPD but does include the square of the TOT and the welfare loss boils down to one derived by most DSGE studies in an open economy in which all goods are tradable. Because our model allows for nontradables, our welfare loss does not necessarily correspond to it in other DSGE literature for an open economy.

4.1 Role of Optimal Fiscal Policy

In this subsection, we investigate the role of optimal fioscal policy by comparing FONCs which clarify the relationship between PPI infaltion and output gap un-

[19] Ferreo (2007) points out that the method to derive second-order approximated utility function launched by Rotemberg and Woodford (1997) cannot be applied to Ferreo (2007)'s and our framework because of a non-negative stock of debt and a non-negative level of government expenditure in the steady state. Thus, Ferrero (2007) follows the method proposed by Sutherland (2002) or Benigno and Woodford (2003) and he can derive second-order approximated utility function without the presence of linear term. Different from Rotemberg and Woodford (1997), our technique to derive second-order approximated utility function do not assume that the steady state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor is zero. Because of this, we can derive second-order approximated utility function correctly, without the presence of linear term. Note that Woodford (2003) discusses how the presence of linear terms generally leads to evaluate incorrectly. A simple enlightening example of this result is proposed by Kim and Kim (2003).

der the case of optimal monetary policy alone and the case of optimal monetary policy and fiscal policy. In both cases, policy authorities minimizethe sum of the discounted value of social losses as follows:

$$\mathcal{L}^W = \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t L_t^W, \tag{44}$$

subject to Eqs.(30), (37), (38) and (39) with commitment. Hereafter, let us assume $\eta = 1$ implying that the elasticity of substitution between tradables and nontradables is unity, which is assumed implicitly by Obstfeld and Rogoff (2000) for simplicity. In the case of optimal monetary policy alone, only the central bank minimizes Eq.(44) by choosing the sequence $\left\{\pi_{P,t}, \pi_{P,t}^*, \tilde{y}_t, \tilde{y}_t^*, \mathsf{n}_t, \hat{r}_t\right\}_{t=0}^{\infty}$ while both the central bank and two local government cooperatively minimize Eq.(44) by choosing $\left\{\pi_{P,t}, \pi_{P,t}^*, \tilde{y}_t, \tilde{y}_t^*, \mathsf{n}_t, \hat{r}_t, b_t, b_t^*\right\}_{t=0}^{\infty}$ in the case of optimal monetary and fiscal policy.

4.1.1 Optimal Monetary Policy Alone

In the case of optimal monetary policy alone, the FONC for union-wide inflation and the output gap is given by:

$$\pi_t^W = -\frac{\bar{\beta}}{\theta} \left(\tilde{y}_t^W - \tilde{y}_{t-1}^W\right), \tag{45}$$

which is a familiar expression in papers discussing optimal monetary policy in an open economy.[20] This implies that local government do not need to dissolve union wide infaltion–output trade-offs. Solitary central bank can stabilize both inflation and output gap simultaneously although nontradables exist

Next, we investigate the relative block of the FONC. We are interested in the effects of nontradables. Thus, we analyze relative block of the FONC in both cases, namely, the case of that all goods are tradable and the case that there are

[20]When $\sigma_G = 0$, Eq.(45) becomes $\pi_t^W = -\frac{1}{\theta}\left(\tilde{y}_t^W - \tilde{y}_{t-1}^W\right)$.

nontradables. When all goods are tradable, namely, $\gamma = 1$, the relative block FONC is given by:

$$\pi^R_{P,t} = -\frac{\bar{\beta}}{\theta}\left(\tilde{y}^R_t - \tilde{y}^R_{t-1}\right) - \frac{\bar{\beta}\varphi\left(1-\sigma_G\right)4}{\theta\left(1+\varphi\right)}\left(\mu_{2,t} - \mu_{2,t-1}\right) \tag{46}$$

where $\mu_{2,t}$ denotes the Lagrange multiplyer associated to the second equality in Eq.(37), relative block of the NKIS. This case corresponds to the case assumed by Ferrero (2007). Eq.(46) implies that inflation–output gap trade-offs no longer dissolved simultaneously. Because of this, Ferrero (2007) insists that fiscal policy is needed to enhance social welfare.

Next, we abandon the assumption that all goods are tradable. In this case, relative block of FONC is given by:

$$\begin{aligned} \pi^R_{P,t} &= -\frac{\bar{\beta}}{\theta}\left(\tilde{y}^R_t - \tilde{y}^R_{t-1}\right) + \frac{\bar{\beta}\varphi\left(1-\sigma_G\right)4}{\theta\left(1+\varphi\right)}\left(\mu_{2,t} - \mu_{2,t-1}\right) + \frac{\bar{\beta}\left(1-\sigma_G\right)4\kappa\varphi}{\theta\left(1+\varphi\right)\left(1+\delta+\kappa\right)}\left(\mu_{5,t} - \mu_{5,t-1}\right) \\ \left(1-\gamma\right)\beta_R \upsilon \mu_{2,t} &= \mu_{5,t} - \frac{1}{1+\delta+\kappa}\mu_{5,t-1} \end{aligned} \tag{47}$$

where $\mu_{5,t}$ denotes the Lagrange multiplyer associated to Eq.(39), NKRD. Two equalities in Eq.(47) imply not only that inflation–output gap trade-offs are not dissolved simultaneously but also that the relationship between inflation and output gap is more diluted.

4.1.2 Optimal Monetary and Fiscal Policy

In this optimal monetary and fiscal policy regime, the FONC for union-wide inflation and the output gap is given by Eq.(45). Thus, that union-wide inflation and the output gap are stabilized by optimal monetary policy and fiscal policy although nontradables exist in a currency union.

The FONC for relative block inflation and the output gap is given by:

$$\pi^R_{P,t} = -\frac{\bar{\beta}}{\theta}\left(\tilde{y}^R_t - \tilde{y}^R_{t-1}\right) \tag{48}$$

This equality also implies that relative inflation and the output gap are stabilized by optimal monetary policy and fiscal policy. Both Eqs.(45) and (48) imply that inflation–output gap trade-offs are dissolved simultaneously not only union wide level but also each country level under the optimal monetary and fiscal policy regime. Note that we have Eq.(47) although nontradables exist in.

Why do we have not Eq.(47) but Eq.(48)? Under the optimal monetary and fiscal policy regime, we have $\mu_{2,t} = 0$ as optimality condition along with optimality conditions derived under the case of optimal monetary policy alone because we have not only nominal interest rate but also government bond as policy function. It is obvious that we have Eq.(48) by plugging $\mu_{2,t} = 0$ along with initial condition $\mu_{5,-1}$ into Eq.(47). Because of this, we have $\mu_{5,t} = 0$ although this equality is not obtained directly by implementation of fiscal policy. This fact implies that optimal fiscal policy shut down the effects of CPI disparity, which makes consumption disparity betwen both countries by dissolving the disparity in the demand block.

4.2 Sensitivity Analysis

In this section, we illustrate the equilibrium behavior of the currency union under the alternative policy regime described above.

4.2.1 Parameterization

We resort to a series of dynamic simulations and adopt the following benchmark parameterization. We assume the inverse of the labor supply elasticity φ, the elasticity of substitution across goods θ, the elasticity of substitution between tradables and nontradables η, the subjective discount factor δ, the steady-state share of government bonds to output σ_B, steady-state share of government expenditure to output σ_G and tax rate τ set equal to 3, 11, 0.75, 0.5, 1, 0.99, 2.4, 0.276 and 0.3 respectively, as if the timing of the model were quarterly.[21] Except for γ, α, γ, φ and η, these parameterizations are used in Ferrero (2007).[22] As mentioned in the introduction, nontradables account for 50.3% of the major Euro area; thus, we set the share of nontradables in the CPI $\gamma = 0.5$. Following Obstfeld and Rogoff (2000), we set $\eta = 1$. We also assume that the government expenditure, productivity and preference shifters are described according to the following AR (1) processes as follows:

$$\begin{aligned} a_{H,t} &= \rho_{\mathcal{T}} a_{H,t-1} + \xi_{H,t} \quad ; \quad a_{F,t} = \rho_{\mathcal{T}} a_{F,t-1} + \xi_{F,t} \\ a_{\mathcal{N},t} &= \rho_{\mathcal{N}} a_{\mathcal{N},t-1} + \xi_{\mathcal{N},t} \quad ; \quad a^*_{\mathcal{N},t} = \rho_{\mathcal{N}} a^*_{\mathcal{N},t-1} + \xi^*_{\mathcal{N},t} \\ g^W_t &= \rho_G g^W_{t-1} + \xi^W_{G,t} \quad ; \quad g^R_t = \rho_G g^R_{t-1} + \xi^R_{G,t} \end{aligned}$$

where $\xi_{H,t}$, $\xi_{F,t}$, $\xi_{\mathcal{N},t}$, $\xi^*_{\mathcal{N},t}$, $\xi^W_{G,t}$ and $\xi^R_{G,t}$ denote the i.i.d. shocks. We set $\rho_{\mathcal{T}}$, $\rho_{\mathcal{N}}$ and ρ_G equal to 0.705, 0.784 and 0.8 following Batini, Harrison and Millard (2001) and Ribeiro (2008). Following Ferrero (2007), we set standard deviation of the innovations equal to 0.01.[23]

As an example of impulse responses, we focus not only on the unit changes in the productivity shifter of tradable goods in country H, $a_{H,t}$, and the productivity shifter of nontradable goods in country H, $a_{\mathcal{N},t}$, to investigate the effects

[21] $\sigma_B = 2.4$ implies that the steady-state debt–output annual ratio is 0.6.

[22] Many DSGE studies use the parameter values in Rotemberg and Woodford (1997). However, to compare our results with those derived by Ferrero (2007) and analyze the Euro area, we use mainly his parameter values which respect for the Euro area. More precisely, parameter values of θ, δ, σ_B, σ_G are set alike as Ferrero (2007). Ferrero (2007) sets different degree of price rigidity in countries H and F. Thus we set α equal to 0.75, which is assumed by Beetsma and Jensen (2005). Because φ do not appear in Ferrero (2007), we set it equal to 3, which is adopted by Gali and Monacelli (2005).

[23] There are few papers which estimate AR (1) processes parameters associated to productivity of tradables and nontradables sectors separately. Following Benigno and Thoenissen (2005), we adopt the result brought about by Batini, Harrison and Millard (2001) who estimate AR (1) processes parameters associated to productivity of tradables and nontradables sectors separately. Note that we recognize that their estimated parameter is smaller than one in much RBC litereture. Ribeiro (2008) is one of few papers which estimate autoregressive processes of government expenditure in Europe. We adopt his estimation result.

of the existence of nontradables but also on the unit changes in the union-wide government expenditure shifter, g_t^W, and the relative government expenditure shifter, g_t^R, to compare with the results of Ferrero (2007).

4.2.2 Optimal Monetary Policy Alone

Figure 1 depicts macroeconomic volatility under optimal monetary policy alone with commitment.[24] First, we consider the occurrence of an innovation in union-wide government expenditure. To secure funds for government expenditure, the nominal interest rate decreases and there is pressure for the output gap to increase (7th panel in Figure 1). As shown in the first equality in Eq.(37), however, an increase in the lagged nominal interest rate decreases the union-wide output gap. Thus, the union-wide output gap is stabilized (1st panel in Figure 1).[25] The union-wide inflation rate is also stabilized through the optimality condition in Eq.(45) (2nd panel in Figure 1). While nontradables account for half of all consumption goods, the output gap and inflation rate are stabilized simultaneously at both the union-wide and country levels. Note that Ferrero (2007) shows that the nominal interest rate increases to a unit innovation in union-wide government expenditure because the tax rate increases simultaneously to secure funds to finance government expenditure, while the tax rate is constant over time in our setting.

That the union-wide shock is absorbed by monetary policy alone can be understood intuitively even when nontradables exist. Next, we consider the occurrence of changes in the productivity shifter of tradables in country H.

[24] See Table 1 for estimates of macroeconomic volatility. Under the bench park parameterization, 4 eigenvalues are larger than one for four forward-looking variables. Thus, Blanchard–Kahn conditions are met.

[25] As mentioned in section 2.3, all interest bearing asset held by households are in the form of government bonds. Lowering interest rate at the previous period reduces the nominal payoff of the bond portfolio purchased by households. Because of budget constraint, this reduces output via a reduce in consumption.

A unit increase in the productivity shifter of tradables in country H causes a decrease in the PPI inflation rate in country H through a decrease in the marginal cost of tradables in country H (5th panel in Figure 1). An increase in real wages in country H stemming from an increase in the productivity of tradables produced in country H causes an increase in the marginal cost of nontradables produced in country H. This is the cause of a relative increase in the CPI in country H, because the price of nontradables produced in country F do not change. As shown in Eq.(35), a relative increase in the CPI in country H causes a decrease in the NPD. An increase in the price of nontradables in country H boosts demand for tradables in both countries. Thus, the output gap in country H decreases while it increases in country F (3rd and 4th panel in Figure 1). Because the logarithmic relative CPI or CPI disparity q_t departs from unity through an increase in the CPI in country H, this can be called the (short-run) Balassa–Samuelson theorem effect. Note that, however, a decrease in output gap in country H increases CPI disparity through lowering CPI in country H (8 th panel in Figure 1). Wherease it is against Balassa–Samuelson theorem, resulting in a real exchange rate depriciation which corresponds to an increase in CPI disparity in our paper is confirmed by Benigno and Thoenissen (2005). A series of mechanism can be confirmed by investigating the model, especially the relative block. The second equality in Eq.(37) shows that the productivity shifter in country H decreases the output gap disparity between both countries. Eqs.(30), (35) and (39) show that a decrease in the output gap disparity between both countries increases the relative CPI. Monetary policy alone can stabilize both the output gap and the inflation rate simultaneously at the union-wide level through increasing nominal interest rate. However, it cannot be stabilized simultaneously at the country level because nontradables create the output gap disparity between countries.

The result of the occurrence of changes in the productivity shifter of nontradables in country H can be explained in the same manner as the occurrence of changes in the productivity shifter of tradables in country H. However, the coefficient of the productivity shifter of nontradables in NKRD Eq.(39) is larger than the coefficient of the productivity shifter of tradables. Thus, the logarithmic relative CPI; i.e., real exchange rate q_t increases and the volatility of other macroeconomic variables is higher in this case than in other cases (8 th panel in Figure 1 and Table 1). Balassa–Samuelson theorem effect appears clearly in this case.

Changes in relative government expenditure mean that the output gap disparity between countries, and the output gap and inflation rate cannot be stabilized at the country level because of the existence of nontradables (3rd to 6th panels in Figure 1). Benigno (2004) implies that monetary policy alone can stabilize both the output gap and the inflation rate simultaneously although asymmetric shocks hit the economy assuming all goods are tradable. However, when nontradables exist in the currency union, monetary policy alone cannot eliminate the trade-offs between the output gap and inflation in each country.

4.2.3 Optimal Monetary and Fiscal Policy Mix

Both the output gap and the PPI inflation rate are stabilized simultaneously at the union-wide level when any shocks occur. However, trade-offs between the output gap and the inflation rate cannot be eliminated in each country. How do fiscal authorities cope with the fact that the trade-offs between the output gap and the inflation rate cannot be eliminated in each country?

Figure 2 depicts macroeconomic volatility under optimal monetary policy alone and fiscal policy with commitment.[26] First, we consider changes in union-wide government expenditure. The central bank decreases the nominal interest rate to finance additional union-wide government expenditure (7th panel in Figure 2). Both fiscal authorities decrease the issue of government bonds (9th and 10th panels in Figure 2). However, decreasing both of them decreases next period union wide output gap. This mechanism is same as the case of optimal monetary policy alone basically. Because of a constant tax rate, our model implies somewhat different behavior of policy authorities from that which is implied by Ferrero (2007). While the effects of a union-wide shock, such as union-wide government expenditure, can be eliminated by monetary policy alone, not only the central bank but also the fiscal authorities cooperate against the union-wide shock. This result is the same as the one derived by Ferrero (2007).

To eliminate the effects of the occurrence of changes in the productivity shifter of tradables in country H, the fiscal authority in country H could increase government bonds on issue by more than the fiscal authority in country F (9th and 10th panels in Figure 2). As mentioned above, an increase in productivity shifter of tradables in country H decreases both PPI inflation rate and output gap in country H while increases both of them in country F. However, An increase in government bond at previous period increases nominal payoff of the bondportfolio purchased by households. Thus, consumption in both countries are boosted up. Because tradables produced in country H is cheaper than it in country F, tradables produced in country H defeat tradables produced in

[26] See Table 2 for estimates of macroeconomic volatility. Under the bench park parameterization, 5 eigenvalues are larger than one for five forward-looking variables. Thus, Blanchard–Kahn conditions are met.

country F. This stabilizes output gap and PPI inflation in both countries (3rd to 6th panels in Figure 2). This series of mechanism can be confirmed by paying attention to the second equality of Eq.(37) and Eq.(38) and its counterpart in country F.

To deal with the effects of changes in the productivity shifter of nontradables in country H, both fiscal authorities have similar behavior in the case of an increase in productivity shifter of tradables in country H (9th and 10th panels in Figure 2). An increase in government bond decreases output gap. This stabilizes output gap in country H (3rd panel in Figure 2). Also, an increase in government bond at previous period increases PPI inflation through an increase in output gap. This stabilizes PPI inflation (5th panel in Figure 2).[27] This is shown as the second equality of Eqs.(37) and (38) and its counterpart in country F.

When the relative government expenditure shifter shocks the economy, the fiscal authority in country F increases government bonds on issue while the fiscal authority in country H decreases government bonds on issue (9th and 10th panels in Figure 2). As shown in the second equality in Eq.(37), an increase in relative government expenditure increases output gap disparity. However, a decrease in government bond on the previous period decreases current output gap disparity. This reduces the effect of relative government expenditure to output gap disparity. Stabilizing output gap makes PPI inflation in both countries stabilizing simultaneously (3rd to 6th panels in Figure 2).

It is noteworthy that a role of fiscal authorities is larger than it of central

[27]Changes in the productivity shifter of nontradables affects both the NKIS and NKRD, and so changes in government bonds on issue are magnified. Stemming from same reason, that changes in the productivity shifter of nontradables affects both the NKIS and NKRD, volatility of CPI disparity is not zero.

bank to stabilize both inflation and output gap. Volatility of nominal interest rate decreases in the case of optimal monetary and fiscal policy (Tables 1 and 2). Also, this tendency do not depend on the degree of share of tradables or nontradables. Volatility of CPI disparity in the case of optimal fiscal and monetary poicy is smaller than it in in the case of optimal monetary poicy alone (Tables 1 and 2).

Result in this sensitivity analysis prove the policy implications that is mentioned by international money and finance authors, such as McKinnon (1963). Cooperative fiscal authorities are essential for stabilizing both the output gap and inflation simultaneously when nontradable goods exist.[28]

4.3 Welfare Analysis

In this section, we analyze social welfare brought about by both regimes, focusing on the share of nontradables. Furthermore, we clarify the role of optimal fiscal policy by comparing the implications, that optimal fiscal policy is essential to minimize social losses although all goods are tradable, derived by Ferrero (2007).

Now, we define the welfare criteria. Taking unconditional expectations on Eq.(44) and setting $\delta \to 1$, the expected welfare losses of any policy that deviates from the optimal cooperative solution can be written in terms of the variances of inflation and the output gap as follows:

$$\begin{aligned}\tilde{\mathcal{L}}^W \equiv\ & \frac{\theta}{4(1-\sigma_G)\kappa}\mathrm{var}(\pi_{P,t}) + \frac{\theta}{4(1-\sigma_G)\kappa}\mathrm{var}\left(\pi^*_{P,t}\right) + \frac{1+\varphi}{4(1-\sigma_G)}\mathrm{var}(\tilde{y}_t) \\ & + \frac{1+\varphi}{4(1-\sigma_G)}\mathrm{var}(\tilde{y}^*_t), \qquad (49)\end{aligned}$$

with $\tilde{\mathcal{L}}^W$ being the expected welfare losses.

[28]Note that as well as other DSGE literature in an open economy, incomplete labor mobility exists as a friction in this model.

Table 3 depicts social losses associated with the two regimes analyzed in the previous section: optimal monetary policy alone and optimal monetary and fiscal policy. As noted above, both regimes are fully committed. When the share of nontradables increases, welfare losses increase under optimal monetary policy alone while optimal monetary and fiscal policy bring about zero welfare losses independent of the share of nontradable goods.[29] The necessity of optimal fiscal policy is clear from this analysis. Note that optimal monetary policy alone cannot result in a zero welfare loss when all goods are tradable, while the welfare losses are minimized among the losses brought about by the optimal monetary policy alone. As mentioned by Benigno (2004), optimal monetary policy alone can eliminate the inflation–output trade-offs simultaneously when all goods are tradable.[30] However, Ferrero (2007) insists that fiscal policy is needed although all goods are tradable. This discrepancy stems from the assumption of steady-state behavior of the fiscal authority. As a major DSGE paper, Benigno (2004) assumes a zero steady-state value of government expenditure and bonds. As in our setting, Ferrero (2007) does not assume zero steady-state value of government expenditure and bonds. Government expenditure and bonds have nonzero values in the steady state. This results in additional elasticities, the steady-state share of government expenditure with respect to output and the steady-state share of government bonds with respect to output, σ_G and σ_B, respectively. These elasticities change the format of the demand block of the economy, which inhibits perfect risk sharing. Thus, the study of Ferrero (2007) corresponds with our study in the case that all goods are tradable, namely $\gamma = 1$.

[29]When share of nontradables excess 0.5, welfare losses decrease under optimal monetary policy alone. In this region, openness in both countries are less. Asymmetric shocks do not affect another country. Thus, welfare losses decrease.

[30]In addition, it is attained when price stickiness is the same in both countries.

Our results insist that the existence of nontradables creates acute losses in the Euro economy not only because of the assumption of a steady state but also because of the Balassa–Samuelson theorem and another side effect. In our benchmark setting where $\gamma = 0.5$, the welfare loss—the percentage deviation of consumption from its steady state, brought about by optimal monetary policy alone—is 8.50%, while it is 3.09% when all goods are tradable, $\gamma = 1$. As noted above, approximately 50.3% of goods are nontradable; thus, the role of optimal fiscal policy is greater than that suggested by Ferrero (2007).

5 Implementing a Cooperative Solution by Self-oriented Fiscal Authorities

Some works, such as Benigno (2002), Obstfeld and Rogoff (2002) and Benigno and Benigno (2008), have shown that self-oriented monetary authorities can replicate the cooperative outcome in a decentralized framework so that there is no need for international monetary policy coordination. Following their context, we investigate whether it is possible that fiscal policies taken in a noncooperative environment can implement the optimal cooperative solution in this section.

While the central bank commits to minimizing union-wide social loss $\mathcal{L}^W$ subject to Eqs.(30), (37), (38), (39) and the counterpart of Eq.(38), we assume that each fiscal authority commits to minimizing its respective losses as follows:

$$\mathcal{L}^{NC} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t L_t^{NC} \ ; \ \mathcal{L}^{NC*} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t L_t^{NC*},$$

subject to structural model with:

$$L_t^{NC} \equiv \frac{1}{2(1-\sigma_G)} \left[\frac{\theta}{\kappa} \pi_{P,t}^2 + \omega (1+\varphi) \tilde{y}_t^2 \right] \ ; \ L_t^{NC*} \equiv \frac{1}{2(1-\sigma_G)} \left[\frac{\theta}{\kappa} \left(\pi_{P,t}^*\right)^2 + \omega^* (1+\varphi) \left(\tilde{y}_t^*\right)^2 \right], \quad (50)$$

where ω and ω^* are nonnegative parameters and L_t^{NC} denotes social losses assigned to fiscal authority to replicate cooperative outcome in country H.[31]

Next, we seek to find ω and ω^*, which satisfy $\mathcal{L}^W = \mathcal{L}^{NCW}$, with $\mathcal{L}^{NCW} \equiv \frac{1}{2}\left[\mathcal{L}^{NC} + \mathcal{L}^{NC*}\right]$ being the union-wide social loss brought about by self-oriented fiscal authorities in both countries. It can be said that $\mathcal{L}^{NCW}$ is the union-wide social loss in the Nash equilibrium under optimal monetary policy. When the noncooperative solution brings about the social loss that is brought about by cooperative solution, $\mathcal{L}^W = \mathcal{L}^{NCW}$ is applied. Let us assume that there are cost push shocks that prevent the central bank from being able to stabilize inflation and the output gap simultaneously and have constant variance. This assumption helps us to calculate the social loss analytically.[32] Note that we still assume $\eta = 1$. With tedious calculations, we have ω and ω^* as follows:

$$\omega = \omega^* = 1.$$

This implies that self-oriented fiscal authority can achieve the cooperative allocation in the Nash equilibrium without any forged work.

Benigno and Benigno (2008) investigate how ω and ω^* bring about the cooperative allocation in the Nash equilibrium. They show that when relative risk aversion multiplied by the elasticity of substitution between home and foreign goods is unity, $\omega = \omega^* = 1$ holds to achieve the cooperative allocation in the Nash equilibrium. Along with the assumption of our model, namely, log utility and the elasticity of substitution between home and foreign goods is unity, $\eta = 1$ is assumed for simplicity. Thus, although the existence of nontradable goods is allowed, our result is the same as that derived by Benigno and Benigno (2008).

[31]Following Beetsma and Jensen (2005), we split the per period union-wide social loss function Eq.(42) as follows: $L_t^W = \frac{1}{2}\left(L_t + L_t^*\right)$ with $L_t \equiv \frac{1}{2(1-\sigma_G)}\left[\frac{\theta}{\kappa}\pi_{P,t}^2 + (1+\varphi)\,\tilde{y}_t^2\right]$ and $L_t^* \equiv \frac{1}{2(1-\sigma_G)}\left[\frac{\theta}{\kappa}\left(\pi_{P,t}^*\right)^2 + (1+\varphi)\left(\tilde{y}_t^*\right)^2\right]$. After this, we introduce ω and ω^*.

[32]Following Walsh (2003) and Monacelli (2004), we calculate period losses.

However, it should be highlighted that the condition $\omega = \omega^* = 1$ always holds independently of the share of nontradables $1 - \gamma$.

Our result differs from the results in the literature such as and McKinnon (1963), who insist on the necessity of moving fiscal policy control from the local or national government to the central government. Furthermore, as a New Keynesian author, Ferrero (2007) suggests a necessity for cooperation between fiscal authorities. Also, our result partially defferent from result of Benigno and Benigno (Forthcoming), Benigno and Benigno (2006) and Beetsma and Jensen (2005) who analyzes necesity of policy coodination under two-country version DSGE model investigate how self-oriented setting bring about the cooperative allocation in the Nash equilibrium. They show that allocation derived by centralized setting can be replicated by self-oriented setting. However, they do not necessarily deny gains from cooperative setting because of TOT externality. Thus, to replicate allocation under cooperative setting, they propose somewhat forged individuall or respective loss function whose sum do not become equal to union-wide loss function. Our result implies that forged respective loss function is not needed. There are contradictory policy implications between our paper and those papers, at glance. However, Benigno and Benigno (Forthcoming) and Benigno and Benigno (2006) show that there are no need to cooperate internationally under special circumstance while some gains from policy cooperaition is proven.[33] Our utility function does not include any exogenous shifter. Because of this, the suare terms of TOT does not appear in the second-order approximated utility function which implies that there is no TOT externality. On the

[33]Benigno and Benigno (2006) show that there are no gains from cooperation when there are no mark-up shocks and government purcahses. Benigno and Benigno (Forthcoming) that sum of respevtive loss functions to replicate cooperative allocation become strictly equal to union-wide loss function when both degree of relative risk sversion and the elasticity of substitution between goods produced in both countries are unity.

contrary, second-order approximated utility function derived by Benigno and Benigno (Forthcoming), Benigno and Benigno (2006) and Beetsma and Jensen (2005) imply TOT externality which dissapear under only special circumstances. Our case corresponds to special cases of these papers. Thus, our policy implication is not necessarily inconsistent with Benigno and Benigno (Forthcoming), Benigno and Benigno (2006).

Our game theoretical analysis following Benigno and Benigno (2008) implies that the national fiscal authority can eliminate welfare losses by having only one central bank when nontradables exist. In this case, the inflation–output tradeoffs in the Euro area may not be an acute problem if Stability and Growth Pact of the Maastricht Treaty is revised.[34]

6 Conclusion

This paper verified the role of optimal fiscal policy in a currency union. Assuming the existence of nontradables, we showed that the role of optimal fiscal policy is more essential than suggested by Ferrero (2007). Furthermore, in contrast to authors such as McKinnon (1963), we showed that there is no need to align fiscal authorities to eliminate social loses through eliminating inflation–output trade-offs. While we assume existence of nontradables, Gali and Monacelli (Forthcoming) assumes currency union which consists of small open economy. Considering the fact that relatively small economies will participate the EMU hereafter, their assumption is plausible. Because the assumption in our paper and Gali and Monacelli (Forthcoming) well reflects a condition in the Euro area, optimal fiscal policy is essential in the Euro area.

[34]According to the Pact, national fiscal policies are bound to respect an upper threshold for the deficit to GDP and the debt to GDP ratios of 3% and 60%, respectively.

While this paper can reconcile some papers that have inconsistencies in their implications, this paper cannot reconcile the implications of Canzoneri, Cumby and Diba (2001), which support the Stability and Growth Pact of the Maastricht Treaty. While we can derive policy implication that support of fiscal policy is essential, plausibility of Stability and Growth Pact is not investigated explicitly. This is a future research agenda.

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Table 1: Macroeconomic Volatility: All Goods are Tradable ($\gamma = 1$)

Variable	Regime	Shocks			
		$a_{H,t}$	$a_{\mathcal{N},t}$	g_t^W	g_t^R
$\tilde{y}_t^W$	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0000	0.0000	0.0000	0.0000
π_t^W	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0000	0.0000	0.0000	0.0000
$\tilde{y}_t$	OMP	0.0066	0.0000	0.0000	0.0083
	OMFP	0.0000	0.0000	0.0000	0.0000
$\tilde{y}_t^*$	OMP	0.0066	0.0000	0.0000	0.0083
	OMFP	0.0000	0.0000	0.0000	0.0000
$\pi_{P,t}$	OMP	0.0082	0.0000	0.0000	0.0150
	OMFP	0.0000	0.0000	0.0000	0.0000
$\pi_{P,t}^*$	OMP	0.0082	0.0000	0.0000	0.0150
	OMFP	0.0000	0.0000	0.0000	0.0000
q_t	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0000	0.0000	0.0000	0.0000
$\hat{r}_t$	OMP	0.0115	0.0000	0.0123	0.0000
	OMFP	0.0008	0.0000	0.0067	0.0000
b_t	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0129	0.0000	0.0057	0.0044
b_t^*	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0081	0.0000	0.0057	0.0044

Notes:
OMP: Optimal Solitary Monetary Policy
OMFP: Optimal Monetary and Fiscal Policy

Table 2: Macroeconomic Volatility: Benchmark ($\gamma = 0.5$)

Variable	Regime	Shocks			
		$a_{H,t}$	$a_{\mathcal{N},t}$	g_t^W	g_t^R
$\tilde{y}_t^W$	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0000	0.0000	0.0000	0.0000
π_t^W	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0000	0.0000	0.0000	0.0000
$\tilde{y}_t$	OMP	0.0015	0.0023	0.0000	0.0042
	OMFP	0.0000	0.0000	0.0000	0.0000
$\tilde{y}_t^*$	OMP	0.0015	0.0023	0.0000	0.0042
	OMFP	0.0000	0.0000	0.0000	0.0000
$\pi_{P,t}$	OMP	0.0076	0.0153	0.0000	0.0236
	OMFP	0.0000	0.0000	0.0000	0.0000
$\pi_{P,t}^*$	OMP	0.0076	0.0153	0.0000	0.0236
	OMFP	0.0000	0.0000	0.0000	0.0000
q_t	OMP	0.0037	0.0084	0.0000	0.0101
	OMFP	0.0004	0.0038	0.0000	0.0001
$\hat{r}_t$	OMP	0.0058	0.0069	0.0130	0.0000
	OMFP	0.0004	0.0004	0.0065	0.0000
b_t	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0067	0.0088	0.0056	0.0046
b_t^*	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0041	0.0036	0.0056	0.0046

Notes:
OMP: Optimal Solitary Monetary Policy
OMFP: Optimal Monetary and Fiscal Policy

Table 3: Effect on Welfare of Varying Share of Tradable goods

γ	$\mathcal{L}^W$ (Percentage)	
	OMP	OMFP
1	-3.0935	0.0000
0.9	-3.7844	0.0000
0.8	-5.9287	0.0000
0.7	-6.5082	0.0000
0.6	-7.5414	0.0000
0.5	-8.4962	0.0000
0.4	-13.0896	0.0000
0.3	-13.9473	0.0000
0.2	-15.6554	0.0000
0.1	-20.3470	0.0000
0	-21.4257	0.0000

Notes:
OMP: Optimal Solitary Monetary Policy
OMFP: Optimal Monetary and Fiscal Policy

Figure 1: Impulse Responses to Shocks under Optimal Monetary Policy Alone: Bench Mark Case

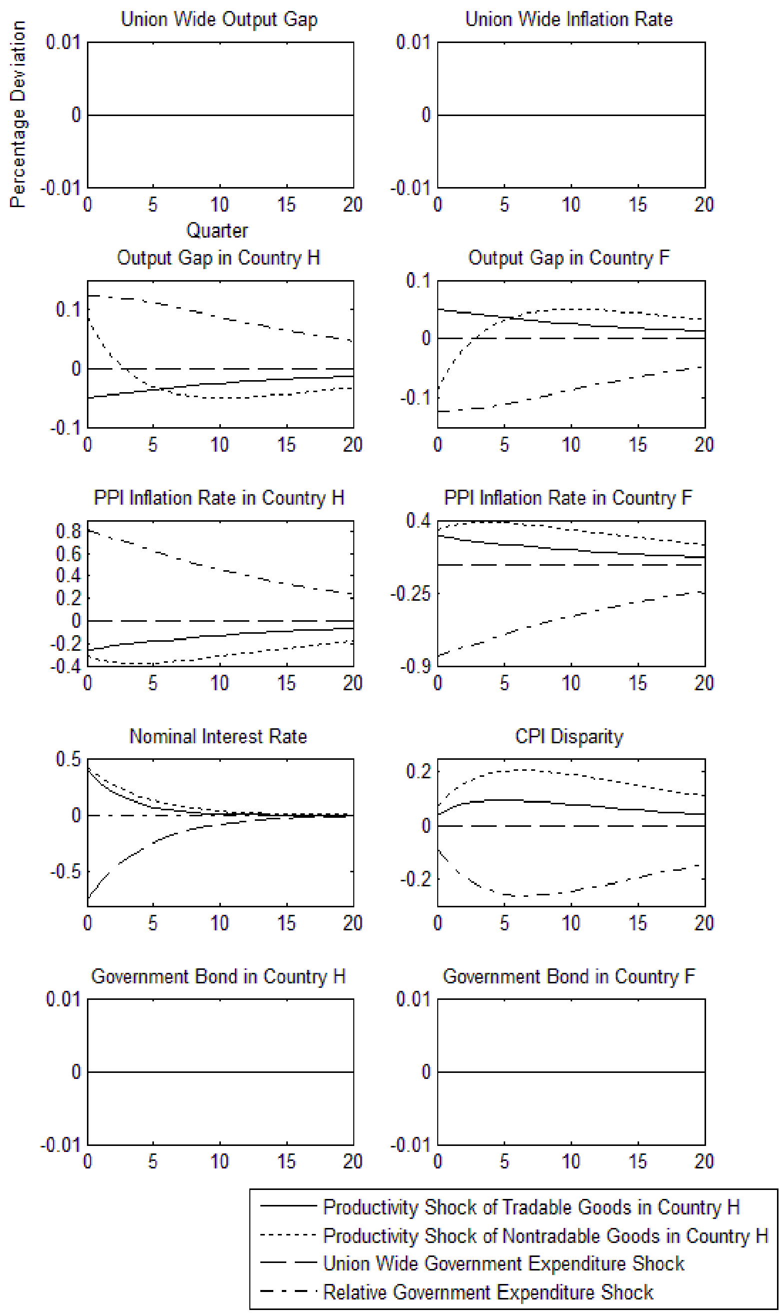

Figure 2: Impulse Responses to Shocks under Optimal Monetary Policy and Fiscal Policy: Bench Mark Case